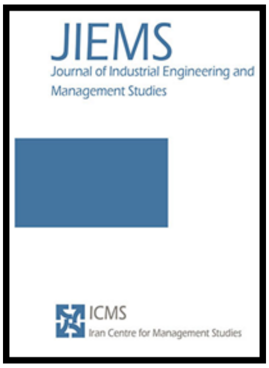
JIEMS

Journal of Industrial Engineering and Management Studies

Vol. 8, No. 1, 2021, pp. 135-151

DOI: 10.22116/JIEMS.2020.218468.1368

www.jiems.icms.ac.ir



Optimizing resiliency of train operations in an underground metro: A hybrid discrete-event simulation and response surface methodology

Ali Shahabi[1], Sadigh Raissi[1,*], Kaveh Khalili-Damghani[1], Meysam Rafei[2]

Abstract

Avoiding the passengers extra waiting time is a vital task for rail planners. The current research focused on minimizing the passenger waiting time on the presence of real frequently random occurred disturbances. Details of the proposed model are on the 1st line of Tehran underground rail rapid transit. All fitness functions are validated using the analysis of variance (ANOVA) by applying the hypothesis testing method. Also, a validated discrete-event computer simulation model is applied to examine the average waiting time per passenger as the key performance measure under different scenarios generated using full factorial design of experiments. The validity of the obtained optimal solution, i.e., train headway times is confirmed at a 95% level of reliability. Also, simulation outcomes indicated that the proposed response surface meta-model could efficiently provide a more reliable train operation plan to ensure a desirable level of system resiliency on the presence of random disturbances. The numerical results indicated that wait time could be reduced by 14.8% for passengers as compared with the baseline train headway plan.

Keywords: rapid transit system; resiliency; discrete-event simulation; design of experiment; response surface methodology.

Received: June 2020-06

Revised: September 2020-10

Accepted: September 2020-14

1. Introduction

Customer satisfaction is a key element and a fundamental measure in the service industry (Ramezani and Naderi, 2018; Basirati et al., 2019; Memarpour et al., 2019). Operating systems with high reliability and resilience to uncertainties and changing conditions can provide better services to users. Urban transportation systems are no exception. The measurement of transport resiliency under random and severe conditions is of great importance for today's economy (Zhou and Chen, 2020). Because of the increasing demand, competition between different

* Corresponding author; raissi@azad.ac.ir

[1] School of Industrial Engineering, Islamic Azad University, South Tehran Branch, Tehran, Iran.

[2] Department of Economics, Kharazmi University, Tehran, Iran.

types of transportation systems to attract more passengers is increasing. Therefore, the policy makers follow the demand based development strategies (Iravani et al., 2020).

In recent decades, rapid rail freight has been widely welcomed by passengers and decision-makers in the field of urban transport due to its concern for the safety and prevention of fossil fuel pollution (Boroun et al., 2020). However, due to the unstable situations and uncertain circumstances caused by the traffic disruptions, improving the resiliency of transit services is of great importance for both passengers and operators (Shakibayifar et al., 2018; Shakibayifar et al., 2017b; Shakibayifar et al., 2017c). In such a transport system, resiliency has definite as to come back to the steady-state condition after any shock or disturbance occurrence (Hassannayebi et al., 2019a). The present study aims to achieve a higher level of resiliency in urban rail transportation systems.

In analyzing a system employing quantitative methods, it is important to determine the behavioral pattern of the state variables of a system (Gholizad et al., 2017). In the real world, the behavioral patterns of the main state variables are often a function of time, and over time their behavioral patterns may change as well (Rastbin et al., 2017; Shafieezadeh et al., 2020). Therefore, the behavior of these systems needs to be dynamically monitored (Ip and Wang, 2011). The analysis of these systems falls within the scope of studying time series patterns and random processes (Eskandari et al., 2013). Usually, the mean variations function, and in some cases the standard deviation variations are estimated in mathematical terms based on empirical data. If at some point in time, these systems cause disruption or random shocks, then the system departs from its stable behavior pattern and requires an indefinite period to return to normal conditions before the change (Yin et al., 2016). For policymakers and planners, it is important to mitigate the effects of shocks that cause disturbance to the state variables and return the studied variable to the mean pre-change pattern, which is called the concept of resiliency (Uday and Marais, 2015).

Resiliency can be quantified by the characteristics of the rail networks and the association between the topological and operational aspects and the corresponding service disruption impact (Wang et al., 2020). The resiliency of rapid rail services, principally as related to operational efficiency, is a key measure of system performance (Rajabighamchi et al., 2019). In a metro system, passengers' count at given station considerers as a main system state variable, which deploys from a complicated random process at over different days and hours (Li and Wang, 2017). However, if disturbances or shocks such as a train crash, disruption operations, escalator failure, blocking off a passenger exit route, anomalous traffic congestion at the city level leading to a significant increase in visitors to the station under study (Lu et al., 2019). This can lead to disruptions in the system that change the behavioral model of the system, such as the number of passengers in the station (Hassannayebi et al., 2019b).

There are different methods for quantifying the robustness and resiliency of the transportation systems, e.g. mathematical models (Chen and Miller-Hooks, 2012; Hassannayebi et al., 2016d), analytical approaches (Duy et al., 2019; Omer et al., 2012), simulation techniques (Hassannayebi et al., 2016a; Sajedinejad et al., 2011), simulation-optimization methods (Tavan and Sajadi, 2015; Gholamian, 2017; Bahramian and Bagheri, 2015; Ilati et al., 2014; Kim et al., 2013), robust optimization (Hassannayebi et al., 2017), stochastic optimization (Hassannayebi et al., 2016b; Khan and Zhou, 2010; Shakibayifar et al., 2017a; Hassannayebi et al., 2014), heuristic algorithms (Pavlov et al., 2017), meta-heuristic algorithms (Hassannayebi and Zegordi, 2017; Hassannayebi et al., 2018; Qi et al., 2018). Lai et al. (2020) proposed a public transport system efficiency metric based on the system efficiency function. They defined resilience as the capability to recover from passenger waiting time and traffic flow fluctuations. In a static approach, the resilience refers to the proficiency of an entity or system to maintain the overall functionality (Cox et al., 2011). Alternatively, in a dynamic approach resiliency signifies the potentiality of a system to a rapid return to the steady-state after disrupted states. Thus far, the topic of transport resiliency assessment has been studied in several articles, and some indicators have been proposed for this purpose. Though, given the fact that the rapid transit networks are vulnerable against disruptive events, and due to its dynamic nature and specific characteristics, providing flexible frameworks to assess the system performance in such conditions is of countless significance. Despite the existing researches in this context, few works have been devoted to quantifying dynamic resiliency and its measurement in metro transportation networks. This study is motivated by the need to enhance the resiliency of the urban metro services as against accidents, technical failures, and severe weather conditions.

2. Literature review

The literature on train service planning is extensive. For a more detailed study of the articles, the models in the research background are divided into two categories: 1. mathematical models for resiliency analysis, 2. simulation approaches to train service optimization.

2.1. Mathematical modeling approaches

Freckleton et al. (2012) addressed the problem of resiliency quantification in transportation networks in the case of disruptions. The proposed modeling framework uses the analysis of a transportation network's resiliency from the perspective of economic stability. Jin et al. (2014) projected an operation research model to optimize a multi-modal transport system, i.e., an integrated bus and underground metro system. The local bus services are synchronized to the regular underground train services to mitigate the risk of disruptions. They modeled the problem using a stochastic programming model and a case study of the Singapore public transport system is employed to verify the method.

Bruyelle et al. (2014) developed a framework related to a European project called SecureMetro that aims to boost the resilience of passenger amenities and the vehicle in a metro system. The multi-layered model was applied to emergency response planning by passenger screening as against to the terrorist occurrences. D’Lima and Medda (2015) studied the resilience of the underground metro line in London using a stochastic model based on a mean-reverting concept. There was an effort to measure the speed of system recovery, i.e. the amount of time required to come back to the equilibrium after a disruption. Chopra et al. (2016) proposed an innovative framework for evaluating the resilience of transport infrastructure and to characterize the critical points, i.e., stations, in an underground metro system. The resiliency of the metro system was quantified in terms of the topological feature of the network and passenger flow. The modeling framework is capable of identifying the most vulnerable stations in London metro system in terms of robustness to random failures.

Sun and Guan (2016) analyzed the resilience and susceptibility of a metro transport system using graph theory. Their methodology was applied to the Shanghai metro system. The critical metro lines were identified using several performance indicators, i.e., average path length, centrality, number of canceled trips, and global network efficiency. The result of the model leads to the generation of a new operating plan based on the rerouting capability of the metro network.

Hua and Ong (2017) studied the resiliency of the urban bus-rail transit systems in terms of survivability and recoverability in the case of unpredicted disruptions. The analysis of system resiliency is conducted to quantify the number of affected passengers who need to be transported by other modes after disruptions. The validity of the model was tested using several disruption scenarios adopted from Singapore's rapid transit system. Zhang et al. (2018) proposed an analytical model to measure and improve the resiliency of large-scale urban rail transport networks. The resiliency of the rail network is measured in terms of the connectivity performance of the transit system and the total loss of the passenger count. The recovery sequence model was validated using real instances of the Shanghai metro network with more than 300 stations and 350 links. Wei et al. (2018) proposed a model to evaluate the resiliency of the metro system in terms of the rate of change in system capacity as against the signaling failures. The effect of signaling failures was measured by train tracking interval under several disruption scenarios. Li et al. (2019) projected a mathematical optimization approach based on a genetic algorithm (GA) to quantify the resiliency of an urban rail subway network and to improve the recovery rapidity of the system. Their graph theory approach considers the distribution of the passenger flows and the topological features of the rail network. The model examined using experimental cases on the Beijing subway network. Shahabi et al. (2019) proposed a framework for optimizing the resiliency of skip-stop schedules in underground rail transport systems. The resiliency of the train service was measured in terms of the variance of the train headway times. Large-size instances of the problem were successfully solved using a genetic algorithm (GA) and a variable neighborhood search (VNS). Li et al. (2020) proposed an Artificial Bee Colony Algorithm for increasing the traveler’s satisfaction by reducing the transfer waiting time. The validity of the solution method was confirmed by real-world instances of the Shanghai metro network.

2.2. Simulation approaches

Simulation can be described as a process of designing a model of a real system and performing experiments with that model to understand the system's behavior (Ehsanifar, et al., 2017). Discrete-event simulation technique is a flexible approach to real-world railway planning problem (Hassannayebi et al., 2014). Also, computer simulation is an effective approach in the timetable design of a rail system in the absence of tractable mathematical models for the complex problem of train timetabling (Gholamian, 2017). The use of discrete-event simulation for railway operation planning and resilience optimization has been presented in a few studies. For example, Greenberg et al. (2013) proposed simulation models for quantifying the resiliency of rail networks under hazard events. Adjetey-Bahun et al. (2016) designed a multi-perspective model to quantify the resiliency of the mass railway system′s capacity in terms of the potential power of recovery, absorption, and adaptation. The developed method is based on the simulation model to measure passenger delay and load balance as the key system′s performance metrics. The model was applied to the Paris railway system and the outcomes indicate the practical advantages of the perturbation control plans. Stroeve and Everdij (2017) applied agent-based simulation for resilience assessment of airline transportation. The result of implementation shows the benefits of the designed model for improvement of the aviation operations.

According to the review of the existing articles, the shortcomings and deficiencies in the research background can be summarized as follows:

1) A model for optimizing the level of resilience has not been observed due to the uncertainty of train travel time and passenger entry rate to the station.
2) Despite the prevailing literature survey on the robust and resilient transport operation, it seems the research gap exists on dynamic resilience assessment of train services remains especially in the cases of general random time between passenger arrivals, stochastic shock occurrences, and random resource breakdowns and repair times.
3) Also, a review of the publication presents the lack of transportation resiliency measurement on the general random processes. Most of them restricted to exponential distributions.

This study aims to fill these research gaps by developing a hybrid response surface methodology and discrete-event simulation for quantifying the inherent randomness in the rail system. The design simulation model is capable of capturing the dynamics of rail operations and uncertainty associated with passenger flows. The primary objective of this study is to design an integrated optimization and statistical framework for quantifying the dynamic resiliency of operations in fast rail transportation systems on the presence of any kind of randomly occurred disturbances, both in time to occurred and their severities.

3. A resiliency optimization framework

This section describes the developed framework for resiliency optimization in a double-track metro line. The proposed approach act based on estimating a meta-model derived from a series of simulation runs of full factorial scenarios. The optimization method applied to the fitness function derived from experiments using statistical regression technique (

Figure 1). The validity of the fitted meta-model is confirmed by conducting simulation experiments.
In this research, it is supposed that the time between the traveler arrivals follows an exponential probability distribution with a time-varying rate. Thus, the arrival pattern is a non-stationary Passion process. The arrival rates and alighting ratios are computed based on the aggregated OD matrices. To assess the resiliency of the train services, it is suitable to measure the train frequency and the headway deviations at stations. Due to the uncertain nature of the rail operations, i.e., running and dwell time randomness, the actual headways may deviate from the baseline plan. One main measure of service regularity is the average passenger waiting time. Thus, this research aims to minimize the average wait time per passenger to achieve a desirable level of service resiliency against random shocks.

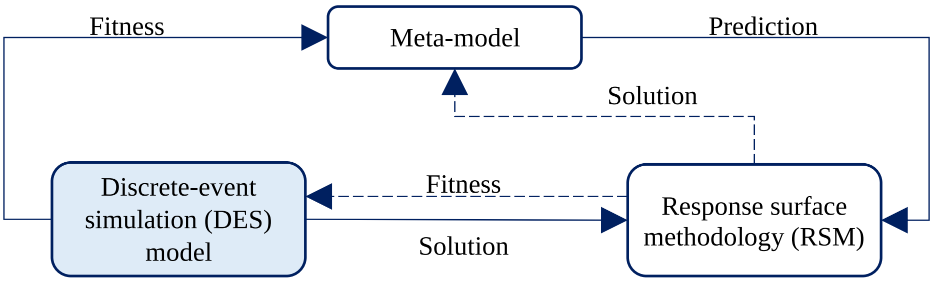


Figure 1. The developed framework for resiliency optimization in the metro line

The designed simulation model works as a solution evaluation module. The detailed description of the developed event-driven simulation model for railway operation is presented in the recently published work of Shahabi et al., (2019). The simulation model is developed based on an object-oriented modeling paradigm using a commercial discrete-event simulator called Enterprise Dynamics (ED).

4. Response surface methodology (RSM)

RSM is a step-by-step statistical method to optimize a response variable. Except for the second-order step, local first-order polynomial regression is fitted in each step. For fitting purposes, RSM uses input/output data obtained from 2k factorial schemes whose variance in the region of interest is minimal and regression coefficients are obtained independently. Also, a third resolution- design plan delivers an unbiased estimate for the first-order polynomial regression coefficients which is sufficient because the number of design points against other types of factor schemes is low. In the case of applying a 2nd order polynomials, we applied the central composite design (CCD) method (Montgomery, 2017).

In each step, except for the last step, the gradient is used to change the input variables. The gradient is a mathematical technique for the sharpest descent for the response variable to reach the extremum point. In the final step, RSM uses the fitted second-order polynomial function derivative to estimate the optimal input composition.

The RSM must initialize and need a starting point. The starting point may be a combination of inputs that are currently in use in the system or need to be set according to prior knowledge. In the nearest region of the starting point, the RSM examines the input/output behavior of the observed system. The RSM estimates the model parameters using a preliminary 1st-order polynomial as the same as a Taylor series, with a residual term of e as follows:

$$Y = \beta_0 + \sum_{j=1}^{k} \beta_j z_j + e \tag{1}$$

Y and z_j represent the response function and the jth input factor, respectively. Here noises should be followed by Gaussian probability function of $e \sim N(0, \sigma^2)$ which is independent of different observations and is uniformly distributed in the solution space. The vector $\beta = (\beta_0, \beta_1, \dots, \beta_k)^T$ is represented by the width of the origin β_0 and k the main effects of β_j ($j = 1, \dots, k$). As the next step jumps to a new solution, hence the variance may be changed. Equation (2) presents a simple regression model for which the best predictor of linear inequality β with the least-squares (LS) method is:

$$\hat{\beta} = (Z^T Z)^{-1} Z^T w \tag{2}$$

where Z is the $N \times (k+1)$ matrix, obtained from the plot with the 2-way interactions (resolution III), m_i denotes the duplicates of point i, and w is the output vector with N simulation replicates. In the context of DOE, design resolutions reflect some extent the effects are aliased with other effects within the fractional factorial framework. Therefore, in resolution III, the main effects are aliased with two-factor interactions. Clearly $N = \sum_{i=1}^{n} m_i$ and the vector Z, $m_i \geq 1$ has the same rows as each row starts with 1 and denotes the width of the origin β_0.

To determine the next subsurface, RSM uses the steepest descending method. If the estimated parameters are $\hat{\beta}_1 \gg \hat{\beta}_2 > 0$, then the RSM reduces the input z_1 further than the input z_2. Unfortunately, the steepest descent method is scale-dependent, as the linear conversion of the inputs affects the search direction. If the step size used causes the output to be significantly larger (rather than lower), the step size may be reduced. Otherwise, it will go the further step trough the steepest current decline direction.

After a few steps of the steepest descent approach, the output has the chance to increase rather than decreasing. The first-order polynomial is just a local estimate of the correct input/output. Once the descent occurs, the RSM detects that the $n > k$ combination specified by the 3rd resolution scheme is centrally located around the best-found point. As a result, RSM possibly will routine the same scheme in step two but converts standardized inputs x_j to different values of initial inputs z_j. The best set has so far found might be on one of the corners of the design. The RSM then estimates the first-order effects using the least-squares method with Equation (2) and the search continues.

As is well known, the scheme used for most preliminary 1st order polynomials cannot adequately signify the hilltop when trying to maximize or minimize a function. Thus, in the neighborhood of the optimum point, the first-order polynomial shows the seriousness of non-fit. A simple and popular method is the coefficient of determination of R^2. The F statistic is applied to examine whether all estimated first-order effects - and therefore the gradient - are equal to zero. RSM may instead use cross-validation. If the first-order polynomial is not sufficient, RSM fits a quadratic polynomial as follows:

$$y = \beta_0 + \sum_{j=1}^{k} \beta_j z_j + \sum_{j=1}^{k} \sum_{j'=j}^{k} \beta_{j;j'} z_j z_{j'} + e \tag{3}$$

If there is no time limit, the RSM may deviate from local optimality and start the search from a different initial local level (which will result in the RSM returning to the initial step). It is recommended that entries that do not have significant effects on first-order fitting polynomials within the test solution space are not eliminated since they may have significant effects on the subsequent test space. The important issue is to determine the number of replications. Determining, where the signal-to-noise ratio will be large enough, is a controversial problem. Also, the Taylor series, which offers higher-order polynomials, is more accurate than lower-order polynomials. Higher-order polynomials may provide a less biased prediction variable, but mean square error (MSE) will increase. However, higher-order polynomials need to be simulated by combining more inputs.

In this study, the average waiting time (AWT) of any passenger is estimated through the performance measure of the resiliency of the train services.

5. Data analysis and discussion

The resiliency estimation model is applied to the 1st line of the Tehran metro. The underground rail rapid line has 36.6 kilometers long, which consists of 30 stops located from south to north of Tehran (

Figur 2). The vehicles are each made up of eight standard wagons, with a maximum capacity of 1,450 persons. Here train speed is less than 85 km/h which will be moderated to an average of 45 km/h as a result of minor stops. The minimum and maximum headways are 4 and 10 minutes, respectively. The required data is adopted from Tehran Urban & Suburban Railway Operation Co (www.metro.tehran.ir). The simulation model is developed using Enterprise Dynamics software and the numerical experiments are performed on a Laptop with an Intel Core i7-6567U processor at 3.60 GHz and 16 gigabytes of RAM.

Figur 2. Route No.1 represented as a red line in the Tehran Metro network

The RSM starts with running a full 2^k factorial design and deriving the first-order model. In this study, k is encoded as two lower and upper levels, i.e., ±1, in which level=0 symbolizes the central point (CP). The first and second input coded variables refer to the headway times (h_1, and h_2) at inbound and outbound routes. It worth mentioning that using additional central points would improve the robustness of the developed meta-model against curvature problems. The experimental model is designed by estimating the sum of squared errors (SSE) and testing the validity of the fitted model. The data of the coded and decoded test points are summarized in Table 1.

Table 1. The test points along with the response values

Simulation replication No.	CP	h_1	h_2	$H_{inbound}$ (minutes)	$H_{outbound}$ (minutes)	AWT (minutes)
1	1	-1	-1	4	4	3.88
2	0	0	0	7	7	4.12
3	0	0	0	7	7	4.10
4	0	0	0	7	7	4.11
5	0	0	0	7	7	4.09
6	0	0	0	7	7	4.16
7	1	-1	+1	4	10	5.89
8	1	+1	-1	10	4	6.58
9	1	+1	+1	10	10	6.27

The result of the first-order model and the coded values are verified using the ANOVA test. Table 2 reports the numerical analysis of ANOVA along with the F-statistic, P-values of the main, and mutual effects with 95% reliability. The fitted value or $\hat{y}$ is obtained by the $n-1$ observation regression model and is a measure of the amount of fitting by the model to each point in the plot. The sequential sum of squares (SS) depends on the order the factors or predictors are entered into the model. The sequential sum of squares is the unique portion of SS Regression explained by a factor, given any previously entered factors. The adjusted sum of squares (Adj SS) is the amount of variation explained by a term, given all other terms in the model, regardless of the order that the terms enter the model. Also, the adjusted sum of squares (Adj MS) is a measure of the amount of variation that a term explains after accounting for the other terms in the model.

Table 2. The result of ANOVA for AWT estimation model in the first-order model

Source of effects	Degree of freedom	Sequential SS	Adjusted SS	Adjusted MS	F-statistic	P-value
Main	2	11126.5	11126.5	5563.2	2346.75	**0.000**
h_1	1	8533.1	8533.1	8533.1	3599.54	**0.000**
h_2	1	2593.4	2593.4	2593.4	1093.96	**0.000**
Mutual	1	4842.1	4842.1	4842.1	2042.53	**0.000**
$h_1 * h_2$	1	4842.1	4842.1	4842.1	2042.53	**0.000**
Curvature	1	18859.8	18859.8	18859.8	7955.65	**0.000**
Error (Residual)	4	9.5	9.5	2.4		
Error (Pure)	4	9.5	9.5	2.4		
Total	8	34837.9				

The result of the ANOVA test indicates that both the main and mutual effects, as well as the curvature, are significant. Therefore, the adequacy of the first-order model is critically questioned. Accordingly, a second-order model is designed to enhance the validity of the meta-model. The quadratic model of the second-order regression system is developed using the central composite design method. In our implementation, the lower and upper limits for the input variables are defined as 4 and 10 minutes. The simulated result of the experimental points with their response values is reported in
Table 3.

Table 3. The experimental points with their response values

Experiment No.	h_1	h_2	Train departure headway (minutes)		AWT (seconds)
			Inbound route	Outbound route	
1	-1	-1	4	4	231.30
2	0	0	7	7	243.61
3	0	0	7	7	242.11
4	-1	0	4	7	251.77
5	-1	-1	4	4	230.08
6	0	0	7	7	252.19
7	1	1	10	10	375.93
8	1	-1	10	4	388.85
9	-1	1	4	10	351.04
10	-1	1	4	10	328.37
11	0	0	7	7	245.80
12	1	1	10	10	375.56
13	-1	0	4	7	243.89
14	0	1	7	10	327.04
15	0	0	7	7	243.97
16	0	0	7	7	249.90
17	0	0	7	7	248.09
18	0	0	7	7	249.16
19	0	0	7	7	245.08
20	0	1	7	10	326.15
21	1	0	10	7	381.98
22	0	0	7	7	240.82
23	0	-1	7	4	251.98
24	0	-1	7	4	247.13
25	1	-1	10	4	390.53
26	1	0	10	7	389.52

Table 4 presents the results on the estimation of the parameters or the quadratic codified fitness function model. As Table 4 reveals that all of the effects at a 95% confidence interval are significant at 95% reliability since the p-value is lower than the α.

Table 4. The result of regression analysis

Element	Coefficient value	Square error coefficient	T-statistics	P-value	Summary result
Constant	249.28	3.242	76.900	0.000	**SD* = 13.52** **R^2 = 95.98%** **R^2 (adjusted) = 95.37%**
h_1	56.43	3.187	17.705	0.000	
h_2	32.67	3.187	10.250	0.000	
h_1^2	54.05	4.698	11.505	0.000	
h_2^2	37.15	4.698	7.907	0.000	
$h_1 * h_2$	-30.10	3.903	-7.711	0.000	

* Standard deviation

Table 5. The result of ANOVA for AWT estimation model in the second-order model

Source of effects	Degree of freedom	Sequential SS	Adjusted SS	Adjusted MS	F-statistic	P-value
Regression	5	143911	143911	28782.2	157.41	**0.000**
Singular	2	76526	76526	38263.2	209.26	**0.000**
h_1	1	57317	57317	57317.1	313.47	**0.000**
h_2	1	19209	19209	19209.3	105.06	**0.000**
Quadratic	2	56512	56512	28256.0	154.53	**0.000**
h_1*h_1	1	45079	24204	24203.8	132.37	**0.000**
h_2*h_2	1	11433	11433	11432.8	62.53	**0.000**
Mutual	1	10873	10873	10872.7	59.46	**0.000**
h_1*h_2	1	10873	10873	10872.7	59.46	**0.000**
RE*	33	6034	6034	182.8		
LOF**	3	5181	5181	1726.9	60.72	**0.000**
PE***	30	853	853	28.4		
Sum	38	149945				

* Error, ** Lack-of-Fit, *** Pure Error

Table 6. Regression coefficients for the AWT estimation

Element	Coefficient
Constant	**374.007**
h_1	**-41.852**
h_2	**-23.481**
h_1^2	**6.005**
h_2^2	**4.127**
$h_1 * h_2$	**-3.344**

The final input/output relationship function based on the pseudo-fitted model and the regression coefficients for the AWT estimation in
Table **6** is expressed as follows:

$$\hat{y} = 374.007 - 41.8525 * h_1 - 23.4816 * h_2 + 6.00529 * h_1^2 + 4.12733 * h_2^2 - 3.34454 * h_1 h_2 \tag{4}$$

Accordingly, the vector of the optimal headway time solution is computed as follows:

$$h = -\frac{1}{2}B^{-1}b = -\frac{1}{2}\begin{bmatrix} 6.00529 & -1.67227 \\ -1.67227 & 4.12733 \end{bmatrix}^{-1}\begin{bmatrix} -41.8525 \\ -23.4816 \end{bmatrix} = \begin{bmatrix} 4.820672 \\ 4.797839 \end{bmatrix} \tag{5}$$

The nature of the obtained optimal solution, i.e. headway times, can be interpreted by calculation of the Eigenvector of matrix *B*. The Eigenvector can be determined as:

$$|B - \lambda I| = 0 \ \rightarrow \ \lambda^2 - 10.1326\lambda + 21.98933 = 0$$
$$\lambda_1 = 6.984, \lambda_2 = 3.148 \tag{6}$$

The obtained solution is minimal since the Eigenvalues are positive. Therefore, the response function can be calculated as follows:

$$\hat{y}_s = \hat{\beta}_0 + \frac{1}{2}h_s' \ b = 216.79 \tag{7}$$

where h_s' refers to the stationary circumstance. To further check the validity of the RSM, the obtained solution is simulated n=30 times which is reported in
Table 7. The average and standard deviation of the response function (AWT) is estimated as 221.17 and 10.94 seconds using simulation experiments, respectively.

Table 7. The simulation outputs for 30 No. of replications

Simulation replicates	AWT (sec)	Simulation replicates	AWT (sec)	Simulation replicates	AWT (sec)
1	235.16	11	234.96	21	202.41
2	236.39	12	211.33	22	207.22
3	227.62	13	237.70	23	204.48
4	213.59	14	220.54	24	227.88
5	229.51	15	219.08	25	219.78
6	229.79	16	215.20	26	219.35
7	236.01	17	219.00	27	229.57
8	221.23	18	211.46	28	210.03
9	235.97	19	202.64	29	207.24
10	221.77	20	219.58	30	228.50

To statistically testify the significance of the difference between the meta-model output and the mean response value, obtained by the simulation experiments, a hypothesis is expressed as follows:

$$\begin{cases} H_0: \bar{Y} = 216.79 \\ H_1: \bar{Y} \neq 216.79 \end{cases} \tag{8}$$

Accordingly, the t-test statistic value is calculated as follows:

$$\left.\begin{aligned} t^* = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{221.17 - 216.79}{10.94/\sqrt{30}} = 2.190 \\ t_{(0.975,29)} = 2.3638 \end{aligned}\right\} \rightarrow\ t^* < t_{(0.975,29)} \tag{9}$$

Due to the statistical analysis, the null hypothesis, i.e., H_0, is not rejected. Thus, the obtained optimal solution is verified.

To further assess the quality of the obtained solution, a comparison with the baseline solution is provided. As can be found in Eq. (5), the optimal train headway times running on inbound and outbound routes are estimated as about 4.8 minutes. To better highlight the advantages of the obtained solution, the AWT is estimated with the setting of $H_{baseline}$=4 minutes.

The statistical analysis of the optimal solution, as well as the baseline solution in terms of 1-α% confidence intervals, is provided in Table 8. The results are summarized in terms of the mean and standard deviation of the response function, i.e. AWT. As reported in this table, in the case when trains depart with minimum safety headways, i.e., $H_{baseline}$=4 minutes, the simulation model would result in higher AWT or equivalently shows less resilience to the random disruptions due to the limited fleet size. According to the result presented in Table 8, the AWT is reduced by about 14.8% as compared with the baseline solution. As the numerical results indicated, the proposed RSM is significantly satisfied with the goal to find optimal headway time for real-world applications of the train timetabling problem.

Table 8. The statistical analysis of the baseline and optimal solutions

Solution	Average	Standard deviation	Lower bound (95%)	Upper bound (95%)
Optimal	225.31	17.16	212.44	238.18
Baseline	264.42	23.89	246.50	282.34

6. Conclusion

Providing cost-effective and efficient transportation services are the two main tasks in public transport systems. In transportation systems, the waiting time of passengers can be greatly increased due to the interaction of various random events, and this can cause inefficiency. The consequence of disruptions leads to a loss of reliability, increased operating costs, and reduced levels of passenger satisfaction. The consequences of accidental malfunctions frequently tend to reduce system reliability as well as customer satisfaction level and increase operating costs.
Also, disruption of the railways directly affects the flow of passengers. Although urban rail systems (such as the underground metro) are fast and secure transport systems, improving passenger flow management in conditions of disruption and uncertainty remains a serious necessity. The scheduling of subway trains in the traditional way is associated with a lack of flexibility. Increasing the speed of the headway time raises the level of passenger satisfaction and, conversely, reducing it will increase operating costs and decrease customer satisfaction level. To achieve the desired level of service resiliency against random disruptions, a meta-model approach is proposed here to optimize the train timetable.
The modeling framework uses a hybrid response surface methodology (RSM) and design of experiment (DOE) to measure and enhance the resiliency of train services in an underground metro line by regulating the headway time while minimizing the passenger wait time under random disturbances. The decision support system is implemented on a real-world case study of the 1st line of the Tehran underground rail rapid transit system. The numerical results confirm that the proposed meta-modeling approach could provide an optimal set of control factors to guarantee a looked-for level of service resiliency in terms of the average waiting time of each passenger. The quality of the obtained solutions was assessed through a comparison with the baseline solution. The statistical analysis of the candidate solutions showed that the baseline headway resulted in a higher AWT which confirms a lower degree of resiliency concerning the random shocks. The simulation outputs revealed that the AWT is reduced by 14.8% as compared with the baseline headway plan.
The theoretical and practical implications of this research can be summarized as follows: a new approach to estimating the resiliency index of a rail transport system based on the integration of RSM and simulation model is developed. Second, from the practical point of view, the model proposed could be applied to determine the headway time intervals of inbound and outbound routes in metro lines to reduce passenger waiting time. So that in a much shorter time than classical methods can guarantee optimal problem solving with statistical tests.
The current research has also several limitations/shortcomings for the proposed simulation-optimization framework. First, the proposed simulation-optimization model is implemented only for the peak hours at short-term horizons. Consequently, the simulation-optimization model has not been developed for optimizing the resiliency along the planning horizon. In future research, the simulation-optimization model might be extended for a longer time horizon to increase the productivity of fleet capacity. Correspondingly, the proposed simulation-optimization model can be extended to address the multi-objective nature of the problem. The extended simulation-optimization model can be used for optimizing both the user preferences and any operator attitudes. In cases where the problem engaged with several simultaneous responses, applying desirability functions could be act efficiently.

References

Adjetey-Bahun, K., Birregah, B., Châtelet, E., and Planchet, J.-L., (2016). "A model to quantify the resilience of mass railway transportation systems", *Reliability Engineering & System Safety*, Vol. 153**,** pp. 1-14.

Bahramian, Z., and Bagheri, M., (2015). "A simulation-based optimization approach for passenger train timetabling with periodic track maintenance and stops for praying", *Journal of Modern Transportation*, Vol. 23, pp. 148-157.

Basirati, M., Jokar, M. R. A., and Hassannayebi, E., (2019). "Bi-objective optimization approaches to many-to-many hub location routing with distance balancing and hard time window", *Neural Computing and Applications*, pp. 1-22.

Boroun, M., Ramezani, S., Vasheghani Farahani, N., Hassannayebi, E., Abolmaali, S., and Shakibayifar, M. (2020). "An efficient heuristic method for joint optimization of train scheduling and stop planning on double-track railway systems", *Infor: Information Systems and Operational Research*, pp. 1-28.

Bruyelle, J.-L., O'neill, C., El-Koursi, E.-M., Hamelin, F., Sartori, N., and Khoudour, L. (2014). "Improving the resilience of metro vehicle and passengers for an effective emergency response to terrorist attacks", *Safety Science*, Vol. 62, pp. 37-45.

Chen, L. and Miller-Hooks, E., (2012). "Resilience: An indicator of recovery capability in intermodal freight transport", *Transportation Science*, Vol. 46, pp. 109-123.

Chopra, S. S., Dillon, T., Bilec, M. M., and Khanna, V., (2016). "A Network-Based Framework For Assessing Infrastructure Resilience: A Case Study of The London Metro System", *Journal of the Royal Society Interface*, Vol. 13, 20160113.

Cox, A., Prager, F., and Rose, A., (2011). "transportation security and the role of resilience: a foundation for operational metrics", *Transport Policy*, Vol. 18, pp. 307-317.

D'lima, M., and Medda, F., (2015). "A new measure of resilience: an application to the london underground", *Transportation Research Part A: Policy and Practice*, Vol. 81, pp. 35-46.

Duy, P. N., Chapman, L., and Tight, M. (2019). "Resilient transport systems to reduce urban vulnerability to floods in emerging-coastal cities: a case study of ho chi minh city, vietnam", *Travel Behaviour and Society*, Vol. 15, pp. 28-43.

Ehsanifar, M., Hamta, N., and Hemesy, M. (2017). "A simulation approach to evaluate performance indices of fuzzy exponential queuing system (an m/m/c model in a banking case study)", *Journal of Industrial Engineering and Management Studies*, Vol. 4, No. 2, pp. 35-51. doi: 10.22116/Jiems.2017.54604

Eskandari, H., Rahaee, M.A., Hasannayebi, E., Memarpour, M., and Malek, S.A., (2013). "Evaluation of different berthing scenarios in shahid rajaee container terminal using discrete-event simulation", IEEE, pp. 3462-3474.

Freckleton, D., Heaslip, K., Louisell, W. and Collura, J. (2012). "Evaluation of resiliency of transportation networks after disasters", *Transportation Research Record*, 2284**,** pp. 109-116.

Gholamian, S., (2017). "A discrete-event optimization framework for mixed-speed train timetabling problem", *Journal of Industrial Engineering and Management Studies*, Vol. 4, No. 2, pp. 64-84. Doi: 10.22116/Jiems.2017.54607

Gholizad, A., Ahmadi, L., Hassannayebi, E., Memarpour, M., and Shakibayifar, M., (2017). "A system dynamics model for the analysis of the deregulation in electricity market", *International Journal of System Dynamics Applications (IJSDA)*, Vol. 6, pp. 1-30.

Greenberg, M., Lioy, P., Ozbas, B., Mantell, N., Isukapalli, S., Lahr, M., Altiok, T., Bober, J., Lacy, C., and Lowrie, K. (2013). "Passenger Rail Security, Planning, And Resilience: Application Of Network, Plume, And Economic Simulation Models As Decision Support Tools.", *Risk Analysis*, Vol. 33, pp. 1969-1986.

Hassannayebi, E., Boroun, M., Jordehi, S. A. and Kor, H., (2019a). "Train Schedule Optimization In A High-Speed Railway System Using A Hybrid Simulation And Meta-Model Approach"; *Computers & Industrial Engineering*, Vol. 138, 106110.

Hassannayebi, E., Memarpour, M., Mardani, S., Shakibayifar, M., Bakhshayeshi, I., and Espahbod, S., (2019b). "A hybrid simulation model of passenger emergency evacuation under disruption scenarios: A case study of a large transfer railway station", *Journal of Simulation*, pp. 1-25.

Hassannayebi, E., Sajedinejad, A. and Mardani, S. (2014). "Urban rail transit planning using a two-stage simulation-based optimization approach", *Simulation Modelling Practice and Theory*, Vol. 49, pp. 151-166.

Hassannayebi, E., Sajedinejad, A., and Mardani, S., (2016a). *Disruption management in urban rail transit system: a simulation based optimization approach*, handbook of research on emerging innovations in rail transportation engineering, Igi-Global, India.

Hassannayebi, E., and Zegordi, S.H., (2017). "Variable and adaptive neighbourhood search algorithms for rail rapid transit timetabling problem", *Computers & Operations Research*, Vol. 78, pp. 439-453.

Hassannayebi, E., Zegordi, S.H., Amin-Naseri, M.R., and Yaghini, M., (2016b). "Demand-oriented timetable design for urban rail transit under stochastic demand", *Journal of Industrial and Systems Engineering*, Vol. 9, pp. 28-56.

Hassannayebi, E., Zegordi, S.H., Amin-Naseri, M.R., and Yaghini, M. (2017). "Train timetabling at rapid rail transit lines: a robust multi-objective stochastic programming approach", *Operational Research*, Vol. 17**,** pp. 435-477.

Hassannayebi, E., Zegordi, S.H., Amin-Naseri, M.R., and Yaghini, M.J.P.T., (2018). "Optimizing headways for urban rail transit services using adaptive particle swarm algorithms", *Public Transport*, Vol. 10, pp. 23-62.

Hassannayebi, E., Zegordi, S. H., and Yaghini, M., (2016d). "Train timetabling for an urban rail transit line using a lagrangian relaxation approach", *Applied Mathematical Modelling*, Vol. 40, pp. 9892-9913.

Hua, W., and Ong, G.P., (2017). "Network survivability and recoverability in urban rail transit systems under disruption", *IET Intelligent Transport Systems*, Vol. 11, pp. 641-648.

Ilati, G., Sheikholeslami, A., and Hassannayebi, E., (2014). "A simulation-based optimization approach for integrated port resource allocation problem", *PROMET - Traffic & Transportation*, Vol. 26, pp. 243-255.

Ip, W.H., and Wang, D., (2011). "Resilience and friability of transportation networks: evaluation, analysis and optimization", *IEEE Systems Journal*, Vol. 5**,** pp. 189-198.

Iravani, M., Bashirzadeh, R., and Tarokh, M., (2020). "Developing an urban congestion pricing model by considering sustainability improvement and using a multi-objective optimization approach", *Journal of Industrial Engineering and Management Studies*, Vol. 7, No. 2, pp. 56-76. doi: 10.22116/Jiems.2019.178753.1262

Jin, J.G., Tang, L.C., Sun, L., and Lee, D.-H., (2014). "Enhancing metro network resilience via localized integration with bus services", *Transportation Research Part E: Logistics and Transportation Review*, Vol. 63, pp. 17-30.

Khan, M.B., and Zhou, X., (2010). "Stochastic optimization model and solution algorithm for robust double-track train-timetabling problem", *IEEE Transactions on Intelligent Transportation Systems*, Vol. 11, pp. 81-89.

Kim, M., Schonfeld, P., and Kim, E. (2013). "Simulation-based rail transit optimization model", *Transportation Research Record*, Vol. 2374, pp. 143-153.

Lai, X., Teng, J., Schonfeld, P., and Ling, L., (2020). "Resilient schedule coordination for a bus transit corridor", *Journal of Advanced Transportation*, pp. 1-12.

Li, M., and Wang, H., (2017). "Resiliency assessment of urban rail transit networks: A case study of shanghai metro", *IEEE*, pp. 620-625.

Li, M., Wang, H., and Wang, H., (2019). "Resilience assessment and optimization for urban rail transit networks: a case study of beijing subway network", *IEEE*, pp. 71221-71234.

Li, X., Yamamoto, T., Yan, T., Lu, L., and Ye, X., (2020). "First train timetabling for urban rail transit networks with maximum passenger transfer satisfaction", *Sustainability*, Vol. 12, 4166.

Lu, J., Ma, X., and Xing, Y., (2019). "Risk factors affecting the severity of disruptions in metro operation in shanghai, 2013-2016", *Journal of Transportation Safety & Security*, pp. 1-24.

Memarpour, M., Hassannayebi, E., Miab, N.F., and Farjad, A., (2019). "Dynamic allocation of promotional budgets based on maximizing customer equity", *Operational Research*, pp. 1-25.

Montgomery, D.C., (2017). *Design and analysis of experiments*, John Wiley & Sons, USA.

Omer, M., Mostashari, A., Nilchiani, R., and Mansouri, M., (2012). "A framework for assessing resiliency of maritime transportation systems", *Maritime Policy & Management*, Vol. 39, pp. 685-703.

Pavlov, A., Ivanov, D., Dolgui, A., and Sokolov, B., (2017). "Hybrid fuzzy-probabilistic approach to supply chain resilience assessment", *IEEE Transactions on Engineering Management*, Vol. 65, pp. 303-315.

Qi, J., Li, S., Gao, Y., Yang, K., and Liu, P., (2018). "Joint optimization model for train scheduling and train stop planning with passengers distribution on railway corridors", *Journal of the Operational Research Society*, Vol. 69, pp. 556-570.

Rajabighamchi, F., Hajlou, E. M. H., and Hassannayebi, E., (2019). "A multi-objective optimization model for robust skip-stop scheduling with earliness and tardiness penalties", Ur*ban Rail Transit*, Vol. 5**,** pp. 172-185.

Ramezani, S., and Naderi, K., (2018). "A multi-objective optimization model for locating the distribution warehouses in the logistic network supply chain: a case study of bonny chow co.", *Journal of Industrial Engineering and Management Studies*, Vol. 5, No. 1, pp. 26-42. doi: 10.22116/Jiems.2018.66502

Rastbin, S., Mozaffar, F., Behzadfar, M., and Gholami Shahbandi, M., (2017). "Designing the optimum plan for regenerating the pedestrian network of historic districts using bi-level programming (case study: historical-cultural district of tehran, Iran)", *Journal of Industrial Engineering and Management Studies*, Vol. 4, No. 1, pp. 34-54. doi: 10.22116/Jiems.2017.51962

Sajedinejad, A., Mardani, S., Hasannayebi, E., Mir Mohammadi K, S., and Kabirian, A., (2011). "SIMARAIL: simulation based optimization software for scheduling railway network", In: *Simulation Conference (WSC)*, Proceedings Of The 2011 Winter, IEEE, pp. 3730-3741.

Shafieezadeh, M., Kalantar Hormozi, M., Hassannayebi, E., Ahmadi, L., Soleymani, M., and Gholizad, A., (2020). "A system dynamics simulation model to evaluate project planning policies", *International Journal of Modelling and Simulation*, Vol. 40, pp. 201-216.

Shahabi, A., Raissi, S., Khalili-Damghani, K., and Rafei, M., (2019). "Designing a resilient skip-stop schedule in rapid rail transit using a simulation-based optimization methodology", *Operational Research*, pp. 1-31. doi: https://doi.org/10.1007/s12351-019-00523-y

Shakibayifar, M., Hassannayebi, E., Jafary, H., and Sajedinejad, A., (2017a). "Stochastic optimization of an urban rail timetable under time-dependent and uncertain demand", *Applied Stochastic Models in Business and Industry*, Vol. 33, pp. 640-661.

Shakibayifar, M., Hassannayebi, E., Mirzahossein, H., Taghikhah, F., and Jafarpur, A.J., (2018). "An intelligent simulation platform for train traffic control under disturbance", *International Journal of Modelling Simulation*, pp. 1-22.

Shakibayifar, M., Hassannayebi, E., Mirzahossein, H., Zohrabnia, S., and Shahabi, A., (2017b). "An integrated train scheduling and infrastructure development model in railway networks", *Scientia Iranica*, Vol. 24, pp. 3409-3422.

Shakibayifar, M., Sheikholeslami, A., Corman, F., and Hassannayebi, E., (2017c). "An integrated rescheduling model for minimizing train delays in the case of line blockage", *Operational Research*, pp. 1-29.

Stroeve, S. H., and Everdij, M. H. C., (2017). "Agent-based modelling and mental simulation for resilience engineering in air transport", *Safety Science*, Vol. 93, pp. 29-49.

Sun, D. J., and Guan, S., (2016). "Measuring vulnerability of urban metro network from line operation perspective", *Transportation Research Part A: Policy and Practice*, Vol. 94, pp. 348-359.

Tavan, F., and Sajadi, S., (2015). "Determination of optimum of production rate of network failure prone manufacturing systems with perishable items using discrete event simulation and taguchi design of experiment", *Journal of Industrial Engineering and Management Studies*, Vol. 2, No. 1, pp. 16-26

Uday, P., and Marais, K., (2015). "Designing resilient systems-of-systems: a survey of metrics, methods, and challenges", *Systems Engineering*, Vol. 18, pp. 491-510.

Wang, Z., Ma, W., and Chan, A., (2020). "Exploring the relationships between the topological characteristics of subway networks and service disruption impact", *Sustainability*, Vol. 12, 3960.

Wei, Q., Niu, R., and Tang, T., (2018). "Assessment of metro signalling system resilience", *IEEE*, pp. 1-5.

Yin, H., Han, B., and Li, D., (2016). "Evaluating disruption in rail transit network: A case study of beijing subway", *Procedia Engineering*, Vol. 137, pp. 49-58.

Zhang, D.-M., Du, F., Huang, H., Zhang, F., Ayyub, B. M., and Beer, M., (2018). "Resiliency assessment of urban rail transit networks: shanghai metro as an example", *Safety Science*, Vol. 106, pp. 230-243.

Zhou, L., and Chen, Z., (2020). "Measuring the performance of airport resilience to severe weather events", *Transportation Research Part D: Transport and Environment*, Vol. 83, P.102362.

This article can be cited: Shahabi, A., Raissi, S., Khalili-Damghani, K., Rafei, M., (2021). "Optimizing resiliency of train operations in an underground metro: A hybrid discrete-event simulation and response surface methodology", *Journal of Industrial Engineering and Management Studies*, Vol. 8, No. 1, pp. 135-151.