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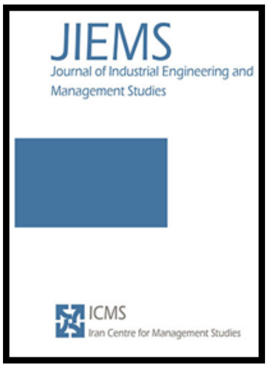

Coordination of the decisions associated with maintenance, quality control and production in imperfect deteriorating production systems

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Abstract
In this paper, a mathematical model is presented for the integrated planning of maintenance, quality control and production control in deteriorating production systems. The simultaneous consideration of these three factors improves the efficiency of the production process and leads to high-quality products. In this study, a single machine produces a product with a known and constant production rate per time unit and the production process has two operational states, i.e. in-control state and out-of-control state, and the probability of the state transition follows a general distribution. To monitor the process, sampling inspection is conducted during a production cycle and a proper control chart is applied. In the developed model, there is no restriction on the type of the control chart. Therefore, different control charts can be applied in practice for quality control. The lot size produced in each production cycle is determined with respect to the production rate of the machine and the proportion of conforming and non-conforming items produced in each cycle. In this study, preventive maintenance and corrective maintenance as perfect maintenance actions and minimal maintenance as imperfect maintenance action are applied to maintain the process in a proper condition. The objective of the integrated model is to plan the maintenance actions, determine the optimal values of the control chart parameters and optimize the production level to minimize the expected total cost of the process per time unit. To evaluate the performance of this model, a numerical study is solved and a sensitivity analysis is conducted on the critical parameters and the obtained results are analyzed.

Keywords: deteriorating production system; imperfect production system; preventive maintenance; control chart; production control.

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1. Introduction

The main aspects in a manufacturing system that affect the performance of that system include production planning, inventory control, quality control, and maintenance planning. Proper and optimal planning of these aspects improves the efficiency of the production

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system by reducing the costs and enhancing the quality of the products, which is the main goal of any manufacturing system.

Competitive condition and rapid changes in the market and increase of product variety have led to increased automation and a need for more complicated equipment such as robotic machines (Xiang, 2013). Therefore, in different manufacturing systems, the condition of the machines used in production process has become more important in quality control due to increased automation. It implies that, the maintenance of the machines should be planned optimally to produce the products efficiently (Ben-Daya and Rahim, 2000).

Complex manufacturing systems are facing a large set of defects. In such systems, managers are investigating large amounts of investment in flexible inspection policies and management issues (Hejazi and Roozkhosh, 2019). The proper operation of machines is a prerequisite for producing high-quality products. In order to control the quality of the produced items in a manufacturing system, the variation of the production process has to be reduced. Statistical process control (SPC) is an effective method to reduce process variation and improve process stability. This is while a proper maintenance planning can reduce the process variation and increase the quality of products (Rasay et al., 2019). Control charts are the most important tools to monitor the production processes, and quality engineers can employ them in manufacturing systems (Salmasnia et al., 2019). Control chart is one of the tools of SPC which can detect the assignable cause of the process and reduce the process variation (Rasay et al., 2017).

The simultaneous consideration of quality control, maintenance and production can reduce operational costs, improve the efficiency of the whole system, and, thus, ensure high-quality products. The integrated planning of these three factors is an important challenge for manufacturing systems (Bahria et al., 2019). Lopes (2018) states that quality inspection, production planning and maintenance scheduling are the most important factors in manufacturing systems, and integrating these approaches improves the efficiency of the systems by reducing the process costs.

Production planning is an important activity in manufacturing systems. The purpose of production planning is to assign the available resources to the operations and make optimal decisions to increase the productivity and the flexibility of the production processes (Tavan and Sajadi, 2014). The availability of machines, their conditions and the time spent on maintenance actions affect production and inventory levels. With respect to the relationship between maintenance planning and production control, production processes can be planned optimally if the amount of production, the level of inventory and the safety stock are planned according to the maintenance plan. The quality of the produced items also affects the amount of production. If defective and non-conforming items are produced, they are discarded, recycled, or sold as low-quality products. Therefore, the amount of defective items should be considered in production control. Since maintenance plans can affect the quality of products, they should also be considered as an influential parameter in the determination of production level and the policies of inventory control.

The deterioration of machines and processes may lead to the producing of non-conforming items. The production process in different manufacturing systems deteriorates due to the occurrence of one or several assignable causes. This implies that production processes should be assumed imperfect and that the items produced in such imperfect processes may be defective and non-conforming (Lopes, 2018). Therefore, studying the behavior of imperfect production systems can help to better plan these systems (Chakraborty and Giri, 2012). In deteriorating production processes, it is assumed that the process has two operational states including in-control and out-of-control states. In an in-control state, the production process is under control and there is no machine failure in the process and the produced items are conforming. In an out-of-control state, however, the machinery may be damaged and

defective and non-conforming items may be produced. In such systems, it is assumed that the process is initially in an in-control state, but, as the time goes on, due to the occurrence of an assignable cause, the machines deteriorate and the process state shifts to an out-of-control state.
In some studies, in order to control the operational state of the production process, it is assumed that the deterioration mechanism of the process follows a specific continuous distribution or a general continuous distribution (Rasay et al., 2018). The failure mechanism of the process depends on how long the process has been in an in-control state; the process can shift to an out-of-control state due to the occurrence of an assignable cause.
In this study, an integrated model is proposed for statistical process control, maintenance planning and production control. The interrelationships among these factors are also investigated. Using the proposed model, the optimal values of control chart parameters as well as maintenance and production parameters are determined. Also, optimal policies are delineated to plan these factors.
The rest of the paper is organized as follows. In Section 2, a literature review is presented about the quality control, maintenance and production control. Section 3 is given to the statement of the problem. Section 4 explains the proposed integrated model. In section 5, a numerical example is presented and analyzed. In Section 6, the results of the sensitivity analysis of the integrated model are presented. Finally, Section 7 concludes the paper.

2. Literature review

Most researchers studied maintenance, production control and quality control, separately. Golpira and Tirkolaee (2019) proposed a bi-objective model for stable maintenance tasks scheduling. In this study, the repair tasks duration was considered uncertain. The purpose of the model was to schedule the maintenance tasks to minimize the total cost. They applied different multi-objective decision-making methods to solve the proposed model and the best solution technique was selected by using Displaced Ideal Solution (DIS) method. Tirkolaee et al. (2019) proposed a fuzzy multi-objective multi-period production planning and fuzzy seasonal demand was taken into account. They proposed a mixed-integer linear programming model to minimize the total cost of the system and maximize the customers' satisfaction level. The model was solved using a weighted goal programming method. Goli et al. (2019) presented a multi-objective mixed-integer linear programming for production planning and scheduling in flow shop manufacturing systems. They considered just-in-time delivery policy and uncertain processing time.
Polotski et al. (2019) integrated production control and maintenance planning for manufacturing-remanufacturing system that consists of a flexible machine and can share its production time between manufacturing mode and remanufacturing mode. The process deterioration was indicated by an increasing failure rate that leads to the loss of machine availability. In this study, preventive maintenance was applied to restore the machine. They presented a mathematical model using stochastic dynamic programming approach.
Integrated planning of maintenance, quality control and production/inventory control can lead to provide more effective mathematical model for production processes' planning. In the following, the studies on the integration of quality control, maintenance, production and inventory control are classified and discussed. The integrated models can be classified based on different factors such as process failure mechanism, quality control policy, maintenance policy, the impact of maintenance action on the system, production control system, type of objective function, and solving method. Some studies considered a specific distribution for process failure mechanism (i.e.; exponential or weibull) (Salmasnia et al., 2018; Pandey et al., 2011) and a few studies considered a general distribution for failure mechanism

(Nourelfath et al., 2016; Fakher et al., 2017) which different distributions can be applied in such mathematical models. Different quality control policies can be applied to monitor the process. Control charts are the most useful statistical process control tools applied in researches (Salmasnia et al., 2017; Bahria et al., 2019). Acceptance sampling, 100% inspection and periodic inspection are the other quality control policies which used in some studies (Radhoui et al., 2010; Bouslah et al., 2015; Wang and Sheu, 2003; Ben-Daya, 2002). In the integrated models, different maintenance policies are applied to maintain the process in a proper condition. PM[1] action and CM[2] action are the most useful policies that used as maintenance actions. The effect of maintenance actions on processes is different. In some studies the maintenance actions are considered perfect which return the process to as-good-as-new condition (Pandey et al., 2011; Salmasnia et al., 2018) and in some studies the maintenance actions are considered imperfect which return the process to as-bad-as-old condition. In joint planning for maintenance, quality control and production/inventory control, different policies are applied for production/inventory control. Many researchers applied EPQ models in their integrated planning (Rahim, 1994; Ben-Daya, 2002; Salmasnia et al., 2018). A few researchers used lot-sizing problem, buffer stock and production scheduling (Nourelfath et al., 2016; Cheng et al., 2018; Lopes, 2018; Bahria et al., 2019; Duffuaa et al., 2019).

Rahim (1994) developed an integrated model for EPQ[3] , an inspection policy and the control chart design of an imperfect production process. The quality of the product was controlled by an $\bar{x}$ control chart. The objective of the model was to determine the optimal control chart parameters and production quantity so as to minimize the expected total cost per unit time. The results obtained showed that a scheme of non-uniform and decreasing process inspection intervals would result in a lower cost than a uniform inspection scheme. Radhoui et al. (2010) studied PM policy and quality control for an imperfect production system. Decisions about PM were based on the proportion of the defective products. Simulations and experiments were conducted to determine the optimal inventory size, the rate of non-conforming items, and the optimal scheduling of PM actions. Pandey et al. (2011) developed a model for the integrated planning of quality control, maintenance policy and production scheduling in a manufacturing system. First, a model was developed to integrate maintenance planning and process quality control policies. It provided an optimal PM interval and control chart parameters to minimize the expected cost per unit time. Subsequently, the optimal PM interval was integrated with the production schedule in order to determine an optimal batch sequence that would minimize the penalty-cost incurred due to schedule delays. In this study, failure mechanism was assumed as a weibull distribution.

Salmasnia et al. (2017) considered EPQ, statistical process control and maintenance simultaneously to optimize the production process. They used the particle swarm optimization (PSO) algorithm to minimize the expected total cost per production cycle. They also used an imperfect production process with two states including in-control and out-of-control states. Then, an $\bar{x}$ control chart was employed to monitor the process quality, and PM and CM served as maintenance actions. In another study, Salmasnia et al. (2018) presented an integrated model for production control and maintenance planning that employed the VP-T^2 Hotelling control chart to monitor multiple quality characteristics. With regard to the complexity of the problem, the expected total cost per time unit was optimized using the PSO algorithm. In this study, failure mechanism was assumed as an exponential distribution.

[1] Preventive maintenance
[2] Corrective maintenance
[3] Economic production quantity

Nourelfath et al. (2016) integrated production, maintenance and quality for an imperfect process in a multi-period multi-product capacitated lot-sizing context. The production system was modeled as an imperfect machine, while the status of the system was considered to be either in-control or out-of-control. To control the quality characteristic, a control chart was used. During each period, the machine was inspected, and imperfect preventive maintenance activities were simultaneously done. The objective was to minimize the total cost while satisfying the demand for all the products. The optimization model allows for the joint selection of the optimal values of the production plan and the maintenance policy while taking into account the quality related costs. Bahria et al. (2019) developed an integrated approach for the joint control of production, maintenance and quality for batch manufacturing systems. The quality control of the produced lots was performed using an $\bar{x}$ control chart. With this graphical tool, the quality of the batch was estimated and perfect preventive or corrective maintenance actions could possibly be taken on the production process. A buffer stock was also built to maintain the continuity of supply during the maintenance actions. The objective of the proposed model was to minimize the expected total cost.

Liao et al. (2009) and Wee and Widyadana (2013) integrated EPQ and PM for an imperfect production process. They assumed that all the defective produced items should be reworked. Radhoui et al. (2010) used a 100% inspection policy to determine the proportion of non-conforming produced items in each produced lot. Then, they took a PM action and performed overhauls by comparing this ratio to a certain limit. Wang and Sheu (2003) studied optimal policies for production, inspection and maintenance with inspection errors. They made periodic inspections to control the quality of the process. Ben-Daya (2002) proposed an integrated model for the joint control of EPQ, quality and maintenance in an imperfect production process with different levels of PM. The produced items were periodically inspected for their quality. Hadidi et al. (2012) reviewed the models of production planning and scheduling integrated with maintenance and quality inspections.

Bouslah et al. (2015) considered the problem of integrated production, preventive maintenance and quality control for a stochastic production system subject to both reliability and quality deteriorations. A make-to-stock production strategy was adopted to protect serviceable stocks against uncertainties. Quality control was also performed using a single acceptance sampling plan. The PM action was imperfect, and the objective of the proposed model was to minimize the total costs. Lopes (2018) studied the influence of a quality inspection policy on an imperfect production system in which a percentage of the produced items was inspected. After each production cycle, preventive maintenance was performed. A buffer stock was established to meet the demand while preventive maintenance was being performed. Lopes also postulated that the defective items detected on inspection had to be sent for reworking. The model was formulated with the objective of minimizing the total expected cost. Bouslah et al. (2017) developed an integrated model for production design, quality control policies and maintenance in multi-stage systems. In this study, the duration of PM and CM actions was considered as a random variable that followed a general distribution. In order to control the quality of the produced items, the optimal level of inspection was determined. The objective of the proposed model was to minimize the total costs and a simulation approach was applied for optimization.

Cheng et al. (2018) developed an integrated model of production lot sizing, quality control and condition-based maintenance for an imperfect production system. It was assumed the system would produce a single type of product to meet a constant demand. To provide protection for the stock against uncertainties, a make-to-stock production policy was adopted. The condition-based maintenance policy consisted of inspections to be done at the end of the production runs to evaluate the system condition and perform imperfect preventive maintenance if the detected degradation level exceeded the threshold. The quality control was performed using a 100% inspection policy to obtain the proportion of defectives. The aim of the research was to jointly optimize the lot size, the inventory threshold, the preventive maintenance and the overhaul thresholds such that the total cost per unit time would be minimized. A stochastic mathematical model was formulated and solved by a simulation-based optimization approach. Pal et al. (2013) assumed that defective products could be produced in both in-control and out-of-control states, and the probability of producing defective items in an in-control state would be less than that in an out-of-control state. The defective items were reworked, and the optimal inventory size and the production plan were determined to minimize the total expected cost. Fakher et al. (2017) investigated a capacitated production system to produce multiple-type products in multiple periods. During each period, the system was inspected, and imperfect PM activities were done to reduce its age, proportional to the PM level. At the end of each period, a complete repair was performed. They developed an integrated optimization model with the objective of maximizing the expected profit. Duffuaa et al. (2019) presented a model to plan the production, maintenance and quality simultaneously. At first, they optimize preventive maintenance schedule. Then, they planned production scheduling and inventory control.

Wang et al. (2019) presented an integrated model for production, maintenance and quality control for a capacitated lot-sizing production system. Imperfect PM and an overhaul were performed after each batch production and non-conforming items were detected using inspection. A genetic algorithm was applied to solve the proposed model. Gomez et al. (2020) presented an integrated policy for production, maintenance and quality control in a deteriorating production system. In this study, dynamic sampling plan was applied to control the quality of the products. The purpose of the model was to determine an optimal production policy, preventive maintenance and quality control rates to minimize the expected total cost of the manufacturing system. A simulation technique was applied to optimize the presented model. For comparison purpose, table 1 summarizes the main features of the most important studies mentioned above.

Bahria et al. (2018) integrated maintenance and quality control in deteriorating manufacturing systems. In this study an $\bar{x}$ control chart was applied for quality control. The objective of the proposed model was to minimize the expected total cost of the process. In our study the proposed model by Bahria et al. (2018) is developed. We developed a mathematical model for joint planning of maintenance, quality control and production control. In the developed model, there is no restriction on the type of the control chart. Therefore, different control charts can be applied in practice to monitor the process for quality control. The lot size produced in each production cycle can be determined with respect to the production rate of the machine and the proportion of conforming and non-conforming items produced in each cycle.

Table 1. Comparison of different studies on integrated models

Author(s)	Failure mechanism	Quality control policy	Maintenance policy	Production/inventory control	Objective function
Rahim (1994)	General distribution	$\bar{x}$ control chart	-	EPQ	Total cost minimization
Radhoui et al. (2010)	-	100% inspection	perfect PM	Determination of buffer stock inventory	Total cost minimization
Pandey et al. (2011)	Weibull distribution	$\bar{x}$ control chart	Perfect PM	Production scheduling	Total cost minimization
Salmasnia et al. (2017)	Weibull distribution	$\bar{x}$ control chart	Perfect PM and CM	EPQ	Total cost minimization and PSO algorithm for solving
Salmasnia et al. (2018)	Exponential distribution	$\bar{x}$ control chart	Perfect PM and RM[1]	EPQ	Total cost minimization and PSO algorithm for solving
Nourelfath et al. (2016)	General distribution	$\bar{x}$ control chart	Periodic PM	Lot-sizing problem	Total cost minimization
Bahria et al. (2019)	-	$\bar{x}$ control chart	Perfect PM and CM	Determination of safety stock level	Total cost minimization
Wang and Sheu (2003)	General distribution	Periodic inspection	Periodic overhaul	EMQ	Total cost minimization and using markov chain
Ben-Daya (2002)	General distribution	Periodic inspection	Periodic PM	EPQ	Total cost minimization
Bouslah et al. (2015)	-	Acceptance sampling and 100% inspection	Perfect PM and CM and overhaul	Determination of inventory level	Total cost minimization
Bouslah et al. (2017)	-	Acceptance sampling and 100% inspection	CM and age-based PM	Determination of buffer stock	Total cost minimization and using simulation
Lopes (2018)	-	Inspecting a percentage of produced items	PM	Determination of buffer stock	Total cost minimization
Cheng et al. (2018)	-	100% inspection	PM, CM and overhaul	Determination of lot-size	Total cost minimization and using simulation
Pal et al. (2013)	Exponential distribution	Inspecting the produced items	PM	EPQ	Total cost minimization
Fakher et al. (2017)	General distribution	Inspecting the produced items	PM	Production planning	Total profit maximization
Duffuaa et al. (2019)	-	$\bar{x}$ control chart	Perfect PM and CM	Production scheduling	Total cost minimization
Wang et al. (2019)	-	100% inspection	Imperfect PM and overhaul	Production lot-sizing problem	Total cost minimization
Gomez et al. (2020)	-	Dynamic sampling plan	Perfect PM	Determination of inventory level	Total cost minimization
Bahria et al. (2018)	-	$\bar{x}$ control chart	Perfect PM and CM	-	Total cost minimization
This paper	General distribution	A proper control chart	Perfect PM and CM, Imperfect minimal maintenance	Determining the production level in each production cycle	Total cost minimization using grid search algorithm

[1] Reactive maintenance

The contributions of the present study are as follows:

- Presenting an integrated mathematical model of maintenance, quality control and production control for deteriorating production systems where the process failure mechanism follows a general continuous distribution at a non-decreasing failure rate
- The applicability of the proposed model for different control charts
- Considering different levels of maintenance including perfect maintenance and imperfect maintenance
- Applying the lot size determination policy to determine the production level in each production cycle according to the production rate of the machine and the proportion of conforming and non-conforming items produced in each production cycle

3. Problem description

Consider an imperfect production system with a single machine that produces a product with production rate R per time unit. This process has an in-control state and an out-of-control state. Each production cycle starts in the in-control state. However, with the use of machine, the process gradually deteriorates. The deterioration of the process is considered as the process state transition from in-control state to the out-of-control state. The process state shifts from the in-control state to an out-of-control state after passing a random time as *t* from the start of the cycle, due to the occurrence of an assignable cause. It is assumed that the occurrence of the assignable cause can be attributed to the machine failure and the time of that occurrence follows a general continuous distribution. The probability of producing defective items in the out-of-control state is higher than that in the in-control state.
To monitor the process and detect the transition of the process state during a production cycle, sampling inspection is conducted at time points $t_1, t_2, \dots, t_{m-1}$, which are decision variables of the model. In this case, *n* units of the items produced in the process are selected, a suitable quality characteristic is measured, a proper statistic is calculated, and then the statistic is plotted on a desired control chart. If the statistic falls within the control limits of the control chart, the process is assumed to be in an in-control state, and it continues its operation without any interruption. If it falls outside the control limits, the process is probably in an out-of-control state, and an alarm is issued from the control chart. After that, an investigation is performed on the system to verify this alarm. If it is concluded that the process is in an in-control state (i.e., the chart signal is incorrect), the process continues its operation without any interruption. However, if it is concluded that the process is in an out-of-control state (i.e., the chart signal is correct), the process is interrupted and a minimal maintenance (MM) is implemented on the system. Henceforth, the investigation performed after the release of the alarm from the control chart is called the maintenance inspection to distinguish it from the sampling inspection. After the MM action, the process returns to the in-control state, and the production cycle continues.
At the end of the production cycle, once a lot with size Q is produced, the process is interrupted and investigated. At this stage, there is no sampling from the produced items, but only maintenance inspection is done to determine the real state of the process. If the maintenance inspection indicates that the system is in an in-control state, PM is conducted on the process. However, if the maintenance inspection indicates that the system state is out-of-control, CM is implemented. The described system is indicated in figure 1.

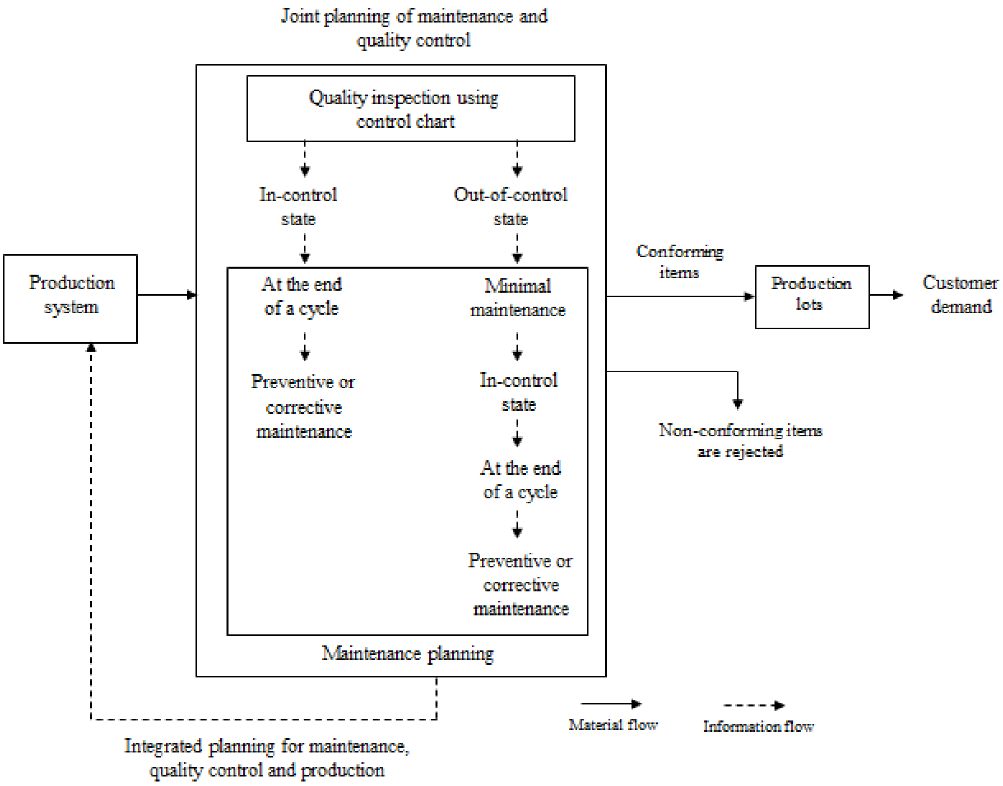


Figure 1. The schema of the manufacturing system and the integrated planning of production, maintenance and quality control

It is assumed that PM and CM actions are perfect and they cause the process to return to as-good-as-new conditions. In contrast, MM action is assumed to be imperfect because they make the process return to the in-control state and after MM action the production cycle continues. Therefore, each production cycle of the process starts in an in-control state and at age zero, but, after a lot with size Q is produced, it is terminated due to the implementation of PM or CM action. Figure 2 illustrates an example of the evolution of a production cycle and the production of a lot with size Q in a production process where two minimal maintenance actions are performed during the cycle. The process produces the items with rate R and then the process state may shift from in-control state to the out-of-control state and the control chart detects the state transition. MM action is conducted on the process and returns the process to the in-control state and the production cycle continues. After producing a lot with size Q, the process is terminated and PM action or CM action is conducted on the process with respect to the process state at the end of the cycle. After that the process returns to as-good-as-new condition and the production cycle is ended.

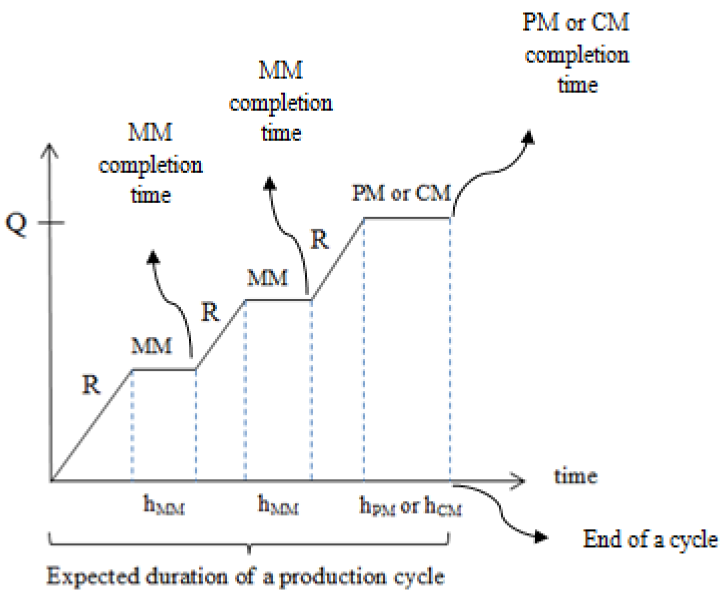


Figure 2. Example of a production cycle

The assumptions considered in this study are as follows.

1. The production rate is equal to the demand rate
2. The process produces one type of product with a single machine.
3. The production process initially is in an in-control state.
4. The quality of the products is associated with the deterioration of the process.
5. The PM action and CM action are perfect and return the process to as-good-as-new condition.
6. The MM action is imperfect and returns the process to an in-control state.
7. The time duration of maintenance actions are deterministic.
8. The values of the unit costs are known and constant.
9. The time of the process state transition follows a general continuous distribution.

The objective of the proposed model is to minimize the expected total cost per unit time through computing the values of such decision variables as sample size, sampling intervals, number of samplings (number of inspection periods), control chart limits, and the lot size produced. The expected total costs in a production cycle are as follows:

- The expected maintenance cost including the cost of performing minimal maintenance, PM action and CM action
- The expected quality control cost including maintenance inspection cost, sampling inspection cost and false alarms cost

Finally, by the calculation of the expected total cost in a production cycle and the expected duration of a production cycle, the expected total cost per unit time is obtained.

4. Developing the integrated model

In this section of the study, an integrated model is presented with the following notations:

Notation	**Description**
Decision variables	
n	Sample size
k	Control chart limit
t_i $(i=1,..,m-1)$	Time points of inspections

m	Maximum number of inspections in each production cycle
t_m	Maximum duration of each production cycle
Q	The size of the lot produced in a production cycle
Parameters	
C_0	Operational cost of the system while the process is in an in-control state
C_1 ($C_1 \geq C_0$)	Operational cost of the system while the process is in an out-of-control state
C_f	Fixed cost of sampling inspection
C_v	Variable cost of sampling inspection
C_{PM}	Preventive maintenance cost
C_{CM}	Corrective maintenance cost
C_{MM}	Minimal maintenance cost ($C_{MM} < C_{PM} < C_{CM}$)
C_I	Cost of investigating the process and specifying the actual state of the process
R	Production rate of the process per unit time
P_0	The probability of producing conforming items in an in-control state
P_1	The probability of producing conforming items in an out-of-control state
h_{PM}	Time duration of performing preventive maintenance action
h_{CM}	Time duration of performing corrective maintenance action
h_{MM}	Time duration of performing minimal maintenance action
t	The time of the occurrence of the assignable cause (random variable)
$f(t)$	Probability density function for the process failure mechanism
$F(t)$	Cumulative distribution function for the process failure mechanism
$\bar{F}(t) = 1 - F(t)$	Complement of the cumulative distribution function
α	The probability of error type I
β	The probability of error type II
$E[T_0]$	Expected time duration that the process is in an in-control state within each production cycle
$E[T_1]$	Expected time duration that the process is in an out-of-control state within each production cycle
$E[QC]$	Expected number of sampling inspections during each production cycle
$E[Q_0]$	Expected number of conforming items produced in a production cycle
$E[Q_1]$	Expected number of non-conforming items produced in a production cycle
E[MM]	Expected number of minimal maintenance actions conducted on the process in each production cycle
$E[N_{FA}]$	Expected number of false alarms in each production cycle
P_{PM}	Probability of performing the preventive maintenance action at the end of a production cycle
P_{CM}	Probability of performing the corrective maintenance action at the end of a production cycle
ARL_0	Lower limit for average run length (ARL) in an in-control state
ARL_1	Upper limit for ARL in an out-of-control state

4.1. Expected total cost per unit time (ETC)

The production cycles of this model can be considered as a renewal reward process consisting of stochastic and independent identical cycles. $E[C]$ is the expected total cost of each production cycle and $E[T]$ is the expected duration of each production cycle. Therefore, the expected total cost of the process per unit time (ETC) can be computed using the following equation:

$$ETC = \frac{E[C]}{E[T]} \tag{1}$$

If $m > 1$, $E[C]$ and $E[T]$ can be obtained from the following equations:

$$E[C] = C_0 E[T_0] + C_1 E[T_1] + (C_f + nC_v)E[QC] + C_I E[N_{FA}] + C_{MM} E[MM] + C_{CM} P_{CM} + C_{PM} P_{PM} \tag{2}$$

$$E[T] = t_m + h_{CM} P_{CM} + h_{PM} P_{PM} \tag{3}$$

If $m = 1$, the values of $E[MM]$, $E[N_{FA}]$, $E[QC]$ are zero. Thus, for the special case of $m = 1$, $E[C]$ and $E[T]$ are computed as follows:

$$E[C] = C_0 E[T_0] + C_1 E[T_1] + C_{PM} P_{PM} + C_{CM} P_{CM} \tag{4}$$

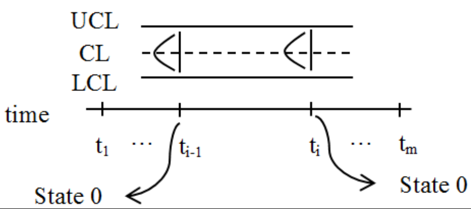
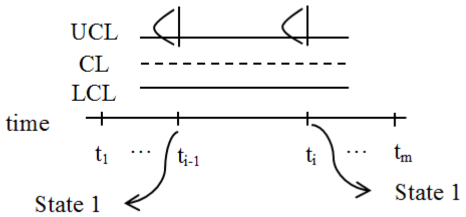
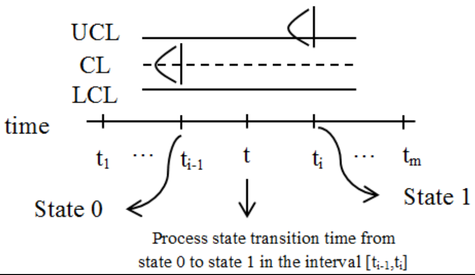
$$E[T] = t_m + h_{CM} P_{CM} + h_{PM} P_{PM} \tag{5}$$

In Equation (2), the first statement calculates the expected operational cost of the process in the in-control state. The second statement calculates the expected operational cost of the system in the out-of-control state. The third statement calculates the expected cost of sampling inspections. The fourth statement calculates the expected cost of false alarms. The fifth, sixth and seventh statements calculate the costs of maintenance actions. In Equation (3), the first statement indicates the time when the process is stopped for conducting PM or CM action, and the second and third statements calculate the time duration of performing maintenance actions at the end of the cycle.

4.2. Different scenarios for the process in an inspection interval

Different scenarios for process evolution in an inspection interval and the probability of occurrence of each scenario are discussed here. Consider a single arbitrary inspection interval (t_{i-1}, t_i). There are three scenarios for the process during each inspection interval. At t_{i-1} and after inspection at this time point, the process can be in the in-control state or the out-of-control state. These scenarios are illustrated in Table 2.

Table 2. Different scenarios for the process evolution in an inspection interval (t_{i-1}, t_i) and their corresponding probabilities

Scenario	Figure*	Probability of occurrence	Out-of-control time duration	In-control time duration
a		$P(a_{t_{i-1}}) = \frac{\bar{F}(t_i)}{\bar{F}(t_{i-1})}$	0	$t_i - t_{i-1}$
b		$P(b_{t_{i-1}}) = 1$	$t_i - t_{i-1}$	0
c		$P(c_{t_{i-1}}) = \int_{t_{i-1}}^{t_i} \frac{f(t)dt}{\bar{F}(t_{i-1})}$	$t_i - t$	$t - t_{i-1}$

* An in-control state is denoted by 0 and an out-of-control state by 1.

Scenario a: In this scenario, the process is in an in-control state at time point t_{i-1} and remains in this state until t_i. Therefore, the probability of occurrence of Scenario a is computed using the following equation:

$$P(a_{t_{i-1}}) = P(t > t_i | X_{t_{i-1}} = 0) = P(t > t_i | t > t_{i-1}) = \frac{\bar{F}(t_i)}{\bar{F}(t_{i-1})} \quad (6)$$

Scenario b: In this scenario, the process is in an out-of-control state at time point t_{i-1} and remains in this state until t_i. Therefore, the probability of occurrence of Scenario b is computed using the following equation:

$$P(b_{t_{i-1}}) = P(X_{t_i} = 1 | X_{t_{i-1}} = 1) = 1 \quad (7)$$

Scenario c: In this scenario, the process is in an in-control state at time point t_{i-1}. At a random time t $(t_{i-1} < t < t_i)$, the process shifts to an out-of-control state and the process is in the out-of-control state at time point t_i . Therefore, the probability of occurrence of Scenario c is obtained as follows:

$$P(c_{t_{i-1}}) = P(t_{i-1} < t < t_i | X_{t_{i-1}} = 0) = \int_{t_{i-1}}^{t_i} \frac{f(t)dt}{\bar{F}(t_{i-1})} \quad (8)$$

4.3. The probability of the process in state 0 or 1 just after the inspection at t_i

The process state just after the inspection at time point t_i *(i = 1,2,…,m-1)* can be 0 or 1. $P_{t_i}^0$ is defined as the probability of operating the process in an in-control state just after the inspection at t_i . $P_{t_i}^0$ is computed using the following equation:

$$P_{t_i}^0 = 1 - F(t_i) = \bar{F}(t_i); i = 1,2,\dots,m \tag{9}$$

This equation derived from this fact that the transition of the state occurs after t_i and the process is in the in-control state at time point t_i .

$P_{t_i}^1$ is defined as the probability of operating the process in an out-of-control state just after the inspection at t_i . $P_{t_i}^1$ is computed using the following recursive formula:

$$\begin{aligned} P_{t_i}^1 &= \beta\left[P_{t_{i-1}}^1 \times P\left(b_{t_{i-1}}\right) + P_{t_{i-1}}^0 \times P\left(c_{t_{i-1}}\right)\right] \\ &= \beta\left[P_{t_{i-1}}^1 + P_{t_{i-1}}^0 \times P\left(c_{t_{i-1}}\right)\right]; \quad i = 1,2,\dots,m-1 \end{aligned} \tag{10}$$

Equation (10) indicates the probability of the process is in an out-of-control state just before the t_i'th inspection period. Therefore, at time point t_{i-1} , the process is in the in-control state with the probability of scenario b or the process is in the out-of--control state with the probability of scenario c. If the process is in the out-of-control state before t_i, the control chart is unable to detect the out-of-control state of the process with probability β; therefore, the process continues its operation after the inspection at time point t_i .

At the last inspection period, there is no sampling from the process, therefore $P_{t_m}^1$ is computed using the following equation:

$$P_{t_m}^1 = P_{t_{m-1}}^1 + P_{t_{m-1}}^0 \times P\left(c_{t_{m-1}}\right) \tag{11}$$

4.4. Expected in-control time duration in each production cycle

T_0^i is defined as the expected time duration of the process in the in-control state at the inspection interval (t_{i-1}, t_i). Therefore, with respect to the number of sampling inspection in a cycle, m, $E[T_0]$ can be obtained using the following equation:

$$E[T_0] = \sum_{i=1}^{m} T_0^i \tag{12}$$

Also, T_0^i is computed using the following equation:

$$T_0^i = P_{t_{i-1}}^0 \left[P\left(a_{t_{i-1}}\right)(t_i - t_{i-1}) + \int_{t_{i-1}}^{t_i} \frac{f(t)dt}{\bar{F}(t_{i-1})}(t - t_{i-1})dt\right]; \tag{13}$$

$i = 1,2,\dots,m$

Equation (13) is derived based on the different scenarios described above. If the process is in an in-control state at time point t_{i-1} and scenario a or scenario c occurs, the process operates in state 0 in one part of interval (t_{i-1}, t_i). If scenario a occurs, the process operates in state 0 in the whole interval (t_{i-1}, t_i). If scenario c occurs, the process operates in state 0 in $t - t_{i-1}$ units of time.

4.5. Expected out-of-control time duration in each production cycle

T_1^i is defined as the expected time duration of the process in an out-of-control state at the inspection interval (t_{i-1}, t_i). Therefore, with respect to the number of sampling inspection in a cycle, m, $E[T_1]$ can be obtained using the following equation:

$$E[T_1] = \sum_{i=1}^{m} T_1^i \tag{14}$$

Also, T_1^i can be computed using the following equation:

$$T_1^i = P_{t_{i-1}}^0 \left[\int_{t_{i-1}}^{t_i} \frac{f(t)dt}{\bar{F}(t_{i-1})}(t_i - t)dt\right] + P_{t_{i-1}}^1 (t_i - t_{i-1}); \quad i = 1,2,\dots,m \tag{15}$$

Equation (15) is derived based on the different scenarios described above. If scenarios b and c occur, the process operates in an out-of-control state in one part of interval (t_{i-1}, t_i). The process operates in state 1 in t_i-t units of time if the process is in an in-control state at time point t_{i-1} and scenario c occurs. The process is in state 1 in t_i-t_{i-1} units of time if, at t_{i-1}, the process is in state 1 and scenario b occurs.

4.6. Expected number of the minimal maintenance (MM) actions performed on the process in a production cycle

P_{MM}^i is defined as the probability of performing minimal maintenance actions after the inspection at time point t_i . Therefore, with respect to the number of sampling inspection in a cycle, *m*, the expected number of the minimal maintenance actions conducted on the process in each production cycle, *E*[*MM*], can be calculated using the following equation:

$$E[MM] = \sum_{i=1}^{m} P_{MM}^i \tag{16}$$

P_{MM}^i is computed by the following equation:

$$P_{MM}^i = (1-\beta)\left[P_{t_{i-1}}^1 + P_{t_{i-1}}^0 \int_{t_{i-1}}^{t_i} \frac{f(t)dt}{\bar{F}(t_{i-1})}\right]; i = 1,2,\dots,m-1 \tag{17}$$

Equation (17) indicates the probability of the process to be in the out-of-control state just before the inspection at t_i . If the process is in state 1 before the inspection at t_i , the control chart detects the out-of-control state of the process with probability 1-β. After a signal is released from the control chart, a minimal maintenance action is conducted.

4.7. Probability of performing CM action in a production cycle

In the last inspection period, t_m, there is no sampling inspection; it is just the process state to investigate. If the process is in the in-control state, a PM action is performed on it. If the process is in the out-of-control state, however, a CM action is performed. Therefore, the probability of performing corrective maintenance action on the process at the end of each production cycle can be obtained by the following equation:

$$P_{CM} = P_{t_{m-1}}^1 + P_{t_{m-1}}^0 \int_{t_{m-1}}^{t_m} \frac{f(t)dt}{\bar{F}(t_{i-1})} \tag{18}$$

This equation derived from this fact that if the process is in the out-of-control state at t_m, CM action is conducted. The process is in the out-of-control state at t_{m-1} if scenario b occurs or is in the in-control state if scenario c occurs.

4.8. Probability of performing PM action in a production cycle

As mentioned before, PM action or CM action is performed on the process at the end of a production cycle. Thus, P_{PM} is computed using the following formula:

$$P_{PM} = 1 - P_{CM} \tag{19}$$

4.9. Expected number of the sampling inspections in each production cycle

Let P_{QC}^idenote the probability of performing a sampling inspection at the end of interval (t_{i-1} , t_i) at t_i . With respect to the number of sampling inspection in a cycle, *m,* the expected number of the sampling inspections during each production cycle is computed using the following equation:

$$E[QC] = \sum_{i=1}^{m-1} P_{QC}^{i} \tag{20}$$

Also, P_{QC}^{i} can be obtained as follows:

$$P_{QC}^{i} = P_{t_{i-1}}^{0} + P_{t_{i-1}}^{1}; i = 1,2, \dots, m-1 \tag{21}$$

4.10. Expected number of the false alarms in each production cycle

P_{FA}^{i} is defined as the probability of a false alarm released from a control chart at time point t_i . Therefore, the expected number of those alarms in each production cycle, $E[N_{FA}]$, can be computed using the following formula:

$$E[N_{FA}] = \sum_{i=1}^{m-1} P_{FA}^{i} \tag{22}$$

The parameter P_{FA}^{i} is computed as follows:

$$P_{FA}^{i} = \alpha \times \bar{F}(t_i); i = 1,2, \dots, m-1 \tag{23}$$

This equation derived from this fact that the process is in the in-control state at time point t_i and the process state transits to the out-of-control after t_i but the control chart releases the out-of-control alarm at t_i with the probability α.

4.11. The integrated model

The integrated model is presented in the following.

$$Minimize\ \ ETC(t_1, \dots t_{m-1}, t_m, m, k, n, Q) = \frac{E[C]}{E[T]}$$

$$Subject\ to\ \ \frac{1}{\alpha} > ARL_0, \quad \frac{1}{1-\beta} \le ARL_1 \tag{24}$$

In this integrated model, the values of $E[C]$ and $E[T]$ are computed using Equations (2) and (3) respectively. The decision variables of the model are sample size (n), control chart limit (k), the inspection time points $(t_1, t_2, \dots, t_{m-1})$, maximum duration of a production cycle (t_m), maximum number of the inspection periods (m) and size of the produced lot in a production cycle (Q). Constraint $\frac{1}{\alpha} > ARL_0$ guarantees that the ARL in the in-control state is greater than ARL_0 and this constraint helps to avoid excessive false alarms in the control chart in quality control of the process. Constraint $\frac{1}{1-\beta} \le ARL_1$ guarantees that the ARL in the out-of-control state is less than ARL_1. This constraint promotes the early detection of an out-of-control condition of the process.

4.12. Computation of a lot size in each production cycle

The optimal lot size in each production cycle is computed with respect to the quality control policy, maintenance planning and the duration of the maintenance actions. The machine produces the products at a specified production rate (R). The expected number of conforming and non-conforming items in each cycle is computed with respect to the probability of their production and the duration of the production cycle. Then, the lot size, Q, can be computed by the sum of the expected numbers of the conforming and non-conforming items.

In each production cycle, the process is in the in-control state in $E[T_0]$ units of time and is in the out-of-control state in $E[T_1]$ units of time. Non-conforming items are produced with the probability 1-p_0 in the in-control state and 1-p_1 in the out-of-control state. Therefore, the expected number of non-conforming items produced in a cycle is obtained through the following equation.

$$E[Q_1] = R\,\{E[T_1](1-p_1) + E[T_0](1-p_0)\} \tag{25}$$

When minimal maintenance actions are performed on the process, the process is stopped. Thus, the maximum duration of the production cycle, the expected number of the minimal maintenance actions performed on the process in each cycle, the duration of the MM actions and the expected number of non-conforming items in the cycle are taken into account to calculate the expected number of the conforming items produced in each production cycle. The calculation is through the following equation:

$$E[Q_0] = \mathrm{R}\,(\mathrm{t_m} - h_{MM}.E[MM]) - \mathrm{E[Q_1]} \tag{26}$$

4.13. Inspection frequency

The inspection time points, t_i (i = 1, 2,…, m – 1), can take any arbitrary values. However, in practice, the inspection frequency and the values of the inspection time points should be designed logically. When the failure mechanism of the process follows a distribution except exponential distribution, 'constant hazard policy' can be applied in practice to determine the sampling inspection frequency in a production system (Panagiotidou and Tagaras, 2012). In this rule, in each inspection interval, the probability of quality shift does not change, given that the process still operates in the in-control state at the start of that interval. According to the rule, the first inspection time point, t_1, takes an arbitrary value and the other inspection time points can be obtained using the following equation:

$$\int_{t_{i-1}}^{t_i} h(t)dt = \int_{t_i}^{t_{i+1}} h(t)dt \;\; ; \;\; i = 1,2,\dots,m-1 \tag{27}$$

In this equation, $h(t)$ is the hazard rate function of the process failure mechanism, and it is obtained as follows:

$$h(t) = \frac{f(t)}{\bar{F}(t)} \tag{28}$$

If the process failure mechanism follows an exponential distribution, Equation (27) leads to a fixed sampling frequency. In the fixed inspection interval policy, once the optimal value of t_1 is determined, the other sampling inspection time points can be obtained using the following equation:

$$t_i = i \times t_1 \quad (i = 1, 2, \dots, m) \tag{29}$$

5. Numerical study

In this section, a numerical example is solved to evaluate the performance of the proposed model. Then, a sensitivity analysis is performed to analyze the effect of the critical parameters on *ETC* and the decision variables of the model. The numerical example is solved by a grid search algorithm in the MATLAB software. It is notable that, through a grid search algorithm, continuous variables are discretized within reasonable ranges.

In the integrated model proposed in this study, there is no restriction on failure mechanisms. It is, however, necessary for numerical examples to assume a specific distribution. In this respect, the Weibull distribution is used for failure mechanism in this numerical example. The density function of this distribution is in the following form:

$$f(t)= v\,\lambda^{v}\,t^{v-1}\,exp(-(\lambda t)^{v}) \quad t,v,\lambda \geq 0 \tag{30}$$

Where v is the shape parameter and λ is the scale parameter of the Weibull distribution. As the mean of the distribution, μ is calculated through the equation below:

$$\mu = \frac{1}{\lambda}\Gamma(1+\frac{1}{v}) \tag{31}$$

Since there is no restriction on the type of the control chart in this study, an $\bar{x}$ control chart is applied to monitor the process in this numerical example. In the $\bar{x}$ control chart, the control limits are as follows:

$$\mu_0 \pm k\frac{\sigma}{\sqrt{n}} \tag{32}$$

k denotes the width of the control limits, and it is a decision variable of the integrated model. Φ is the cumulative distribution function of the normal distribution. In the $\bar{x}$ control chart, α and β are obtained through the following equations:

$$\alpha = 2\emptyset(-k)$$
$$\beta = \emptyset\left(k-\delta\sqrt{n}\right) - \emptyset(-k-\delta\sqrt{n}) \tag{33}$$

The data are exploited from references Sett et al. (2016) and Linderman et al. (2005). The values of the input parameters of the numerical example are illustrated in Table 3.

Table 3. The values of the input parameters

δ	v	μ	C_0	C_1	C_{PM}	C_{CM}	C_{MM}	C_v	C_f	C_I	h_{PM}	h_{CM}	h_{MM}	R	P_0	P_1
1	2	20	100	500	2400	5000	500	0.2	10	200	4	4	2	100	0.85	0.65

The obtained results and the optimal values of the decision variables are indicated in Table 4.

Table 4. The optimal values of the objective function and the decision variables

ETC	t_1	k	n	m	t_m	Q
241.23	3.9	2.9	27	6	8.72	860.86

The results indicate that the process inspections should start at the time point of 3.9; the other time points for the inspections can be obtained using Equation 27. At each time point, a sample of 27 is taken from the items produced, and the quality characteristics of the products are measured. The control limit parameter of the $\bar{x}$ control chart is 2.9, and the maximum duration of a production cycle is 8.72. The optimal size for the lot produced in each production cycle is 860.86.

6. Sensitivity analysis

A series of sensitivity analyses have been conducted of some important parameters of the integrated model. The results are presented.

Based on Table 5, larger values of δ lead to an increase in the value of k, but it decreases the sample size. This is because, for the larger values of δ, the control chart has more power to detect the shift and it is easier for it to recognize the out-of-control state of the process. This fact can be seen in Figure 3.

Table 5. The obtained results for δ changes

δ	ETC	t_1	k	n	m	t_m	Q
0.5	242.72	3.9	2	40	6	8.72	861.5
1	241.23	3.9	2.9	27	6	8.72	860.86
1.5	240.86	2.3	3.3	8	50	16.1	1541.7

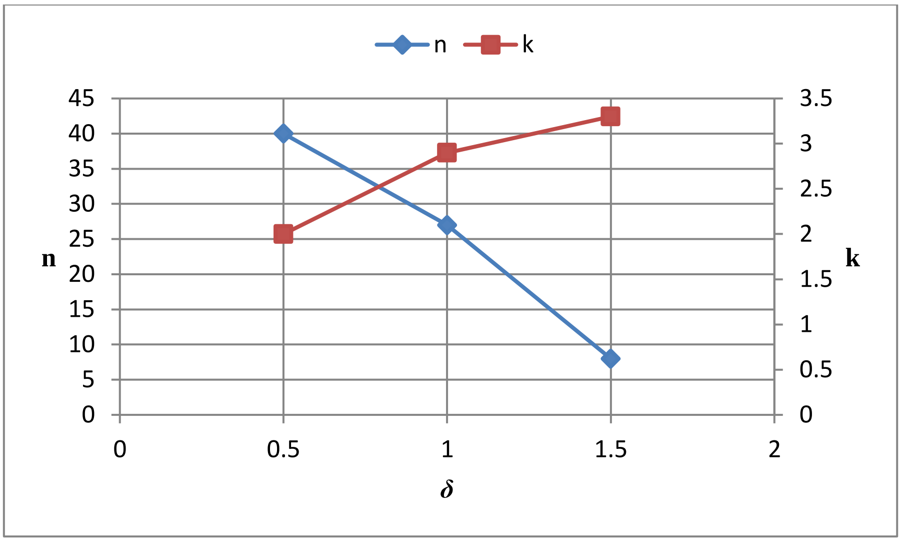


Figure 3. The effect of δ on the decision variables

The effect of the shape parameter (v) is shown in table 6 and Figure 4. As it can be seen, an increase in the value of parameter v, decreases the values of *ETC* and n but increases the values of t_1, t_m and Q. The increase in the value of t_1 can be explained based on the fact that, in the Weibull distribution, the increase of the shape parameter leads to the reduction of the variance of the distribution; therefore, with larger values of parameter v, it is easier to predict the failure time. In addition, the lot size, Q, increases due to an increase in the maximum duration of the cycle.

Table 6. The obtained results for *v* changes

V	ETC	t_1	k	n	m	t_m	Q
2	241.23	3.9	2.9	27	6	8.72	860.86
2.5	251.03	3.9	3	18	13	10.53	1033.2
3	250.37	4.9	3	17	13	11.21	1102.4
3.5	250.19	6.5	3	19	9	11.77	1159.6
4	245.82	6.5	3.1	15	12	11.83	1169.6

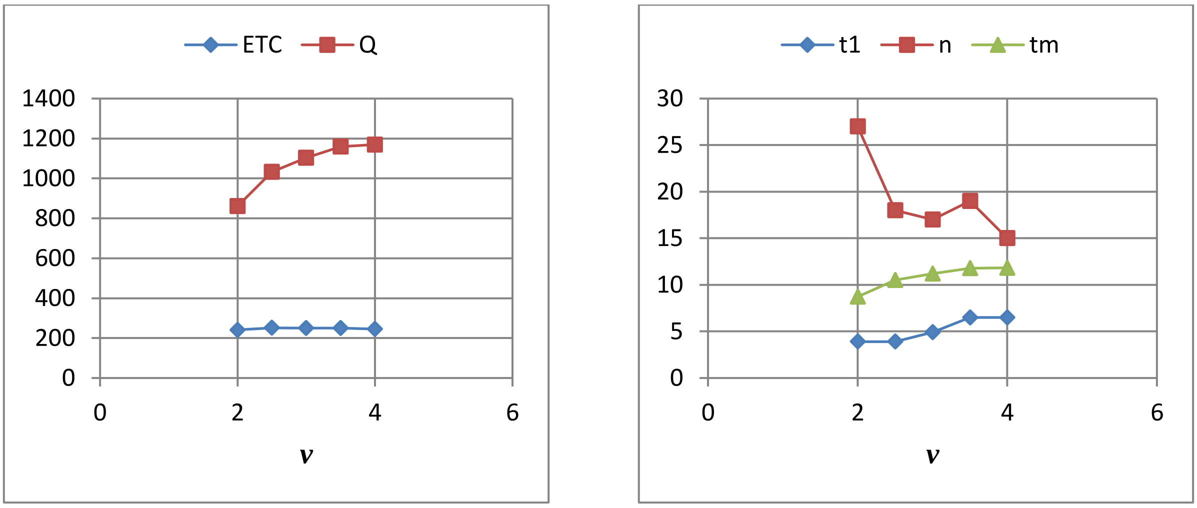


Figure 4. The effect of the shape parameter on *ETC* and the decision variables

The effects of the mean of the Weibull distribution (μ) on *ETC* and the decision variables are shown in table 7 and Figure 5. Based on the figure, an increase in μ decreases the values of *ETC* and n but increases the values of m and t_m. This is because, an increase in the value of μ leads to a decrease in the value of the failure rate of the process; therefore, it is predictable for the maximum duration of the production cycle to increase and for *ETC* to decrease. Also, the value of *Q* increases due to an increase in the maximum duration of the cycle.

Table 7. The obtained results for *μ* changes

μ	ETC	t_1	k	n	m	t_m	Q
20	241.23	3.9	2.9	27	6	8.72	860.86
22	238.80	3.3	3	21	10	9.9	973.96
25	233.03	3.3	3	18	16	12.78	1251.4
30	221.44	3.3	3.1	14	50	23.1	2242.6

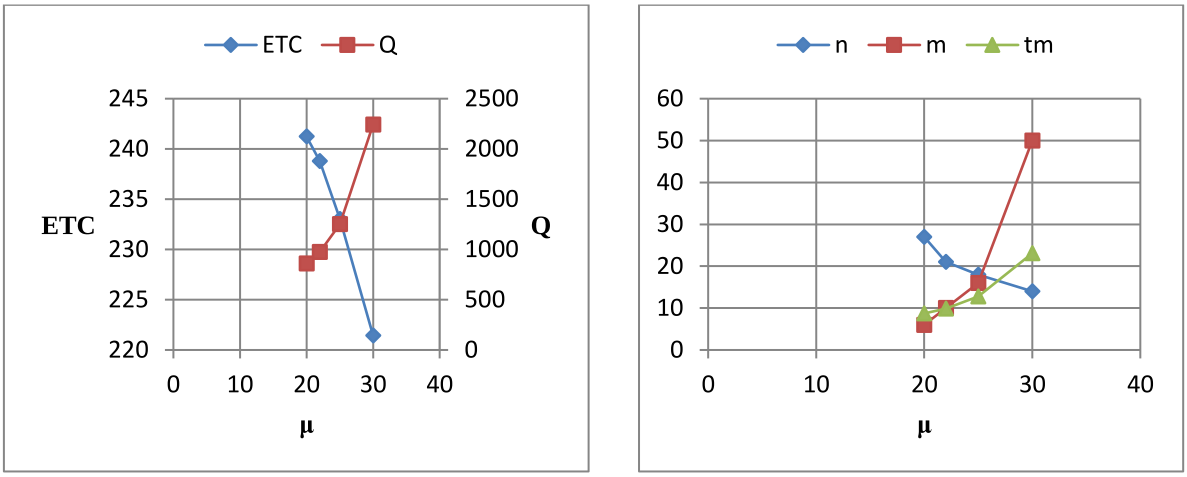


Figure 5. The effects of μ on *ETC* and the decision variables

Larger values of the variable sampling cost led to a decrease in the sample size (*n*) and the control limit (*k*). The sample size is decreased to reduce the total inspection cost. Also, the decrease in the control limit compensates the effect of the decrease in the sample size. Thus, the sensitivity of the control chart is preserved to recognize the out-of-control state of the process. This point is illustrated in table 8 and Figure 6. The effect of this change on the other decision variables is insignificant.

Table 8. The obtained results for C_V changes

C_V	ETC	t_1	k	n	m	t_m	Q
0.2	241.23	3.9	2.9	27	6	8.72	860.86
0.5	241.75	3.9	2.6	21	6	8.72	860.93
1	242.43	3.9	2.4	17	6	8.72	861.04
2	243.47	3.9	2.2	13	6	8.72	861.25

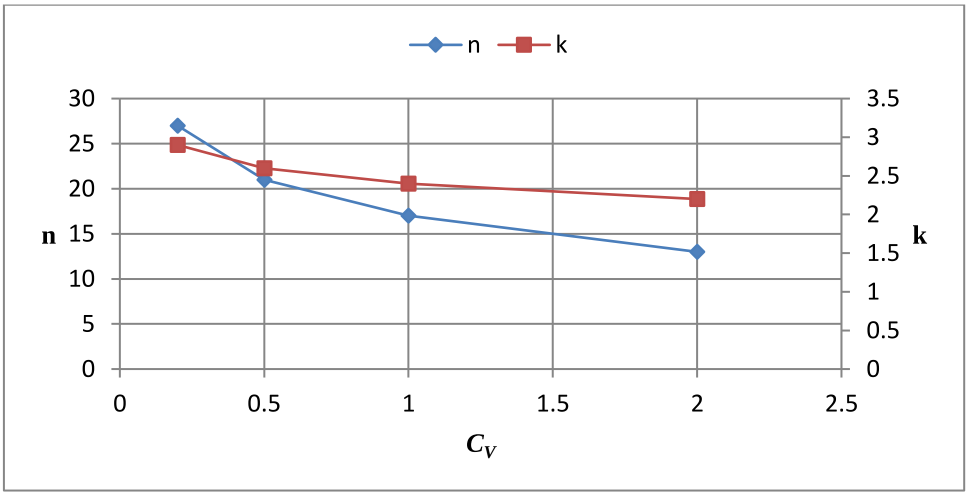


Figure 6. The effect of the variable sampling cost on *ETC* and the decision variables

The effects of the corrective maintenance cost and the preventive maintenance cost on the decision variables and the total cost of the process are displayed in table 9 and Figure 7. As expected, an increase in the maintenance costs leads to an increase in the value of *ETC*. However, an increase in the preventive maintenance cost (C_{PM}) leads to a decrease in the sample size and an increase in m and t_m. The increase of t_m causes an increase in the lot size, Q.

Table 9. The obtained results for C_{CM} and C_{PM} changes

		ETC	t_1	k	n	m	t_m	Q
C_{CM}	5000	241.23	3.9	2.9	27	6	8.72	860.86
	6000	243.07	3.3	2.9	27	7	8.08	800.26
	8000	246.25	3.3	2.9	29	7	8.08	800.24
	10000	249.41	3.3	2.9	30	7	8.08	800.23
C_{PM}	2400	241.23	3.9	2.9	27	6	8.72	860.86
	2700	250.41	4.9	3	23	50	34.3	3259.6
	3000	252.25	4.9	3	23	50	34.3	3259.6
	4000	258.36	4.9	3	22	50	34.3	3259.6

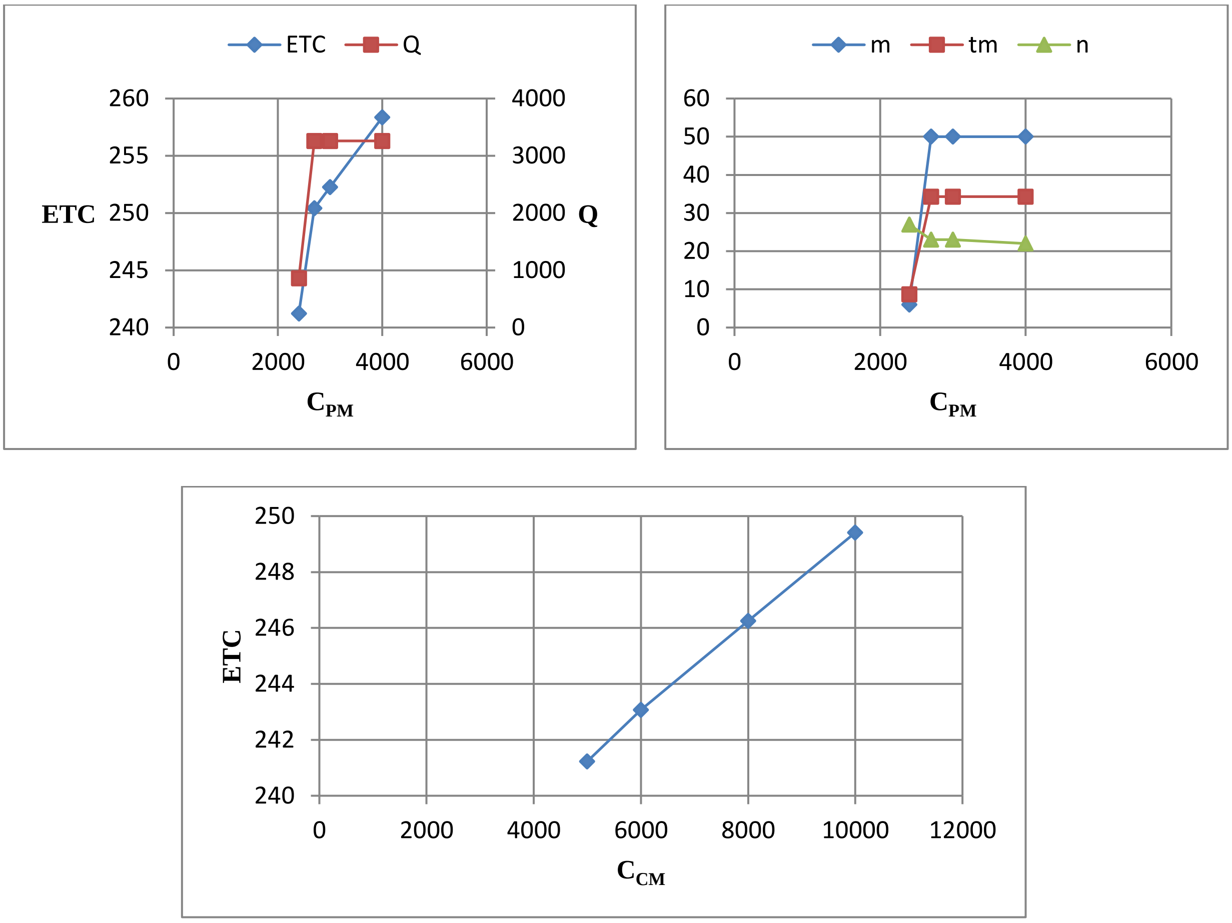


Figure 7. The effect of CM and PM costs on *ETC* and the decision variables

As the minimal maintenance cost increases, *ETC* and k increase too, but the sample size decreases. Also, the increase in the cost of the minimal maintenance cost leads to the decrease of the frequency to perform MM actions. Therefore, the control limits increase and more points fall between the control limits, which leads to a reduction in the optimal sample size. This point is illustrated in table 10 and Figure 8.

Table 10. The obtained results for C_{MM} changes

C_{MM}	ETC	t_1	k	n	m	t_m	Q
500	241.23	3.9	2.9	27	6	8.72	860.86
700	242.11	3.9	2.9	27	6	8.72	860.86
1000	243.14	3.3	3	26	7	8.08	800.29
1500	244.80	3.3	3	24	7	8.08	800.33

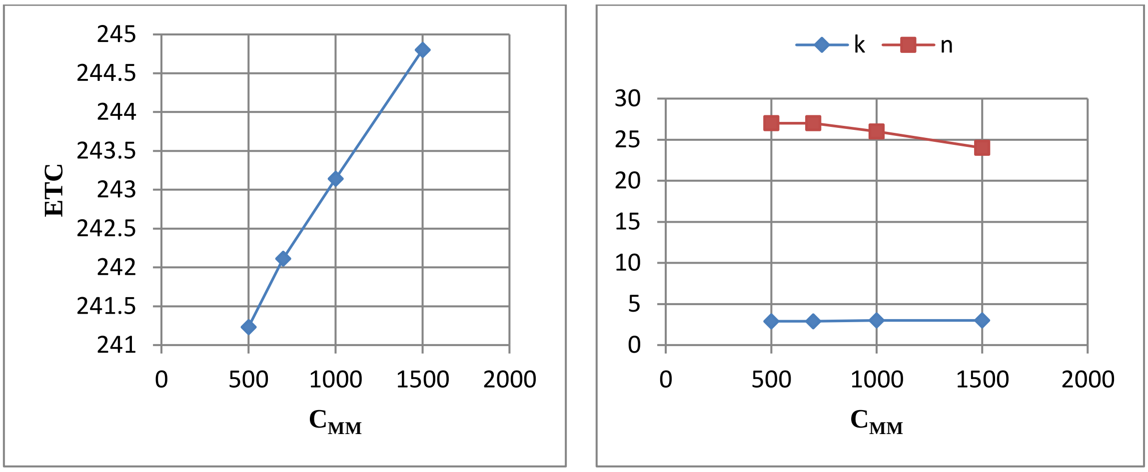


Figure 8. The effect of the minimal maintenance cost on *ETC* and the decision variables

The obtained results indicate the performance and the efficiency of the proposed model. With respect to the market conditions and the production process in manufacturing systems and the variability of different parameters such as failure rate, unit costs of maintenance and quality control and the other parameters used in this model, the obtained results for sensitivity analysis can be applied in practice to analyze the impacts of such parameters changes on the mathematical model solutions. The production processes can be planned optimally and dynamically by applying the model and the obtained results and the impacts of the parameters changes on final solutions can be controlled.

7. Conclusion

In this study, an integrated model of quality control, maintenance planning and production control was developed for a deteriorating imperfect production system. It was assumed that the production process deteriorates due to the occurrence of an assignable cause and that the failure mechanism of the process follows a general distribution. The process has two operational states including in-control and out-of-control states. With respect to the process state, a proper maintenance action is taken on the process. In order to monitor the process, a proper control chart can be used. During each inspection, a random sample is taken and the quality characteristics of the products are evaluated. The integrated model can determine the optimal values of sample size, control limits and inspections intervals; thus, an optimal policy can be adopted for quality control and maintenance. The optimal value of the lot size is determined too with respect to the duration of each production cycle. A numerical example was solved with a grid search algorithm, and the results were analyzed. Finally, a series of sensitivity analyses was conducted on some important parameters. The obtained results indicate the performance of the model in real situations. The managers in manufacturing systems can plan dynamically and optimally for the production processes by joint planning of maintenance, quality control and production. The machine, human resources and other production resources can be applied in an optimal manner and finally the productivity of the production process can be improved by integrated planning. The integrated models can be applied in manufacturing systems with JIT (Just In Time) policy. In such manufacturing

systems, the products are produced with respect to the demand level and the managers' purpose is to eliminate the inventory. Therefore, the integrated model can be applied in such production processes to jointly determine the production plans, maintenance schedules and quality control policies with respect to the demand level and production capacity. Also, the integrated models can be applied in continuous manufacturing systems. In such systems, the process should operate without interruptions during the production process. Therefore, optimal planning for maintenance scheduling and quality control policy is important. The efficiency of production processes can, indeed, be improved by the use of an integrated model that enables the joint planning of production, maintenance and quality control.
This research can be extended in several directions. Application of different types of control charts such as multivariate charts and attribute control charts and a combination of control charts to monitor the mean and the variance of the process are interesting subjects to extend the model. Also, evaluating the effects of different distributions of failure mechanisms on the performance of the model can be considered as another extension of this study. The insights provided by this study may be utilized to develop integrated models for multi-stage systems and series production systems. Considering failure mode and analyzing the impact of this state on joint planning is another suggestion to develop this paper. Applying different solving methods such as meta-heuristic algorithms and compare the obtained results, is one of the suggestions to develop the integrated model proposed in this paper.

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