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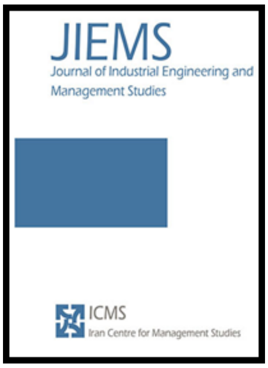

Optimization of supply chain finance based on Stackelberg model

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Abstract
In recent years, the relationship between the concepts of operations management and finance management has been an attractive area of research among researchers. One of the emerging areas at the beginning of the 21st century in the literature of operations and supply chain management is the topic of supply chain finance (SCF). SCF is a new concept that provides efficient financing of the supply chain, where all parties can balance the working capital and improve cash flow at a reduced cost by utilizing the buyer's or other parties' credit rating. Hence, in this study, an approach to optimize financing based on the Stackelberg model in a three-level supply chain, considering the circumstances in which the supplier is financially constrained for fulfillment the buyer's order and funded by the bank as another member of the supply chain based on the purchase order financing (POF) is discussed. For this purpose, a nonlinear mathematical programming model has been developed to maximize the payoff function of the partners.

Keywords: optimization; supply chain finance (SCF); purchase order financing (POF); Stackelberg model.

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1. Introduction

The funding and capital requirements of firms can be provided in a variety of ways. The ability of the firm to fund and prepare the right financial plans are key factors in the growth of a business. This ability, if formed within the framework of communication and interactions between different businesses across a chain, can provide lasting competitive advantages for that much more valuable business chain. Firms can finance their operations differently. A firm's finances can be provided through its owners and shareholders (internal sources of undivided profits) or through external sources that are sources other than shareholder capital and through money and capital markets. Therefore, adopting appropriate policies for the effective management of financial flow throughout the supply chain is a subject that should be considered professionally (Ross, 2005).

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As we know, the effectiveness of supply chain management is the result of the coordination of different flows of goods, information, and, ultimately, financial flows. Over the past three decades, many efforts have been made to improve and improve supply-chain efficiency (such as significantly reducing delay time, lower inventories, more accountability, increasing product diversity, more cooperation in planning and forecasting, and improving customer service). However, the supply chain financial flows are still managed in the manner of the 1970s. The current economic situation has created a lot of fragile risks in business and business. These risks are often financial, especially after the recent economic and financial crisis. The unpredictability of the global economy in the current situation and the tightening of financing has made it possible to affect trade flows and widespread financial pressures not only on global buyers but also on a growing number of suppliers (Cronie and Sales, 2008).

Financial Supply Chain Management (FSCM) has different meanings from different stakeholders in the supply chain, including logistics service providers (LSPs), information service providers (ISPs), financial service providers (FSPs). This concept starts from a simple definition of cash flow management (whether financing or short-term financing) and, in some other terms, managing investment costs or financing for the entire supply chain and how to reduce it. These costs are concentrated. The FSCM encompasses many approaches and services to accelerate the flow of funding and information among business partners in supply chains (Hausman, 2005).

To optimize financial processes, supply chain financial management helps companies to focus on the whole chain beyond their enterprise boundaries. This systematic approach focuses on working with other members within the chain and is generalized as an appropriate way to manage financial flow to safeguard the strategic components of the supply chain. Therefore, new considerations have emerged in aligning financial management with logistics and supply chain management that have created new business areas, especially for banks and financial service providers (Hofman and Belin, 2011).

Optimizing the cash flow in supply chain operations processes not only enhances the satisfaction of chain stakeholders but also enhances supply chain efficiency and results in a win-win approach for both financial service institutions and supply chain companies (Hausman, 2005).

Hereto after a limited number of financiers and banks have addressed this issue. While the concept of supply chain financial management is still unknown to many companies and banks, its benefits are unclear, and solutions are complex. Although numerous financial metrics have been proposed and capable of assisting managers in the link between financial status and operational performance, these are lacking the flexibility to cover financial flow management issues throughout the chain (Bærentsen and Thorstenson 2012).

Due to the importance of the SCF and its position in the academic and industrial sectors, as well as the many gaps in this area, especially in the literature on operations management, this study aims to design a model for supply chain financing taking into account the considerations that Less attention has already been given to focusing on the interactions of part of the supply chain members to achieve a win-win situation, as a proposed structure for financing Iranian companies, as in many developed countries.

The rest of the paper is organized as follows: Section 2 presents a literature review for the application of operations research methods in SCF. Problem definition, as well as the proposed mathematical model, is described in Section 3. The proposed solution approach is discussed in Section 4. Section 5 is devoted to computational experimentations. Finally, Section 6 provides concluding remarks and discusses further research opportunities.

2. Literature review

Much research has been done on the application of game theory in the supply chain, such as Seyyed Esfahani et al (2015), Pakdel Mehrabani and Seifi (2018) and etc., but not so much research has been done in the optimization of the supply chain finance area due to the novelty of the SCF issue by applying game theory. In this section, the most related studies of the current research are reported.

Zhang et al. (2020) investigated the optimal operational strategies for the SCF system with the 3PL financing service when the retailer or the manufacturer is stuck with capital constraint, respectively, in their study. For analyzing the problem, they used a combinational of Stackelberg model concept with designing some experiments for sensitivity analysis in the paper. By constructing and solving Stackelberg game models, they obtained the optimal operational strategies of the above two scenarios, and combined with the sensitivity analysis of relevant parameters; they captured the following conclusions: (1) For the SCF system with a capital-constrained retailer, except the manufacturer's optimal wholesale price remains unchanged, other participants' optimal decisions and the optimal profits of each participant increase with the logistics service sensitivity coefficient; except the manufacturer's optimal wholesale price remains unchanged, other participants' optimal decisions and the optimal profits of each participant decrease with the logistics service cost efficiency. (2) For the SCF system with a capital-constrained manufacturer, the optimal decisions and profits of supply chain participants increase with the logistics service sensitivity coefficient; the optimal decisions and profits of supply chain participants decrease with the logistics service cost efficiency. Their analysis suggests that the retailer and manufacturer must take into account 3PL enterprise's decisions (logistics service level and logistics service price) under the 3PL financing service mode before making decisions.

Li et al. (2020) have studied the efficiency of buyer lending for supporting financially constrained suppliers under spectral risk conditions. For this purpose, the retailer and the supplier play a Stackelberg game, where the retailer is the leader, and the supplier is the follower. In this paper, researchers investigated the efficiency of buyer lending in a supply chain with a retailer and a capital-constrained supplier subject to spectral risk measures with providing a comprehensive comparative statics analysis. The findings of the paper show that the retailer should always charge the supplier the lowest possible interest rate under the buyer lending scheme. The retailer is better off in the buyer lending scheme than in the bank lending scheme if he is risk-neutral or risk-seeking, regardless of the supplier's risk attitude, internal capital, and bankruptcy costs while the retailer's preference switches to bank lending as his risk-aversion increases to a certain level.

Liu et al. (2020) have presented a paper it is called "Multiparty game research and example analysis in supply chain finance system based on MPDE theory" .in this paper, researchers have considered members of the supply chain as nodes. In this supply chain finance system, each node on the chain can pursue its own best interests. The core nodes (commercial banks, small and medium-sized enterprises, third-party logistics companies) can coordinate and monitor their behavior with each other, and can achieve mutual benefit and efficient operation of the supply chain. This paper studies the theory of meshless partial differential equations (MPDE) and uses game theory and information economics theories and methods to establish corresponding multiparty game models. The application of this game model to the members of supply chain finance was explored. Mathematical derivation and example analysis prove that the model is reasonable, and it also provides a way to make the supply chain operate most efficiently and maximize the benefits.

Kouvelis and Zhao (2018) presented a game-theoretical modeling framework to answer the question of who should finance the inventories at what interest rate in terms of a different credit-rating hole of members of the supply chain. A key feature associated with operations management in this study is the wholesale price contract between the buyer and supplier and the ordering decisions. The financial characteristic of this study relates to the use of credit and bank loans. Bank loans are used by both the buyer and the lender. The bank's financing conditions are closed in a competitive balance based on the credit rating mark that was visible to all members of the chain. They showed that there is a threshold for a credit rating of the supplier that in high amounts, the lender offers commercial credit at the rate of zero percent, and the buyer only uses this credit to finance his inventory. Otherwise, the lender would consider a rigid, positive interest rate for lending, in which case a combination of bank lending and lending would be a cheaper way to finance the buyer.

Lee et al. (2018) examined a study of commercial credit financing under competitive conditions and its impact on firm performance in the supply chain. They were found that suppliers with smaller market share and buyers with larger market share are associated with larger commercial credit amounts. This study appropriately complemented the results of previous papers by Burkart and Ellingsen (2004), Kouvelis and Zhao (2012) and explored the role of supply chains in asset pricing and risk management. Examples of these studies in the literature include the work of Cohen and Frazzini (2008), Wu and Birge (2014), Wu (2015), and Agca et al. (2017).

Deng et al. (2018) studied the financing problem of several heterogeneous suppliers in assembly systems. In this study, buyer financing versus bank financing has been studied, and it has been examined whether, in a system of assembly of different players, they must be financed through bank loans or by buyer-centered financial services. In this study, the authors obtained equilibrium solutions for different modes of financing through a theoretical game model. They presented the necessary conditions in which buyer financing was preferable than the bank loan. Under the buyer financing, the assembler pays the lowest possible interest rate, even when the opportunity cost of capital is higher than the risk of the loan rate. The study showed that order quantities depend on the cost difference between supplier, bank, and buyer capital.

Tang et al. (2018) compared two forms of financing in a study of sourcing from providers with financial constraints and performance risk. First, purchase order financing allows banks to provide loans to suppliers, taking into account the value of purchase orders issued by reputable buyers, and assessing the risk of the supplier delivering the order successfully. Another form of financing involves direct buyer financing, where the buyer (producer) sends sourcing contracts and loans directly to the supplier. In fact, in this study, the supplier can use bank financing or buyer financing, depending on the circumstances. The authors found that when information about the cost of the supplier effort between the bank and the buyer is symmetrical, the buyer can guarantee that the supplier makes his first-best effort. Even when the information is asymmetric, and the buyer is more aware, buyer financing is not better than a bank loan every time. If the funding required is small, the use of both sources of funding is no different. This paper shows that buyer-centered financing often derives its advantages through the producer's information advantage over the bank about the supplier's delivery performance, not to control the interest rate offered. This is an interesting finding in recent attempts to analyze asymmetric information and consider the ethical risk involved in supply chain financing issues.

Chen et al. (2017), in a study of pricing/ordering models for a binary supply chain with guaranteed financing, redefined and considered fairness in a binary supply chain that mainstreamed its products through budget-constrained retailing. The retailer faces random demand and fairness considerations. If needed, the retailer can obtain financial support from the bank using BGF, which is common in China. By presenting the Nash bargaining solution as a point of reference for fairness, the authors formulated the utility function with consideration of retailer fairness considerations. They developed a two-stage pricing/ordering game model. They next examined the combined effect of fairness and BGF considerations on two-member equilibrium strategies and supply chain performance. The results showed that the two-player equilibrium strategies are influenced mainly by the behavior of fairness and initial budget considerations; compared to the unbounded state, BGF can make the whole supply chain perform better, meaning that BGF can be of value. Add-ons, and when the retailer alone accepts the risk of market uncertainty, considerations of retailer fairness will be beneficial to improve supply chain performance.

Xiao et al. (2017) studied coordinating financing contracts for a finite supply chain. They considered a supply chain in which a supplier sells its products to a retailer who has no access to banking resources due to low credit ratings and has financial constraints. However, the lender provides commercial credit to the retailer to deal with financial constraints by obtaining a loan from a bank. They considered non-repayment of loans or loans as variable costs and analyzed a centralized supply chain to obtain new coordination requirements. Finally, a public revenue sharing contract that coordinates the supply chain with flexible profit allocation was designed and illustrated with numerical examples of model superiority over revenue sharing and buyback contracts.

Yang et al. (2017), in a study, analyzed the equilibrium of a two-stage supply chain with financially constrained retailers by examining the impact of capital financing on a two-stage supply chain with restricted retailers and suppliers. They show that as competition intensifies, the supplier may merge with a retailer to avoid doubling profit margins and rising prices. They also conducted a sensitivity analysis concerning the structure of retail capital and the intensity of its competition.

Yan et al. (2016) presented a study of a partial credit guarantee contract in a capital-constrained supply chain with a balance of financing and coordination strategy. To this end, a supply chain financing system consisting of a retailer, a manufacturer, and a commercial bank with capital constraints is designed and a Stockholm game model in which the bank acts as a leader. Second, they designed a Partial Credit Guarantee (PCG) contract to finance the supply chain, which includes bank credit financing (BCF) and manufacturer commercial credit guarantees for analyzing their equity financing strategies. They then examined the interdependence between operational and financial decisions and analyzed the terms of PCG contract coordination. The findings indicated that, based on the relevant guarantee factor, the PCG contract might achieve maximum profit and coordination in the supply chain finance system.

Wuttke et al. (2016) examined a supply chain optimization study. Experimental evidence suggests that some members of the supply chain are skeptical of SCF approval so that there is a difference between the time of acceptance of the SCF financing proposal by the buyer and the acceptance of it by all suppliers. Based on their observations, they incorporated the decisions made on SCF into a mathematical model to optimize insights about consumer decisions in terms of timing and payment conditions. They found that initial payment terms and volume of procurement strongly influenced the optimal timing of SCF use and increased optimal payment terms. Their results showed that despite the apparent benefits of using the SCF approach, many supply chain buyers might delay the recommendation to implement the SCF.

Jing and Seidmann (2014), in a study, compared the relative advantage of banks' credit versus the method of trade credit in a production supply chain. They showed that trade credit is much more effective than bank credit when the costs of production are relatively low, and vice versa when credit costs are relatively high. Also, they found that when banks finance a supply chain, members of the chain store the same amount of inventory they would have in the absence of working capital, and when financed by a trade financing method. It distributes

low demand to the distributor and maintains a higher inventory than bank financing, which would lower the profit margin when the cost of manufacturing the product is high.
Some of the reviewed studies are summarized in Table 1.

Table 1. Summary of literature review

Researcher/ Year	Purpose	Type of game an equilibrium	Supply chain		
			Financial contract	**Players**	**level**
Chen et al. (Chen, Zhou, & Zhong, 2017)	A pricing/ordering model for a dyadic supply chain with buyback guarantee financing and fairness considerations	The Stackelberg model and the Nash bargaining equilibrium / Stackelberg equilibrium and the Nash equilibrium	BGF contract for financing the retailer by the buyer	A major supplier and a retailer with a limited budget and a bank	Two-level
Yang et al. (H. Yang, Zhuo, & Shao, 2017)	Evolution of equilibrium in Two-echolen Supply Chain with Retailers with Financial Constraints: The Impact of Financing Capital	Stackelberg Model/ Stackelberg equilibrium	Financing by the supplier in three forms	A major supplier in the presence of two competing retailers in a Cournot market	Two-level
Xiao et al (Xiao, Sethi, Liu, & Ma, 2017)	Coordinating contracts for a supply chain with financial constraints	Stackelberg Model/ Stackelberg equilibrium	Revenue Sharing Contracts and Order Size Contracts for Supply Chain Coordination	A leader supplier and a follower retailer with financial constraint	Two-level
Yan et al. (Yan, Sun, Zhang, & Liu, 2016)	A partial credit guarantee contract in a supply chain with a capital constraint: Financing equilibrium and coordinating strategy	Stackelberg Model/ Stackelberg equilibrium	The PCG contract, which includes part of the financing by the bank and the rest t by the manufacturer's credit.	The supply chain includes a manufacturer, retailer with capital constraints and a bank for financing	Two-level
(Tang, Yang, & Wu, 2018)	Sourcing from suppliers with financial constraints and performance risks	Stackelberg Model and signaling game/ perfect Bayesian Nash equilibrium	BDF and POF financing agreement financed by the manufacturer and the bank, respectively	Sourcing from suppliers with financial constraints and performance risks	Three-level
Kouvelis and Zhao (Kouvelis & Zhao, 2018)	Who should finance the supply chain? The Impact of Credit Ratings on Supply Chain Decisions	Stackelberg model/ Stackelberg equilibrium	The retailer is financed by short-term bank loans and bank credit, and the supplier is financed by a short-term bank loan or early payment by the retailer.	A supplier and a retailer both have constraint capital	Two-level
Deng et al.	Financing multiple heterogeneous suppliers in assembly systems: Buyer financing versus bank financing	Stackelberg model/ Nash equilibrium Optimal Pareto	Buyer financing, bank financing and without financing	An assembler and several suppliers	Two-level
Jin et al.	cooperative and non-cooperative financing in a two-level supply chain with capital constraints	Stackelberg model, cooperative and non-cooperative games / Stackelberg equilibrium	Financing includes bank loan, bank loan based on trade credit and loan based on supplier guarantee	Supplier, a retailer, and a bank	Two-level

3. Proposed model

This study optimizes multilevel supply chain finance under the Purchase Order Financing (POF) arrangement with the presence of the manufacturer or buyer (M), supplier (S), and bank (B). In this model, the supplier has financial constraints to fulfill the order of the buyer (manufacturer) so based on a POF contract, the buyer introduces the supplier to the bank for financing, so in this Stackelberg model the buyer is the leader and the supplier is the follower. The bank, as a third party, finances the cost of preparing the order (c) in the form of a loan with an agreed interest rate (i). It is the leading manufacturer of Stackelberg, which offers a price contract (p) to supply the goods it needs to be supplied by the supplier (Steckelberg leader). If the supplier accepts the price offer, the contract is made visible to the bank, and the bank offers the borrower the value of a (c) at the rate of interest (i). Thus, the partnership between the members of the supply chain is formed. Both parties will accept the contract if the payoff is greater than zero. It should be noted that this study assumes that if the manufacturer wishes to obtain the goods he needs from the free market, he must pay a v value of money, which is $v \geq p$, so that the rest of the Initial assumptions will be as follows:

$$0 \leq a \leq c < p$$

To illustrate the mathematical model of the problem, it is first necessary to present the payoff functions of each of the partners.

Manufacturer's (buyer) payoff

The payoff of the manufacturer resulting from the probability of timely delivery of the order by the supplier (e) in the difference of market price (v) with the contract price (p) and in the event of timely delivery with a probability of 1-e in the market price. So the manufacturer payoff can be expressed in Equation (1):

$$\pi_M = v - [ep + (1 - e)v] = e(v - p) \quad (1)$$

Bank's payoff

If the order is timely delivered by the supplier, with the e probability the bank will receive a loan payment (c) with an interest rate (i_B), otherwise, with the probability of 1-e, the bank takes supplier's asset (a) instead of the supplier's debt, so the bank's payoff function as follow:

$$\pi_B = e[(1 + i_B)c] + (1 - e)a - c \quad (2)$$

Supplier payoff

If the order is delivered on time with the probability of e, the supplier will recover the difference between the order price (p) and the cost of the order and the amount of bank interest. If not delivered on time, with the probability of 1-e, the supplier will lose his asset (a). While there is an effort cost of $\frac{1}{2}ke^2$ for the supplier, the supplier's payoff function can be expressed in Equation (3):

$$\pi_S = e[p - (1 + i_B)c] - (1 - e)a - \frac{1}{2}ke^2 \quad (3)$$

Figure 1 illustrates how each partner interacts and sequence of events corresponding to POF contract terms described earlier.

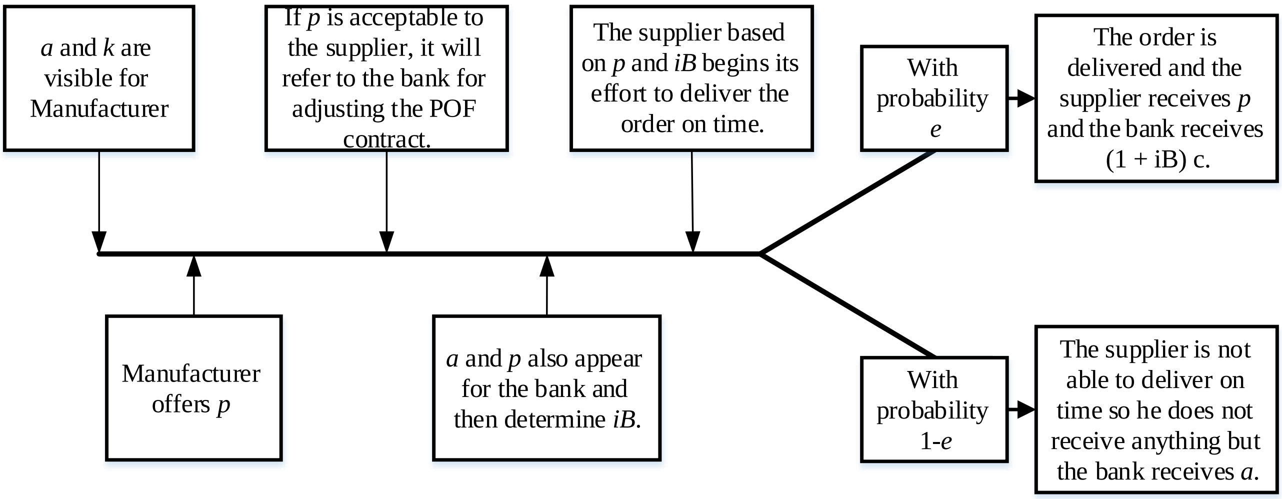

Figure 1. Sequence of events under the POF

4. Proposed solutions approach

Centralized supply chain as a primary solution

To get an initial solution to the model, we first consider a centralized supply chain, assuming what the supplier will be able to make the order. For this, we assume that the manufacturer will be sourcing from an internal supplier, in which case the payoff will be in Equation (4):

$$\pi_{Cen} = v - \left[c + \frac{1}{2}ke^2 + (1-e)v\right] \quad (4)$$

The best response of internal supplier will be obtained by deriving π_C as follow:

$$e_{Cen} = \frac{v}{k} \quad (5)$$

$$0 \le e_{Cen} \le 1 \Rightarrow v \le k \quad (6)$$

By placing e_{Cen} in the manufacturer's payoff in the context of a centralized supply chain, the following constraints will be achieved:

$$\pi_C(e_{Cen}) \ge 0 \Rightarrow \frac{v^2}{2k} \ge c \quad (7)$$

Calculates the best response of players in the Stackelberg model

To obtain the best supplier response, we derive the supplier payoff function relative to e:

$$\pi_S^{POF} = e[p - (1+i_B)c] - (1-e)a - \frac{1}{2}ke^2 \xrightarrow{(\pi_S^{POF})'=0} e_{(p,i_B)} = \frac{p + a - (1+i_B)c}{k} \quad (8)$$

By inserting (8) into the supplier's payoff function, it can be overwritten as follows:

$$\pi_S^{POF}\left(e_{(p,i_B)}\right)=\frac{p+a-(1+i_B)c}{k}\,[p-(1+i_B)c]-\left(1-\frac{p+a-(1+i_B)c}{k}\right)a-\frac{1}{2}k\left(\frac{p+a-(1+i_B)c}{k}\right)^2 \tag{9}$$

Now to obtain the best response of the bank which is the best bank interest rate, for this we put $e_{(p,i_B)}$ into the bank payoff function and by derive relative to i_B:

$$\pi_B=e[(1+i_B)c]+(1-e)a-c\xrightarrow{e\to e_{(p,i_B)}}$$
$$\pi_B\left(e_{(p,i_B)}\right)=\frac{p+a-(1+i_B)c}{k}(1+i_B)c+\left(1-\frac{p+a-(1+i_B)c}{k}\right)a-c \tag{10}$$

$$\xrightarrow{(\pi_B(e_{(p,i_B)}))'=0}\quad i_B=\frac{2a+p}{2c}-1 \tag{11}$$

$$\xrightarrow{0\le i_B\le 1} p\le 4c-2a \tag{12}$$

The bank consequence function can be calculated by placing (11) in (10):

$$\pi_B=\frac{p^2+4ak-4ck}{4k} \tag{13}$$

By placing i_B in $e_{(p,i_B)}$and the payoff function of the supplier (9) the following equations will be obtained:

$$e(p)=\frac{p}{2k} \tag{14}$$

$$\pi_S^{POF}(p)=\frac{p^2-8ak}{8k} \tag{15}$$

By placing $e(p)$ in (1), the payoff of the manufacturer in terms of variable p is as follows:

$$\pi_M^{POF}(p)=\frac{p(v-p)}{2k} \tag{16}$$

Consequently, to obtain the appropriate strategy for each player in the Stackelberg model, we must solve the following mathematical model with the specified constraints:

$$Max\ \pi_M=\frac{p(v-p)}{2k} \tag{17}$$
$$\sqrt{8ak}-p\le 0 \tag{18}$$
$$p-4c+2a\le 0 \tag{19}$$
$$c<p<v \tag{20}$$
$$0<a\le c \tag{21}$$
$$c\le\frac{v^2}{2k} \tag{22}$$
$$v\le k \tag{23}$$
$$\pi_M,\pi_B,\pi_S\ge 0 \tag{24}$$
$$0\le e\le 1 \tag{25}$$
$$0\le i_M\le 1 \tag{26}$$

Since the above objective function is nonlinear, one of the appropriate methods for solving this model is using Karush–Kuhn–Tucker (KKT) conditions (Ji, Li, and Qu, 2018).

Solve by KKT conditions

To use the KKT conditions, we rewrite the model according to the variable p and based on the KKT method.

$$Max\ Z = \frac{p(v-p)}{2k} + \mu_1\left(\sqrt{8ak} - p\right) + \mu_2(p - 4c + 2a) + \mu_3(c - p) + \mu_4(p - v) \quad (27)$$

$$\sqrt{8ak} - p \leq 0 \quad (28)$$
$$p - 4c + 2a \leq 0 \quad (29)$$
$$c - p \leq 0 \quad (30)$$
$$p - v \leq 0 \quad (31)$$
$$\mu_1, \mu_2, \mu_3, \mu_4 \geq 0 \quad (32)$$

$$\frac{\partial Z}{\partial p} = \frac{v - 2p}{2k} - \mu_1 + \mu_2 - \mu_3 + \mu_4 = 0 \quad (33)$$

Given the above Equation, four possible cases are considered to obtain the optimal solution.
Case 1)

$$\begin{cases} \mu_3 = \mu_4 = 0 \\ \mu_1 = \mu_2 = 0 \end{cases} \rightarrow \frac{v - 2p}{2k} = 0 \rightarrow p = v/2 \quad (34)$$

$$i_B = \frac{v + 4a}{4c} - 1 \quad (35)$$
$$e = v/4k \quad (36)$$
$$\pi_M = v^2/8k \quad (37)$$
$$\pi_B = (p^2 + 4ak - 4ck)/4k \quad (38)$$
$$\pi_S = (v^2 - 32ak)/32k \quad (39)$$

Case 2)

$$\begin{cases} \mu_1 = \mu_3 = \mu_4 = 0 \\ \mu_2 \neq 0 \end{cases} \rightarrow \mu_2 = \frac{2p - v}{2k} \geq 0 \rightarrow 2p - v \geq 0 \rightarrow p \leq v/2 \quad (40)$$

From this, it can be concluded that the maximum value of *p* is *v*/2 so that the first case has a higher value for the payoff.

$$p = 4c - 2a \quad (41)$$
$$i_B = 1 \quad (42)$$
$$e = (2c - a)/k \quad (43)$$
$$\pi_M = \frac{(4c - 2a)(v + 2a - 4c)}{2k} \quad (44)$$
$$\pi_B = a - c + \frac{(2a - 4c)^2}{4k} \quad (45)$$
$$\pi_S = \frac{(2a - 4c)^2}{8k} - a \quad (46)$$

Case 3)

$$\begin{cases} \mu_2 = \mu_3 = \mu_4 = 0 \\ \mu_1 \neq 0 \end{cases} \rightarrow \mu_1 = \frac{v - 2\sqrt{8ak}}{2k} \geq 0 \rightarrow v - 2\sqrt{8ak} \geq 0 \rightarrow v \geq 2\sqrt{8ak} \quad (47)$$

$$p = \sqrt{8ak} \tag{48}$$

$$i_B = \frac{a + \sqrt{2ak}}{c} - 1 \tag{49}$$

$$e = \frac{\sqrt{2ak}}{k} \tag{50}$$

$$\pi_M = \frac{v\sqrt{2ak}}{k} - 4a \tag{51}$$

$$\pi_B = 3a - c \tag{52}$$

$$\pi_S = 0 \tag{53}$$

Case 4)

$$\begin{cases} \mu_3 = \mu_4 = 0 \\ \mu_1, \mu_2 \neq 0 \end{cases} \rightarrow \frac{v - 2p}{2k} - \mu_1 + \mu_2 = 0 \rightarrow \mu_1 = \frac{v - 2p}{2k} + \mu_2 \tag{54}$$

$$p = 4c - 2a = \sqrt{8ak} \tag{55}$$

$$i_B = 1 = \frac{a + \sqrt{2ak}}{c} - 1 \tag{56}$$

$$e = \frac{2c - a}{k} = \frac{\sqrt{2ak}}{k} \tag{57}$$

$$\pi_M = \frac{(4c - 2a)(v + 2a - 4c)}{2k} = \frac{v\sqrt{2ak}}{k} - 4a \tag{58}$$

$$\pi_B = a - c + \frac{(2a - 4c)^2}{4k} = 3a - c \tag{59}$$

$$\pi_S = \frac{(2a - 4c)^2}{8k} - a = 0 \tag{60}$$

After solving the model, the best response and payoff of each partner in terms of parameters and according to the problem conditions are obtained as Table 2:

Table 2. The optimal solutions

Row	Player	Best response	Payoff
1	Manufacturer (Buyer)	$p^* = v/2$	$\boldsymbol{\pi_M^* = v^2/8k}$
2	Bank	$i_B{}^* = \frac{v + 4a}{4c} - 1$	$\boldsymbol{\pi_B^* = (p^2 + 4ak - 4ck)/4k}$
3	Supplier	$e^* = v/4k$	$\boldsymbol{\pi_S^* = \frac{v^2}{32k} - a}$
$\boldsymbol{\sqrt{8ak} - p \leq 0}$ $\boldsymbol{p - 4c + 2a \leq 0}$ $\boldsymbol{c < p < v}$ $\boldsymbol{0 < a \leq c}$ $\boldsymbol{c \leq \frac{v^2}{2k}}$ $\boldsymbol{v \leq k}$ $\boldsymbol{\pi_M, \pi_B, \pi_S \geq 0}$			

5. Sensitivity analysis

In this section, some experiments have designed based on Zhang et al. (2020) to analyze the changes in optimal operational decisions and payoffs for the SCF system that the supplier has capital-constrained. The sensitivity analysis of optimal solutions (p^*, $\boldsymbol{\pi_M^*}$, ${i_B}^*$, $\boldsymbol{\pi_B^*}$, e^*, $\boldsymbol{\pi_S^*}$) concerning any parameters (v, k, c, a) is conducted, In this section, we perform a series of sensitivity analyses to explore changes of operational solutions in two scenarios (i.e., capital-constrained retailer and capital-constrained manufacturer). Assuming that sourcing from an internal supplier is not possible for the manufacturer we set the values of relevant parameters, respectively, $v = 30, k = 8, c = 8, a = 3$). The results of sensitivity analysis are shown in Table 3.

Table 3. The results of sensitivity analysis

	Parameter value	p	π_M	i_B	π_B	e	π_S
v	30	15	900	0.31	2.03	0.94	0.52
	35	17.5	1225	0.47	4.57	1.09	1.79
	40	20	1600	0.63	7.5	1.25	3.25
k	8	15	900	0.31	2.03	0.94	0.52
	10	15	1125	0.31	0.63	0.75	-0.19
	12	15	1350	0.31	-0.31	0.63	-0.66
c	8	15	900	0.31	2.03	0.94	0.52
	10	15	900	0.05	0.03	0.94	0.52
	12	15	900	-0.125	-1.97	0.94	0.52
a	3	15	900	0.31	2.03	0.94	0.52
	4	15	900	0.44	2.03	0.94	-0.48
	5	15	900	0.56	2.03	0.94	-1.48

Sensitivity analysis results and economic interpretations are given below:

- Assuming other parameters remain the same, the optimal values of all variables and payoffs in equilibrium conditions increase with *v*. It means when the market price of order increases, the buyer (manufacturer) will offer a higher price under the same conditions and will also get a higher payoff from this contract. An increase in the market price will also increase the interest rate offered by the bank, and with the increase in the proposed interest rate, the bank's payoff will be more. As the market price rises, the supplier will have to work harder to deliver the buyer's order timely, so the supplier's payoff will increase in this situation. Figure 2 shows the three players' best payoff in terms of the change of *v*.

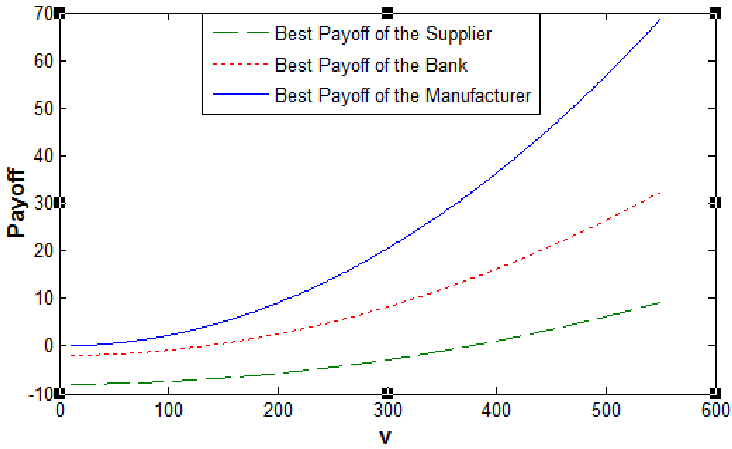

Figure 2. Players' best payoff in terms of the change of *v* under POF contract

- Assuming other parameters remain the same, the variable *p* will remain constant with *k*; it means when the cost of the supplier's effort increases, the price offered by the buyer will not change. On the other hand, increasing the cost coefficient of the supplier's effort does not affect the bank's proposed interest rate in equilibrium conditions. As the supplier's cost effort increases, the probability of delivery from the supplier to the buyer during the order decreases, thus reducing the supplier payoffs, the buyer, and the bank. Figure 3 shows the three players' best payoff in terms of the change of *k*.

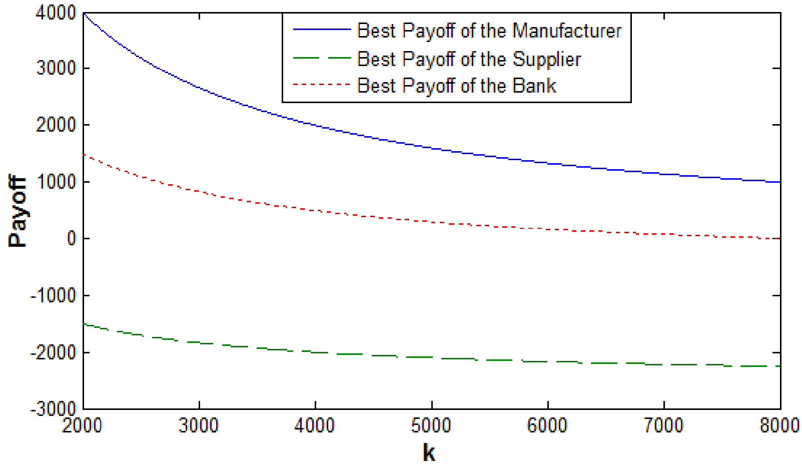

Figure 3. Players' best payoff in terms of the change of *k* under POF contract

- Assuming other parameters are constant and the parameter *c* increases, we see the buyer's proposed price will not change in equilibrium conditions, so the proposed price does not depend on the value of the order supply cost parameter. On the other hand, with the increase in the cost of ordering in the equilibrium conditions, the bank's proposed interest rate will also decrease, i.e., to achieve the bank's equilibrium conditions for financing the supplier by higher loans, lower interest rates will be offered. Also, with the increase in the cost of ordering, the probability of timely delivery of the order is the same. Figure 4 shows the three players' best payoff in terms of the change of *c*.

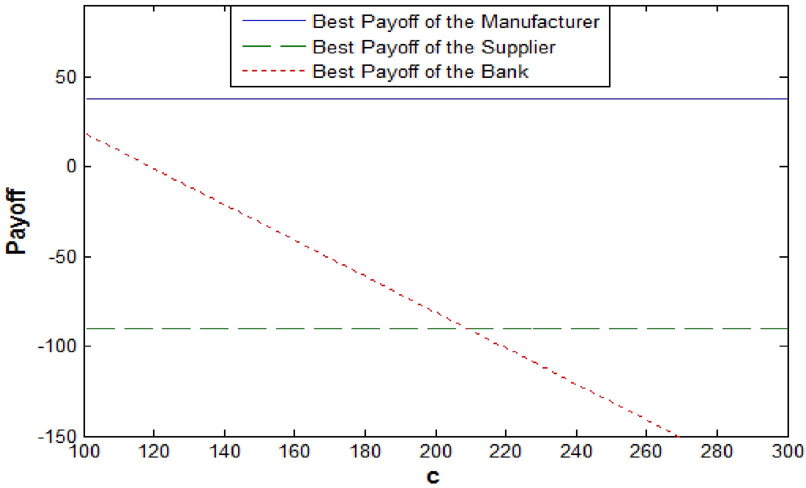

Figure 4. Players' best payoff in terms of the change of *c* under POF contract

- By increasing the parameter *a* and assuming other parameters remain the same, there will be no change in the buyer's proposed price in equilibrium conditions so the buyer's payoff will remain constant. Interestingly, an increase in the asset level, the bank's proposed interest rate would increase in the equilibrium condition. However, at the same time, the bank's payoff would not change under these circumstances. Also, with the increase of the supplier's asset, the probability of timely delivery of the order by the supplier will remain constant. However, the supplier's payoff will be reduced in equilibrium conditions. Figure 5 shows the three players' best payoff in terms of the change of *a*.

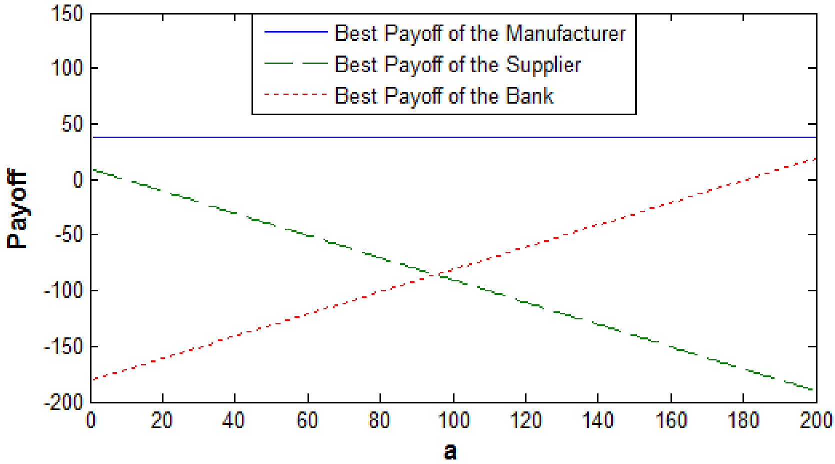

Figure 5. Players' best payoff in terms of the change of *a* under POF contract

6. Discussion

As shown by Equation 16, the value of buyer payoff (π_M) is directly related to the market price (v); that is, the higher the difference between the market price and the proposed price by the buyer resulting in the higher payoff. Given the optimal price (p^*) the buyer will get the highest utility when it offers half the market price. Also, π_M is inversely related to k, which means that as k increases, the supplier payoff will decrease. This means that the supplier with higher efficiency (the smaller the k), resulting from increasing buyer payoff. It is important to note that in general, since v has the power of 2 in π_M Increasing or decreasing the market price will make a far greater difference to the value of the buyer's payoff. This sentence means proposed price (p) is more important to gain more profit for the buyer, so in the real world, the buyer is likely to go to a lower-performing supplier to obtain higher results, because in this condition accept a lower p by the supplier is more probable and leads to more profit for the buyer.

Also the optimal bank payoff (π_B^*) indicates that when the proposed price (p) is increased, and the supplier is more efficient (k is small), the bank's willingness to accept the contract also grows. Interestingly, the value of the supplier's assets (a) does not have a significant impact on the bank's willingness (payoff) to accept the contract. It also follows that when the bank makes higher financing (c is big value), the bank should offer higher interest rates (higher i_B) to increase its profit.

The optimal supplier payoff (π_S^*) also indicates that increasing the value of the v parameter has a significant effect on increasing supplier utility, and by decreasing supplier's efficiency (higher k), the supplier payoff will decrease. In general, increasing v increases the probability of delivery to the buyer on time. However, k has an opposite effect on the probability of delivery to the buyer timely. It means the higher value of the supplier's commodity market price and the higher quality of its performance will increase the profits of the supplier in this financing contract.

7. Conclusions and future studies

Combining operations research and supply chain finance is one of the topics that has received less attention in the literature on operations management. Recent studies have seen attempts to fill this theoretical and literary divide. To develop the literature in this field, based on the game theory concept and the Leader-Follower Stackelberg model, a mathematical model for supply chain modeling consisting of three levels was developed. The supply chain under study involves the manufacturer (buyer), the supplier (with financial constraints to supply buyer order), and a bank (financial institution) interacting with each other in the context of a purchase order financing (POF) contract. The buyer submits an order to the supplier at a specified price to supply the goods he needs, if the offer is accepted, supplier goes to the bank with this contract, and the bank offers the interest rate to lend to the supplier. By modeling these interactions, a nonlinear mathematical model was designed based on the Stackelberg model and solved using the KKT method to optimize the payoff of each partner and determine the optimal strategy of each partner, the optimal values of the buyer proposed p, supplier effort (probability of timely delivery) and interest rate of bank's loan as each player's equilibrium strategies. The results showed that market price parameters (v) and supplier performance (expressed as k) had the most significant impact on the payoff of each supply chain member.

Like all scientific research, this study has limitations that can be developed and improved by offering suggestions for future studies. These include considering more than one supplier in the chain with varying levels of performance, as well as model assumptions of the existence of symmetric information conditions between players that can be asymmetric between players to approach real-world conditions. This means that each member does not have the same information about the other member's conditions and strategies. One can also consider several sources of financing in the model or compare the performance of different financing agreements or contracts for interaction between supply chain partners.

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