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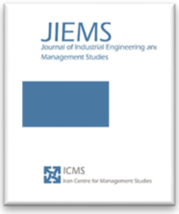

Location-routing in competition environment using game theory

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Abstract

Competitive advantage in features, number of branches, or location of any company enables it to provide better services to customers than competitors. In this article, the issue of location in a situation where competitors can decide based on competitor conditions to maximize their profits is examined. First, based on the conditions and characteristics of each competitor, including the number of branches and budget limit, the performance range of each competitor is determined as the radius of effect. Two mathematical formulas are presented for the player and using the concepts of game theory, each player's market share in the competitive environment is determined to earn maximum profit. To solve the problem, first, the initial answers were obtained through the ant colony algorithm, then these answers were entered as input to the Simulated Annealing algorithm, which has a high speed to obtain the answer. The models developed for the two supermarkets have been evaluated and the results have been approved by experts.

Keywords: competitive location-routing problem; competitive environment; simultaneous game; decision making.

Paper Type: Original Research

1. Introduction

Naturally, the market environment is affected by competition, and ignoring the conditions of competition in decision-making may lead to wrong and irreparable decisions. The decision- maker may be faced with a situation in which he or she is unable to respond appropriately. Therefore, to make decisions in a competitive environment; It is necessary to examine how competitors influence each other's results and achievements and make decisions based on a specific roadmap. In other words, in today's competitive marketplace, companies seek an efficient structure of methods to provide customers with the highest value and achieve competitive advantage. This requires a broader perspective than just the borders of an individual company (Mokhtari, 2015). Productivity is crucial to a competitive business. Logistics processes play a significant role among factors affecting the productivity of distribution systems since firms spend a large portion of budget savings on logistics costs (Prodhon and Prins, 2014). Decision support systems can play an effective role in improving the ability of decision-makers to evaluate and make decisions (Hadiguna, 2012). In the current competitive environment, the focus will not be solely on the cost factor, but studies show that achieving competitive logistics capabilities can lead to increased organizational performance. (Rajagopal et al., 2018).

Lack of attention to decision dynamics, and competition for location-routing decisions and market share, diverts the model from the real situation. One of the most important effects of competition between firms is the division of market share between them so, when firms are in competition but this competition is ignored, the classic location-routing decisions are based on inaccurate information and will not have the necessary credibility. By properly determining the market share of each firm, and determining the customers, each firm can allocate the resources at its disposal more appropriately and succeed in achieving the intended goals. One of the most commonly used issues in distribution systems is the location-routing problem (LRP). It consists of two main functions of logistics planning. In LRP, decisions on locating branches (including factories, branches, warehouses, hubs, cross-docks, etc. are combined with decisions related to vehicle routing (Drexl and Schneider, 2015). The issue of location-routing (LRP) is one of the most important issues for many production and service units that can play an important role in the costs and profitability of the firm. Adding competition to the LRP issue increases its importance, and in this case, competition requirements must also be considered.

In recent years, research in the LRP field has been increasingly developed and researchers have investigated various aspects of LRP. Some scholars have addressed its theoretical aspects and some others have tried to find ways to solve the problem.

In conclusion, it can be said that the issue of location-routing in competitive conditions has not received much attention. Although it is mentioned in some articles (Nagy and Salhi, 2007; Prodhon and Prins, 2014; Drexl and

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Schneider, 2015; Schneider and Drexl, 2017), it still needs further study. In other words, no focused study has been conducted on the subject of competitive LRP. In this paper, the LRP in a competitive condition is studied considering the dynamic decisions and behavior of competitors. In this regard, a new algorithm is developed to process the dynamic behavior of competitors and LRP in a competitive environment that includes two operational and strategic levels.

In section 2, previous researches in the field are categorized and discussed. Next, the mathematical model of the problem is developed and then the solution method is proposed in sections 3, and 4. In section 5, the problem is examined in a real-world case study and finally, the results are discussed in section 6.

2. Review of literature

For many years, location and routing decisions were made separately until advances in optimization techniques made it possible to integrate the two decisions under the LRP. The idea of combining depot location and vehicle routing dates back to about 50 years ago.

Fazayeli et al. (2018) have studied the location-routing problem on the multimodal transport network. In this study, four objectives have been sought simultaneously. They have applied linear integer programming to model the problem and a genetic algorithm with a different chromosomal structure to solve it. This chromosomal structure consists of two parts encompassing multimodal transport and LRP. To ensure the functionality of the proposed model, the results of these methods have been examined on samples of different dimensions. Martínez-Salazar et al. (2014) have developed a bi-objective Two-Echelon LRP containing limited-capacity branches with opening costs on both levels. Direct shipping at level one and routing decisions at level two with a homogeneous fleet have been examined. The first objective function in this problem is to minimize the total fixed costs of opening facilities and variable costs of fleet routing and the second objective is to balance the path length. Other researchers have examined location-orientation issues based on the type of data. In their research, data related to customer demand, costs, and other items are considered fuzzy, random, etc. for example Fazel Zarandi, Hemmati, and Davari (2011) have investigated an LRP with capacitated fleet and facilities within time window constraint. There are uncertain travel times for this problem presented with triangular fuzzy variables. The authors have presented a Simulated Annealing (SA) algorithm for solving the problem. In another study, (Fazel Zarandi et al., 2013) examined the problem with customers' fuzzy demand using the SA algorithm to solve the problem.

Zare Mehrjerdi and Nadizadeh (2013) have investigated an LRP with capacitated fleet and facilities, uncertain demand, and triangular fuzzy variables. In this study, a heuristic method for fuzzy data is proposed to solve the clustering problem, and random simulation is applied to determine the actual customer demand. The researchers have discussed the risks associated with fleet and route failure. Then, the results obtained from the proposed method have been compared with the solutions from a commercial solver. Following this study, Nadizadeh and Hosseini Nasab (2014) have developed the same problem for multi-period vehicle routing.

Hamidi et al. (2012a; 2012b; 2014) in their three articles have studied a Three- Echelon LRP in which diverse goods can be transmitted at all levels. There are branches in level zero, and there are fixed opening costs for branches in levels 1&2. In these articles, customers can receive service at any level. The routes are allowed only for the delivery of goods to customers and the transport between branches must be direct.

Ahmadi Javid and Seddighi (2013) have devoted the LRP in a stochastic environment. They have assumed that during the planning phase, customers should be met several times, and each visit should be done with the same vehicle through a similar route. The problem is modeled by discrete random variables to minimize the total fixed opening costs of facilities, variable routing costs, and penalties. The authors have developed a MIP model and used a metaheuristic method to solve it.

Several researchers have examined different solution methods, such as; Ambrosino, Sciomachen, and Scutellà (2009) have examined a two-phase hybrid algorithm for a Two- Echelon LRP. In this case, customers are divided into predetermined clusters. There is a central facility and there is a need for a local facility in each area to open. Customer demand is divided into two parts, one part is responded by the central facility, and the other part is supplied by the local facilities. They used integer programming to generate an initial solution and use the branch and bound method to determine the sequence of visits in each cluster.

Perboli, Tadei, and Vigo (2011) have presented a formulation for two-level LRP that contains a zero-level facility and level one facilities with capacity constraints. In this paper, two heuristics are presented. Whenever level one customers are assigned, the problem is divided into a zero-level VRP and a VRP for each level one facility.

Schwengerer et al. (2012) presented a Variable Neighborhood Search (VNS) algorithm for 2- Echelon LRP. First, in this method, an initial solution is determined, and each customer is assigned to the closest open facility. Then, the routes are calculated for each facility of the first level, that if the number of routes exceeds the fleet, customers should be reduced through reassignment.

Crainic et al. (2008) have studied a heuristic way to solve the two-level VRP problem, which has one facility on the zero levels. In their model, the number of the fleet is known but, for the potential facility, fixed opening costs and capacity constraints are not taken into account.

Leksakul et al. (2017) have compared different methods for solving LRP. They have studied the actual data related to the transfer of employees of an industrial plant in Thailand, employing four techniques of Maxmin, K-means, Fuzzy C-means, and Competitive Learning. The researchers have mentioned that the combination of K-means with Maxmin will be more appropriate from the qualitative aspect. Then ant colony optimization (ACO) has been used to route between 300 up to 700 assigned stations in the previous phases. Since determining the proposed number of buses (optimal answer in this phase) requires chief investment, the constraint of bus route numbers has been placed in this problem.

Ferreira and Queiroz (2018) have provided two new methods based on Simulated Annealing to solve the capacitated LRP. Their proposed model consists of four components: the initial solution generation, the use of four neighboring operators according to displacement and replacement, finding the best answer by solving the knapsack problem and improving the achieved solutions. Computational experimental studies on three samples have shown that their method is more effective than other methods.

The growing trend of LRP studies has continued in recent years and many of these studies have focused on solutions and metaheuristics. Such as Hadiana et al. (2019) and Yu et al. (2019) Hadiana et al. (2019) present a multi-objective location-routing problem with capacitated vehicles that minimizes the total system cost in the first objective and the difference of distances traveled by different vehicles in the second objective. To solve the problem, it uses a Multi-Objective Imperialist Competitive Algorithm. Finally, it has compared the performance of the proposed method with NSGA-II.

And Yu et al. (2019) focus on the capacitated LRP with the capacity constraint on both branches and vehicles. And to process these constraints, it uses a hybrid genetic algorithm that, in addition to searching on the feasible solutions space, also searches on the infeasible solution space. This proposed solution uses the capabilities of GA exploration and exploitation to exit local optimum with innovations in genetic algorithm components.

In another category, case study-based articles are categorized as: Rabbani et al. (2020) studied the management of wastes produced in the automotive industry. This paper covered an industrial waste transportation system in the automotive industry by presenting a capacitated location routing model with a heterogeneous vehicle fleet. In this study, the waste collection network was considered based on a mixed-integer bi-objective optimization model. Decisions on the transportation route of various collection vehicles, the location of collection centers, the level of inventory in these centers, and the reduction of costs have been made.

Fakhrzad and Ali Doosti have dealt with the inventory model of perishable goods and have added the aspects of product corruption and variable prices to the LRP model. Then, considering the NP-hard nature of the problem, they present a genetic algorithm to achieve a quasi-optimal solution in a reasonable time.

Rath and Gutjahr (2014) have investigated the problem of locating branches to distribute essential goods in disaster. The problem is similar to the LRP for the warehouse, where goods should be shipped from supply locations (such as an airport or port) to the warehouse (mid branches). Minimizing mid-term costs, minimizing short-term costs, and maximizing covered demand have been discussed.

Sivakumar et al. (2008) addressed the issue of routing-allocation with a case study on the blood chain in the Indian private and public health sector. In this research, a multi-device problem with multiple warehouses and multiple criteria has been studied (multiple-vehicle, multi-depot, multi-criteria allocation-routing problem). To solve the models, the hierarchical process and the integer linear programming model are combined. And on random datasets, the quality of solutions has been examined.

Reddy et al. (2010) focus on the issue of multi-vendors using the traveling salesman problem. In their problem, instead of solving the problem of a traveling salesman, the number of cities is distributed among the number of tours, and the authors have used a clustering algorithm (k- means) for this problem.

Considering simultaneous objectives is a common way to reach the goals in supply chain network design. For example, Khorshidvand et al. (2021a) present a hybrid approach for economic and environmental objectives improving through optimal coordinating decisions.

Also, Khorshidvand et al. (2021b) offer a two-stage approach to model and solve a sustainable Closed-Loop Supply Chain, taking into account pricing, green quality, and advertising.

Babaee Tirkolaee, Hadian, and Golpira (2019) develops a multi-objective mixed-integer linear programming model for a two-echelon green capacitated vehicle routing problem in which environmental issues and time windows constraints are considered for the perishable products delivery phase.

A review of LRP studies shows that much of the conducted research covers the standard LRP problem. Although studies have examined the complexity and integrity of the various domains, including multi-level LRPs, multi-objective LRPs, and LRP combination with inventory decisions and integration with other domains, the competitive LRP problem has not been investigated seriously. Among the research done by Bozkaya et al. (2010) and Burkat et al. (2017) is close to the competitive LRP field but the structure of the proposed LRP in these two articles differs from standard LRP. Furthermore, the reaction and dynamics of competitors' behavior have not been investigated in these two reteaches. In this paper, in a competitive environment, the customer demand is divided based on customer perception of facilities among active firms, and also the allocated demand to each market firm is estimated. It will be described in more detail in the following. The novelty of the paper among related researches is shown in Table 1.

Table 1. The novelty of this research

Decision		Dynamic behavior of competitors	Competition	Routing	Location	Researchers	row
Operational	Strategic						
	*		*		*	Burkart et al, 2017	1
*		*	*			Drezner et al,2015	2
	*		*			Drezner et al,2012	3
	*		*		*	Bozkaya, et al, 2010	4
*	*	*	*	*	*	This Research	5

3. Problem definition and model development

Location of facilities and routing of customer service are important factors in the competitive environment. In many firms such as supermarkets and other stores with multiple branches (i.e. Whole Food with 279 stores and Trader Joe's with 300 stores in the United States.) the issue of competition seriously affects the profitability of each of them. In such a competitive environment, every firm will try to attract maximum customers. In some cases, firms are forced to open new branches or expand existing ones to maximize market share. On the other hand, constraints such as firms' budgets may limit their ability to expand or create new branches. Therefore, each firm can operate within a certain range and maximize its profits in that range. When a firm starts operating in a competitive environment, other competing firms also undertake other activities in order not to lose their profits. The activity of each firm can affect the activity of another firm and make them react until an equilibrant happened. This type of problem can be considered as a simultaneous game between different players. In this paper, a new method is developed for investigating the location-routing problem in a competitive environment. The considered model consists of two sections: estimating the share of each player through the competition modeling and formulating an LRP for minimizing the total cost. In the proposed competitive LRP, there are two main players in the market, each player may have several branches to perform activities. The problem approach includes 3 phases. In the first phase, the required information is entered. This information includes the geographic location, the capacity of the branches, the characteristics of the fleet (such as the number of vehicles for each branch and the capacity of each vehicle), cost information (including fixed and variable costs of branch opening, fixed cost of vehicles and the unit cost of transport, and budget constraints) as well as the primary radius of influence and the allowed improving radius of the facility.

In the second phase, the highest market share in response to the activities of other competitors is calculated. Accordingly, all possible cases for the opening and the improvement of the radius of influence for all potential branches per each competitor (player) are taken into consideration. These developments are presumed as potential game scenarios. The value of allocated demand for each firm is determined depending on the selected scenario by each player. These values are assumed as Pay-Offs for each player. Possible game scenarios and Pay-Offs associated with interactive scenarios of the players are the elements needed to run a simultaneous game. To find a solution, determining the Nash equilibrium is necessary. The Pay-Off related to Nash equilibrium can be regarded as an estimate of the market share of each player. In this phase, game theory is used to estimate the contribution of each player. To model a game, the following must be identified:

1- Players and their goals 2- Possible scenarios of each player 3- Pay-off of each player.

In a competitive environment, one player is defined as the main player and the other as the rival player. Each player will have branches that can be closed, increasing the radius of influence of opened branches or be opened with an increase in the radius of influence. So, each player will have 3 possible scenarios for each branch, which combines different modes to determine the total possible scenarios of each player. Depending on the choice of the main player scenario, the status of the rival player will change. Accordingly, the amount of demand allocated to each player is determined and considered as the Pay-Off of that player. These pay- offs will change based on the choice of different scenarios so, it is necessary to prepare a table in this field and determine the amounts of Pay-Offs for each selected scenario. Given that each player's goal is to maximize Pay-Off, Nash equilibriums must be specified for them. Each Nash equilibrium represents a situation in which both players are satisfied and are reluctant to change their decisions. Therefore, Pay-Off related to this balance can be considered as an estimate of the market share of each player.

In the third phase, each Nash equilibrium is considered as a scenario and an LRP is solved for each player in each equilibrium point to minimize the total service cost of each player. To solve LRP, a mathematical model is needed. As the nature of an LRP problem is NP-hard, with the increase in the number of branches and customers, it is impossible to solve it with exact methods in a reasonable time. We proposed a new metaheuristic algorithm based on the combination of SA and ant colony. In this algorithm, the first many repetitions are done with ACO to achieve the initial answer and then, this initial answer is placed in the simulated annealing algorithm and the search for optimality is followed. According to the above algorithm, the optimal or near-optimal location of branches, as well as the appropriate routes and travel sequences of each player, are determined. Ultimately, the decision variables related to the branch opening, the assigned customers to each branch, and the customer coverage paths are determined with the lowest cost. The competition phase and LRP phase are described in more detail in the following subsections.

3.1. Developing a model for the competitive phase

Different methods can be used to analyze competition problems based on the different conditions that occur in a competitive environment. If the competition is constant, it can be formulated by mathematical programming. If the competition is accompanied by predictions, it can be modeled using the Stackelberg game. If competition is in a dynamic environment, it can be examined with Nash equilibrium. Dynamic competition is considered in this paper in which the ability to change competitive features in response to market changes is allowed. Dynamic competition can be modeled by the Nash equilibrium since both players can modify their competitive features at the same time.
In uncertain situations, demands divide between players in terms of competitive features. In this study, dynamic competition and uncertain customer behavior are investigated. The proposed structure to formulate competition and estimate the demand share is shown in Figure 1.

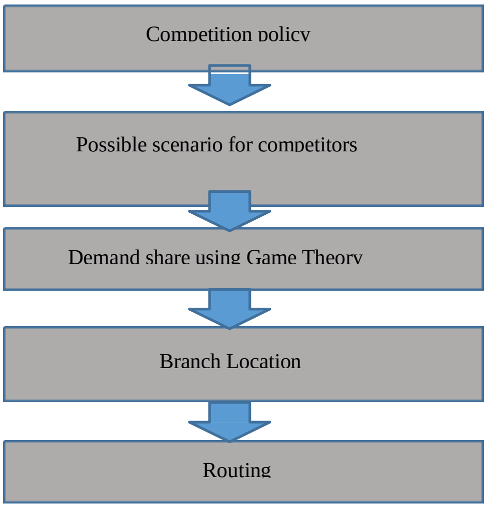

Figure 1: The proposed competitive LRP model

In this figure, the most important inputs and outputs of the two main parts of the model are identified. In the first part, estimating the market share and identifying the amount of demand allocated to each player is determined. In this section, the scenarios of each player, including the location of customers and the amount of demand of each customer, budget constraints are considered as input. The output of this part will be the market share of each player, in other words, the amount of cumulative demand allocated to each player is determined in this stage. Also, according to the model intended for estimating market share, the best scenario of each player and the pay-off related to that scenario will be determined.
In the second part, location-routing decisions are made. Inputs to this section include the position of customers assigned to each firm, and the amount of demand allocated, the position and capacity of potential branches of each firm, and reopening costs and vehicle capacity, and other costs associated with LRP. Also, the information of the initial radius of each branch and the amount of possible increase in the radius and related costs are other inputs. Decisions to the reopening of each branch of the firm, the radius intended for each branch, and the determination of vehicle routes are the outputs of this section. This subject is described in more detail in subsection 3.4.
In the competition phase, the decisions related to the opening or improving the radius of influence of the branches determine the game scenarios and determine the allocated cumulative demand to each player. Cumulative demand values are considered as the Pay-Off for the players. These elements are preliminaries for the game that estimate the demand or market share.
In the considered model, there are a certain number of customers with given demand and location.
For each branch of a firm, based on the facilities and equipment it has, a radius of influence can be considered. If the customer distance to the branch is less than the radius of influence of that branch, this customer can be assigned to it and its demand will be considered as the market share of that branch. If the customer is within the radius of the influence of several branches, its demand will be divided equally between those branches.

3.2. Game theory

According to the definitions, game theory can be used wherever limited resources and different decision options lead to different results due to different choices. To develop the proposed model, the simplest case for a game is described here. The simplest case happens when there are only two active players; each player has only one

branch and the competition is for getting a larger share of customer's demands. The following points need to be noticed:
rf: radius of influence of main player F.
rc: radius of influence of rival player C.
lfj: distance between customer/demand point j and main player F
lcj: distance between customer/demand point j and rival player C.

Table 2 shows the ratio of the market share of each customer's belonging to the main player or the rival player. It should be noted that the scenarios of each player are determined concerning the overall game environment and the results of the LRP model. When there are n players, an n-dimensional matrix is needed. Moreover, if a demand point falls in the coverage range of more than one branch, the Pay-Off of each player will be proportional to the number of branches covered by each player and its rivals.

Table 2. A simple explanation for the distribution of customer's demand

Players		Rival player (C)	
		lcj <= rc	lcj > rc
Main player (F)	lfj<= rf	0.5, 0.5	1, 0
	lfj> rf	0, 1	0, 0

As shown in Table 2, when the customer is within the radius of influence of the main and the rival players, the demand will be 50% for each of them. If the distance of the customer from the main player is more than the radius of influence and it is within the radius of the rival player, all the demand will be allocated to the rival. If the customer is not in the radius of influence of any of them, no demand will be assigned to any of them.
Generally, to calculate the market share of the main player, for the number of branches of the main player "F", the number of branches for the rival "C", the customer demand "d", and this demand is within the range of the radius of influence of F and C, then the main player share is equal to (d F/(F+C)).
Also, if the demand is within the radius of influence q (additional branch of the main player), this player's share is determined by equation (1).

$$d\frac{F+q}{F+C+q} \tag{1}$$

For estimating the allocation of demands to each player, a new approach is presented in this paper. In the competition section, we improve the competition model of Drezner, (Drezner and Kalczynski 2012). The developed equations are as follows:

$$D_{Fj}^{o} = d_j \frac{F_j}{(F_j + C_j)} \tag{2}$$

Where D_{Fj}^{o} refers the assigned demand to the main player at demand point j. d_j is the amount of demand at demand point j, F_j & C_j are considered as number of existing branches covering demand point j for main player and rival player.

$$D_{Fj}^{f} = d_j \frac{F_j + q_{Fj}}{F_j + [C_j + q_{Cj}] + q_{Fj}} \tag{3}$$

In equation 3, D_{Fj}^{f} is the Final share of assigned demand to the main player at demand point j. And q_{Fj} & q_{Cj} defines the number of new branches for main player and rival player for demand point j. When the number of branches covered by each player is known, Equation 3 shows the main player's demand share at demand point j.
The share of the main player in demand of customer j is determined by equations (2) and (3), and the number of covered branches is calculated based on the radius of influence associated with each branch. Each player tends to maximize his share of the entire market and decides to pay attention to which customers and pass which ones concerning its constraints. Cumulative demand (obtained from all customers) is considered as the Pay-Off value related to the decision made by each player. In this way, the objective function of the competition model is created for the main player. Taking into account all the demand points, the objective function of the competition model is equal to Equation (4). where DF is the total demand assigned to the main player.

$$\max D^{F} = \sum_{j\in J} d_j \left[\frac{F_j + q_{Fj}}{F_j + [C_j + q_{Cj}] + q_{Fj}}\right] \tag{4}$$

In a monopolistic market, facing mathematical optimization, the decision-makers only consider their constraints to reach objectives. In this situation, the players will have the two given formulas and the competitor variables are assumed to be constant (for example, in the objective function of the main player (C_j+q_Cj) is assumed constant). In the general case of competition, there is more than one decision-maker, so solving models is not possible with the usual methods of mathematical programming. In such cases, game theory can be used. For this purpose, mathematical models have been developed in the next section.

3.3 Developing mathematical model for the main player

For determining the demand share of the main player necessary decision variables and notations are:

D^T : Total demand in the market

Oi : Fixed cost for establishing a branch in i.

$\bar{c}_{ri}$: Cost of improving the radius of influence.

r_i^f : Assumed radius of influence for a branch located in site i.

r_i^0 : Starting radius of influence for a branch located in site i.

BF : Main player's budget for location and routing development

N : the number of potential and existing points for branches.

M : a big number.

D_{Fj}^f : final share of assigned demand to branch i at demand point j.

y_{ij} : demand point j is covered by potential location i or not

ZFi : main player's new branch is located at candidate point i or not.

IMFi : the radius of influence of the main player's branch is improved at site i or not.

So, the mathematical model for the Main Player' demand share can be written as:

$$\text{Maximize } D^F = \sum_{j\in J} D_{Fj}^f = \sum_{j\in J} d_j \frac{F_j + q_{Fj}}{F_j + [C_j + q_{Cj}] + q_{Fj}} \qquad \text{F-1}$$

subject to:

$$q_{Fj} = \sum_{i=1}^{N} y_{ij} Z_{Fi} + \sum_{i=1}^{N} y_{ij} IM_{Fi} \qquad ; \ \forall j \in J \qquad \text{F-2}$$

$$(r_i^F - r_i^0) \le IM_{Fi} M \qquad ; \ \forall i \in I \qquad \text{F-3}$$

$$\sum_{i=1}^{N} O_i Z_{Fi} + \bar{c}_{ri} IM_{Fi} \le B_F \qquad \text{F-4}$$

$$l_{ij} \le r_i^F + M(1 - y_{ij}) \qquad ; \forall i \in I \text{ و } j \in J \qquad \text{F-5}$$

$$y_{ij} \text{ و } Z_{Fi} \text{ و } IM_{Fi} \in [0 \text{ و } 1] \qquad ; \forall i \in I \text{ و } j \in J \qquad \text{F-6}$$

The maximization of cumulative demand (as the objective function) is defined for the main player in (F-1). This equation shows the total demand of customers assigned to the main player. This value is proportional to the branches that cover customer j. Equation (F-2) shows the total branches of the main player (which cover the demand point j). It involves building new branches or improving the radius of influence of the existing ones. In F-3 when the radius of influence is greater than the initial radius for each branch, the radius improvement variable takes the value of 1, otherwise, it takes 0. F-4 shows the main player budget constraint. F-5 assures that any demand point is covered by a potential site, only when its distance with the corresponding branch is less than the radius of influence of that branch. Finally, F-6 illustrates binary variables.

3.4 Developing mathematical model for rival player

The mathematical model for Rival Player' demand share can be defined as:

$$\text{Maximize } D^C = \sum_{j\in J} D_{Cj}^f = \sum_{j\in J} d_j \frac{C_j + q_{Cj}}{F_j + [C_j + q_{Cj}] + q_{Fj}} \div \qquad \text{C-1}$$

subject to:

$$q_{Cj} = \sum_{i=1}^{N} y_{ij} Z_{Ci} + \sum_{i=1}^{N} y_{ij} IM_{Ci} \qquad ; \ \forall j \in J \qquad \text{C-2}$$

$$(r_i^C - r_i^0) \le IM_{Ci} M \qquad ; \ \forall i \in I \qquad \text{C-3}$$

$$\sum_{i=1}^{N} O_i Z_{Ci} + \bar{c}_{ri} IM_{Ci} \le B_C \qquad \text{C-4}$$

$$l_{ij} \le r_i^F + M(1 - y_{ij}) \qquad ; \forall i \in I \text{ و } j \in J \qquad \text{C-5}$$

$$y_{ij} \text{ و } Z_{Ci} \text{ و } IM_{Ci} \in [0 \text{ و } 1] \qquad ; \forall i \in I \text{ و } j \in J \qquad \text{C-6}$$

Where

BC : Rival player's budget for location and routing development

D_{Cj}^{f} : Final share of assigned demand to branch i at demand point j.

ZCi : rival's new branch is located at candidate point i or not.

IMCi : the radius of influence of rival's branch is improved at site i or not.

For Rival's model, the description of each equation is similar to the corresponding equation in the proposed model for the Main Player.

3.5 Developing mathematical model for location routing problem

Given that the LRP model is comprised of both strategic (location) and operational (routing) aspects of decisions. It is necessary to formulate a suitable mathematical model for the LRP phase. It is assumed that there is a set of locations for potential branches. Cost factors such as costs associated with the opening of branches, costs for improving the radius of influence, and costs related to vehicle routing are known. Delivery costs depend on the distance traveled by the vehicle. The LRP phase makes decisions involving customers that should be assigned to each branch that needs to be either opened or changed, branches whose radius must be improved, and the distribution routs to customers. The LRP model and the model of competition are highly interdependent. The main competition between firms occurs for the customer i.e. determining which firm the customer will be assigned to. For developing a mathematical model, several assumptions are considered, namely:

- The total customer demand is fixed.
- Each customer is served by one vehicle of each player.
- The path to the vehicle ends on the same branch as the start of its journey, and at a distance from the start to the end of the route, a set of customers will meet this path.

In the proposed model, all vehicles are assumed homogeneous with capacity constraints. Each path is exactly connected to a branch and, each player may have several branches for customer service.

Notation:

n : Number of demand points.

N : Number of the considered site for the branch.

S : Set of a potential site for the branch.

I : {i | i = 1, . . ., N} set of potential branch locations,

J : {j | j = 1, . . ., N + M} set of customers(N+M=n)

V : I ∪ J

K : {k | k = 1, . . ., P} set of vehicle routes,

cijk : the unit cost of shipment from point i to j by vehicle k, (i, j ∈ I ∪ J, k ∈ K),

$\bar{c}_{ri}$: the unit cost of increasing the radius of influence for main player branches at i∈{1,…,N}

Oi : fixed cost of establishing a branch at site i,

Pi : the capacity of the branch at i,

Qk : the capacity of vehicle k,

Vk : fixed cost of using vehicle k,

Lij : length of the distance between point i and point j.

dj : the amount of demand in demand point j

D^F : total assigned demand to the main player.

d_{ij}^{F} : assigned demand to main player's branch i from demand point j.

d_{Fj}^{o} : assigned demand to the main player from demand point j.

r_i^0 : primitive radius of influence for the main player at location i

r_i^f : final radius of influence for the branch located in site i.

M : a big number.

U_{jk} : auxiliary variables used in sub-tour elimination constraints.

Let yij for j=1, ..., n and i ∈ S be a binary variable such that;

$$\begin{cases} y_{ij} = 1 \text{ if demand point j is covered by potential location i. and} \\ \qquad y_{ij} = 0 \text{ otherwise} \end{cases}$$

The value of Zi (i is part of the branch set) is a binary variable in such a way that Zi = 1 in the case that the new branch is built on the point i, otherwise Zi = 0.

IM_{Fi} is a binary variable in such a way that;

IM_{Fi} =1 if the radius of influence of the branch is improved at point i, otherwise it equals 0.

x_{ijk} for i and j, members of the set of customers and the set of branches, because the tour points are selected from the collection of customers and branches

x_{ijk}=1 if edge (i,j) is traversed in a tour with the vehicle k.

The developed mathematical model for the location routing problem can be defined as:

$$\text{Minimize Cost} = \sum_{i\in I} O_i Z_i + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} Fx_{ijk} + \sum_{i\in \mathrm{I}\cup \mathrm{J}}\sum_{j\in \mathrm{I}\cup \mathrm{J}}\sum_{k\in K} c_{ijk}x_{ijk} + \sum_{i\in I}\bar{C}_{ri} IM_{Fi} \qquad \text{b-1}$$

Subject to

$$\textstyle\sum_{i\in \mathrm{I}\cup \mathrm{J}}\sum_{j\in \mathrm{I}\cup \mathrm{J}} d^{o}_{Fj}x_{ijk} \le Q_k \qquad \forall\, k\in K \qquad \text{b-2}$$

$$\sum_{j\in J} d^{o}_{Fj}y_{ij} \le P_i Z_i \qquad \forall\, i\in I \qquad \text{b-3}$$

$$\textstyle\sum_{i\in \mathrm{I}\cup \mathrm{J}}\sum_{k\in \mathrm{I}\cup \mathrm{J}} x_{ijk} = 1 \qquad \forall\, j\in J \qquad \text{b-4}$$

$$U_{ik} - U_{jk} + Nx_{ijk} \le N-1 \qquad \forall\, i و j\in J\,;\ \forall\, k\in K \qquad \text{b-5}$$

$$\sum_{i\in \mathrm{I}\cup \mathrm{J}} x_{ijk} - \sum_{j\in \mathrm{I}\cup \mathrm{J}} x_{ijk} = 0 \qquad \forall i\in \mathrm{I}\cup \mathrm{J}\,;\ \forall k\in K \qquad \text{b-6}$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \le 1 \qquad \forall k\in K \qquad \text{b-7}$$

$$\sum_{u\in J} x_{iuk} - \sum_{u\in V\setminus\{j\}} x_{ujk} \le 1 + Y_{ij} \qquad \forall i\in I;\ \forall j\in J\,;\ \forall k\in K \qquad \text{b-8}$$

$$l_{ij} \le r^{F}_{i} + M\left(1-y_{ij}\right) \qquad \forall\, i\in I\ و\ j\in J \qquad \text{b-9}$$

$$x_{ijk} \in \{0 و 1\} \qquad \forall i\in I; \forall j\in \mathrm{I}\cup \mathrm{J}\,;\ \forall k\in K \qquad \text{b-10}$$

$$y_{ij} \in \{0 و 1\} \qquad \forall i\in I; \forall j\in J \qquad \text{b-11}$$

$$Z_i و IM_{Fi} \in \{0 و 1\} \qquad \forall i\in I \qquad \text{b-12}$$

$$U_{jk} \ge 0\,;\ integer \qquad \forall j\in J\,;\ \forall k\in K \qquad \text{b-13}$$

The objective function (b-1) minimizes the total branch setup costs, the fixed costs of using vehicles, shipping costs, and radius of influence improvement costs. Constraints (b-2) and (b-3) guarantee capacity constraints of vehicles and branches, respectively. The assignment of a vehicle to a path is made by constraint (b-4). The term (b-5) is considered for sub-tour elimination. The (b-6) and (b-7) guarantee the continuity of each route, and that each route terminates at the branch where the route starts. The constraint (b-8) indicates that the customer can be assigned to the branch to which the vehicle path is connected. The expression (b-9) guarantees that a demand point can be allocated to a branch only when their distance is less than the radius of influence of the branch. Binary variables are also shown in relations (b-10), (b-11), and (b-12). Finally, the auxiliary variables of the model with integer and non-negative values are mentioned in (b-13).

4. Solving the problem

To solve the problem, it is first assumed that all the branches of the players are closed, so for each branch of the player, there can be three cases, which are: open the branch without increasing the radius of influence, open the branch by increasing the radius of influence, and keep closed Branch. Therefore, each actor has three to the power of the number of branches scenario to decide. Given that in a competitive environment, it is not clear which customer is assigned to which player and how much is the demand share of each player from each customer, it is necessary to solve the models under the previous sections 3.2 and 3.3 to reach a Nash equilibrant in the competitive environment based on the limitation of each player.

Solving these models, the market share of each player is calculated. By determining the market share of each player, and determining the customers assigned to them, the problem model will become a typical LRP model, which can be solved to achieve the lowest total cost.

From the mathematical aspect, LRP is a combination of two NP-hard problems (branch location and vehicle routing). Adding the radius of influence to this problem will increase the complexity. To overcome this problem a hybrid metaheuristic algorithm based on the combination of Ant Colony Optimization (ACO) and Simulated Annealing (SA) and has been developed.

4.1 Ant colony optimization

The problem of finding the shortest path is an optimization problem that can be compared to the behavior of ants. Ant colony optimization method inspired by the behavior of ants in finding the path between the nest and food; first introduced in 1992 by Marco Dorigo. How to find the shortest path between food sources and the nest is an activity that every ant does daily. The ants first randomly go around to find food and then leave a trail called a pheromone on the way back to the nest. Seeing this effect, other ants follow it and improve it until they reach the shortest path.

4.2 Simulated annealing method

The simulated annealing algorithm is an optimization algorithm for solving optimization problems in large search spaces. This algorithm is mostly used when the search space is discrete. The gradual annealing technique is used by metallurgists to achieve a state of matter that is well-organized and energy-minimized. The aim is to maximize the size of the solid crystals of the solid state. This technique involves placing the material at a high temperature and then gradually lowering that temperature. At each cooling step, the algorithm considers a neighboring state and possibly decides whether to move to the new state or stay in the previous state. These probabilities eventually lead the system to less energetic states. This step is usually repeated until the system reaches a certain state, or the number of computational operations exceeds a certain limit. Neighbors of a state are new states of the problem that are created by changing the current state according to a predetermined method.

4.3 Hybrid method (ACO-SA)

In the proposed hybrid algorithm, first, several repetitions are done with the ant colony optimization (ACO) to achieve the initial answer and then, this initial answer is placed in the simulated annealing (SA) algorithm and the search for optimality is followed. According to this algorithm, the optimal or near-optimal location of branches, as well as the appropriate routes and travel sequences of each player, are determined. Ultimately, the decision variables related to the branch opening, the assigned customers to each branch, and the customer coverage paths are determined by the lowest total cost. The hybrid model needs to be run several times to achieve the best solution. More description of the proposed hybrid algorithm is explained in section 5. The pseudo-code of the proposed hybrid algorithm is also attached. It is worth mentioning that the hybrid method is used after finding the minimum cost values calculated by each method for different reference samples as shown in Table 3.

Table 3. The minimum cost values calculated by each method for different reference samples

ROW	BARRETO	ACO (500&80)	SA (300&1000)	ACO-SA (50,300)	SA-ACO (300,50)	ACO (1500&60)	SA (1500&60)	ACO-SA (50,1500)
1	582.70	754.59	778.70	736.96	768.25	732.42	640.72	627.32
2	886.30	1320.83	1124.86	1271.03	1209.23	1447.41	996.86	989.57
3	889.40	1366.12	1475.51	1460.54	1447.98	1265.66	1087.05	1021.93
4	384.90	1158.63	684.08	730.81	807.61	1149.91	450.27	451.26
5	46642.70	82871.17	61347.91	60809.84	60171.66	81627.29	63351.47	57786.25
6	435.90	494.49	438.19	438.67	440.71	486.03	435.17	440.82
7	591.50	725.10	585.11	586.70	585.11	738.21	586.70	586.70
8	512.10	606.60	531.72	545.37	543.79	594.47	529.39	529.90
9	571.70	630.72	609.74	623.99	624.96	638.67	610.68	588.51
10	511.40	594.27	566.27	534.20	576.67	593.66	585.01	504.33
11	470.70	619.38	556.53	520.99	514.24	597.18	514.09	495.35
12	3,062.00	3794.31	3063.22	3062.02	3065.24	3783.71	3068.38	3065.24
13	6,238.00	8450.02	6657.20	7012.27	6627.20	8046.02	8830.78	7529.97
14	204.00	386.94	381.58	381.58	382.87	424.35	381.58	381.58
15	1136.20	4500.14	2212.34	2268.74	2242.23	4454.35	2228.50	1993.29
16	1656.90	4437.39	3240.53	3243.06	3342.68	4475.34	3301.38	3354.96
17	580680.20	752148.31	970647.07	718479.31	963473.82	724740.66	1236124.83	1050999.46
18	747,619.00	1183044.28	1074391.46	930775.88	1075494.80	1183044.28	1181830.61	1333059.28
19	12474.20	21438.22	13875.15	14388.12	14040.52	20490.36	16609.64	15638.02
20	----	449.37	469.81	448.40	448.40	480.45	448.40	448.40

5. Case study

Food safety and health are one of the most important issues in food supply chain management. Maintaining safety, quality, and standards is critical to all components of the food supply chain, including producers, retailers, distributors, and the government. Which are affected during distribution (Trivedi et al.,2019; Behzadi et al.,2013).

Due to the competition between service providers, the type of these services and their quality in the business environment is constantly changing. Providing door-to-door delivery service from supermarkets is commonplace. With the outbreak of COVID 19 viruses, this type of service has become more popular. In this study, we examine the service of 2 large stores of Kourosh and Shahrvand for delivering items to customers, in the 11th, 9th, and 17th districts of Mashhad. There are several potential locations for stores, 7 of which were examined in this case study. In the study area, there are 20 main customers, including student dormitories and residential complexes, etc., who receive a significant portion of their demand at home. On average, each customer demands 700 different items. It is assumed that this amount of demand is evenly distributed among all centers of 20 sectors of these districts. So, there are 20 centers with equal 700 demands annually. Other required input information is shown in Table 4.

Table 4. Assumptions and Input information

row	Explanation	Amount
1	Number of demand points	20
2	Estimated demand of each point	700
3	Max No of the vehicle for each shop	4
4	Capacity of vehicle	10000
5	Maximum Capacity of each shop	20000
6	Transportation cost per unit	37900
7	Fixed cost of a vehicle	6000000
8	Cost of radius change (*10*IRR*)	5112900
9	Length of the radius of influence	0/0008

Two firms A with 4 shops and B with 3 shops are considered as the main player and rival player respectively. The geographic coordinates of the potential locations for branches as well as the latitude and longitude of the demand centers are known for both players. The annual cost information for investigating the market share of each firm is shown in table 5.

Table 5. Annual cost information (Iranian currency: IRR)

	Row	Monthly Rent	Decoration & equipment	Total annual cost
Main Player	1	15000000	25000000	205000000
	2	35000000	25000000	445000000
	3	23000000	25000000	301000000
	4	18000000	25000000	241000000
Rival Player	1	35000000	25000000	445000000
	2	28000000	25000000	361000000
	3	17000000	25000000	229000000

The location of the customers and the potential branches of the players are shown in Figure 2. In this figure, the customers' locations, the potential stores corresponding to the main player, and the rival stores are shown by green circles, blue squares, and red squares, respectively.

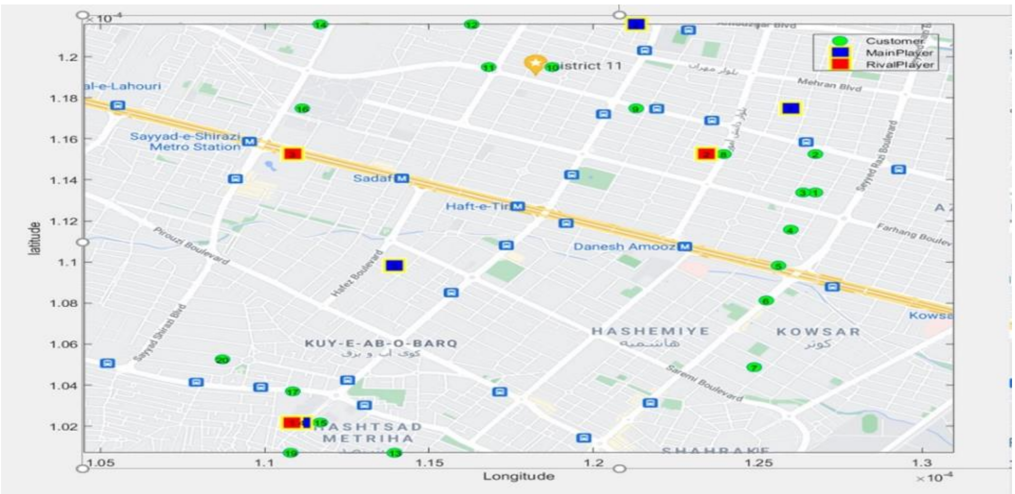

Figure 2. The situation of the study area in Mashhad

The objective function of each player is, to find the location of the shops, the routing decisions, and opening/closing shops for increasing the market share in the competition environment. Each firm may apply three decisions for each branch, which are: opening a branch, keeping the branch closed, and increasing the branch effect radius. A complete counting algorithm is used to estimate the market share of each player. Given that the main player has 4 branches, the number of possible scenarios will be 3 to the power of 4 modes, i.e. 81 scenarios. The number of possible scenarios for the rival player will be 27. Since the reopening of each branch, or increasing the radius of influence, has a cost, each player cannot implement all the scenarios due to budget constraints. Due to the amount of 16 billion Rials for the main player, the number of possible scenarios will be reduced to 63 scenarios. Also, the number of possible scenarios of the rival player will be reduced to 21, considering the budget amount of 15 billion Rials. The number of possible scenarios for each player is shown in table 6.

Table 6. The number of a possible scenario for each player

Main Player	Rival Player
63	19

For each scenario, there is a Pay-Off for the main player and a Pay-Off for the rival player which is calculated using the developed mathematical models. This process continues to achieve Nash equilibrium. For example, when the main player decides to open branches 1, 2, and 4 and increase their radius of influence, he will cover customers located in places 1 to 5, 8 to 13, 15, 17, 19, and 20.

Also, based on the decision of the main player, the rival player will open branches 1 and 2 and will increase their radius of influence. This player will cover customers located in places 1 to 6, 8 to 11, 13, 15, 17, 19, and 20. Customer 12 is only within the radius of coverage of the main player and devotes all his demand to it. Also, customers 6 are only within the radius of the effect of the branches related to the rival player and allocate all their demand to this player. The rest of the customers, who are affected by both players, allocate their demand to them in proportion to the players' branches. Customers 7, 14, 16, and 18 are not within the radius of the effect of the branches of any of the players and as a result, their demand will be lost.

Figure 3 shows the results for the proposed case study. The opened branches and covering circles for the main players are shown with blue squares and blue circles respectively. The opened branches and covering circles for the rival players are shown with red squares and red circles respectively. In this figure, the customers are shown with green circles.

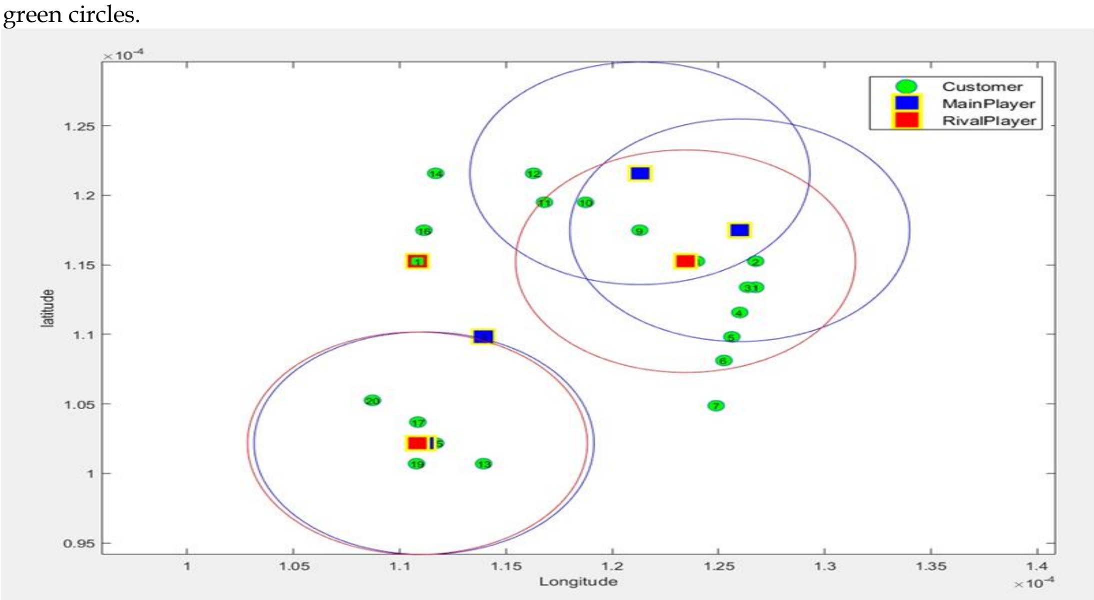

Figure 3. Opened branches and covering circles for each player

The share value of each player in the total demand of the population area is shown in Table 7.

Table 7. The amount of market share for each player

Main player's Pay-Off	Rival player's Pay-Off
5951	5249

As can be seen in Table 7, out of the total demand of 14000 of the studied market, 5951 will be the share of the main player and 5249 will be the share of the rival player. In this case, some demands are not covered. Regarding the basic data of the case study and the obtained data from the customers' location and covering circles, the total market share of the players in total demand of (14000) has been equal to 80%, with a main player's share of 42.51% and a rival player's share of 37.5%.

Depending on the market share gained for each customer and the identification of opened branches, a typical LRP problem can be solved for each player to get the minimum total cost. It should be noted that the LRP problem must be solved for all main player and rival player scenarios. For this purpose, the hybrid algorithm presented in Section 4.3 is used.

To solve the LRP problem, according to the information in Table 4, it is necessary to convert the basic information, including the location of customers and branches into an array. This array is considered as the input of the ant colony algorithm. The primary array is shown in Table 8.

Table 8. The primary location of customers and branches

location	1	2	3	4	………..	29	30	31	32
Number	1	2	3	4	………..	29	30	31	32

In the case study, after 50 repetitions in ACO, the resulting answer was entered into the hybrid algorithm as the initial answer. The final answer of the ACO-SA algorithm, which is an improved array of locations of customers and branches, was considered as visitable points in the problem.

There are 20 demand points in this example. According to the 3 branches opened by the main player, and the use of 4 homogeneous vehicles of transportation, a total of 20 numbers were assigned to the routes related to the branches. All 4 consecutive numbers will belong to an opened branch. Therefore, the main player will have 32 points to visit, so numbers 1 to 20 related to customers and numbers 21 to 32 related to branches and routes were considered.

In this way, the required array of the ACO-SA hybrid algorithm was created. The process of creating an array for the rival player is the same, so a 28-array is created for the rival. After executing the hybrid algorithm with 1550 repetitions, the results of Table 9 are obtained. After obtaining the improved array by the hybrid algorithm, by decoding the numbers, the customer service paths are determined.

Table 9. The final location of customers and branches

location	1	2	3	4	5	6	7	8	9	….
Number	21	1	3	2	22	9	10	21	4	….

The customer numbers, in the order in which they are placed, will represent the sequence in the path and end with the first number that belongs to a branch and route. For example, the customer located in positions 5 and 8 is assigned to the branch, and the path corresponding to the number 21, and this number corresponds to the second path from the first branch of the main player. Other information of branches and routes and customers assigned to the main player can be seen in Figure 4. The result of the proposed hybrid algorithm (ACO-SA) for the Rival player is shown in figure 5.

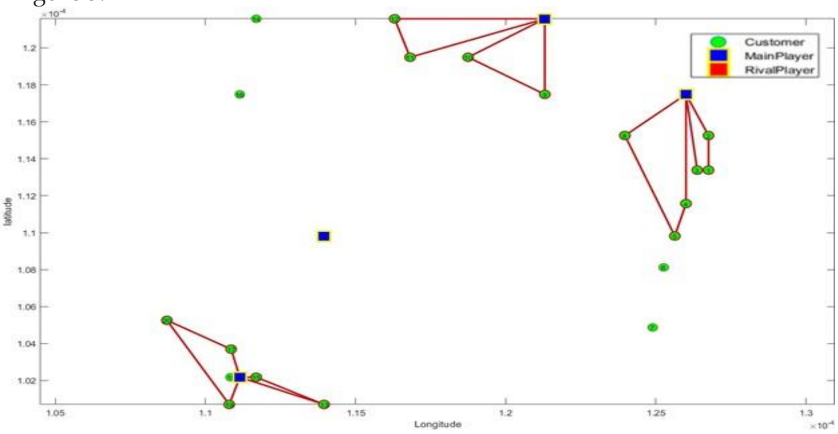

Figure 4. Location and Routing decisions for the Main-Player

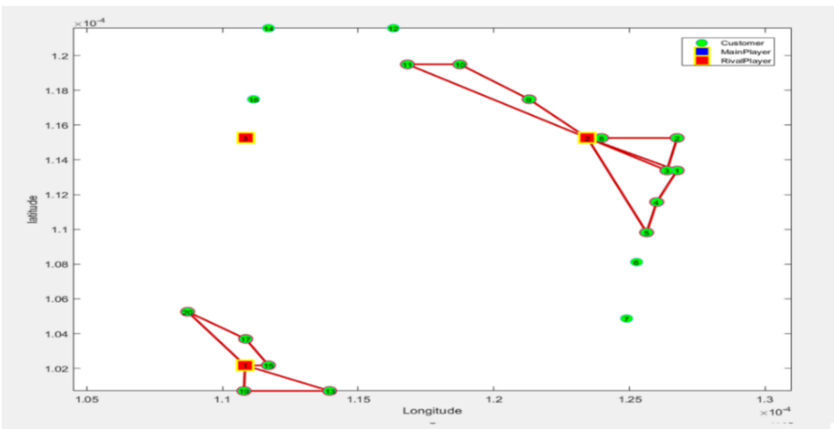

Figure 5. Location and Routing decisions for the Rival player

Figure 4 and 5 show the best result for the main and rival players. Opened branches are shown with blue squares. The opened branches and the related tours indicate how each branch serves its customers. The results of this step are shown in Tables 10 and 11 for the Main and Rival players.

Table 10. Location and routing decisions for the main player

Player	Opened Branch	Path Number	The sequence of Customers in Route
Main Player	1	1	1-3-2
		2	4-5-8
	2	1	9-10
		2	11-12
	4	1	13-15
		2	19-20-17
	Closed Branch:	3	

Table 11. Location and routing decisions for the main player

Player	Opened Branch	Path Number	The sequence of Customers in Route
Rival Player	1	1	19-13
		2	15-17-20
	2	1	8-2-3
		2	1-4-5
		3	9-10-11
	Closed Branch:		3

As shown in Tables 10 and 11 the Main player opens branches 1, 2, and 4 to compete with the rival player which opens branches 1 and 2 while branches 3 from the Main player and 3 of the rival players are kept close.

6. Conclusion and future research

In this paper, the problem of LRP was discussed in a competitive environment and the concept of coverage was used to determine the share of each firm. A two-section model was developed to formulate the LRP and estimate the market share of each participant. The concept of simultaneous games was used to estimate the market share in the competition section and an LRP was implemented for each player per each Nash equilibrium. Finally, the developed method was implemented on the problem of the market with good delivery in the city of Mashhad. In this study, according to the scope of the research, it has been presumed that the variables related to the radius of influence of the branches (improving the radius or maintaining the current status) are exclusively binary variables. To solve the problem in less time, the ACO-SA hybrid algorithm was proposed. The model was solved for a real problem with small dimensions. In the example provided, customers were evenly distributed in specific areas, so that in reality the focus could be different. The current study opens new horizons for competitive LRP. The integration of different LRPs with a competitive environment can be an attractive area for the adaptation of the existing models to the real world. For example, considering the presented approach in Ayough et al. (2020) in the routing section could improve the total result in the economic dimension. Naturally, research like Sayyah, Larki, and Yousefi-Khoshbakht (2016), which studied Simultaneous Pickup and Delivery by an ACO, could improve the routing method of the present study.

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Appendix A

The pseudo-code of the proposed hybrid algorithm for LRP:

Start

```
Input requirements of LRP []
Run The ACO
        Input the ACO Parameters
        BestAnt.Cost= ∞
        ACO Main Loop
                FOR 1 through Max-Iteration
                        FOR 1 through number of ant
                        Ant-tour ← create Random solution based on ACO method
                        Ant.Cost ← calculate the cost of Ant-tour
                        BestAnt.Cost THEN >IF Ant.Cost
                                BestAnt.Cost = Ant.Cost
                                BestAnt = Ant
                        ENDIF
                        ENDFOR
                        Update Pheromones
                                Increase Pheromones in best path
                                Decrease Pheromones from other path
                        Store Best tour
                                Best tour ← Tour of the BestAnt
                                Best tour.cost ← BestAnt.Cost
                ENDFOR
END ACO
Run The SA
        Input The SA Parameters
        MaxItSA ← Maximum Number of Iterations
        Initial Solution ← solution that corresponded whit the Best tour
        SA Main Loop
                Best Solution ← Initial Solution
                FOR 1 through MaxItSA
                        New solution ← create different neighbor solution based on SA method
                        New solution. Cost ← calculate the cost of New solution
                        Best solution. Cost THEN > IF New solution. Cost
                            Best solution. Cost = New solution. Cost
                            Best solution = New solution
                        ENDIF
        ENDFOR
END SA
Extract answers for LRP decisions & De-coding
  Finish
```