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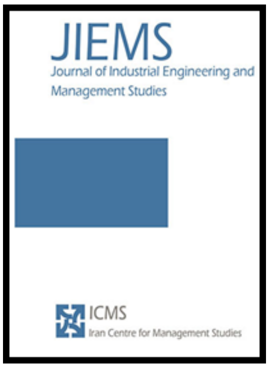

Hybrid genetic algorithm for the economic-statistical design of variable sample size and sampling interval x-bar control chart

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Abstract

The design of control chart has economic consequences that pure statistical viewpoint does not consider them. The economic-statistical design of control chart, attends not only statistical properties such as average time to signal (ATS) but also economic consequences like hourly expected total cost. The x-bar control chart dominates others if the quality is measured by continuous scale. This paper has considered the economic-statistical design of variable sample size and sampling interval (VSSI) x-bar control chart with multiple assignable causes. Using three sample sizes and three sampling intervals to construct the VSSI x-bar control chart and considering possible combination of design parameters as a decision-making unit, are part of novelty of this research. The problem is formulated as multiple objective decision making (MODM). Also, one capable hybrid meta-heuristic based on genetic algorithm is developed in this research and it was compared with some approaches extracted from the literature and it is found that it can be competitive based on economic and statistics factors.

Keywords: economic-statistical design; x-bar control chart; variable sample size; variable sampling interval; genetic algorithm.

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1. Introduction

X-bar chart is the most widely used control chart in variable quality characteristic condition and this chart is easy to understand and use. Duncan considered the economic design of the x-bar control chart for the first time in 1956. He determined the parameters of control chart n, h and k (sample size, sampling interval and coefficient of control limits respectively) by formal optimization methodology. Duncan (1956) maximized the expected net income per unit of time of the process by using this approach.

After Duncan, during the last decades the design of control charts with respect to economic criteria has been an interest field of study. Comparing to Shewhart, economic design of x-bar control charts, can help us to manage total cost of process. Pure economic design of control

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charts considers costs only and does not focus on statistical properties, such as type I or type II error and average time to signal (ATS), and thus it is entirely possible to produce designs that are optimal in an economic sense but which may have very poor statistical performance (Woodal, 1986).
This research considers special kind of economic-statistical design for a VSSI x-bar control chart with multiple assignable causes for a continuous-flow production process. The main questions are:

- How can we construct the VSSI chart to reach a good result?
- What are the main attributes in VSSI chart that can be considered as a decision-making criteria?
- How can we solve the real sized VSSI chart in short time and with good result?

In this paper we use three sample sizes and three sampling intervals to construct the VSSI x-bar control chart for the first time. Also, a possible combination of design parameters is considered as a decision-making unit which is identified by two attributes: hourly expected cost and average time to signal (ATS). Therefore, optimal design of control charts is formulated as multiple objective decision making (MODM). In this research for ATS's criterion, we set an upper limit and formulate it as a constraint of mathematical model. For the problems in the real-world scale, the core question is how to solve the problem in a short time period and with good result. In fact, in this research we deal with the problem by a new approach and also a new solve method. Meta-heuristics, mainly are the best tools in these situations especially in big size problems. Therefore, to solve the model and extract its parameters, we propose one capable hybrid meta-heuristic based on genetic algorithm and demonstrate that it is very competitive in comparison to other approaches.
The rest of paper is organized as follows. Section 2 is devoted to literature review. Section 3 introduces VSSI x-bar control cart. Formulations and loss function are given in section 4. Section 5 discusses about the proposed genetic algorithm applied to solve the problem. Section 6 is devoted to comparison study between proposed method and others in the literature. Finally, Section 7 concludes the paper.

2. Literature review

The first research on economic-statistical design is due to Saniga (1977) since he considers economic factors and statistical properties to develop control chart. McWilliams (McWilliams, 1994) provided a FORTRAN program and user can determine economic-statistical x-bar chart design by that.
Yu et al (2010) considered an economic statistical design of control charts for one assignable cause. However, in the real world, there are multiple assignable causes like machine problems, material deviation, human errors, etc. In order to have a real application, in this paper, we consider the economic-statistical model for the x-bar control chart with multiple assignable causes.
Main focus in economic design of control chart' research is on a fixed-sampling interval (FSI), but care must take into account variable-sampling-interval (VSI) control charts have more speed in detecting shifts in the process (Prasath, 2011).
Das and Jain (1997), Das et al. (1997), Bai and Lee (1998) considered variable sampling interval (VSI) control charts for the first time. Yu and Wu (2004) proposed an economic design of VSI moving average control charts. Yu and Chen (2005) considered an economic design for a VSI x-bar control chart for a continuous-flow process. Yu and Hou (2006) developed an economic design for a VSI x-bar control chart with multiple assignable causes. Christopher et al. (2010) proposed the economic design of x-bar control charts with continuously variable

sampling intervals to the field in that it allows the sampling interval to be determined by the extremity of the most recent sample. Fallahnezhad and Golbafial (2016) presented an economic model in the presence of inspection errors to design control chart to investigation of the effect of inspection errors on the formulation of chart. Oprine et al. (2018) suggested the possibility of re-sampling the process and a new chart design was proposed when the conditions "criticity depending on the deviation sign" and "control process cost" were considered. Salmasnia et al. (2019) determined the parameters of control chat in a way that the expected total cost is minimized. They suggested employing exponentially weighted moving average (*EWMA*) and range (*R*) control charts for profile monitoring, simultaneously. Finally, Niaki and Toosheghanian (2019) developed a methodology to economically design a variable sampling interval X-bar control chart that takes into consideration correlated non-normal sample data.
Prabhu considered adaptive x-bar control chart with variable sample sizes (VSS) for the first time in 1993 and also Costa focused on the same area in 1994. Zimmer et al. (1998) investigated VSS control chart with three different sample sizes. Daudin (1992) investigated double sampling x-bar control chart. Prabhu et al (2001) and Costa (1997) combined VSI and VSS features and proposed variable sample sizes and sampling intervals (VSSI) x-bar control charts. Zimmer et al (2000) and Mahadik et al (2009) constructed a three stage VSSI control charts. Costa & Rahim (2001) and Chen (2004, 2007), considered an economic design of VSSI x-bar chart with assuming non-normal data. Kosztyan and Katona (2018) proposed a risk-based concept for the design of an X-bar chart with variable sample size and sampling interval. Finally, Khoo et al. (2019) proposed an upper-sided improved variable sample size S-chart by improving the existing upper-sided variable sample size S-chart through the inclusion of an additional sampling interval. For more previous researches we refer readers to one literature review on adaptive control charts done by Tagaras (1998).
One of the interesting and capable algorithms to solve real-world size problems is genetic that almost hybridized with other approaches. For example, Hejazi and Roozkhosh (2019) is one of the recent researches that used Monte-Carlo optimization method and applied double sampling method for inspection. In their research, decision variables are the sample size per sampling time and the maximum number of defective items in the first and second samples in each stage. Genetic algorithm is a part of main approach in presented work. By a fast glance, it can be seen a huge attention to genetic algorithm during the last decades by researchers. This stimulated us to develop a new genetic algorithm to solve the economic-statistical design of VSSI x-bar chart.

3. VSSI x-bar control chart

Major assumptions for presented VSSI x-bar control chart are as follows:
1) The process is either in-control or out-of-control state only and is in-control state at the beginning.
2) The process is characterized by an in control state μ_0
3) k^{th} assignable cause of magnitude δ_k, which occurs at random, results in a shift in the mean from μ_0 to either $\mu_0 + \delta_k \sigma$ or $\mu_0 - \delta_k \sigma$, and is assumed to occur according to a Poisson process with an intensity of λ_k occurrences per hour (σ is the standard deviation of x-bar control chart).

Therefore it is assumed that the quality characteristic (QC) has normal distribution with mean μ_0 and a standard deviation σ. When the mean of the QC is at its target value, μ_0, the process is in control state, but when kth assignable cause is occurred, the mean changes from μ_0 to $\mu_1 = \mu_0 + \delta_k \sigma$ the process is out of control state (δ_k represents the ratio of the change with standard deviation units). In this case when the control chart produced an alarm, the process will be stopped and finding related assignable causes and removing them is started. z_i is the value of standardized $\bar{X}_i$ ($i = 1,2,...$) and defined as:

$$Z_i = \frac{\bar{X}_i - \mu}{\frac{\sigma}{\sqrt{n(i)}}}$$

$\bar{X}_i, i = 1,2,...$ is the mean of i^{th} subgroup calculated by $n(i)$ and $t(i)$ as sample size and sampling interval, respectively. It is clear that if $\mu = \mu_0$ then $Z_i \approx N(0,1)$ and otherwise $Z_i \approx N(n\delta_k, 1)$. For VSSI model, we utilize three different sample sizes ($n_1 < n_2 < n_3$) and three sampling intervals ($h_1 > h_2 > h_3$). Therefore there are four zones using three threshold limits ($k_1 < k_2 < k_3$) as follows:

$$I_1 = (-k_1, k_1)$$

$$I_2 = (-k_2, -k_1) \cup (k_1, k_2)$$

$$I_3 = (-k_3, -k_2) \cup (k_2, k_3)$$

$$I_4 = (-\infty, -k_3) \cup (k_3, \infty)$$

In VSS, based on the previous plotted point, sample size and sampling interval is chosen as follow:

$$(n(i), h(i)) = \begin{cases} (n_1, h_1) if Z_{i-1} \in I_1 \\ (n_2, h_2) if Z_{i-1} \in I_2 \\ (n_3, h_3) if Z_{i-1} \in I_3 \end{cases}$$

Figure 1 portraits presented VSSI.

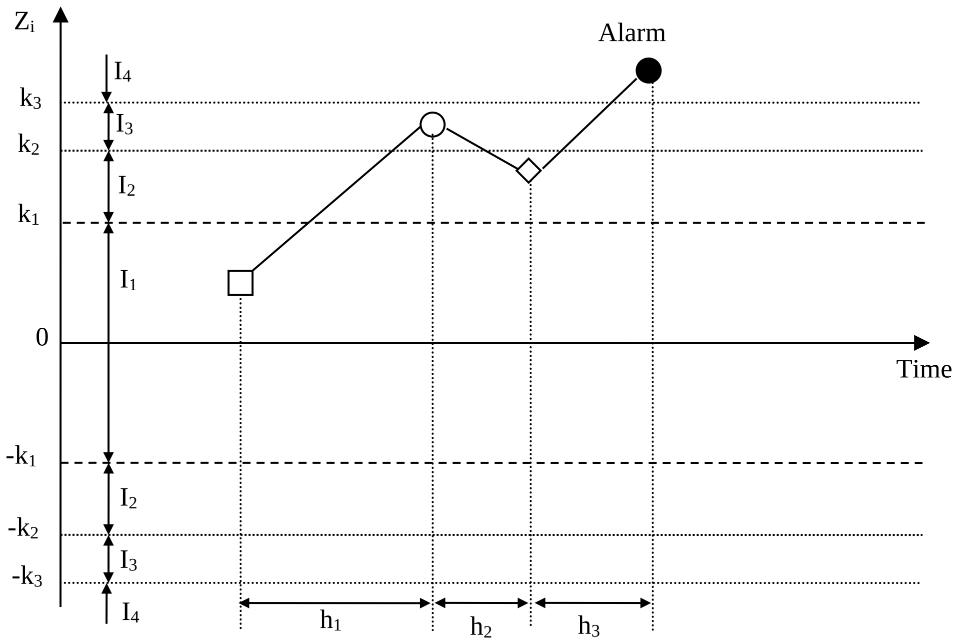

Figure 1. VSSI $\bar{x}$ chart

Fast detection of mean shifts is the major statistical property of a control chart. When sampling interval is constant, we use ARL for this property. However, in VSI it has to be measured by an average time to signal (ATS) that is defined as the average time needed to detection of shift in the process's parameters (Niaki et al., 2012).

The transition probability matrix of the Markov chain model is defined as:

$$P = \begin{bmatrix} p_{11}, p_{12}, p_{13}, p_{14} \\ p_{21}, p_{22}, p_{23}, p_{24} \\ p_{31}, p_{32}, p_{33}, p_{34} \\ p_{41}, p_{42}, p_{43}, p_{44} \end{bmatrix}$$

Where p_{ij} denotes the transitional probability that *i* is the prior state and *j* is the new state (Noorossana, et al., 2002). Therefore the average time to signal (ATS) as a performance measure can be calculated by can be computed by the Markov chain approach for computing average run length (ARL) only with minor modifications. For control charts with VSI, it is shown that:

$$ATS = b' \times (I - Q)^{-1} \times H'$$

Where $b' = (b_1, b_2, b_3)$ is the vector of initial or starting probabilities (calculate as follows), I is the identity matrix of order 3, Q is the transition matrix with eliminated elements corresponding to the absorbing state, and $H' = (h_1, h_2, h_3)$ is the vector of the sampling intervals corresponding to the transient states (Noorossana, et al., 2002).

$$b_1 = \frac{(2\varphi(k_1) - 1)}{(2\varphi(k_3) - 1)}$$

$$b_2 = \frac{(2\varphi(k_2) - 2\varphi(k_1))}{(2\varphi(k_3) - 1)}$$

$$b_3 = \frac{(2\varphi(k_3) - 2\varphi(k_2))}{(2\varphi(k_3) - 1)}$$

4. The loss function

The average time cycle (ATC) includes four time intervals as (1) the interval during which the process is in control; (2) the interval during which the process is out-of-control but still undetected; (3) the time required to sample, inspect, evaluate and plot a sample mean; (4) the time required to search and repair for the assignable cause. Based on Duncan’s (1971) multiple assignable cause's model, it is assumed that the process starts with mean μ_0 and standard deviation σ. The mean time for occurrence of k^{th} assignable cause is equal to $\frac{1}{\lambda_k}$ (k=1, 2,…,m).

Therefore ATC can be calculated as follow:

$$ATC = ATS + \frac{1}{\sum_{k=1}^{m} \lambda_k}$$

As a result, the expected length of a production cycle is:

$$E(T) = ATC + T_1 \times E(false) + \sum_{k=1}^{m} T_{2k} \times P_k$$

T_1 is the average time to finding assignable cause when the process is in control state and T_{2k} is the average time to find and repair the k^{th} assignable cause and correct the process when it is in out of control state and finally is the occurrence probability of k^{th} assignable cause in each time cycle. Finally $E(false)$ is the average number of false alarm in each production cycle and is calculated as follow (Niaki et al., 2012):

$$E(false) = b' \times (I - Q)^{-1} \times \alpha$$

Where $\alpha = (P[Z > k_3] + P[Z < -k_3], 0, 0)$ is the vector of false alarms probabilities in each transition state and Q is the transition matrix without elements associated with the absorbing state.
Let $E(BC)$ be the expected benefits minus costs incurred during a cycle. We have:

$$E(BC) = B_1 \times \left(\frac{1}{\sum_{k=1}^{m} \lambda_k} \right) + B_2 \times ATS - C_1 \times E(false) - \sum_{k=1}^{m} C_{2k} \times P_K - \sum_{i=1}^{3} \left(\frac{E(T) \times b_i}{h_i} \times n_i \times a_1 + a_2 \right)$$

Where
B_1 is the benefit per hour when the process is in control state
B_2 is the benefit per hour when the process is out of control state
C_1 is the cost of false alarm
C_{2k} is the cost of searching and removing k^{th} assignable cause
a_1 is the variable cost of sampling and testing
a_2 is the fixed cost of sampling and testing
Then the $E(BC)$ per unit time is:

$$E(A) = \frac{E(BC)}{E(T)}$$

Now we can earn loss function as follow:

$$E(L) = B_1 - E(A)$$

Therefore the loss function is a function of cost parameters (B_1, B_2, C_1, C_{2k}), the process parameters ($T_1, T_{2k}, \lambda_k, \delta$) and the design parameters ($n_1, n_2, n_3, h_1, h_2, h_3, k_1, k_2, k_3$).
To solve VSSI x-bar control chart statistically and economically we need to solve the following mathematical model (Model 1):

$$\begin{aligned} &Min \quad E(L) \\ &S.T. \quad 0 < n_1 < n_2 < n_3 \\ &\qquad h_1 > h_2 > h_3 > 0 \\ &\qquad 0 < k_1 < k_2 < k_3 \\ &\qquad n_1, n_2, n_3 \in Z^+ \\ &\qquad ATS \le ATS_0 \end{aligned}$$

5. Hybrid genetic algorithm

Genetic algorithm (GA) as a class of evolutionary algorithms (EA) proposed by John Holland in the 1960s for the first time. EA class of algorithm generate solutions by techniques inspired by natural evolution, such as mutation, selection, and crossover. The main concept of GA is to evolve a population of candidate solutions (called individuals, creatures, or phenotypes) to an optimization problem toward better solutions. Today, GA has wide and successful applications to solve NP-hard and completely NP-hard problems. The success is mainly due to its easy to understand and use and good flexibility. These reasons are enough to us to develop an approach based on GA to solve the presented problem.
At first so many individual solutions (called chromosomes) are (usually) randomly generated to generate an initial population. The population size depends on the problem, but generally contains several hundreds or thousands of possible solutions. The chromosomes evolve through successive iterations, called generations. During each generation, the chromosomes are evaluated through a fitness-based process where fitter solutions (as measured by a fitness function) are typically more likely to be selected. After that in the next step of GA, it generates a second generation population of solutions by using genetic operators: crossover and mutation. To generate of each new solution, a pair of "parent" solutions is selected for breeding from the

pool selected previously. By producing a "child" solution by operators like crossover and mutation, a new solution is created which typically shares many of the characteristics of its "parents". For each new child, new parents are selected and the process continues until a new population of solutions of appropriate size is generated.
After predetermined number of generations the algorithm converges to the best chromosome, which probably is the optimal solution or may be a near-optimal solution of the problem (William et al., 2008).
GA is a strong approach to solve the real word problems and we select it to solve the Model 1. To enrich the capability of the proposed GA in this paper it is hybridized with some other approaches.
Fig.2 shows the flowchart of HGA for the VSSI. HGA hybridizes with a heuristic called the swap procedure (SP). Also the author uses two genetic operators to make good new offspring.

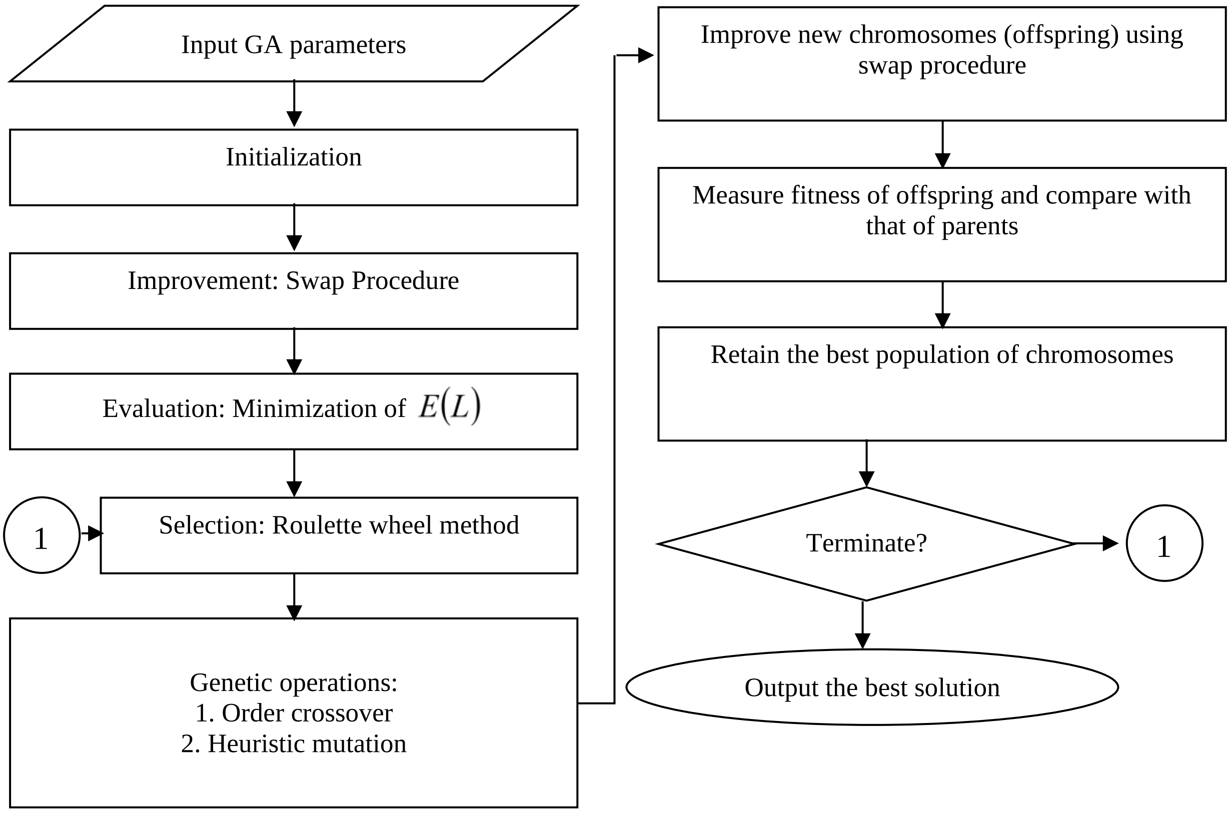

Figure 2. The flowchart of the HGA

For a fast glance it is clear after determination of the GA parameters (such as the population size, the iteration number, the crossover rate, and the mutation rate) and also generates the initial population the SP is applied to improve each chromosomes. Each chromosome is then evaluated by an evaluation function. To extract some solutions for the genetic operations (crossover and mutation) the roulette wheel selection operation is applied.
One child may become a part of population if has good quality based on the fitness function. After a fix number of repetitions the HGA will be stop.

In the proposed GA there are three chromosomes with 3 genes, each gene represents a decision variable of the model. Figure 3 shows a typical chromosome.

n_1	N_2	n_3

h_1	H_2	h_3

k_1	K_2	k_3

Figure 3. Typical chromosome of *n*, *h* and *k*

5.1. Initialization

The initial solution for HGA is ideally generated using Uniform pseudo random numbers is equal to population size. based on our best experience good result is gained when n_1, n_2 and n_3 are given from Uniform random U(1, 5), U(max(n_1,3), 15) and U(max(n_2,10), 30) discrete distributions, h_1, h_2 and h_3 are given from Uniform random U(1, 8), U(0.15, min(h_1,3)) and U(0, min(h_2,0.5)) continuous distributions k_1, k_2 and k_3 are given from Uniform random U(0, 1.8), U(max(k_1,1.5), 2.8) and U(max(k_2,2.5), 3.5) continuous distributions respectively.

5.2. Improvement

One effective method to improve the solutions of the hard optimization problems is the local search heuristic. If a new solution generated is better than the previous solution, or parent, based on evaluation function, it will replace and become the new parent. Swap procedure (SP) is used to improve the links of each initial solution. The procedure of the SP is as follows:
For each typical chromosome of *n*, *h* and *k*:
Step 1: Select one or two genes randomly from a link of a parent.
Step 2: Replace related gene(s) with the gene(s) from one other parent.
Step 3: Sort genes in ascending order for chromosome of *n* and *k* and in descending order for chromosome of *h*.
Step 4: Repeat step 1 to form four offspring for each typical chromosome.
Step 5: Evaluate all offspring and find the best one.
Step 5: If the best offspring is better than the parent, replace the parent with the best offspring and go back to Step 1; otherwise, stop.

5.3. Evaluation

As mentioned before, the fitness function is minimizing the objective function of Model 1 or $E(L)$.

5.4. Selection

The commonly-used genetic operator is the roulette wheel selection operation (Goldberg, 1989). It is the proportionate reproduction operator where a string is selected for the mating pool with a probability proportional to its fitness. Thus, the i^{th} string in the population is selected with a probability proportional. The fitter the chromosome, the higher the probability of being selected. Although one chromosome has the highest fitness, there is no guarantee it will be selected. Since the population size is usually kept fixed in a simple GA, the sum of the probability of each string being selected for the mating pools must be one. Suppose the population size is P_{size}, then the selection procedure is as follows:
Step 1: Calculate the total fitness of the population:

$$F = \sum_{h=1}^{Psize} E(L)_h$$

Step 2: Calculate the selection probability P_h for each chromosome X_h:

$$P_h = \frac{F - E(L)_h}{F * (P_{size} - 1)} \qquad h=1, 2, \ldots, P_{size}$$

Step 3: Calculate the cumulative probability Q_h for each chromosome X_h:

$$Q_h = \sum_{j=1}^{h} P_j \qquad h=1, 2, \ldots, P_{size}$$

Step 4: Generate a random number r in the range (0, 1].
Step 5: If Q_{h-1}<r≤Q_h, then chromosome X_h is selected.

5.5. Genetic operation

The genetic search progress is obtained by two essential genetic operations, including exploitation and exploration. Generally, the crossover operator exploits a better solution while the mutation operator explores a wider search space (Mirabi, 2012). The genetic operators used in the algorithms for the VSSI x-bar chart are one crossover and two mutations, which are called the heuristic mutation and the inversion mutation, respectively.

5.5.1. The order crossover

The procedure of the order crossover operation is (Mirabi, 2012):
Step 1: Select a substring from the first parent randomly.
Step 2: Produce a protochild by copying the substring into the corresponding positions in the protochild.
Step 3: Delete those genes in the substring from the second parent. The resulting genes form a sequence.
Step 4: Place the genes into the unfilled positions of the protochild from left to right according to the resulting sequence of genes in Step 3 to produce an offspring, shown in Fig. 4.
Step 5: Sort genes in ascending order for chromosome of *n* and *k* and in descending order for chromosome of *h*.
Step 6: Repeat Steps 1–4 to produce another offspring by exchanging the two parents.

5.5.2. The heuristic mutation

The procedure of the heuristic mutation operation, shown in Fig. 5, is taken as follows:
Step 1: Select one gene in a parent at random.
Step 2: Generate four other values for the related gene and with proper sorting generates four offspring
Step 3: Extract the best offspring.

	Selected substring		
Parent 1	n_1=3	n_2=5	n_3=18
Parent 2	n_1=2	n_2=8	n_3=17
Offspring 1	n_1=3	n_2=5	n_3=17
Sorting	n_1=3	n_2=5	n_3=17

Fig. 4. The order crossover operator

		Select one gene randomly	
Parent	n_1=2	n_2=5	n_3=11
Offspring 1	n_1=2	n_2=11	n_3=12
Offspring 2	n_1=2	n_2=8	n_3=11
Offspring 3	n_1=2	n_2=4	n_3=11
Offspring 4	n_1=2	n_2=11	n_3=14
Best Offspring			

Figure 5. The heuristic mutation operator

6. Comparison study

At first let B_1=150, B_2=10, C_1=100, $C_{2k} \in$U(50, 150), T_1=5, $T_{2k} \in$U(2, 8), $\lambda_k \in$U(0.01, 0.05) a_1=0.1 and a_2=1. In genetic algorithm the quality of the final solution is strongly depends on four parameters: the population size (P_{size}), the crossover rate (CR), the mutation rate (MR) and the number of generation (GN) (Niaki et al., 2012). In this research the following parameters are selected:

P_{size}=20, CR=0.5, MR=0.2 and GN=1000.

When the GA algorithm repeats GN times, the chromosome with the lowest fitness value is reported as the solution of the problem (Niaki et al., 2012).

Computational study is carried out to compare the presented VSSI (called VSSI33) with economic-statistical design (ESD) proposed by Fong et al. (2010). And also with modified variable sample size and sampling interval X (MVSSI) chart proposed by Shashibhushan et al. (2007). All methods are programmed in Matlab 7 and on a PentiumVI 2 MHz personal computer.

Tables 1 and 2 demonstrate based on ATS index both methods of VSSI33 and ESD work better than MVSSI. But in accordance to E(L), VSSI33 and MVSSI outperform EDS. In detail comparison it can be seen VSSI33 outperform both MVSSI and ESD based on the both criteria. Based on ATS index VSSI33 outperforms ESD and MVSSI in 70 and 100 percent of all cases respectively. About E(L) index (loss function) VSSI33 outperforms ESD and MVSSI in 86 and 60 percent of all cases respectively.

Table 1. Comparison study between VSSI33 and ESD

Problem No.	ESD					VSSI33										
	n	h	K	ATS	E(L)	n_1	n_2	n_3	h_1	h_2	h_3	k_1	k_2	k_3	ATS	E(L)
1	11	2.28	2.63	3.93	127	2	7	10	1.45	0.58	0.25	0.84	2.39	3.12	1.64	106
2	13	2.76	2.60	2.60	118	4	15	15	7.25	0.57	0.31	0.73	1.50	3.28	1.86	108
3	15	2.75	2.66	2.88	117	1	3	16	7.59	2.19	0.19	1.50	1.50	2.50	2.01	102
4	16	2.72	2.62	3.23	93	1	12	18	2.11	0.26	0.11	1.40	1.78	2.50	1.50	93
5	15	2.06	2.60	3.57	64	1	4	10	7.96	2.32	0.34	1.07	1.50	2.50	3.60	59
6	13	2.35	2.51	4.30	80	1	4	12	7.18	2.41	0.30	0.97	2.11	2.50	3.97	52
7	14	2.34	2.65	3.65	138	3	15	15	1.60	0.72	0.30	1.34	2.27	2.50	2.74	126
8	13	2.48	2.55	3.81	136	2	11	11	4.04	2.98	0.18	0.73	1.50	2.50	1.98	102
9	15	2.55	2.59	3.32	153	2	13	25	5.05	2.92	0.38	0.55	2.56	2.56	3.77	129
10	14	2.06	2.66	2.03	107	3	14	22	1.83	0.62	0.08	0.67	1.50	2.50	3.48	79
11	16	2.04	2.59	4.21	118	1	3	10	7.71	2.70	0.41	0.97	1.50	2.50	1.92	115
12	14	2.90	2.68	4.14	91	2	14	28	8.00	1.83	0.30	0.38	1.78	2.50	1.50	80
13	15	2.88	2.53	2.16	90	2	8	13	6.05	1.67	0.31	0.30	1.79	2.50	4.36	87
14	14	2.07	2.57	2.48	101	2	4	20	2.39	2.04	0.18	0.71	2.70	2.86	2.50	101
15	14	2.86	2.55	4.16	124	1	4	10	4.22	1.64	0.28	0.89	1.50	2.50	1.77	94
16	14	2.81	2.69	3.01	112	5	5	28	1.76	1.76	0.46	0.88	2.39	2.50	2.25	112
17	15	2.96	2.56	3.99	113	2	6	25	1.43	0.27	0.27	0.89	1.50	3.36	2.40	113
18	14	2.45	2.57	4.49	79	4	15	15	2.53	1.32	0.38	1.54	1.86	2.98	1.68	78
19	14	2.02	2.51	3.20	142	4	14	14	6.97	1.44	0.13	1.30	1.50	2.50	1.58	111
20	15	2.24	2.57	4.30	66	3	4	10	2.26	1.22	0.30	0.93	1.98	2.57	3.21	62
21	15	2.65	2.56	3.77	63	2	13	26	3.77	1.10	0.08	1.52	1.52	2.50	4.36	55
22	14	2.81	2.69	4.35	158	5	5	16	5.51	0.18	0.08	1.54	1.96	2.50	1.66	129
23	15	2.75	2.57	4.06	78	1	8	27	2.20	1.63	0.39	0.62	1.93	2.50	2.14	77
24	15	2.20	2.66	3.14	97	2	8	10	3.01	0.46	0.37	1.21	1.50	2.50	2.79	95
25	14	2.32	2.67	4.03	111	1	12	12	2.81	2.73	0.28	1.70	2.09	2.60	4.23	95
26	16	2.49	2.53	3.26	115	4	8	14	7.57	1.61	0.08	1.22	2.61	2.73	1.62	103
27	16	2.95	2.55	2.51	159	5	5	23	6.12	2.05	0.09	1.75	1.75	2.50	2.24	144
28	12	2.82	2.69	3.10	141	3	10	26	4.54	1.70	0.08	1.15	1.93	3.06	4.37	116
29	13	2.13	2.63	3.82	71	5	12	20	1.67	0.24	0.24	0.59	2.46	2.50	4.01	48
30	14	2.34	2.66	3.52	127	1	13	16	6.47	1.22	0.34	0.52	2.33	2.50	2.44	118

7. Conclusion

In this paper, we derive a statistical-economic objective function for a VSSI X-bar chart with multiple assignable causes called VSSI33. Using three sample sizes and three sampling intervals to construct the VSSI x-bar control chart have strong effect to increase the capability of the chart in economic and statistical viewpoints. Also a possible combination of design parameters is considered as a decision making unit which is identified by two attributes: hourly expected cost and average time to signal (ATS).

Besides modeling of the proposed situation, because of large size of real-world problems, one novel genetic algorithm (GA) is developed to determine the value of chart parameters ($n_1, n_2, n_3, h_1, h_2, h_3, k_1, k_2, k_3$). Using swap procedure order crossover and also heuristic mutation are some issues of developed GA. Two main criteria to challenge a new method in this area are loss function and average time to signal. Comparing presented GA based method (VSSI33) with economic-statistical design (ESD) and also modified variable sample size and sampling interval X (MVSSI) by using these criteria, demonstrates it is competitive and superior based on both economic (Loss function index) and statistics (average time to signal) factors.

There are other meta-heuristic approaches that can be attended to solve economic-statistical design of x-bar chart like ant colony, electromagnetism, simulated approaches, etc. also these algorithms can be used to extract parameters of other kinds of control charts like R, S, np etc. These areas can be so attractive and ca be seen as future researches.

Table 2. Comparison study between VSSI33 and MVSSI

Problem No.	MVSSI										VSSI33										
	n_1	n_2	n_3	t_1	t_2	w_1	w_2	L	ATS	E(L)	n_1	n_2	n_3	h_1	h_2	h_3	k_1	k_2	k_3	ATS	E(L)
1	4	13	18	9.97	0.35	0.19	1.03	2.41	2.94	117	2	9	26	6.15	2.28	0.14	0.27	1.67	2.50	2.78	117
2	6	10	15	9.95	0.46	0.11	0.62	2.58	3.88	148	5	7	10	7.19	2.92	0.13	1.61	1.61	2.50	1.81	146
3	7	10	15	9.96	0.42	0.26	1.07	2.23	4.34	117	2	15	16	5.58	2.37	0.29	0.47	1.73	2.50	3.65	117
4	5	13	15	9.95	0.39	0.18	1.06	2.21	3.41	116	1	11	26	2.80	0.99	0.47	0.64	1.50	2.50	2.50	115
5	8	8	17	9.98	0.27	0.08	0.68	2.29	2.26	110	3	10	26	2.56	1.30	0.10	0.19	1.50	2.95	1.70	108
6	6	13	16	9.99	0.29	0.11	1.02	2.55	3.41	82	5	9	10	7.73	1.72	0.13	0.10	1.50	2.50	2.95	80
7	6	8	15	9.96	0.23	0.24	0.82	2.21	4.50	128	4	8	22	3.69	1.78	0.38	1.57	1.57	3.23	2.63	127
8	8	9	14	9.97	0.16	0.17	0.69	2.44	4.78	107	3	12	19	4.30	0.89	0.08	0.86	1.50	2.50	3.59	106
9	9	9	16	9.99	0.37	0.25	0.85	2.24	4.98	74	4	12	12	5.45	2.44	0.31	1.69	1.77	2.71	3.15	73
10	7	13	17	10.00	0.48	0.15	1.13	2.41	4.04	76	5	15	15	5.41	1.73	0.19	1.24	1.52	2.50	3.98	75
11	7	9	16	9.98	0.27	0.22	1.01	2.10	4.37	70	2	9	10	3.18	1.24	0.44	1.26	1.50	2.50	3.53	69
12	9	11	17	9.99	0.48	0.13	0.98	2.13	3.72	66	3	4	10	7.11	0.65	0.44	0.74	1.50	2.50	2.08	64
13	9	9	15	9.97	0.33	0.22	1.17	2.38	4.86	82	5	6	10	7.25	2.05	0.36	1.10	1.96	2.50	4.27	82
14	7	8	16	9.97	0.45	0.21	0.77	2.55	4.27	90	2	11	11	3.80	2.92	0.28	0.95	1.76	2.50	2.83	88
15	10	12	16	9.97	0.45	0.14	0.77	2.58	4.77	79	3	12	28	3.81	1.67	0.41	0.63	1.64	2.50	2.00	79
16	6	10	16	9.98	0.39	0.13	0.76	2.57	2.50	91	2	5	16	6.17	2.61	0.50	0.73	2.58	2.58	1.73	89
17	6	11	13	9.99	0.34	0.25	0.84	2.18	4.00	66	4	4	28	6.90	0.84	0.15	0.38	1.50	2.71	2.81	66
18	5	12	17	9.96	0.17	0.21	1.14	2.47	6.40	148	3	14	14	3.17	2.55	0.33	0.35	1.50	2.50	3.14	146
19	4	8	13	9.97	0.37	0.26	1.14	2.13	6.03	78	2	3	10	3.00	2.36	0.09	1.50	1.62	2.50	3.91	77
20	5	10	16	9.98	0.43	0.21	0.99	2.31	2.37	108	2	13	16	2.16	1.32	0.39	1.32	1.50	2.50	2.49	108
21	4	12	18	9.96	0.27	0.20	0.78	2.43	3.54	118	4	10	23	4.00	2.29	0.43	1.73	1.81	2.51	1.88	118
22	6	10	15	9.97	0.17	0.14	0.72	2.33	4.69	120	1	14	14	7.79	2.67	0.15	0.61	1.89	2.64	2.70	120
23	5	11	18	9.97	0.17	0.23	0.94	2.23	4.94	76	5	9	10	2.16	0.41	0.20	0.53	2.04	2.50	3.63	75
24	6	12	16	9.95	0.23	0.18	0.82	2.24	6.44	132	4	5	10	3.98	2.19	0.48	1.13	1.50	2.50	1.80	132
25	7	10	17	9.96	0.19	0.16	1.08	2.39	4.94	65	1	13	27	2.46	0.74	0.22	1.57	1.57	2.50	4.50	64
26	9	12	18	9.96	0.45	0.14	1.00	2.35	5.53	138	5	5	10	3.17	1.67	0.39	0.55	2.51	2.51	2.05	138
27	10	11	18	9.96	0.23	0.10	0.67	2.34	2.05	109	5	5	29	5.55	1.44	0.16	0.86	1.74	2.50	4.00	109
28	9	10	14	9.96	0.20	0.28	0.82	2.15	2.04	70	2	6	16	7.81	1.26	0.23	1.21	1.64	2.65	3.62	68
29	9	9	15	10.00	0.27	0.17	1.15	2.47	2.34	139	3	15	18	5.98	0.27	0.27	0.20	1.50	2.51	3.07	137
30	7	8	16	10.00	0.16	0.28	1.21	2.30	4.98	154	4	7	10	4.28	1.33	0.08	1.22	1.50	2.50	3.33	152

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