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Measuring performance of a hybrid system based on imprecise data: Modeling and solution approaches

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Abstract

Data Envelopment Analysis (DEA) is one of the methods most widely used for measuring the relative efficiency of DMUs in the world today. The efficiency evaluation of the network structure opens the “black box” and considers the internal structure of systems. In this paper, a three-stage network model is considered with additional inputs and undesirable outputs and obtains the efficiency of the network, as interval efficiency in presence of the imprecise datum. The proposed model of this paper simulates a factory in the factual world with a production area, three warehouses and two delivery points. This factory is taken into consideration as a dynamic network and a multiplicative DEA approach is utilized to measure efficiency. Given the non-linearity of the models, a heuristic method is used to linearize the models. Ultimately, this paper concentrates on the interval efficiency to rank the units. The results of this ranking demonstrated that the time periods namely, (24) and (1) were the best and the poorest periods, respectively, in context to the interval efficiency within 24 phases of time.

Keywords: Network DEA; Three-stage processes; Interval data; Additional inputs; Undesirable outputs; minimax regret-based approach.

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1. Introduction

Evaluation and the measurement of performance will lead to smart or intelligent systems with incentives for individuals for the desired behavior. A measurement of performance is one of the fundamental managerial processes, for analyzing one’s own performance and likewise, surveying the conformity between the performance and the set of goals. The outcome of the evaluation can provide the grounds for taking the correct measures in decision-making for the future. Performance appraisal is a key part in the formulation and implementation of organizational policies (Hajijabbari & Sarabadani, 2008).

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The data envelopment analysis (DEA) is utilized as a non-parametric mathematical programming method, to evaluate the relative efficiency of entities, capable of being compared to multiple inputs and several outputs. These comparable entities are called decision-making units (DMUs). (Khalili-Damghani et al., 2015). Farrell (1957) considered a model for performance evaluation with an input and an output for the first time. Charnes, Cooper and Rhodes developed Farrell's model for multiple inputs and outputs and dubbed it as "CCR" in honor of its makers (Charnes et al., 1978). Charnes, Cooper and Banker extended the DEA models and registered this model as their own and known as "BCC" (Banker et al., 1984). A failure to consider the internal correlations and the intermediate variables of complex systems (*i.e.* two or multiple stage processes), was a major flaw or defect, which the classic DEA models, such as, the CCR and BCC were labeled with. In actual fact, these models considered the systems as black boxes and would only be satisfied with the initial inputs and final outputs (Tone and Tsutsui, 2009). In order to overcome this problem, Fare and Grosskopf (2000) introduced the Network Data Envelopment Analysis (NDEA) models. With the assistance of the sub-series and parallels, including the intermediate variables, complex systems were simulated and a more genuine evaluation was computed (Chen and Yan, 2011). From Kao's viewpoint, NDEA models can be categorized into three groups, namely, series, parallel and hybrid models. When the activities of a system are prolonged in respect to each other, the network has a serial structure; and at times when the activities are parallel to each other, the network has a parallel structure. Similarly, it maintains a hybrid mode, when there is a combination between the series and parallel set-up (Kao, 2009). Generally, the multiplicative and additive approaches are used to compute the network performance in the parallel and serial mode, respectively, (kao, 2006). After introducing the NDEA models, numerous studies took place in this field; and a combination of this science with branches of the game theory, brought about NDEA studies in cooperative and non-cooperative modes, which can be mentioned as hereunder. Li et al. (2012) presented a model for a two-stage structure, a phase which holds a more important standpoint for managers; and they have named this phase as "leader" and the other is known as "follower". In order to calculate the efficiency, initially, the efficiency of the leader phase was maximized to the optimum and thus, the efficiency of the follower phase was secured by maintaining a constant efficiency in the phase of the leader. This exemplary, was designated as a decentralized control or a Stackelberg Game, which has been widely utilized by researchers in recent years. Du et al. (2015) analyzed a parallel network in the cooperative and non-cooperative modes and stated that, the efficiency of the former was more than that of the latter mode. An et al. (2017) considered a network, comprising of two stages, which interacted with each other; and computed the efficiency of this network in a cooperative and non-cooperative mode or condition. In another research Zhou et al. (2018), evaluated a multi-stage network in the non-cooperative and black box mode and compared the results. In recent years, the Stackelberg Game was utilized by several researchers such as, Fard and Hajaghaei-Keshteli (2018), Fathollahi-Fard et al. (2018), including Hajaghaei-Keshteli and Fathollahi-Fard (2018).

The role of undesirable factors has been extremely crucial in NDEA, in the recent years, such that, Liu et al. (2016) utilized the clustering methods and described this sphere as one of the four critical spheres or domains of DEA, from the researchers' viewpoint. Undesirable factors are one of the critical arenas that are accounted for DEA. For the first time, Fare et al. (1989) took the undesirable factors under consideration to evaluate the efficiency in DEA models. Lu and Lo (2007) classified the undesirable outputs within a framework of three modes; the first method was to overlook all the undesirable outputs. The second method was to restrict the expansion of the undesirable outputs, or that, these undesirable outputs were to be considered as a nonlinear DEA model; whereas, the third technique, which was taken under contemplation, for the undesirable outputs, was as an input, signified with a negative sign, as an output and or was handled by imposing a single downward conversion. In the past few years, the role of the undesirable factors in DEA models has made considerable progress and the tasks of Wang et al. (2013) and Wu et al. (2016) are significant in this respect.

Classic DEA models, such as, (CCR and BCC models), were proposed for certainty in data as it does not deal with datum uncertainty. In fact, the actual, fundamental assumption in these models is that the amount of data in relevance with the inputs and outputs is an accurate numerical value. Though, in most cases, in the business environment, determining values for inputs and outputs is not feasible in reality (Khalili-Damghani et al., 2015). Ben-Tal and Nemirovski (2000), have demonstrated that an extremely slight deviation in the data leads to an unjustified response or a considerable change in the efficiency results. Hence, they proclaimed that the results of the classical DEA methods with determined parameters are not reliable. Kao (2006) stated that the reason for the absence of the presence of reliability, in terms of human judgment and concept, DEA models with imprecise data can play a more important role in evaluating efficiency in factual issues. Wang et al. (2009) expressed that in the presence of imprecise data, DEA models are capable of drawing insights for companies in variable and ambiguous conditions, in order to have a more realistic assessment of their own. Thereby, it is extremely essential to consider the uncertainty in the data available and the manner of dealing with it during the evaluation of efficiency by utilizing DEA methods. In most of the initial DEA tasks, uncertainty was ignored, but in recent years, several models have been under discussion for imprecise or inaccurate data (Azizi, 2013). Cooper et al. (1999) utilized a technique for DEA to confront imprecise data. Cooper et al. (2001) developed a method for converting a nonlinear planning problem into linear programming, by taking the alternative variables into consideration for determining the efficiency of the DMUs in presence of imprecise data. Imprecise data have several criterions and varied models have been designed to oppose this aspect (Amirteimoori and Kordrostami, 2014). One of the most widespread approaches in context with data uncertainty conditions is utilizing an interval DEA model (Farzipoor Saen, 2009). Despotis and Smirlis (2002) developed the CCR model and rendered a model, in which the efficiency evaluation is calculated by taking the interval data into account. Entani et al. (2002) used DEA with a pessimistic approach in the presence of interval data. Despotis et al. (2006) rendered a method in which the unspecified and imprecise values were replaced with a series of intervals and utilized the DEA intervals for evaluating the efficiencies of units. Aghayi et al. (2013) modeled a two-stage network to consider the uncertainty in input and output data. In this research, uncertainty was modeled intermittently and the efficiency results were expressed in terms of the intervals with the upper and lower bounds. Khalili-Damghani (2015) ushered a model which calculated the efficiency evaluation in the presence of interval data and undesirable outputs.

The researches which were carried out utilized and were based on DEA, which were mainly in static environments. For the initial time, Sengupta (1995), dealt with efficiency evaluations in dynamic environments. Dynamic models are models where, data is continuously changing over several incessant periods or cycles; and each time period is considered as a DMU. Similarly, the correlation between the periods in these models, utilizes additional inputs and outputs amid these periods (Jafarian Moghaddam and Ghoseiri, 2011). Since (the epoch of) Sengupta's task, several articles have been published in the sphere of dynamic networks, which differ in relevance to case studies and the manner in which the efficiency of the DMUs are calculated. In other words, models in relative to Kawaguchi et al. (2014) and Wang et al. (2014) can be mentioned respectively, for performance or efficiency evaluation in hospital environments and banks in a dynamic genre. Table (1) reviews the studies which have applied the game theory methods in DEA. The last row of Table (1) presents characteristics of the current paper.

Table 1. Classification of Studies on DEA-Game Theory method

Reference	Type of game	Structure of network	Additional inputs	Undesirable output	Type of modelling	Type of data
Hwang et al. (2013)	Cooperative	One-stage	-	✓	Linear programming	Precise
Azizi (2013)	Cooperative	One-stage	-	-	Linear programming	Imprecise
Despotis and Smirlis (2002)	Cooperative	One-stage	-	-	Linear programming	Imprecise
Shabanpour et al. (2017)	Cooperative	one-stage	-	-	Linear programming	Precise
Kao and Hwang (2008)	Cooperative	Two-stage	-	-	Linear programming	Precise
Aghayi et al. (2013)	Cooperative	Two-stage	-	-	Linear programming	Imprecise
Wang et al. (2014)	Cooperative	Two-stage	-	-	Linear programming	Imprecise
Kou et al. (2016)	Cooperative	Two-stage	✓	-	Linear programming	Precise
Li et al. (2012)	Non-cooperative	Two-stage	✓	-	Linear programming	Precise
Liang et al. (2008)	Cooperative and Non-cooperative	Two-stage	-	-	Linear programming	Precise
Wu et al. (2015)	Cooperative	Two-stage	✓	✓	Linear programming	Precise
Zhou et al. (2018)	Non-cooperative	Two -stage	-	-	Non- linear programming	Precise
An et al. (2017)	Cooperative and Non-cooperative	Two-stage	✓	-	Non- linear programming	Precise
Wu et al. (2016)	Cooperative and Non-cooperative	Two -stage	✓	✓	Non- linear programming	Precise
Du et al. (2015)	Cooperative and Non-cooperative	Three -stage	-	-	Linear programming	Precise
Yousefi et al. (2017)	Cooperative	Three -stage	✓	-	Non- linear programming	Imprecise
Badiezadeh et al. (2018)	Cooperative	Three -stage	✓	✓	Linear programming	Precise
current paper	Cooperative	Three -stage	✓	✓	Non- linear programming	Imprecise

According to the abovementioned, in major, the researches performed in the network, focused on two stages. But the current research considers the three-stage processes, which in addition to intermediate measures, also has additional inputs and undesirable outputs. In actual fact, DEA signifies a theoretical framework in the way of analyzing efficiency, but its application in the grounds of production planning and inventory control is observed as being extremely low. In this paper we simulate a factory producing dairy products with a production area, three warehouses, two delivery points for goods and we analyze this network in a dynamic condition. Therefore, we are faced with a hybrid system, including a complex internal structure with three stages, six sub-DMUs, and undesirable outputs in the first stage, additional inputs and outputs in the second stage and additional inputs and undesirable outputs in the third stage. Interval data is utilized to evaluate efficiency, in order to make results more realistic. Likewise, in this paper, the cooperative approach is used to evaluate efficiency and a heuristic method is applied to convert nonlinear models to linear models. As a summarization, contributions of this paper are as follows:

- A three-stage network is taken under consideration in respect to the additional desirable and undesirable inputs and outputs
- A hybrid system with a complex internal structure is developed by a DEA approach.
- Interval data is utilized to evaluate efficiency, in order to make results more realistic
- A heuristic technique is proposed to convert non-linear models into linear models

- Implementation of the suggested model on an authentic example. (We simulate a factory in a real world that has a production area and three warehouses for goods and two delivery points for first time .
- The said factory is considered as a dynamic network.
- In this simulation, all costs are considered, including, production costs, setup cost, maintenance costs of the products, warehouse reservation costs, transportation costs, delay penalty costs and the profit obtained from the sale of products.

The remainder of the paper is organized as follows: Section (2) describes the model and introduces the network and modeling from the cooperative perspective for interval data. Section (3) resolves the model and in this section a heuristic approach is used to solve the model. Section (4) describes a case-study, where, a factory is elaborated upon as an example in the factual world and ultimately Section (5) renders the results of the paper.

2. Model description

We consider a set of n homogeneous DMUs that are denoted by DMU_j (j=1,..., n), and the each DMU_j (j=1,…,n) has a three-stage with a complex internal structure, as shown in Fig. 1. We denote, the inputs to sub-DMU_{1j}, sub-DMU_{2j}, sub-DMU_{3j}, sub-DMU_{4j}, sub-DMU_{5j} and sub-DMU_{6j} by $x^1_{i_1j}$ (i_1=1,…,I_1), $x^2_{i_2j}$ (i_2=1,…,I_2), $x^3_{i_3j}$ (i_3=1,…,I_3), $x^4_{i_4j}$ (i_4=1,…,I_4), $x^5_{i_5j}$ (i_5=1,…,I_5) and $x^6_{i_6j}$ (i_6=1,…,I_6), respectively. We denote, the intermediate measures between stage 1 and 2 by $z^1_{d_1j}$ (d_1=1,…,D_1), $z^2_{d_2j}$ (d_2=1,…,D_2) and $z^3_{d_3j}$ (d_3=1,…,D_3), and between stage 2 and 3 by $z^4_{d_4j}$ (d_4=1,…,D_4), $z^5_{d_5j}$ $(d_5 = 1, \dots, D_5)$, $z^6_{d_6j}$ $(d_6 = 1, \dots, D_6)$, $z^7_{d_7j}$ $(d_7 = 1, \dots, D_7)$, $z^8_{d_8j}$ (d_8=1,…,D_8) and $z^9_{d_9j}$ (d_9=1,…,D_9). We denote, the undesirable output of the first stage by $y^1_{r_1j}$ (r_1=1,…,R_1). The outputs of sub-DMU_{2j}, sub-DMU_{3j} and sub-DMU_{4j} are denoted by $y^2_{r_2j}$(r_2=1,…,R_2), $y^3_{r_3j}$ (r_3=1,…,R_3) and $y^4_{r_4j}$ (r_4=1,…,R_4), respectively. Finally, the outputs of sub-DMU_{5j} and sub-DMU_{6j} are denoted by $y^5_{r_5j}$ (r_5=1,…,R_5), $y^6_{r_6j}$ (r_6=1,…,R_6), $y^7_{r_7j}$ (r_7=1,…,R_7) and $y^8_{r_8j}$ (r_8=1,…,R_8), respectively where $y^6_{r_6j}$ and $y^8_{r_8j}$ are undesirable outputs.
We adopt $v^1_{i_1}$, $v^2_{i_2}$, $v^3_{i_3}$, $v^4_{i_4}$, $v^5_{i_5}$ and $v^6_{i_6}$ as the weights of the inputs to sub-DMU_{1j}, sub-DMU_{2j}, sub-DMU_{3j}, sub-DMU_{4j}, sub-DMU_{5j} and sub-DMU_{6j}, respectively. We adopt $u^1_{r_1}$ as the weights of the outputs to sub-DMU_{1j} in the first stage and $u^2_{r_2}$, $u^3_{r_3}$ and $u^4_{r_4}$ as the weights of the outputs to sub-DMU_{2j}, sub-DMU_{3j} and sub-DMU_{4j}, in the second stage respectively. Kao and Hwang (2008) used the same weights for the intermediate measures. In accordance with this, we value the intermediate measures in this research, irrespective of its dual role (as an input in one stage or as an output in the next stage). We assume that the weights relative to the intermediate measures between stages 1 and 2 and similarly, weights in relevance with the intermediate measures between stages 2 and 3 are uniform. Therefore, we adopt $w^1_{d_1}$, $w^2_{d_2}$ and $w^3_{d_3}$ as the weights of the intermediate measures between stage 1 and stage 2. The weights of the intermediate measures between stage 2 and stage 3 are denoted by $w^4_{d_4}$, $w^5_{d_5}$, $w^6_{d_6}$,$w^7_{d_7}$, $w^8_{d_8}$ and $w^9_{d_9}$. Finally, we adopt $u^5_{r_5}$, $u^6_{r_6}$, $u^7_{r_7}$and $u^8_{r_8}$ as the weights of the outputs to sub-DMU_{5j} and sub-DMU_{6j}, respectively.

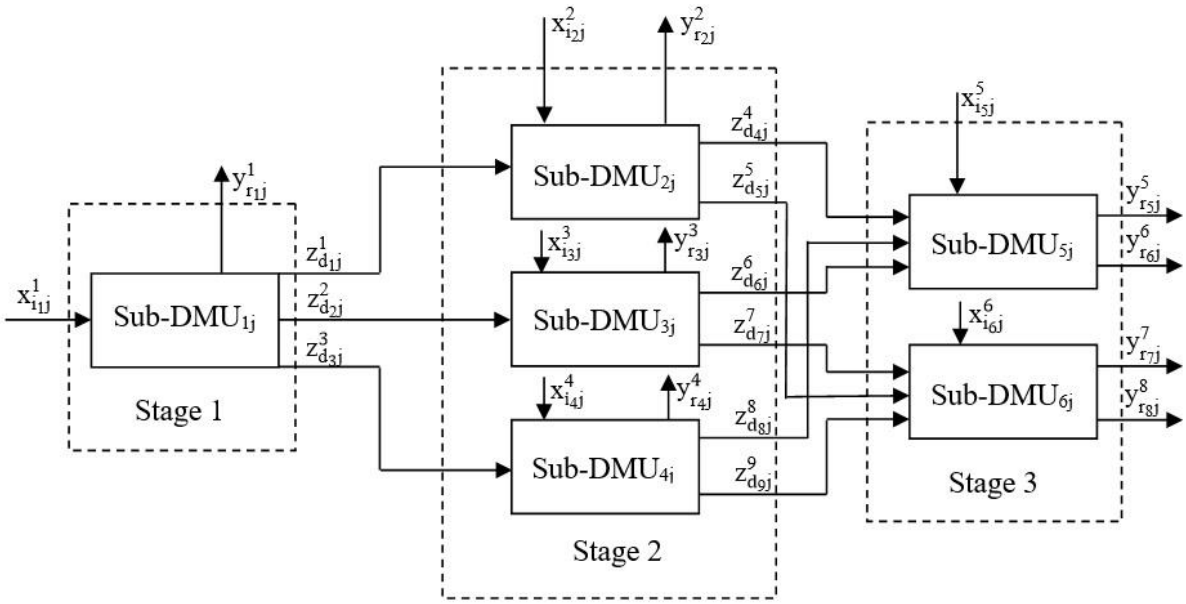

Figure. 1 The structure of three-stage network system with additional inputs and undesirable outputs

It is mainly for three reasons that researchers are most likely to use input-oriented models for efficiency analysis. Primarily, demand is in a state of growth and the estimation of demand is a complex issue. Secondly, managers have more control over inputs than outputs; and thirdly, this model reflects the initial goals or objectives of policy-makers, in being responsible for responding to the demands of the people and that the units should reduce costs and or limit the use of resources. Thereby, we utilize the input-axis model in this research. In accordance with Korhonen and Luptacik (2004), we signify the undesirable outputs in the models with a negative mark. Therefore, the efficiency of sub-DMU$_{1o}$ in the first stage can be calculated using the following model (1):

$$\theta_o^1 = \max \ \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 + \sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^3 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^1}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^1}$$

$$\text{s.t.} \quad \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 + \sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^3 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1} \leq 1, \quad j=1,\dots,n \tag{1}$$

$$u_{r_1}^1, v_{i_1}^1, w_{d_1}^1, w_{d_2}^2, w_{d_3}^3 \geq \varepsilon;\ r_1=1,\dots,R_1;\ i_1=1,\dots,I_1;\ d_1=1,\dots,D_1;\ d_2=1,\dots,D_2;\ d_3=1,\dots,D_3.$$

In the second stage we have three sub-DMUs form a parallel structure and the sub-DMUs are independent. The efficiencies of sub-DMU$_{2o}$, sub-DMU$_{3o}$ and sub-DMU$_{4o}$ are defined, respectively, as follows:

$$\theta_o^2 = \max \ \frac{\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4o}^4 + \sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5o}^5 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1}$$

$$\text{s.t.} \quad \frac{\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^4 + \sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^5 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1} \leq 1, \quad j=1,\dots,n \tag{2}$$

$$u_{r_2}^2, v_{i_2}^2, w_{d_1}^1, w_{d_4}^4, w_{d_5}^5 \geq \varepsilon;\ r_2=1,\dots,R_2;\ i_2=1,\dots,I_2;\ d_1=1,\dots,D_1;\ d_4=1,\dots,D_4;\ d_5=1,\dots,D_5.$$

$$\theta_o^3 = \max \ \frac{\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6o}^6 + \sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7o}^7 + \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3o}^3}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2}$$

$$\text{s.t.}\quad \frac{\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^6+\sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^7+\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2}\le 1,\quad j=1,\dots,n \tag{3}$$

$$u_{r_3}^3, v_{i_3}^3, w_{d_2}^2, w_{d_6}^6, w_{d_7}^7 \ge \varepsilon;\ r_3=1,\dots,R_3;\ i_3=1,\dots,I_3;\ d_2=1,\dots,D_2;\ d_6=1,\dots,D_6;\ d_7=1,\dots,D_7.$$

$$\theta_o^4=\max\ \frac{\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8o}^8+\sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9o}^9+\sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4o}^4}{\sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4o}^4+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^3}$$

$$\text{s.t.}\quad \frac{\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^8+\sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^9+\sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4j}^4}{\sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4j}^4+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^3}\le 1,\quad j=1,\dots,n \tag{4}$$

$$u_{r_4}^4, v_{i_4}^4, w_{d_3}^3, w_{d_8}^8, w_{d_9}^9 \ge \varepsilon;\ r_3=1,\dots,R_3;\ i_4=1,\dots,I_4;\ d_4=1,\dots,D_4;\ d_8=1,\dots,D_8;\ d_9=1,\dots,D_9.$$

Kao (2009) used the additive approach for the overall efficiency of a parallel structure where sub-DMUs are independent. We then define the efficiency of the second stage as: $\theta_o^{234}=\max(w_1.\theta_o^2+w_2.\theta_o^3+w_3.\theta_o^4)$, where w_1, w_2 and w_3 are weights specified by experts such that, $w_1+w_2+w_3=1$. Chen et al. (2009), show that, the relative size of the inputs of a stage expresses the importance of that stage. Hence, based on the fact that our model is input-oriented, we compute the weights determined by the experts from the relative input value of each sub-DMU to implicit the value of inputs in the second stage. Thereby, we define w_1, w_2 and w_3 as follows:

$$w_1=\frac{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2+\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2+\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1+\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2+\sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4o}^4+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^3}, \tag{5}$$

$$w_2=\frac{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2+\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1+\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2+\sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4o}^4+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^3},$$

$$w_3=\frac{\sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4o}^4+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^3}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2+\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1+\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2+\sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4o}^4+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^3}$$

We defined w_1, w_2 and w_3 as the parts of total input resources devoted to the sub-DMU_{2o}, sub-DMU_{3o} and sub-DMU_{4o}, respectively. In order to make the models more convenient, we define I_o^{234} and O_o^{234}, as inputs and outputs to the second stage, respectively, as follows:

$$(I_o^{234}=\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2+\sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1o}^1+\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3+\sum_{d_2=1}^{D_2} w_{d_2}^2\, z_{d_2o}^2+\sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4o}^4+\sum_{d_2=1}^{D_2} w_{d_3}^3\, z_{d_3o}^3)\ \text{ and}$$

$$(O_o^{234}=\sum_{d_4=1}^{D_4} w_{d_4}^4\, z_{d_4o}^4+\sum_{d_5=1}^{D_5} w_{d_5}^5\, z_{d_5o}^5+\sum_{d_6=1}^{D_6} w_{d_6}^6\, z_{d_6o}^6+\sum_{d_7=1}^{D_7} w_{d_7}^7\, z_{d_7o}^7+\sum_{d_8=1}^{D_8} w_{d_8}^8\, z_{d_8o}^8+\sum_{d_9=1}^{D_9} w_{d_9}^9\, z_{d_9o}^9$$

$$+\sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2+\sum_{r_2=1}^{R_2} u_{r_3}^3 y_{r_3o}^3+\sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4o}^4) \tag{6}$$

Then, with models (2), (3) and (4) and formulas (5) and (6), the efficiency of the second stage is defined as follows:

$$\theta_o^{234}=\max\ \frac{O_o^{234}}{I_o^{234}}$$

$$\text{s.t.}\quad \frac{\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^4+\sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^5+\sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2+\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1}\le 1,\quad j=1,\dots,n$$

$$\frac{\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^6+\sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^7+\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2} \leq 1, \quad j=1,\ldots,n \tag{7}$$

$$\frac{\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^8+\sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^9+\sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4j}^4}{\sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4j}^4+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^3} \leq 1, \quad j=1,\ldots,n$$

$$u_{r_2}^2, u_{r_3}^3, u_{r_4}^4, v_{i_2}^2, v_{i_3}^3, v_{i_4}^4, w_{d_1}^1, w_{d_2}^2, w_{d_3}^3, w_{d_4}^4, w_{d_5}^5, w_{d_6}^6, w_{d_7}^7, w_{d_8}^8, w_{d_9}^9 \geq \varepsilon;$$

$$r_2=1,\ldots,R_2;\ r_3=1,\ldots,R_3;\ r_4=1,\ldots,R_4;\ i_2=1,\ldots,I_2;\ i_3=1,\ldots,I_3;\ i_4=1,\ldots,I_4;\ d_1=1,\ldots,D_1;\ d_2=1,\ldots,D_2;$$

$$d_3=1,\ldots,D_3;\ d_4=1,\ldots,D_4;\ d_5=1,\ldots,D_5;\ d_6=1,\ldots,D_6; d_7=1,\ldots,D_7;\ d_8=1,\ldots,D_8;\ d_9=1,\ldots,D_9.$$

In the model (7) we measured the efficiency of the second stage, based on the efficiency of each sub-DMU in the second stage being less than one. In the third stage we have two sub-DMUs from a parallel structure. In the same way we define the efficiency of the third stage, as we did in the second stage, as follows:

$$\theta_o^{56} = \max \ \frac{O_o^{56}}{I_o^{56}}$$

$$\text{s.t.} \quad \frac{\sum_{r_5=1}^{R_5} u_{r_5}^5 y_{r_5j}^5 - \sum_{r_6=1}^{R_6} u_{r_6}^6 y_{r_6j}^6}{\sum_{i_5=1}^{I_5} v_{i_5}^5 x_{i_5j}^5+\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^4+\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^6+\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^8} \leq 1, \quad j=1,\ldots,n$$

$$\frac{\sum_{r_7=1}^{R_7} u_{r_7}^7 y_{r_7j}^7 - \sum_{r_8=1}^{R_8} u_{r_8}^8 y_{r_8j}^8}{\sum_{i_6=1}^{I_6} v_{i_6}^6 x_{i_6j}^6+\sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^5+\sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^7+\sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^9} \leq 1, \quad j=1,\ldots,n$$

(8)

$$u_{r_5}^5, u_{r_6}^6, u_{r_7}^7, u_{r_8}^8, v_{i_5}^5, v_{i_6}^6, w_{d_4}^4, w_{d_5}^5, w_{d_6}^6, w_{d_7}^7, w_{d_8}^8, w_{d_9}^9 \geq \varepsilon;$$

$$r_5=1,\ldots,R_5;\ r_6=1,\ldots,R_6;\ r_7=1,\ldots,R_7;\ r_8=1,\ldots,R_8;\ i_5=1,\ldots,I_5;\ i_6=1,\ldots,I_6;\ d_4=1,\ldots,D_4;\ d_5=1,\ldots,D_5;$$

$$d_6=1,\ldots,D_6; d_7=1,\ldots,D_7;\ d_8=1,\ldots,D_8;\ d_9=1,\ldots,D_9.$$

Model (8) gains the performance of the third step and to alleviate the input and output values of the third stage we have denoted it in formula (9) as follows:

$$(I_o^{56}=\sum_{i_5=1}^{I_5} v_{i_5}^5 x_{i_5o}^5+\sum_{i_6=1}^{I_6} v_{i_6}^6 x_{i_6o}^6+\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4o}^4+\sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5o}^5+\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6o}^6+\sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7o}^7$$

$$+\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8o}^8+\sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9o}^9) \ \text{and} \ (O_o^{56}=\sum_{r_5=1}^{R_5} u_{r_5}^5 y_{r_5o}^5+\sum_{r_7=1}^{R_7} u_{r_7}^7 y_{r_7o}^7-\sum_{r_6=1}^{R_6} u_{r_6}^6 y_{r_6o}^6-\sum_{r_8=1}^{R_8} u_{r_8}^8 y_{r_8o}^8)$$

(9)

For the network structure as shown in Fig.1, stages 1, 2 and 3 are connected in series. Kao and Hwang (2008) used the multiplicative approach to measure the overall efficiency of a series structure. We define then the overall efficiency of integrated system shown in Fig.1 as $\theta_o^{overall}=\max \theta_o^1 . \theta_o^{234} . \theta_o^{56}$ Thus:

$$\theta_o^{overall}=\max\frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^3-\sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^1}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^1}\cdot\frac{O_o^{234}}{I_o^{234}}\cdot\frac{O_o^{56}}{I_o^{56}}$$

$$\begin{aligned}
\text{s.t.}\quad & \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^3-\sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1}\leq 1, \quad j=1,\dots,n \\
& \frac{\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^4+\sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^5+\sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2+\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1}\leq 1, \quad j=1,\dots,n \\
& \frac{\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^6+\sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^7+\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2}\leq 1, \quad j=1,\dots,n \qquad (10)\\
& \frac{\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^8+\sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^9+\sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4j}^4}{\sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4j}^4+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^3}\leq 1, \quad j=1,\dots,n \\
& \frac{\sum_{r_5=1}^{R_5} u_{r_5}^5 y_{r_5j}^5-\sum_{r_6=1}^{R_6} u_{r_6}^6 y_{r_6j}^6}{\sum_{i_5=1}^{I_5} v_{i_5}^5 x_{i_5j}^5+\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^4+\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^6+\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^8}\leq 1, \quad j=1,\dots,n \\
& \frac{\sum_{r_7=1}^{R_7} u_{r_7}^7 y_{r_7j}^7-\sum_{r_8=1}^{R_8} u_{r_8}^8 y_{r_8j}^8}{\sum_{i_6=1}^{I_6} v_{i_6}^6 x_{i_6j}^6+\sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^5+\sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^7+\sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^9}\leq 1, \quad j=1,\dots,n \\
& u_{r_1}^1,u_{r_2}^2,u_{r_3}^3,u_{r_4}^4,u_{r_5}^5,u_{r_6}^6,u_{r_7}^7,u_{r_8}^8,v_{i_1}^1,v_{i_2}^2,v_{i_3}^3,v_{i_4}^4,v_{i_5}^5,v_{i_6}^6,w_{d_1}^1,w_{d_2}^2,w_{d_3}^3,w_{d_4}^4,w_{d_5}^5,w_{d_6}^6,w_{d_7}^7,w_{d_8}^8,w_{d_9}^9 \geq \varepsilon; \\
& r_1=1,\dots,R_1;\ r_2=1,\dots,R_2;\ r_3=1,\dots,R_3;\ r_4=1,\dots,R_4;\ r_5=1,\dots,R_5;\ r_6=1,\dots,R_6;\ r_7=1,\dots,R_7;\ r_8=1,\dots,R_8; \\
& i_1=1,\dots,I_1;\ i_2=1,\dots,I_2;\ i_3=1,\dots,I_3;\ i_4=1,\dots,I_4;\ i_5=1,\dots,I_5;\ i_6=1,\dots,I_6;\ d_1=1,\dots,D_1;\ d_2=1,\dots,D_2;\ d_3=1,\dots,D_3; \\
& d_4=1,\dots,D_4;\ d_5=1,\dots,D_5;\ d_6=1,\dots,D_6; d_7=1,\dots,D_7;\ d_8=1,\dots,D_8;\ d_9=1,\dots,D_9.
\end{aligned}$$

In model (10) we measure the overall efficiency based on the efficiencies of the all sub-DMUs being less than one.

2.1. Interval DEA models

In view of not evading the entire subject, it shall be assumed that some of the data in model (10) due to being unreliable and are incapable of being accurately determined and we only know that they are within their upper and lower bounds. Despotis and Smirlis (2002) have calculated the efficiency of DMUs in the presence of intervals and have proposed models for the upper and lower bound efficiencies. Therefore, by developing the task of Despotis and Smirlis we shall modify model (10), with the assumption that the variables are bounded in the presence of undesirable outputs as in the figure below:

$$\theta_o^{overall(U)}=\max\frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^{1U}+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^{2U}+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^{3U}-\sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^{1L}}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^{1L}}\cdot\frac{O_o^{234U}}{I_o^{234L}}\cdot\frac{O_o^{56U}}{I_o^{56L}} \qquad (11)$$

$$\begin{aligned}
\text{s.t.}\quad & \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^{1L}+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^{2L}+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^{3L}-\sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^{1U}-\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^{1U}\leq 0, \quad \forall j\neq o \\
& \sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^{4L}+\sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^{5L}+\sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^{2L}-\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^{2U}-\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^{1U}\leq 0, \quad \forall j\neq o \\
& \sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^{6L}+\sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^{7L}+\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^{3L}-\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^{3U}-\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^{2U}\leq 0, \quad \forall j\neq o \\
& \sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^{8L}+\sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^{9L}+\sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4j}^{4L}-\sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4j}^{4U}-\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^{3U}\leq 0, \quad \forall j\neq o \\
& \sum_{r_5=1}^{R_5} u_{r_5}^5 y_{r_5j}^{5L}-\sum_{r_6=1}^{R_6} u_{r_6}^6 y_{r_6j}^{6U}-\sum_{i_5=1}^{I_5} v_{i_5}^5 x_{i_5j}^{5U}-\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^{4U}-\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^{6U}-\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^{8U}\leq 0, \quad \forall j\neq o
\end{aligned}$$

$$\sum_{r_7=1}^{R_7} u_{r_7}^7\, y_{r_7j}^{7L} - \sum_{r_8=1}^{R_8} u_{r_8}^8\, y_{r_8j}^{8U} - \sum_{i_6=1}^{I_6} v_{i_6}^6 x_{i_6j}^{6U} - \sum_{d_5=1}^{D_5} w_{d_5}^5\, z_{d_5j}^{5U} - \sum_{d_7=1}^{D_7} w_{d_7}^7\, z_{d_7j}^{7U} - \sum_{d_9=1}^{D_9} w_{d_9}^9\, z_{d_9j}^{9U} \leq 0, \quad \forall j\neq o$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1o}^{1U} + \sum_{d_2=1}^{D_2} w_{d_2}^2\, z_{d_2o}^{2U} + \sum_{d_3=1}^{D_3} w_{d_3}^3\, z_{d_3o}^{3U} - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^{1L} - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^{1L} \leq 0$$

$$\sum_{d_4=1}^{D_4} w_{d_4}^4\, z_{d_4o}^{4U} + \sum_{d_5=1}^{D_5} w_{d_5}^5\, z_{d_5o}^{5U} + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^{2U} - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^{2L} - \sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1o}^{1L} \leq 0$$

$$\sum_{d_6=1}^{D_6} w_{d_6}^6\, z_{d_6o}^{6U} + \sum_{d_7=1}^{D_7} w_{d_7}^7\, z_{d_7o}^{7U} + \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3o}^{3U} - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^{3L} - \sum_{d_2=1}^{D_2} w_{d_2}^2\, z_{d_2o}^{2L} \leq 0$$

$$\sum_{d_8=1}^{D_8} w_{d_8}^8\, z_{d_8o}^{8U} + \sum_{d_9=1}^{D_9} w_{d_9}^9\, z_{d_9o}^{9U} + \sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4o}^{4U} - \sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4o}^{4L} - \sum_{d_3=1}^{D_3} w_{d_3}^3\, z_{d_3o}^{3L} \leq 0$$

$$\sum_{r_5=1}^{R_5} u_{r_5}^5\, y_{r_5o}^{5U} - \sum_{r_6=1}^{R_6} u_{r_6}^6\, y_{r_6o}^{6L} - \sum_{i_5=1}^{I_5} v_{i_5}^5 x_{i_5o}^{5L} - \sum_{d_4=1}^{D_4} w_{d_4}^4\, z_{d_4o}^{4L} - \sum_{d_6=1}^{D_6} w_{d_6}^6\, z_{d_6o}^{6L} - \sum_{d_8=1}^{D_8} w_{d_8}^8\, z_{d_8o}^{8L} \leq 0$$

$$\sum_{r_7=1}^{R_7} u_{r_7}^7\, y_{r_7o}^{7U} - \sum_{r_8=1}^{R_8} u_{r_8}^8\, y_{r_8o}^{8L} - \sum_{i_6=1}^{I_6} v_{i_6}^6 x_{i_6o}^{6L} - \sum_{d_5=1}^{D_5} w_{d_5}^5\, z_{d_5o}^{5L} - \sum_{d_7=1}^{D_7} w_{d_7}^7\, z_{d_7o}^{7L} - \sum_{d_9=1}^{D_9} w_{d_9}^9\, z_{d_9o}^{9L} \leq 0$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3, u_{r_4}^4, u_{r_5}^5, u_{r_6}^6, u_{r_7}^7, u_{r_8}^8, v_{i_1}^1, v_{i_2}^2, v_{i_3}^3, v_{i_4}^4, v_{i_5}^5, v_{i_6}^6, w_{d_1}^1, w_{d_2}^2, w_{d_3}^3, w_{d_4}^4, w_{d_5}^5, w_{d_6}^6, w_{d_7}^7, w_{d_8}^8, w_{d_9}^9 \geq \varepsilon;$$

$r_1=1,\dots,R_1$; $r_2=1,\dots,R_2$; $r_3=1,\dots,R_3$; $r_4=1,\dots,R_4$; $r_5=1,\dots,R_5$; $r_6=1,\dots,R_6$; $r_7=1,\dots,R_7$; $r_8=1,\dots,R_8$;

$i_1=1,\dots,I_1$; $i_2=1,\dots,I_2$; $i_3=1,\dots,I_3$; $i_4=1,\dots,I_4$; $i_5=1,\dots,I_5$; $i_6=1,\dots,I_6$; $d_1=1,\dots,D_1$; $d_2=1,\dots,D_2$; $d_3=1,\dots,D_3$;

$d_4=1,\dots,D_4$; $d_5=1,\dots,D_5$; $d_6=1,\dots,D_6$; $d_7=1,\dots,D_7$; $d_8=1,\dots,D_8$; $d_9=1,\dots,D_9$.

$$\theta_o^{overall(L)} = \max \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^{1L} + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^{2L} + \sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^{3L} - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^{1U}}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^{1U}} \cdot \frac{O_o^{234L}}{I_o^{234U}} \cdot \frac{O_o^{56L}}{I_o^{56U}} \tag{12}$$

$$\text{s.t.}\ \sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1j}^{1U} + \sum_{d_2=1}^{D_2} w_{d_2}^2\, z_{d_2j}^{2U} + \sum_{d_3=1}^{D_3} w_{d_3}^3\, z_{d_3j}^{3U} - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^{1L} - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^{1L} \leq 0, \quad \forall j\neq o$$

$$\sum_{d_4=1}^{D_4} w_{d_4}^4\, z_{d_4j}^{4U} + \sum_{d_5=1}^{D_5} w_{d_5}^5\, z_{d_5j}^{5U} + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^{2U} - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^{2L} - \sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1j}^{1L} \leq 0, \quad \forall j\neq o$$

$$\sum_{d_6=1}^{D_6} w_{d_6}^6\, z_{d_6j}^{6U} + \sum_{d_7=1}^{D_7} w_{d_7}^7\, z_{d_7j}^{7U} + \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^{3U} - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^{3L} - \sum_{d_2=1}^{D_2} w_{d_2}^2\, z_{d_2j}^{2L} \leq 0, \quad \forall j\neq o$$

$$\sum_{d_8=1}^{D_8} w_{d_8}^8\, z_{d_8j}^{8U} + \sum_{d_9=1}^{D_9} w_{d_9}^9\, z_{d_9j}^{9U} + \sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4j}^{4U} - \sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4j}^{4L} - \sum_{d_3=1}^{D_3} w_{d_3}^3\, z_{d_3j}^{3L} \leq 0, \quad \forall j\neq o$$

$$\sum_{r_5=1}^{R_5} u_{r_5}^5\, y_{r_5j}^{5U} - \sum_{r_6=1}^{R_6} u_{r_6}^6\, y_{r_6j}^{6L} - \sum_{i_5=1}^{I_5} v_{i_5}^5 x_{i_5j}^{5L} - \sum_{d_4=1}^{D_4} w_{d_4}^4\, z_{d_4j}^{4L} - \sum_{d_6=1}^{D_6} w_{d_6}^6\, z_{d_6j}^{6L} - \sum_{d_8=1}^{D_8} w_{d_8}^8\, z_{d_8j}^{8L} \leq 0, \quad \forall j\neq o$$

$$\sum_{r_7=1}^{R_7} u_{r_7}^7\, y_{r_7j}^{7U} - \sum_{r_8=1}^{R_8} u_{r_8}^8\, y_{r_8j}^{8L} - \sum_{i_6=1}^{I_6} v_{i_6}^6 x_{i_6j}^{6L} - \sum_{d_5=1}^{D_5} w_{d_5}^5\, z_{d_5j}^{5L} - \sum_{d_7=1}^{D_7} w_{d_7}^7\, z_{d_7j}^{7L} - \sum_{d_9=1}^{D_9} w_{d_9}^9\, z_{d_9j}^{9L} \leq 0, \quad \forall j\neq o$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1o}^{1L} + \sum_{d_2=1}^{D_2} w_{d_2}^2\, z_{d_2o}^{2L} + \sum_{d_3=1}^{D_3} w_{d_3}^3\, z_{d_3o}^{3L} - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^{1U} - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^{1U} \leq 0$$

$$\sum_{d_4=1}^{D_4} w_{d_4}^4\, z_{d_4o}^{4L} + \sum_{d_5=1}^{D_5} w_{d_5}^5\, z_{d_5o}^{5L} + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^{2L} - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^{2U} - \sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1o}^{1U} \leq 0$$

$$\sum_{d_6=1}^{D_6} w_{d_6}^6\, z_{d_6o}^{6L} + \sum_{d_7=1}^{D_7} w_{d_7}^7\, z_{d_7o}^{7L} + \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3o}^{3L} - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^{3U} - \sum_{d_2=1}^{D_2} w_{d_2}^2\, z_{d_2o}^{2U} \leq 0$$

$$\sum_{d_8=1}^{D_8} w_{d_8}^8\, z_{d_8o}^{8L} + \sum_{d_9=1}^{D_9} w_{d_9}^9\, z_{d_9o}^{9L} + \sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4o}^{4L} - \sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4o}^{4U} - \sum_{d_3=1}^{D_3} w_{d_3}^3\, z_{d_3o}^{3U} \leq 0$$

$$\sum_{r_5=1}^{R_5} u_{r_5}^5\, y_{r_5o}^{5L} - \sum_{r_6=1}^{R_6} u_{r_6}^6\, y_{r_6o}^{6U} - \sum_{i_5=1}^{I_5} v_{i_5}^5 x_{i_5o}^{5U} - \sum_{d_4=1}^{D_4} w_{d_4}^4\, z_{d_4o}^{4U} - \sum_{d_6=1}^{D_6} w_{d_6}^6\, z_{d_6o}^{6U} - \sum_{d_8=1}^{D_8} w_{d_8}^8\, z_{d_8o}^{8U} \leq 0$$

$$\sum_{r_7=1}^{R_7} u_{r_7}^7\, y_{r_7o}^{7L} - \sum_{r_8=1}^{R_8} u_{r_8}^8\, y_{r_8o}^{8U} - \sum_{i_6=1}^{I_6} v_{i_6}^6 x_{i_6o}^{6U} - \sum_{d_5=1}^{D_5} w_{d_5}^5\, z_{d_5o}^{5U} - \sum_{d_7=1}^{D_7} w_{d_7}^7\, z_{d_7o}^{7U} - \sum_{d_9=1}^{D_9} w_{d_9}^9\, z_{d_9o}^{9U} \leq 0$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3, u_{r_4}^4, u_{r_5}^5, u_{r_6}^6, u_{r_7}^7, u_{r_8}^8, v_{i_1}^1, v_{i_2}^2, v_{i_3}^3, v_{i_4}^4, v_{i_5}^5, v_{i_6}^6, w_{d_1}^1, w_{d_2}^2, w_{d_3}^3, w_{d_4}^4, w_{d_5}^5, w_{d_6}^6, w_{d_7}^7, w_{d_8}^8, w_{d_9}^9 \geq \varepsilon;$$

$r_1=1,\dots,R_1$; $r_2=1,\dots,R_2$; $r_3=1,\dots,R_3$; $r_4=1,\dots,R_4$; $r_5=1,\dots,R_5$; $r_6=1,\dots,R_6$; $r_7=1,\dots,R_7$; $r_8=1,\dots,R_8$;

$i_1=1,\dots,I_1$; $i_2=1,\dots,I_2$; $i_3=1,\dots,I_3$; $i_4=1,\dots,I_4$; $i_5=1,\dots,I_5$; $i_6=1,\dots,I_6$; $d_1=1,\dots,D_1$; $d_2=1,\dots,D_2$; $d_3=1,\dots,D_3$;

$d_4=1,\dots,D_4$; $d_5=1,\dots,D_5$; $d_6=1,\dots,D_6$; $d_7=1,\dots,D_7$; $d_8=1,\dots,D_8$; $d_9=1,\dots,D_9$.

Model (11) measures the efficiency under the most desirable conditions which is known as the 'upper bound efficiency'; whereas, model (12) measures the efficiency under the most undesirable conditions and is known as the 'lower bound efficiency'. It should be noted that the undesirable output have similar characteristics to that of the inputs, thereby, their boundaries are also considered alike the inputs. Models (11) and (12) are nonlinear and are obtained by multiplying their objective function. In the third section of this paper, an innovative approach is used to solve them. Let us assume that the models (11) and (12) are resolved, the interval efficiency of the network illustrated in Fig. 1 for DMU_0 in the $\left[\theta_o^{\text{overall(L)}}, \theta_o^{\text{overall(U)}}\right]$ manner. In order to compare the interval efficiency and the ranking of DMUs, a minimax regret-based approach was proposed by Wang et al. (2005) and we shall use this approach to rank the units. Wang et al. has also stated that, the interval efficiency with the slightest waste of efficiency would be the optimal efficiency interval. They defined the criteria for the minimax regret-based approach for the efficiency interval as $= A_i[a_i^L, a_i^U], (i = 1 \dots n)$ which has been described in formula (13) as below:

$$R(A_i) = max\left(\max_{j\neq i}(a_j^U) - a_i^L, 0\right), \quad (i = 1 \dots n) \tag{13}$$

On the basis of which, we first calculate the maximum loss for each interval and consider the minimum loss, as the most desirable or favorable interval. In the next stage we eliminate the desirable interval and repeatedly, in the same manner, from the remaining n-1 select the optimal interval and this is iteratively performed until only one interval efficiency remains; and the lowest position is assigned to this.

3. Model solution

Models (11) and (12) cannot be turned into linear models because of the additional outputs in the first stage in relative to sub-DMU_{1o}, the additional inputs and outputs in the second stage related to sub-DMU_{2o}, sub-DMU_{3o} and sub-DMU_{4o} and the additional inputs in the third stage in relevance to sub-DMU_{5o} and sub-DMU_{6o} respectively. Thus, we propose the heuristic approach given hereunder, for solving models (11) and (12). This approach shall be founded on model (11). We are aware that the objective function of model (11) is the product of the efficiency of the three phases *i.e.* $\theta_o^{\text{overall(U)}}=\max \theta_o^{1U} . \theta_o^{234U} . \theta_o^{56U}$. First, we measure the maximum efficiency of each stage provided that the efficiency of each sub-DMU in stage 1, stag 2 and stage 3 is less than one. Therefore, we define $\theta_o^{1U\text{-max}}$, $\theta_o^{234U\text{-max}}$ and $\theta_o^{56U\text{-max}}$ maximum efficiencies for stage 1, stage 2 and stage 3, respectively as follows:

$$\theta_o^{1U\text{-max}}=\max\left\{\begin{matrix}\theta_o^{1U} \mid \theta_j^{1L} \le 1, \theta_j^{2L} \le 1, \theta_j^{3L} \le 1, \ \theta_j^{4L} \le 1, \theta_j^{5L} \le 1, \theta_j^{6L} \le 1, \forall j\neq o, \\ \theta_o^{1U} \le 1, \theta_o^{2U} \le 1, \theta_o^{3U} \le 1, \theta_o^{4U} \le 1, \theta_o^{5U} \le 1, \theta_o^{6U} \le 1\end{matrix}\right\} \tag{14}$$

$$\theta_o^{234U\text{-max}}=\max\left\{\begin{matrix}\theta_o^{234U} \mid \theta_j^{1L} \le 1, \theta_j^{2L} \le 1, \theta_j^{3L} \le 1, \ \theta_j^{4L} \le 1, \theta_j^{5L} \le 1, \theta_j^{6L} \le 1, \forall j\neq o, \\ \theta_o^{1U} \le 1, \theta_o^{2U} \le 1, \theta_o^{3U} \le 1, \theta_o^{4U} \le 1, \theta_o^{5U} \le 1, \theta_o^{6U} \le 1\end{matrix}\right\}$$

$$\theta_o^{56U\text{-max}}=\max\left\{\begin{matrix}\theta_o^{56U} \mid \theta_j^{1L} \le 1, \theta_j^{2L} \le 1, \theta_j^{3L} \le 1, \ \theta_j^{4L} \le 1, \theta_j^{5L} \le 1, \theta_j^{6L} \le 1, \forall j\neq o, \\ \theta_o^{1U} \le 1, \theta_o^{2U} \le 1, \theta_o^{3U} \le 1, \theta_o^{4U} \le 1, \theta_o^{5U} \le 1, \theta_o^{6U} \le 1\end{matrix}\right\}$$

All the variables are non-negative in models (14). As in the objective function of model (11) the upper limit variables are in the form of fractions and the lower limit of variables are the fraction denominators. Hence, the fractions secure their maximum value, which we have demonstrated in model (14) as θ_o^{1U} , θ_o^{234U} and θ_o^{56U}. Similarly, the other restrictions have also been briefly outlined. With the Charnes–Cooper (1962) converted models (14) can be turned into a linear model. We can solve the models (14) and measure $\theta_o^{1U\text{-max}}$, $\theta_o^{234U\text{-max}}$ and $\theta_o^{56U\text{-max}}$, respectively. The objective function of model (11) is $\theta_o^{1U} \,.\, \theta_o^{234U} \,.\, \theta_o^{56U}$ and an upper bound can be obtained for each one. We suggest a heuristic method to solve model (11) by using two models as follows:

Step 1: In the first model let us consider two stages for example θ_o^{1U} and θ_o^{234U} as two variables that change between intervals $[0, \theta_o^{1U\text{-max}}]$ and $[0, \theta_o^{234U\text{-max}}]$, respectively. Then the maximal efficiency of model (11) and the maximal and minimal efficiency of θ_o^{1U} and θ_o^{234U} are obtained, provided that the efficiency of model (11) is fixed or constant.

Step 2: In the second model consider another two stages for example θ_o^{234U} and θ_o^{56U} as two variables and as in the first model, it obtains the maximal efficiency of model (11) and the maximal and the minimal efficiency of θ_o^{234U} and θ_o^{56U}.

By using two abovementioned steps we obtain the maximal efficiency of the model (11) and the maximal and minimal efficiency of one stage twice, that is, with a very agreeable approximation, which is equivalent. We perform the two abovementioned steps three times and consider all the modes. In the continuation of this paper, we shall consider one of the modes. First, we convert model (11) to model (15) and use a heuristic method to solve it as follows:

$$\theta_o^{\text{overall(U)}} = \max \left\{ \theta_o^{1U} . \theta_o^{234U} . \theta_o^{56U} \;\middle|\; \begin{array}{c} \theta_j^{1L} \le 1,\ \theta_j^{2L} \le 1,\ \theta_j^{3L} \le 1,\ \theta_j^{4L} \le 1,\ \theta_j^{5L} \le 1,\ \theta_j^{6L} \le 1,\ \forall j \neq o, \\ \theta_o^{1U} \le 1,\ \theta_o^{2U} \le 1,\ \theta_o^{3U} \le 1,\ \theta_o^{4U} \le 1,\ \theta_o^{5U} \le 1,\ \theta_o^{6U} \le 1 \\ \theta_o^{1U} = \frac{O_o^{1U}}{I_o^{1L}},\ \theta_o^{234U} = \frac{O_o^{234U}}{I_o^{234L}},\ \theta_o^{1U} \in [0, \theta_o^{1U\text{-max}}], \\ \theta_o^{234U} \in [0, \theta_o^{234U\text{-max}}] \end{array} \right\} \tag{15}$$

In the models (15) all the variables are non-negative. It should be noted that in the model (15), we consider θ_o^{1U} , θ_o^{234U} as two variables in the objective function and the two constraints which specify these two variables, together with the interval modifications are supplemented to the model by us. In model (1), we have described the efficiency of the first stage and in model (15) have briefly demonstrated by outputs to inputs. In model (15) let us consider stage 1 and 2 as two variables θ_o^{1U} and θ_o^{234U} that change between intervals $[0, \theta_o^{1\text{-max}}]$ and $[0, \theta_o^{234\text{-max}}]$, respectively. We should fix θ_o^{1U} and θ_o^{234U} until model (15) becomes a linear programing model and we can solve it. For this purpose, we define θ_o^{1U} and θ_o^{234U} as follows:

$$\theta_o^{1U} = \theta_o^{1U\text{-max}} - k_1 \Delta\varepsilon, \qquad k_1 = 0,1,\ldots, \left[\frac{\theta_o^{1U\text{-max}}}{\Delta\varepsilon}\right] + 1 \tag{16}$$

$$\theta_o^{234U} = \theta_o^{234U\text{-max}} - k_2 \Delta\varepsilon, \qquad k_2 = 0,1,\ldots, \left[\frac{\theta_o^{234U\text{-max}}}{\Delta\varepsilon}\right] + 1$$

In the model (16), $\Delta\varepsilon$ is a step size and we consider $\Delta\varepsilon = 0.01$. With the Charnes–Cooper (1962) converted models, model (15) can be turned into a linear model as follows:

Let $T = \dfrac{1}{\sum_{i_5=1}^{I_5} v_{i_5}^5 x_{i_5o}^{5L} + \sum_{i_6=1}^{I_6} v_{i_6}^6 x_{i_6o}^{6L} + \sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4o}^{4L} + \sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5o}^{5L} + \sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6o}^{6L} + \sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7o}^{7L} + \sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8o}^{8L} + \sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9o}^{9L}}$, thus:

$$\theta_o^{\text{overall(U)}} = \max \theta_o^{1U} . \theta_o^{234U} . \left(\sum_{r_5=1}^{R_5} u_{r_5}^5 y_{r_5o}^{5U} + \sum_{r_7=1}^{R_7} u_{r_7}^7 y_{r_7o}^{7U} - \sum_{r_6=1}^{R_6} u_{r_6}^6 y_{r_6o}^{6L} - \sum_{r_8=1}^{R_8} u_{r_8}^8 y_{r_8o}^{8L} \right) \tag{17}$$

$$\text{s.t.} \quad \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^{1L} + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^{2L} + \sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^{3L} - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^{1U} - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^{1U} \le 0, \qquad \forall j \neq o$$

$$\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^{4L} + \sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^{5L} + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^{2L} - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^{2U} - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^{1U} \le 0, \qquad \forall j \neq o$$

$$\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^{6L} + \sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^{7L} + \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^{3L} - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^{3U} - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^{2U} \le 0, \qquad \forall j \neq o$$

$$\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^{8L} + \sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^{9L} + \sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4j}^{4L} - \sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4j}^{4U} - \sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^{3U} \le 0, \qquad \forall j \neq o$$

$$\sum_{r_5=1}^{R_5} u_{r_5}^5\, y_{r_5 j}^{5L} - \sum_{r_6=1}^{R_6} u_{r_6}^6\, y_{r_6 j}^{6U} - \sum_{i_5=1}^{I_5} v_{i_5}^5 x_{i_5 j}^{5U} - \sum_{d_4=1}^{D_4} w_{d_4}^4\, z_{d_4 j}^{4U} - \sum_{d_6=1}^{D_6} w_{d_6}^6\, z_{d_6 j}^{6U} - \sum_{d_8=1}^{D_8} w_{d_8}^8\, z_{d_8 j}^{8U} \le 0, \quad \forall j \neq o$$

$$\sum_{r_7=1}^{R_7} u_{r_7}^7\, y_{r_7 j}^{7L} - \sum_{r_8=1}^{R_8} u_{r_8}^8\, y_{r_8 j}^{8U} - \sum_{i_6=1}^{I_6} v_{i_6}^6 x_{i_6 j}^{6U} - \sum_{d_5=1}^{D_5} w_{d_5}^5\, z_{d_5 j}^{5U} - \sum_{d_7=1}^{D_7} w_{d_7}^7\, z_{d_7 j}^{7U} - \sum_{d_9=1}^{D_9} w_{d_9}^9\, z_{d_9 j}^{9U} \le 0 \quad \forall j \neq o$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1 o}^{1U} + \sum_{d_2=1}^{D_2} w_{d_2}^2\, z_{d_2 o}^{2U} + \sum_{d_3=1}^{D_3} w_{d_3}^3\, z_{d_3 o}^{3U} - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1 o}^{1L} - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1 o}^{1L} \le 0$$

$$\sum_{d_4=1}^{D_4} w_{d_4}^4\, z_{d_4 o}^{4U} + \sum_{d_5=1}^{D_5} w_{d_5}^5\, z_{d_5 o}^{5U} + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2 o}^{2U} - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2 o}^{2L} - \sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1 o}^{1L} \le 0$$

$$\sum_{d_6=1}^{D_6} w_{d_6}^6\, z_{d_6 o}^{6U} + \sum_{d_7=1}^{D_7} w_{d_7}^7\, z_{d_7 o}^{7U} + \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3 o}^{3U} - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3 o}^{3L} - \sum_{d_2=1}^{D_2} w_{d_2}^2\, z_{d_2 o}^{2L} \le 0$$

$$\sum_{d_8=1}^{D_8} w_{d_8}^8\, z_{d_8 o}^{8U} + \sum_{d_9=1}^{D_9} w_{d_9}^9\, z_{d_9 o}^{9U} + \sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4 o}^{4U} - \sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4 o}^{4L} - \sum_{d_3=1}^{D_3} w_{d_3}^3\, z_{d_3 o}^{3L} \le 0$$

$$\sum_{r_5=1}^{R_5} u_{r_5}^5\, y_{r_5 o}^{5U} - \sum_{r_6=1}^{R_6} u_{r_6}^6\, y_{r_6 o}^{6L} - \sum_{i_5=1}^{I_5} v_{i_5}^5 x_{i_5 o}^{5L} - \sum_{d_4=1}^{D_4} w_{d_4}^4\, z_{d_4 o}^{4L} - \sum_{d_6=1}^{D_6} w_{d_6}^6\, z_{d_6 o}^{6L} - \sum_{d_8=1}^{D_8} w_{d_8}^8\, z_{d_8 o}^{8L} \le 0$$

$$\sum_{r_7=1}^{R_7} u_{r_7}^7\, y_{r_7 o}^{7U} - \sum_{r_8=1}^{R_8} u_{r_8}^8\, y_{r_8 o}^{8L} - \sum_{i_6=1}^{I_6} v_{i_6}^6 x_{i_6 o}^{6L} - \sum_{d_5=1}^{D_5} w_{d_5}^5\, z_{d_5 o}^{5L} - \sum_{d_7=1}^{D_7} w_{d_7}^7\, z_{d_7 o}^{7L} - \sum_{d_9=1}^{D_9} w_{d_9}^9\, z_{d_9 o}^{9L} \le 0$$

$$\sum_{i_5=1}^{I_5} v_{i_5}^5 x_{i_5 o}^{5L} + \sum_{i_6=1}^{I_6} v_{i_6}^6 x_{i_6 o}^{6L} + \sum_{d_4=1}^{D_4} w_{d_4}^4\, z_{d_4 o}^{4L} + \sum_{d_5=1}^{D_5} w_{d_5}^5\, z_{d_5 o}^{5L} + \sum_{d_6=1}^{D_6} w_{d_6}^6\, z_{d_6 o}^{6L} + \sum_{d_7=1}^{D_7} w_{d_7}^7\, z_{d_7 o}^{7L}$$

$$+ \sum_{d_8=1}^{D_8} w_{d_8}^8\, z_{d_8 o}^{8L} + \sum_{d_9=1}^{D_9} w_{d_9}^9\, z_{d_9 o}^{9L} = 1$$

$$O_o^{1U} = I_o^{1L}.\, \theta_o^{1U}$$

$$O_o^{234U} = I_o^{234L}.\, \theta_o^{234U}$$

$$\theta_o^{1U} \in [0, \theta_o^{1U\text{-}max}]$$

$$\theta_o^{234U} \in [0, \theta_o^{234U\text{-}max}]$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3, u_{r_4}^4, u_{r_5}^5, u_{r_6}^6, u_{r_7}^7, u_{r_8}^8, v_{i_1}^1, v_{i_2}^2, v_{i_3}^3, v_{i_4}^4, v_{i_5}^5, v_{i_6}^6, w_{d_1}^1, w_{d_2}^2, w_{d_3}^3, w_{d_4}^4, w_{d_5}^5, w_{d_6}^6, w_{d_7}^7, w_{d_8}^8, w_{d_9}^9 \ge \varepsilon;$$

$r_1=1, \dots, R_1$; $r_2=1,\dots,R_2$; $r_3=1,\dots, R_3$; $r_4=1, \dots,R_4$; $r_5=1,\dots,R_5$; $r_6=1,\dots, R_6$; $r_7=1, \dots,R_7$; $r_8=1,\dots,R_8$;

$i_1=1,\dots,I_1$; $i_2=1,\dots,I_2$; $i_3=1,\dots,I_3$; $i_4=1,\dots,I_4$; $i_5=1,\dots,I_5$; $i_6=1,\dots,I_6$; $d_1=1,\dots,D_1$; $d_2=1,\dots,D_2$; $d_3=1,\dots,D_3$;

$d_4=1,\dots,D_4$; $d_5=1,\dots,D_5$; $d_6=1,\dots,D_6$; $d_7=1,\dots,D_7$; $d_8=1,\dots,D_8$; $d_9=1,\dots,D_9$.

In model (17) we increase k_1 and k_2 from zero to the upper bound of each one of them independently and solve each linear model with both k_1 and k_2 and show the value of the objective function with $\theta_o^{overall(U1)}(k_1, k_2)$. In comparing all the values of the objective function in model (17), we define the maximal overall efficiency as $\theta_o^{overall(U1)}$=max $\theta_o^{overall(U1)}(k_1, k_2)$. The maximal and minimal efficiencies of the first stage are defined as θ_o^{1U+}=max $\theta_o^{1U}(k_1)$ where, k_1=min $\left(k_1 \,\middle|\, \theta_o^{overall(U1)}(k_1, k_2) = \theta_o^{overall(U1)}\right)$ and θ_o^{1U-}=min $\theta_o^{1U}(k_1)$ where, k_1=max $\left(k_1 \,\middle|\, \theta_o^{overall(U1)}(k_1, k_2) = \theta_o^{overall(U1)}\right)$, respectively. Similarly, the maximal and minimal efficiencies of the second stage are defined as $\theta_o^{234(U1)+}$=max $\theta_o^{234U}(k_2)$ where, k_2=min $\left(k_2 \,\middle|\, \theta_o^{overall(U1)}(k_1, k_2) = \theta_o^{overall(U1)}\right)$ and $\theta_o^{234(U1)-}$=min $\theta_o^{234U}(k_2)$ where, k_2=max $\left(k_2 \,\middle|\, \theta_o^{overall(U1)}(k_1, k_2) = \theta_o^{overall(U1)}\right)$, respectively. If $\theta_o^{1U+}=\theta_o^{1U-}$ and $\theta_o^{234(U1)+}=\theta_o^{234(U1)-}$ we obtain a unique efficiency for the third stage as follows: $\theta_o^{56U}=\frac{\theta_o^{overall(U1)}}{\theta_o^{1U}\,.\,\theta_o^{234(U1)}}$. Or else, for measuring the maximal and minimal efficiency of the third stage we convert model (11) into the following model:

$$\theta_o^{overall(U)}=\max\left\{\theta_o^{1U}.\,\theta_o^{234U}.\,\theta_o^{56U}\;\middle|\;\begin{array}{c}\theta_j^{1L}\le 1,\theta_j^{2L}\le 1,\theta_j^{3L}\le 1,\ \theta_j^{4L}\le 1,\theta_j^{5L}\le 1,\theta_j^{6L}\le 1,\forall j\neq o,\\ \theta_o^{1U}\le 1,\theta_o^{2U}\le 1,\theta_o^{3U}\le 1,\ \theta_o^{4U}\le 1,\theta_o^{5U}\le 1,\theta_o^{6U}\le 1\\ \theta_o^{234}=\frac{O_o^{234}}{I_o^{234}},\ \theta_o^{56}=\frac{O_o^{56}}{I_o^{56}},\\ \theta_o^{234U}\in[0,\theta_o^{234U\text{-}max}],\theta_o^{56U}\in[0,\theta_o^{56U\text{-}max}],j=1,\dots,n\end{array}\right\} \quad (18)$$

In model (18) we consider stages 2 and 3 as two variables θ_o^{234U} and θ_o^{56U} that change between intervals $[0, \theta_o^{234U\text{-}max}]$ and $[0, \theta_o^{56U\text{-}max}]$, respectively. We define θ_o^{234U} and θ_o^{56U} as follows:

$$\theta_o^{234U}=\theta_o^{234U\text{-}max}-k_2\Delta\varepsilon,\qquad k_2=0,1,\dots,\left[\frac{\theta_o^{234U\text{-}max}}{\Delta\varepsilon}\right]+1 \quad (19)$$

$$\theta_o^{56U}=\theta_o^{56U\text{-}max}-k_3\Delta\varepsilon,\qquad k_3=0,1,\dots,\left[\frac{\theta_o^{56U\text{-}max}}{\Delta\varepsilon}\right]+1$$

In model (19) we consider $\Delta\varepsilon$ =0.01. With the Charnes–Cooper (1962) converted model (18) can be turned into a linear model as follows:

Let $T=\frac{1}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^{1L}}$, thus:

$$\theta_o^{overall(U)}=\max\ \theta_o^{234U}.\,\theta_o^{56U}.\left(\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^{1U}+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^{2U}+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^{3U}-\sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^{1L}\right) \quad (20)$$

$$\begin{aligned}
\text{s.t.}\ &\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^{1L}+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^{2L}+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^{3L}-\sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^{1U}-\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^{1U}\le 0, && \forall j\neq o\\
&\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^{4L}+\sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^{5L}+\sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^{2L}-\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^{2U}-\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^{1U}\le 0, && \forall j\neq o\\
&\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^{6L}+\sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^{7L}+\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^{3L}-\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^{3U}-\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^{2U}\le 0, && \forall j\neq o\\
&\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^{8L}+\sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^{9L}+\sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4j}^{4L}-\sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4j}^{4U}-\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3j}^{3U}\le 0, && \forall j\neq o\\
&\sum_{r_5=1}^{R_5} u_{r_5}^5 y_{r_5j}^{5L}-\sum_{r_6=1}^{R_6} u_{r_6}^6 y_{r_6j}^{6U}-\sum_{i_5=1}^{I_5} v_{i_5}^5 x_{i_5j}^{5U}-\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4j}^{4U}-\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6j}^{6U}-\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8j}^{8U}\le 0, && \forall j\neq o\\
&\sum_{r_7=1}^{R_7} u_{r_7}^7 y_{r_7j}^{7L}-\sum_{r_8=1}^{R_8} u_{r_8}^8 y_{r_8j}^{8U}-\sum_{i_6=1}^{I_6} v_{i_6}^6 x_{i_6j}^{6U}-\sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5j}^{5U}-\sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7j}^{7U}-\sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9j}^{9U}\le 0, && \forall j\neq o\\
&\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^{1U}+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^{2U}+\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^{3U}-\sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^{1L}-\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^{1L}\le 0\\
&\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4o}^{4U}+\sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5o}^{5U}+\sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^{2U}-\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^{2L}-\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^{1L}\le 0\\
&\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6o}^{6U}+\sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7o}^{7U}+\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3o}^{3U}-\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^{3L}-\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^{2L}\le 0\\
&\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8o}^{8U}+\sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9o}^{9U}+\sum_{r_4=1}^{R_4} u_{r_4}^4 y_{r_4o}^{4U}-\sum_{i_4=1}^{I_4} v_{i_4}^4 x_{i_4o}^{4L}-\sum_{d_3=1}^{D_3} w_{d_3}^3 z_{d_3o}^{3L}\le 0\\
&\sum_{r_5=1}^{R_5} u_{r_5}^5 y_{r_5o}^{5U}-\sum_{r_6=1}^{R_6} u_{r_6}^6 y_{r_6o}^{6L}-\sum_{i_5=1}^{I_5} v_{i_5}^5 x_{i_5o}^{5L}-\sum_{d_4=1}^{D_4} w_{d_4}^4 z_{d_4o}^{4L}-\sum_{d_6=1}^{D_6} w_{d_6}^6 z_{d_6o}^{6L}-\sum_{d_8=1}^{D_8} w_{d_8}^8 z_{d_8o}^{8L}\le 0\\
&\sum_{r_7=1}^{R_7} u_{r_7}^7 y_{r_7o}^{7U}-\sum_{r_8=1}^{R_8} u_{r_8}^8 y_{r_8o}^{8L}-\sum_{i_6=1}^{I_6} v_{i_6}^6 x_{i_6o}^{6L}-\sum_{d_5=1}^{D_5} w_{d_5}^5 z_{d_5o}^{5L}-\sum_{d_7=1}^{D_7} w_{d_7}^7 z_{d_7o}^{7L}-\sum_{d_9=1}^{D_9} w_{d_9}^9 z_{d_9o}^{9L}\le 0\\
&\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^{1L}=1\\
&O_o^{234U}=\theta_o^{234U}.I_o^{234L}\\
&O_o^{56U}=\theta_o^{56U}.I_o^{56L}\\
&\theta_o^{234U}\in[0,\theta_o^{234U\text{-}max}]\\
&\theta_o^{56U}\in[0,\theta_o^{56U\text{-}max}]
\end{aligned}$$

$u^1_{r_1}, u^2_{r_2}, u^3_{r_3}, u^4_{r_4}, u^5_{r_5}, u^6_{r_6}, u^7_{r_7}, u^8_{r_8}, v^1_{i_1}, v^2_{i_2}, v^3_{i_3}, v^4_{i_4}, v^5_{i_5}, v^6_{i_6}, w^1_{d_1}, w^2_{d_2}, w^3_{d_3}, w^4_{d_4}, w^5_{d_5}, w^6_{d_6}, w^7_{d_7}, w^8_{d_8}, w^9_{d_9} \geq \varepsilon;$

$r_1=1, \ldots, R_1;\ r_2=1,\ldots,R_2;\ r_3=1,\ldots, R_3;\ r_4=1, \ldots, R_4;\ r_5=1,\ldots,R_5;\ r_6=1,\ldots, R_6;\ r_7=1, \ldots, R_7;\ r_8=1,\ldots,R_8;$

$i_1=1,\ldots,I_1;\ i_2=1,\ldots,I_2;\ i_3=1,\ldots,I_3;\ i_4=1,\ldots,I_4;\ i_5=1,\ldots,I_5;\ i_6=1,\ldots,I_6;\ d_1=1,\ldots,D_1;\ d_2=1,\ldots,D_2;\ d_3=1,\ldots,D_3;$

$d_4=1,\ldots,D_4;\ d_5=1,\ldots,D_5;\ d_6=1,\ldots,D_6; d_7=1,\ldots,D_7;\ d_8=1,\ldots,D_8;\ d_9=1,\ldots,D_9.$

We increase k_2 and k_3 from zero to the upper bound of each one of them, independently and solve each linear model with both k_2 and k_3 and show the value of the objective function with $\theta_o^{overall(U2)}(k_2, k_3)$. In comparing all the values of the objective function in the model (20), we define the maximal overall efficiency as $\theta_o^{overall(U2)}$=max $\theta_o^{overall(U2)}(k_2, k_3)$. The maximal and minimal efficiencies of the second stage are defined as $\theta_o^{234(U2)+}$=max $\theta_o^{234}(k_2)$ where, k_2=min $\left(k_2 \left|\theta_o^{overall(U2)}(k_2, k_3) = \theta_o^{overall(U2)}\right.\right)$ and $\theta_o^{234(U2)-} = \min \theta_o^{234}(k_2)$ where, k_2=max $\left(k_2 \left|\theta_o^{overall(U2)}(k_2, k_3) = \theta_o^{overall(U2)}\right.\right)$, respectively. Similarly, the maximal and minimal efficiencies of the third stage are defined as θ_o^{56U+}=max $\theta_o^{56U}(k_3)$ where, k_3=min $\left(k_3 \left|\theta_o^{overall(U2)}(k_2, k_3) = \theta_o^{overall(U2)}\right.\right)$ and θ_o^{56U-}=min $\theta_o^{56U}(k_3)$, where, k_3=max $\left(k_3 \left|\theta_o^{overall(U2)}(k_2, k_3) = \theta_o^{overall(U2)}\right.\right)$, respectively. Note, that the efficiency of structure as shown in Fig. 1. is unique and with a very agreeable approximation we obtain: $\theta_o^{overall(U)}=\theta_o^{overall(U1)}= \theta_o^{overall(U2)}$; $\theta_o^{234U+}=\theta_o^{234(U1)+}= \theta_o^{234(U2)+}$; $\theta_o^{234L-}=\theta_o^{234(L1)-}= \theta_o^{234(L2)-}$. We obtain the maximal efficiency of the system $\theta_o^{overall(U)}$ and the maximal and minimal efficiencies for the stages θ_o^{1U+}, θ_o^{1U-}, θ_o^{234U+}, θ_o^{234U-}, θ_o^{56U+} and θ_o^{56U-} respectively. Note, that if ($\theta_o^{1U+}=\theta_o^{1U-}$ and $\theta_o^{234U+}=\theta_o^{234U-}$) or ($\theta_o^{1U+}=\theta_o^{1U-}$ and $\theta_o^{56U+}=\theta_o^{56U-}$) or ($\theta_o^{234U+}=\theta_o^{234U-}$ and $\theta_o^{56U+}=\theta_o^{56U-}$) then we have a unique efficiency for each stage based on $\theta_o^{overall(U)}= \theta_o^{1U}.\,\theta_o^{234U}.\,\theta_o^{56U}$. We tested our proposed approach in three modes and each time denoted two different stages as variables. Given the fact that, the efficiency of a network is unique, hence, the results of these three methods were approximately very close and we took advantage of one of these three abovementioned conditions to describe our approach. It should be observed that, the objective function of model (12) is as rendered, $\theta_o^{overall(L)}$=max $\theta_o^{1L}\,.\,\theta_o^{234L}\,.\,\theta_o^{56L}$ For which, we also implemented the heuristic approach for model (12). These results have been illustrated in the next part of the paper.

4. Case study description

In the past two decades, the manufacturing or production sector has grown significantly and being attentive towards production is one of the key goals of Iran's programs. An increment in the importance of the production sector, during the recent years and anxiety as to efficiency growth in this sphere, has a direct correlation with the economic system. A rise in costs, has led to pressurizing the production units to increase their organizational efficiency. A rise in costs, has led to haul, the production units towards incrementing their organizational performance. The best manner to ensure an efficiency increase would be to carry out a correct and logical use of the resources available. This could only be accomplished by ensuring a correct managerial performance, including a coherent evaluation of the returns attained. The DEA is a theoretical framework which discusses the analyzing of efficiency and its application in the arena of production planning and inventory control is observed very poorly. Hence, in this paper, an example from the world of reality in the field of production planning and inventory control has been described as follows. Let us consider a dairy factory a production area, a warehouse premises and a delivery point. Each of which, is considered as a stage. The warehouse premises comprises of three warehouses within distances from each other, but in the same area; and a delivery point consisting of two locations, but situated at long distances from the warehouses. This factory produces three products and stores each product in a warehouse. We take this factory into consideration as a dynamic network for duration of 24 time periods and assume each time period as a DMU. In this network a number of outputs in the time period t in stage 2 are converted to a number of inputs to stage 2 during the time period t+1. The inputs-outputs of each DMU are defined as follows: the first stage is the production area and the inputs of the first stage are production costs (x_1^1) for three goods. The intermediate measures between stage 1 and 2 are the quantity of each of the goods produced (z_1^1, z_1^2, z_1^3). The undesirable output of the first stage is the cost of moving the goods from the production area to the warehouses (y_1^1). In the second stage we have three warehouses and additional inputs of each warehouse, which is the cost for reserving storage location (x_1^2, x_1^3, x_1^4), the cost of holding goods in the warehouse (x_2^2, x_2^3, x_2^4) and goods that have remained in the warehouse from the last period (x_3^2, x_3^3, x_3^4). The desirable outputs of the warehouses in the second stage are defined by the goods remaining in each of the warehouses for the next period (y_1^2, y_1^3, y_1^4). The intermediate measures between stages 2 and 3 are the quantity of goods delivered from each warehouse to every delivery point (z_1^4, z_1^5, z_1^6, z_1^7, z_1^8, z_1^9). The additional inputs in the third stage denotes the cost of moving goods from the warehouses to the delivery points (x_1^5, x_1^6). Finally, in the third stage the desirable and undesirable outputs are profits due to the sale of the goods (y_1^5, y_1^7) and the delay penalty (y_1^6, y_1^8), respectively. The inputs-outputs for each stage are summarized in Table (2).

Table 2. Variables of inputs and outputs

Stage-SubDMU	Input-Output	Variable	Symbol
Stage1- sub-DMU_1	Input	Production cost	x_1^1
Stage1- sub-DMU_1	Output	Quantity of each of the goods produced Cost of Transport goods to warehouses	z_1^1, z_1^2, z_1^3 y_1^1
Stage2- sub-DMU_2	Input	Quantity of goods 1 produced Cost of reserving storage location 1 Cost of holding 1 goods Goods 1 remaining from last period	z_1^1 x_1^2 x_2^2 x_3^2
Stage2- sub-DMU_2	Output	Quantity of goods 1 delivered Goods 1 remaining for next period	z_1^4, z_1^5 y_1^2
Stage2- sub-DMU_3	Input	Quantity of goods 2 produced Cost of reserving storage location 2 Cost of holding goods 2 Goods 2 remaining from last period	z_1^2 x_1^3 x_2^3 x_3^3
Stage2- sub-DMU_3	Output	Quantity of goods 2 delivered Goods 2 remaining for next period	z_1^6, z_1^7 y_1^3
Stage2- sub-DMU_4	Input	Quantity of goods 3 produced Cost of reserving storage location 3 Cost of holding goods 3 Goods 3 remaining from last period	z_1^3 x_1^4 x_2^4 x_3^4
Stage2- sub-DMU_4	Output	Quantity of goods 3 delivered Goods 3 remaining for next period	z_1^8, z_1^9 y_1^4
Stage3- sub-DMU_5	Input	Quantity of each of the goods delivered Cost of Transport goods to delivery points	z_1^4, z_1^6, z_1^8 x_1^5
Stage3- sub-DMU_5	Output	Profit Delay Penalty	y_1^5 y_1^6
Stage3- sub-DMU_6	Input	Quantity of each of the goods delivered Cost of Transport goods to delivery points	z_1^5, z_1^7, z_1^9 x_1^6
Stage3- sub-DMU_6	Output	Profit Delay Penalty	y_1^7 y_1^8

In this paper, in order to specify the variables and perform data collection, observance, interviews and questionnaires were utilized for a number of variables and the interval modifications came to hand. Initially, indexes were collected in the form of documents and library studies. Some of the indexes which we required were in the form of documents and reports; whereas, others, in connection with production planning and inventory control, were identified by domestic and foreign articles. So as to seek the most effective indexes and factors, we observed the organization under consideration and were present at the factory site, where, information as to the manner of their functions and services were attained. In the next phase, we dealt with gathering information by interviewing the factory managers and likewise, by questionnaire design. Tables (3) and (4) provide the data for the factory for 24 intervals or periods in 2016. The inputs of the factory are shown in Table (3) and the outputs and the intermediate measures of the factory are illustrated in Table (4).

Table 3. The inputs of the factory for 24 period in 2016

DMU	Production cost	Cost of reserving storage location			Cost of holding goods			Goods remaining from last period			Cost of Transport goods to delivery points	
	x_1^1	x_1^2	x_1^3	x_1^4	x_2^2	x_2^3	x_2^4	x_3^2	x_3^3	x_3^4	$[x_1^{5L}, x_1^{5U}]$	$[x_1^{6L}, x_1^{6U}]$
1	11680	65	60	45	50	48	45	0	0	0	[150,165]	[218,240]
2	19152	65	60	45	50	48	45	0	0	0	[254,264]	[369.5,384]
3	30832	65	60	45	50	48	45	0	0	0	[404,420]	[587.5,610]
4	40880	65	60	45	50	48	45	0	0	0	[525,546]	[763,793]
5	17520	65	60	45	50	48	45	0	0	0	[225,234]	[327,340]
6	21728	65	60	45	50	48	45	0	0	0	[271,282]	[393.5,408]
7	37576	65	60	45	50	72	45	0	0	0	[479,498]	[696.5,724]
8	58400	65	60	45	113	144	105	0	2	0	[629,654]	[914.5,950]
9	58400	65	60	45	92	120	105	6	8	4	[779,810]	[1132.5,1177]
10	58400	65	60	45	50	48	75	4	6	4	[854,888]	[1201.5,1249]
11	58400	65	60	45	50	48	45	0	0	2	[683,710]	[1239,1288]
12	58400	65	60	45	50	48	45	0	0	0	[857,891]	[824.5,857]
13	31560	65	60	45	50	48	45	0	0	0	[381,400]	[648,680]
14	19152	65	60	45	50	48	45	0	0	0	[254,266]	[369.5,388]
15	21728	65	60	45	50	48	45	0	0	0	[271,284]	[393.5,412]
16	17520	65	60	45	50	48	45	0	0	0	[225,236]	[327,343]
17	30832	65	60	45	50	48	45	0	0	0	[404,424]	[587.5,617]
18	36672	65	60	45	50	48	45	0	0	0	[479,503]	[696.5,730]
19	29200	65	60	45	50	48	45	0	0	0	[375,401]	[545,583]
20	35040	65	60	45	50	48	45	0	0	0	[450,481]	[654,699]
21	32464	65	60	45	50	48	45	0	0	0	[433,463]	[630,674]
22	42512	65	60	45	134	144	105	0	0	0	[404,432]	[587.5,628]
23	58400	65	60	45	50	48	45	8	8	4	[900,980]	[1308,1430]
24	58400	65	60	45	50	48	45	0	0	0	[750,825]	[1090,1200]

Table (3) shows values of zero, which indicate each period, in which, the goods have not remained in the warehouse (Columns 9 to 11). Similarly, in Table (4), the values of zero also demonstrate the fact that, the goods have not remained in the warehouse for the next period (Columns 3 to 5) or that, a delay penalty, for that specific period of time, has not occurred (Columns 8 and 9). The cost of transporting the goods from the warehouses to the delivery points in Table (3), have been demonstrated inaccurately and from the interval viewpoint (Columns 12 and 13). Likewise, the transporting costs for goods from the production area to the warehouse have been rendered in the second column of Table (4). This has been shown imprecisely and from the interval standpoint.

Table 4. The outputs and the intermediate measures of the factory for 24 period in 2016

DMU	Cost of Transport goods to warehouses	Goods remaining for next period			Profit		Delay Penalty		Quantity of each goods produced			Quantity of goods delivered					
	$[y_1^{1L}, y_1^{1U}]$	y_1^2	y_1^3	y_1^4	y_1^5	y_1^7	y_1^6	y_1^8	z_1^1	z_1^2	z_1^3	z_1^4	z_1^5	z_1^6	z_1^7	z_1^8	z_1^9
1	[196,215]	0	0	0	1590	1590	0	0	8	8	4	4	4	4	4	2	2
2	[331,344]	0	0	0	2555.5	2555.5	0	0	14	14	6	7	7	7	7	3	3
3	[527,548]	0	0	0	4145.5	4145.5	0	0	22	22	10	11	11	11	11	5	5
4	[686,713]	0	0	0	5565	5565	0	0	28	28	14	14	14	14	14	7	7
5	[294,305]	0	0	0	2385	2385	0	0	12	12	6	6	6	6	6	3	3
6	[355,369]	0	0	0	3009.5	3009.5	0	0	14	14	8	7	7	7	7	4	4
7	[646,672]	0	2	0	4940.5	4940.5	0	0	26	28	12	13	13	13	13	6	6
8	[980,1019]	6	8	4	6530.5	6530.5	0	0	40	40	20	17	17	17	17	8	8
9	[980,1019]	4	6	4	8120.5	8120.5	0	0	40	40	20	21	21	21	21	10	10
10	[980,1019]	0	0	2	8915.5	8842.5	0	20	40	40	20	23	21	23	23	11	11
11	[980,1019]	0	0	0	7990	8818	120	0	40	40	20	16	24	18	22	11	11
12	[980,1019]	0	0	0	9686.5	5994.5	60	270	40	40	20	17	17	27	13	12	8
13	[543,570]	0	0	0	2823	5541	0	0	24	22	10	18	6	6	16	3	7
14	[331,348]	0	0	0	2555.5	2555.5	0	0	14	14	6	7	7	7	7	3	3
15	[355,373]	0	0	0	3009.5	3009.5	0	0	14	14	8	7	7	7	7	4	4
16	[294,309]	0	0	0	2385	2385	0	0	12	12	6	6	6	6	6	3	3
17	[527,554]	0	0	0	4145.5	4145.5	0	0	22	22	10	11	11	11	11	5	5
18	[625,656]	0	0	0	4940.5	4940.5	0	0	26	26	12	13	13	13	13	6	6
19	[490,524]	0	0	0	3975	3975	0	0	20	20	10	10	10	10	10	5	5
20	[588,629]	0	0	0	4770	4770	0	0	24	24	12	12	12	12	12	6	6
21	[564,603]	0	0	0	4316	4316	0	0	24	24	10	12	12	12	12	5	5
22	[723,774]	8	8	4	4145.5	4145.5	0	0	30	30	14	11	11	11	11	5	5
23	[980,1069]	0	0	0	9540	9540	0	0	40	40	20	24	24	24	24	12	12
24	[980,1078]	0	0	0	7950	7950	0	0	40	40	20	20	20	20	20	10	10

A failure in controlling the weight of factors, could possibly lead to assigning extremely minute weights to important issues or, (in the contrary), allotting very high weights to factors of insignificant importance. This feature shall hinder the evaluation validity to a great extent. Hence, in this paper, so as to survey the importance of the input, intermediate and output indexes, a questionnaire was used, which ensued the management of weights according to Table (5). The questionnaires were completed by a number of factory managers.

Table 5. Constraints to control weights

Inputs		Intermediate measures	Outputs
$\frac{v_1^1}{v_3^4} \geq 1.74$	$\frac{v_1^5}{v_1^4} \geq 1.4$	$\frac{w_1^8}{w_1^9} \geq 1.03$	$\frac{u_1^5}{u_1^7} \geq 1.03$
$\frac{v_3^4}{v_3^3} \geq 1.1$	$\frac{v_1^4}{v_1^3} \geq 1.07$	$\frac{w_1^9}{w_1^6} \geq 1.12$	$\frac{u_1^7}{u_1^6} \geq 1.03$
$\frac{v_3^3}{v_3^2} \geq 1.17$	$\frac{v_1^3}{v_1^2} \geq 1.16$	$\frac{w_1^6}{w_1^7} \geq 1.04$	$\frac{u_1^6}{u_1^8} \geq 1.04$
$\frac{v_3^2}{v_2^4} \geq 1.2$		$\frac{w_1^7}{w_1^3} \geq 1.15$	$\frac{u_1^8}{u_1^4} \geq 1.17$
$\frac{v_2^4}{v_2^3} \geq 1.2$		$\frac{w_1^3}{w_1^2} \geq 1.05$	$\frac{u_1^4}{u_1^3} \geq 1.10$
$\frac{v_2^3}{v_2^2} \geq 1.09$		$\frac{w_1^2}{w_1^4} \geq 1.05$	$\frac{u_1^3}{u_1^2} \geq 1.17$
$\frac{v_2^2}{v_1^6} \geq 1.05$		$\frac{w_1^4}{w_1^5} \geq 1.06$	$\frac{u_1^2}{u_1^1} \geq 1.29$
$\frac{v_1^6}{v_1^5} \geq 1.05$		$\frac{w_1^5}{w_1^1} \geq 1.21$	

We implemented our heuristic approach, for the two models (11 and 12) by taking into consideration the aspects of weight restrictions for Table (5). This illustrates the maximum efficiency of the stages, in the condition of the upper and lower bounds, which have been demonstrated in Table (6).

Table 6. Results of the maximum efficiencies of the stages in Upper and Lower bound

DMU	Upper bound			Lower bound		
	$\theta_o^{1U\text{-max}}$	$\theta_o^{234U\text{-max}}$	$\theta_o^{56U\text{-max}}$	$\theta_o^{1L\text{-max}}$	$\theta_o^{234L\text{-max}}$	$\theta_o^{56L\text{-max}}$
1	0.68207	0.32199	1.00000	0.67855	0.32199	0.98907
2	0.66416	0.48223	1.00000	0.66175	0.48223	0.99275
3	0.62509	0.67994	1.00000	0.62120	0.67994	0.98355
4	0.59117	0.81444	0.99862	0.58616	0.81444	0.97234
5	0.66389	0.44855	1.00000	0.66185	0.44855	0.99349
6	0.64629	0.53261	1.00000	0.64369	0.53261	0.99240
7	0.60493	0.79623	0.99702	0.60010	0.79623	0.97407
8	0.53662	0.97676	0.98736	0.52938	0.97676	0.95577
9	0.53662	0.66100	0.97752	0.52938	0.66100	0.93785
10	0.53662	0.65969	0.97952	0.52938	0.65969	0.93706
11	0.53662	0.87195	0.97805	0.52938	0.87195	0.95172
12	0.53662	0.99999	0.99564	0.52938	0.99999	0.97299
13	0.62313	0.71847	1.00000	0.61812	0.71847	1.00000
14	0.66416	0.48223	1.00000	0.66100	0.48223	0.99102
15	0.64629	0.53261	1.00000	0.64295	0.53261	0.99067
16	0.66389	0.44855	1.00000	0.66111	0.44855	0.99202
17	0.62509	0.67994	1.00000	0.62008	0.67994	0.98024
18	0.60621	0.76170	0.99702	0.60045	0.76170	0.97066
19	0.62753	0.65432	1.00000	0.62122	0.65432	0.97894
20	0.60935	0.73908	1.00000	0.60174	0.73908	0.97050
21	0.62221	0.70460	1.00000	0.61498	0.70460	0.97882
22	0.58751	0.99999	1.00000	0.57805	0.99999	0.97440
23	0.53662	0.59207	0.97590	0.52011	0.59207	0.89251
24	0.53662	0.99876	0.98581	0.51844	0.99876	0.91106

Table (7), shows the optimal values for k_1, k_2 and k_3. Given that, the second stage in every step is considered as a variable, thereby, in each model two values shall come to hand for k_2, which has been specified in the Table hereunder.

Table 7. Results of the k values

DMU	Upper bound				Lower bound			
	Step 1		Step 2		Step 1		Step 2	
	k_1	k_2	k_2	k_3	k_1	k_2	k_2	k_3
1	0	0	1	6	0	1	1	9
2	0	1	1	11	0	0	1	14
3	0	1	0	9	0	1	0	12
4	0	2	1	6	0	0	1	7
5	0	1	0	6	0	0	0	10
6	0	0	1	4	0	1	1	5
7	0	1	2	8	16	0	2	10
8	0	5	3	7	0	5	3	8
9	2	0	0	6	0	13	0	6
10	37	0	1	4	32	14	1	4
11	0	0	0	5	10	1	0	5
12	0	10	5	4	9	3	5	4
13	39	3	3	14	38	3	3	15
14	0	1	1	11	0	0	1	14
15	0	0	1	4	0	1	1	5
16	0	1	0	6	0	0	0	9
17	0	1	0	9	0	1	0	11
18	0	1	1	8	0	1	1	10
19	0	1	1	6	0	0	1	8
20	0	1	1	6	0	0	1	7
21	0	1	0	12	0	0	0	14
22	0	8	11	9	7	4	11	11
23	0	0	0	4	0	2	0	0
24	0	1	1	5	0	0	1	1

By studying the values of k, we observed that, in this case study, the overall efficiency is optimized when the values of k are low, which means that the optimal efficiency values of the stages are close to the maximum limit and their minimum limit value. Table (8) renders the maximal overall efficiency and the maximal and minimal efficiencies of stages, based on upper bound by considering constraints to control weights.

Table 8. Results based on the Upper bound

DMU	$\theta_o^{overall(U1)}$	θ_o^{1U+}	θ_o^{1U-}	$\theta_o^{234(U1)+}$	$\theta_o^{234(U1)-}$	$\theta_o^{overall(U2)}$	$\theta_o^{234(U2)+}$	$\theta_o^{234(U2)-}$	θ_o^{56U+}	θ_o^{56U-}
1	0.2037	0.68207	0.68207	0.32199	0.32199	0.20003	0.31199	0.31199	0.94	0.94
2	0.29122	0.66416	0.66416	0.47223	0.47223	0.27914	0.47223	0.47223	0.89	0.89
3	0.39459	0.62509	0.62509	0.66994	0.66994	0.38677	0.67994	0.67994	0.91	0.91
4	0.45717	0.59117	0.59117	0.79444	0.79444	0.44637	0.80444	0.80444	0.93862	0.93862
5	0.26605	0.66389	0.66389	0.43855	0.43855	0.27992	0.44855	0.44855	0.94	0.94
6	0.32192	0.64629	0.64629	0.53261	0.53261	0.32425	0.52261	0.52261	0.96	0.96
7	0.44854	0.60493	0.60493	0.78623	0.78623	0.4306	0.77623	0.77623	0.91702	0.91702
8	0.47034	0.53662	0.53662	0.92676	0.92676	0.46606	0.94676	0.94676	0.91736	0.91736
9	0.26839	0.51662	0.51662	0.661	0.661	0.32545	0.661	0.661	0.91752	0.91752
10	0.09389	0.16662	0.16662	0.65969	0.65969	0.32755	0.64969	0.64969	0.93952	0.93952
11	0.36235	0.53662	0.53662	0.87195	0.87195	0.43424	0.87195	0.87195	0.92805	0.92805
12	0.41361	0.53662	0.53662	0.89999	0.89999	0.48717	0.94999	0.94999	0.95564	0.95564
13	0.1404	0.23313	0.23313	0.68847	0.68847	0.36894	0.68847	0.68847	0.86	0.86
14	0.29122	0.66416	0.66416	0.47223	0.47223	0.27914	0.47223	0.47223	0.89	0.89
15	0.32192	0.64629	0.64629	0.53261	0.53261	0.32425	0.52261	0.52261	0.96	0.96
16	0.26605	0.66389	0.66389	0.43855	0.43855	0.27992	0.44855	0.44855	0.94	0.94
17	0.39459	0.62509	0.62509	0.66994	0.66994	0.38677	0.67994	0.67994	0.91	0.91
18	0.4305	0.60621	0.60621	0.7517	0.7517	0.41787	0.7517	0.7517	0.91702	0.91702
19	0.39561	0.62753	0.62753	0.64432	0.64432	0.38007	0.64432	0.64432	0.94	0.94
20	0.43357	0.60935	0.60935	0.72908	0.72908	0.41761	0.72908	0.72908	0.94	0.94
21	0.39361	0.62221	0.62221	0.6946	0.6946	0.3858	0.7046	0.7046	0.88	0.88
22	0.44701	0.58751	0.58751	0.91999	0.91999	0.47073	0.88999	0.88999	0.91	0.91
23	0.30529	0.53662	0.53662	0.59207	0.59207	0.29735	0.59207	0.59207	0.9359	0.9359
24	0.51249	0.53662	0.53662	0.98876	0.98876	0.49653	0.98876	0.98876	0.93581	0.93581

In Table (8), the results of the two models (18 and 21) are compared. The second and the seventh columns of this Table illustrate the overall upper bound efficiency of the network, shown in Fig. 1. These are obtained from two different approaches for a network and are similar in expectation with an extremely sound approximation. The other columns of the Table render the minimal and maximal efficiency levels of the stages; which, for this factory, in particular, we found the efficiency of all the stages to be unique. Similarly, the upper limit of the performance of the second stage is also attained by both models which also display a comparable approximation. Table (9) demonstrates the minimal overall efficiency and the minimal and maximal efficiencies of stages, based on the lower bound, in consideration with constraints to control weights.

Table 9. Results based on the lower bound

DMU	$\theta_o^{overall(L1)}$	θ_o^{1L+}	θ_o^{1L-}	$\theta_o^{234(L1)+}$	$\theta_o^{234(L1)-}$	$\theta_o^{overall(L2)}$	$\theta_o^{234(L2)+}$	$\theta_o^{234(L2)-}$	θ_o^{56L+}	θ_o^{56L-}
1	0.17033	0.67855	0.67855	0.31199	0.31199	0.19132	0.31199	0.31199	0.89907	0.89907
2	0.25413	0.66175	0.66175	0.48223	0.48223	0.26745	0.47223	0.47223	0.85275	0.85275
3	0.36214	0.6212	0.6212	0.66994	0.66994	0.36703	0.67994	0.67994	0.86355	0.86355
4	0.36872	0.58616	0.58616	0.81444	0.81444	0.42911	0.80444	0.80444	0.90234	0.90234
5	0.24943	0.66185	0.66185	0.44855	0.44855	0.26607	0.44855	0.44855	0.89349	0.89349
6	0.30205	0.64369	0.64369	0.52261	0.52261	0.3183	0.52261	0.52261	0.9424	0.9424
7	0.30436	0.4401	0.4401	0.79623	0.79623	0.41043	0.77623	0.77623	0.87407	0.87407
8	0.38274	0.52938	0.52938	0.92676	0.92676	0.44494	0.94676	0.94676	0.87577	0.87577
9	0.24043	0.52938	0.52938	0.531	0.531	0.31138	0.661	0.661	0.87785	0.87785
10	0.09059	0.20938	0.20938	0.51969	0.51969	0.31275	0.64969	0.64969	0.89706	0.89706
11	0.32459	0.42938	0.42938	0.86195	0.86195	0.42192	0.87195	0.87195	0.90172	0.90172
12	0.37112	0.43938	0.43938	0.96999	0.96999	0.47562	0.94999	0.94999	0.93299	0.93299
13	0.14028	0.23812	0.23812	0.68847	0.68847	0.36465	0.68847	0.68847	0.85	0.85
14	0.25154	0.661	0.661	0.48223	0.48223	0.26691	0.47223	0.47223	0.85102	0.85102
15	0.30022	0.64295	0.64295	0.52261	0.52261	0.31772	0.52261	0.52261	0.94067	0.94067
16	0.24781	0.66111	0.66111	0.44855	0.44855	0.26861	0.44855	0.44855	0.90202	0.90202
17	0.35656	0.62008	0.62008	0.66994	0.66994	0.36987	0.67994	0.67994	0.87024	0.87024
18	0.31913	0.60045	0.60045	0.7517	0.7517	0.39675	0.7517	0.7517	0.87066	0.87066
19	0.31762	0.62122	0.62122	0.65432	0.65432	0.36347	0.64432	0.64432	0.89894	0.89894
20	0.34017	0.60174	0.60174	0.73908	0.73908	0.40006	0.72908	0.72908	0.9005	0.9005
21	0.313	0.61498	0.61498	0.7046	0.7046	0.36775	0.7046	0.7046	0.83882	0.83882
22	0.3901	0.50805	0.50805	0.95999	0.95999	0.44714	0.88999	0.88999	0.8644	0.8644
23	0.23268	0.52011	0.52011	0.57207	0.57207	0.28357	0.59207	0.59207	0.89251	0.89251
24	0.41732	0.51844	0.51844	0.99876	0.99876	0.47809	0.98876	0.98876	0.90106	0.90106

Table (9), illustrates the lower limit of the overall network efficiency performance and the minimal and maximal lower bound efficiency of the stages. In comparing the second and the seventh columns of the Table (9), it can be observed that, the difference in efficiency attained at the lower level is more than the upper limit results. In this paper, $\Delta\varepsilon$ =0.01 is considered as a step size; and by taking the smallest of this value into consideration, the rate of error of the two models reduces, though the solving time shall increase. Likewise, results indicate that the stages have unique efficiency aspects and the efficiency value of the second stage, which has come to hand from two different models, as per expectations is extremely perfect and close in approximation. The results of Tables (8) and (9) represent the network range of modifications in efficiency in Fig. 1 and which is in the presence of imprecise data. We shall integrate the results of Tables (8) and (9) to increase the accuracy of the calculations and define the upper and lower bound of the network in the context of efficiency interval modifications given in Fig. 1 and defined by formula (23) as follows:

$$\theta_j^{overall(L)} = \min\left(\theta_j^{overall(L1)}, \theta_j^{overall(L2)}\right), \quad \theta_j^{overall(U)} = \max\left(\theta_j^{overall(U1)}, \theta_j^{overall(U2)}\right), \; j=1,\ldots,24 \tag{23}$$

In order to compare and perform the ranking of the interval efficiencies, we shall utilize Wang's minimax regret-based approach, which we have explained in Section (2) of this paper. Table (10), displays the range of efficiency changes and the ratings obtained for the DMUs

Table 10. Results based on the interval efficiency

DMU	$[\theta_o^{overall(L)},\theta_o^{overall(U)}]$	RANK	DMU	$[\theta_o^{overall(L)},\theta_o^{overall(U)}]$	RANK
1	[0.17033 0.2037]	24	13	[0.14028 0.36894]	22
2	[0.25413 0.29122]	16	14	[0.25154 0.29122]	17
3	[0.36214 0.39459]	6	15	[0.30022 0.32425]	15
4	[0.36872 0.45717]	5	16	[0.24781 0.27992]	19
5	[0.24943 0.27992]	18	17	[0.35656 0.39459]	7
6	[0.30205 0.32425]	14	18	[0.31913 0.4305]	11
7	[0.30436 0.44854]	10	19	[0.31762 0.39561]	12
8	[0.38274 0.47034]	4	20	[0.34017 0.43357]	8
9	[0.24043 0.32545]	20	21	[0.313 0.39361]	13
10	[0.09059 0.32755]	23	22	[0.3901 0.47073]	2
11	[0.32459 0.43424]	9	23	[0.23268 0.30529]	21
12	[0.37112 0.48717]	3	24	[0.41732 0.51249]	1

Therefor the performance of 24 DMUs is rated as follows:

$$\mathrm{DMU}_{24} > \mathrm{DMU}_{22} > \mathrm{DMU}_{12} > \mathrm{DMU}_{8} > \mathrm{DMU}_{4} > \mathrm{DMU}_{3} > \mathrm{DMU}_{17} > \mathrm{DMU}_{20} > \mathrm{DMU}_{11} >$$
$$\mathrm{DMU}_{7} > \mathrm{DMU}_{18} > \mathrm{DMU}_{19} > \mathrm{DMU}_{21} > \mathrm{DMU}_{6} > \mathrm{DMU}_{15} > \mathrm{DMU}_{2} > \mathrm{DMU}_{14} > \mathrm{DMU}_{5} >$$
$$\mathrm{DMU}_{16} > \mathrm{DMU}_{9} > \mathrm{DMU}_{23} > \mathrm{DMU}_{13} > \mathrm{DMU}_{10} > \mathrm{DMU}_{1},$$

Where symbol " $>$ " means that the interval performance is better than.

The black box approach neglects the internal activities of systems and evaluates performance based on the final inputs and outputs. According to the belief of many researchers, this task causes a lack of confidence in the evaluation results. In this research, we tried to pay attention to the intra-system activities using the proposed model, which was based on our knowledge, It has not been performed so far in the area of production planning and inventory control. Several researchers have taken advantage of the additive approach to avoid the complexity of the model, as well as its non-linearization. A additive or a multiplicative performance approach has been utilized in a serial mode. The problem lies in the fact that, as this approach comprises of stages, which are in consecutive series and the failure of a stage, will devastate the entire system. But in the case of utilizing the additive approach, this problem is not considered, as it has a compensatory property. Hence, for a more accurate analysis, we took advantage of the multiplicative approach in a serial mode.

This paper presents the modeling method and solution for evaluating the efficiency of a hybrid complex system with additional inputs and undesirable outputs in the presence of imprecise data. A majority of the manufacturing centers always comprise of a production area, warehouse area and a delivery point for products. Therefore, the proposed model can be used for numerous manufacturing centers.

5. Conclusions

It could be possible that, access to accurate data, relative to inputs and outputs, may not be available always in the factual world. Hence, the manner of dealing with the presence of uncertainty in data, when evaluating efficiency, utilizing the DEA method, is absolutely necessary, as it will secure more realistic results. Thence, in this paper, a model for evaluating efficiency and ranking complex networks, in the presence of imprecise datum, from the interval criteria has been presented. This is capable of bringing the network efficiency interval and the efficiency of the stages into hand; and then rates the units, on the basis of these intervals achieved.

This system allow us to take the intermediate variables under consideration and to open the structure of the black box and gain crucial information, from the various efficient and inefficient points of the system, to be put at the disposal of managers. DEA is considered as a theoretical framework for efficiency analysis under discussion; and its application, in the spheres of production planning and inventory control has been observed to be, an extremely insignificant amount. In this paper, we present an example from the authentic world, in the grounds of, production planning and inventory control. A hybrid system with a complex internal structure having three stages, six sub-DMUs, additional inputs and undesirable outputs is studied in this paper. We simulated a factory in a real world with a production area and three warehouses for goods, including two delivery points. In this simulation we have taken into consideration aspects such as, the overall costs consisting of production, storage and inventory costs, including costs relative to transportation from the production area to the warehouses and from the warehouses to the delivery points, delay penalties and the profits gained from the sale of goods have been taken into consideration. We deliberated on this factory as a dynamic network and obtained its maximal and minimal efficiency and the intervals of the efficiencies of the stages that modify within them in the presence of interval data. The cooperative approach and multiplicative model to measure the efficiency of the network structure, as well as a heuristic method was used to convert the nonlinear models into linear models.
The heuristic approach proposed in this paper is of two stages, the smaller the selection of a $\Delta\varepsilon$ step size will assist in attaining closer results between the two stages, but the time taken to resolve the problem shall increment. Moreover, we implemented this approach for three stages and due to the increase in stages and the presence of additional inputs and outputs, this model becomes more complex, resulting in an extremely high resolution time. We can change the size step ($\Delta\varepsilon$) so as to reduce the resolving period. Therefore the value of the step size ($\Delta\varepsilon$), which identifies the resolution accuracy and solving period, should be considered by managers.
In this paper we utilize the Wang (2005) minimax regret-based approach to compare and rank the interval efficiencies which come to hand. The results of the ranking illustrated that, the time periods namely, (24) and (1) were the best and the poorest periods, respectively, in relative to interval efficiencies between the 24 periods. Similarly, we observed that between the time periods (1) and (24), a fluctuating condition has occurred and no specific system is available to alleviate efficiency. The results obtained from the model are valid empirical researches, for in Iran and based on the Hegira-Solar calendar, period (1) comprised of the Nowrooz or (Iranian New Year) vacations and period (24) was prior to these vacations, when demand and consumption reaches its peak. The results assisted the factory in improving performance. For tasks in the future we recommend a model to be developed for fuzzy data.

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