A green transportation location-inventory-routing problem by dynamic regional pricing

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Abstract
Non-uniform distribution of customers in a region and variation of their maximum willingness to pay at distinct areas make regional pricing a practical method to maximize the profit of the distribution system. By subtracting the classic objective function, which minimizes operational costs from revenue function, profit maximization is aimed. A distribution network is designed by determining the number of trucks to each established distribution center, allocating customers in routes, and inventory levels of customers. Also, environmental impacts, including fuel consumption and CO2 emission, aimed to be minimized. So, a new quadratic mixed-integer programming model is presented for the Green Transportation Location-Inventory-Routing Problem integrated with dynamic regional pricing problem (GTLIRP+DRP). The model is applied to the real case study, to show its competent application. To tackle this problem, a Hybrid Bees Algorithm (HBA) is developed and verified by the genetic algorithm. Finally, managers suggested using HBA that achieves better solutions in the less computational time.

Keywords: transportation location-inventory-routing problem; dynamic pricing; regional pricing; green objectives; metaheuristic algorithms.

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1. Introduction
Even though, in designing a distribution system, which is a small part of supply chain, the main goal is to minimize operational costs of the system (such as fixed cost for operating distribution centers, vehicle routing costs, inventory holding costs at retailers, etc.) traditionally, it is not the only way to obtain more profit. It is implicated from the simple formulation of profit (i.e., profit = revenue – costs). Therefore, the other way to increase the profit of the system is to maximize the revenue of the company. This will not be achieved except through demand management methods. Mostly the amount of product’s demand except core products depends on its price. Therefore, pricing techniques as one of the demand management tools are an excellent means to achieve this goal. Managing the price in a way that enhances the product demands could cause gaining more revenue and net profit increasing.

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Since the city traffic regulations for heavy transportation fleets like trucks avoid them to link planet to its customers directly (Martínez-Salazar et al., 2014) the plant needs to locate some facilities called Distribution Centers (DCs) in the border of the cities in order to facilitate distribution process. To complete the chain, customers spread in different zones of the city need to be allocated to DCs. In the next level, the sequence of customers in the routes of vehicles will be determined by routing decisions. Also, the number of delivered products to each customer specifies the inventory level of each customer at the end of each period with regards to inventory holding cost. In conclusion, in the current study, three decision levels should be decided.

Decision levels in supply chain design involve strategic, tactical, and operational levels (Prodhon and Prins 2014). This study addresses a facility location problem (FLP) for DCs, in strategic level, inventory control problem for customers, in tactical level, and vehicle routing problem (VRP), at the operational level. A holistic perspective achieves optimal decisions and integrates these three decision levels. As a result, the location-inventory-routing problem (LIRP) was introduced as a combination of the three mentioned problems (Nekooghadirli et al., 2014). The genesis of LIRP refers to the study of Ahmadi Javid and Azad (2010). One of the integration reasons is that solving them separately may cause sub-optimal solutions (Ghorbani and Akbari Jokar, 2016). Since the number of trucks from plant to operated DCs should be determined in the first stage, a new extension of LIRP called the transportation location-inventory-routing problem (TLIRP) is developed.

The spread of customers in different areas of the city by different economic conditions and different maximum willingness to pay (MWP) result in maximizing the revenue of the company using regional pricing techniques. Regional pricing is one of the price differentiation techniques (Philips, 2005), and the other is seasonal pricing, which is applicable for goods like ice cream, apparel, etc. (Etebari and Dabiri, 2016). Based on various MWP of different regions, the pricing mechanism may offer different prices, which may cause different demand levels. So, the location, routing, and especially inventory decisions got influenced (Etebari and Dabiri, 2016). As such, regional pricing is integrated by TLIRP in this study. Solving two problems simultaneously increases the profitability of the supply chain (Etebari and Dabiri, 2016). To the best of our knowledge, it is the first time that a TLIRP problem is solved simultaneously with a dynamic regional pricing problem (DRP) in the literature.

Intensifying the global warming phenomenon attracts social concerns about the environment more and more (Govindan et al., 2014). Never the less by maximizing the company's profit, designers will obtain the primary goal of designing a distribution system, but it is not sufficient in this century. Nowadays, the interest of other stakeholders such as customers, the community must be taken into account to design a sustainable distribution network (Eskandarpour et al., 2015). There are environmental regulations such as ISO 14000 (environmental management), ISO 50001 (energy management) forcing companies to pay attention to the impact of distribution activities on the environment (Rabbani et al., 2018). Also, Navazi et al. (2019) claimed that these days end-consumers prefer to buy products with a lower carbon emission on their carbon footprint (CFP) label. So, they added a green objective function to their problem. Accordingly, in this study, a green objective is added to the model. This objective seeks to minimize the fuel consumption of vehicles in the transportation and routing phase to palliate the energy consumption. Hence, it tries to deduct the CO2 emission costs, which is one of the greenhouse gasses and the main reason for global warming (Bektas and Laporte 2011).

To handle the aforementioned needs, a comprehensive problem is modeled in this study. In the proposed model, regional pricing, which is one of the demand management methods is integrated with a location-inventory-routing problem that is one of the main contributions of this study. Since the problem is multi-period, the other contribution is that the regional pricing is dynamic as well. Adding transportation stage from plant to the LIRP is another contribution
of this study. Furthermore, a new green TLIRP model is solved using a developed hybrid meta-heuristic algorithm in this study for the first time. Because of the NP-hard nature of the location-inventory-routing problem (Ahmadi Javid and Azad, 2010), the proposed model in this paper, which is a combination of LIRP with regional pricing problem will be NP-hard, too. Since combinations of the location-inventory-routing problem and regional pricing problem make it even more complicated.

The remainder of this paper is organized as follows: In the next section, the close literature to this paper is reviewed. Then, the problem is described, and the mathematical formulation is presented. Section 4 includes the explanation of metaheuristic approaches that are used for solving the problem. The results are reported in section 5, and in section 6, the application of the model is shown in a case study from the real world, which results in some managerial insights. Finally, there are conclusions and future research in the last section.

2. Literature review

The lack of papers maximizing profit using differentiation pricing makes us review the location-inventory-routing problem (LIRP) at the beginning of this section. As aforementioned, LIRP is introduced by Ahmadi Javid and Azad (2010). It is the outcome of incorporating location, routing, and inventory problems (Gholamian, M.R., Heydari, 2017). In their problem, the demand has a normal distribution, and the DCs maintain the determined amount of safety stock (SS). Both exact and heuristic methods were used to solve the problem. The heuristic method was the hybridization of Tabu Search (TS) and Simulated Annealing Algorithm (SA). In fact, LIRP is a comprehensive view of the Location-Routing, Location-Inventory, and Inventory Routing problems. For instance, A location routing problem (LRP) locates the DCs in the outskirt of city and determines the order of customers at each vehicle’s route (Navazi et al., 2019), a location-inventory problem (LIP) decides about the location of ambulance stations and the quantity of ordered perishable drug (Navazi et al., 2018) and an inventory routing problem specifies the permutation of customer in routes and inventory levels of them (Crama et al., 2018), but none of them considered the location, inventory, and routing decisions simultaneously like LIRP. A customized LIRP for perishable products with a shelf life is proposed by Hiassat and Diabat (2011). GAMS software is used for solving small size problems in their study. Later, a genetic algorithm (GA) with customized chromosome is developed for the LIRP for perishable products by Hiassat et al. (2017). The multi-source version of LIRP is investigated by Taylor et al., Seddighi (2013) in which a depot could supply from more than one plant. A heuristic approach is used to solve this problem. Guerrero et al. (2013) focused on a two-echelon LIRP with deterministic demand, which was multi-depot and also, multi-retailer. In their model, both depots and retailers are able to keep inventory, and the stock-out situation is not allowed (W. j. Guerrero et al. 2013). Since an exact method has excessively long computational time, they improved it by a heuristic method and proposed a mat heuristic method for solving the problem (W. j. Guerrero et al., 2013). A heuristic method named relax and the price is formed by combining the set of exact methods, including column generation, Lagrangian relaxation, and local search is developed for the problem (W. J. Guerrero et al., 2015). Nekooghadirli et al., (2014) extend the LIRP problem to multi-product model with probabilistic travel times. The complexity of this problem is tackled by four metaheuristic algorithms, such as Multi-Objective Imperialist Competitive Algorithm (MOICA) (Nekooghadirli et al., 2014). The shortage situation has also studied in LIRP models. The multi-product LIRP by fuzzy demands with the allowable lost sales, which is a kind of shortage is modeled by Tavakkoli-moghaddam and Raziei (2016). The fuzzy mathematical programming proposed by Lin (2012) is used to solve the problem. Also, the other kind of shortage, namely back-ordering, in a multi-resource multi-product, LIRP is overhauled (Ghorbani and Akbari Jokar, 2016). The backlog could not exceed
A green transportation location-inventory-routing problem by dynamic regional pricing

A predefined fraction of demand. To solve the problem, a new hybrid heuristic algorithm based on the imperialist competitive algorithm and SA is developed. Moreover, the hybrid metaheuristics for LIRP models are reviewed by (Zhang et al., 2014). A LRP for perishable products solved by Lagrangian Relaxation Method in a stochastic environment by Rafie-Majd et al. (2018). Also, location, inventory, and routing problems are optimized in supply chain network design level using the Generalized Benders Decomposition (GBD) method (Zheng, Yin and Zhang, 2019). Additionally, LIRP is used for designing a closed-loop network on a stochastic possibilistic environment (Zhalechian et al., 2016). In addition to fuel consumption and CO2 emission, they try to minimize wasted energy by DCs as another environmental factor. In the future directions of this study, challenges such as pricing decisions in the designing supply chain network recognized as an interesting topic. There are few papers in the literature which have considered pricing problem in their models. A closed-loop location inventory problem is studied by Ahmadzadeh and Vahdani (2017) in a three-level supply chain by the correlated demand of regions assumption. Besides determining the price of new products to maximize revenue, the incentive price of returned products should be specified in their study. Finally, three metaheuristic algorithms are compared for problem optimization, containing: GA, ICA, and firefly algorithm (FA). Leastways, the appearance of pricing in IRP problems dates back to the study by Liu and Chen (2011). They relinquished some common assumptions such as known demand and ignoring the impact of price on the volume of demand by considering a price response demand function. They proposed a non-linear model for the problem and expanded a heuristic approach based on a TS adopting different neighborhood search. Also, an inventory routing problem considering regional pricing is solved using a heuristic method with simulated annealing framework by Etebari and Dabiri (2016). They applied a dynamic pricing approach instead of static pricing, which was used before by Liu and Chen (2011). This is the most similar article to the current study, but the idea of Rabbani et al. (2016) in constructing multiple middle depots is used here. Therefore, locating some distribution centers in the boundary of the city is considered in this study. Plus, regional pricing, the effect of supplying demand of a region in different periods is incorporated into this study. Since global warming is one of the significant challenges of the current century (Farrokhi-Asl et al., 2018). Adding green objectives besides economic objectives to the conventional problems like the inventory routing problem (IRP) is a new aspect, which tries to design cleaner distribution system for having a greener environment. Due to the environmental consideration in this study, interested readers can refer to Lin et al. (2014) as a survey on green vehicle routing problems. Cheng et al. (2017) developed a green IRP with heterogeneous fleets. To the best of our knowledge, this study is one of the rare green LIRP papers. The aim of this paper is making several necessary decisions about the number and location of distribution centers required to be operated by renting the existing regional DCs, the number of trucks, which deliver products from plant to each operated DC due to the capacity limitation of trucks, allocating customers to operated DCs, their optimal permutation in the routing phase, the inventory level at them and the offering price at each customer area, periodically. All in all, the main noticeable contributions of this study versus the others are as follows:

- Considering a transportation stage in addition to the location-inventory-routing problem.
- Considering the regional pricing problem in LIRP for the first time, to the best of our knowledge.
- Considering the location of multi DCs problem like multi-depot problems.
- Considering the routing problem with heterogeneous vehicles by different capacities.
• Minimizing environmental side effects as a social objective function. Since it maintains not only the interest of the company but also the interest of other stakeholders, including customers, surrender population, etc.
• Tailoring powerful and efficient evolutionary algorithm for the problem by customized solution representation.

### Table 1. Significant features of this study in the opposite of other related articles

<table>
<thead>
<tr>
<th>Articles</th>
<th>Transportation</th>
<th>Locating DCs</th>
<th>Inventory routing</th>
<th>Pricing</th>
<th>Heterogeneous vehicles</th>
<th>Multi-objective</th>
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### 3. Problem description and mathematical formulation

In this section, the mathematical formulation for a two-stage distribution system is presented. Because of traffic regulation for cities, which avoids entrance of heavy trucks to the city (Khalili-Damghani, Abtahi, and Ghasemi 2015), some intermediate facilities need to be established among a set of potential places for distribution centers, which receive the product from the plant and deliver them to customers. Due to different land acquisition and its capacity, these potential DCs have different operating costs. Identical trucks by limited capacity are utilized to deliver products from a plant to the operated DCs, which creates a transportation problem in the first stage. After the location problem, customers, which are distributed in different geographical areas of a region (i.e., different cities of a state), should be allocated to the operated DCs. A routing problem should determine the optimal sequence of customers in created routes of heterogeneous vehicles. It should be noted that vehicles should return to their departing DC, periodically. The model also finds the optimal inventory level of each customer at the end of each period due to the limited holding capacity.

Regarding the price dependent nature of the product’s demand, a linear price-response function is applied to show the relationship between price and demand of customers at each period (i.e. \( D_i = \alpha_i - \beta_i p_i \)). If the price in an area increases, the corresponding demand of area will plunge by the specific slope (\( \beta_i \)). The model efforts to find the optimal regional price in a way that the total revenue of the company is maximized during all periods. Because of that, the first objective function, which is used to maximize the profit of the company is quadratic, and the implementation costs of the distribution system are subtracted to calculate net profit. The implementation costs involve customer’s inventory holding cost, the heterogeneous fleet rental cost for visiting all customers in the second stage, and traveling cost for the traveled distance between nodes, as well as truck transportation costs in the first stage. The green objective function minimizes the energy consumption and CO2 emission of trucks and vehicles. The cost

¹ I: Inventory problem
² R: Routing problem
of fuel consumption is calculated by having the Fuel Consumption Rate (FCR) at 100 kilometers of each transportation commodity, which is expressed in vehicle characteristics. A sample solution for the transportation problem of the first stage associated with the vehicle routing inventory problem in the second stage through a facility location problem for DCs is shown in Figure. 1. The following notations are applied in presenting the mathematical formulation of TLIRP+DRP.

3.1. Assumptions
- The capacity of the supplier is infinitive, and the plant could supply all demands.
- According to the budget limitation of the organization for operating the distribution centers and limitation in the number of available workforces, there could be open just a maximum number of DCs at each period.
- DCs are not holding inventory; they just have the product distribution duty.
- Each customer could be assigned to just one DC.
- Demand leakage is not going to happen (reach population regions will not buy their needs from poor population regions with the lower maximum willingness to pay.).
- The demands of different customers from various areas are independent.
- Inventory shortage in customer’s place is not allowed.

3.2. Notations

Sets:
- $M$ Set of potential distribution centers \{1,2,...,m\}
- $N$ Set of customers \{1,2,...,n\}
- $K$ Set of possible vehicles for rent \{1,2,...,k\}
- $T$ Set of Different time periods \{1,2,...,TT\}
- $p$ A plant \{1\}

Parameters:
- $f_{mt}$ Cost of operating distribution center $m$ in period $t$
\(q_{dc_m}\) Capacity of distribution center \(m\)

\(nm\) Maximum number of distribution centers to open

\(qt\) Capacity of truck

\(Cc_m\) Cost of traveling from plant to DC \(m\) per truck

\(q_k\) Capacity of vehicle \(k\)

\(g_{tk}\) Fix cost for renting vehicle \(k\) which is used in period \(t\)

\(c_{ij}\) Cost of traveling from customer \(i\) to customer \(j\)

\(D_i\) Total amount of potential demand for customer \(i\) in period \(t\)

\(s_{it}\) Absolute value of the slope of demand function of customer \(i\) in period \(t\)

\(cap_i\) Capacity of customer \(i\)

\(h_i\) Holding cost of each product unit in customer \(i\) in each period

\(A_i\) Ordering cost of each turn in customer \(i\) in each period

\(R_m\) Distance from plant to potential DC \(m\)

\(r_{ij}\) Distance from customer/DC \(i\) to DC/customer \(j\)

\(ff\) Fixed cost of 1-liter fuel

\(Ce\) CO2 emission cost of 1 liter fuel consumption

\(FCR_k\) Fuel consumption rate of vehicle \(k\) in 1 kilometer

\(FCRt\) Fuel consumption rate of the truck in 1 kilometer

\(BM\) A very big number

Variables:

\[x_{ijt}^k = \begin{cases} 1 & \text{If vehicle } k \text{ goes directly from customer/DC } i \text{ to DC/customer } j \text{ at period } t \\ 0 & \text{Otherwise} \end{cases}\]

\[y_{ijt}^k \] The quantity of the product on vehicle \(k\) that goes directly from customer/DC \(i\) to customer \(j\) at period \(t\)

\(p_{it}\) The product offering price in customer \(i\) at period \(t\)

\(I_{it}\) Inventory level in customer \(i\) at the end of period \(t\)

\(z_{mt}\) = \(1\) If distribution center \(m\) is operated in period \(t\)

\(0\) Otherwise

\(u_{mit}\) = \(1\) If customer \(i\) assigned to distribution center \(m\) in period \(t\)

\(0\) Otherwise

\(nt_{ma}\) Number of trucks sent from plant to DC \(m\) in period \(t\)

\(vd_{mt}\) Amount of product sent from plant to DC \(m\) in period \(t\)

### 3.3. Mathematical formulation

#### Objective Functions:

1. **Economic objectives**

\[
\begin{align*}
\text{Max } O1 &= \sum_{t=1}^{TT} \left( \sum_{i=1}^{n} p_{it} (D_{it} - s_{it}) - \sum_{m=1}^{m+n} Cc_m \sum_{mt}^{nt_{mt}} - \sum_{i=1}^{m+n} \sum_{j=1}^{n} \sum_{k=1}^{k} g_{tk} x_{ijt}^k \right) \\
&\quad - \sum_{i=1}^{m+n} \sum_{j=1}^{n} \sum_{k=1}^{k} c_{ij} x_{ijt}^k - \sum_{m=1}^{m+n} \sum_{mt}^{f_{mt}z_{mt}} - \sum_{i=1}^{n} h_i / 2 \right) \\
&\quad - \sum_{i=1}^{m+n} \sum_{j=1}^{n} \sum_{k=1}^{k} c_{ij} x_{ijt}^k - \sum_{m=1}^{m+n} \sum_{mt}^{f_{mt}z_{mt}} - \sum_{i=1}^{n} h_i / 2 \right) \\
\end{align*}
\]

2. **Green objectives**

\[
\begin{align*}
\text{Min } O2 &= \sum_{t=1}^{TT} \left( \sum_{i=1}^{m} FCRt (ff + Ce) 2R_m \sum_{mt}^{nt_{mt}} + \sum_{i=1}^{m+n} \sum_{j=1}^{n} \sum_{k=1}^{k} (FCR_k (ff + Ce) r_{ij} x_{ijt}^k) \right)
\end{align*}
\]
Constraints:

\[ n_{mt} \geq \frac{vd_{mt}}{qt} \quad \forall t \in T, \forall m \in M \]  
(3)

\[ vd_{mt} \leq z_{mt}qdcm \quad \forall t \in T, \forall m \in M \]  
(4)

\[ vd_{mt} = \sum_{i=1}^{n} (D_{it} - p_{it}s_{it})u_{mit} \quad \forall t \in T, \forall m \in M \]  
(5)

\[ \sum_{j=1}^{n+m} x_{ijt}^k = \sum_{i=1}^{n+m} x_{lit}^k \quad \forall i \in V, \forall t \in T, \forall k \in k \]  
(6)

\[ \sum_{j=1}^{n+m} x_{ijt}^k \leq 1 \quad \forall i \in V, \forall t \in T, \forall k \in k \]  
(7)

\[ \sum_{l=1}^{n+m} u_{lit} = 1 \quad \forall t \in T, \forall i \in N \]  
(8)

\[ y_{ijt}^k \leq x_{ijt}^k q_k \quad \forall i \in V, \forall j \in V \quad \forall t \in T, \forall k \in k \]  
(9)

\[ \sum_{i=1}^{n+m} y_{lit}^k \geq \sum_{j=1}^{m} y_{ijt}^k \quad \forall i \in N, \forall t \in T, \forall k \in k \]  
(10)

\[ \sum_{i=1}^{n+m} x_{ijt}^k \leq 1 \quad \forall t \in T, \forall k \in k \]  
(11)

\[ \sum_{l=1}^{n} x_{ijt}^k + \sum_{l=1}^{n} x_{lit}^k \leq 1 + u_{mit} \quad \forall i \in N, \forall j \in M \quad \forall t \in T, \forall k \in k \]  
(12)

\[ \sum_{j=1}^{m+n} x_{ijt}^k = 1 \quad \forall t \in T, \forall i \in N \]  
(13)

\[ l_{it-1} - l_{it} + \sum_{k=1}^{k} (\sum_{l=1}^{m+n} y_{lit}^k - \sum_{j=1}^{m+n} y_{ijt}^k) = D_{it} - s_{it}p_{it} \quad \forall t \in T, \forall i \in N \]  
(14)

\[ l_{it} \leq \text{cap}_i \quad \forall t \in T, \forall i \in N \]  
(15)

\[ \sum_{l=1}^{m} u_{mit} \leq BMz_{mt} \quad \forall t \in T, \forall m \in M \]  
(16)

\[ \sum_{l=1}^{n} z_{lt} \leq nm \quad \forall t \in T \]  
(17)

\[ l_{it} \geq 0 \quad \forall t \in T, \forall i \in N \]  
(18)

\[ p_{it} \geq 0 \quad \forall t \in T, \forall i \in N \]  
(19)

\[ vd_{mt} \geq 0 \quad \forall t \in T, \forall m \in M \]  
(20)

\[ y_{ijt}^k \geq 0 \quad \forall t \in T, \forall k \in k \]  
(21)

\[ nt_{mt} \in Z^+ \quad \forall t \in T, \forall m \in M \]  
(22)

\[ x_{ijt}^k \in \{0,1\} \quad \forall i \in V, \forall j \in V \quad \forall t \in T, \forall k \in k \]  
(23)

\[ u_{mit} \in \{0,1\} \quad \forall i \in N, \forall m \in M, \forall t \in T \]  
(24)
\[ z_{mt} \in \{0,1\} \quad \forall t \in T, \forall m \in M \]  

The first objective function aims to maximize the profit; it is the minus of operational costs from the selling revenue. The quadratic concave nature of revenue function is leading to a convex maximization problem. The first cost term is the truck transportation cost; the second one is the cost of renting the heterogeneous vehicles, the traveling cost is calculated by third cost term, and the next one is the fixed cost of operating DCs; the last one is the inventory holding cost for each customer. The green objective optimizes energy consumption and environmental impacts by minimizing the cost of fuel consumption and CO2 emission. The first term measures the fuel consumption and CO2 emission of trucks from plant to DCs and the second one for vehicles in the routes. The first constraints evaluate the optimal number of trucks from plant to operated DC. The violation of DC’s capacity is prohibited by constraints (4). Equations (5) illustrate the amount of assigned demand to each DC. Constraints (6) ensure the continuity of vehicle routes to each node. Constraints (7) guarantee that less than one route can be chosen between two nodes by each vehicle. Constraints (8) ensure that each customer supplied from just one DC. The observance of the limited capacity of heterogeneous vehicles is guaranteed by Constraints (9). Constraints (10) indicate that output freight from node \( i \) must be less than the input freight to customer \( i \). Constraints (11) guarantee that each vehicle being used at each period just once, and assigned to at most one DC. Constraints (12) ensure that a customer could be assigned to a DC only if there is a route linking the DC to the customer and also cause the sub-tours elimination. Visiting all customers is ascertained by Constraint (13). Balancing the inventory levels of customers between periods is the duty of Constraint (14). Constraints (15) keep the capacity limitation of customer \( i \). The fact that the allocation of customers is just allowed to open DCs is stated by constraints (16). The maximum number of DCs which could be operated is determined by constraints (17). Finally, constraints (18-25) specify the types of variables.

4. Methodology
Since the cost of CO2 emission and fuel consumption is calculated, the nature of the green objective function is the same as the first objective. By adding the second objective function to the first one and deducting costs from the revenue, a single objective problem should be solved, which maximizes the profit of the company. Because it takes much time to solve the problem with BARON solver of GAMS software, especially for larger instances, the need for an approximation algorithm to solve the problem in a reasonable time becomes clear. So, the metaheuristic algorithm is developed in the next section.

4.1. Developing metaheuristic
Two powerful evolutionary algorithms are developed to cope with the NP-hard complexity of the presented model for finding optimal solutions at an acceptable time. Since in the presented model regional pricing is integrated with LIRP and according to (Ahmadi Javid and Azad, 2010) LIRP has NP-hard complexity; our model is Np-hard too. Plus, the high reported solving time of exact method in subsections 5.1 and 5.3 proves the NP-hard complexity of the model. In the next sections these algorithms are described briefly.

4.1.1. Solution representation
Solution representation is the most important part of proposing an evolutionary algorithm because of its effect on finding near-optimum solutions and the duration of computing time. The solution representation of the proposed model includes two steps. In the first step, a matrix which every row is filled by permutation of integer numbers between 1 to \( N+M-1 \) is generated.
that is shown for $N=9$ and $M=3$ in Figure. 2. The rows of the matrix show planning horizons and the columns of matrix illustrate the minus one of the sum of customers and distribution centers. In the second step, according to the elements of the matrix, the customers are assigned to DC for each period that is shown in Figure. 3.

![Figure 2. First step of solution representation](image)

![Figure 3. Second step of solution representation](image)

### 4.1.2. Genetic Algorithm
Genetic Algorithm (GA) is the population-based well-known algorithm that can find the best solutions in a reasonable time developed by Gen and Cheng (1997) for the first time. GA generates a random initial population and improved them by crossover, mutation, and selection operators during the iterations (Deb et al., 2002). Since this well-known algorithm widely used for LIRP in the literature (Saif-Eddine et al., 2019) chosen here as a base meta-heuristic algorithm. The steps of this algorithm are shown in Figure. 4.

![Figure 4. Pseudocode of the GA](image)

Since the initial matrix is filled by integer numbers, single-point crossover, which is proper for individuals with integer numbers is used here. For each period, which has a row in the initial matrix $|T| \times |N + M - 1|$, a single point crossover is applied on the $1 \times |N + M - 1|$ matrix.
In a single-point crossover, a random point is selected, and the tails of its two parents are swapped to generate two new children. The single point crossover mechanism for \( T=1 \) is shown in Figure 5.

![Figure 5. Single point crossover operator](image)

Also, inversion mutation, which is appropriate for individuals with integer numbers, is used for \( 1 \times |N + M - 1| \) matrix of each period in this study. In the inversion mutation, a subset of genes is selected, and the selected string is inverted to generate muted individual. The inversion mutation mechanism on the \( 1 \times |N + M - 1| \) matrix is shown in Figure 6.

![Figure 6. Inversion mutation operator](image)

### 4.1.3. Hybrid Bees Algorithm

The bees algorithm (BA) is a nearly new evolutionary algorithm developed by Pham et al. (2006) inspired by honey bees. BA is a proper algorithm for problems with permutation-based solution representation (Nemmich et al., 2019). This population-based algorithm could be put in swarm intelligence algorithms category since the information exchange is happening between scout bees and worker bees. The operation of BA is based on food source foraging behavior of swarms of honey bees. At first, scout bees are sent from hive to selected sites for evaluating the conditions of the food source on that site. It is equivalent to the function evaluation in metaheuristic algorithm. After data gathering, they will return to the hive and exchange gathered information on the dance floor directly by the mechanism of waggle dance with other bees. Then the elite sites are chosen among the selected sites. In the next turn, the number of bees to each elite and the none-elite site is determined. When they fly to the food sources because of misinterpretation from the observed waggle dance, the neighborhood solutions are produced. Bees with new positions are generated in this step of the algorithm. After reevaluating, the bees with better objective function value (OFV) become responsible for next execution of waggle dance. This cycle for food source foraging would continue until the stop criterion is achieved.
The parameters of the algorithm include: number of scout bees, number of selected sites, number of elite selected sites, number of bees assign to elite selected sites, number of bees assign to selected sites. The steps of this algorithm are shown in the pseudo-code of Figure 7. For bee colony initialization, the presented solution representation in section 4.1.1 is used. So, Bees are generated by $|T| \times (N + M - 1)$ matrix, which a feasible solution could extract from it by proposed representation in section 4.1.1. Each of the bees is a structure that has a position and OFV.

As mentioned before, the most important operator of the BA is waggle dance, which causes producing neighborhoods. Typically, in continuous problems, two kinds of operators are used for waggle dance, including the uniform and the normal distribution for bees. However, because of the discrete nature of this problem, they could not be used directly. The contribution of the proposed hybrid bee algorithm (HBA) is the bees dancing function that has a new approach to generate neighborhood solutions. So, the uniform distribution is combined with an interchanging mutation in this HBA algorithm that hybrids one of the GA operators with BA. In interchanging mutation, the parent is the previous bee, and the child is the newly generated bee in the neighborhood. In the interchanging mutation, two random positions of the string are chosen, and the genes corresponding to those positions are interchanged. The mechanism of interchanging mutation is illustrated in Figure 8.

1. Initialize random individual and evaluate fitness function
   For (Iteration=1: Max-iteration) do
     2. Select sites and assign bees to them and evaluate fitness function
     3. Return bees to hive and dance
     4. Assign more bees to the best site according to the dancing of bee
     5. Select the fittest bee from each site
     6. Assign remaining bees to a random site and evaluate fitness function
     7. Store and return the best solution
   End

Figure 7. Pseudocode of the HBA

Figure 8. Interchanging Mutation operator

4.2. Parameters tuning
The efficiency and quality of evolutionary algorithms depend on the values of algorithms’ parameters. Therefore, tuning the parameters is a vital part of the proposed algorithms. In this paper, the Taguchi method is used to tune the parameters of GA and HBA. Because Taguchi design achieves the most substantial information by generating the least number of experiments. The Medium-sized problem and three levels of parameters are considered for the Taguchi method in Minitab 17 software that is illustrated in Table 2. The number of parameters in different levels is taken from the literature. The results of Minitab for the Taguchi Design of Experiment (DOE) method based on the mean of means are shown in Figure 9, and Figure 10. for GA, and HBA algorithm, respectively. The level of parameter which has minimum mean of means is a desirable level. The desirable levels of both algorithms’ parameters are reported in Table 3 and set for the rest of study.
Table 2. The levels of parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of population</td>
<td>50</td>
<td>70</td>
<td>100</td>
</tr>
<tr>
<td>Rate of Crossover</td>
<td>0.3</td>
<td>0.5</td>
<td>0.8</td>
</tr>
<tr>
<td>Rate of Mutation</td>
<td>0.2</td>
<td>0.5</td>
<td>0.7</td>
</tr>
<tr>
<td>Iteration</td>
<td>100</td>
<td>150</td>
<td>200</td>
</tr>
</tbody>
</table>

GA

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of scout bees</td>
<td>30</td>
<td>50</td>
<td>80</td>
</tr>
<tr>
<td>Number of selected sites based on the percentage of scout bees</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
</tr>
<tr>
<td>Number of elite selected sites based on the percentage of selected site bees</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>Number of bees assign to selected sites based on the percentage of scout bees</td>
<td>0.3</td>
<td>0.5</td>
<td>0.7</td>
</tr>
<tr>
<td>Iteration</td>
<td>50</td>
<td>100</td>
<td>150</td>
</tr>
</tbody>
</table>

HBA

Figure 9. The result of Taguchi design for GA

Figure 10. The result of Taguchi design for HBA
Table 3. GA and HBA parameters settings

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Parameters</th>
<th>Value</th>
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<tr>
<td>GA</td>
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<tr>
<td></td>
<td>Rate of Crossover</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>Rate of Mutation</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Iteration</td>
<td>150</td>
</tr>
<tr>
<td>HBA</td>
<td>Number of scout bees</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Number of selected sites</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>Number of elite selected sites</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>Number of bees assign to selected</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>sites</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Iteration</td>
<td>50</td>
</tr>
</tbody>
</table>

5. Computational results

For comparison, some test problems are needed. The initial data which is used for generating test problems is shown in Table 4. No benchmark is found for the proposed model because it is the first time that a LIRP is considered by regional pricing. So, test problems are generated randomly in two categories: small-sized problems and large-sized problems. The small-sized problems are used to compare metaheuristic algorithms with the results of BARON solver of GAMS, which used exact methods.

Table 4. Random input data for test problems

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum demands of customers</td>
<td>U(200,400)</td>
</tr>
<tr>
<td>Coordinate of nodes</td>
<td>U(1,100)</td>
</tr>
<tr>
<td>Operating cost of the distribution center</td>
<td>U(10,100)</td>
</tr>
<tr>
<td>Cost of renting a vehicle</td>
<td>U (50,75)</td>
</tr>
<tr>
<td>Collection time in nodes</td>
<td>Depending on the capacity U(1,10)</td>
</tr>
<tr>
<td>Capacity of vehicles</td>
<td>U(300,700)</td>
</tr>
<tr>
<td>Capacity of trucks</td>
<td>1000</td>
</tr>
<tr>
<td>Holding cost</td>
<td>U(20,35)</td>
</tr>
<tr>
<td>Slope of the demand pattern</td>
<td>U(0.3,0.8)</td>
</tr>
<tr>
<td>Fixed cost of 1-liter fuel</td>
<td>1</td>
</tr>
<tr>
<td>CO2 emission cost of 1-liter fuel consumption*</td>
<td>46</td>
</tr>
<tr>
<td>Fuel consumption rate of vehicle k in 1 kilometer*</td>
<td>0.06</td>
</tr>
<tr>
<td>Fuel consumption rate of the truck in 1 kilometer*</td>
<td>0.1</td>
</tr>
</tbody>
</table>

*(Rabbani et al., 2018)*

5.1. Results for small-sized test problems

Some small-sized test problems are solved by BARON solver of GAMS 23.6 software, which gains the exact solution for the problem to validate the developed model. The performance of two proposed evolutionary algorithms (GA and HBA) are verified by comparison with GAMS results in small-sized test problems. Table 5 shows the comparison between evolutionary algorithms solutions and GAMS solution in 6 small-sized problems by using GAP formulation for maximization problems that is calculated as \[
\left(\frac{S_{GAMS} - S_{Alg}}{S_{Alg}}\right),
\]
where \(S_{GAMS}\) and \(S_{Alg}\)
are the objective function value of the exact method and proposed evolutionary algorithms, respectively. Also, the CPU time of the exact method and metaheuristic algorithms for the small-sized problem is reported in Table 5, which shows a faster speed of metaheuristic algorithms in achieving solutions. According to Table 5, the average GAP of HBA and GA are 0.00, 0.1, respectively. Figure 11 illustrates the HBA and GA algorithm GAP with exact method for 6 small-sized problems. The graph shows that the HBA has lower GAP with exact method in comparison with GA. So, HBA performs more efficient for small-sized problems in a shorter time.

Table 5. GAP and CPU time of exact method, HBA and GA for small-sized problem

<table>
<thead>
<tr>
<th>Problem</th>
<th>Data set</th>
<th>Exact method</th>
<th>HBA</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T  M  N</td>
<td>CPU Time</td>
<td>OFV</td>
<td>CPU Time</td>
</tr>
<tr>
<td>1</td>
<td>2  3  3</td>
<td>2780.39</td>
<td>311110.88</td>
<td>5.79</td>
</tr>
<tr>
<td>2</td>
<td>2  3  4</td>
<td>2840.93</td>
<td>312324.69</td>
<td>6.35</td>
</tr>
<tr>
<td>3</td>
<td>2  3  5</td>
<td>2850.49</td>
<td>427607.82</td>
<td>6.91</td>
</tr>
<tr>
<td>4</td>
<td>2  4  5</td>
<td>2894.10</td>
<td>477865.68</td>
<td>7.35</td>
</tr>
<tr>
<td>5</td>
<td>2  4  6</td>
<td>3104.44</td>
<td>488166.90</td>
<td>7.88</td>
</tr>
<tr>
<td>6</td>
<td>2  5  8</td>
<td>3170.75</td>
<td>595068.52</td>
<td>9.59</td>
</tr>
</tbody>
</table>

Figure 11. GAP of HBA and GA from the exact method for small-sized problems

5.2. Results for large-sized test problems

In this subsection, the validation and verification of proposed algorithms are considered by applying the 24 more test problems with random input data. In addition, verification and validation of HBA in the large-sized problem is evaluated by GA that is a well-known algorithm. Table 6 shows the CPU time, objective function, and comparison between HBA and GA in large-sized test problems. GAP formulation for comprising metaheuristic algorithms is computed as \( \frac{S_{HA} - S_{GA}}{S_{GA}} \), where \( S_{HA} \) and \( S_{GA} \) are the function value. The numbers in Table 6 are visualized by Figures. 12 to 14. Figure. 12 shows that on average the HBA reaches higher values of the objective function compared to GA for 30 test problems including both small-sized and large-sized. When the problem size increases, the difference between HBA and GA objective function values becomes considerable. While it was negligible for small-sized test problems.
Also, Figure 13 depicted the GAP between HBA and GA in medium-sized and large-sized problems, which increased for the last three large-sized test problems. Since GA is a well-known algorithm in the literature, it is used for comparison with HBA to show the acceptable performance of the HBA. Due to Figure 14, the HBA running times are always shorter than GA running times on average. So, HBA is able to achieve almost better solutions faster than GA. Therefore, the results show a better performance of HBA for the proposed model.

The abstract of the comparison between HBA and GA were shown in Figure 12 and Figure 14. Regarding the results of test problems, the proposed evolutionary HBA algorithm is an efficient algorithm for this model.

<table>
<thead>
<tr>
<th>Problem</th>
<th>T</th>
<th>M</th>
<th>N</th>
<th>OFV</th>
<th>CPU Time</th>
<th>Gap</th>
<th>OFV</th>
<th>CPU Time</th>
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</table>
Figure 12. Objective function value of HBA and GA

Figure 13. GAP between GA and HBA for large-sized problem

Figure 14. CPU time of HBA and GA
5.3. Application and Model verification by a small-sized case study
With solving a small-sized case taken from the real world using BARON solver as a powerful solver in solving Mixed Integer None-Linear Problems (MINLP) by GAMS software version 23.6, the model is validated, and its application is shown. The following section explains the features of the case study.

5.3.1 Case study explanation
Since the observed real-world problem inspires the proposed problem, in this part, the real case study is introduced. A company in the Semnan province of the Islamic Republic of Iran wants to decide about renting and operating, which DC is economical for the company. Also, determining the number of product trucks to each operated DC, allocating customers to them in the routing decisions, and inventory levels of each customer, periodically, are the goals of designing distribution system for this company. Minimization of the environmental impacts, including fuel consumption and CO2 emission, besides maximizing the company's profit by specification the optimal product price for customers from different regions is considered. The geographical position of Semnan province on the map of the Islamic Republic of Iran is shown in Figure 15.

The characteristics of the small case study (P1 problem) is presented in Table 7. The detailed parameters of the P0 problem, which is also used for validating the model with GAMS 23.6 are available at https://www.dropbox.com/s/dakzkmskhq00nfe/fn2e.xlsx?dl=0. Because the solving time for software lasts more than one hour, a relative gap is set equal to 0.001, and the maximum time limit for running is set equal to 2700 seconds. After this time limitation, GAMS 23.6 achieve a solution with a relative gap equal to 0.04478. The value of the objective function is 397272.940. The results of GAMS 23.6, including the number of trucks, opened DCs, and second stage routes are reported here in Table 8. The GAMS 23.6 solution is marked by green and blue color in Figure 15 and 16, respectively.

<table>
<thead>
<tr>
<th>Name</th>
<th>P</th>
<th>DC</th>
<th>N</th>
<th>T</th>
</tr>
</thead>
<tbody>
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<td>4</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period</th>
<th>factory</th>
<th>NT</th>
<th>Distribution centers</th>
<th>Order of customers in routs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3(opened)</td>
<td>3\rightarrow 5\rightarrow 7\rightarrow 8\rightarrow 9\rightarrow 6\rightarrow 3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3(opened)</td>
<td>3\rightarrow 9\rightarrow 7\rightarrow 8\rightarrow 5\rightarrow 6\rightarrow 3</td>
</tr>
</tbody>
</table>
Figure 15. The position of Semnan province in the country and the GAMS solution for period 1

Figure 16. The GAMS solution for period 2
The results of metaheuristic algorithms for the case study are shown in Figure.17, Figure. 18. The metaheuristic results are compared with exact method results for the case study in Table 9. Although the case study is in a small category, as mentioned previously, the exact method takes 45 minutes to find a solution with 0.04478 relative gaps. This is a long time, which is not acceptable in real situations. However, as can be seen in Table 9, both developed metaheuristic algorithms break the computation time to a few seconds with tiny tolerable gaps from the GAMS solution. Here is a suitable reason for developing metaheuristic algorithms for this problem. By ignoring the tiny gap from the GAMS solution, it can be observed that the HBA algorithm decreases running time by a factor of approximately 230.26, and the GA algorithm shortens it almost 395.95 times.

As managerial insights, it is better to use metaheuristic algorithms because they are more applicable in the real world with an ignorable gap and proper computation time. Since the value of the metaheuristic's objective function has little gaps with the value of GAMS objective function, the performance accuracy of metaheuristic algorithms is verified as a result.

### Table 9. The comparison between GAMS and metaheuristic algorithms for the case study

<table>
<thead>
<tr>
<th>GAMS</th>
<th>HBA</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective function value</td>
<td>Objective function value</td>
<td>Gap</td>
</tr>
<tr>
<td>397272.940</td>
<td>397154.866</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

![Figure 17. The GA process for the case study](image1)

![Figure 18. The HBA process for the case study](image2)

### 6. Conclusions and future research

In this study, the paradigm of designing a distribution network, which always has tried to minimize the operational costs, has been changed. Herein, the endeavor was maximizing the company's profit, by maximizing its revenue employing specifying regional prices for customers at different geographical areas with different economic conditions and also variations in their maximum willingness to pay for the product, then subtracting operational
costs from the revenue. The questions of this study during consecutive periods were: which DCs to operate, the number of trucks to each operated DC, in which order allocating customers to routes and the second stage vehicles, inventory levels at each customer, and optimal prices. This distribution system was designed in a way that environmental impacts, including fuel consumption and CO2 emission, are minimized as well. A quadratic mixed-integer programming model was presented for the green transportation location-inventory-routing problem integrated with dynamic regional pricing problem (GTLIRP+DRP) for a single product in this paper. Because solving the problem will take much time for the larger cases, two metaheuristic algorithms were customized for the problem, involving the genetic algorithm and the Hybrid Bees algorithm. According to the comparison of algorithms to GAMS result, the proposed HBA is more efficient than GA. Also, the CPU time of GA is greater than HBA, so the results showed that HBA is more powerful to find near-optimal solutions.

After that, by solving the case study from the real world by GAMS software, the application of the model was shown. However, since GAMS had long execution time even in the small size problem, distribution system managers are suggested that even in small real cases utilize customized metaheuristic algorithms which have significantly shorter running time without any remarkable difference in the objective function values. Some directions for further research could be considering other types of price response demand functions except linear price response demand function (\( D = \alpha P - \beta \)) for customers.

References


A green transportation location-inventory-routing problem by dynamic regional pricing


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