



Measuring the efficiency of organizations with hierarchical structures using DEA and game theory

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Abstract

Most organizations and companies have hierarchical structures, and appropriate models are required to measure the efficiency of this kind of network systems. Data envelopment analysis (DEA) is a well-known method introduced for measuring the relative efficiency of a set of decision-making units (DMUs) that use multiple inputs to produce multiple outputs. Conventional DEA models usually generate misleading results while evaluating the performance of network systems. The present study aims at developing suitable models for measuring the efficiency of hierarchical structures using the centralized and non-cooperative leader-follower game models. In the proposed method, the divisional efficiencies (within an organization) and the overall efficiency of the organization are calculated. The proposed models are applied to assess the performance of 20 schools in Iran. The results of the two proposed models show that none of the schools are efficient, suggesting that these schools do not optimally utilize their resources. The application of the results of the proposed models enables managers to identify inefficient sub-units and develop strategies to improve their performance.

Keywords: hierarchical structure; efficiency; data envelopment analysis; centralized model; leader-follower model.

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1. Introduction

Measuring the efficiency of systems has been an important task in management for control, planning, and other purposes. One of the most useful methodologies for measuring the relative efficiency of a set of decision-making units (DMUs) which utilizes multiple inputs to generate multiple outputs is data envelopment analysis (DEA) developed first by Charnes, Cooper, and Rhodes, (1978). The conventional DEA models for measuring the relative efficiency of a set of DMUs, without considering the operations of the component processes, often produce misleading results, and thus network models have been recommended (Kao, 2015). Network data envelopment analysis models have been introduced for calculating the efficiencies of complex systems and networks. They take a decision-making unit into account

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along with its entire sub-units and the interconnections which exist between the components in the form of a network structure. Unlike traditional models, network DEA models do not have a fixed formula, and different models have been proposed so far based on the nature of studied procedures and the network's structure. In general, network data envelopment analysis models have two major structures: series structures and parallel structures. The network data envelopment analysis technique is capable of fully representing the connections and dependencies among the internal procedures, and assessing the overall efficiency and the efficiencies of different phases accurately. Furthermore, by employing these models, the sources of inefficiency will be precisely identified within inefficient units.

One particular type of network systems is the hierarchical structure where an organization or a DMU can be composed of a set of independent sub-units, with each consuming the same set of inputs to generate the same set of outputs as is true of the entire system (Kao, 2009). In addition, many systems such as universities, government ministries, and agencies have hierarchical structures (Figure 1), which require suitable models for efficiency calculations. Kao (2015) developed a network DEA model for systems with a hierarchical structure. It was shown that the hierarchical structure is equivalent to the parallel structure, with the components being the units at the bottom of the hierarchy. In addition, Cooperative model such as centralized model and non-cooperative model are game theoretic approaches to evaluate network systems. Non-cooperative model supposes that one of the stages is the leader and another stage is the follower, whereas in the centralized model, all stages are evaluated simultaneously. In the context of game-theoretic approaches, Du et al. (2015) investigated parallel structure by using the cooperative (or centralized) and non-cooperative (Stackelberg or leader-follower) game theory concepts. Jahangoshai Rezaee, Izadbakhsh, and Yousefi (2016) combined DEA and Nash bargaining game as a cooperative game theory approach to evaluate the performance of transportation systems by a large scale of measures. In Tavana et al. (2018), a fuzzy two-stage Game-DEA (FTSGDEA) approach was proposed using a bargaining game model. Mahmoudi, Emrouznejad, and Rasti-Barzoki (2018) proposed a novel game-DEA model for efficiency assessment of network structure DMUs. Two-stage modeling was proposed, where in the first stage, the network was divided into several sub-networks; input variables categorized to measure the efficiency of sub-networks within each input category. In the second stage, the efficiency of the network calculated by aggregating efficiency scores of sub-networks within each category. Omrani, Fahimi, and Mahmoodi (2018) applied the game theory approach in order to increase distinguish power in the DEA model and find out the fair weights in the cross-efficiency DEA context. Sun, Li, and Lim (2020) proposed a comprehensive model using game theory and DEA method to improve resource utilization efficiencies and reduce pollutant emissions in the circular economic system.

The main objective of this paper is to develop two models for measuring the efficiencies of systems with a hierarchical structure through DEA and using game-theory ideas. In this research, two game approaches are presented to measure the efficiency of hierarchical structures. One is a centralized model based upon the theory of a cooperative game where the overall efficiency is maximized to optimize all stages' efficiency scores simultaneously. The other is a non-cooperative game model which supposes that one stage is the leader with action priority to optimize its efficiency first, and then the efficiency of the follower stage is measured subject to maintaining the first-derived leader's efficiency. The presented models are applied to measure the efficiencies of the educational institutions in Iran, which have hierarchical structures.

Educational institutions play an important role in the development of a country in this age of the knowledge economy. In addition, a considerable portion of the government's budget in many countries including Iran is spent on educational institutions at different levels ranging

from elementary school to higher education. Thus, employing patterns and procedures which are compatible with the present structure of these institutions is of paramount importance. Furthermore, analyzing the efficiency of educational systems is one of the main focuses of the policy debate to promote national competitiveness and future economic growth. Therefore, the available data and information should be used for evaluating the educational institutions' efficiency to be further employed in planning and budgeting their resources. The remainder of the paper is organized as follows. Section 2 reviews the relevant literature. In section 3, the centralized and leader-follower approaches for measuring the hierarchical structure's efficiency are developed. In section 4, we test the proposed models on the data presented by Chen and Yan (2011), and in section 5 these models are applied to measure the efficiencies of the educational institutions. Section 6 presents concluding remarks.

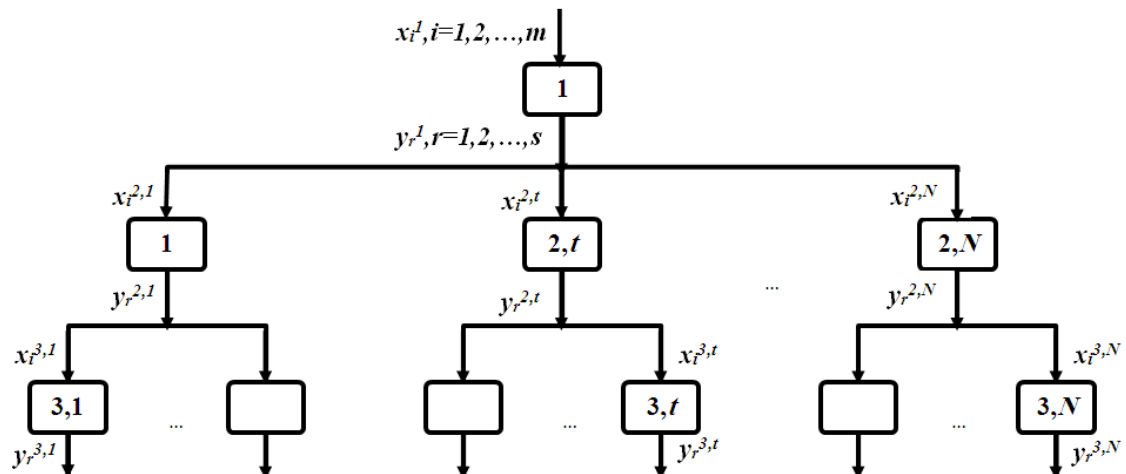


Figure 1. A hierarchical structure with 3 levels

2. Literature review

Conventional DEA models view the system as a black-box, ignoring the operations and interdependence of the internal processes. As a result, a system may be evaluated as efficient even when all its component processes are not. However, in network models, which were developed by Faïre (1991), it is possible to look inside a black-box as well as its components and sub-units for evaluating the performance of a decision making unit. Although many organizations have a hierarchical structure, little attention has been paid to this structure so far. Cook et al. (1998) introduced the concept of hierarchical DEA, where efficiency can be viewed at various levels. Castelli, Pesenti, and Ukovich, (2004) presented a model to measure the performance of a hierarchical system with one and two levels, where the units at a higher level can have common subordinate units at a lower level. Cook and Green (2005) developed a CCR-type model to measure the efficiencies of ten Canadian power plants, with 40 subordinate power units. In these examples, a unit at a higher level does not necessarily have the same number of subordinate units at a lower level.

Some researchers have used the game theory approach to measure efficiency in network systems. Chen, Liang, and Yang (2006) developed the DEA model based on the game approach to evaluate supply chain efficiency. Liang, Cook, and Zhu (2008) extend the data envelopment analysis method for two-stage processes using game theory concepts. Du et al. (2011) applied directly Nash bargaining game theory to the efficiency of DMUs that have the afore-mentioned two-stage processes. Li et al. (2012) extended Liang et al. (2008) by assuming that the inputs to the second stage include both the outputs from the first stage and additional inputs to the second stage. Two models proposed to evaluate the performance of

this type of general two-stage network structures. One is a non-linear centralized model whose global optimal solutions can be estimated using a heuristic search procedure. The other is a non-cooperative model, in which one of the stages is regarded as the leader and the other is the follower. Jalali Naini, Moini, and Jahangoshai Rezaee_(2013) proposed a game-theoretic model based on the Nash bargaining game to calculate weights when parallel stages with shared inputs compete to reach high efficiency in the competitive strategy. Two data sets including the bank branches and thermal power plants in Iran were used to show the abilities of the proposed model. Kou et al. (2016) presented a model to measure the efficiencies of multi-period and multi-division systems using game theory and DEA. Wu et al. (2016) provided an approach for analyzing the reuse of undesirable intermediate outputs in a two-stage production process with a shared resource. Additive efficiency measures and non-cooperative efficiency measures were proposed to illustrate the overall efficiency of each DMU and the respective efficiency of each sub-DMU. An et al. (2017) proposed a centralized model and two leader-follower game models to measure the efficiency of a parallel system with two components that have an interactive relationship in a centralized mode and non-centralized mode, respectively.

In recent years, several studies have been undertaken to analyze the efficiency of the educational institutions. Haelermans and Ruggiero (2017) introduced a non-parametric measure of the cost of adequacy that controls for the socio-economic environment and resource prices to analyze the estimates of the cost of the adequacy of Dutch schools. Cordero, Santin, and Simancas, (2017) used a fully nonparametric framework to assess the efficiency of primary schools using data about schools in 16 European countries. Yang, Fukuyama, and Song (2018) investigated the inefficiency and productivity of 64 Chinese research universities and their evolution over the recent period of 2010–2013, where the production process of each research university was described as a general two-stage network process. Cordero et al. (2018) assessed the performance of secondary schools from 36 countries. For this purpose, they applied a robust conditional nonparametric approach that allows them to incorporate the effect of contextual factors at both school and country level in the estimation of efficiency measures. Masci, Witte, and Agasisti (2018) investigated the influence of school size, principal characteristics, and school management practices on educational performance. Santín and Sicilia (2018) introduced a new methodology that combines production frontier and impact evaluation insights that allows using DEA as an identification strategy of a treatment with high and low quality teachers within schools to assess their performance. The authors used a unique database of primary schools in Spain that, for every school, supplies information on two classrooms in 4th grade where students and teachers were randomly assigned to the two classrooms.

There are many studies in the context of efficiency measurement. Among them, some recent studies are briefly reviewed below. Yurdakul and İç (2018) developed a comprehensive hierarchical performance measurement model to determine a manufacturing company's overall performance within its industry as well as its strengths and weaknesses in critical activities. It lets one to combine a company's performance scores in seventeen critical activities with important industry-specific objectives to obtain a single overall performance score by using a 4-Point Fuzzy Scale and a modified fuzzy version of the Technique for Order Preference by Similarity to Ideal Solution approach. The calculated overall performance scores provide a ranking order among manufacturing companies within their industry. Arab Momeni, Ebrahimi Arjestan, and Yaghoubi (2018) presented a two-stage DEA in order to evaluate the efficiency of enterprise resources planning (ERP) systems in such a way that the operational and the technical aspect are evaluated in the first stage and the second stage of DEM model, respectively. Vaezi et al. (2019) considered a three-stage network model with additional inputs and undesirable outputs and obtains the efficiency of

the network, as interval efficiency in the presence of the imprecise datum. Zhang and Shi (2019) applied DEA to measure the efficiency of 24 colleges and universities. Cook et al. (2019) extended DEA to the evaluation of performance in the specific context of pay-for-performance incentive plans. Their approach ensures that the evaluation of the performance of DMUs that follow the implementation of incentive plans is made in terms of targets that are attainable, as well as representing best practices. The proposed approach was applied to evaluate the performance of public Spanish universities. Tavana et al. (2019) defined a fuzzy multi-objective multi-period network DEA model customized to evaluate the dynamic performance of oil refineries in the presence of undesirable outputs. A standard fuzzy operator has used to define the efficiency levels to integrate multiple objectives and periods within a unique maximization framework. Zhou et al. (2019) proposed a multi-period three-stage DEA model to measure the efficiency of banking systems under uncertainty. Adabavazeh and Nikbakht (2020) evaluated the performance of an organization based on the main factors of the reverse supply chain with the service quality approach using the DEA model. Firstly, performance indicators were identified and then the efficiency of the 24 main factors of reverse supply chain success in the airline industry was determined by the output-oriented DEA-BCC model. Nemati, Kazemi Matin, and Toloo (2020) presented a two-stage DEA model with partial impacts between inputs and outputs. The authors formulated a couple of new mathematical programming models in the DEA framework with the aim of considering partial impact between inputs and outputs for calculating aggregate, overall, and sub-unit efficiencies along with resource usage by production lines for a two-stage production system. Table 1 provides a summarization of the related literature. According to Table 1, in major, the researches performed in the network systems focused on two and three stages networks. However, the current research considers the hierarchical structure and applies the game theory approach to measure the efficiency of this kind of network systems.

Table 1. Summarized related literature

Reference	Methodology	Type of game	Structure of network	Application
Castelli, Pesenti, and Ukovich (2004)	DEA	-	Hierarchical structure	University
Cook and Green (2005)	DEA	-	Hierarchical structure	Power plants
Li et al. (2012)	DEA and game	Non-cooperative	Two-stage	China Construction Bank
Jalali Naini, Moini, and Jahangoshai Rezaee (2013)	DEA and game	Cooperative	Two-stage	Bank branches
Kao (2015)	DEA	-	Hierarchical structure	University
Du et al. (2015)	DEA and game	Cooperative and Non-cooperative	Parallel network	National forests of Taiwan
Kou et al. (2016)	DEA and game	Cooperative	Two-stage	OECD countries
Jahangoshai Rezaee, Izadbakhsh, and Yousefi (2016)	DEA and game	Cooperative	-	Transportation systems
Wu et al. (2016)	DEA and game	Cooperative and Non-cooperative	Two-stage	Industrial production processes
An et al. (2017)	DEA and game	Cooperative and Non-cooperative	Two-stage	Chinese “985 Project” universities’ performance

Reference	Methodology	Type of game	Structure of network	Application
Yousefi et al. (2017)	DEA and game	Cooperative	Three-stage	Supply chain
Badiezadeh, Saen, and Samavati, (2018)	DEA and game	Cooperative	Three-stage	Supply chain
Tavana et al. (2018)	DEA and game	Cooperative	Two-stage	Bank branches
Mahmoudi, Emrouznejad, and Rasti-Barzoki (2018)	DEA and game	Cooperative	Three-stage	Urban road construction projects
Zhou et al. (2018)	DEA and game	Non-cooperative	Two-stage	Supply chain
Vaezi et al. (2019)	DEA and game	Cooperative	Three-stage	A factory's efficiency
Sun, Li, and Lim (2020)	DEA and game	Non-cooperative	Two-stage	Circular economic systems
Current paper	DEA and game	Cooperative and Non-cooperative	Hierarchical structure	Educational institutions

3. Proposed game models for hierarchical structures

An organization usually has a hierarchical structure with one unit at the first level possessing sub-units at the second level. Some of the units at the second level may possess sub-units at the third level, and this classification can be extended to other levels. Figure 1 displays a balanced hierarchical structure with three levels. The first unit at the first level, which has been numbered 1, has N sub-units at the second level, with each of them possessing their own sub-units at the third level and so forth.

As mentioned above, considering the system as a black-box may lead to incorrect results in efficiency measurement. Therefore, in the following, we develop some network DEA models based on the game theory which take the operation of individual sub-units into account in calculating the efficiency of the hierarchical structure.

In sections 3.1 and 3.2, we develop the centralized and non-cooperative leader-follower game theory for efficiency measurement and decomposition in the hierarchical structure. The models that are developed here are based on the approaches presented by Liang et al. (2008) and Du et al. (2015). First, we consider n levels in the hierarchical structure as a series network with n stages (Figure 2). Next, each level of this series network is considered as a parallel network. For calculating the efficiency of every level, centralized and leader-follower game approaches will be used.

3.1. The centralized approach

Consider there are q systems with the structure displayed in Figure 2. In this system with a hierarchical structure, suppose that there are n levels. We assume that these n levels form a series network with n stages and N sub-units in each stage. These N sub-units compose a parallel network. Suppose that decision making unit j is denoted by DMU_j ($j=1, \dots, q$), and the i th input and r th output of DMU_j denoted by x_{ij} ($i = 1, \dots, m$) and y_{rj} ($r = 1, \dots, s$), respectively. Also, X_j^p, Y_j^p denote all inputs and outputs of Level p for DMU_j and X_j^{pt}, Y_j^{pt} are inputs and outputs of sub-unit t in Level p of the system in DMU_j . Consider v_i^p and u_r^p as the relative importance of i th input and r th output in level p , respectively, to be

determined by the mathematical program. Besides, v_i^{pt} and u_r^{pt} show the relative importance of i th input and r th output of sub-unit t in Level p of the system, respectively. Based on Du et al. (2015), the efficiency score of Level p in DMU_k using the centralized model is calculated from the model (1).

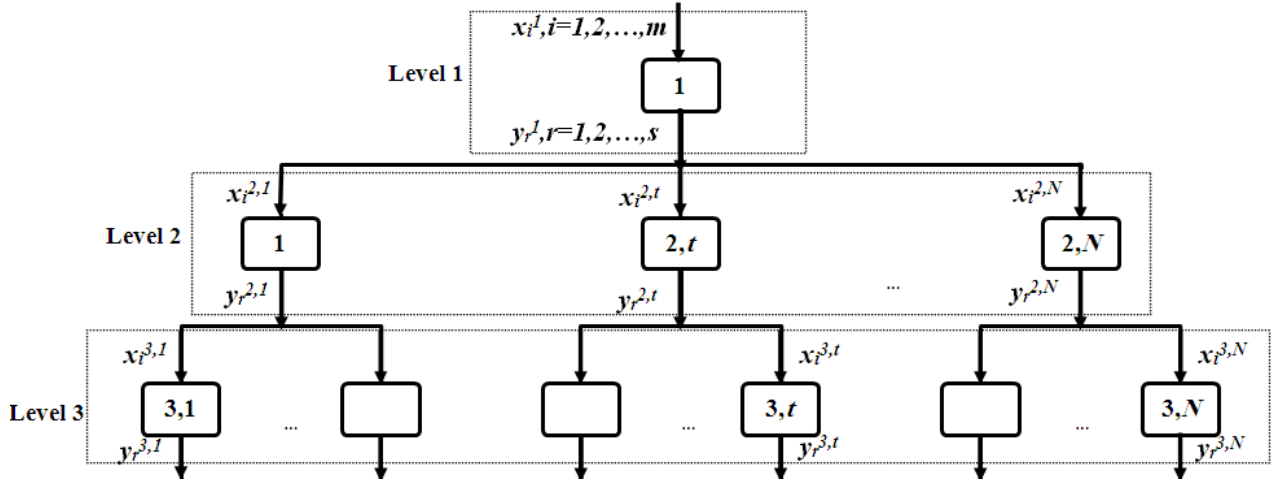


Figure 2. A hierarchical structure as a series network with 3 levels

$$E_k^p = \max \frac{\sum_{r=1}^s u_r^p Y_{rk}^p}{\sum_{i=1}^m v_i^p X_{ik}^p} \tag{1}$$

s.t :

$$\frac{\sum_{r=1}^s u_r^p Y_{rj}^p}{\sum_{i=1}^m v_i^p X_{ij}^p} \leq 1$$

$$\frac{\sum_{r=1}^s u_r^{pt} Y_{rj}^{pt}}{\sum_{i=1}^m v_i^{pt} X_{ij}^{pt}} \leq 1 \quad t = 1, \dots, N$$

$$u_r^p, v_i^p, u_r^{pt}, v_i^{pt} \geq \varepsilon; r = 1, \dots, s; i = 1, \dots, m; j = 1, 2, \dots, q$$

Note that the objective function maximizes the ratio of outputs to inputs at the level p of DMU_k . The first constraint sets correspond to the level p and the second constraint sets correspond to the sub-unit t ($t=1, \dots, N$) in Level p of the system. Since

$$X_j^p = \sum_{t=1}^N X_j^{pt}, \quad Y_j^p = \sum_{t=1}^N Y_j^{pt}$$

it is obvious that the first set of constraints in the model (1) is redundant. Using the Charnes–Cooper transformation (1978), we achieve the following linear program (2) equivalent to the fractional program (1):

$$\begin{aligned}
 E_k^p &= \max \sum_{r=1}^s u_r^p Y_{rk}^p \\
 \text{s.t. :} & \\
 \sum_{i=1}^m v_i^p X_{ik}^p &= 1 \\
 \sum_{r=1}^s u_r^{pt} Y_{rj}^{pt} - \sum_{i=1}^m v_i^{pt} X_{ij}^{pt} &\leq 0 \\
 u_r^p, v_i^p, u_r^{pt}, v_i^{pt} &\geq \varepsilon; r = 1, \dots, s; i = 1, \dots, m; j = 1, 2, \dots, q
 \end{aligned}
 \tag{2}$$

The efficiencies of all n levels of DMU_k in the hierarchical structure are obtained using the model (2) for $p=1, \dots, n$. Based on the model proposed by Du et al. (2011) for series systems, the overall efficiency of DMU_k is obtained from the product of the efficiency scores of all levels.

$$E_k = \prod_{p=1}^n E_p = E_1 \cdot E_2 \cdot \dots \cdot E_n
 \tag{3}$$

3.2. The Leader-Follower Approach

Similar to the previous section, we suppose the hierarchical structure as a series network with n levels and each level of this network as a parallel system with N sub-units. Next, we develop the leader-follower approach for calculating the efficiency score of each stage. The efficiency of the first stage i.e., first level, is determined via the following CCR model.

$$\begin{aligned}
 E_k^{1*} &= \max \sum_{r=1}^s u_r^1 Y_{rk}^1 \\
 \text{s.t. :} & \\
 \sum_{i=1}^m v_i^1 X_{ik}^1 &= 1 \\
 \sum_{r=1}^s u_r^1 Y_{rj}^1 - \sum_{i=1}^m v_i^1 X_{ij}^1 &\leq 0 \\
 u_r^1, v_i^1 &\geq \varepsilon; r = 1, \dots, s; i = 1, \dots, m; j = 1, 2, \dots, q
 \end{aligned}
 \tag{4}$$

Where, X_j^1, Y_j^1 denote all inputs and outputs of Level 1 for DMU_j. For calculating the efficiencies of other levels, the non-cooperative leader-follower model will be developed and used here. At Level p ($p > 1$) of the system, there are N sub-units. If we assume that the sub-unit 1 is the leader, then its efficiency score is determined via the following model.

$$\begin{aligned}
 E_k^{p1*} &= \max \sum_{r=1}^s u_r^{p1} Y_{rk}^{p1} \\
 \text{s.t. :} & \\
 \sum_{i=1}^m v_i^{p1} X_{ik}^{p1} &= 1 \\
 \sum_{r=1}^s u_r^{p1} Y_{rj}^{p1} - \sum_{i=1}^m v_i^{p1} X_{ij}^{p1} &\leq 0 \\
 u_r^{p1}, v_i^{p1} &\geq \varepsilon; r = 1, \dots, s; i = 1, \dots, m; j = 1, 2, \dots, q
 \end{aligned} \tag{5}$$

Here, X_j^{p1} , Y_j^{p1} show inputs and outputs of sub-unit 1 at Level p of the system for DMU_j. According to the leader's efficiency results, the efficiency for follower 1 (sub-unit 2) is derived by solving the following linear model (6).

$$\begin{aligned}
 E_k^{p2*} &= \max \sum_{r=1}^s u_r^{p2} Y_{rk}^{p2} \\
 \text{s.t. :} & \\
 \sum_{i=1}^m v_i^{p2} X_{ik}^{p2} &= 1 \\
 \sum_{r=1}^s u_r^{p2} Y_{rj}^{p2} - \sum_{i=1}^m v_i^{p2} X_{ij}^{p2} &\leq 0 \\
 \sum_{r=1}^s u_r^{p1} Y_{rk}^{p1} - E_k^{p1*} \sum_{i=1}^m v_i^{p1} X_{ik}^{p1} &\leq 0 \\
 u_r^{p2}, v_i^{p2}, u_r^{p1}, v_i^{p1} &\geq \varepsilon; r = 1, \dots, s; i = 1, \dots, m; j = 1, 2, \dots, q
 \end{aligned} \tag{6}$$

Where, X_j^{p2} , Y_j^{p2} refer to the inputs and outputs of sub-unit 2 at Level p of the system for DMU_j.

Based on the efficiency results of the leader and all the previous followers, the efficiency for the follower ($w - 1$) (sub-unit w) at Level p is derived from the following relation:

$$\begin{aligned}
 E_k^{pw*} &= \max \sum_{r=1}^s u_r^{pw} Y_{rk}^{pw} \\
 \text{s.t. :} & \\
 \sum_{i=1}^m v_i^{pw} X_{ik}^{pw} &= 1 \\
 \sum_{r=1}^s u_r^{pw} Y_{rj}^{pw} - \sum_{i=1}^m v_i^{pw} X_{ij}^{pw} &\leq 0 \\
 \sum_{r=1}^s u_r^{pt} Y_{rk}^{pt} - E_i^{pt*} \sum_{i=1}^m v_i^{pt} X_{ik}^{pt} &\leq 0 \quad t = 1, \dots, w-1 \\
 u_r^{pw}, v_i^{pw}, u_r^{pt}, v_i^{pt} &\geq \varepsilon; r = 1, \dots, s; i = 1, \dots, m; j = 1, 2, \dots, q
 \end{aligned} \tag{7}$$

Once the efficiency scores for all sub-units at Level p are obtained as $E_k^{pt^*}$, $t = 1, \dots, N$, we calculate the overall efficiency of Level p for DMU $_k$ as

$$E_k^{p^*} = \max \sum_{r=1}^s u_r^p Y_{rk}^p$$

s.t :

$$\sum_{i=1}^m v_i^p X_{ik}^p = 1$$

$$\sum_{r=1}^s u_r^p Y_{rj}^p - \sum_{i=1}^m v_i^p X_{ij}^p \leq 0$$

$$\sum_{r=1}^s u_r^{pt} Y_{rk}^{pt} - E_k^{pt^*} \sum_{i=1}^m v_i^{pt} X_{ik}^{pt} \leq 0 \quad t = 1, \dots, N$$

$$u_r^p, v_i^p, u_r^{pt}, v_i^{pt} \geq \varepsilon; r = 1, \dots, s; i = 1, \dots, m; j = 1, 2, \dots, q$$
(8)

Finally, based on the model proposed by Du et al. (2011) for series systems, the overall efficiency score of DMU $_k$ is obtained from the multiplication of the efficiency scores of all stages i.e.,

$$E_k = \prod_{p=1}^n E_p = E_1 \cdot E_2 \cdot \dots \cdot E_n.$$

4. Pre-calibration of the proposed models

Before applying the introduced models for evaluating the performance of the educational institutions, to prove their capability, we tested and run them on the data presented by Chen and Yan (2011). Based on Chen and Yan (2011), consider a network system with two levels (Figure 3). Referring to Table 2, suppose that there are nine DMUs. The efficiency scores of the nine DMUs as calculated from our proposed models and the model presented by Chen and Yan (2011) are displayed in Table 2 and Figure 4. For testing the reliability of the efficiency scores of the proposed models, we used Spearman's rank correlation test. Table 3 reports Spearman's rank correlation test between the proposed models and Chen and Yan (2011)'s model. As can be seen, Spearman's rho for CCR is 0.844 with the p-value 0.002. Further, for the centralized model, it is 0.812 with the p-value is equal to 0.004 and for the leader-follower model, it is 0.786 with the p-value 0.012. Therefore, the results of the proposed models are very close to Chen and Yan's (2011) findings and there is no significant difference between them. It should be noted that the difference between efficiency scores obtained from the proposed models and the Chen and Yan's (2011) model at points such as G and E is due to the difference between the magnitude of the deviation of these data points from the efficiency frontier compared to Chen and Yan's (2011) model. The high correlation between the efficiency scores of Chen and Yan's (2011) model and the proposed models proved that these models are suitable for measuring the efficiency of network structures.

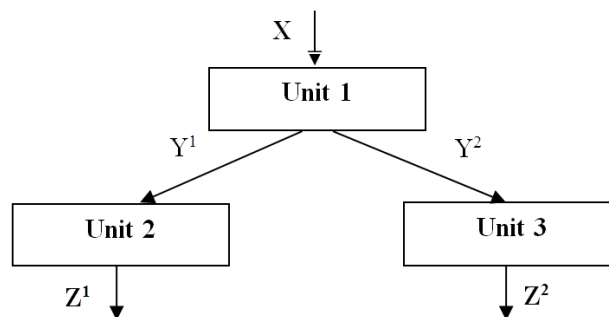


Figure 3. A network structure with three units (Chen and Yan, 2011)

Table 2. Input-output data, efficiency scores, and ranks of nine DMUs for testing of the proposed models

DMU	X	Y ¹	Y ²	Z ¹	Z ²	CCR (Rank)	The Centralized Model (rank)	The Leader-Follower Model (rank)	Chen and Yan 2011 (Rank)
A	5	4	4	4	3	0.5988 (5)	0.4706 (7)	0.25 (9)	0.3697 (7)
B	6	4	1	8	2	0.5382 (6)	0.5154 (6)	0.3651 (7)	0.4893 (5)
C	5	1	3	2	6	0.9197 (2)	0.7059 (2)	0.5000 (4)	0.5000 (4)
D	12	8	2	8	4	0.3840 (7)	0.3651 (9)	0.3651 (8)	0.2764 (8)
E	6	4	2	8	4	0.7680 (3)	0.6051 (4)	0.4286 (5)	0.5528 (3)
F	6	2	3	2	6	0.7664 (4)	0.4167 (8)	0.4167 (6)	0.4167 (6)
G	6	8	1	8	2	0.5382 (6)	0.6667 (3)	0.6667 (2)	0.4893 (5)
H	6.67	1	3	2	9	1 (1)	0.5622 (5)	0.5622 (3)	0.5625 (2)
I	6	8	1	17	3	1 (1)	1 (1)	1 (1)	1 (1)

Table 3. Spearman's rank correlation test between proposed models and Chen and Yan (2011)

	CCR (p-value)	The Centralized Model (p-value)	The Leader-Follower Model (p-value)
Chen and Yan (2011)	0.844 (0.002)	0.812 (0.008)	0.786 (0.012)

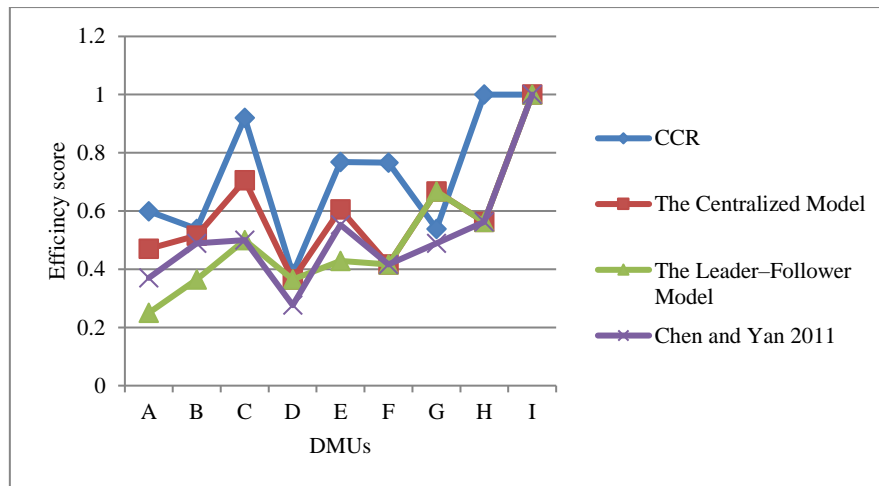


Figure 4. Comparison of efficiency scores between developed models and Chen and Yan (2011)

5. Case study description

Educational institutions play a vital role in the development of countries by training efficient, competent, and skilled human resources for meeting the needs of the society in different areas because they are practically moving in the direction of development by offering their outputs to the society. Every system receives certain inputs from the society based on its nature and philosophy, and after processing them in a specific way, some outputs are offered to the society as final products. With regards to the key role of the educational institutions and people's sensitivity about their performance, they must develop symmetrically both in terms of their quality and their quantity. Further, nowadays, financial limitations and limitations of human resources render the performance appraisal of the organizations necessary. The developments of the educational system of Iran within two last decades, specifically in terms of the number of students, suggest that the growth has been primarily in terms of quantity, and that little attention has been paid to improving the quality of the educational institutions. Quality improvement requires establishing an appropriate system for assessment and evaluation. For precise identification of the current state and improving it, such a system in the educational institutions of Iran should comply with specific features, conditions, and the context that exists. If the efficiency measurement of the educational institutions is performed in a systematic manner, it enables the relevant authorities to constantly control and improve their activities with a higher accuracy level. As such, in this section, we apply the proposed DEA models to measure the efficiency of some schools in Iran. Note that previous researches on measuring the efficiency of schools have not paid attention to the hierarchical structure of these institutions. In this paper, we are going to consider the hierarchical structure as an important feature in measuring the efficiencies of schools.

5.1. Data

Since the final results of a DEA model depend on the type of variables, their selection is of paramount importance. Indeed, it is so critical that if one of the output or input variables changes, the final results of the model will fluctuate accordingly. As such, if the variables are selected carefully, DMU's efficiency can be evaluated in a more realistic manner. After searching the domestic and foreign sources concerning this field, and consulting the experts and authorities of the field, the variables of this research were selected. More information on selecting input and output variables is available in Abbott and Doucouliagos (2003), Daraio,

Bonaccorsi, and Simar (2015), Katharaki and Katharakis (2010), and Sohn and Kim (2012). The selection of input and output variables depends on the purpose of the efficiency evaluation aimed by the author (Ramzi et al., 2016). Through this study, we aim to analyze the effect of academic degree and work experience of principal, as well as institution expenditures on the institution performance. Therefore, we select the specific inputs and outputs for each sub-unit of the DMUs as follows.

Academic degree: The average academic degree (Bachelor's degree=2, Master's degree=3, Ph.D., and above =4).

Work experience: Number of years of work experience.

Education vice-presidency unit's expenditures: Total annual costs of the education vice-presidency unit (million Rials).

Cultural and Health vice-presidency unit's expenditures: Total annual costs of the cultural and health vice-presidency unit (million Rials).

Cultural preceptor's expenditures: Total annual costs of the cultural preceptor unit (million Rials).

Health preceptor's expenditures: Total annual costs of the health preceptor unit (million Rials).

Overall performance: The overall institution's performance throughout the year (from one up to a hundred).

Student satisfaction: Satisfaction level of students with the presidency unit (from one up to a hundred).

The average result of Science: The average grade of all students in Science (from one up to a hundred).

The average result of English: The average grade of all students in English (from one up to a hundred).

Results of competitions: The average score of students in competitions which have been held between institutions (from one up to a hundred).

Cultural program 1: The result of the first cultural program that has been implemented throughout institutions (from one up to a hundred).

Cultural program 2: The result of the second cultural program that has been implemented throughout institutions (from one up to a hundred).

Health program 1: The result of the first health program that has been implemented throughout institutions (from one up to a hundred).

Health program 2: The result of the second health program that has been implemented throughout institutions (from one up to a hundred). Here, Health programs 1 and 2 are programs implemented by the Ministry of Education in the field of health among schools in two different time periods. Besides, Cultural programs 1 and 2 are plans performed by the Ministry of Education in the field of culture and education among schools.

These variables are presented in Table 4. The input and output data of each sub-unit are reported in Tables 5 and 6. Table 7 and Table 8 present the descriptive statistics of inputs and outputs for the institutions. The data are for 20 schools and have been derived from the Department of Education in Zanjan province for the academic year 2018-2019.

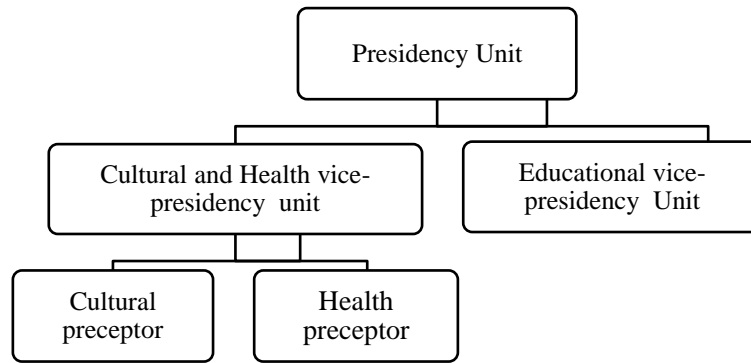


Figure 5. The hierarchical structure of institutions

Table 4. Input and output variables allocated to each sub-unit

	Input variables	Output variables
Presidency unit	Academic degree of dean	Overall performance
	Work experience of dean	Student satisfaction
Education vice-presidency unit	Education vice-presidency unit's expenditures	The average result of science
		The average result of English
Cultural and Health vice-presidency unit	Cultural and Health vice-presidency unit's expenditures	Overall performance
		Results of competitions
Cultural preceptor	Cultural preceptor's expenditures	Cultural program 1
		Cultural program 2
Health preceptor	Health preceptor's expenditures	Health program 1
		Health program 2

Table 5. Input data of DMUs

Dmus	Presidency unit		Education vice-presidency unit	Cultural and Health vice-presidency unit	Cultural preceptor	Health preceptor
	Academic degree of dean	Work experience of dean	Education vice-presidency unit's expenditures	Cultural and Health vice-presidency unit's expenditures	Cultural preceptor's expenditures	Health preceptor's expenditures
1	2	17	42	44	47	48
2	2	15	45	41	47	49
3	3	19	41	47	40	40
4	2	9	42	38	52	52
5	3	14	41	37	48	52
6	2	10	58	52	41	41
7	2	18	82	78	71	78
8	3	17	41	45	41	42
9	2	20	78	72	81	80
10	2	11	44	41	48	50
11	3	21	47	40	49	51
12	2	12	43	39	42	40
13	3	14	44	49	61	52
14	2	18	43	45	50	54
15	2	8	61	52	43	43
16	3	13	84	72	71	80
17	3	9	42	40	45	44
18	2	12	80	71	87	82
19	3	13	42	41	51	52
20	2	17	48	42	51	50

Table 6. Output data of DMUs

Dmus	Presidency unit		Education vice-presidency unit		Cultural and Health vice-presidency unit		Cultural preceptor		Health preceptor	
	Overall performance	Student satisfaction	The average result of science	The average result of English	Overall performance	Results of competitions	Cultural program 1	Cultural program 2	Health program 1	Health program 2
1	42	44	55	48	47	48	45	49	43	44
2	45	41	52	48	47	49	42	44	45	50
3	41	47	47	42	40	40	40	47	42	39
4	42	38	44	48	52	52	50	57	54	51
5	41	37	40	48	48	52	44	50	51	50
6	58	52	52	52	41	41	39	42	41	40
7	82	78	62	78	71	78	65	72	75	46
8	41	45	38	42	41	42	40	43	42	44
9	78	72	81	75	81	80	72	80	85	76
10	44	41	57	50	49	50	42	51	45	52
11	47	40	54	50	49	51	47	46	47	53
12	43	39	47	42	42	40	44	41	44	41
13	44	49	41	47	61	55	55	62	58	50
14	43	45	42	50	50	54	57	54	53	51
15	61	52	52	51	43	43	48	44	41	52
16	84	72	68	82	71	80	68	74	75	78
17	42	40	39	42	45	44	42	45	41	41
18	80	71	81	78	87	82	81	84	85	80
19	42	41	61	51	51	52	53	51	42	51
20	48	42	57	51	51	50	47	49	51	50

Table 7. Descriptive statistics for the selected inputs

	Presidency unit		Education vice-presidency unit	Cultural and Health vice-presidency unit	Cultural preceptor	Health preceptor
	Academic degree of dean	Work experience of dean	Education vice-presidency unit's expenditures	Cultural and Health vice-presidency unit's expenditures	Cultural preceptor's expenditures	Health preceptor's expenditures
Mean	2.4	14.35	52.4	49.3	53.3	54
Std. dev.	0.49	3.80	15.23	12.70	13.32	13.70
Maximum	3	21	84	78	87	82
Minimum	2	8	41	37	40	40

Table 8. Descriptive statistics for the selected outputs

	Presidency unit		Education vice-presidency unit		Cultural and Health vice-presidency unit		Cultural preceptor		Health preceptor	
	Overall performance	Student satisfaction	The average result of science	The average result of English	Overall performance	Results of competitions	Cultural program 1	Cultural program 2	Health program 1	Health program 2
Mean	52.4	49.3	53.5	53.75	53.35	54.15	51.05	54.25	53	51.95
Std. dev.	15.63	13.03	12.53	13.03	13.65	14.05	11.89	13.15	14.85	12.09
Maximum	84	78	81	82	87	82	81	84	85	80
Minimum	41	37	38	42	40	40	39	41	41	39

5.2. Measuring the efficiency

In this section, the centralized and non-cooperative leader-follower models are applied to measure the efficiencies of 20 institutions with the structure illustrated in Figure 5. In the hierarchical structure of the institutions, the presidency unit located at top of the hierarchical structure and education vice-presidency unit with cultural and health vice-presidency unit is at the second level. Further, the cultural preceptor and health preceptor are at the third level of the system.

If we ignore the internal structure of the hierarchical system and use the conventional black-box CCR model to measure the efficiency, we will get the results shown in the second column of Table 9, in which there are 3 efficient and 17 inefficient institutions. This information is not very informative, as it does not distinguish the order of efficient DMUs, nor is it able to decompose the efficiency of a DMU into that of its functions. In order to obtain this information, the centralized approach and non-cooperative leader-follower approach are applied. In the non-cooperative model, the education vice-presidency unit at the second level of the hierarchical structure and the cultural preceptor at the third level are considered as the leader.

The third to sixth columns of Table 9 reports the results obtained from applying the centralized model. The efficiency score of each DMU from the centralized model based on the data of the first, second, third levels is shown in the third, fourth, fifth columns of Table 9, respectively. For example, these results show that the efficiency scores of levels 1, 2, and 3 for DMU 1 are 0.7154, 0.9412, and 0.913. Also, DMUs 7, 15, and 18 are efficient when we measure the efficiency of DMUs based on level 1 using the centralized model. This means that the performance of the presidency unit in DMUs 7, 15, and 18 is appropriate; however, other DMUs need to improve the performance of the presidency unit. Further, the overall efficiency score of each DMU using the centralized model is presented in the sixth column of Table 9.

In addition, the results of efficiency decomposition of each DMU using the non-cooperative leader-follower approach based on each sub-unit, and each level are shown in Table 10. For instance, the efficiency scores of levels 1, 2, and 3 for DMU 1 are 0.7154, 0.9152, and 0.9127 by using the non-cooperative leader-follower model. This indicates that the activities of these units for DMU 1 are not efficient. For another example, in comparing DMUs based on education vice-presidency unit, only DMU 19 has been efficient, therefore, other DMUs should take suitable actions to increase the efficiency of the education vice-presidency unit.

The ninth column of Table 10 displays the overall efficiency score of DMUs using the non-cooperative leader-follower approach. The overall efficiency score and rank of each DMU using all three models are reported in Table 11. Based on the results of Tables 9, 10, and 11,

the DMU 18 is in the first rank with efficiency scores 1, 0.8474, and 0.8435 using the proposed models. On the other hand, DMUs 3, 11, and 8 were the last in the ranking based on three models. We note that the black-box CCR model results in three efficient DMUs; however, none of the DMUs is efficient when we apply the centralized or non-cooperative leader-follower approaches. Moreover, every time a given DMU is inefficient according to the black-box model, it is also inefficient with regard to both centralized and leader-follower models. Moreover, Table 12 presents the descriptive statistics of the efficiency scores obtained by three models. From the analysis of the results of Table 12, it emerges that the average efficiency in the black-box model is higher than both centralized and leader-follower models. This proves that both centralized and leader-follower models are more strict and reliable than the black-box model. Also, Figure 6 displays that the efficiency scores of all DMUs in both centralized and leader-follower models are less than those obtained from the black-box model. In addition, the efficiency scores of DMUs in the centralized and leader-follower models are very close together.

Table 9. Efficiency scores based on black-box and centralized models

DMUs	Black-box	Centralized model (Level 1)	Centralized model (Level 2)	Centralized model (Level 3)	Centralized model (Overall)
1	0.679	0.7154	0.9412	0.913	0.6148
2	0.642	0.5556	0.8784	0.9032	0.4408
3	0.3868	0.4365	0.8436	1	0.3682
4	0.9048	0.6696	1	0.9495	0.6358
5	0.5357	0.4229	1	0.9114	0.3854
6	0.8	0.8455	0.7383	0.9273	0.5789
7	1	1	0.7834	0.8822	0.6911
8	0.3882	0.4423	0.8436	0.9572	0.3572
9	1	0.9512	0.8221	0.9587	0.7497
10	0.7677	0.6189	0.9358	0.9043	0.5237
11	0.4444	0.3884	0.9192	0.9127	0.3258
12	0.5802	0.5493	0.8044	1	0.4419
13	0.6327	0.56	0.9097	1	0.5094
14	0.7037	0.5769	0.9576	1	0.5524
15	0.9808	1	0.6885	1	0.6885
16	0.9704	0.8727	0.8039	0.9129	0.6405
17	0.7179	0.6838	0.8235	0.8812	0.4962
18	1	1	0.8954	0.9464	0.8474
19	0.6952	0.4955	1	0.9199	0.4558
20	0.7037	0.5878	0.8874	0.9478	0.4944
Average	0.7267	0.6686	0.8738	0.9414	0.5399

Table 10. Efficiency scores and efficiency decomposition based on non-cooperative leader-follower model

DMUs	leader-follower (Level 1)	Level 2-leader (Cultural and Health vice-presidency unit)	Level 2-follower (Education vice-presidency unit)	Level 3-leader (Cultural preceptor)	Level 3-follower (Health preceptor)	Level 2 (overall)	Level 3 (overall)	leader-follower (overall)
1	0.7154	0.7915	0.9152	0.913	0.8376	0.9152	0.9127	0.5976
2	0.5556	0.8612	0.8262	0.8322	0.8863	0.8611	0.8863	0.4240
3	0.4365	0.6219	0.8063	1	0.9533	0.8063	1	0.3519
4	0.6696	1	0.8352	0.9434	0.9486	1	0.9486	0.6352
5	0.4229	1	0.833	0.8976	0.9079	1	0.9079	0.3840
6	0.8455	0.5762	0.6552	0.9009	0.9206	0.6552	0.9206	0.5100
7	1	0.7115	0.6768	0.8822	0.7792	0.7115	0.8822	0.6277
8	0.4423	0.6765	0.7425	0.923	0.939	0.7425	0.939	0.3084
9	0.9512	0.8221	0.741	0.8583	0.9533	0.8221	0.9533	0.7455
10	0.6189	0.8851	0.907	0.9043	0.8918	0.907	0.904	0.5075
11	0.3884	0.9192	0.8223	0.8576	0.8973	0.9192	0.8973	0.3204
12	0.5493	0.787	0.7754	0.919	0.9649	0.7869	0.9647	0.4170
13	0.56	0.9097	0.8657	0.8784	0.9881	0.9097	0.9881	0.5034
14	0.5769	0.8547	0.8282	1	0.8688	0.8547	1	0.4931
15	1	0.6043	0.6187	0.9792	1	0.6187	1	0.6187
16	0.8727	0.7906	0.6946	0.9129	0.8809	0.7906	0.9129	0.6299
17	0.6838	0.8221	0.7474	0.8812	0.8631	0.8221	0.881	0.4953
18	1	0.8954	0.7559	0.8617	0.9429	0.8946	0.9429	0.8435
19	0.4955	0.9207	1	0.9199	0.8238	1	0.9199	0.4558
20	0.5878	0.8874	0.838	0.8557	0.9359	0.8873	0.9359	0.4881
Average	0.6686	0.8169	0.7942	0.9060	0.9092	0.8452	0.9349	0.5179

Table 11. Efficiency scores and rank of each institution based on three models

DMUs	Black-box model	rank	DMUs	Centralized model	rank	DMUs	Leader-follower model	rank
18	1	1	18	0.8474	1	18	0.8435	1
9	1	1	9	0.7497	2	9	0.7455	2
7	1	1	7	0.6911	3	4	0.6352	3
15	0.9808	2	15	0.6885	4	16	0.6299	4
16	0.9704	3	16	0.6405	5	7	0.6277	5
4	0.9048	4	4	0.6358	6	15	0.6187	6
6	0.8	5	1	0.6148	7	1	0.5976	7
10	0.7677	6	6	0.5789	8	6	0.5100	8
17	0.7179	7	14	0.5524	9	10	0.5075	9
14	0.7037	8	10	0.5237	10	13	0.5034	10
20	0.7037	8	13	0.5094	11	17	0.4953	11
19	0.6952	9	17	0.4962	12	14	0.4931	12
1	0.679	10	20	0.4944	13	20	0.4881	13
2	0.642	11	19	0.4558	14	19	0.4558	14
13	0.6327	12	12	0.4419	15	2	0.4240	15
12	0.5802	13	2	0.4408	16	12	0.4170	16
5	0.5357	14	5	0.3854	17	5	0.3840	17
11	0.4444	15	3	0.3682	18	3	0.3519	18
8	0.3882	16	8	0.3572	19	11	0.3204	19
3	0.3868	17	11	0.3258	20	8	0.3084	20

Table 12. Descriptive statistics of the efficiency scores

	Black-box model	Centralized model	Leader-follower model
Mean	0.7267	0.5399	0.5179
Std. dev.	0.1968	0.1368	0.1353
Maximum	1.0000	0.8474	0.8435
Minimum	0.3868	0.3258	0.3084

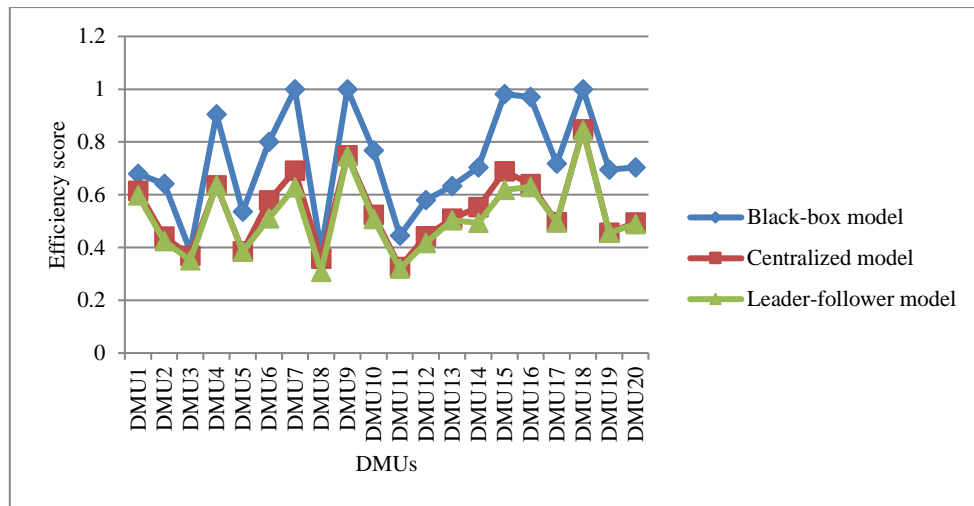


Figure 6. Comparison of efficiency scores: black-box, centralized and non-cooperative leader-follower models

5.3. Managerial insight

The black-box CCR, centralized, and non-cooperative leader-follower models were applied to measure the efficiency of 20 schools. The results show that although the black-box, centralized and non-cooperative leader-follower can be applied to measure the efficiencies of schools, the findings of the centralized and leader-follower models are more informative than those of the black-box model, as the formers can both provide the efficiency score of each sub-unit in institutions and distinguish the order of the DMUs based on the efficiency scores of the sub-units. The proposed method provides a means for managers to monitor and measure the efficiency of their institutions including whole system and its components. Using the results of centralized and non-cooperative leader-follower models, an institution is able to identify the functions requiring greater efforts in order to improve the overall efficiency of the institution. Moreover, the non-cooperative leader-follower model is able to measure the efficiency of each function, which provides valuable information for recognizing the areas of weakness that require more endeavors to be devoted to them. Based on the efficiency scores in Tables 9 and 10, the head of an institution is able to identify the areas in which they should use more efforts to improve their overall performance. For example, the low-efficiency score of the DMU 9 is due to the poor performance of the second level (education vice-presidency unit and cultural and health vice-presidency unit) and if this institution wants to improve its performance, then these two units must be strengthened. The results of Table 10 show that the mean efficiency score of level 1 i.e., the presidency unit for all DMUs is 0.6686 which has the lowest efficiency score in contrast to other divisions. This means that all institutions should pay more attention to the presidency unit to increase the overall efficiency score. Other results pertaining to the comparison of campuses based on their sub-units are available in Table 10.

6. Conclusion

The objective of efficiency measurement is to detect the weaknesses of processes so that managers can improve the performance of the systems. On the other hand, most organizations in the real world have a hierarchical structure, with units at different levels. Managers are always looking for suitable methods that can calculate the efficiencies of complex systems such as hierarchical structures. The conventional DEA models for measuring the efficiency of network systems, which ignore the internal structure of a DMU, may produce misleading

results. The previous researches on the field of measuring the efficiencies of network systems using game theory focused on two and three stages networks. However, in this study, the hierarchical structure was considered and the game theory approach was applied to measure the efficiency of this kind of network systems. This paper developed two suitable models in order to provide more meaningful evaluations for DMUs with the hierarchical structure. These two models were developed for measuring the efficiency and efficiency decomposition in the hierarchical structures based on the centralized and non-cooperative leader-follower game theory.

On the other hand, analyzing the efficiency of educational systems is one of the main focuses of the policy debate to promote national competitiveness and future economic growth. In particular, policymakers and researchers alike are concerned with developing guidelines for educational institutions to encourage improvements in system outcomes given their system factors. Educational institutions in Iran like many organizations have a hierarchical structure, which are similar to network structures and require suitable models for efficiency calculations. Therefore, the models, which proposed in this paper, have been employed for calculating the efficiency of the educational institutions in Iran.

The results obtained from applying the aforementioned models made it clear that although some of the institutions had high efficiency, none of them is efficient, suggesting that these institutions were not optimally utilizing their resources. Moreover, the findings of the case study show that the presidency unit has the lowest efficiency score compared to other divisions. Therefore, all schools should take appropriate plans to improve its performance.

Furthermore, the findings of this research can be helpful in budget allocation for the institutions so as to provide a larger budget for those which have succeeded in offering the best performance with a limited level of resources, and to offer improvement plans for those institutions which have been classified as inefficient. Further, making inefficient institutions efficient can save the input resources, enhance the output, and finally reduce the per capita expenditure of the institutions.

Finally, our models were presented for the hierarchical structures. It is desirable to improve these approaches to measure and decompose efficiencies for other complex network structures and other organizations in future researches. Besides, considering the proposed models in case of uncertainty and dynamic situation that deals with efficiency change over time can be potential issues for future studies. Moreover, the calculation of the Malmquist index for the hierarchical structure will be another challenge for future researches.

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