Three-stage mining metals supply chain coordination and air pollutant emission reduction with revenue sharing contract

Hamed Homaei[1], Iraj Mahdavi[1], Ali Tajdin[1,*]

Abstract

One of the main concerns of all industries such as mine industries is to increase their profit and keep their customers through improving quality level of their products; but increasing the quality of products usually releases air pollutants. Nowadays the management of air pollutant emissions with harmful environmental and health effects is one of the most pressing problems. In this paper, authors study the decision behaviour and coordination issue of a mining metal three-level supply chain with one supplier (extractor), one mineral processor and one manufacturer. We compare the decentralized and the centralized systems and reduce air pollutant emission by designing a revenue sharing contract for the mentioned decentralized supply chain under cap-and-trade regulation. Finally, a numerical example shows that the designed contract not only provides win-win condition for all supply chain members and increases whole supply chain profit but also reduces harmful air pollutant emissions in the supply chain.

Keywords: Mining metals supply chain; Channel coordination; Emissions reduction; Revenue sharing contract.

1. Introduction

In today's competitive market, supply chain management is one of the most useful management practices for industries to increase their profit and competitiveness. Since rapid economic development brings huge amounts of pollutants emissions, governmental pressures such as cap-and-trade regulation are made to force companies to find new methods to reduce these emissions across all the stages of their supply chains. Under a cap-and-trade regulation, companies get predetermined free emission credits from the government (Xu et al., 2016a). They could sell/buy credits in the air pollutants trading market when they have surplus/lack credits; this emission credit price is determined by market.

There are two decision making systems in a supply chain: centralized and decentralized.

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* Corresponding author; ali_tajdin@yahoo.com

[1] Industrial Engineering Department, Mazandaran University of Science and Technology, Babol, Iran.

 In the centralized system, supply chain members operate jointly as a single firm and make their decisions to maximize the total profit of the system; but in the decentralized system, supply chain members make their decisions separately to maximize their own profits. The decision making system in most supply chain models such as this study is assumed to be decentralized. To improve the overall performance of the supply chain, a coordination mechanism is needed. Different definitions and perspectives on the supply chain coordination exist in the literature (Arshinder & Deshmukh, 2008a, Arshinder & Deshmukh, 2008b) for the comprehensive review of supply chain coordination. A supply chain is coordinated when the members make the decisions that are optimal for the whole supply chain. For coordinating a supply chain, contracts are designed to reduce the difference between the outcome of a centralized system and a decentralized system. Different kinds of contracts such as commitment order (Cai et al., 2019), option(Biswas & Avittathur, 2019), two-part tariff (Bai et al., 2017), revenue sharing (Heydari & Ghasemi, 2018, Zhao et al., 2019), return policy (Radhi & Zhang, 2019), cost sharing (Li et al., 2019), mail-in-rebate (Saha et al., 2015), have been used in supply chains as the ways improving supply chain performance. Revenue sharing is one of the widely used contracts in the supply chain that is between an upper and lower level of supply chain, where the upper level provides better selling condition to the lower level and then the lower level shares a fraction of its revenue with upper level.

There are two streams of literatures related to the research in this paper. The first stream focuses on operational decisions in supply chain under the cap-and-trade regulation in order to reduce harmful gases emission. Cap-and-trade regulation is proved to be one of the most effective mechanisms to control air pollutants emissions (Zhang & Xu, 2013). Many researches have studied the problems in supply chains considering the cap-and-trade regulation and it has been recommended by many senior researchers such as (Hua et al., 2011) and (Du et al., 2016) and implemented in many parts of the world. Xu et al studied the joint production and pricing problem of a manufacturer under cap-and-trade and carbon tax policies. They illustrated that, under the cap-and-trade system, both the emission trading prices and the cap play crucial role in the optimal manufacturing quantity (Xu et al., 2016b). Gong and Zhou proposed an optimal manufacturing strategy under carbon trading policy through a dynamic model (Gong & Zhou, 2013). Hua et al explored how companies manage carbon footprints in inventory management under the carbon-trading regulation. They showed that both the carbon price and carbon cap have a major effect on the retailer's order decisions, carbon footprints, and total costs (Hua et al., 2011). Xu et al investigated the production and pricing problems in make-to-order supply chain under cap-and-trade regulation. They analysed the impact of emission trading price on the production decisions and company's profits (Xu et al., 2017). He et al considered the impact of cap-and-trade regulation on company's carbon emission decisions. They showed that the differentiated permits trading prices play a crucial role in company's permits trading and decisions of optimal emissions (He et al., 2015). Song and Leng discussed the optimal manufacturing quantities of a company under cap-and-trade regulation. They pointed out that there are specific conditions to increase the company's profit and decrease carbon emissions (Song & Leng, 2012). Zhang and Xu investigated a company's optimal manufacturing quantities under cap-and-trade regulation. They found that cap-and-trade regulation can force the company to produce more carbon efficient products (Zhang & Xu, 2013). Benjaafar et al studied the multi-period operational decision-making of a company under cap-and-trade regulation; they illustrated how adjustments in procurement, production and inventory decisions can decrease carbon emissions.

The second subset of literature related to this research is the supply chain coordination under revenue sharing contract. Le et al provided a good survey on this contract (Li et al., 2019). Arani and Rabbani developed a new mixed revenue-sharing option contract to coordinate the supply chain and modelled that through a game theoretic approach to obtain the order quantity

of the retailer and the production quantity of the manufacturer (Arani et al., 2016). Qian and Guo proposed a revenue-sharing bargaining model between Energy Service Company and an Energy-Using Organization to analyse the impact on energy prices, risk-adjusted discount rates and accidents on the ESCO's bargaining strategies (Qian & Guo, 2014). Hsueh presented a new revenue sharing contract embedding corporate social responsibility to coordinate a two level supply chain (Hsueh, 2014). Yao et al proposed a revenue sharing contract to coordinate a two stage supply chain with one manufacturer and two competing retailers. They illustrated that the provision of revenue sharing in the contract can increase supply chain performance more than a price-only contract (Yao et al., 2008). Palsule-Desai proposed a game theory model for revenue-dependent revenue sharing contracts in which the supply chain revenue is shared among the members depending on the quantum of revenue generated (Palsule-Desai, 2013). Zhang et al discussed the revenue sharing contracts for coordinating a supply chain with one manufacturer and two competing retailers in which demands are disrupted. They showed that it is necessary to adjust the original revenue-sharing contracts to demand disruptions (Zhang et al., 2012). Hu and Feng modelled the supply chain with revenue sharing contract and service requirement under supply and demand uncertainty. They found that in the coordinated supply chain under supply and demand uncertainty, the revenue sharing ratio for the supplier will be higher if the wholesale price remains the same, or the wholesale price will be higher if the revenue sharing ratio for the supplier keeps the same (Hu & Feng, 2017). Hu et al studied supply chain coordination via revenue sharing contracts in two different supply chain structures. First, for a three-echelon supply chain with a loss-averse retailer, a loss-neutral distributor, and a loss-neutral manufacturer and second, for a two-echelon supply chain consisting of a loss-averse retailer and a loss-neutral distributor (Hu et al., 2016).

However, a few researches have been done on the three level supply chain coordination with revenue sharing contract considering environmental aspects under cap-and-trade regulation; also the three-level supply chain coordination research literatures mentioned above have not paid attention to environmental issues. Therefore the main purpose of this study is to design a revenue sharing contract for a mining metal three level supply chain in order to 1- coordinate three-stage mining metals supply chain 2- provide a win-win condition for all supply chain members 3- decrease the difference between the outcome of a centralized system and a decentralized system 4- reduce air pollutant emissions in the supply chain under cap-and-trade regulation 5- increase whole supply chain profit.

The rest of this paper proceeds as follows. In section 2 the notations will be defined. Section 3 presents the supply chain descriptions and assumptions used in this paper. We analyses decision behaviour the decentralized and centralized supply chain in Section 4. Section 5 develops a new revenue sharing contract for coordinating decentralized supply chain. Section 6 provides numerical example to illustrate the proposed contract performance. Conclusions are provided in Section 7.

2. Notations

The following notations are used to describe the proposed model.

i	Index for supply chain levels; S for supplier, P for processor and M for manufacturer.
j_0	Minimum acceptable product quality level in considered supply chain.
P_{ij}	selling price of unit product produced at the supply chain level i with quality level j
D_i	Amount of product quality level improvement in supply chain level i

C_i	Constant production cost for a unit product in supply chain level i
Cd_{ij}	Cost coefficient for increasing product quality level with quality level j in supply chain level i
α_i	Price increasing coefficient for product produced in supply chain level i per unit product quality improvement in supply chain level i
β_i	Price increasing coefficient for product produced in supply chain level i per unit product quality improvement in supply chain levels before level i
γ_i	Quality improvement cost increasing coefficient in supply chain level i per unit product quality improvement in supply chain levels before level i
π_i	Supply chain level i profit
ϕ_1	Processor's revenue share, $0 < \phi_1 < 1$
ϕ_2	Manufacturer's revenue share, $0 < \phi_1 < 1$
Cp_i	unit air pollutant emissions trading price for supply chain level i
K_i	air pollutant emissions cap for supply chain level i
G_i	amount of air pollutant emission for a unit product quality level improvement in supply chain level i

In this paper d_i, ϕ_1 and ϕ_2 are decision variables.

3. Model description and assumptions

A decentralized mining metal three-stage supply chain in which minerals will convert to concentrate after extraction is assumed in this paper. The considered supply chain is consists of a supplier (extractor), a processor, and a manufacturer. The first level extracts minerals and sells them to second level and then he processes minerals and sells the mineral concentrate after processing to manufacturer and then he produces mineral products such as pellets and ingots and sells them to the customers. The product price of each supply chain level depends on the quality of that product. Therefore all of these supply chain members try to increase their product quality.

Product quality level improvement at the supplier level doesn't emit air pollutants because improving product quality at that level is done by some activities such as more samplings for accurate identification of underground mineral veins and performing explosive operation optimally ($Cp_S = 0$). But product quality improvement in supply chain levels 2 and 3 emit air pollutants; the emitted air pollutant type at the processor level is usually dust because of the physical processes at this level and the emitted air pollutant at the manufacturer level is of the chemical type, such as SO2, due to chemical processes. That is why the parameter Cp for the manufacturer is higher than the processor ($Cp_M > Cp_P$). The government monitors pollutant emissions of the supply chain members by online measuring equipment (Figure 1). Product quality level improvement is not mandatory for supply chain members but supplier must supplies raw material with minimum quality level j_0. Product quality improvement for each supply chain member requires more operating costs but these cost enhancements are different for each member because of different production processes in each supply chain level and it is assumed to be a nonlinear ascending.

Figure 1. Air pollutant online monitoring equipment in Golgohar mining & Industrial Co.

The product quality improvement cost increases from supplier to manufacturer due to the increasing complexity of production processes from supplier to manufacturer ($Cd_{Sj} < Cd_{Pj} < Cd_{Mj}$). It is a mention worthy assumption that in this type of supply chain, increasing the quality of the product at each supply chain level creates an added value for both that level (with coefficient α) and the next levels (with coefficient β), but increases next levels' product quality improvement costs (with coefficient γ), for example the cost for increasing 5 quality levels from level 70 to level 80 is more than the cost for increasing 10 quality levels from level 20 to level 30.

In our study we assume that all members are in full capacity production and all their products will be sold. Therefore, the consideration of the demand parameter in the problem is neglected. Also the shipment costs are not considered in this model due to equality in centralized and decentralized system.

4. Decision analysis

In this part, we propose the decision model for the decentralized and centralized systems. After solving both the system models, we obtain the quantitative relationships among the profits and the decision variables under the centralized and decentralized systems.

4.1. Analysis of the decentralized system

We assume that all members in the considered supply chain try to improve their products quality in order to increase their profit, but in the decentralized supply chain they try to maximize their own profit. Considering the model assumptions the supplier's profit for each unit product extraction in the decentralized system is

$$\pi_S = P_{S_{j_0}}(1 + \alpha_S D_S) - C_S - Cd_{S_{j_0}} D_S^2 \tag{1}$$

where the first part denotes a unit extracted material selling price with minimum quality level plus the selling price enhancement due to product quality level improvement by the supplier (D_S). The second part is the constant extraction cost for a unit product at the supplier level. Similar to previous studies (Gavious & Lowengart, 2012, Kopalle & Winer, 1996) the third part shows the supplier's cost for increasing product quality level. As mentioned before, at this supply chain level we don't have environmental costs for product quality improvement.

The processor's profit for processing one unit product in the decentralized system is

$$\pi_P = P_{P_{j_0}}(1 + \beta_P D_S + \alpha_P D_P) - P_{S_{j_0}}(1 + \alpha_S D_S) - C_P - Cd_{P_{j_0}} D_P^2(1 + \gamma_P D_S) - cp_P(G_P D_P - K_P) \tag{2}$$

where similar to equation (1), the first part shows a unit processed material selling price with minimum quality level plus the selling price enhancement due to product quality level improvement by processor (D_P) and supplier (D_S). The second part is the purchasing price of a unit extracted product from the supplier. The third part is the constant processing cost for a unit product in processor and the fourth term shows the processor's cost for increasing product quality level and the last part is the cost (income) from buying (selling) extra dust emission permits for the processor.

The manufacturer's profit for manufacturing one unit product in the decentralized system is

$$\pi_M = P_{Mj_0}(1 + \beta_M(D_S + D_P) + \alpha_M D_M) - P_{Pj_0}(1 + \beta_P D_S + \alpha_P D_P) - C_M - Cd_{Mj_0}D_M^2(1 + \gamma_M(D_S + D_M)) - Cp_M(G_M D_M - K_M) \tag{3}$$

where the first part represents a unit manufactured product selling price with minimum quality level plus the selling price enhancement due to product quality level improvement by the manufacturer (D_M) and its previous levels ($D_S + D_P$). The second part is the purchasing price of a unit processed product from the processor. The third part is constant manufacturing cost for a unit product at the manufacturer level. The fourth part shows the manufacturer's cost for increasing product quality level and the last term is the cost (income) from buying (selling) extra chemical pollutant emission permits for the manufacturer.

As mentioned before, all members in the decentralized supply chain try to maximize their own profit so the members' optimal decision will be as follows.

Proposition 1. The optimal product quality level improvement by the supplier in considered decentralized supply chain is

$$D_S^* = \frac{\alpha_S P_{Sj_0}}{2Cd_{Sj_0}} \tag{4}$$

Proposition 2. The optimal product quality level enhancement by the processor in considered decentralized supply chain is

$$D_P^* = \frac{\alpha_P P_{Pj_0} - Cp_P G_P}{2Cd_{Pj_0}(1 + \gamma_P D_S^*)} \tag{5}$$

Proposition 3. The optimal product quality level improvement by the manufacturer in considered decentralized supply chain is

$$D_M^* = \frac{\alpha_M P_{Mj_0} - Cp_M G_M}{2Cd_{Pj_0}(1 + \gamma_M(D_S^* + D_P^*))} \tag{6}$$

Therefore the optimal value of the whole decentralized supply chain profit without coordination can be written as

$$\pi_T^* = \pi_S^* + \pi_P^* + \pi_M^* \tag{7}$$

And optimal product quality improvement for final product without coordination can be calculated as follows

$$D_T^* = D_S^* + D_P^* + D_M^* \tag{8}$$

4.2. Analysis of the centralized system

In the centralized system, all supply chain members operate jointly as a single company and determine the optimal value of product quality level improvement to maximize the total profit of the whole supply chain. In this scenario, the total supply chain profit function can be formulated as:

$$\pi_{TC} = -Cd_{Sj_0}D_S^2 - Cd_{Pj_0}d_P^2(1+\gamma_P D_S) - Cp_P(G_P D_P - K_P) + P_{Mj_0}(1+\beta_M(D_S+D_P)+\alpha_M D_M)$$
$$-C_M - C_S - C_P - Cd_{Mj_0}D_M^2(1+\gamma_M(D_S+D_M)) - Cp_M(G_M D_M - K_M) \tag{9}$$

Proposition 4. The optimal product quality level enhancement by the supplier in considered centralized supply chain is

$$D_S^* = \frac{\beta_M P_{Mj_0} - Cd_{Pj_0}\gamma_P D_P^{\,2} - Cd_{Mj_0}D_M^{\,2}\gamma_M}{2Cd_{Sj_0}} \tag{10}$$

Proposition 5. The optimal product quality level improvement by the processor in considered centralized supply chain is

$$D_P^* = \frac{\beta_M P_{Mj_0} - Cd_{Mj_0}d_M^{\,2}\gamma_M - Cp_P G_P}{2Cd_{Pj_0}(1+\gamma_P D_S)} \tag{11}$$

Proposition 6. The optimal product quality level improvement by the manufacturer in considered centralized supply chain is

$$D_M^* = \frac{\alpha_M P_{Mj_0} - Cp_M G_M}{2Cd_{Mj_0}(1+\gamma_M(D_S+D_P))} \tag{12}$$

It is mention worthy that unlike the decentralized system, in centralized system we have to obtain optimal value of D_S, D_P and D_M by solving the systems of three equations.

5. Supply chain coordination with revenue sharing contract

Since the product quality level improvement by the supplier increases the processor and manufacturer's profit, they share a portion of this profit enhancement with the supplier. Based on the designed revenue sharing contract, whenever the supplier increases the quality of his product (D_S') he will receive more profit from the processor. Considering this revenue sharing contract, the supplier's profit for each unit product extraction is

$$\pi_S' = \phi_1 P_{Pj_0}\beta_P D_S' + \phi_1\phi_2 P_{Mj_0}\beta_M D_S' + P_{Sj_0}(1+\alpha_S D_S') - C_S - cd_{Sj_0}D_S'^2 \tag{13}$$

where the first and second terms show a portion of processor's profit which the supplier receives from the processor due to increasing the product quality improvement (d_S'). It is clearly understandable that if the supplier doesn't increase his product quality level he will receive no shared profit from the processor. The other parts of equation (13) are similar to equation (1). According to the presented revenue sharing contract, when the processor delivers product with higher quality to the manufacturer, he will share his profit more with the processor. But some percent of this product quality improvement is done by supplier and the rest of product quality improvement is done by the processor. Therefore the processor shares a portion of the profit received from the manufacturer which is related to the supplier's product quality improvement with the supplier. Hence, considering the above contract descriptions, the processor's profit for processing one unit product under the proposed revenue sharing contract is

$$\pi_P' = \phi_2 P_{Mj_0}\beta_M\big((1-\phi_1)D_S + D_P\big) + P_{Pj_0}\big(1+(1-\phi_1)\beta_P D_S + \alpha_P D_P\big) - P_{Sj_0}(1+\alpha_S D_S)$$
$$- C_P - Cd_{Pj_0}D_P^2(1+\gamma_P D_S) - Cp_P(G_P D_P - K_P) \tag{14}$$

Where the first part is a portion of the manufacturer's profit which the processor receives from manufacturer minus a part of it that the processor gives to supplier. The second part denotes a unit processed material selling price with minimum quality level plus the selling price enhancement due to product quality level improvement by the processor (D_P') and the supplier (D_S'), minus a part of it that the processor gives to the supplier proportionate to (D_S'). Based on

the designed contract even if the processor doesn't like to increase his product quality, it's beneficial for him to motivate the supplier to improve product quality. The other parts of equation (14) are similar to equation (2).

According to the proposed contract, the manufacturer will share a part of his profit caused by product quality improvement in previous supply chain levels with the processor, so the manufacturer's profit for manufacturing one unit product based on the presented contract can be written as

$$
\begin{aligned}
\pi'_M = &P_{Mj_0}\left(1+\left(1-\phi_2\right)\beta_M\left(D_S+D_P\right)+\alpha_M D_M\right)-P_{Pj_0}\left(1+\beta_P D_S+\alpha_P D_P\right)-C_M \\
&-Cd_{Mj_0}D_M^2\left(1+\gamma_M\left(D_S+D_M\right)\right)-Cp_M\left(G_M D_M-K_M\right)
\end{aligned}
\tag{15}
$$

where the first term shows a unit manufactured product selling price with minimum quality level plus the selling price enhancement because of product quality level improvement by the manufacturer (D'_M) and its previous levels ($D'_S+D'_P$) minus a part of it that manufacturer gives to the processor proportionate to ($D'_S+D'_P$). The other parts of equation (15) are similar to equation (3).

After considering the proposed revenue sharing contract in the supply chain all members still try to maximize their own profit due to the decentralization of the supply chain. Therefore the members' optimal decisions can be written as follows.

Proposition 7. The optimal product quality level improvement by the supplier after considering revenue sharing contract in supply chain will be

$$
D'^{*}_S = \frac{\phi_1\beta_P P_{Pj_0}+\phi_1\phi_2\beta_M P_{Mj_0}+\alpha_S P_{Sj_0}}{2Cd_{Sj_0}}
\tag{16}
$$

Proposition 8. The optimal product quality level enhancement by the processor in assumed decentralized supply chain after considering revenue sharing contract is

$$
D'^{*}_P = \frac{\phi_2\beta_M P_{Mj_0}+\alpha_P P_{Pj_0}-Cp_P G_P}{2Cd_{Pj_0}\left(1+\gamma_P d'^{*}_S\right)}
\tag{17}
$$

Proposition 9. The optimal product quality level improvement by the manufacturer in considered decentralized supply chain based on designed contract is

$$
D'^{*}_M = \frac{\alpha_M P_{Mj_0}-Cp_M G_M}{2Cd_{Pj_0}\left(1+\gamma_M\left(D'^{*}_S+D'^{*}_P\right)\right)}
\tag{18}
$$

Proposition 10. The other decision variables in considered contract are ϕ_1 and ϕ_2, whose optimal values can be calculated as follows

$$
\phi_1^{*} = \frac{2\alpha_S P_{Sj_0}+Cd_{Pj_0}D'^{2}_P\gamma_P-\phi_2\beta_M P_{Mj_0}-\beta_P P_{Pj_0}}{-2\left(\phi_2\beta_M P_{Mj_0}+\beta_P P_{Pj_0}\right)}
\tag{19}
$$

And

$$
\phi_2^{*} = \frac{1}{2}-\frac{Cd_{Mj_0}D'^{2}_M\gamma_M}{2\beta_M P_{Mj_0}}-\frac{Cd_{Pj_0}\left(1+\gamma_P D'_S\right)\left(2\phi_1\beta_P P_{Pj_0}+\alpha_S P_{Sj_0}\right)+Cd_{Sj_0}\left(2\alpha_P P_{Pj_0}-Cp_P G_P\right)}{2Cd_{Pj_0}\left(1+\gamma_P D'_S\right)\phi_1\beta_M P_{Mj_0}+2Cd_{Sj_0}\beta_M P_{Mj_0}}
\tag{20}
$$

It is necessary to note that we have to obtain the optimal values of ϕ_1 and ϕ_2 by solving the systems of two equations.

Therefore, the optimal value of total supply chain profit under revenue sharing contract can be written as

$$
\pi'^{*}_T = \pi'^{*}_S+\pi'^{*}_P+\pi'^{*}_M
\tag{21}
$$

Also optimal product quality improvement for final product can be calculated as follows:

$$D_T'^* = D_S'^* + D_P'^* + D_M'^* \tag{22}$$

The amount of air pollutant emissions depends on product quality improvement at the processor and manufacturer levels. In other words, the more product quality improvement at the processor and manufacturer levels, the greater air pollutant emissions. The propositions 11 and 12 obtain the upper bound and the lower bound for the decision variable ϕ_2 respectively and guarantee that processor and manufacturer's air pollutant emissions after considering proposed revenue sharing contract is less than that without the coordination case.

Proposition 11. The optimal product quality level improvement by the processor in considered decentralized supply chain without coordination is more than the coordinated with designed revenue sharing contract case ($D_P'^* < D_P^*$) if

$$\phi_2 < \frac{\phi_1 \beta_P P_{Pj_0} \left(\alpha_P P_{Pj_0} - Cp_P G_P \right)}{\beta_M P_{Mj_0} \left(2Cd_{Sj_0} + \alpha_S P_{Sj_0} \right) - \phi_1 \beta_M P_{Mj_0} \left(\alpha_P P_{Pj_0} - Cp_P G_P \right)} \tag{23}$$

Proposition 12. The optimal product quality level improvement by the manufacturer in considered decentralized supply chain without coordination is more than the coordinated with proposed revenue sharing contract case ($D_M'^* < D_M^*$) if

$$\phi_2 > \frac{\left(2Cd_{Sj_0} + \gamma_P \left(\phi_1 \beta_P P_{Pj_0} + \phi_1 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0} \right) \right) \left(\frac{2Cd_{Sj_0}^2 \left(\alpha_P P_{Pj_0} - Cp_P G_P \right)}{2cd_{Sj_0} + \gamma_P \alpha_S P_{Sj_0}} - Cd_{Pj_0} \phi_1 \beta_P P_{Pj_0} \right) - 2Cd_{Sj_0}^2 \left(\alpha_P P_{Pj_0} - Cp_P G_P \right)}{\beta_M P_{Mj_0} \left(2Cd_{Sj_0}^2 + \phi_1 Cd_{Pj_0} \left(2Cd_{Sj_0} + \gamma_P \left(\phi_1 \beta_P P_{Pj_0} + \phi_1 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0} \right) \right) \right)} \tag{24}$$

6. Numerical example

In this section, we provide a numerical example in order to illustrate the designed revenue sharing contract performance by using the below parameters: $C_S = 150$; $C_P = 250$; $C_M = 350$; $P_{Sj0} = 20$; $P_{Pj0} = 30$; $P_{Mj0} = 40$; $\alpha_S = 3$; $\alpha_P = 5$; $\alpha_M = 7$; $\beta_P = 6$; $\beta_M = 20$; $\gamma_P = \gamma_M = 10$; $Cd_{Sj_0} = 4$; $Cd_{Pj_0} = 5$; $Cd_{Mj_0} = 6$; $Cp_P = 2$; $Cp_M = 5$; $G_P = 3$; $G_M = 4$; $K_P = 50$ and $K_M = 40$.

The MATLAB software is used to solve the numerical example considering mentioned parameters and its results are presented in the Tables 1-3 and Figures 2 and 3.

Table 1 shows the optimum value of key variables for centralized supply chain. It is necessary to note that the negative profit of the supplier in the centralized supply chain is not important because all members in the centralized supply chain operate jointly as a single company and achieving the win-win condition for supply chain members is not important in this case.

**Table 1. Optimum value of key variables for centralized supply chain**

Key Variables	g_P^*	g_M^*	π_S^*	π_P^*	π_M^*	π_T^*
Optimum Value	0.0793	0.214	-34093	11834	61908	39649

From Table 2 and Figure 2 we observe that the profit of all supply chain members and whole supply chain profit in decentralized supply chain without coordination are is much lower than centralized case; But the pollutant that emissions by processor and manufacturer in decentralized supply chain is higher than centralized system. Therefore we can say that air pollutant emissions in the decentralized supply chain is higher than centralized system.

As mentioned before, we obtain the optimal values of ϕ_1 and ϕ_2 by solving the systems of two equations using equations 19 and 20. In this example, the lower bound and upper bound for the decision variable ϕ_2 are obtain -0.102 and 2.879 respectively and the conditions mentioned in propositions 11 and 12 are satisfied because the optimal values of ϕ_1 and ϕ_2 are obtained 0.438 and 0.190 respectively.

Table 2. Optimum value of key variables for decentralized supply chain without coordination

| Key Variables	g_P^*	g_M^*	π_S^*	π_P^*	π_M^*	π_T^*
Optimum Value	0.1894	0.2781	95	773	4669	5537
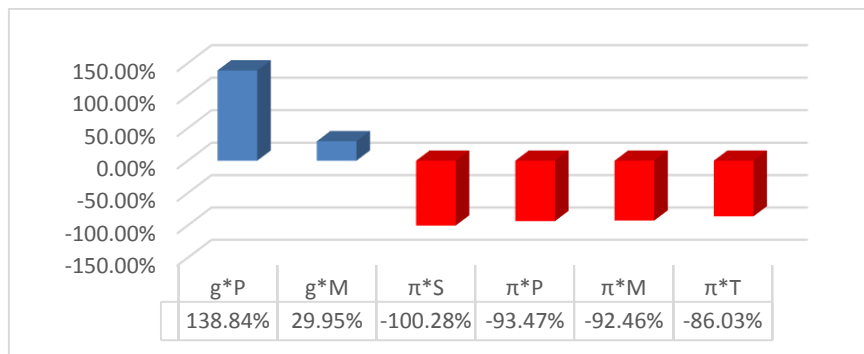
Figure 2. Key variables change percentage of the decentralized supply chain without coordination compared to centralized supply chain

As it is shown in Table 3 and Figure 3, the designed revenue sharing contract increases the whole supply chain profit by 217.48%; also the proposed revenue sharing contract increases the supplier, processor and manufacturer's profit by 2538.95%, 304.53% and 155.81% respectively so we can say that this contract provides a win-win condition for all supply members. It should be mention that we can never increase the whole supply chain profit of decentralized supply chain to its centralized case due to the necessity of the win-win condition for all members in the decentralized supply chain. Also we can say that the designed revenue sharing contract decreases 39.39% and 69.94% air pollutant emissions in processor and manufacturer supply chain level respectively.

Table 3. Optimum value of key variables for coordinated decentralized supply chain with designed revenue sharing contract

Key Variables	g_P^*	g_M^*	π_S^*	π_P^*	π_M^*	π_T^*
Optimum Value	0.1148	0.0836	2507	3127	11944	17579
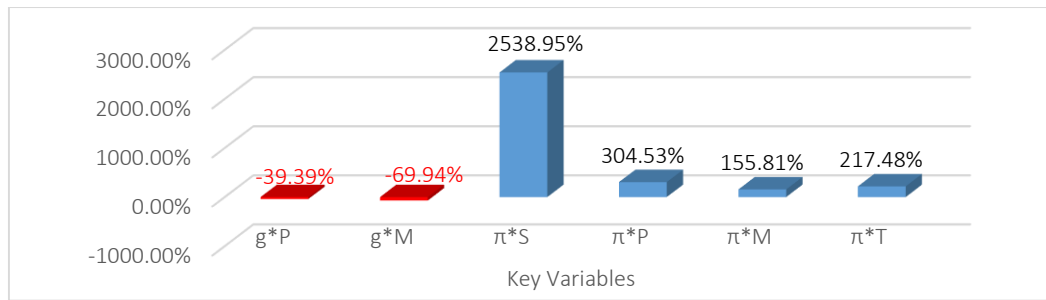
Figure 3. Key variables change percentage of the coordinated decentralized supply chain compared to without coordination case

7. Conclusions

One of the main concerns of miners is to increase their profit without enhancing air pollutant emissions. This paper studied the coordination issue of a decentralized three-level mining metal supply chain with one supplier (extractor), one processor and one manufacturer under cap-and-trade regulation and compared it with the centralized system. There are two common practices for reducing air pollutant emissions in industries: 1- Technology changing and 2- Practical policies. Since the first method is very costly, the second method is a competitive advantage for miners; therefore, due to financial limitation of industries, one of the most important concern of miners is how to decrease air pollutant emissions by operational approaches. In this study a revenue sharing contract is designed in order to coordinating mentioned supply chain and decrease air pollutant emissions. Finally, the numerical example illustrated that the proposed a new revenue sharing contract can 1- coordinates three-stage mining metals supply chain 2- provides a win-win condition for all supply chain members 3- decreases the difference between the outcome of a centralized system and a decentralized system 4- reduces air pollutant emissions in the supply chain under cap-and-trade regulation and 5- increases whole supply chain profit. The authors' suggestions for future researches is to use other coordination mechanisms and consider stochastic demand in the model also use more techniques such as robust optimization (Goli et al., 2019, Sangaiah et al., 2019), multi-objective optimization (Roy et al., 2017).

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Appendix A.

Proof of Proposition 1

Since the π_S is concave in D_S, there exists a unique optimal product quality level improvement D_S that maximizes supplier's profit because the second derivative of equation π_S is negative

$$\frac{\partial \pi_S^2}{\partial^2 D_S} = -2Cd_{Sj_0} < 0$$

Therefore the optimal value of d_S can be obtained as follows

$$\frac{\partial \pi_S}{\partial D_S} = 0 \rightarrow \alpha_S P_{Sj_0} - 2Cd_{Sj_0} D_S = 0 \rightarrow D_S^* = \frac{\alpha_S P_{Sj_0}}{2Cd_{Sj_0}}$$

This completes the proof.

Appendix B.

Proof of Proposition 2

Since the π_P is concave in D_P, there exists a unique optimal product quality level improvement D_P that maximizes the processor's profit because second derivative of the function π_P is negative

$$\frac{\partial \pi_P^2}{\partial^2 D_P} = -2Cd_{Pj_0}(1 + \gamma_P D_S) < 0$$

Therefore the optimal value of D_P can be obtained as follows

$$\frac{\partial \pi_P}{\partial D_P} = 0 \rightarrow \alpha_P P_{Pj_0} - 2Cd_{Pj_0} d_P(1 + \gamma_P D_S) - Cp_P G_P = 0 \rightarrow D_P^* = \frac{\alpha_P P_{Pj_0} - Cp_P G_P}{2Cd_{Pj_0}(1 + \gamma_P D_S^*)}$$

This completes the proof.

Appendix C.

Proof of Proposition 3

Since the π_M is concave in D_M, there exists a unique optimal product quality level improvement D_M that maximizes the manufacturer's profit because second derivative of the function π_M is negative

$$\frac{\partial \pi_M^2}{\partial^2 D_M} = -2Cd_{Mj_0}(1 + \gamma_M(D_S + D_P)) < 0$$

Therefore the optimal value of d_M can be obtained as follows

$$\frac{\partial \pi_P}{\partial D_P} = 0 \rightarrow \alpha_M P_{Mj_0} - 2Cd_{Mj_0} d_M(1 + \gamma_M(D_S + D_P)) - Cp_M G_M = 0 \rightarrow D_M^* = \frac{\alpha_M P_{Mj_0} - Cp_M G_M}{2Cd_{Mj_0}(1 + \gamma_M(D_S^* + D_P^*))}$$

This completes the proof.

Appendix D.

Proof of Proposition 4

Since the π_{TC} is concave in D_S, there exists a unique optimal product quality level improvement D_S that maximizes whole centralized supply chain profit because the second derivative of equation π_{TC} is negative

$$\frac{\partial \pi_{TC}^2}{\partial^2 D_S} = -2Cd_{Sj_0} < 0$$

Therefore the optimal value of D_S in centralized system can be obtained as follows

$$\frac{\partial \pi_{TC}}{\partial D_S} = 0 \rightarrow \beta_M P_{Mj_0} - Cd_{Pj_0} \gamma_P D_P{}^2 - Cd_{Mj_0} D_M{}^2 \gamma_M - 2Cd_{Sj_0} D_S = 0 \rightarrow$$

$$D_S^* = \frac{\beta_M P_{Mj_0} - Cd_{Pj_0} \gamma_P D_P{}^2 - Cd_{Mj_0} D_M{}^2 \gamma_M}{2Cd_{Sj_0}}$$

This completes the proof.

Appendix E.

Proof of Proposition 5

Since the π_{TC} is concave in D_P, there exists a unique optimal product quality level improvement D_P that maximizes whole centralized supply chain profit because second derivative of function π_{TC} is negative

$$\frac{\partial \pi_{TC}^2}{\partial^2 D_P} = -2Cd_{Pj_0}(1 + \gamma_P D_S) < 0$$

Therefore the optimal value of d_P can be obtained as follows

$$\frac{\partial \pi_{TC}}{\partial D_P} = 0 \rightarrow \beta_M P_{Mj_0} - Cd_{Mj_0} D_M{}^2 \gamma_M - 2Cd_{Pj_0} D_P(1 + \gamma_P D_S) - Cp_P G_P = 0 \rightarrow$$

$$D_P^* = \frac{\beta_M P_{Mj_0} - Cd_{Mj_0} D_M{}^2 \gamma_M - Cp_P G_P}{2Cd_{Pj_0}(1 + \gamma_P D_S)}$$

This completes the proof.

Appendix F.

Proof of Proposition 6

Since the π_{TC} is concave in D_M , there exists a unique optimal product quality level improvement D_M that maximizes whole centralized supply chain profit because second derivative of function π_{TC} is negative

$$\frac{\partial \pi_M^2}{\partial^2 D_M} = -2Cd_{Mj_0}(1 + \gamma_M(D_S + D_P)) < 0$$

Therefore the optimal value of D_M can be obtained as follows

$$\frac{\partial \pi_{TC}}{\partial D_P} = 0 \rightarrow \alpha_M P_{Mj_0} - 2Cd_{Mj_0} D_M(1 + \gamma_M(D_S + D_P)) - Cp_M G_M = 0 \rightarrow D_M^* = \frac{\alpha_M P_{Mj_0} - Cp_M G_M}{2Cd_{Mj_0}(1 + \gamma_M(D_S + D_P))}$$

This completes the proof.

Appendix G.

Proof of Proposition 7

Since the π_S' is concave in D_S' and there exists a unique optimal product quality level improvement D_S' that maximizes supplier's profit because second derivative of equation π_S' is negative

$$\frac{\partial \pi_S'^2}{\partial^2 D_S'} = -2Cd_{Sj_0} < 0$$

Therefore the optimal value of d_S' can be obtained as follows

$$\frac{\partial \pi'_S}{\partial D'_S} = 0 \rightarrow \phi_1 \beta_P P_{Pj_0} + \phi_1 \phi_2 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0} - 2Cd_{Sj_0} D'_S = 0 \rightarrow D'^*_S = \frac{\phi_1 \beta_P P_{Pj_0} + \phi_1 \phi_2 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0}}{2Cd_{Sj_0}}$$

This completes the proof.

Appendix H.

Proof of Proposition 8

Since the π_P is concave in D'_P and there exists a unique optimal product quality level improvement D'_P that maximizes processor's profit because second derivative of function π'_P is negative

$$\frac{\partial \pi'^2_P}{\partial^2 D'_P} = -2Cd_{Pj_0}(1 + \gamma_P D'_S) < 0$$

Therefore the optimal value of D'_P can be obtained as follows

$$\frac{\partial \pi'_P}{\partial D'_P} = 0 \rightarrow \phi 2 \beta_M P_{Mj_0} + \alpha P P_{Pj_0} - 2Cd_{Pj_0} D'_P(1 + \gamma_P D'_S) - Cp_P G_P = 0 \rightarrow D'^*_P = \frac{\phi_2 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0} - Cp_P G_P}{2Cd_{Pj_0}(1 + \gamma_P D'^*_S)}$$

This completes the proof.

Appendix I.

Proof of Proposition 9:

Proof of proposition 9 is similar to proof of proposition 3.

Appendix J.

Proof of Proposition 10

Since the π'_P is concave in ϕ_1 and there exists a unique optimal value for ϕ_1 that maximizes processor's profit because second derivative of function π'_P is negative. It should be noted that before the derivation of π'_P we should replace equations (16) in equation (14)

$$\frac{\partial \pi'^2_P}{\partial^2 \phi_1} = -2(\phi_2 \beta_M P_{Mj_0} + \beta_P P_{Pj_0}) < 0$$

Therefore the optimal value of ϕ_1 can be obtained as follows

$$\frac{\partial \pi'_P}{\partial \phi_1} = 0 \rightarrow 2\alpha_S P_{Sj_0} + Cd_{Pj_0} D'^2_P \gamma_P - \phi_2 \beta_M P_{Mj_0} - \beta_P P_{Pj_0} + 2\phi_1(\phi_2 \beta_M P_{Mj_0} + \beta_P P_{Pj_0}) = 0$$

$$\rightarrow \phi^*_1 = \frac{2\alpha_S P_{Sj_0} + Cd_{Pj_0} D'^2_P \gamma_P - \phi_2 \beta_M P_{Mj_0} - \beta_P P_{Pj_0}}{-2(\phi_2 \beta_M P_{Mj_0} + \beta_P P_{Pj_0})}$$

This proofs equation (19).

In the following, π'_M is concave in ϕ_2 and there exists a unique optimal value for ϕ_2 that maximizes processor's profit because second derivative of function π'_M is negative. It should be noted that before the derivation of π'_M we should replace equation (16) and (17) in equation (15)

$$\frac{\partial \pi'^2_M}{\partial^2 \phi_2} = -\beta_M P_{Mj_0} \left(\frac{\phi_1 \beta_M P_{Mj_0}}{Cd_{Sj_0}} + \frac{\beta_M P_{Mj_0}}{Cd_{Pj_0}(1 + \gamma_P D'_S)} \right) < 0$$

Therefore the optimal value of ϕ_2 can be calculated as follows

$$\frac{\partial \pi'_M}{\partial \phi_2} = 0 \rightarrow \left(\frac{\phi_1 \beta_M P_{Mj_0}}{2Cd_{Sj_0}} + \frac{\beta_M P_{Mj_0}}{2Cd_{Pj_0}(1+\gamma_P D'_S)} \right) \left(\beta_M P_{Mj_0} - Cd_{Mj_0} D'^2_M \gamma_M \right) - \frac{\phi_1 P_{Pj_0} \beta_P \beta_M P_{Mj_0}}{2Cd_{Sj_0}} - \frac{\alpha_P P_{Pj_0} \beta_M P_{Mj_0}}{2Cd_{Pj_0}(1+\gamma_P D'_S)}$$

$$-2\phi_2 \beta_M P_{Mj_0} \left(\frac{\phi_1 \beta_M P_{Mj_0}}{2Cd_{Sj_0}} + \frac{\beta_M P_{Mj_0}}{2Cd_{Pj_0}(1+\gamma_P D'_S)} \right) - \beta_M P_{Mj_0} \left(\frac{\phi_1 P_{Pj_0} \beta_P + \alpha_S P_{Sj_0}}{2Cd_{Sj_0}} + \frac{\alpha_P P_{Pj_0} - cp_P g_P}{2Cd_{Pj_0}(1+\gamma_P D'_S)} \right) = 0$$

$$\rightarrow \phi_2^* = \frac{1}{2} - \frac{cd_{Mj_0} D'^2_M \gamma_M}{2\beta_M P_{Mj_0}} - \frac{Cd_{Pj_0}(1+\gamma_P D'_S)(2\phi_1 \beta_P P_{Pj_0} + \alpha_S P_{Sj_0}) + Cd_{Sj_0}(2\alpha_P P_{Pj_0} - Cp_P G_P)}{2Cd_{Pj_0}(1+\gamma_P D'_S)\phi_1 \beta_M P_{Mj_0} + 2Cd_{Sj_0} \beta_M P_{Mj_0}}$$

This completes the proof.

Appendix K.

Proof of Proposition 11.

$$D'^*_P < D^*_P \rightarrow \frac{\phi_2 \beta_M P_{Mj_0} + \alpha_P P_{Pj_0} - Cp_P G_P}{2Cd_{Pj_0}(1+\gamma_P D'^*_S)} < \frac{\alpha_P P_{Pj_0} - Cp_P G_P}{2Cd_{Pj_0}(1+\gamma_P D^*_S)}$$

By replacing equation (16) in equation (17) and replacing equation (4) in equation (5) we have

$$\frac{\phi_2 \beta_M P_{Mj_0} + \alpha_P P_{Pj_0} - Cp_P G_P}{2Cd_{Sj_0} + \phi_1 \beta_P P_{Pj_0} + \phi_1 \phi_2 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0}} < \frac{\alpha_P P_{Pj_0} - Cp_P G_P}{2Cd_{Sj_0} + \alpha_S P_{Sj_0}} \rightarrow$$

$$\rightarrow \phi_2 < \frac{\phi_1 \beta_P P_{Pj_0}(\alpha_P P_{Pj_0} - Cp_P G_P)}{\beta_M P_{Mj_0}(2Cd_{Sj_0} + \alpha_S P_{Sj_0}) - \phi_1 \beta_M P_{Mj_0}(\alpha_P P_{Pj_0} - Cp_P G_P)}$$

This completes the proof.

Appendix L.

Proof of Proposition 12.

$$d'^*_M < d^*_M \rightarrow \frac{\alpha_M P_{Mj_0} - cp_M g_M}{2cd_{Pj_0}(1+\gamma_M(d'^*_S + d'^*_P))} < \frac{\alpha_M P_{Mj_0} - cp_M g_M}{2cd_{Pj_0}(1+\gamma_M(d^*_S + d^*_P))}$$

Comparing equations (18) and (6) it is clear that numerator in both equations is the same so we have

$$\rightarrow 2cd_{Pj_0}(1+\gamma_M(d'^*_S + d'^*_P)) > 2cd_{Pj_0}(1+\gamma_M(d^*_S + d^*_P)) \rightarrow (d'^*_S + d'^*_P) > (d^*_S + d^*_P) \rightarrow (d'^*_S - d^*_S) > (d^*_P - d'^*_P)$$

$$\rightarrow \frac{\phi_1 \beta_P P_{Pj_0} + \phi_1 \phi_2 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0}}{2cd_{Sj_0}} - \frac{\alpha_S P_{Sj_0}}{2cd_{Sj_0}} > \frac{\alpha_P P_{Pj_0} - cp_P g_P}{2cd_{Pj_0}(1+\gamma_P d^*_S)} - \frac{\phi_2 \beta_M P_{Mj_0} + \alpha_P P_{Pj_0} - cp_P g_P}{2cd_{Pj_0}(1+\gamma_P d'^*_S)} \rightarrow$$

$$\phi_2 > \frac{\left(2cd_{Sj_0} + \gamma_P(\phi_1 \beta_P P_{Pj_0} + \phi_1 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0})\right) \left(\frac{2cd^2_{Sj_0}(\alpha_P P_{Pj_0} - cp_P g_P)}{2cd_{Sj_0} + \gamma_P \alpha_S P_{Sj_0}} - cd_{Pj_0} \phi_1 \beta_P P_{Pj_0} \right) - 2cd^2_{Sj_0}(\alpha_P P_{Pj_0} - cp_P g_P)}{\beta_M P_{Mj_0}\left(2cd^2_{Sj_0} + \phi_1 cd_{Pj_0}(2cd_{Sj_0} + \gamma_P(\phi_1 \beta_P P_{Pj_0} + \phi_1 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0}))\right)}$$

This completes the proof.