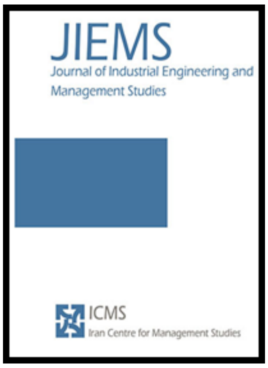
JIEMS

Journal of Industrial Engineering and Management Studies

Vol. 8, No. 1, 2021, pp. 240-260

DOI: 10.22116/jiems.2020.215206.1330

www.jiems.icms.ac.ir



JIEMS
Journal of Industrial Engineering and Management Studies
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Iran Centre for Management Studies

Sustainable closed-loop supply chain network: Mathematical modeling and Lagrangian relaxation

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Abstract

This paper addresses a novel two-stage model for a Sustainable Closed-Loop Supply Chain (SCLSC). This model, as a contribution, provides a balance among economic aims, environmental concerns, and social responsibilities based on price, green quality, and advertising level. Therefore, in the first stage, the optimal values of price are derived by considering the optimal level of advertising and greening. After that, in the second stage, multi-objective Mixed-Integer Linear Programming (MOMILP) is extended to calculate Pareto solutions. The objectives are include maximizing the profit of the whole chain, minimizing the environmental impacts due to CO2 emissions, and maximizing employee safety. Besides, a Lagrangian relaxation algorithm is developed based on the weighted-sum method to solve the MOMILP model. The findings demonstrate that the proposed two-stage model can simultaneously cope with coordination decisions and sustainable objectives. The results show that the optimal price of the recovered product equals 75% of the new product price which considerably encourages customers to buy it. Moreover, to solve the MOMILP model, the proposed algorithm can reach to exact bound with an efficiency gap of 0.17% compared to the optimal solution. Due to the use of this algorithm, the solution time of large-scale instances is reduced and simplified by an average of 49% in comparison with the GUROBI solver.

Keywords: sustainable closed-loop supply chain; pricing; Lagrangian relaxation.

Received: January 2020-01

Revised: March 2020-20

Accepted: July 2020-24

1. Introduction

Nowadays, Sustainable Closed-Loop Supply Chain (SCLSC) is an essential mark for competitive companies and helps them to cope with the economic, environmental, and social issues, simultaneously (Sauer and Seuring, 2019). Therefore, this field is known as one of the interesting subject areas for the academic world and the real world researches. Closed-Loop

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Supply Chains (CLSCs) consist of supply, manufacturing, distribution, and retail centers in forwarding logistics. They also include collection, recovery, disassembly, and disposal centers in reversing logistics. These networks focus on improving the created value of products over their life-cycle (Govindan and Soleimani, 2017). In addition, environmental concerns have been increasingly raised by considering governmental regulations. Thus, Green Supply Chain Management (GSCM) considers these aspects to satisfy stakeholder and consumer considerations (Hong and Guo, 2019). These considerations can increase the final demand while decreasing environmental impacts (Zhen et al., 2019). As another vital mark, the advertisement has a pivotal role in the growth of Supply Chains (SCs). Thus, SCs have increasingly focused on promoting advertising policies to gain more attractive demands. They are also investing in different selling channels, such as online delivery channels (Heydari et al., 2018). Besides, the social issues are increasingly developed along with environmental issues. As a result, international enterprises such as ISO are introducing the fundamental principles to make SCs closer to the desired goals. These principles are included human rights, labor practices, environment, customer satisfaction, operating practices, organizational governance, and social development. Above all, the financial concerns are still playing the most crucial role for the entire competitive SCs. Thus, the Revenue Management (RM) is designated as a complementary skill to meet the economic objectives (Talluri and Van Ryzin, 2006).

Consequently, this study provides a multi-stage approach to reach the optimal values of price, greenness, and advertisement, and then give the optimal balance among economic, environmental, and social objectives. For this purpose, the first stage yields the equilibrium value of the new and recovered products' price, in different channels, regarding the optimal green quality level of product, and the optimal level of advertising. Afterward, a MOMILP model is formulated to maximize the profit and the social satisfaction and minimize the total CO_2 emissions of the whole chain.

The current paper is organized as follows. The related literature and the novelty of the study are presented in Section 2. Section 3 declares the problem statement in detail. The mathematical models are presented in Section 4. In Section 5, the solution methods are defined for solving the proposed two-stage approach. The numerical example is implemented to validate the proposed model and solution method in Section 6. The managerial implications and sensitivity analyses are provided in Section 7. Finally, the conclusions and future studies are provided in Section 8.

2. Literature review

According to the procedure of modeling for the proposed study, the related literature is categorized into two principal sections. The first section refers to design green SCLSC networks, and the second section refers to coordinate the pricing, advertising, and greening decisions.

In the recent decade, the SCLSC problems have increasingly attracted many academic pieces of research. These studies have provided different mathematical models and solution methods to improve products' created values. Zhen et al. (2019) presented an SCLSC network considering the environmental issues and the various range of customers' demands. They modeled through a MOMILP formulation to minimize total costs and CO_2 emissions. Taleizadeh et al. (2018) represented an SCLSC model considering the pricing decisions. They considered a reverse flow to collect the returned products for recovering and selling them to consumers at the same price to new products. By increasing the importance of sustainability concerns, many studies are tending to incorporate the financial, environmental, and social objectives, simultaneously. Ansari and Kant (2017) have introduced a comprehensive review of the most important aspects of Sustainable Supply Chain (SSC) management. As well, Badi and Murtagh (2019) have comprehensively described the environmental impacts of SCs regarding the critical success factors like greenhouse emissions. Hussain et al. (2019) have presented an SSC by considering the conformity factors. Mardani et al. (2020) deeply surveyed green and sustainable concerns to handle SCs. They used the implementation of Structural Equation Modelling (SEM) to evaluate this field and clarify interesting and novel future subjects. Soleimani et al. (2017) extended a green SCLSC under uncertainty conditions. Their multi-objective model contains three objectives, including economic, environmental, and social goals. Besides, a variety of applied solution methods have been extended. These methods can generally be categorized into exact solutions, decomposition solutions, and meta-heuristics solutions (Soleimani and Govindan, 2015). Many researches have been done under uncertainty conditions in the field of SC.

Also, several studies in this field have developed different and applied solution methods. As a whole, They are categorized into exact solutions, decomposition solutions such as benders decomposition and Lagrangian relaxation, and meta-heuristic solutions (Zhen et al., 2019).

On the other hand, many studies have focused on coordinating all the decisions of SCs. Wang et al. (2019) studied the coordination policies under the different structures of SCs. They introduced pricing models by considering greening and selling efforts assumptions. Madani and Rasti-Barzoki et al. (2017) represented the pricing decisions based on green policies. They incorporated the green quality levels of products in the demand function model. Heydari et al. (2018) defined a multi-channel SC model to determine the optimal level of pricing and greening decisions. They considered price elasticity and cross-price effects in the modeling of customers' demand function for two different selling channels. Khorshidvand et al. (2021) preented a revenue management model by considering pricing, greening and advertising decisions. Alamdar et al. (2018), have provided coordination contracts in a CLSC to reach the optimal value of the wholesale and retail price. They also considered the effort of selling as a pivotal decision. As well, they developed the models considering fuzzy circumstances. A revenue-sharing strategy has presented by Ranjan and Jha (2019) for incorporating the green quality level of products and sales efforts. They extended the proposed models for a dual-channel SC under centralized, decentralized, and collaborated structures. Raza and Govindaluri (2019) described a price and greening differentiation coordination based on partial demand information and cannibalization. They showed that selling green and regular products considering differentiated prices can raise the profitability of all the entities. Khorshidvand et al. (2021) developed a two-stage model for a sustainable closed-loop supply chain by focusing pricing and advertising. Khorshidvand et al. (2021) also extend a hybrid model to consider supply chain coordination decisions and supply chain network design decisions under the uncertainty of demands at the same time.

3. Problem definition

This study presents a multi-objective, multi-echelon, multi-channel, multi-period, SCLSC dealing with financial, environmental, and social goals. This network includes supplier centers, manufacturing centers, distribution centers, and retail centers in a forward chain. These centers are responsible for the supply of components, production of a new product, and distribution of new and recovered products. Also, in a return chain, it consists of collection centers, recovery centers, disassembly centers, and disposal centers. Three different selling channels are incorporated to satisfy the different types of demand. The retailer's channel is assigned to sell the new product, the distributor's channel is assigned to sell the recovered product, and the secondary channel satisfies the components' demand. A simple schematic of the investigated network with component and (recovered) new product's flows is demonstrated in Figure 1.

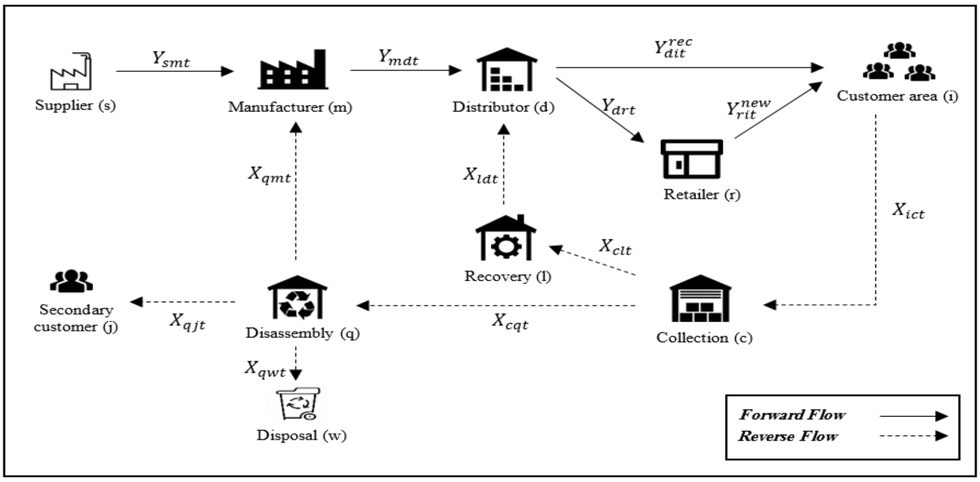

Figure 1. Investigated SC overview

As indicated in Figure 1, the disassembly centers provide a given share of required materials for the manufacturing centers, and the external suppliers prepare the remaining of demand. Also, the disassembly centers sell a defined share of components to secondary customers. Clearly, the scrapped components are transported to the disposal centers. Then, the manufacturers decide on the optimal level of green quality and advertising. Afterward, they produce a new product to satisfy the demands of distribution centers. The distribution centers have two primary responsibilities. The first one is distributing the new product to the retailers, and the second one is selling the recovered product to the customers through their online channels. They can hold the inventory to cope with the variations of demand. In the last echelon, the retail centers satisfy the demands of the new product. In addition, the returned product is collected by the collection centers. These centers deliver the recoverable product to recovery centers and transport the rest of the returned product to disassembly centers.

Accordingly, the proposed network is modeled using a novel integrated hybrid approach. This approach contains two-stage modeling. In the first stage, under a centralized structure, the optimal values of the new and recovered products are derived; plus, the green quality and advertising level are optimized. Then, in the second stage, a multi-objective SCLSC is developed under the optimal decisions of the first stage. The objectives involve economic, environmental, and social concerns.

Furthermore, the assumptions of this study are stated as below:

- There exists at least one facility at each echelon in both forward and reverse logistics.
- The centralized structure is considered for the studied SCLSC.
- The new product has more price than the recovered product because of its higher quality.
- The price of products is not floating.
- The number of vehicles is not limited.
- Shortage costs can be neglected.
- All new products are produced in a predefined green quality level.
- The advertisement policy for all the new and recovered products is the same.

3.1. Notations

The notations of this study are described as follows.

3.1.1. Indices, sets, and superscripts

s	Indicator of supplier centers
m	Indicator of manufacturing centers
d	Indicator of distribution centers
r	Indicator of retail centers
i	Indicator of customer areas
c	Indicator of collection centers
l	Indicator of recovery centers
q	Indicator of disassembly centers
j	Indicator of secondary customer areas
w	Indicator of disposal centers
t, t'	Indicators of periods
S	Set of supplier centers
M	Set of manufacturing centers
D	Set of distribution centers
R	Set of retail centers
I	Set of customer areas
C	Set of collection centers
L	Set of recovery centers
Q	Set of disassembly centers
J	Set of secondary customer areas
W	Set of disposal centers
T	Set of periods
new	Superscript of new product
rec	Superscript of recovered product
ret	Superscript of returned product
com	Superscript of components

3.1.2. Parameters

d_{sm}	Distance between supplier s and manufacturer m (in km)
d_{md}	Distance between manufacturer m and distributor d (in km)
d_{dr}	Distance between distributor d and retailer r (in km)
d_{di}	Distance between distributor d and customer i (in km)
d_{ri}	Distance between retailer r and customer i (in km)
d_{cl}	Distance between collection c and recovery l (in km)
d_{cq}	Distance between collection c and disassembly q (in km)
d_{ld}	Distance between recovery l and distributor d (in km)
d_{qm}	Distance between disassembly q and manufacturer m (in km)
d_{qj}	Distance between disassembly q and secondary customer j (in km)
d_{qw}	Distance between disassembly q and disposal w (in km)
CT	Transportation cost rate (per km)

CF_m	Fixed cost of opening manufacturer *m* in each period
CF_d	Fixed cost of opening distributor *d* in each period
CF_r	Fixed cost of opening retailer *r* in each period
CF_c	Fixed cost of opening collection *c* in each period
CF_l	Fixed cost of opening recovery *l* in each period
CF_q	Fixed cost of opening disassembly *q* in each period
CP_m	Cost rate of producing new product by manufacturer *m*
CP_d	Cost rate of processing new and recovered product by distributor *d*
CP_r	Cost rate of processing new product by retailer *r*
CP_c	Cost rate of processing returned product by collection *c*
CP_l	Cost rate of processing recovered product by recovery *l*
CP_q	Cost rate of decomposing components by disassembly *q*
CH	Holding cost rate per unit of product
ET	Unit CO_2 emissions of transporting one truck-load of products per kilometer (in g)
CAP_m	Capacity of producing new product manufacturer *m*
CAP_d	Capacity of processing new product and recovered product distributor *d*
CAP_r	Capacity of processing new product retailer *r*
CAP_c	Capacity of processing returned product collection *c*
CAP_l	Capacity of recovering returned product recovery *l*
CAP_q	Capacity of disassembling return product disassembly *q*
D_{it}	Customer demand *i* for new product in period *t*
D'_{it}	Customer demand *i* for recovered product in period *t*
D_{jt}	Secondary demand *j* for components in period *t*
γ	Factor for converting a unit of product to the unit capacity in facilities
γ'	Factor for converting a unit of component to the unit capacity in facilities
η	Factor for converting a unit of component to new product
η'	Factor for converting rate of return product to component
α	Rate of recoverable products
β	Rate of returning components to manufactures
β'	Rate of selling components to secondary customers
ϖ	Vehicle capacity occupied by a unit of (return) product
ϖ'	Vehicle capacity occupied by a unit of component
$Perc$	Percentage of returned products
MD_{mt}	Number of missed days in case of opening manufacturer *m* in period *t*
P^{buy}	Buying price of components from external suppliers
P^{com}	Selling price of components to secondary customers
ρ	Share of new product from potential demand
U	Potential demands (new product + recovered product)
υ	Cross-price sensitivity in recovered product channel
υ'	Cross-price sensitivity in new product channel

φ	Price elasticity of demand for recovered product channel
φ'	Price elasticity of demand for new product channel

3.1.3. Decision variables

Y_{smt}	Volume of components transported from supplier *s* to manufacturer *m* in period *t*
Y_{mdt}	Volume of new product transported from manufacturer *m* to distributor *d* in period *t*
Y_{drt}	Volume of product transported from distributor *d* to retailer *r* in period *t*
Y_{rit}^{new}	Volume of new product transported from retailer *r* to customer *i* in period *t*
Y_{dit}^{rec}	Volume of recovered product transported from distributor *d* to customer *i* in period *t*
X_{imt}	Volume of returned product transported from customer *i* to collection *m* in period *t*
X_{clt}	Volume of returned product transported from collection *c* to recovery *l* in period *t*
X_{cqt}	Volume of returned product transported from collection *c* to disassembly *q* in period *t*
X_{ldt}	Volume of returned product transported from recovery *l* to distributor *d* in period *t*
X_{qmt}	Volume of components transported from disassembly *q* to manufacturer *m* in period *t*
X_{qwt}	Volume of components transported from disassembly *q* to disposal *w* in period *t*
X_{qjt}	Volume of components transported from disassembly *q* to secondary customer *j* in period *t*
INV_{dt}	Inventory level of distributor *d* at the end of period *t*
Z_{mt}	Binary variable equals “1” if manufacturer *m* is established in period *t*, and “0” otherwise
Z_{dt}	Binary variable equals “1” if distributor *d* is established in period *t*, and “0” otherwise
Z_{rt}	Binary variable equals “1” if retailer *r* is established in period *t*, and “0” otherwise
Z_{ct}	Binary variable equals “1” if collection *c* is established in period *t*, and “0” otherwise
Z_{qt}	Binary variable equals “1” if recycling *q* is established in period *t*, and “0” otherwise
Z_{lt}	Binary variable equals “1” if recovery *l* is established in period *t*, and “0” otherwise
λ_t	Membership degree for number of missed working days due to occupational accidents for each worker resulting from establishing manufacturers in period *t*
P^{ret}	Buying price of return product
P^{new}	Selling price of new product
P^{rec}	Selling price of recovered product

4. Mathematical models

4.1. Pricing models

The studied SC is considered under a centralized approach. This means that a single decision-making unit handles all the entities. Therefore, we can formulate the entire financial flows using a Non-Linear Programming (NLP) model as follows:

$$\Pi = [(P^{new} - \omega) \times DR] + [(P^{rec} - \omega) \times DD] \tag{1}$$

Where the demand function of new product (DR) and the demand function of the recovered product (DD) are given as:

$$DR = [U\rho - (\varphi' \times P^{new}) + (\upsilon' \times P^{rec})] \tag{2}$$

$$DD = [U(1-\rho) - (\varphi \times P^{rec}) + (\upsilon \times P^{new})] \tag{3}$$

According to Eq. (1), the first term calculates the profit of the retailer's selling channel, the second one determines the distributor's selling channel, the third term defines the advertising costs, and the last term refers to green quality costs. Note that the price in the secondary selling channel is as same as the buying cost of components from external suppliers. Plus, DR and DD functions (Eqs. (2) and (3)) indicate the direct relation of attracted demands to the price in another channel, and advertising and greening levels. In contrast, the attracted demand has an indirect relation to the price in its channel. It should be noted that the proposed NLP model is solved based on a running time $T(n) = O(n^k)$ for some constant k so that $T(n)$ is bounded by a polynomial function of n; thus, this model, as stated in the related literature, is not NP-hard.

Corollary 1. Eq. (1) is concave. It satisfies definite negative conditions (refer to Appendix A).

Theorem 1. The optimal values of new and recovered products.

The optimal prices of the distributor's and retailer's channel are defined as:

$$P^{rec} = \frac{U(1-\rho) + \left(\frac{\left(U\rho + \omega(\varphi - \upsilon)\right)(\upsilon + \upsilon')}{2\varphi'}\right) + \omega(\varphi - \upsilon)}{2\varphi - \frac{(\upsilon + \upsilon')^2}{2\varphi'}}$$

$$P^{new} = \frac{U\rho + \left(\frac{U(1-\rho) + \left(\frac{\left(U\rho + \omega(\varphi - \upsilon)\right)(\upsilon + \upsilon')}{2\varphi'}\right) + \omega(\varphi - \upsilon)}{2\varphi - \frac{(\upsilon + \upsilon')^2}{2\varphi'}}\right)(\upsilon + \upsilon') + \omega(\varphi - \upsilon)}{2\varphi'} \tag{4}$$

Proof. Appendix A. ■

4.2. Missed days uncertainty

In this subsection, we propose fuzzy modeling to minimize the total missed days in the manufacturing, recovery, and disassembly centers. Therefore, the lower missed days limits (LMD) and the upper missed days limits (UMD) are considered using ISO26000, which is an international standard for social responsibilities. Figure 2 shows the fuzzy membership function.

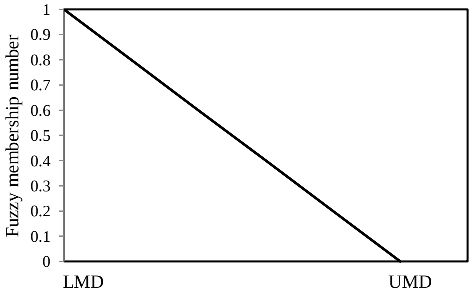

Figure 2. The fuzzy membership function for missed days

According to Figure 3, the membership number for the manufacturers can be formulated as Eq. (5).

$$\lambda_t = \begin{cases} \dfrac{UMD - \sum_{m\in M}\sum_{t\in T} MD_{mt} \times Z_{mt}}{UMD} & \forall\, LMD \le \sum\limits_{m\in M}\sum\limits_{t\in T} MD_{mt} \times Z_{mt} \le\ UMD \\ 0 & otherwise \end{cases} \tag{5}$$

4.3. Objective functions

Now, a multi-objective, multi-level, multi-period, multi-channel model is formulated regarding the sustainable aims. The SCLCS has been already known as an NP-Hard problem (Zhen et al., 2019; Soleimani et al., 2017). Therefore, MILP modeling is developed to reach the three objectives, including financial goals, environmental concerns, and social responsibilities. The first objective function is to maximize the total profit of the chain. The second objective function is to minimize the total amount of CO_2 emissions. The third objective function is to maximize the total social responsibilities.

The first objective function includes the total income of selling the products and components (TREV), the total fixed costs of establishing the facilities (TFC), the total transporting cost (TTC), the total processing cost (TPC), the total buying cost (TBC), the total inventory cost (TIC), and the total advertising cost (TAC). As a result, the Objective Function 1 (OF1) can be presented as follows.

$$OF1 = Min\,[TREV - TFC - TTC - TPC - TBC - TIC - TAC] \tag{6}$$

The total income of the entire network is calculated as:

$$TREV = P^{new}\sum_{r\in R}\sum_{i\in I}\sum_{t\in T} Y_{rit}^{new} + P^{rec}\sum_{d\in D}\sum_{i\in I}\sum_{t\in T} Y_{dit}^{rec} + P^{com}\sum_{q\in Q}\sum_{j\in J}\sum_{t\in T} X_{qjt} \tag{7}$$

From Eq. (7), the first term defines the income of the retailers' channel, the second term also determines the income of distributors' channel, and the last term calculates the income of secondary customers' channel.

The total fixed cost of establishing the facilities is:

$$\begin{aligned} TFC = &\sum_{m\in M}\sum_{t\in T} CF_m Z_{mt} + \sum_{d\in D}\sum_{t\in T} CF_d Z_{dt} + \sum_{r\in R}\sum_{t\in T} CF_r Z_{rt} + \sum_{c\in C}\sum_{t\in T} CF_c Z_{ct} \\ &+ \sum_{l\in L}\sum_{t\in T} CF_l Z_{lt} + \sum_{q\in Q}\sum_{t\in T} CF_q Z_{qt} \end{aligned} \tag{8}$$

In Eq. (8), the fixed cost of opening the entities are defined.

Also, the total transportation cost is given as:

$$\begin{aligned} TTC = {} & \varpi' \times CT \left[\sum_{s\in S} \sum_{m\in M} \sum_{t\in T} d_{sm} Y_{smt} + \sum_{q\in Q} \sum_{j\in J} \sum_{t\in T} d_{qj} X_{qjt} + \sum_{q\in Q} \sum_{m\in M} \sum_{t\in T} d_{qm} X_{qmt} \right. \\ & \left. + \sum_{q\in Q} \sum_{w\in W} \sum_{t\in T} d_{qw} X_{qwt} \right] \\ & + \varpi \\ & \times CT \left[\sum_{m\in M} \sum_{d\in D} \sum_{t\in T} d_{md} Y_{mdt} + \sum_{d\in D} \sum_{r\in R} \sum_{t\in T} d_{dr} Y_{drt} + \sum_{d\in D} \sum_{i\in I} \sum_{t\in T} d_{dI} Y_{dit}^{rec} \right. \\ & + \sum_{r\in R} \sum_{i\in i} \sum_{t\in T} d_{ri} Y_{rit}^{new} + \sum_{c\in C} \sum_{l\in L} \sum_{t\in T} d_{cl} X_{clt} + \sum_{c\in C} \sum_{q\in P} \sum_{t\in T} d_{cq} X_{cqt} \\ & \left. + \sum_{l\in L} \sum_{d\in D} \sum_{t\in T} d_{ld} X_{ldt} \right] \end{aligned} \tag{9}$$

From Eq. (9), the total transportation cost is computed considering truck-load vehicles, city-block distance, and capacity converting rate.

The total processing cost is determined as:

$$\begin{aligned} TPC = {} & \sum_{m\in M} \sum_{d\in D} \sum_{t\in T} CP_m Y_{mdt} + \sum_{d\in D} \sum_{i\in I} \sum_{t\in T} CP_d Y_{dit} + \sum_{d\in D} \sum_{i\in I} \sum_{t\in T} CP_d Y_{dit}^{rec} \\ & + \sum_{r\in R} \sum_{i\in I} \sum_{t\in T} CP_r Y_{rit}^{new} + \sum_{c\in C} \sum_{l\in L} \sum_{t\in T} CP_c X_{clt} + \sum_{c\in C} \sum_{q\in Q} \sum_{t\in T} CP_c X_{cqt} \\ & + CP_l \sum_{l\in L} \sum_{d\in D} \sum_{t\in T} X_{ldt} + \sum_{q\in Q} \sum_{m\in M} \sum_{t\in T} CP_q X_{qmt} + \sum_{q\in Q} \sum_{j\in J} \sum_{t\in T} CP_q X_{qjt} \\ & + \sum_{q\in Q} \sum_{w\in W} \sum_{t\in T} CP_q X_{qwt} \end{aligned} \tag{10}$$

From Eq. (10), the total processing cost is calculated for all the chain, considering a different process rate at each entity.

Besides, the total buying cost is formulated as:

$$TBC = P^{buy} \sum_{s\in S} \sum_{m\in M} \sum_{t\in T} Y_{smt} + P^{ret} \sum_{i\in I} \sum_{c\in C} \sum_{t\in T} X_{ict} \tag{11}$$

According to Eq. (11), the first term is buying cost from the external suppliers, and the second one is buying cost of returned products from the customers.

The total inventory cost is considered by:

$$TIC = CH \left[\frac{\sum_{d\in D} \sum_{t\in T} INV_{dt}}{|T|} \right] \tag{12}$$

In Eq. (12), the inventory cost is derived using the average inventory level for the distributors.

From Eq. (13), the advertising cost is obtained considering the level of advertising which defined by the manufacturer.

The second objective function includes environmental concerns. This objective minimizes the total CO_2 emissions due to transportation (TTE). Thus, the Objective Function 2 (OF2) can be modeled as follows.

$$OF2 = Min\,[TTE] \tag{13}$$

Now, the total amount of emitted CO_2 from transshipment is derived as:

$$\begin{aligned} TTE = \varpi' \times e^t \Bigg[& \sum_{s\in S}\sum_{k\in K}\sum_{t\in T} d_{sk}Y_{skt} + \sum_{p\in P}\sum_{q\in Q}\sum_{t\in T} d_{pq}X_{pqt} + \sum_{p\in P}\sum_{k\in K}\sum_{t\in T} d_{pk}X_{pkt} \\ & + \sum_{p\in P}\sum_{d\in D}\sum_{t\in T} d_{pd}X_{pdt} \Bigg] \\ & + \varpi \times e^t \Bigg[\sum_{k\in K}\sum_{j\in J}\sum_{t\in T} d_{kj}Y_{kjt} + \sum_{j\in J}\sum_{l\in L}\sum_{t\in T} d_{jl}Y_{jlt} + \sum_{r\in R}\sum_{i\in i}\sum_{t\in T} d_{ri}Y_{rit}^{new} \\ & + \sum_{d\in D}\sum_{i\in i}\sum_{t\in T} d_{di}Y_{dit}^{rec} + \sum_{m\in M}\sum_{n\in N}\sum_{t\in T} d_{mn}X_{mnt} + \sum_{m\in M}\sum_{p\in P}\sum_{t\in T} d_{mp}X_{mpt} \\ & + \sum_{n\in N}\sum_{l\in L}\sum_{t\in T} d_{nl}X_{nlt} \Bigg] \end{aligned} \tag{14}$$

Eq. (14) computes the CO_2 emissions from transporting the truck-load vehicles between the facilities.

The third objective function assigns to maximize total social concerns. Therefore, total employee satisfaction (TES) and total customer satisfaction (TCS) are contemplated. Thus, the Objective Function 3 (OF3) can be defined as follows:

$$OF3 = Max\,[TES] \tag{15}$$

The total employee satisfaction is obtained by:

$$TES = \sum_{t\in T} \lambda_t \tag{16}$$

From Eq. (16), the fuzzy membership for missed days should be maximized.

4.4. Constraints

The constraints of this study are classified into five categories, including establishing, capacity, balancing, social, and sign constraints.

4.4.1. Establishing constraints

$$\sum_{t\in T} Z_{mt} \le 1 \qquad \forall\, m \in M \tag{17}$$

$$\sum_{t\in T} Z_{dt} \le 1 \qquad \forall\, d \in D \tag{18}$$

$$\sum_{t\in T} Z_{rt} \le 1 \qquad \forall\, r \in R \tag{19}$$

$$\sum_{t\in T} Z_{ct} \le 1 \qquad \forall\, c \in C \tag{20}$$

$$\sum_{t\in T} Z_{qt} \le 1 \qquad \forall\, q \in Q \qquad (21)$$

$$\sum_{t\in T} Z_{lt} \le 1 \qquad \forall\, l \in L \qquad (22)$$

$$1 \le \sum_{m\in K}\sum_{t\in T} Z_{mt} \le UB \qquad (23)$$

$$1 \le \sum_{d\in J}\sum_{t\in T} Z_{dt} \le UB \qquad (24)$$

$$1 \le \sum_{r\in R}\sum_{t\in T} Z_{rt} \le UB \qquad (25)$$

$$1 \le \sum_{c\in C}\sum_{t\in T} Z_{ct} \le UB \qquad (26)$$

$$1 \le \sum_{q\in Q}\sum_{t\in T} Z_{qt} \le UB \qquad (27)$$

$$1 \le \sum_{l\in L}\sum_{t\in T} Z_{lt} \le UB \qquad (28)$$

Constraints (17)-(22) guarantee that each facility can be established once in all periods. Constraints (23)-(28) ensure that there is at least one facility in every echelon. Besides, the number of facilities should be equal to or lower than an upper bound limit.

4.4.2. Capacity constraints

$$\gamma \sum_{d\in D} Y_{mdt} \le \sum_{t'=1}^{t} Z_{mt} \times CAP_m \qquad \forall\, t \in T; m \in M \qquad (29)$$

$$\gamma \left[\sum_{r\in R} Y_{drt} + \sum_{i\in I} Y_{dit}^{rec}\right] \le \sum_{t'=1}^{t} Z_{dt} \times CAP_d \qquad \forall\, t \in T; d \in D \qquad (30)$$

$$\gamma \sum_{i\in I} Y_{rit}^{new} \le \sum_{t'=1}^{t} Z_{rt} \times CAP_r \qquad \forall\, t \in T; r \in R \qquad (31)$$

$$\gamma \left[\sum_{l\in L} X_{clt} + \sum_{q\in Q} X_{cqt}\right] \le \sum_{t'=1}^{t} Z_{ct} \times CAP_c \qquad \forall\, t \in T; c \in C \qquad (32)$$

$$\gamma \sum_{d\in D} X_{ldt} \le \sum_{t'=1}^{t} Z_{lt} \times CAP_l \qquad \forall\, t \in T; l \in L \qquad (33)$$

$$\gamma' \left[\sum_{j\in J} X_{qjt} + \sum_{m\in M} X_{qmt} + \sum_{w\in W} X_{qwt}\right] \le \sum_{t'=1}^{t} Z_{qt} \times CAP_q \qquad \forall\, t \in T; q \in Q \qquad (34)$$

Constraints (29)-(34) guarantee that the product flows from each facility is not more than its capacity.

4.4.3. Balanced constraints

$$\sum_{r\in R} Y_{rit}^{new} \le D_{it} \qquad \forall\, t\in T; i\in I \qquad (35)$$

$$\sum_{d\in D} Y_{dit}^{rec} \le D'_{it} \qquad \forall\, t\in T; i\in I \qquad (36)$$

$$\sum_{c\in C} X_{ict} \le Perc \times (D_{it} + D'_{it}) \qquad \forall\, t\in T; i\in I \qquad (37)$$

$$\sum_{m\in M} Y_{mdt} + INV_{dt} \ge \sum_{r\in R} Y_{drt} \qquad \forall\, t\in T; d\in D \qquad (38)$$

$$\sum_{l\in L} Y_{ldt} = \sum_{i\in I} Y_{dit}^{rec} \qquad \forall\, t\in T; d\in D \qquad (39)$$

$$INV_{dt} = INV_{d,t-1} + \sum_{m\in M} Y_{mdt} - \sum_{r\in R} Y_{drt} \qquad \forall\, t\in T; d\in D \qquad (40)$$

$$\sum_{s\in S} Y_{smt} + \sum_{q\in Q} X_{qmt} = \eta \sum_{d\in D} Y_{mdt} \qquad \forall\, t\in T; m\in M \qquad (41)$$

$$\sum_{d\in D} Y_{drt} = \sum_{i\in I} Y_{rit}^{new} \qquad \forall\, t\in T; r\in R \qquad (42)$$

$$\sum_{i\in I} X_{ict} = \sum_{l\in L} X_{clt} + \sum_{q\in Q} X_{cqt} \qquad \forall\, t\in T; c\in C \qquad (43)$$

$$\sum_{l\in L} X_{clt} = \alpha \sum_{i\in I} X_{ict} \qquad \forall\, t\in T; c\in C \qquad (44)$$

$$\sum_{q\in Q} X_{cqt} = (1-\alpha) \sum_{i\in I} X_{ict} \qquad \forall\, t\in T; c\in C \qquad (45)$$

$$\sum_{c\in C} X_{clt} = \sum_{d\in D} X_{ldt} \qquad \forall\, t\in T; l\in L \qquad (46)$$

$$\sum_{c\in C} X_{cqt} = \eta' \left[\sum_{j\in J} X_{qjt} + \sum_{m\in M} X_{qmt} + \sum_{w\in W} X_{qwt} \right] \qquad \forall\, t\in T; q\in Q \qquad (47)$$

$$\eta' \sum_{m\in M} X_{qmt} = \beta \sum_{c\in C} X_{cqt} \qquad \forall\, t\in T; q\in Q \qquad (48)$$

$$\eta' \sum_{j\in J} X_{qjt} = \beta' \sum_{c\in C} X_{cqt} \qquad \forall\, t\in T; q\in Q \qquad (49)$$

$$\eta' \sum_{w\in W} X_{qwt} = (1-\beta-\beta') \sum_{c\in C} X_{cqt} \qquad \forall\, t\in T; q\in Q \qquad (50)$$

$$\sum_{q \in Q} X_{qjt} \leq D_{jt} \qquad \forall\, t \in T; j \in J \tag{51}$$

Constraints (35)-(37) calculate met demands for each selling channel and collected returns. Constraints (38)-(39) confirm that the product flows, leaving from a distributor is less than or equal to product flows going to it for both the new and the recovered products. Then, constraint (40) calculates the inventory levels of distributors. Constraints (41) describe that the volume of the product's flow is less than or equal to components' flows. Constraints (42)-(50) guarantee the uniformity of product flows. Constraints (51) consider secondary customers' demand.

4.4.4. Social constraints

$$\lambda_t \leq \frac{UMD - \sum_{m \in M} \sum_{t'=1}^{t} MD_{mt} \times Z_{mt}}{UMD} \qquad \forall\, t \in T \tag{52}$$

Constraints (52) define the fuzzy membership number of missed working days.
Finally, the binary and non-negative constraints are explained as:

$Y_{smt}, Y_{mdt}, Y_{drt}, Y_{rit}^{new}, Y_{dit}^{rec}, X_{ict}, X_{clt}, X_{cqt}, X_{ldt}, X_{qmt}, X_{qwt}, X_{qjt}, INV_{dt} \geq 0;\ \lambda_t, \delta_t \in [0, 1];$
$Z_{mt}, Z_{dt}, Z_{rt}, Z_{ct}, Z_{qt}, Z_{lt} \in \{0,1\};$

5. Solution approach

5.1. Multi-objective functions conversion

In many real-world situations, the problems are dealing with several conflicting objectives (Mavrotas, 2009). Therefore, multi-objective decision making (MODM) problems are designed to get a balance among various concerns (Diabat et al., 2019). The solution methods of MODM problems can be solved using some different methods such as weighted sum, ε-constraint, LP metric, goal programming, and fuzzy optimization (Soleimani et al., 2017). In this study, we imply the weighted sum method because it can decrease the complexity of the running time of the proposed Lagrangian relaxation method.

5.2. Weighted sum method

The weighted sum method has been widely used to solve multi-objective problems. This approach converts all the objectives to a unified objective by assigning the proper weight coefficients to each objective (Xiang et al., 2017). Now, the weighted sum reformulation of the proposed model is represented as below:

$$Max\ (\theta 1 \times OF1) + (-\theta 2 \times OF2) + (\theta 3 \times OF3) \tag{53}$$

Subject to:

Constraints (17)-(52)

Note that the coefficients $\theta 1$, $\theta 2$, and $\theta 3$ are weighting coefficients which their ranges are between 0 and 1. As before mentioned, the objectives are not consistent; thus, their magnitude of objectives should be closed to each other. Consequently, the units should be consistent through suitable converting rates. Due to the use of several numerical experiments, we employ OF1×0.5×10^{-3}, OF2×10^{-3}, and OF3×10^{3}, respectively.

5.3. Lagrangian relaxation

As mentioned before, the SCLCS has been already known as an NP-Hard problem, a Lagrangian relaxation algorithm is developed to cope with the large-scaled instances within a reasonable time. This solution method can reach to an exact upper-bound (for maximization problems) and an exact lower-bound (for minimization problems). Also, it can reduce the complexity of the problem by using the dual form of side constraints (Fisher, 2004). Here, we relax Constraints (41), which leads to the dual Lagrangian model $\mathrm{L}(\pi)$. This model is formulated as:

$$L(\pi) = Max\ (\theta 1 \times OF1) + (-\theta 2 \times OF2) + (\theta 3 \times OF3) + \sum_{i \in I} \sum_{t \in T} \pi_{it} \left(D_{it} - \sum_{r \in R} Y_{rit}^{new} \right) \quad (54)$$

Subject to:

Constraints (18)-(35)

Constraints (37)-(53)

Where π_{it} represents non-negative Lagrange multiplier. Eq. (54) obtains an exact upper-bound for the studied model. Besides, a lower-bound is going to reach using the results of the dual Lagrangian model, if the results are feasible. Obviously, the infeasible solutions violate Constraint (36). As a whole, the optimal value of the original model provides an exact lower-bound for the dual Lagrangian model, if its elapsed time be located within a predefined time interval (7200 seconds).

6. Numerical examples

Now, the numerical examples are presented to show: (1) validation of the proposed model, (2) accuracy of the introduced solution. Thus, the first stage of the model (NLP model) is coded in MATLAB R2012a, and the second stage is formulated using GAMS 24.1.2 and solved by GUROBI solver. Note, these software packages are run by a PC with an Intel Core i5 processor and 4 GB of RAM.
The initial dataset for solving the model is generated randomly.

The results of pricing, greening, and advertising decisions, in the first stage, are given as follows:

Table 2. The results of the first stage

Variable	P^{new}	P^{rec}
Optimum value	759.32	565.87

According to Table 2, the optimal price of the new product is obtained at a higher level than the optimal price of the recovered product. Also, the manufacturer should fulfill the optimal levels of greening quality and advertising.
Now, the second stage can be performed by using the optimal pricing, greening, and advertising decisions. Therefore, the MOMILP model is solved regarding different scales. To simplify, the scale of each problem is shown as $(|S|,|M|,|D|,|R|,|C|,|L|,|Q|,|W|,|I|,|J|,|T|)$. For instance, the size of (1,2,3,5,4,2,3,1,10,7,6) explains one potential supplier centers, two

potential manufacture center, three potential distribution centers, five potential retail centers, four potential collection centers, two potential recovery centers, three potential disassembly centers, one potential disposal centers, ten customer areas, seven secondary markets, and six periods.

The results of the second stage are presented in Table 3, as follows:

Table 3. The results of the second stage

Instance	**Scale**	**Size**	**Lower Bound**	**Upper Bound**	**Bound gap** (%)	**GUROBI time** (s)	**Lagrangian time** (s)	**Time gap** (%)
1	Small	(5,3,4,6,3,2,2,1,5,3,6)	8783.29	8798.15	0.17	0.619	0.499	24.05
2		(4,5,5,7,3,3,2,1,7,4,6)	12253.47	12294.41	0.33	1.418	0.951	32.93
3		(5,6,8,6,4,4,5,3,8,5,6)	14830.74	14837.44	0.05	6.869	10.126	32.16
4		(3,7,6,9,5,3,5,4,10,9,9)	26995.79	27025.30	0.11	10.200	4.675	54.17
5		(6,4,9,10,8,4,4,2,12,8,9)	27476.84	27567.61	0.33	99.417	7.258	92.70
6	Large	(14,25,30,32,21,18,15, 8,30,22,12)	53095.43	53107.16	0.02	2195.974	943.646	57.03
7		(24,40,38,47,42,35,21, 10,70,40,12)	NA	24962.28	-	>7200	4876.667	-
8		(30,45,43,48,37,30,22, 9,80,45,12)	NA	30914.81	-	>7200	5867.576	-
9		(37,42,36,55,41,27,29, 13,90,51,18)	NA	43597.18	-	>7200	6166.591	-
10		(28,37,41,53,44,34,23, 9,100,38,24)	NA	34246.03	-	>7200	7089.278	-
Average					**0.17**			**48.84**

Note: “NA” means that GUROBI could not reach to a feasible solution within 2 hours.

From Table 2, the proposed algorithm obtains an exact upper bound in a reasonable time interval. Equally important, the upper bounds are very close to the exact lower bounds. The average gap between the upper and lower bounds is 0.17. Note that the lower bounds for instances 1 to 6 are the optimal global solutions. As well, GORUBI solver cannot obtain an optimal or feasible solution for instances 7 to 10. However, the Lagrangian relaxation method especially shows its efficiency for large-scale instances. Indeed, the proposed algorithm calculates the exact upper bound for all the instances in a reasonable time. The average time gap between GUROBI and Lagrangian is 48.84, which is more applied for (large-scale) real-world problems.

Figures 3 and 4 depict the bound gaps and time gaps, respectively.

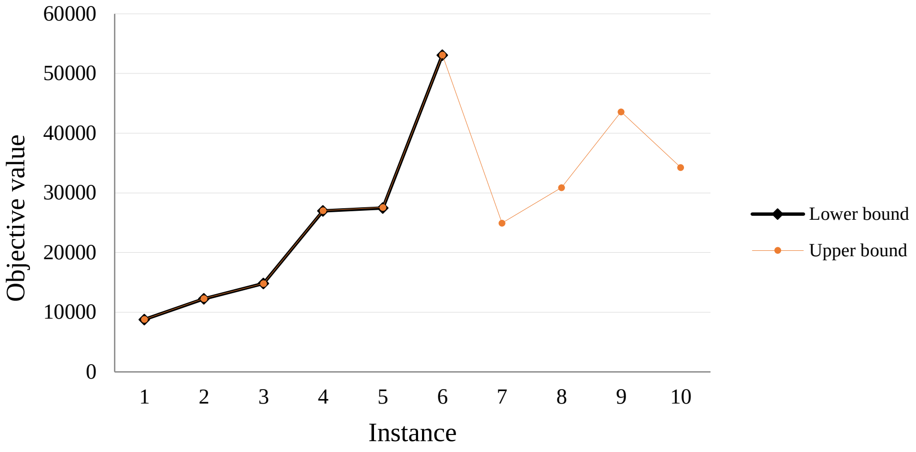

Figure 3. Bound gaps

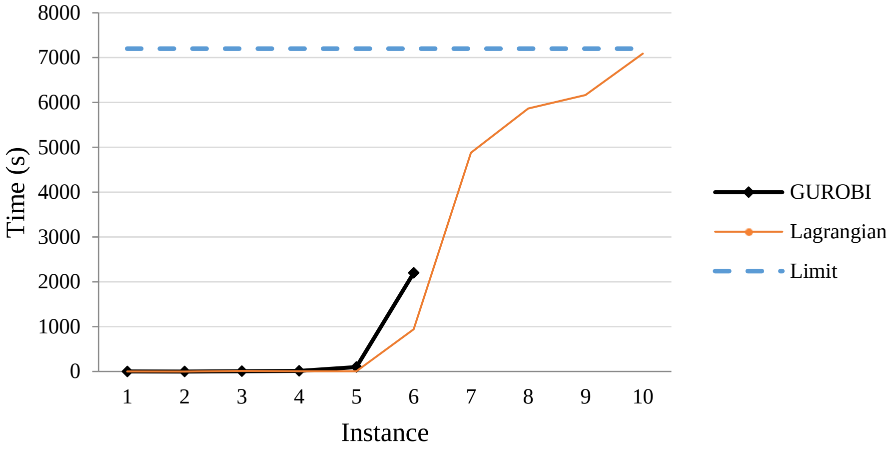

Figure 4. Time gaps

According to Figures 3-4, the bound gaps for instances 1 to 6 are tiny. These results approve the efficiency of the relaxed solution. In other words, the Lagrangian algorithm practically improves the upper bounds to reach an acceptable gap considering the stop condition. Plus, this algorithm provides the optimal upper bound for the large-scale instances while GORUBI solver cannot even give a feasible solution. Moreover, the Lagrangian algorithm significantly decreases the solution time that makes it a practical method to cope with large-scale instances.

7. Conclusions

This paper presented a new two-stage hybrid model for designing an SCLSC network. As a novelty, the coordination decisions such as the price of new and recovered products, greening level, and advertising level are made in the first stage based on a nonlinear profit equation. The results of this stage were as the initialization phase for formulating the MOMILP model in stage two. Also, the MOMILP model developed in terms of economic, environmental, and social objectives. The social objective has been defined under the fuzzy circumstance, as another contribution. The numerical examples showed that the proposed procedure is valid and applied. Also, the Lagrangian relaxation algorithm could significantly decrease the difficulty of the NP-hard problem by relaxing the rough constraint of demand. This algorithm gave the exact upper bounds for large-scale instances within 2 hours. As well, the average gap between the bounds is calculated by about 0.17%, which shows the efficiency of the proposed solution. Further, the average gap for running time (about 49%) is also shown the implication of the Lagrangian algorithm in real-world situations.

As a limitation of the study, the SC has been model under a centralized decision-making structure. Also, in some real cases, the SCs are following decentralized structures. As well, the optimal solutions have been obtained due to the goals of the whole chain, as another limitation. The members of SC (e.g., retailers and manufacturers) could request a deterministic share of profit under a profit-sharing contract.

Finally, future studies can develop the hybrid model considering different structures of SC. Also, the environmental objectives can be extended based on the quality of the establishing facilities. The risk of satisfying demands is also an interesting field of study. Plus, other solution methods such as meta-heuristics and benders decomposition could be used to compare the quality of the results.

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This article can be cited: Khorshidvand, B., Soleimnai, H., Seyyed Esfahani, M.M., Sibdari, S., (2021). "Sustainable closed-loop supply chain network: Mathematical modeling and lagrangian relaxation", *Journal of Industrial Engineering and Management Studies*, Vol. 8, No. 1, pp. 240-260.

Appendix A.

As before mentioned, the studied SC structure is centralized. Thus, to check the concavity rules, the Hessian matrix of Eq. (1) is firstly calculated as below:

$$H=\begin{bmatrix}-2\varphi' & \upsilon'+\upsilon\\ \upsilon'+\upsilon & -2\varphi\end{bmatrix} \tag{A1}$$

Then, Eq. (1) is going to be concave if the sign of determinant of matrix H is the same to its scale. The results are shown as follows:

$$|H_{1\times1}|=-2\varphi'<0$$

$$|H_{2\times2}|=4\varphi'\varphi-(\upsilon'+\upsilon)^2>0 \tag{A2}$$

Thus, Eq. (1) is concave.

Then, the first order optimally conditions for Eq. (1) is derived as:

$$\frac{\partial\Pi}{\partial P^{new}}=0$$

$$P^{new}=\frac{U\times\rho+\upsilon'\times P^{rec}+\omega\times(\varphi'-\upsilon)+\upsilon\times P^{rec}}{2\varphi'} \tag{A3}$$

$$\frac{\partial\Pi}{\partial P^{rec}}=0$$

$$P^{rec}=\frac{U\times(1-\rho)+\upsilon\times P^{new}+\omega\times(\varphi-\upsilon')+\upsilon'\times P^{new}}{2\varphi} \tag{A4}$$

Accordingly, the optimal values of coordination decisions are derived by solving a system of the Eqs. (A3) and (A4). The final results are given as follows:

$$P^{rec}=\frac{U(1-\rho)+\left(\frac{\left(U\rho+\omega(\varphi-\upsilon)\right)(\upsilon+\upsilon')}{2\varphi'}\right)+\omega(\varphi-\upsilon)}{2\varphi-\frac{(\upsilon+\upsilon')^2}{2\varphi'}} \tag{A5}$$

$$P^{new}=\frac{U\rho+\left(\frac{U(1-\rho)+\left(\frac{\left(U\rho+\omega(\varphi-\upsilon)\right)(\upsilon+\upsilon')}{2\varphi'}\right)+\omega(\varphi-\upsilon)}{2\varphi-\frac{(\upsilon+\upsilon')^2}{2\varphi'}}\right)(\upsilon+\upsilon')+\omega(\varphi-\upsilon)}{2\varphi'} \tag{A6}$$

Hence, Theorem 1 is approved.