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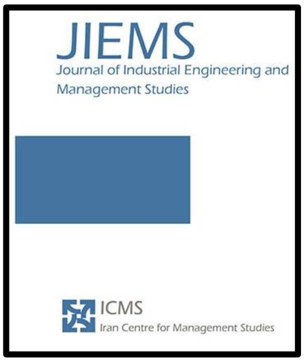

The Optimal Age-based Replacement Policy for Systems Subject to Shocks

B. Khorshidvand[1], A. Ayough[2,*], A. Alem Tabriz[3]

Abstract

In this article, two different systems subject to shocks occurring based on a non-homogeneous Poisson process (NHPP) are analyzed. Type –I system is consisted of a single unit and type –II system is consisted of two parallel units in which both units operate identically and simultaneously. In type –I system occurrence of a shock causes system stopping and consequently will be received minimal repairs. Also this system is replaced preventively at time Ψ, or at time less than Ψ due to probable failure. In type –II system a shock to each units leads to unit stopping and accordingly the unit receives minimal repairs and other unit receives preventive maintenance services with no system stop. Simultaneously, this system is replaced at time Ψ or at times less than Ψ preventively, due to failure of both units. Systems will be replaced with new and the same types when minimizes total expected cost.

Keywords: Age based replacement, Non-homogeneous Poisson process (NHPP), Shock process, Log linear process (LLP), Minimal repairs

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1. Introduction

Replacement policy is an important issue that has been widely applied to the actual production settings. In general, there are two major policies, maintenance policy and replacement policy. Maintenance policy implies that an optimal maintenance time is based on mean down time or average working time. Replacement policy implies that an optimal replacement time interval exists after the long-time running of the system. Thus this means that in the real production, we should find out the optimal change time to make the loss of the system minimum. Age based policy is quite common and easy-to-implement in practice. Under a basic age based replacement an operating system is replaced at particular age or at failure, whichever occurs first. In the literature, developments on replacement models based on the shock have provided satisfactory

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results for the maintenance operation. But in these researches, shock is not considered as an independent parameter of failure. In this article, shock process is unavoidable and plays an important role in determining the optimum replacement age. The main contribution of this article is developing shock process with age based models by considering the structure of systems. These models are applied to industrial equipment like cutting tool, hydraulic structure, compressor blades and electronic board and other similar equipment. This article constructed as follows: literature are briefly reviewed in section 2. Model description such as shock process and maintenance model is given in the section 3. Assumptions and notations are explained in the section 4. Numerical examples for proofing model optimization and analysis cumulative functions are given in the section 5. Finally conclusions are drawn in the section 6.

2. Literature

The classical replacement policies proposed by Barlow and Hunter (1960), first researches following this work studied total replacement models such as constant interval replacement and age based replacement. Regarding a system subject to shocks which may cause system failures; Esary et al. (1973) studied the survival probability of a system with homogeneous Poisson process (HPP) shocks. A-Hameed and Proschan (1973) further considered a non-homogeneous Poisson process (NHPP) case. Boland and Proschan (1983) proposed the periodic replacement policy for a system subject to shocks and derived the sufficient conditions for the existence of the optimal policy. Yusuf and Ali have studied structural dependence replacement model for parallel system. Guo et al. (2013) have proposed a maintenance optimization for systems exposed to risks by using of log linear process. Sheu et al. (2013) have studied an extended replacement policy for a system subject to non-homogeneous pure birth shocks by using non-homogeneous pure birth Poisson process, and developed the model to optimize the number of repairs before replacement, as well. Marquez (2007) in his comprehensive review have explained total replacement models such as constant interval replacement and age based replacement models as well as shock based replacement, partial replacement, replacement with imperfect maintenance and inspection models. Huynh et al. (2011) have studied age based maintenance strategies with minimal repairs for systems considering degradation and shocks. There are other studies in the literature on the optimal replacement policies based on age and number of failures or combination age and failures number; Guan Jun Wang and Yuan Lin Zhang (2007) have proposed an optimal replacement policy by considering a series structure and assuming geometric process and developed this model to k dissimilar components, K. Yao and D. A. Ralescu (2013) have combined age based replacement policy with uncertainty considerations and treated the age of the system as uncertain variable instead of random variable, Yu-Hung Chien (2009) considered a system with continuous preventive maintenance and random lead times and provided a number-dependent replacement policy, Xu-Feng Zhao et al. (2010) have studied three kinds of replacement models and combined it with additive and independent damages, Ling Wang et al. (2008) have proposed a condition based order replacement policy for a gradually and stochastically deteriorating single unit system inspected periodically, Yen-Luan Chen (2012) presented an optimal bivariate preventive maintenance policy with NHPP shock model and cumulative damage model, David D. Hanagal and Rupali A. Kanade (2011) have studied optimal replacement policies based on number of downtimes for cold standby system when the lifetime and the repair time are dependent, also cold standby repairable system with priority in repair and use by considering an α-series process repair model have studied by M. Sreedhar et al. (2013), Yu-Hung Chien and Jih-An Chen (2011) have studied optimal maintenance policy for a system suffered damage in discrete time process and provided a preventive maintenance policy for a continuously running system over indefinitely long operation cycle, Ming Xu et al. (2012) have presented a replacement model for a non-repairable safety related system, Sophie Mercier and Hai Ha PHAM (2012) have proposed a preventive maintenance policy for a continuously monitored system modeled by a bivariate subordinator, A. Pak et al. (2007) have provided maintenance and replacement policies for protective devices with imperfect repairs, M. Yasin Ulukus et al. (2012) have provided a frame work for optimally replacing a system that degrades due to the influence of its random environment, where the objective is to minimize the total expected discounted cost over an infinite horizon, Chin-Chih Chang (2012) have studied a preventive replacement policy and imperfect maintenance model by considering random operating time.

3. Model description

3.1. Importance of model

The systems that are subject to external and internal shocks by occurring any of these shocks can be failure and then the minimal repairs to prevent larger and more costly downtime should be done. Given that the shocks have cumulative damage property and occur randomly, thus the minimal repairs with lower cost can be done.

3.2. Shock process

In this article, a cumulative shock model is considered to explain the shock process. The probabilities for the shock damages to occur in different time intervals are assumed to be independent. The log linear process (LLP) is very flexible and has been widely used to describe the occurrence of random events.

N (Ψ) is the number of events by time Ψ. A stochastic process is a non-homogeneous Poisson process for some small value h if:

(i) $N(0) = 0$

(ii) $\text{Non} - \text{overlaping increments are independent}$

(iii) $P(N(\Psi + h) - N(\Psi) = 1) = \Lambda_k(\Psi)h + o(h)$

(iv) $P(N(\Psi + h) - N(\Psi) > 1) = o(h)$

For all Ψ and where in little o notation $\lim_{h\to 0}\frac{o(h)}{h} = 0$

Thus, we consider stochastic shocks occur in a non-homogeneous Poisson process (NHPP).

To simulate this process by using of LLP, intensity function is

$$\Lambda(\Psi) = ke^{c\Psi};\;\; k \in (0,\infty), c \in (-\infty,\infty)$$

Let N (Ψ) demonstrate the number of shocks until time Ψ, then the expected number of shocks until time Ψ, denoted by D (Ψ) is given by

$$D(\Psi) = E[N(\Psi)] = \int_0^{\Psi} ke^{cs}ds = \begin{cases} \frac{k}{c}\left(e^{c\Psi} - 1\right) & \forall\, c \neq 0 \\ k\Psi & \forall\, c = 0 \end{cases}$$

Also, the probability distribution of N (Ψ) is

$$P(N(\Psi) = n) = \frac{(D(\Psi))^n e^{-D(\Psi)}}{n!}$$

3.3. Age based replacement policy

In this case, the preventive replacement is done after equipment reaches a certain operating time-age Ψ. In case of equipment failure a corrective replacement (CR) is done and the next preventive replacement (PR) is scheduled after Ψ units of time. We again want to calculate the best Ψ which minimizes total expected cost per unit time.

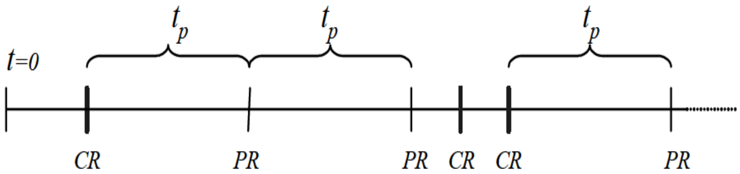

Figure1: Age based policy over time

4. Model assumptions and notations

4.1. Assumptions

a) When Ψ=0, the systems begin to work and under internal and external shocks constantly.
b) If the systems work until time Ψ properly, replace preventively.
c) If the systems break before time Ψ, replace correctively.
d) If in time interval [0, Ψ] unit suffers any internal or external shocks, minimal repairs done.
e) Certainly for type –II system, when a unit receives minimal repairs other unit receives preventive maintenance.
f) Preventive and corrective replacement, minimal repairs and preventive maintenance time are negligible.

4.2. Notations

R (Ψ): The survival or reliability function and means that probability of operation until Ψ.
F (Ψ): The failures function and mean that probability of failure until Ψ.
f(x): The Probability density function of failure.
D (Ψ): The expected number of shocks until time Ψ.
μ (Ψ): Certainly for type –I system, the expected length of the failure cycle.
V (Ψ): The expected time of system operation until time Ψ.
CR: The preventive replacement cost rate.
CF: The corrective replacement cost rate.
CM: The minimal repairs cost rate.
CP: The preventive maintenance cost rate.
K (Ψ): The expected cost of system until time Ψ.
C (Ψ): Total expected cost per unit time.

For type –I system with a single unit, long term average cost rate is

$$C(\Psi) = \frac{K(\Psi)}{V(\Psi)} \tag{1}$$

In this system, the expected cost until time Ψ is

$$K(\Psi) = [CR(R(\Psi))] + [CF(F(\Psi))] + [CM(D(\Psi))]$$

$$K(\Psi) = \left[CR\left(\int_{\Psi}^{\infty} f(x)dx\right)\right] + \left[CF\left(\int_{0}^{\Psi} f(x)dx\right)\right] + \left[CM\left(\int_{0}^{\Psi} ke^{cs}ds\right)\right] \tag{2}$$

Also, the length of failure cycle can be estimated calculating the expected value of the failure distribution now truncated in Ψ as follows

$$\mu(\Psi) = \frac{\int_0^{\Psi} xf(x)dx}{F(\Psi)}$$

Thus, expected time of system operation until time Ψ is

$$V(\Psi) = [\Psi(R(\Psi))] + [\mu(\Psi)(F(\Psi))]$$

$$V(\Psi) = \left[\Psi\left(\int_{\Psi}^{\infty} f(x)dx\right)\right] + \left[\frac{\int_0^{\Psi} xf(x)dx}{F(\Psi)}\left(\int_0^{\Psi} f(x)dx\right)\right] \tag{3}$$

Finally, taking into account equations (1), (2) and (3) average cost rate is

$$C(\Psi) = \frac{\left[CR\left(\int_{\Psi}^{\infty} f(x)dx\right)\right] + \left[CF\left(\int_0^{\Psi} f(x)dx\right)\right] + \left[CM\left(\int_0^{\Psi} ke^{cs}ds\right)\right]}{\left[\Psi\left(\int_{\Psi}^{\infty} f(x)dx\right)\right] + \left[\frac{\int_0^{\Psi} xf(x)dx}{F(\Psi)}\left(\int_0^{\Psi} f(x)dx\right)\right]}$$

And, for type –II system with two parallel units, we have

$$C(\Psi) = \frac{K(\Psi)}{V(\Psi)}$$

By considering two parallel units, the expected cost of system until time Ψ is

$$K(\Psi) = CR[R(\Psi)]^2 + [2(D(\Psi))(CM + CP)] + CF[F(\Psi)]^2$$

$$K(\Psi) = CR\left[\int_{\Psi}^{\infty} f(x)dx\right]^2 + [2\int_0^{\Psi} ke^{cs}ds\,(CM + CP)] + CF\left[\int_0^{\Psi} f(x)dx\right]^2 \tag{4}$$

Also, in this case the system works if at least one of the units works, thus the expected time of system operation until time Ψ

$$V(\Psi) = \int_0^{\Psi}\left[1 - (F(t))^2\right]dt$$

$$V(\Psi) = \int_0^{\Psi}\left[1 - \left(\int_0^{t} f(x)dx\right)^2\right]dt \tag{5}$$

Finally, taking into account equations (1), (4) and (5) average cost rate is

$$C(\Psi) = \frac{CR\left[\int_{\Psi}^{\infty} f(x)dx\right]^2 + [2\int_0^{\Psi} ke^{cs}ds(CM + CP)] + CF\left[\int_0^{\Psi} f(x)dx\right]^2}{\int_0^{\Psi}\left[1 - \left(\int_0^{t} f(x)dx\right)^2\right]dt}$$

5. Numerical examples

5.1. Type –I system

In type –I system a shock causes system will be stopped and received minimal repairs. Also this system replaces preventively at time Ψ, or at time less than Ψ due to failure with a same system. In this system, assume the cumulative function followed the Weibull distribution. In this distribution λ is scale parameter and β is shape parameter. If β>1 there is an aging process, if β=1 there is a constant process and if β<1 there is a mortality process. We consider λ=1 and β=2, β=1

and β=0.5 respectively. Also we have CR=20, CF=35, CM=8 and shock parameters are k=0.5 and c=0.07.
Weibull probability density function is

$$f(x,\lambda,\beta)=\frac{\beta}{\lambda}\left(\frac{x}{\lambda}\right)^{\beta-1}e^{-\left(\frac{x}{\lambda}\right)^{\beta}}$$

By using MATLAB we have

Table1: Total expected Cost per unit time for β=2

Ψ	0.1	0.2	0.3	0.4	0.5	0.6
C(Ψ)	206.192	108.397	77.269	62.795	54.963	50.427
Ψ	0.7	0.8	0.9	1.0	**1.1**	1.2
C(Ψ)	47.746	46.196	45.371	45.024	**44.998**	45.186

Table2: Total expected Cost per unit time for β=1

Ψ	0.1	0.2	0.3	0.4	0.5	0.6
C(tp)	229.385	129.778	96.845	80.587	71.003	64.760
Ψ	0.7	0.8	0.9	1.0	1.1	1.2
C(tp)	60.429	57.286	54.964	53.194	51.835	50.786
Ψ	1.3	1.4	1.5	1.6	1.7	1.8
C(tp)	49.976	49.356	48.888	48.545	48.306	48.154
Ψ	1.9	**2**	2.1	2.2	2.3	2.4
C(tp)	48.076	**48.062**	48.102	48.192	48.323	48.492

Table3: Total expected Cost per unit time for β=0.5

Ψ	0.1	0.2	0.3	0.4	0.5	0.6
C(Ψ)	301.280	175.612	131.132	107.966	93.631	83.847
Ψ	0.7	0.8	0.9	1.0	1.1	1.2
C(Ψ)	76.728	71.313	67.056	63.626	60.808	58.456
Ψ	1.3	1.4	1.5	1.6	1.7	1.8
C(Ψ)	56.467	54.769	53.306	52.036	50.928	49.957
Ψ	1.9	2.0	2.1	2.2	2.3	2.4
C(Ψ)	49.102	48.347	47.678	47.086	46.560	46.093
Ψ	2.5	2.6	2.7	2.8	2.9	3.0
C(Ψ)	45.679	45.312	44.987	44.700	44.448	44.227
Ψ	3.1	3.2	3.3	3.4	3.5	3.6
C(Ψ)	44.036	43.871	43.730	43.612	43.515	43.437
Ψ	3.7	3.8	3.9	**4.0**	4.1	4.2
C(Ψ)	43.378	43.335	43.308	**43.296**	43.298	43.313

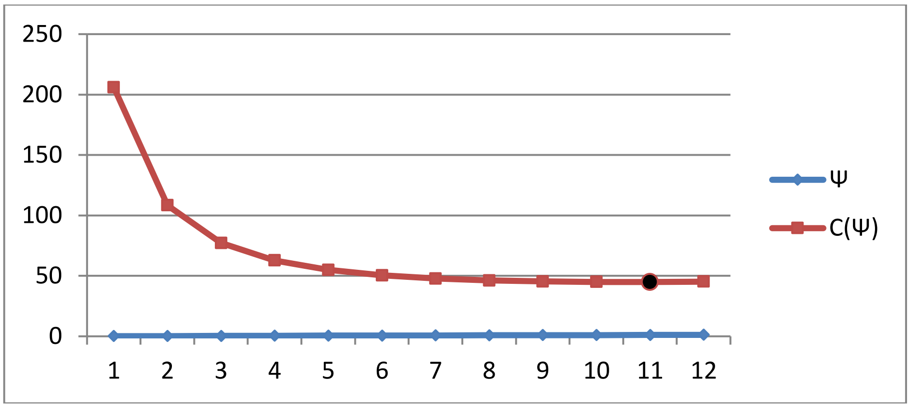

Figure2: Total expected Cost per unit time for β=2

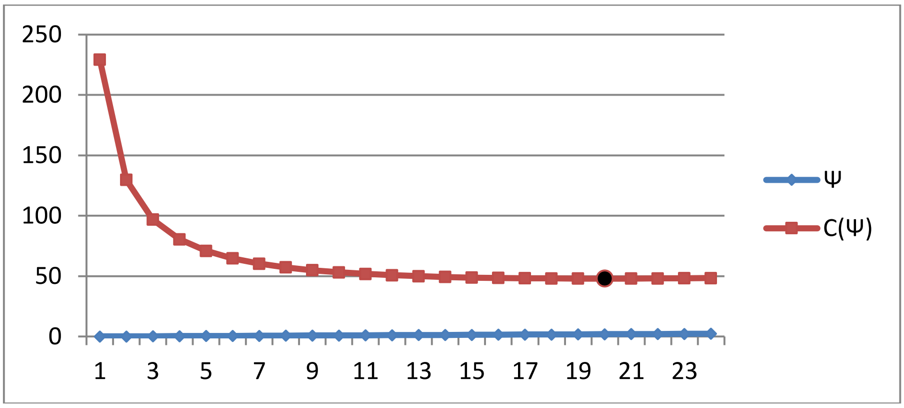

Figure3: Total expected Cost per unit time for β=1

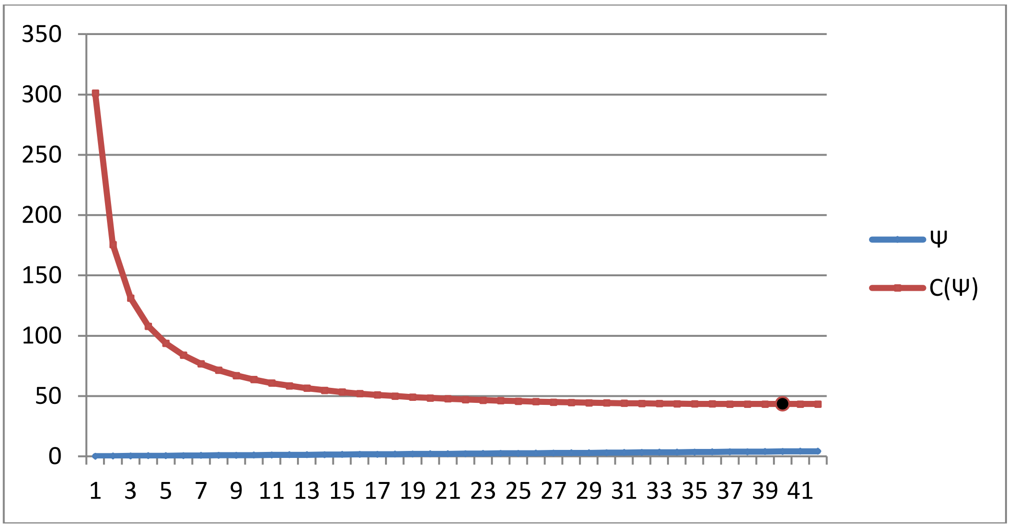

Figure4: Total expected Cost per unit time for β=0.5

- Shows the optimal point in the figures.

In type –I system, when β=2 (aging), the optimal replacement time is 1.1 and cost rate is 44.998 and reliability at this time is 0.298, when β=1 (constant), the optimal replacement is 2.0 and cost rate is 48.062 and reliability at this time is 0.135 and when β=0.5 (mortality), optimal replacement time is 4.0 and cost rate 43.296 and reliability at this time is 0.135. When equipment is in the constant state, replacement cycle is nearly twice of aging state and when it is in the mortality state is twice of constant state, reliability in mortality state is as equal as constant state and both states are less then aging state, as well cost rates differences are not much in all states.

5.2. Type –II system

In type –II system a shock for each unit causes that unit will be stopped and received minimal repairs and other unit will be received preventive maintenance but system won't be stopped. In addition system replaces with a same type at time Ψ preventively or at times less than Ψ due to failure both units simultaneously.

In this model, assume once the system followed the Erlang distribution. This means that operation and failure time have exponential distribution with same rate parameter; through the sum of these two distributions we have Erlang distribution. In this distribution α is rate parameter and ω is shape parameter. In the other hand, assume the system followed the exponential distribution and its rate parameter is same to Erlang distribution. We have α=0.5, ω=2 and CR=10, CF=100, CM=5, CP=5. Shock parameters are k=0.5 and c=0.05. Also we consider $C1 = \frac{CF}{CR}$ and $C2 = \frac{CM}{CP}$

C1=10, 30, 50, 70, 90. And C2=1, 3, 5, 7, 9.

Exponential probability density function is

$$f(x) = \alpha\, e^{-\alpha x}$$

And, Erlang probability density function is

$$f(x) = \frac{\alpha^{k} x^{\omega-1} e^{-\alpha x}}{(\omega-1)!}$$

By using MATLAB we have

Table4: Total expected Cost per unit time for exponential distribution

(C1,C2) / Ψ	(10,1)	(30,3)	(50,5)	(70,7)	(90,9)	MEAN
0.1	102.970	117.764	132.558	147.352	162.146	132.558
0.2	55.687	74.852	94.018	113.183	**132.349**	94.0178
0.3	41.515	64.681	**87.847**	**111.013**	134.178	**87.8468**
0.4	35.482	**62.320**	89.159	115.998	142.837	89.1592
0.5	32.608	62.830	93.052	123.274	153.496	93.052
0.6	31.247	64.594	97.941	131.288	164.635	97.941
0.7	30.702	66.943	103.185	139.427	175.668	103.185
0.8	**30.629**	69.557	108.488	147.418	186.348	108.488
0.9	30.842	72.274	113.706	155.138	196.571	113.706
1	31.234	75.001	118.768	162.535	206.302	118.768

Table5: Total expected Cost per unit time for Erlang distribution

(C1,C2)	(10,1)	(30,3)	(50,5)	(70,7)	(90,9)	MEAN

Ψ						
0.1	109.873	119.909	129.945	139.981	150.017	129.945
0.2	59.780	69.883	79.987	90.090	100.193	79.9866
0.3	43.058	53.272	63.485	73.699	83.912	63.4852
0.4	34.712	45.087	55.462	65.836	76.211	55.4616
0.5	29.744	40.337	50.931	61.524	72.118	50.9308
0.6	26.489	37.263	48.237	59.110	69.984	48.2166
0.7	24.233	35.453	46.672	57.891	**69.110**	46.6718
0.8	22.621	34.252	45.883	**57.515**	69.144	45.883
0.9	21.454	33.565	**45.676**	57.787	69.898	**45.676**
1	20.612	33.272	45.931	58.591	71.250	45.9312
1.1	20.020	**33.297**	46.573	59.850	73.126	46.5732
1.2	19.627	33.589	47.551	61.513	75.475	47.551
1.3	19.399	34.114	48.829	63.544	78.259	48.829
1.4	**19.308**	34.844	50.381	65.917	81.454	50.3808
1.5	19.338	35.764	52.189	68.614	85.040	52.189

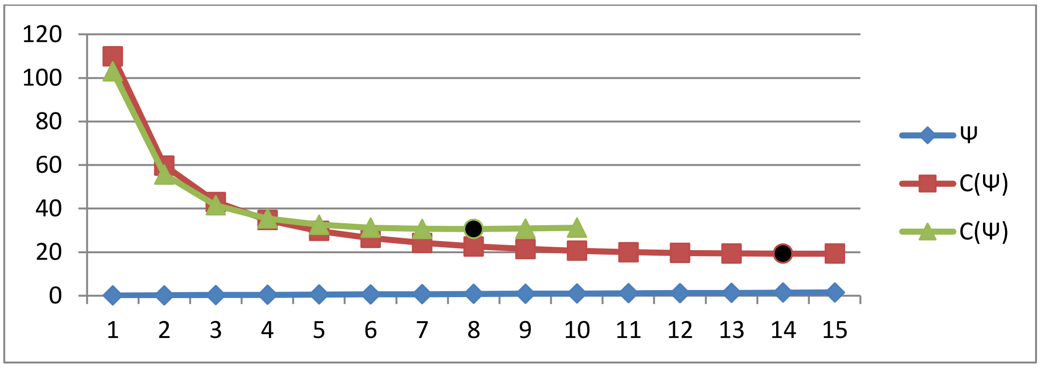

Figure5: Total expected Cost per unit time for Exponential and Erlang C1=10, C2=1

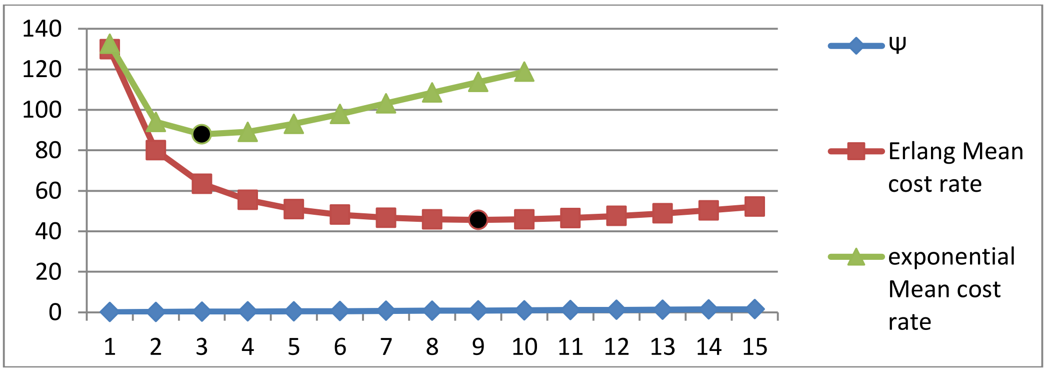

Figure6: Total expected Cost per unit time for Exponential and Erlang Mean

In type –II system, when cumulative function is exponential, mean optimal replacement time is 0.3 with cost rate 87.847, also when cumulative function is Erlang, mean optimal replacement time is 0.9 and cost rate at this time is 45.676. Unlike the results from exponential distribution, in Erlang with increasing C1 and C2, optimal cycle changes are less than exponential. Also cost rates in Erlang significantly less than exponential. Indeed, Erlang distribution increases the accuracy of the modeling by considering the operation and failure distributions.

5.3. Comparison of models

Now, it is assumed type –I and type –II systems are followed exponential distribution with α=0.6 and have the same parameters, CR=20, CF=30, CM=12, CP=10, k=0.4 and c=0.01. In this case the optimal replacement time for type –I system is 2.7 with cost rate 30.787 and reliability 0.198 and for type –II system, optimal replacement time is 1.5 with cost rate 42.370 and reliability at this time is 0.407. By comparing two structures, it can be said reliability in parallel structure is more than single unit structure, but because of the increased risk of shocks in parallel structure and an increase in the number of repairs and preventive maintenance subsequently, system average cost is more than single unit structure.

6. Conclusions

In this paper, an optimal replacement policy by considering shock process are presented and studied. This policy is extended for two systems with different structures. We have shown optimal replacement time through numerical examples. Also, analyses are provided by considering cumulative functions and structures. The models in this article will assist in replacement policies decision making by considering internal and external shocks and will help managers to achieve the lowest cost. Future research can formulate models with the addition parameters such as corrosion and delay; also expand models application by considering other structure of components.

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