Vol. 9, No. 2, 2022, Pages 49-62

JIEMS

Journal of Industrial Engineering and Management Studies

journal homepage: www.jiems.icms.ac.ir

Providing a Lagrangian relaxation algorithm to solve a reliable location- inventory model by considering disruption

Somayeh Najafi- Ghobadi[1*], Mahtab Sherafati[1]

[1] Department of Industrial Engineering, Faculty of Engineering, Kermanshah Branch, Islamic Azad University, Kermanshah, Iran.

Received: May 2021-10/ Revised: June 2021-25/ Accepted: December 2021-14

Abstract

Nowadays, supply chains have been facing significant economic forfeitures because of unpredicted disruptions. Furthermore, managers try to design sustainable and reliable supply chains. In this paper, we present an inventory-location model to propound a reliable three echelon supply chain which includes a production plant, distribution centers, and retailers. The production plant distributes a single product to retailers through distribution centers that are at risk of disruption. We considered reactive (consider backup distribution center for each retailer) and proactive (distribution center fortification) activates to enhance the supply chain's reliability. The proposed model indicates the location of distribution centers (DCs), the DCs that must be fortified, the allocation of retailers to DCs, and the inventory policy of DCs. The problem is formulated as a nonlinear integer programming model. Since our model is an NP-hard problem, we provide a Lagrangian relaxation algorithm to solve it. Numerical examples demonstrate the computational efficiency of the proposed solution algorithm. Results show that, with increasing the budget of fortification, the total expected cost will decrease. A higher inventory cost leads to an increase in the number of opened DCs, while higher ordering cost and the transportation cost from production plant to DCs decrease the number of opened DCs. Among other results, the number of opened DCs is positively affected by the cost of transporting from DCs to retailers.

Keywords: disruption; location; inventory; Lagrangian relaxation; facility fortification.

Paper Type: Original Research

1. Introduction

Increasing competition in global markets makes cost reduction and customer service improvement two critical challenges for firms. In this regard, firms try to design a more efficient supply chain by integrating tactical and strategical decisions. Inventory control and facility location are the two most important tactical and strategical decisions, respectively. These decisions are interconnected because a shift in the number or location of facilities can affect inventory costs and delivery time. Also, changing the inventory policy of facilities can influence location costs and allocation decisions. Hence, modeling the location-inventory problems has always been attractive for researchers, and many studies are being conducted in this field (Amiri-Aref et al., 2018; Bagherinejad and Najafi-Ghobadi, 2019; Liu et al. 2020; Wu et al., 2021). In classical inventory-location models, it is assumed that the facilities will always work as planned, and they have never been disrupted. In the real world, however, firms face several unexpected events in their supply chain each year. In March 2000, a fire at Philips's semiconductor plant in New Mexico shut down the company for nine months. The shutdown caused $ 40 million in damage to Philips direct sales and $ 2.34 billion in losses to Ericsson's mobile division (Sheffi, 2005). The following year, the United States banned meat imports due to a possible outbreak of the foot-and-mouth disease in the United Kingdom. The ban affected four percent of US pork imports (Marquis, 2001; Reuters, 2001). In the events of 11 September 2001, US borders were closed due to the terrorist attack. As a result, Ford Motors was forced to close several assembly lines due to a shortage of overseas parts (Sheffi et al., 2005; Cundari et al., 2008). In 2003, the deadly SARS outbreak disrupted China's furniture production and reduced its exports to the United States by about 15 percent (Koncius, 2003). Following Hurricane Katrina in 2005, crude oil production in the Gulf of Mexico was disrupted by 1.4 million barrels per day (Kotak, 2005; Mouawad and Romero, 2005; Strahan et al., 2005).

The mentioned and other examples (Sheffi, 2001; Christopher and Peck, 2004; Wilson, 2005; Carpenter, 2010) reveal the severe need to design supply chains by considering the risk of disruption. There are very few resources and time to rebuild and repair the strategic network infrastructure when a network disruption occurs. Studies have shown that additional investment in the initial design can lead to optimal supply chain performance when disruption occurs. Disruption makes the strategic levels of the supply chain vulnerable and, of course, severely

*Corresponding Author: s.najafi2010@gmail.com

DOI: 10.22116/jiems.2021.296138.1449

overshadows the operational and tactical decisions. Therefore, considering disruption in designing supply chains are critical, and the questions that arise here are:

- What factors and activities increase the supply chain reliability under disruption risk?
- How to design a reliable supply chain taking into account inventory and location decisions?
- How should firms manage sudden disruptions by making appropriate inventory and location decisions to reduce costs?

Answering the above questions was our primary motivation to conduct this study. We design a reliable three echelon supply chain, including a production plant, distribution centers, and retailers. The production plant distributes a single product to retailers through distribution centers that are at risk of disruption. Our model optimizes distribution center (DC) locations to be opened, the DCs to be fortified, the allocation of retailers to opend DCs, and inventory policy to minimize inventory and location costs. We assume one primary opened DC and one reliable opened backup DC for each retailer. If the primary DC fails, the retail will be served by the backup DC. Because our proposed model is NP-hard, a Lagrangian relaxation algorithm is developed to solve it.

2. Review of literature

In the literature, two kinds of policies were proposed to increase the reliability of supply chains: considering some backup facilities for failed facilities and using a fortification budget to fortify facilities. Thus, we classified the reviewed studies into two categories: studies considering backup facilities and researches on allocating fortification budget.

2.1. Studies with considering backup facilities

Facility location models have attracted more attention in researches of the supply chain under disruption (Berman et al., 2007; Cui et al., 2010; Liberatore et al., 2011; Lim et al., 2010; Peng et al., 2011; Snyder & Daskin, 2005). At first, Drezner (1987) studied the facility location problem under disruption. He provided a neighborhood search heuristic algorithm to solve the proposed model. Snyder and Daskin (2005) modeled a facility location problem by assuming a primary server and some backup servers for each customer. If the primary server fails, the customer receives the service from the next server. They assumed that at least one of the assigned servers was reliable. The failure probabilities of facilities were supposed to be equal and independent. Berman et al. (2009) solved a location-allocation model by providing three heuristic algorithms: simulated annealing, tabu search, and ascent algorithm. Cui et al. (2010) expanded the Snyder and Daskin model by considering heterogeneous failure probabilities. Chen et al. (2011) investigated a reliable joint inventory-location model. Their goal was to determine distribution centers' locations, inventory management decisions and allocate customers to DCs in a situation where DCs were at disruption risks. They considered R backup facilities for each customer. Peng et al. (2011) proposed a mixed-integer programming model to minimize the total cost in situations where there are no disruptions. They decreased the disruption risk by applying the p-robustness criterion. Rayat et al. (2017) studied a multi-period location-inventory-routing problem by considering disruption risks. The problem was presented as a bi-objective mixed-integer nonlinear model assuming stochastic demand. They provided a modified multi-objective simulated annealing algorithm for solving the model and numerical examples. Yun et al. (2017) proposed a "trial-and-error" strategy for modeling a reliable facility location problem. They assumed that facility failure probabilities varied across the space. Azizi (2017) modeled a hub location problem when hubs were subject to failure. In their model, each demand point was allocated to a primary hub and one backup hub. They formulated the problem as a mixed integer quadratic program and proposed three-particle swarm optimization-based meta-heuristics. Zhang et al. (2016) formulized a competitive location model that facilities were subject to disruption. They considered two players that competed with each other to capture market share. The competition was modeled as a Stackelberg game, and a variable neighborhood decomposition search heuristic was presented. Jabbarzadeh et al. (2018) studied the effect of disruption in the design of a close-loop supply by allowing lateral transshipment between facilities. Their model determined the location of facilities and the lateral transshipment quantities. A Lagrangian relaxation algorithm was provided to solve the model. Eskandari-Khanghahi et al. (2018) proposed a multi-period and multi-objective model for a sustainable blood supply chain under the risk of disaster. Yahyaei and Bozorgi-Amiri (2019) modeled the design of relief logistics by considering the disruption of facilities. They assumed that the number of customers affected by the disruption of facilities is an uncertain parameter and used a robust approach to solve the model. The problem of designing a transportation network for hazardous materials under disruption was investigated by Ghaderi and Burdett (2019). The aim was to minimize the cost of transportation and risk. Diabat et al. (2019) provided a supply chain model for perishable products. They considered facilities' location and routes under distribution risk, and a Lagrangian relaxation algorithm was presented to solve the proposed model. Shen et al. (2020) proposed a mixed-integer linear programming model to address a hub location problem. They showed that although considering backup servers would slightly increase costs, they significantly increase the supply chain reliability.

2.2. Researches on allocating fortification budget

In the event of a disaster, allocating customers to other servers may result in customers' dissatisfaction. Customers expect that the servers have acceptable stability. Thus, managers must accomplish proactive activates such as investment in reliability improvement of existing facilities. Church and Scaparra (2007) modeled a location problem by considering a fortification budget. They aimed to allocate a fortification budget among a set of facilities to minimize the impact of disruption. Scaparra and Church (2008) also studied an r-interdiction median problem. The goal was to minimize the effect of the disruptive attacks on facilities through the most cost-effective ways to allocate protective resources among them. Li et al. (2013) investigated a location model under disruption by considering a finite fortification budget. Jabbarzadeh et al. (2016) designed a resistant supply chain when disruption probability was a function of fortification investment. They used a Monte Carlo simulation method to investigate the performance of the proposed model. Afify et al. (2019) developed an evolutionary learning algorithm to solve a location problem where distribution centers were under disruption risk. Their model indicated the location of facilities and a subset of facilities that should be fortified. Haghjoo et al. (2020) modeled a blood supply chain where the facility disruption depended on the initial budget allocated to open it. Two meta-heuristic algorithms, including invasive weed optimization and self-adaptive imperialist competitive algorithms, were provided. The reviewed literature is summarized in Table 1.

2.3. Contributions of this study

Based on reviewed literature and Table 1, we can mention our novelties as follows:

1. A novel inventory-location model is proposed to determine the location of distribution centers, the distribution centers that must be fortified, the allocation of retailers to opened DCs, and inventory policy of DCs.
2. Both fortification budget constraints and backup facilities are incorporated to increase the reliability of the supply chain. As a result, our model provides a more realistic supply chain and vouches that the obtained optimal solutions are consistent with the available reliability improvement resources.
3. The proposed model can help the strategic decision-makers to evaluate the rate of return on investment in fortifications. It can help companies to determine the right amount of investment to strengthen their supply chain.
4. A Lagrangian relaxation algorithm is developed to solve the model and numerical examples.

Table 1: The summary of the reviewed literature

Study	Decision variable		Type of increasing reliability		Number of opened facilities		Solution approach	
	Location	Inventory	Backup facilities	Fortification budget	Limited	Unlimited	Heuristic	Lagrangian relaxation
Drezner (1987)	*		*			*	*	
Snyder and Daskin (2005)	*		*		*	*		*
Church and Scaparra (2007)	*			*	*			
Scaparra and Church (2008)	*			*	*		*	
Berman et al. (2009)	*				*		*	
Cui et al. (2010)	*		*			*		*
Chen et al. (2011)	*	*	*			*		*
Peng et al. (2011)	*		*		*		*	
Li et al. (2013)	*			*	*	*		*
Jabbarzadeh et al. (2016)	*			*		*	*	
Zhang et al. (2016)	*		*			*	*	
Rayat et al. (2017)	*	*	*			*	*	
Yun et al. (2017)	*		*			*	*	
Azizi (2017)	*		*		*		*	
Jabbarzadeh et al. (2018)	*		*			*		*
Eskandari-Khanghahi et al. (2018)	*	*				*	*	
Yahyaei and Bozorgi-Amiri (2019)	*		*			*		
Ghaderi and Burdett (2019)	*		*			*	*	
Diabat et al. (2019)	*	*	*			*		*
Afify et al. (2019)	*			*	*		*	
Shen et al. (2020)	*		*		*		*	
Haghjoo et al. (2020)	*			*		*	*	
Current paper	*	*	*	*	*			*

3. Problem description and notations

We consider a set of retailers with deterministic demand for a single product and a reliable production plant. The production plant provides the retailer's demand through DCs that are subject to heterogeneous disruption. Each retailer is allocated to a primary DC and a different backup DC to improve the supply chain's reliability. Also, according to the available budget, some of opened DCs are fortified. If a DC is fortified, it becomes reliable. It is desired to select DCs in the candidate locations to be opened, indicate which of DCs must be fortified, assign retailers to opened DCs and determine inventory policy in the opened DCs so that the sum of expected costs of inventory and transportation will be minimized (Fig. 1).

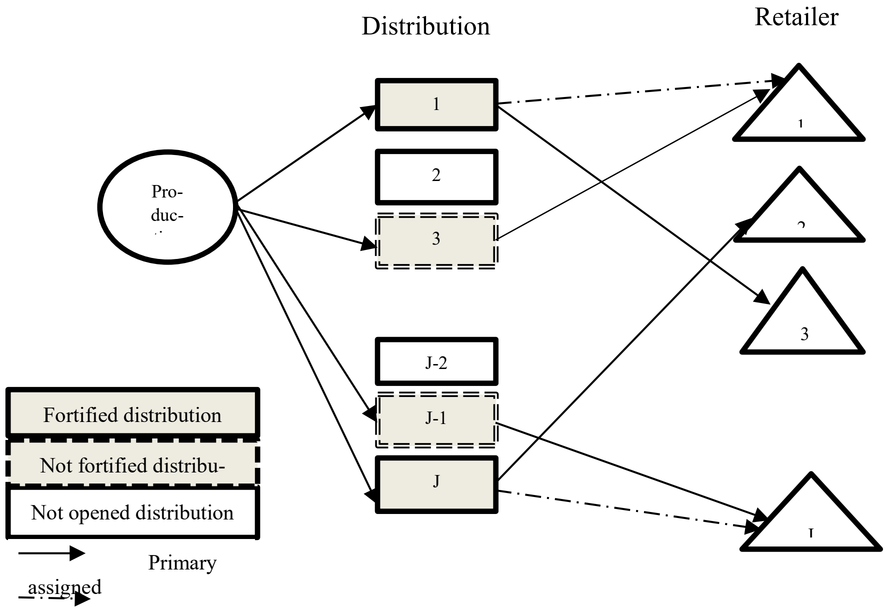

Figure 1: A sample solution of the proposed model

The assumptions, notations, input parameters, and decision variables of the model are described in the following.

Assumptions

1. The location of the retailers is fixed and known.
2. The retailer's demand must be satisfied with just one primary or backup DC.
3. If the primary DC fails, the backup DC is reliable and will serve the assigned retailer.
4. There is not any backlog at DCs.
5. The number of DCs that can be opened is given (Li et al., 2013; Snyder and Daskin, 2005).
6. The failures of DCs are independent of each other (Cui et al., 2010; Lim et al., 2010 and Snyder and Daskin, 2005).
7. If a DC fails, it becomes unavailable

Sets

i: The set of retailers {1, 2... I}
j: The set of potential DCs nods {1, 2…J}
Parameters
di: Demand of retailer i
hi: Unit holding cost in DC j
cj: Ordering cost of DC j
qj: The probability that DC j failed
sji: Unit transportation cost from DC j to retailer i
sj: Unit transportation cost from production plant to DC j
f_j: Fortification fixed cost of each facility, including the costs of research and developments (R&D), personnel training, and contract negotiation
rj: Variable fortification cost to decreases one unit of the failure probability of DC j
B: Total budget that is considered to fortify DCs

P: Number of DCs that must be opened

Decision variables

$$Y_j = \begin{cases} 1 & \text{If a DC is opened at node j} \\ 0 & \text{Otherwise} \end{cases}$$

$$X_{ij0} = \begin{cases} 1 & \text{If DC j is considered as the primary server of retailer i} \\ 0 & \text{Otherwise} \end{cases}$$

$$X_{ij1} = \begin{cases} 1 & \text{If DC j is considered as the backup server of retailer i} \\ 0 & \text{Otherwise} \end{cases}$$

$$W_j = \begin{cases} 1 & \text{If DC j is fortified} \\ 0 & \text{Otherwise} \end{cases}$$

4. Model formulation

Let a distribution center that follows continuous inventory policy is located at node j. For this DC, its expected annual demand is the sum of retailers' demand that are assigned to it and is:

$$\sum_{i=1}^{I}\left[d_i X_{ij0}(1-q_j(1-W_j)) + d_i X_{ij1} \sum_{\substack{r\in J \\ r\neq j}} X_{ir0} q_r (1-W_r) \right] \quad (1)$$

Where $(1-q_j(1-W_j))$ and $\sum_{r\in J, r\neq j} X_{ir0} q_r (1-W_r)$ are the probability that the DC j is available as the primary and the backup server, respectively.
By considering ordering and holding costs as the annual inventory cost, we have:

$$TIC_j = c_j \frac{\sum_{i=1}^{I}[d_i X_{ij0}(1-q_j(1-W_j)) + d_i X_{ij1} \sum_{r\neq j} X_{ir0} q_r (1-W_r)]}{Q_j} + h_j \frac{Q_j}{2} \quad (2)$$

For any given DC and its retailer assignments, according to the EOQ trade-off, the optimal ordering quantity can gain as follows:

$$Q^* = \sqrt{\frac{2cD}{h}} \Rightarrow Q_j^* = (\frac{2c_j}{h_j} \sum_{i=1}^{I}[d_i X_{ij0}(1-q_j(1-W_j)) + d_i X_{ij1} \sum_{r\neq j} X_{ir0} q_r (1-W_r)])^{0.5} \quad (3)$$

Thus, under the optimal ordering quantities, the total expected inventory cost at node j by substituting (3) in (2) is:

$$TIC_j^* = (2c_j h_j \sum_{i\in I}[d_i X_{ij0}(1-q_j(1-W_j)) + d_i X_{ij1} \sum_{r\neq j} X_{ir0} q_r (1-W_r)])^{0.5} \quad (4)$$

Then, the cost of transportation from DC j to assigned retailers is:

$$\sum_{i\in I}[d_i s_{ij} X_{ij0}(1-q_j(1-W_j)) + d_i s_{ij} X_{ij1} \sum_{\substack{r\in J \\ r\neq j}} Y_{ir0} q_r (1-W_r)] \quad (5)$$

So, we have the total transportation cost from DCs to retailers as follows:

$$\sum_{j\in J}\sum_{i\in I}[d_i s_{ij} X_{ij0}(1-q_j(1-W_j)) + d_i s_{ij} X_{ij1} \sum_{\substack{r\in J \\ r\neq j}} X_{ir0} q_r (1-W_r)] \quad (6)$$

Also, the cost of transportation from production plant to DCs is:

$$\sum_{j\in J} s_j (\frac{2c_j}{h_j} \sum_{i=1}^{I}[d_i X_{ij0}(1-q_j(1-W_j)) + d_i X_{ij1} \sum_{r\neq j} X_{ir0} q_r (1-W_r)])^{0.5} \quad (7)$$

According to equations (4), (6), and (7), we can present the objective function in equation (8).

$$(M): Minimize = \sum_{j\in J}\sum_{i\in I}[d_i s_{ij} X_{ij0}(1-q_j(1-W_j)) + d_i s_{ij} X_{ij1}\sum_{r\neq j} q_r X_{ir0}(1-W_r)] +$$
$$\sum_{j\in J}[(s_j(\frac{2c_j}{h_j})^{0.5} + (2c_j h_j)^{0.5})(\sum_{i\in I} d_i X_{ij0}(1-q_j(1-W_j)) + d_i X_{ij1}\sum_{\substack{r\neq j\\ r\in J}} q_r X_{ir0}(1-W_r))^{0.5}]$$

$$(M): Minimize = \sum_{j\in J}\sum_{i\in I}[d_i s_{ij} X_{ij0}(1-q_j(1-W_j)) + d_i s_{ij} X_{ij1}\sum_{r\neq j} q_r X_{ir0}(1-W_r)] +$$
$$\sum_{j\in J}[(s_j(\frac{2c_j}{h_j})^{0.5} + (2c_j h_j)^{0.5})(\sum_{i\in I} d_i X_{ij0}(1-q_j(1-W_j)) + d_i X_{ij1}\sum_{\substack{r\neq j\\ r\in J}} q_r X_{ir0}(1-W_r))^{0.5}]$$

(8)

$$\sum_{j\in J} X_{ij0} = 1 \qquad \forall i \in I \tag{9}$$

$$\sum_{j\in J} X_{ij1} = 1 \qquad \forall i \in I \tag{10}$$

$$X_{ij0} + X_{ij1} \leq Y_j \qquad \forall i \in I,\, j \in J \tag{11}$$

$$\sum_{j\in J} Y_j = P \tag{12}$$

$$\sum_{j\in J} (f_j + r_j q_j) W_j \leq B \tag{13}$$

$$Y_j, W_j \in \{0,1\} \qquad \forall j \in J \tag{14}$$

$$X_{ij0}, X_{ij1} \in \{0,1\} \qquad \forall i \in I,\, j \in J \tag{15}$$

Equations (9) and (10) reveal that each retailer is allocated to only one primary and one backup DC, respectively. Two purposes are shown in equations (11). First, it ensures that each retailer is assigned to only an opened DC. Second, it also selects different primary and backup servers for each retailer. Equation (12) presents the maximum number of DCs that can be opened. The total available budget to fortify DCs is presented in equation (13). Finally, the variables Yj, Wj, Xij0, and Xij1 are binary and are expressed by (14) and (15).

5. Solution approach

Previous researches demonstrated that both the location-inventory and the location-allocation problems are NP-hard (Daskin et al., 2002; Cooper, 1963). So, our model that is a combination of them belongs to NP-hard problems. Heuristic and Meta-heuristic algorithms are primarily used procedures to solve NP-hard problems. In literature, Lagrangian relaxation is the best approach for location models (Lee et al., 1996; Miranda and Garrido, 2008; Diabat et al., 2015; Beltran-Royo et al., 2012; Nezhad et al., 2013; Diabat et al., 2013). Thus, we motivate to provide a Lagrangian relaxation algorithm to solve the proposed model. The details of this algorithm proceed as follows.

5.1. Lagrangian relaxation

One of the most widely used methods to solve constrained optimization problems, especially integer problems, is the Lagrangian relaxation (LR) method (Khorshidvand et al., 2021 a, b, and c). Held and Karp (1970-1971) first proposed an LR algorithm for solving the traveling salesman problem. The main idea of LR is to relax complicating constraints and multiply them into a factor called the Lagrange multiplier, and add them to the objective function. It is expected that solving the relaxed model will be easier than solving the primary model. By considering a fixed value for the Lagrange multipliers, the optimal solution of the relaxed model will be a lower bound for the primary model (in the minimization problem). On the other hand, if the obtained solution is a feasible solution to the main problem, it will be the upper bound. For this purpose, a heuristic algorithm is usually proposed to obtain a feasible solution (upper bound) from the lower bound solution. As a result, by maximizing the minimum of the relaxed model, a lower bound is obtained for the main model, and in a repetitive process, the resulting solution will be pushed to reach an optimal solution for it.

5.1.1 Lower bound

Using Lagrange multipliers λ_{ij}, we relax the set of constraints (11) to reduce the complexity of the proposed model and remove the decision variable Yj. After relaxing these constraints, the following sub-model has been obtained.

$$(RM) = \min \sum_{j\in J}\sum_{i\in I} -\lambda_{ij} Y_j +$$
$$\sum_{i\in I}\sum_{j\in J}(d_i s_{ij}(1-q_j(1-W_j))+\lambda_{ij})X_{ij0}+(d_i s_{ij}\sum_{\substack{r\neq j\\ r\in J}} q_r X_{ir0}(1-W_r)+\lambda_{ij})X_{ir1}+$$
$$\sum_{j\in J}[(s_j(\frac{2c_j}{h_j})^{0.5}+(2c_j h_j)^{0.5}(\sum_{i\in I} d_i X_{ij0}(1-q_j(1-W_j))+d_i X_{ij1}\sum_{\substack{r\neq j\\ r\in J}} q_r X_{ir0}(1-W_r))^{0.5}]$$

Subject to (9), (10), (12)–(15), and Xij0+Xij1≤1.
For a fixed value of λij, we can find an optimal value for Yj by sorting $(-\sum_{i\in I}\lambda_{ij})$. If $(-\sum_{i\in I}\lambda_{ij})$ belongs to the P smallest sorted values, we set Yj=1; otherwise, Yj=0.
In the following, we indicate which opened DCs must be fortified. First, we assume that the budget for fortification is zero by considering B=0. So, we have Wj=0 for all j, and constraint (13) will be omitted. The simplified model is:

$$(M1): Min\sum_{i\in I}\sum_{j\in J}([d_i s_{ij}(1-q_j)+\mu_{ij}]X_{ij0}+[d_i s_{ij}\sum_{\substack{r\neq j\\ r\in j}} q_r X_{ir0}+\mu_{ij}]X_{ij1}+$$
$$\sum_{j\in J}[(s_j(\frac{2c_j}{h_j})^{0.5}+(2c_j h_j)^{0.5})(\sum_{i\in I} d_i X_{ij0}(1-q_j)+d_i X_{ij1}\sum_{\substack{r\neq j\\ r\in J}} q_r X_{ir0})^{0.5}]$$

Subject to (9), (10) and (15).
It is noticeable that when constraint (11) is relaxed, a retailer may be allocated to a DC that is not opened. However, it is still guaranteed that each retailer is allocated to only one primary and one backup DC considering constraints (9) and (10). To allocate a primary and backup server to each retailer optimally, we separate the problem M1 in i. For retailer i, we select DCs v and t as the primary and backup server, respectively. Thus, the objective function of (M1) related to retailer i will be:

$$\phi_i(v,t) = d_i s_{iv}(1-q_v)+\lambda_{iv}+d_i s_{it}q_v+\lambda_{iv}+(s_v(\frac{2c_v}{h_v})^{0.5}+(2c_v h_v)^{0.5})(d_i(1-q_v))^{0.5}+ \quad (16)$$
$$(s_t(\frac{2c_t}{h_t})^{0.5}+(2c_t h_t)^{0.5})(d_i\times q_v)^{0.5}$$

We calculate $\emptyset_i(v,t)$for all v, t ∈J and then compute $\emptyset_i^* = min_{v,t}\{\emptyset_i(v,t)\}$ to obtain the optimal allocation.
In the following, we suppose B≠0. We again suppose that retailer i is allocated to DC v as primary DC and t as the backup DC. If we assume that DC v is fortified (i.e., Wv=1), the objective function of M1 (for retailer i) is:

$$\Psi_i(v,t) = d_i s_{iv}+\lambda_{iv}+\lambda_{it}+(S_v(\frac{2c_v}{h_v})^{0.5}+(2c_v h_v)^{0.5})d_i^{0.5} \quad (17)$$

Consider $\Psi_i^*(v) = Min_t(\Psi_i(v,t))$ and Ei(v)=max $\{\emptyset_i^* - \Psi_i^*(v), 0\}$. Ei(v) is the improvement for retailer i if DC v is fortified. Thus, our objective is to spend the fortification budget in a way that maximizes Ei(v) for all v∈J, for all retailers i∈I. In this line, we define the variable Kij as follows:

$$K_{ij} = \begin{cases} 1 & \text{If a fortified DC j is considered as the primary server of retailer i} \\ 0 & \text{Otherwise} \end{cases}$$

By considering K_{ij}, we can propose the following model.

$$(M2): Max\sum_{i\in I}\sum_{j\in J} E_i(j)k_{ij} \quad (18)$$

Subject to

$$K_{ij}\le W_j \qquad \forall i\in I, j\in J \quad (19)$$
$$K_{ij}\le 1 \qquad \forall i\in I \quad (20)$$
$$\sum_{j\in J}(f_j+r_j q_j)W_j\le B \quad (21)$$
$$K_{ij}\in\{0,1\} \qquad \forall i\in I, j\in J \quad (22)$$
$$W_j\in\{0,1\} \qquad \forall j\in J \quad (23)$$

The objective function (M2) defined in (18) maximizes the total improvement from the fortification of DCs. To actualize improvement Ei (j), DC j as the primary server of retailer i must be fortified that is shown in constraint (19). Each retailer i must allocated to no more than one fortified server and is presented at constraint (20). The total

available budget to fortify DCs is presented in constraint (21). Kij and Wj are binary variables and are expressed by (22) and (23).
It is noticeable that lots of software like GAMS can quickly solve M2 and the difference between the optimal objective function values of M2 and M1 is the optimal value of RM.

5.1.2 Upper bound

In each LR iteration, a lower bound and an upper bound for M need to be provided. In the above section, the solution of RM is a lower bound, and it is also an upper bound if it is feasible. Then, this solution is the optimal solution of M. If the lower bound solution is infeasible, we obtain an upper bound by creating a feasible solution. For this reason, the following heuristic is proposed.
The solution of RM indicates exactly P distribution centers that must be opened. We choose the nearest opened DC as the primary and the second nearest opened DC as the backup server for each retailer. To indicate which DCs should be fortified, we show the set of retailers who have DC j as the primary server with g (j). If DC j is not fortified, the expected cost for each retailer i∈g(j) by considering r as its backup server is:

$$d_i s_{ij}(1-q_j) + d_i s_{ir} q_j + (s_j(\frac{2c_j}{h_j})^{0.5} + (2c_j h_j)^{0.5})(d_i(1-q_j))^{0.5} + q_j(s_r(\frac{2c_r}{h_r})^{0.5} + (2c_r h_r)^{0.5})d_i^{0.5} \tag{24}$$

If DC j is fortified, the expected cost is:

$$d_i s_{ij} + (s_j(\frac{2c_j}{h_j})^{0.5} + (2c_j h_j)^{0.5})d_i^{0.5} \tag{25}$$

Thus, by fortifying DC j, we have the total expected cost reduction:

$$\Phi_j = \sum_{i\in\Phi(j)} -d_i s_{ij} q_j + d_i s_{ir} q_j + (s_j(\frac{2c_j}{h_j})^{0.5} + (2c_j h_j)^{0.5})(d_i(1-q_j))^{0.5} + (q_j - 1)(s_j(\frac{2c_j}{h_j})^{0.5} + (2c_j h_j)^{0.5})d_i^{0.5} \tag{26}$$

So, we should solve the following model to maximize the fortification budget over P opened DCs. (M3):

$$Max\left\{\sum_{j\in J}\Phi_j W_j : \sum_{j\in J} c_j W_j \le B, \quad W_j \in \{0,1\}\right\} \tag{27}$$

The above model is a knapsack problem, and software solutions such as GAMS can solve it.

5.1.3. Multiplier choosing

The choice of initial multipliers can influence the performance of LR (Snyder, 2006). So, we examine the final multipliers of the situations that RM is solved optimality to get a good initial multiplier. For our problem, λ1ij=hi/||I|| generate efficient initial multipliers. We use the sub-gradient optimization proposed by Fisher (2004) to update λij as follows:
$\lambda_{ij}^{K+1} = \lambda_{ij}^{K} + t_k(X_{ij0} + X_{ij1} - Y_j)$,
tk is defined as the step size and we have:

$$t_k = \frac{\beta_k(Z^* - Z(\lambda^k))}{||X_{ij0} + X_{ij1} - Y_j||^2}$$

Where βk is a constant at iteration k. Like Fisher (2004), we set β_0 =2. When the multipliers are equal to λk, Z (λ^k) is the lower bound, and Z* is the best known upper bound.
The algorithm ends by meeting any of the following criteria:

- For a specified tolerance ε, we have (Z*-Z (λ^k))/ Z*≤ε.
- For some iteration limit kmax, we have K>kmax

The pseudo-code of the proposed algorithm is provided in Figure. 2.

{Input}

Define a feasible lower- bound (Z1) and a best bound (Z*)
Set k= 1, kmax, ε, λij= λ1ij=hi/||I|| (initial vector of Lagrangian multipliers)
{Initialization}

While (Z*-Z (λ^k))/ Z*>ε & K>kmax. **Do**

Sort the values of $((-\sum_{i\in I}\lambda_{ij}^{k}))$;
If $(-\sum_{i\in I}\lambda_{ij}^{k})$ belongs to the P smallest sorted values
Yj=1;
Else
Yj=0;
{Lower bound}
Set B=0;
For i=1 to I
Consider v and t as the primary and backup DC for retailer i and calculate $\emptyset_i(v,t)$ by using equation (16) ∀ v and t= 1, 2…J;
Compute $\emptyset_i^* = min_{v,t}\{\emptyset_i(v,t)\}$;
End
Set $M1 = \sum_{i=1}^{n} \emptyset_i^*$;
Set B≠0
For i=1 to I
Consider v and t as the primary and backup DC for retailer i and calculate $\Psi_i(v,t)$ in the situation that v is fortified by using equation (17) ∀ v and t= 1, 2…J;
Compute $\Psi_i^*(v) = Min_t(\Psi_i(v,t))$;
End
Calculate the improvement for the retailer i if DC v is fortified Ei(v)=max $\{\emptyset_i^* - \Psi_i^*(v), 0\}$;
Solve the sub problem M2 (equations 18-23) by Games software;
Set Lower bound= Z (λ^k) = M2-M1;
{Upper bound}
Choose the nearest opened DC as the primary and the second nearest opened DC as the backup server for each retailer;
Define the set of retailers who have DC j as the primary server with g (j);
If DC j is not fortified
Calculate the expected cost for retailer i with considering r as the backup server by using equation (24);
Else
Calculate expected cost for retailer i by using equation (25);
Compute the total expected cost reduction by fortifying DC j by using equation (26);
Update the upper bound by solving M3 using equation (27);
If upper bound< the best bound (Z*)
Z*= upper bound;
{Multiplier choosing}
Update
$\lambda_{ij}^{K+1} = \lambda_{ij}^{K} + t_k(X_{ij0} + X_{ij1} - Y_j)$;
$t_k = \frac{\beta_k(Z^* - Z(\lambda^k))}{||X_{ij0}+X_{ij1}-Y_j||^2}$;
End while

Figure. 2: The pseudo-code of the proposed algorithm

6. Numerical examples

We provide a numerical study to examine the proposed model and solution algorithm. Numerical examples are solved on one data set containing 88 nodes from Snyder and Daskin (2005). We use the Euclidean distance between the production plant and DC j and between retailer i and DC j as the transportation cost sj and sij, respectively. The holding cost (hj) is randomly generated from U ~(0, 50), and the ordering cost (cj) is equal to a constant value 5. We generate the probability of failure (qj) and the variable fortification cost (rj) randomly from U ~(0, 0.5) and U~(0, 3000), respectively. We change the values B between 0 and 450. The proposed algorithm is coded in matlab15 and run on a Windows 10 with a 2.5GHz Core i7 CPU and 8 GB of physical RAM. For solving M2 and the knapsack problems in the algorithm, we use GAMS. We adopt 300 and 0.5% as the maximum number of iterations and the gap tolerance, respectively.

Table 2: The result of numerical examples to teste the proposed LR

Number of nodes	P	B	LB*	UB**	Location of DCs to be opened	Location of DC to be fortify	Time (s)	Gap (in %)
5	2	0	868.3	872.21	1,3	-------	0.9	0.45
5	2	30	729.47	732.46	1,3	3	0.11	0.41
5	2	120	698	700.23	3,4	3,4	0.22	0.32
10	2	0	1424.13	1429.98	1,3	-------	0.99	0.41
10	2	60	1358.66	1365.27	2,4	4	0.86	0.48
10	2	120	1258.02	1262.36	3,5	3,5	1.20	0.34
15	5	0	2565.11	2577.45	1,3,7,9,11	-------	4.89	0.48
15	5	60	2498.79	2502	1,3,7,9,15	3	9.96	0.12
15	5	120	2427.45	2437.43	1,3,5,8,13	3,8	5.68	0.41
15	5	240	2291.18	2301.76	3,5,9,11,15	5,9,15	6.23	0.46
15	5	360	2026.97	2036.64	1,3,8,11,13	1,3,8,13	10.14	0.47
20	5	0	3742.76	3762.14	1,4,6,11,17	--------	40.63	0.50
20	5	60	3561.48	3572.9	1,4,7,13,17	7	50.29	0.32
20	5	120	3432.64	3449.95	1,4,8,11,18	8,11	70.45	0.50
20	5	240	3394.29	3401.53	1,4,8,11,18	8,11,18	100.02	0.21
20	5	360	3339.12	3346.47	1,4,10,13,17	4,10,13,17	123.78	0.22
25	7	0	4978.25	4994.32	1,3,8,11,13,17,20	---------	150.39	0.32
25	7	60	4896.33	4917.81	1,2,3,8,11,18,21	3	169.68	0.44
25	7	120	4785.45	4802.44	1,2,3,15,18,20,23	3,18	135.12	0.35
25	7	240	4639.62	4662.16	1,2,4,5,9,11,19	1,4,9,19	163.54	0.48
25	7	360	4599.8	4621.57	1,2,4,9,11,19,23	1,4,9,19,23	189.20	0.47
30	7	0	6852.80	6882.95	1,2,3,8,15,18,25	---------	210	0.44
30	7	60	6800.40	6810.33	1,2,3,8,11,17,23	3	279.14	0.14
30	7	120	6672.15	6695.92	1,2,3,8,9,15,19	3,8	263	0.36
30	7	240	6577.92	6607.51	1,2,3,8,11,15,25	1,2,3,8,11	228.27	0.45
30	7	360	6393.41	6425.26	1,2,4,5,6,11,17	1,2,4,5,6,11,17	309.49	0.49
40	10	0	7489.67	7506.29	1,4,3,7,11,16,23,27,32,38	----------	315.56	0.22
40	10	120	7346.32	7354.31	1,4,3,8,11,17,23,27,35,39	8, 23	395	0.11
40	10	240	7149.46	7172.9	1,2,3,8,13,19,25,27,35,39	1,2,8,19	420.61	0.33
40	10	360	7039.23	7064.62	1,2,3,8,11,17,25,27,32,39	1,2,8,17,27	487.12	0.36
40	10	450	6815.88	7837.24	1,2,4,8,11,17,19, ,27,35,39	1,2,4,8,17,27	533.84	0.27
50	11	0	8997.57	9042.24	1,2,4,8,17,25, 27,32,40,43,45	-----------	579.75	0.49
50	11	120	8863.26	8906.31	1,2,3,7,17,23, 25,32,40,43,45	2,3	620	0.48
50	11	240	8694.65	8711.76	1,3,4,7,19,25, 27,32,42,44,49	1,3,4,19,42	661.49	0.20
50	11	360	7984.93	8018.22	1,3,4,8,11,17,25,27,38,45,49	3,4,8,17,45	693.36	0.41
50	11	450	7541.33	7645.33	1,2,3,8,11,17,25,27,38,43,45	1,3,4,8,17,25,45	728.74	0.50
70	11	0	10631.44	10654.21	1,4,7,13,27,34,38,45,53,61,69	----------	994.22	0.21
70	11	120	10439.57	10487.84	3,4,7,9,13,25,34,39,43,64,70	4,43	1010.07	0.46
70	11	240	10231.92	10281.65	3,4,7,9,13,25,32,40,43,64,70	4,13,34,43	1264.67	0.48
70	11	360	10094.87	10125.79	3,4,8,9,17,25,32,40,44,64,69	4,9,25,40,64	1166.32	0.30
70	11	450	9976.07	9987.64	1,4,8,11,17,27,32,40,44,64,70	1,4,17,11,40,64	1329.19	0.11
88	12	0	13828.75	13894.32	1,4,9,11,23,27,39,43,65,72,79,83	-----------	1691.04	0.47
88	12	120	13634.83	13651.85	1,3,7,11,25,27,39,45,67,75,79,86	11,45	1706.33	0.13
88	12	240	13197.06	13246.44	1,4,9,17,25,27,40,43,65,74,79,86	4,17,25,43,65	1574.51	0.37
88	12	360	13007.53	13071.91	1,3,7,17,23,27,32,40,65,72,79,81	3,17,27,40,65	1852.68	0.49
88	12	450	12899	12946.67	3,4,11,19,25,27,32,43,66,72,75,86	3,4,19,25,43,72	1883.95	0.37

*Lower bound, **Upper bound

To show the performance of the proposed LR, we consider gap (in %) and CPU time (s). The gap (in %) is considered as the difference between the upper and lower bounds. The result of these measures is shown in Table 2. Results reveal that the proposed algorithm solved all of the examples to 0.50% optimality (not optimal). According to Table 2, it can be said that when the size of the problem increases, the CPU time enhances. This possibly confirms that by increasing the number of DCs and retailers, the computational effort to indicate the decision variables is more complex, and the algorithm needs more time to solve them.

The optimal solutions exhibit a decrease in the total expected cost by increasing the budget of fortification. The total cost reduction is higher than the used budget. Thus, managers can use supply chain fortification as an effective way to reduce supply chain costs.

6.1 Sensitivity analysis

In this section, the sensitivity analysis is done to gain a profound insight into the proposed model and algorithm. We vary the value of one parameter by +50% and -50% at a time and hold the other parameter values unchanged (Table 3). The sensitivity analysis is done by considering the number of nodes equal to 15, P=5, and B=120. We drive the following results based on Table 3.

Table 3: Sensitivity analysis with respect to the model's parameters

Parameter	% Change	Location of DCs to be opened	Location of DCs to be fortified	Upper bound	Lower bound
	+50	1, 3, 6, 11, 13, 15	3, 6, 13	2693.84	2634.62
h_i	0	1, 3, 5, 8, 13	3,8	2437.43	2427.45
	-50	1,3,8,12	3,6	2381.64	2315.11
	+50	3, 4, 8, 13	3,8	2567.92	2522. 18
c_j	0	1, 3, 5, 8, 13	3,8	2437.43	2427.45
	-50	1, 3, 5, 7,8,13	3,7,8	2256.55	2148.37
	+50	1, 3, 5, 7, 8, 12, 13	1, 3, 7, 8	2794.06	2700.45
s_{ji}	0	1, 3, 5, 8, 13	3,8	2437.43	2427.45
	-50	1,3,8, 13	1,3	2018.28	1996.46
	+50	1,3,8	3,8	2522.51	2493.86
s_j	0	1, 3, 5, 8, 13	3,8	2437.43	2427.45
	-50	1, 3, 5, 8, 9, 13	3, 8	2294.64	2200.44
	+50	1, 3, 5, 8, 13	3	2499	2454.17
f_j	0	1, 3, 5, 8, 13	3,8	2437.43	2427.45
	-50	1, 3, 5, 8, 13	3, 8, 13	2386.77	2341.14

- When the values of parameter hi increase, the number of opened DCs and costs will increase. This means that the inventory should be decreased in DCs in the situation that the holding cost is high.
- In the case in which the ordering cost (cj) shifts up, the number of opened DCs should be decreased to reduce the total cost. It is reasonable that by increasing the ordering cost, fewer DCs are established, and more inventory is kept in them to meet the retailers' demand.
- The results show that if the cost of transporting from DCs to retailers (sji) increases, managers must create more DCs. In this case, the number of fortified DCs must also be increased to reduce the displacements.
- Findings reveal that an increase in the transportation cost from the production plant to DCs leads to a decrease in the number of opened DCs. This is expected because this procedure reduces the shipping cost and, therefore, the total cost.
- As shown in Table 3, the number of fortified DCs is relatively insensitive to changes in fortification fixed coefficient.

7. Conclusion

This paper provided an inventory-location model for a three-level supply chain by considering the disruption risk of distribution centers. We assumed that distribution centers are heterogeneous with independent failure probabilities. Each retailer was allocated
to two distribution centers, one as a primary server and another as a backup server. Also, we considered the fortification budget to increase the supply chain reliability. When a distribution center was fortified, it became totally reliable. The model indicated the location of distribution centers, distribution centers that must be fortified, the allocation of retailers to distribution centers and inventory policy at distribution centers. The proposed model was a nonlinear integer programming model that we proved it is NP-hard. Thus, a Lagrangian relaxation algorithm was developed to solve it. Our findings revealed that the proposed algorithm is computationally efficient. The average time and gap to identify optimal bounds was 487.67 s and 0.36%, respectively.
Furthermore, numerical examples showed that the utilization of fortification budget can reduce the cost of supply chain and it is beneficial for firm that using fortification budget to increase the reliability. Results highlighted that the number of opened distribution centers would be increased by increasing the inventory cost. When the cost of ordering and transportation from production plant to distribution centers are high, it is better to decrease the number of opened distribution centers. A higher transportation cost from distribution centers to retailers leads to an increase in the number of opened DCs.
We recommend the followings issues for future researches. The proposed model can become more realistic by expanding it to a case with considering routing problem. We suppose that the demand of retailers is certain; however, it can be considered uncertain. Another way to increase the reliability of distribution network is placing safety stock at distribution centers, this research can be developed by considering this issue.

Acknowledgment
We would like to thank the supports from Islamic Azad University, Kermanshah Branch.

References

Afify, B., Ray, S., Soeanu, A., Awasthi, A., Debbabi, M., & Allouche, M. (2019). Evolutionary learning algorithm for reliable facility location under disruption. Expert Systems with Applications, 115, 223-244.

Amiri-Aref, M., Klibi, W., & Babai, M. Z. (2018). The multi-sourcing location inventory problem with stochastic demand. European Journal of Operational Research, 266(1), 72-87.

Azizi, N. (2019). Managing facility disruption in hub-and-spoke networks: formulations and efficient solution methods. Annals of Operations Research, 272(1), 159-185.

Bagherinejad, J., & Najafi-Ghobadi, S. (2019). Two Meta-heuristic Algorithms for a Capacitated Inventory-location Problem in Multi-echelon Supply Chain. International Journal of Supply and Operations Management, 6(4), 334-348.

Beltrán-Royo, C., Vial, J. P., & Alonso-Ayuso, A. (2012). Semi-Lagrangian relaxation applied to the uncapacitated facility location problem. Computational Optimization and Applications, 51(1), 387-409.

Berman, O., Drezner, T., Drezner, Z., & Wesolowsky, G. O. (2009). A defensive maximal covering problem on a network", International Transactions in Operational Research, 16(1), 69-86.

Berman, O., Krass, D., & Menezes, M. B. (2007). Facility reliability issues in network p-median problems: Strategic centralization and co-location effects. Operations research, 55(2), 332-350.

Carpenter, C. (2010). Copper advances as Chile earthquake cuts power, halts mines. Business Week.

Chen, Q., Li, X., & Ouyang, Y. (2011). Joint inventory-location problem under the risk of probabilistic facility disruptions. Transportation Research Part B: Methodological, 45(7), 991-1003.

Christopher, M., & Peck, H. (2004). Building the resilient supply chain.

Church, R. L., & Scaparra, M. P. (2007). Protecting critical assets: the r-interdiction median problem with fortification. Geographical Analysis, 39(2), 129-146.

Cooper, L. (1963). Location-allocation problems. Operations research, 11(3), 331-343.

Cui, T., Ouyang, Y., & Shen, Z. J. M. (2010). Reliable facility location design under the risk of disruptions. Operations research, 58(4-part-1), 998-1011.

Cui, T., Ouyang, Y., & Shen, Z. J. M. (2010). Reliable facility location design under the risk of disruptions. Operations research, 58(4-part-1), 998-1011.

Cundari, S., Czerwinski, J. J., Kitfield, J., Mason, D., & Sands, C. (2002). The US-Canada Strategic Partnership in the War on Terrorism. Center for the Study of the Presidency.

Daskin, M. S., Coullard, C. R., & Shen, Z. J. M. (2002). An inventory-location model: Formulation, solution algorithm and computational results. Annals of operations research, 110(1), 83-106.

Diabat, A., Battaïa, O., & Nazzal, D. (2015). An improved Lagrangian relaxation-based heuristic for a joint location-inventory problem. Computers & Operations Research, 61, 170-178.

Diabat, A., Jabbarzadeh, A., & Khosrojerdi, A. (2019). A perishable product supply chain network design problem with reliability and disruption considerations. International Journal of Production Economics, 212, 125-138.

Diabat, A., Richard, J. P., & Codrington, C. W. (2013). A Lagrangian relaxation approach to simultaneous strategic and tactical planning in supply chain design. Annals of Operations Research, 203(1), 55-80.

Drezner, Z. (1987). Heuristic solution methods for two location problems with unreliable facilities. Journal of the Operational Research Society, 38(6), 509-514.

Eskandari-Khanghahi, M., Tavakkoli-Moghaddam, R., Taleizadeh, A. A., & Amin, S. H. (2018). Designing and optimizing a sustainable supply chain network for a blood platelet bank under uncertainty. Engineering Applications of Artificial Intelligence, 71, 236-250.

Fisher, M. L. (1981). The Lagrangian relaxation method for solving integer programming problems. Management science, 27(1), 1-18.

Ghaderi, A., & Burdett, R. L. (2019). An integrated location and routing approach for transporting hazardous materials in a bimodal transportation network. Transportation research part E: logistics and transportation review, 127, 49-65.

Haghjoo, N., Tavakkoli-Moghaddam, R., Shahmoradi-Moghadam, H., & Rahimi, Y. (2020). Reliable blood supply chain network design with facility disruption: A real-world application. Engineering Applications of Artificial Intelligence, 90, 103493.

Haraguchi, M., & Lall, U. (2015). Flood risks and impacts: A case study of Thailand's floods in 2011 and research questions for supply chain decision making. International Journal of Disaster Risk Reduction, 14, 256-272.

Held, M., & Karp, R. M. (1970). The traveling-salesman problem and minimum spanning trees. Operations Research, 18(6), 1138-1162.

Held, M., & Karp, R. M. (1970). The traveling-salesman problem and minimum spanning trees. Operations Research, 18(6), 1138-1162.

Jabbarzadeh, A., Fahimnia, B., Sheu, J. B., & Moghadam, H. S. (2016). Designing a supply chain resilient to major disruptions and supply/demand interruptions. Transportation Research Part B: Methodological, 94, 121-149.

Jabbarzadeh, A., Haughton, M., & Khosrojerdi, A. (2018). Closed-loop supply chain network design under disruption risks: A robust approach with real world application. Computers & industrial engineering, 116, 178-191.

Khorshidvand, B., Soleimani, H., Seyyed Esfahani, M. M., & Sibdari, S. (2021). Sustainable closed-loop supply chain network: Mathematical modeling and Lagrangian relaxation. Journal of Industrial Engineering and Management Studies, 8(1), 240-260.

Khorshidvand, B., Soleimani, H., Sibdari, S., & Esfahani, M. M. S. (2021). Developing a two-stage model for a sustainable closed-loop supply chain with pricing and advertising decisions. Journal of Cleaner Production, 309, 127165.

Khorshidvand, B., Soleimani, H., Sibdari, S., & Esfahani, M. M. S. (2021). A hybrid modeling approach for green and sustainable closed-loop supply chain considering price, advertisement and uncertain demands. Computers & Industrial Engineering, 157, 107326.

Koncius J. (2003). SARS hits the furniture industry; China is crucial to the U.S. Market. The Washington Post; May 29.

Koncius J. (2003). SARS scope extends to 'Made in China' furniture industry /http:// articles.sfgate.com/2003-06-04/homeand-garden/17494564_1_furniture-experts-high-end-furniture-sars.

Lee, Y., Lim, B. H., & Park, J. S. (1996). A hub location problem in designing digital data service networks: Lagrangian relaxation approach. Location Science, 4(3), 185-194.

Kotak R (2005). Oil prices drift lower as demand wanes after-Katrina/http://www.forbes.com/feeds/afx/2005/09/12/afx2219126.html/S.

Li, Q., Zeng, B., & Savachkin, A. (2013). Reliable facility location design under disruptions. Computers & Operations Research, 40(4), 901-909.

Liberatore, F., Scaparra, M. P., & Daskin, M. S. (2011). Analysis of facility protection strategies against an uncertain number of attacks: The stochastic R-interdiction median problem with fortification. Computers & Operations Research, 38(1), 357-366.

Lim, M., Daskin, M. S., Bassamboo, A., & Chopra, S. (2010). A facility reliability problem: Formulation, properties, and algorithm. Naval Research Logistics (NRL), 57(1), 58-70.

Liu, Y., Dehghani, E., Jabalameli, M. S., Diabat, A., & Lu, C. C. (2020). A coordinated location-inventory problem with supply disruptions: A two-phase queuing theory–optimization model approach. Computers & industrial engineering, 142, 106326.

Marquis C (2001). Meat from Europe is banned by U.S. as illness spreads. The New York Times; March 14.

Miranda, P. A., & Garrido, R. A. (2008). Valid inequalities for Lagrangian relaxation in an inventory location problem with stochastic capacity. Transportation Research Part E: Logistics and Transportation Review, 44(1), 47-65.

Mouawad J, Romero S. (2005). Hurricane Katrina: the oil supply; gas prices surge as supply drops. The New York Times; September 1.

Nezhad, A. M., Manzour, H., & Salhi, S. (2013). Lagrangian relaxation heuristics for the uncapacitated single-source multi-product facility location problem. International Journal of Production Economics, 145(2), 713-723.

Park, Y., Hong, P., & Roh, J. J. (2013). Supply chain lessons from the catastrophic natural disaster in Japan. Business horizons, 56(1), 75-85.

Peng, P., Snyder, L. V., Lim, A., & Liu, Z. (2011). Reliable logistics networks design with facility disruptions. Transportation Research Part B: Methodological, 45(8), 1190-1211.

Qi, L., Shen, Z. J. M., & Snyder, L. V. (2010). The effect of supply disruptions on supply chain design decisions. Transportation Science, 44(2), 274-289.

Rayat, F., Musavi, M., & Bozorgi-Amiri, A. (2017). Bi-objective reliable location-inventory-routing problem with partial backordering under disruption risks: A modified AMOSA approach. Applied Soft Computing, 59, 622-643.

Reuters (2001). UK's foot-and-mouth crisis deepens /http://tvnz.co.nz/view/ tvnz_smartphone_story_skin/30727/S.

Scaparra, M. P., & Church, R. L. (2008). A bilevel mixed-integer program for critical infrastructure protection planning. Computers & Operations Research, 35(6), 1905-1923.

Sheffi, Y., & Rice Jr, J. B. (2005). A supply chain view of the resilient enterprise. MIT Sloan management review, 47(1), 41.

Sheffi, Y. (2001). Supply chain management under the threat of international terrorism. The International Journal of logistics management, 12(2), 1-11.

Sheffi, Y., & Rice Jr, J. B. (2005). A supply chain view of the resilient enterprise. MIT Sloan management review, 47(1), 41.

Shen, H., Liang, Y., & Shen, Z. J. M. (2021). Reliable hub location model for air transportation networks under random disruptions. Manufacturing & Service Operations Management, 23(2), 388-406.

Snyder, L. V. (2006). Facility location under uncertainty: a review. IIE transactions, 38(7), 547-564.

Snyder, L. V., & Daskin, M. S. (2005). Reliability models for facility location: the expected failure cost case. Transportation science, 39(3), 400-416.

Stecke, K. E., & Kumar, S. (2009). Sources of supply chain disruptions, factors that breed vulnerability, and mitigating strategies. Journal of Marketing Channels, 16(3), 193-226.

Strahan A, Smith G. (2005) Gulf oil, Gas output to take 3 months to recover 90%. Bloomberg; September 16.

Wilson, M. C. (2007). The impact of transportation disruptions on supply chain performance. Transportation Research Part E: Logistics and Transportation Review, 43(4), 295-320.

Wu, W., Zhou, W., Lin, Y., Xie, Y., & Jin, W. (2021). A hybrid metaheuristic algorithm for location inventory routing problem with time windows and fuel consumption. Expert systems with applications, 166, 114034.

Yahyaei, M., & Bozorgi-Amiri, A. (2019). Robust reliable humanitarian relief network design: an integration of shelter and supply facility location. Annals of Operations Research, 283(1), 897-916.

Yun, L., Wang, X., Fan, H., & Li, X. (2017). A reliable facility location design model with site-dependent disruption in the imperfect information context. Plos one, 12(5), e0177104.

Zhang, Y., Snyder, L. V., Ralphs, T. K., & Xue, Z. (2016). The competitive facility location problem under disruption risks. Transportation Research Part E: Logistics and Transportation Review, 93, 453-473.

This article can be cited: Najafi-Ghobadi, S., Sherafati, M., (2022). Providing a Lagrangian Relaxation algorithm to solve a reliable location- inventory model by considering disruption. Journal of Industrial Engineering and Management Studies, Vol.9, No. 5, pp. 49-62.