

An inventory model for non-instantaneous deteriorating items with imperfect quality, delay in payments and time value of money

M. Mahmoudinezhad¹, M. Ghoreishi¹, A. Mirzazadeh^{1,*}, A. Ghodrathnama¹

Abstract

In this paper, Economic Order Quantity (EOQ) based model for non-instantaneous deteriorating items with imperfect quality, permissible delay in payments and inflation is proposed. We adopt a time-dependent demand function. Also, the effects of time value of money are studied using the Discounted Cash Flow approach. Moreover, we assume that orders may contain a random proportion of defective items, which follow a known distribution and an inspection process is utilized to describe the defective proportion of the received lot. The mathematical model have been derived for obtaining the optimal number of cycle and the optimal inspection time so that the present value of total cost in a finite time horizon is minimized. An algorithm has been presented to find the optimal solution. Finally, numerical examples are provided to illustrate the solution procedure.

Keywords: EOQ, Non-instantaneous deteriorating items; Imperfect items; Inflation; permissible delay in payments.

Received: April 2014-04

Revised: July 2014-12

Accepted: August 2014-07

1. Introduction

In recent years, inventory problems for deteriorating items have been widely studied. In daily life, the deteriorating of goods is a frequent and common phenomenon. Fruit, foods, vegetables, cakes, sweets, and pharmaceuticals are a few examples of such goods. In general, deterioration is defined as the damage, spoilage, dryness, vaporization, and and loss of utility of the product, that result in the decrease of usefulness of the commodity. The first attempt to describe the optimal ordering policies for such items was made by Ghare and Schrader (1963). Deb and Chaudhri (1986) proposed inventory model with time-dependent deterioration rate. Goyal and Giri (2001) provided a detailed review of deteriorating inventory literatures. Chang et al. (2003) improved an EOQ model for deteriorating items under supplier trade credits linked to order quantity. Wu et al. (2006) proposed an inventory model for non-instantaneous deteriorating items considering stock-

* Corresponding Author.

¹ Department of Industrial Engineering, Karazmi University, Mofatteh Avenue, Tehran, Iran

dependent demand and partial backlogging. Yang et al. (2009) developed an inventory model for non-instantaneously deteriorating items with price dependent demand and shortages. Nakhai and Maihami (2011) developed the joint pricing and ordering policies for deteriorating items considering partial backlogging in which the demand is considered as a function of both price and time. Uthavakumar and Rameswari (2012) improved economic order quantity model for deteriorating items with time discounting. Das et al. (2012) considered a production-inventory model for a deteriorating item with stock-dependent demand under two storage facilities over a random planning horizon, which is assumed to follow exponential distribution with known parameter. Shah et al. (2013) considered an inventory system with non-instantaneous deteriorating items in which demand rate is a function of advertisement of an item and selling price. The objective is maximizing the total profit by determining optimal inventory and marketing parameters. Mishra and Singh (2013) improved an economic order quantity model for deteriorating items with power demand pattern and shortages partially backlogged. Shilpi et al. (2014) improved an economic production model for deteriorating items with ramp type demand rate and Weibull deterioration under inflationary conditions. Sicilia et al. (2014) developed a deterministic inventory system for items with a constant deterioration rate, time-dependent time and shortages.

Inflation and time value of money issues will have main effects in financial markets. Inflation also plays an important role for the optimal order policy and influences the demand of certain products. As inflation increases, the value of money goes down and erodes the future worth of saving and forces one for more current spending. As a result, the effect of inflation and time value of the money cannot be ignored in global economics. Buzacott (1975) first derived an EOQ model considering the inflationary effect on costs. Then consequently in the subsequent years, Misra (1979), Padmanavan and Vrat (1990), Hariga and Ben-daya (1996), Chen (1998), Moon and Lee (2000), Dey et al. (2004) and others worked in this area. Dey et al. (2008) improved the model for deteriorating items with time-dependent demand rate and interval valued lead-time under inflationary conditions. Mirzazadeh et al. (2009) consider stochastic inflationary conditions with variable probability density functions (pdfs) over the time horizon and the demand rate is dependent on the inflation under a finite replenishment rate, finite time horizon, deteriorating items and shortages. Yang (2010) considered an integrated inventory model with crashing cost and inflation with the objective of minimizing present value of the joint expected total cost. Mirzazadeh (2010) discussed an inventory model with time dependent inflation and demand rate is assumed to be inflation-proportional. Jaggi et al. (2011) considered a two-warehouse inventory model for deteriorating items with linear trend in demand and shortages under inflationary conditions. Roy and Chaudhuri (2011) investigated an EOQ model with ramp type demand under finite time horizon and inflation. Guria et al. (2013) considered an Inventory policy for an item with inflation induced purchase cost and selling price. Singh and Sharma (2013) discussed an integrated model under inflation condition in which production rate is assumed as a function of demand rate and customer demand rate and time-dependent. Taleizadeh and Nematollahi (2014) investigated the effects of time value of money and inflation on the optimal ordering policy in an inventory control system with back-ordering and delayed payments.

In framing the traditional inventory model, it was assumed that the retailer must be paid for the items as soon as the items were received. However, in practice, the supplier may provide the retailer many incentives such as a cash discount to motivate faster payment and stimulate sales, or a permissible delay in payments to attract new customers and increase sales. Usually, there is no charge if the outstanding amount is settled within the permitted fixed settlement period. Goyal (1985) improved a single-item inventory model considering a permissible delay in payments. Aggarwal and Jaggi (1995) extended Goyal's (1985) model to consider the deteriorating items. Salameh et al. (2003) examined the continuous review inventory model under permissible delays

in payments. Ouyang et al. (2006) presented the inventory model for non-instantaneous deteriorating items considering permissible delay in payments. Sana and Chaudhuri (2008) discussed a production–inventory model with quadratic demand pattern. Liao (2008) investigated the impact of the trade credit policy on the classical Economic Production Quantity (EPQ) model for an item subject to exponential decays. Chung and Liao (2009) discussed the optimum order quantity of the EOQ model with trade credit depending on the quantity ordered that using a discounted cash-flows (DCF) approach. Geetha and Uthayakumar (2010) proposed economic order quantity (EOQ) based model for non-instantaneous deteriorating items with permissible delay in payments and shortages. Sarkar (2012) extended an EOQ model for finite replenishment rate where demand and deterioration rate are both time-dependent under delay in payments. Recently Ouyang et al. (2012) studied the EOQ model with defective items and partially permissible delay in payments linked to order quantity using the AGM inequality to determine the optimal solution under various situations. Taleizadeh (2013) considered an EOQ problem under partial delayed payment and shortages that is combination of backorders and lost sales. Chen et al (2013) extended an economic order quantity model under conditionally permissible delay in. Bhunia et al. (2014) considered an inventory model for single deteriorating item with two separate warehouses, shortages and permissible delay in payments.

In the traditional economic order quantity models, the basic assumption is that 100% of received items are perfect. However, the lot sizes ordered may contain some defective items. Then the screening process is adopted to identify the imperfect items. It is supposed that a fraction, say α , of items received from a supplier or manufacturer are imperfect. The parameter α was either assumed to be of a fixed value or a random variable with probability distribution function, $f(\alpha)$, where $f(\alpha)$, can be either known, known but dependent on other unknown parameters, or unknown. There are several studies that consider the effect of imperfect quality on the inventory policy. Porteus (1986) and Rosenblatt and Lee (1986) Are some of the earliest ones that considered imperfect quality. Cheng (1991) investigated an EOQ model with demand-dependent unit production cost and imperfect production processes. Chan et al. (2003) proposed an economic production quantity (EPQ) model in which the imperfect (not necessarily defective) items could be sold at a lower price and the defective items could be either reworked or rejected. Liao (2007) studied imperfect production processes that require maintenance. They considered two states, namely the in-control and the out-of-control state of the production process. Jabber et al. (2009) discussed the classical EOQ model considering the concept of entropy cost, perfect and imperfect quality. Khan et al. (2011) proposed an EPQ model considering random defective rate and inspection errors with known probability distribution function. Hsu^a and Hsu^b (2013) developed an economic order quantity model with imperfect quality items, inspection errors, shortage backordering, and sales returns. Chung (2013) established the EOQ model considering defective items and partially permissible delay in payments linked to order quantity. It also uses the rigorous method of mathematics to derive the optimal solution procedure to locate the optimal solution. Moussawi-Haidar et al. (2014) considered a modified EOQ-type inventory model for a deteriorating item with unreliable supply and imperfect quality. As soon as an order is received, a retailer conducts a screening process to identify imperfect quality items.

In this article, we develop an appropriate inventory model for an economic ordering quantity (EOQ) model with non-instantaneous deteriorating items taking into account imperfect quality, permissible delay in payments and inflation. We assume that demand is time-dependent. In the traditional inventory model, it was assumed that the payment must be made to the manufacturer for items immediately after receiving the items. In practice, the manufacturer hopes to stimulate his products and so he will offer the retailer a delay period. During the permissible delay period, there is no interest charge. Hence, the retailer can earn the interest from sales revenue. Therefore, in order to incorporate the realistic conditions, the delay in payment should be considered. One

of the unrealistic assumptions of the EOQ model is that items stocked preserve their physical characteristics during their stay in inventory. Items in stock are subject to several possible risks, such as pilferage, breakage, evaporation, and obsolescence. In general, deterioration is defined as decay, damage, or spoilage. Moreover, in practice, most goods would have a span of maintaining quality or original condition, so, during this period, the deterioration does not occur. As a result, in the field of inventory management, it is necessary to incorporate the inventory problems for non-instantaneous deteriorating items. Moreover, in order to address the realistic situations, the effect of time value of money should be considered.

In addition, it is assumed that the manufacturer's production processes are imperfect and may produce defective items. Once the buyer receives the lot, a 100% screening process is conducted at a rate $\mu > D$. The screening process and demand proceed simultaneously. At the end of screening process, imperfect quality items are eliminated. Hence, a finite planning horizon inventory model for non-instantaneous deteriorating items with imperfect quality, permissible delay in payments and time-dependent demand rate is developed. The major objective is to determine the optimal screening time and the optimal number of cycle. This is the first work that has the above assumptions. An appropriate algorithm has been presented to find the optimal solution. Finally, a numerical examples is provided to illustrate the solution procedure.

The rest of the paper is organized as follows. In section 2, assumptions and notations are presented. We adopted the mathematical model in section 3. Next, section 4 presented an algorithm to find the optimal inventory variables. In section 5, the numerical examples are provided. Finally, discussion of the results as well as directions for future study is presented in section 6.

2. Notation and assumptions

The following notation and assumptions are used throughout the paper:

2.1. Notation

θ	deteriorating rate of the items $0 < \theta < 1$
D	demand rate
$a, b > 0$	Constant parameters of demand
$I_{ir}(t)$	inventory level i at time t , $i = 1, 2, 3$
I_e	interest earned per dollar per unit time
I_p	interest charged per dollar per unit time
p_1	selling price per unit, where $p > c_1$
t_2	replenishment time
t_1	inspection time
t_d	length of time in which the product exhibits no deterioration
M	trade credit period
q	order quantity
c_1	purchasing cost per unit
s	unit inspection cost
μ	inspection rate of item, $(1 - \alpha\gamma)\mu > D(t_1)$
h_r	inventory holding cost per unit
A	ordering cost
n	number of deliveries from the manufacturer
α	defective proportion, $0 < \alpha < 1$
r	constant representing the difference between the discount (cost of capital) and the inflation rate
H	length of planning horizon

T	duration of inventory cycle
N	number of cycles during the time horizon H
ETC	Expected value of total cost of the retailer

2.2. Assumptions

1. The supplier or manufacturer proposes a certain credit period M . During the time the account is not settled, the retailer deposits his/her generated sales revenue in an interest-bearing account with rate I_e . At the end of the trade credit period, the account is settled and the retailer starts paying for the interest charges on the items in stock with rate I_p .
2. $p_1 \geq c_1, I_p \geq I_e, c_1 I_p \geq p I_e$.
3. It is assumed that deterioration rate is non-instantaneous for the retailer and instantaneous for the manufacturer.
4. Demand rate is time dependent and linear, .e., $D(t) = a + bt; a, b > 0$.
5. Shortages are not allowed.
6. The defective items exist and the percentage defective (α) is a random variable having uniform *pdf* as $f(\alpha)$ with expected value $E(\alpha) = \int f(\alpha) d\alpha$.
7. The defective items treated as a single batch at the end of the inspection of retailer's 100% screening process and eliminated from inventory.
8. The screening and demand proceeds simultaneously.
9. The planning horizon is finite.
10. A constant deterioration rate is a fraction of the on-hand inventory. It deteriorates per unit time and there is no repair or replenishment of the deteriorated inventory during a replenishment cycle.
11. The retailer received the items in n equal size shipments from the manufacturer or supplier in each cycle.

The planning horizon divided to N equal cycle of T ($T = \frac{H}{N}$) and T divided to n equal replenishment time of t_2 ($t_2 = \frac{T}{n}$).

3. The model formulation

Here, an inventory model for deteriorating items with imperfect quality and permissible delay in payments under inflationary conditions is formulated. . Each received lot is having α percent defective items with a known probability density function, $f(\alpha)$. As soon as the retailer receives the order quantity, the 100% inspection process is done in which there is no inspection errors. During the screening process the demand occurs parallel to the screening process and is satisfied from the items which are found to be of perfect quality by the screening process. The defective items of αq are eliminated after the screening process at time t_1 as a single batch.

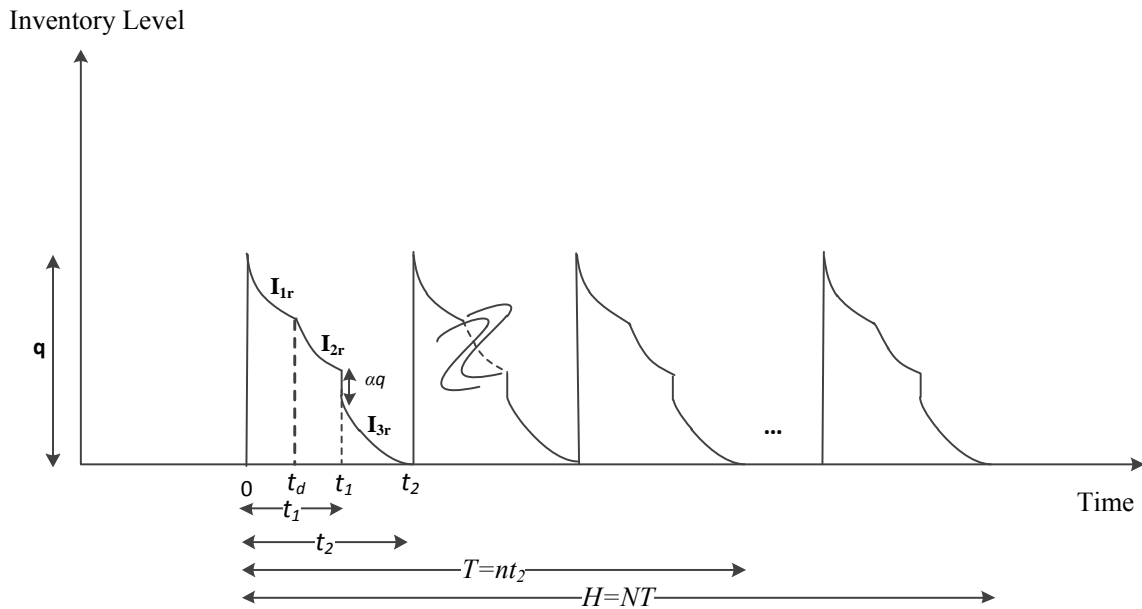


Figure 1: Behavior of the inventory level for the retailer.

Here, we considered an EOQ model for non-instantaneous deteriorating items, which will be described as follows. During the time interval $[0, t_d]$, the inventory level decreases due to demand only. At time t_d , deterioration starts, and thus, during the interval $[t_d, t_1]$, the inventory level decreases due to the demand and deterioration. Subsequently the inventory level drops to zero due to both demand and deterioration during the time interval $[t_1, t_2]$. The graphical representation of the model is shown in Figure 1.

Let $I_{ir}(t)$ be the inventory level at time t , ($0 \leq t \leq t_d$). The differential equation that describes the instantaneous states of $I_{ir}(t)$ over the period $[0, t_d]$ is given by

$$\frac{d I_{1r}(t)}{dt} = -D = -(a + bt) \quad (0 \leq t \leq t_d). \quad (1)$$

With the condition $I_{1r}(0) = q$, solving Equation (1) yields:

$$I_{1r}(t) = -\frac{1}{2}bt_d^2 - at_d + q \quad (0 \leq t \leq t_d). \quad (2)$$

In the time interval $[t_d, t_1]$, the system is affected by the combination of the deterioration and demand. Hence, the change of the inventory level at time t , $I_{2r}(t)$, is given by

$$\frac{d I_{2r}(t)}{dt} + \theta I_{2r}(t) = -D = -(a + bt) \quad (t_d \leq t \leq t_1). \quad (3)$$

With the condition $I_{2r}(t_d) = I_{1r}(t_d)$ solving Equation (3) yields

$$I_{2r}(t) = -\frac{a}{\theta} + \frac{b}{\theta^2} - \frac{bt}{\theta} - \frac{e^{-\theta t}(\frac{1}{2}bt_d^2 + at_d - q - \frac{a}{\theta} + \frac{b}{\theta^2} - \frac{bt_d}{\theta})}{e^{-\theta t_d}} \quad (t_d \leq t \leq t_1). \quad (4)$$

The differential equation below represents the inventory status in the time interval $[t_1, t_2]$

$$\frac{d I_{3r}(t)}{dt} + \theta I_{3r}(t) = -D = -(a + bt) \quad (t_1 \leq t \leq t_2). \quad (5)$$

With the condition $I_{3r}(t_1) = I_{2r}(t_1) - \alpha q$, solving Equation (5) yields

$$I_{3r}(t) = \frac{1}{2} \frac{1}{\theta^2 e^{-\theta t_d} e^{-\theta t_1}} \left(\left((-bt_d^2 - 2at_d + 2q)\theta^2 + (2bt_d + 2a)\theta - 2b \right) e^{-\theta t} - 2((a + bt)\theta - b)e^{-\theta t_d} \right) e^{-\theta t_1} - 2e^{-\theta t} \alpha q \theta^2 e^{-\theta t_d} \quad (t_1 \leq t \leq t_2). \quad (6)$$

At $t = t_2$, $I_{3r}(t) = 0$, Equation (6) gives order quantity as follows:

$$q = \frac{1}{\theta^2 e^{-\theta t} (e^{-\theta t_1} - \alpha e^{-\theta t_d})} \left(\left(t_d \left(\frac{1}{2} bt_d + a \right) \theta^2 + (-bt_d - a)\theta + b \right) e^{-\theta t} + e^{-\theta t_d} ((a + bt)\theta - b) \right) e^{-\theta t_1}. \quad (7)$$

Now, we can obtain the present value of inventory costs, which consists of the following elements:

1. *OC*. The present worth of the ordering cost

The ordering cost in each cycle time is done at the beginning of each cycle T . The present value of ordering cost for the first cycle is k , which is a constant value.

$$OC = k. \quad (8)$$

2. *IC*. The present worth of the inspection cost

The present worth of inspection cost occurs during the time interval $[0, t_1]$, but we assume it is at the middle of t_1 and is given by

$$IC = sq \cdot e^{-\left(\frac{1}{2}\right)r \cdot t_1}. \quad (9)$$

3. *PC*. The present worth of the purchasing cost

We assume this cost occurs at the beginning of t_2 .

$$PC = c_1 q. \quad (10)$$

4. *HC*. The present worth of holding cost

The present worth of holding cost can be expressed as

$$HC = h_r \left[\int_0^{t_d} I_{1r}(t) \cdot e^{-r \cdot t} dt + e^{-r \cdot t_d} \int_{t_d}^{t_1} I_{2r}(t) \cdot e^{-r \cdot t} dt + e^{-r \cdot t_1} \int_{t_1}^{t_2} I_{3r}(t) \cdot e^{-r \cdot t} dt \right]. \quad (11)$$

5. The present value of interest payable.

We need to consider cases where the length of the credit period is longer or shorter than the length of time in which the product exhibits no deterioration (t_d) and the length of the inspection time (t_1). Thus, we calculate the present value of interest payable for the items kept in stock under the following three cases.

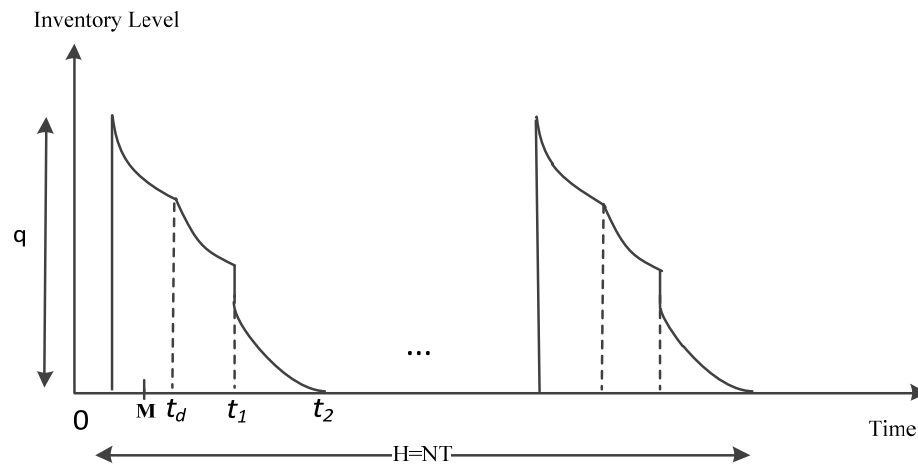


Figure 2: Case 1, $0 < M \leq t_d$

Case 1: The delay time of payments occurs before deteriorating time or $0 < M \leq t_d$ (see Figure 2).

In this case, payment for items is settled and the retailer starts paying the interest charged for all unsold items in inventory with rate I_p . Thus, the present value of interest payable during $[0, t_2]$, is given as follows

$$IP_1 = c_1 I_p \left\{ e^{-r \cdot M} \int_M^{t_d} I_{1r}(t) \cdot e^{-r \cdot t} dt + e^{-r \cdot t_d} \int_{t_d}^{t_1} I_{2r}(t) \cdot e^{-r \cdot t} dt + e^{-r \cdot t_1} \int_{t_1}^{t_2} I_{3r}(t) \cdot e^{-r \cdot t} dt \right\}. \quad (12)$$

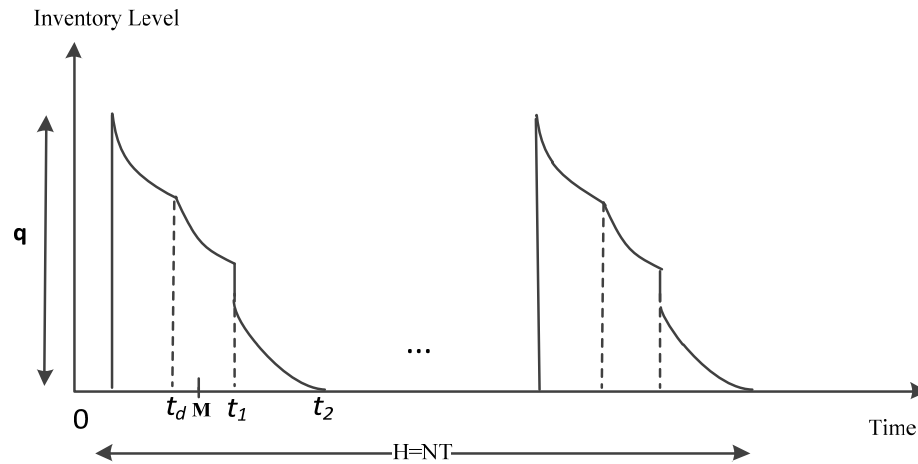


Figure 3: Case 2, $t_d < M \leq t_1$

Case 2: The delay time of payments occurs after deteriorating time and before inspection time; that is, $t_d < M \leq t_1$ (see Figure 3).

The conditions of this case are similar to those for case 1. Thus, the present value of interest payable during $[0, t_2]$, is given as follows

$$IP_2 = c_1 I_p \left\{ e^{-r \cdot M} \int_M^{t_1} I_{2r}(t) \cdot e^{-r \cdot t} dt + e^{-r \cdot t_1} \int_{t_1}^{t_2} I_{3r}(t) \cdot e^{-r \cdot t} dt \right\}. \quad (13)$$

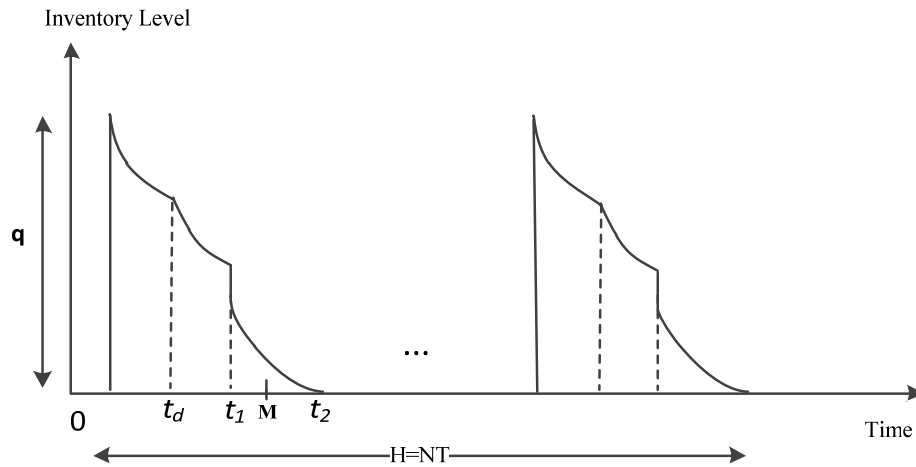


Figure 4: Case 3, $t_1 < M \leq t_2$

Case 3: The delay time of payments occurs after inspection time and before the replenishment time or $t_1 < M \leq t_2$ (see Figure 4).

In this case, the retailer starts paying the interest for the items in stock from time M to t_2 with rate I_p . Hence, the present value of interest payable during $[0, t_2]$, is as given below

$$IP_3 = c_1 I_p \left\{ e^{-r \cdot M} \int_M^{t_2} I_{3r}(t) \cdot e^{-r \cdot t} dt \right\}. \quad (14)$$

6. The present value of interest earned.

There are different ways to tackle the interest earned. Here we use the approach used by Geetha and Uthayakumar (2010). We assume that during the time when the account is not settled, the retailer sells the goods and continues to accumulate sales revenue and earns interest with rate I_e . Therefore the interest earned during each $[0, t_2]$, is as given below for the three different cases.

Case 1: The delay time of payments occurs before deteriorating time or $0 < M \leq t_d$.

$$IE_1 = p_1 I_e \int_0^M t \cdot D(t) \cdot e^{-r \cdot t} dt. \quad (15)$$

Case 2: The delay time of payments occurs after deteriorating time and before production period time; that is $t_d < M \leq t_1$.

$$IE_2 = p_1 I_e \int_0^M t \cdot D(t) \cdot e^{-r \cdot t} dt. \quad (16)$$

Case 3: The delay time of payments occurs after production period time and before duration of inventory cycle or $t_1 < M \leq t_2$.

$$IE_2 = p_1 I_e \int_0^M t \cdot D(t) \cdot e^{-r \cdot t} dt. \quad (17)$$

Therefore, the present value of the retailer's total cost over finite planning horizon, denoted by $TC(t_1, t_2, T, N)$, is given by

$$TC(t_1, t_2, T, N) = \begin{cases} \sum_{i=0}^N (OC + \sum_0^n (IC + PC + HC + IP_1 - IE_1) \cdot e^{-r \cdot i \cdot t_2}) \cdot e^{-r \cdot i \cdot T} & 0 < M \leq t_d. \\ \sum_{i=0}^N (OC + \sum_0^n (IC + PC + HC + IP_2 - IE_2) \cdot e^{-r \cdot i \cdot t_2}) \cdot e^{-r \cdot i \cdot T} & t_d < M \leq t_1. \\ \sum_{i=0}^N (OC + \sum_0^n (IC + PC + HC + IP_3 - IE_3) \cdot e^{-r \cdot i \cdot t_2}) \cdot e^{-r \cdot i \cdot T} & t_1 < M \leq t_2. \end{cases} \quad (18)$$

The value of the variables t_2, T can be replaced by the equations $T = H/N$ and $t_2 = T/n$ and we will use Maclaurin's approximation for $\sum_{i=0}^n e^{-r \cdot i \cdot t_2} \cong \frac{(1 - e^{-r \cdot n \cdot t_2})}{(1 - e^{-r \cdot t_2})}$ and $\sum_{i=0}^N e^{-r \cdot i \cdot T} \cong \frac{(1 - e^{-r \cdot N \cdot T})}{(1 - e^{-r \cdot T})}$. Thus, the objective of this article is to determine the values of t_1 and N , that minimize $TC(t_1, N)$ subject to $0 \leq t_1 \leq t_2$, where N is a discrete variable and t_1 is continuous variable. Since α is a random variable with known *pdf* of $f(\alpha)$, the expressed value of TC , i.e., ETC , becomes:

$$\text{Min } ETC(t_1, N) \quad \text{st: } 0 \leq t_1 \leq t_2. \quad (19)$$

However, since $ETC(t_1, N)$ is a very complicated function due to high-power expressions in the exponential function, it is difficult to show analytically the validity of the sufficient conditions. Hence, if more than one local minimum value exists, we would attain lowest of the local minimum values over the regions subject to $0 \leq t_1 \leq t_2$. The lowest value is referred to as the global minimum value of $ETC(t_1, N)$. So far, the procedure is to locate the optimal values of t_1 when N is fixed. Since N is a discrete variable, the following algorithm can be used to determine the optimal values of t_1 , and N .

4. The optimal solution procedure

The objective function has two variables. The number of cycles (N) is a discrete variable and the length of the inspection time (t_1) is a continuous variable. We have used the following algorithm to obtain the optimal amount of N and t_1 for case 1, $0 < M \leq t_d$.

Step 1 Let $N = 1$.

Step 2 Take the partial derivatives of $ETC(t_1, N)$ with respect to t_1 , and equate the results to zero. Then calculate the ETC .

Step 3 Add one unit to N and repeat *step 2* for new N . If there will be no decrease in the new ETC , then show the previous one which has the minimum value.

The necessary condition for minimizing $ETC(t_1, N)$ for a given positive integer of N is:

$$\frac{\partial ETC}{\partial t_1} = 0. \quad (20)$$

If the objective function was concave, the following sufficient conditions must be satisfied:

$$\frac{\partial^2 ETC}{\partial^2 t_1} > 0. \quad (21)$$

It is difficult to show the validity of the above sufficient conditions, analytically, due to involvement of a high-power expression of the exponential function. However, it can be assessed numerically in the following illustrative example.

5. Numerical examples

The preceding algorithm can be illustrated using the numerical example. The results can be found by applying the Maple 16.

Example 1. The parameters of this example, are given as follows:

$D(t) = a + bt = 1 + 6t$, $\theta = 0.04$, $c_1 = \$30/\text{per unit/per unit time}$, $h_r = \$4/\text{per unit/per unit time}$, $A = \$300/\text{per order}$, $\alpha \sim u(0, 0.04)$, $\mu = 10 \text{ units/per unit time}$, $s = \$6/\text{per unit/per unit time}$, $h_m = \$9/\text{per unit/per unit time}$, $r = 0.04$, $H = 10 \text{ unit time}$, $n = 4 \text{ delivery number/per order}$, $p_1 = 80$, $I_p = 0.06$, $I_e = 0.03$, $M = 0.1$, $t_d = 0.2$,

$$f(\alpha) = \begin{cases} 25, & 0 \leq \alpha \leq 0.04 \\ 0, & \text{otherwisw} \end{cases}$$

From Table 1, if all the conditions and constraints are satisfied, the optimal solution can be derived. In this example, the minimum present value of total cost is found in the 3th cycle. Therefore, the optimal solution is as follows:

$$t_1^* = 0.3293, N^* = 3, t_2^* = 0.84$$

By substituting the optimal values of N^* and t_1^* to Equation (21), it will be shown that PWTP is strictly concave:

$$\frac{\partial^2 ETC}{\partial^2 t_1} = 9.632754572$$

Table 1: Optimal solution of numerical the example.

N	ETC	q	t_1	t_2
2	1971.2099	6.26011	0.4777	1.25
3*	1965.4894*	3.04046*	0.3293*	0.84*
4	1975.7171	1.862665	0.2536	0.625

*Optimal solution

Example 2 This example is based on the following data.

$D(t) = a + bt = 2 + 4t$, $\theta = 0.06$, $c_1 = \$30/\text{per unit/per unite time}$, $h_r = \$4/\text{per unit/per unite time}$, $A = \$200/\text{per order}$, $\alpha \sim u(0, 0.04)$, $\mu = 10 \text{ units/per unit time}$, $s = \$8/\text{per unit/per unite time}$, $h_m = \$9/\text{per unit/per unite time}$, $r = 0.04$, $H = 10 \text{ unit time}$, $n = 4 \text{ delivery number/per order}$, $p_1 = 70$, $I_p = 0.06$, $I_e = 0.03$, $M = 0.1$, $t_d = 0.2$

$$f(\alpha) = \begin{cases} 25, & 0 \leq \alpha \leq 0.04 \\ 0, & \text{otherwisw} \end{cases}$$

According to the computational results shown in Table 2, the optimal solution is as follows:

$$t_1^* = 0.3271, N^* = 4, t_2^* = 0.625$$

By substituting the optimal values of N^* and t_1^* to Equation (21), it will be shown that PWTP is strictly concave:

$$\frac{\partial^2 ETC}{\partial^2 t_1} = 7.497892702$$

Table 2: Optimal solution of numerical the example.

N	ETC	q	t_1	t_2
3	1820.1797	3.2126	0.3985	0.84
4*	1805.6314*	2.1187*	0.3271*	0.625*
5	1867.4719	1.5573	0.2838	0.5

*Optimal solution

6. Conclusion

In this context, we studied an *EOQ* model for non-instantaneous deteriorating items with imperfect quality, permissible delay in payments and inflation. The demand is time-dependent. Also, if we ignored inflation and time value of money the optimal present value of total cost is overstated. In the competitive marketing environment, every supplier wants to sell more items to earn more revenues, as a result, most of the suppliers offer a delay period to encourage the retailer to buy more items. Before the end of the delay period, the retailer can sell his products, accumulates revenue and earns interest. Thus, the delay in payment by the supplier is a kind of price discount which encourages the retailers to increase their order quantity. In addition, it is assumed that the manufacturer's production processes are imperfect and may produce defective

items. To the author's best knowledge, such a type of model has not yet been discussed in the existing literature.

An algorithm is presented for deriving the optimal replenishment policy that wants to minimize the present value of total cost. Finally, numerical examples are provided to illustrate the algorithm and the solution procedure. This paper can be extended in several ways. For instance: price-dependent demand rate, variable deterioration rate, shortages with full or partial backlogging, quantity discounts, inspection errors and multiple products.

References

1. Ghare P.M., Schrader G.H., A model for exponentially decaying inventory system, *International Journal of Productions Research*, 21: 449-460.
2. Deb M., Chaudhuri K.S., 1986, An EOQ model for items with finite rate of production and variable rate of deterioration, *Opsearch* 23: 175-181.
3. S.K. Goyal, 2001, B.C. Giri, Recent trends in modeling of deteriorating inventory, *European Journal of Operational Research* 134 1-16
4. Chang, C.T., Ouyang, L.Y., Teng, J.T., 2003. An EOQ model for deteriorating items under supplier credits linked to ordering quantity. *Applied Mathematical Modeling* 27, 983–996.
5. Wu K.-S., Ouyang L.-Y., Yang C.-Te., 2006, An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging, *International Journal of Production Economics* 101 : 369-384.
6. Yang C.T., Ouyang L.Y., Wu H.H., 2009, Retailers optimal pricing and ordering policies for non-instantaneous deteriorating items with price-dependent demand and partial backlogging, *Mathematical Problems in Engineering* 1-18.
7. Nakhai K. I., Maihami R., 2011 “Joint pricing and inventory control for deteriorating items with partial backlogging,” *International Journal of Industrial Engineering and Production Management*, 21: 167–177.
8. Das D., Kar MB., Roy A., Kar S., 2012, Two-warehouse production model for deteriorating inventory items with stock-dependent demand under inflation over a random planning horizon. *Central European Journal of Operational Research*, 20(2), 251-280.
9. Uthavakumar, R., Rameswari, M. 2012. Economic order quantity for deteriorating items with time discounting. *The International Journal of Advanced Manufacturing Technology*, 58(5-8), 817-840.
10. Shah N.H., Soni H.N., Patel K.A., 2013 Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. *Omega* 41:421–430.
11. Mishra S.S., Singh P. K., 2013. Partial Backlogging EOQ Model for Queued Customers with Power Demand and Quadratic Deterioration: Computational Approach. *American Journal of Operational Research*, 3(2), 13-27.
12. Shilpi P., Mahapatra G.S., Samanta G.P., 2014. An EPQ model of ramp type demand with Weibull deterioration under inflation and finite horizon in crisp and fuzzy environment. *International Journal of Production Economics*. 156: 159-166.
13. Sicilia J., Rosa M., Acosta J., Pablo D., 2014, An inventory model for deteriorating items with shortages and time-varying demand. *Journal of Industrial Engineering International*. 9:4, 1-5.
14. Buzacott J.A., 1975, Economic order quantities with inflation, *Operational Research Quarterly*. 26 553–558.
15. Moon I, Lee S., 2000 The effects of inflation and time-value of money on an economic order quantity model with a random product life cycle. *European Journal of Operational Research* 125: 140–153.
16. Chen J.M., 1998, An Inventory model for deteriorating items with time-proportional demand and shortages under inflation and time discounting, *International Journal of production Economics*. 55 21–30.
17. Dey J.K., Kar S., Maiti M., 2004, An EOQ model with fuzzy lead time over a finite time horizon under inflation and time value of money, *Tamsui Oxford Journal of Management Sciences* 20: 57–77.
18. Padmanavan G., Vrat P., 1990, Analysis of multi-item inventory systems under resource constraints; a non-linear goal programming approach, *Engineering Costs and production Economics*. 20 : 121–127.

19. Hariga M., Ben-daya M., 1996, Optimal time-varying lot sizing models under inflationary conditions, *European Journal of Operational Research*, 89 : 313–325.
20. Misra RB. 1979, A note on optimal inventory management under inflation. *Naval Research Logistics*. Q., 26:161–165.
21. Dey J.K., Mondal S.K., Matti M., 2008, Two shortage inventory problem with dynamic demand and interval valued lead time over finite time horizon under inflation and time-value of money. *European Journal of Operational Research*, 185:170–194.
22. Mirzazadeh A., Seyed-Esfehani M.M., Fatemi-Ghomi S.M.T., 2009, An inventory model under uncertain inflationary conditions, finite production rate and inflation-dependent demand rate for deteriorating items with shortages, *International Journal of Systems Science* 40: 21–31.
23. Yang M.F., 2010 Supply chain integrated inventory model with present value and dependent crashing cost is polynomial. *Mathematical and Computer Modeling* 51: 802-80.
24. Mirzazadeh A., 2010, Effects of variable inflationary conditions on an inventory model with inflation-proportional demand rate, *Journal of Applied Sciences* 10: 551–557.
25. Jaggi C.K., Khanna A., Verma P., 2011, Two-warehouse partial backlogging inventory model for deteriorating items with linear trend in demand under inflationary conditions, *International Journal of Systems Science*. 42: 1185–1196.
26. Roy T., Chaudhuri K.S., 2011. A finite time horizon EOQ model with ramp type demand under inflation and time-discounting, *International Journal of Operational Research*, 11(1), 100-118.
27. Taheri-Tolgari J, Mirzazadeh A, Jolai F. 2012, An inventory model for imperfect items under inflationary conditions with considering inspection errors. *Comput. Math. Appl.*;63:1007–1019.
28. Guria, A., Das, B., Mondal, S., Maiti, M. 2013. Inventory policy for an item with inflation induced purchasing price, selling price and demand with immediate part payment. *Applied Mathematical Modelling*, 37 (1-2), 240-257.
29. Singh S.R., Sharma S., 2013 An integrated model with variable production and demand rate under inflation. *Procedia Technology* 10:381 – 391.
30. Taleizadeh A.A., Nematollahi M., 2014, An inventory control problem for deteriorating items with back-ordering and financial considerations. *Applied Mathematical Modelling* 38:93–109.
31. Goyal, S.K., 1985. EOQ under conditions of permissible delay in payments. *Journal of the Operational Research Society* 36, 335–338.
32. Aggarwal, S.P., Jaggi, C.K., 1995. Ordering policies of deteriorating items under permissible delay in payments. *Journal of the Operational Research Society* 46, 658–662.
33. Salameh, M.K., Abboud, N.E., El-Kassar, A.N., Ghataas, R.E., 2003. Continuous review inventory model with delay in payments. *International Journal of Production Economics* 85, 91–95.
34. Ouyang L.Y., Wu K.S., Yang C.T., 2006, A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments. *Computer and Industrial Engineering*, 51: 637–651.
35. S. Sana, K.S. Chaudhuri, 2008, A deterministic EOQ model with delays in payments and price-discount offers, *European Journal of Operational Research*. 184: 509–533.
36. Liao JJ., 2008, An EOQ model with non-instantaneous receipt and exponentially deteriorating items under two-level trade credit. *International Journal of Production Economics* 113,852–861.
37. Chung K.J., Liao JJ. 2009. The optimal ordering policy of the EOQ model under trade credit depending on the ordering quantity from the DCF approach. *European Journal of Operational Research* 196: 563–568.
38. Geetha K.V., Uthayakumar R., 2010. Economic design of an inventory policy for non-instantaneous deteriorating items under permissible delay in payments. *Journal of Computational and Applied Mathematics* 233: 2492-2505.
39. Sarkar B., 2012, An EOQ model with delay in payments and time varying deterioration rate. *Mathematical and Computer Modeling* 55:367–377.
40. Ouyang L.Y., Chang C.T., Shum P., 2012, The EOQ with defective items and partially permissible delay in payments linked to order quantity derived algebraically, *Central European Journal of Operations Research*. 20 (1): 141–160.
41. Taleizadeh A.A., Pentico D.W., Jabalameli M.S., Aryanezhad M., 2013, An EOQ model with partial delayed payment and partial backordering. *Omega* 41: 354–368.

42. Chen S.C., Cárdenas-Barrón L.E., Jinn-Tsair Teng J.T., 2013, Retailer's economic order quantity when the supplier offers conditionally permissible delay in payments link to order quantity. *International journal of production Economics*. 155:284-291.
43. Bhunia A.K., Jaggi C.K., Sharma A., Sharma R., 2014. A two-warehouse inventory model for deteriorating items under permissible delay in payment with partial backlogging. *Applied Mathematics and Computation* 232: 1125–1137.
44. Porteus E., 1986. Optimal lot sizing, process quality improvement, and setup cost reduction. *Operations Research* 34: 137–144.
45. Rosenblatt M.J., Lee H.L., 1986. Economic production cycles with imperfect production processes. *IIE Transactions* 18, 48–55.
46. Cheng C.E., 1991. An economic order quantity model with demand-dependent unit production cost and imperfect production processes. *IIE Transactions* 23(1) 23–28.
47. Chan W.M., Ibrahim R.N., Lochert P.B., 2003. A new EPQ model: integrating lower pricing, rework and reject situations. *Production Planning and Control* 14 (7), 588–595.
48. Liao G.L., 2007. Optimal production correction and maintenance policy for imperfect process. *European Journal of Operational Research* 182 (3), 1140–1149.
49. Jaber M.Y., Bonney M., Moualek I., 2009. An economic order quantity model for an imperfect production process with entropy cost. *International Journal of Production Economics*, 118, 26–33.
50. Khan M., Jaber M.Y., Bonney M., 2011. An economic order quantity (EOQ) for items with imperfect quality and inspection errors. *International Journal of Production Economics* 133: 113–118.
51. Chung K.J., 2013, The EOQ model with defective items and partially permissible delay in payments linked to order quantity derived analytically in the supply chain management. *Applied Mathematical Modeling* 37: 2317–2326.
52. Hsu J.T., Hsu L.F., 2013. An EOQ model with imperfect quality items, inspection errors, shortage backordering, and sales returns. *International Journal of Production Economics* 143: 162–170.
53. Moussawi-Haidar L., Salameh M., Nasr W. 2014. Effect of deterioration on the instantaneous replenishment model with imperfect quality items. *Applied Mathematical Modeling*, 38(24): 5956-5966.