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A revised model for solving the Cell formation problem and solving by gray wolf optimization algorithm

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Abstract

The Cellular Manufacturing System (CMS) is one of the most efficient systems for production environments with high volume and product variety which takes advantage of group technology. In the cellular production system, similar parts called part families are assigned to a production cell having similar production methods, and the needed machines are dedicated to cells. Determining part families and allocating the necessary machines to the production cell is known as the Cell Formation Problem (CFP) which is known as an NP-Hard problem. Safaei and Tavakkoli-Moghaddam (2009a) proposed a model that is widely used in literature which suffers some killer weaknesses highly affecting subsequent researches. In this paper, the mentioned model is modified and revised to fix these major issues. Besides, due to the NP-Hard nature of the problem, a meta-heuristic algorithm based on Gray Wolf Optimization (GWO) approach is also developed for solving the revised model on the sample examples and the results are compared. Simulation results indicated that the proposed method can reduce the total cost of the manufacturing system by 3% in comparison with the base model. Furthermore, simulation results of five sample problems indicate the better performance of the proposed method comparing with Lingo and PSO.

Keywords: cellular manufacturing systems; cell formation problem; gray wolf optimization algorithm; PSO.

Paper Type: Original Research

1. Introduction

Production planning is regarded as one of the significant operations in each production unit that may be carried out formally or informally. The purpose of production planning is the optimal use of factory resources. In recent years, due to intense competitions on providing new products and the wide and diverse tastes and demands of consumers, producers are turning to production systems that are more efficient, more economical, and less time-consuming; in this way, they can meet the demands of the consumers and the market. The cellular production system is considered to be one of the production systems. This system provides an opportunity for the organizations so that they can be transformed from the traditional and operation-oriented mode to a product-oriented mode. In this system, products are sorted and categorized according to the similarities they may have in their production process or in their external form. Machinery and system resources are located in separate sections based on the needs and the roles they play in production operations. Each group or classification is referred to as a part family and production shops are known as cells. The success of such a production system depends on the following: proper division of cells and the machines inside them, part family, and the precise planning of their tasks. A planning and control system should maintain complete coordination among cells, production sections, material transportation, and production volume.

Traditional production systems are categorized into the following two groups in terms of equipment arrangement and the nature of planning and control tasks: flow systems and workshop systems. Flow systems are apt for the mass-production of a limited number of products and the workshop system is appropriate for producing diverse and various productions in small numbers. The cellular production system is a relatively new system for filling the gap and the working space between traditional flow systems and workshop systems, i.e., where the variation and demand for products are average.

In this paper, the cell formation problem is considered and the base mathematical model proposed by Safaei and Tavakkoli-Moghaddam (2009a) is deeply studied. The investigation of the model shows the lack of some critical constraints and reporting incorrect results which none of the previous researches has noticed before. It means that the gap in the literature was wrongly covered and treatment for filling the gap is required. So, this paper aims to refine the model and prevent the propagation of the mistake. At the first phase of this research, the base model is revised, modified, and completed, and at the second phase, a meta-heuristic algorithm based on the GWO approach is developed for solving the model and it is simulated in Matlab over the same test cases using a PC with

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4.2 GHz processor and running Microsoft Windows 10. The simulation results indicate that the objective function is reduced by %3, and better cell formation is achieved. The proposed model is also solved by Lingo and implemented using Particle Swarm Optimization (PSO) approach. The simulation results revealed the superiority of the proposed GWO obtaining a better objective function value.
The rest of the paper is organized as follows: the literature review and related work are discussed in section 2; the mathematical model is described in section 3; the proposed GWO algorithm is stated in section 4; findings and simulation results are revealed in section 5, and the conclusions are stated in section 6.

2. Literature review and related work

Several research studies have been conducted on cellular production and its relevant issues and problems. Designing cellular production systems encompasses a wide research domain. The majority of conducted studies in this area deal with cell formation procedures and issues. Effective factors and parameters on cell formation such as production planning of static and dynamic environments have led to the application of various methods for this stage of cell formation. Hence, several studies have been done on this issue. Identifying parts and allocating the required machinery to the production cell is considered as an NP-Hard problem (Ballakur and Steudel, 1987), hence, researchers have used various meta-heuristic methods for achieving optimal solutions. Some of these methods are discussed below.
Ahi et al. (2009) considered and investigated the concept of multi-criteria decision-making; also, they proposed a two-stage method for the arrangement of cells, intra-cell layout of machines, and setting of cells as three fundamental specifications in designing CMS. Kioon et al. (2009) formulated a hybrid method in designing CMS as a mixed non-linear integer model along with production planning and system reconfiguration decisions which are based on alternative process routing, operation sequence, number of machines, machine capacity, and inter-cell movement of parts. Bajestani et al. (2009) proposed a mixed non-linear integer programming for the problem of dynamic cell arrangement based on machine flexibility, alternative operation scheme, and machine movement. The multi-objective purposes of the model were to reduce the total workload of the cells, reduce the costs of the machine investment, reduce inter-cell material movements and reduce machinery movement at the same time. By capitalizing on the similarities among production parts, Noktehdan et al. (2009) grouped parts into part families and allocated machines within cells. By doing so, they intended to minimize intracellular movements that were inspired by Genetic Algorithm grouping.
Safaei and Tavakkoli-Moghaddam (2009b) proposed a multi-period cellular arrangement and production planning in a dynamic cellular production system. They aimed to reduce the cost of the machinery, reduce inter-cellular and intra-cellular movements, and reduce reconfiguration and subcontracting, and inventory maintenance. They investigated the impacts of production costs and outsourcing costs on cell reconfiguration. Kioon et al. (2009) carried out a study on the comprehensive designing of a cellular production system in a dynamic manner. By capitalizing on an integrated planning horizon, they focused on issues such as system rearrangement under dynamic conditions, alternative routes, and production planning. Their objective function was to minimize intra-cellular and inter-cellular transportation costs, material maintenance, internal production, re-arrangement, and variable and overhead costs of machines. They proposed a mixed integer programming model to achieve these objectives. Deljoo et al. (2010) optimized the shortcomings of the previously proposed models. Then, they demonstrated that Lingo software cannot solve large-sized problems within a logical and acceptable time. However, the Genetic Algorithm is capable of solving large-scale problems within an acceptable time in comparison with Lingo.
Mahdavi et al. (2010) proposed an integer mathematical planning model for designing cellular production systems considering the following factors: multi-period production planning, dynamic system configuration, machine capacity, available time, and the allocation of workers. The rationale behind this model was to reduce the costs of cellular production systems. For solving the proposed model regarding large-sized problems, a heuristic approach was proposed. Aghajani et al. (2010) proposed one of the most exhaustive models with an integrated approach with respect to a content system for designing CMS. This model considers the following factors: dynamic cellular structure, alternative process route, part capability, the sequence of operations, machine capacity, operation costs, setup costs, cell size, and production planning. This model was designed in such a way that it selects the best route instead of the predetermined routes by the user.
Rafiee et al. (2011) developed a comprehensive mathematical model for cell formation issues. This model was aimed at reducing cell formation costs such as production-related costs under dynamic conditions, costs of machinery capacity limitation by considering operation sequence, cell size limitation, and machinery breakdown. Next, the PSO algorithm was used for solving the proposed model and the obtained results were discussed. Rafiee and Godsi (2012) proposed a comprehensive mathematical model concerning cell formation issues under dynamic conditions. In this model, the first objective function was concerned with reducing the costs of machine investment, locating, variable machinery costs, intra/inter-cellular movement costs, overtime costs, and the cost of labor force movement between cells. On the other hand, the second objective function was concerned with enhancing labor productivity and efficiency. Then, the ACO meta-heuristic algorithm was proposed for solving the developed model.

Solimanpur et al. (2010) developed a meta-heuristic algorithm based on Ant Colony Optimization (ACO) method for solving the CFP to minimize inter-cellular movements and voids. Kia et al. (2012) proposed a mixed-integer non-linear programming model for designing the Dynamic Cellular Manufacturing System (DCMS). The purpose was to minimize intracellular and extra-cellular costs and the costs of buying machinery. Kiaa et al. (2016) developed a comprehensive mathematical model for dynamic cell formation where inter-cellular movement costs and the costs of moving in different factory floors were taken into consideration and solved the model using a Genetic Algorithm. Saeedi et al. (2010) used ACO, GA, and Simulated Annealing (SA) methods for solving the CFP and reported that the GA-based algorithm is more efficient in solving the proposed model. A comprehensive multi-objective mathematical model for solving the CFP is developed by Saeedi et al. (2014). The proposed model considered machine redundancy, operation sequence, and alternative process routes. The objective function included four different objectives that were aggregated using the fuzzy goal programming approach. The authors developed a GA-based meta-heuristic algorithm for solving the model and compared the results with the NSGA-II approach (Saeedi et al., 2014).
Hafezalkotob et al. (2015) proposed a hybrid meta-heuristic named DPSO-SA for solving the CFP. Doulabi et al. (2009) developed a two-phase algorithm for solving the CFM based on heuristic methods. The machine cells are created in the first phase, and the part-families are assigned in the second phase. Kheirkhah and Ghajari (2018) proposed a nonlinear mathematical programming model for solving the CFP and developed a three-phase algorithm. Hamza and Jahed (2019) used the hamming distance concept for solving the CFP. Rajesh et al. (2018) proposed a new approach for solving this problem considering the voids of the cells based on the rank order clustering method.
Karim and Biswas (2015) performed a literature review on cellular manufacturing methods and categorized existing approaches into four different groups. Dmytryshyn et al. (2018) proposed a novel approach based on progressive modeling for solving the CFP and solved a benchmark problem and demonstrated the efficiency of their method. Zhu and Li (2018) developed an algorithm for solving the CFP using similarity coefficients assessment.
Almonacid (2019) used a global search method for analyzing the solutions in a three-cell dataset. Ayough and Khorshidvand (2019) proposed a new model for solving the CFP. The authors consider a dynamic situation and demand uncertainty. They concluded that the PSO was more successful in obtaining the final solution than the Simulated Annealing (SA) approach. Dehnavi et al. (2020) proposed a bi-objective mixed-integer programming model for solving the cell formation problem considering the transferring of the semi-manufactured parts between the machines by automated vehicles.
Nagaraj et al. (2020) developed meta-heuristic algorithms based on butterfly and firefly approaches for enhancing cell formation. Golmohammadi et al. (2020) proposed a bi-objective mathematical model under fuzzy conditions for solving the cell formation problem.
The review of the literature and considering the previous work referenced by authors, it can be concluded that none of the previous researches has distinguished the missing constraints and weak points of the based model widely referenced in most papers. Besides, the GWO approach has not been used for solving the cell formation problem before.

3. The mathematical model

The most significant features of the problems of dynamic cellular production systems are as follows: variable demands, multiple paths, machine costs, tool consumption costs, material transportation costs, machine iteration, production planning costs, and the costs of controlling the degree of outsourcing parts. Here, by expanding the developed model by Safaei and Tavakkoli-Moghaddam (2009a), we proposed a non-linear integer model for the problem of integrating the production planning and dynamic cell formation. In this model, cost function consists of machinery-related costs (variable and fixed), transportation costs (intracellular and intercellular costs), production planning costs (inventory maintenance, the cost of part shortage, and part outsourcing), and re-deployment costs (machinery setup).

3.1. Model assumptions

In this study, besides the common hypotheses of classic cellular production models, the following assumptions were investigated and examined:

1. The demand for each part in each period is variable and known.
2. Each part requires several operations for being processes on different machines.
3. The fixed costs of a machine include overhead maintenance costs and rental costs.
4. Variable costs of a machine depend on the workload allocated to that machine.
5. Each machine can perform one or more operations without additional costs (machine flexibility).
6. Fixed construction and cell formation costs (for setting up cells) are known at the beginning of each period for each cell.

7. The costs of inventory maintenance, shortage, and outsourcing are known at the beginning of periods for all parts regarding different cells.
8. Operation times regarding all operations of different parts on different machines are known.

3.2. The Objective functions

The purpose of the model is to reduce different costs which are listed below:

1. Machine investment costs: it refers to the costs of purchasing a new machine in each period.
2. Operational costs: operation cost denotes the cost required for producing parts which depends on the machine type and the processing time needed for each operation.
3. Inter-cellular and intra-cellular movement cost: this cost is applied when all the operations of a part on a machine are not completed the part is transmitted on another machine in the current cell or in another cell.
4. Production planning cost: consists of the inventory maintenance cost, shortage costs, and the cost of out-sourcing parts.
5. The cost of moving machines: it refers to the cost of changing the location of a machine from one cell to another cell. During different periods, machines may have to be moved and replaced

3.3. Indexes

p: the index of parts (p=1,2, …, P)
j: the index for the operation (j=1, 2, …, OP)
m: the index of the machines (m=1,2, …, M)
c: the index of the cells (c=1,2, …, C)
h: the index of the periods (h=1, 2, …, H)

3.4. Parameters

P: total number of different parts.
C: the maximum number of cells.
M: total number of different machines.
H: total number of periods.
Dph: the demand for the part p at the h period.
Bintra: the size of the intra-cellular production batch for part p.
Binter: the size of the inter-cellular production batch for part p.
γintra: intra-cellular movement cost for each production batch.
γinter: inter-cellular movement cost for each production batch.
αm: fixed cost of machine type m in each period.
βm: variable cost of machine type m for each time unit.
Tm: time capacity of the machine type m in each period in terms of the hour.
ajpm: if the j operation of the part p can be performed on machine type m, it will be equal to one; otherwise, it is zero.
Tjpm: process time for the j operation of part p on machine type m.
λp: the unit cost of outsourcing of part p.
ηp: inventory maintenance unit cost of part p in each period.
ρp: the unit cost of tardy orders of part p in each period.
Hm: the cost of installing the machine type m in a cell.
Rm: the cost of taking or removing the machine type m from a cell.
Uc: the upper bound of the cell.
M: A big positive number.

3.5. Decision variables

Nmch: the number of m-type machines allocated to cell c at the period h.
K+mch: the number of m-type machines added to the cell c at the period h.
K-mch: the number of m-type machines deducted from cell c at the period h.
Qph: the demands for the p part regarding internal production in period h.
Sph: the demands for the p part for outsourcing in period h.
Iph: net inventory level (I+ph the remaining inventory for part p at the end of period h; I-ph the shortage of part p)
Xjpmch: if the operation j regarding the part p is carried out on m machine in the cell c at the period h, it will be equal to one; otherwise, it will be equal to zero.
Yph: if Qph≥0, it will be equal to one; otherwise, it will be equal to zero.
Y'ph: if I+ph≥0, it will be equal to one; if Iph->0, it will be zero.

3.6. The mathematical model

$$\mathrm{MIN}\ Z=\sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}\alpha_m N_{mch} \tag{1}$$

$$+\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{OP}\sum_{m=1}^{M}\beta_m Q_{ph} t_{jpm} x_{jpmch} \tag{2}$$

$$+\frac{1}{2}\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{O_{P-1}}\left[\frac{Q_{ph}}{B_p^{inter}}\right]\gamma^{inter}\times\left|\sum_{m=1}^{M}x_{(j+1)pmch}-\sum_{m=1}^{M}x_{jpmch}\right| \tag{3}$$

$$+\frac{1}{2}\sum_{h=1}^{H}\sum_{c=1}^{C}\sum_{p=1}^{P}\sum_{j=1}^{O_{P-1}}\left[\frac{Q_{ph}}{B_p^{intra}}\right]\gamma^{intra}\left(\sum_{m=1}^{M}\left|X_{(j+1)pmch}-X_{jpmch}\right| - \left|\sum_{m=1}^{M}x_{(j+1)pmch}\sum_{m=1}^{M}x_{jpmch}\right|\right) \tag{4}$$

$$+\sum_{h=1}^{H}\sum_{p=1}^{P}\left(\eta_p I_{ph}^{+}+\rho_p I_{ph}^{+}+\lambda_p S_{ph}\right) \tag{5}$$

$$+\sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{c=1}^{C}\left(H_m K_{mch}^{+}+R_m K_{mch}^{-}\right) \tag{6}$$

Subject to:

$$\sum_{c=1}^{C}\sum_{m=1}^{M}a_{jpm}x_{jpmch}=y_{ph} \qquad \forall\, j,p,h \tag{7}$$

$$\sum_{p=1}^{P}\sum_{j=1}^{OP}Q_{ph}t_{jpm}\le T_m N_{mch} \qquad \forall\, m,c,h \tag{8}$$

$$x_{jpmch}\le a_{jpm} \qquad \forall\, j,p,h,m,c \tag{9}$$

$$\sum_{m=1}^{M}N_{mch}\le U_c \qquad \forall\, h,c \tag{10}$$

$$N_{mch}=N_{mc(h-1)}+K_{mch}^{+}-K_{mch}^{-} \qquad \forall m,c,h \tag{11}$$

$$I_{ph}=I_{p(h-1)}+Q_{ph}+S_{p(h-1)}-D_{ph} \qquad \forall\, p,h \tag{12}$$

$$I_{ph}=I_{ph}^{+}-I_{ph}^{-}=0 \qquad \forall\, p \tag{13}$$

$$Q_{ph}\le MY_{ph}\,,Q_{ph}\ge Y_{ph} \qquad \forall\, p,h \tag{14}$$

$$I_{ph}^{+}\le M*Y_{ph}^{'}\,,I_{ph}^{-}\le M\left(1-Y_{ph}^{'}\right) \qquad \forall p,h \tag{15}$$

$$x_{jpmch}, Y_{ph}, Y'_{ph} \in \{0,1\},$$
$$N_{mch}, K^{+}_{mch}, K^{-}_{mch}, Q_{ph}, I^{+}_{ph}, I^{-}_{ph}, S_{ph} \geq 0 \text{ , Integer} \tag{16}$$

In this model, the objective function minimizes the sum of different cost clauses. The expressions (1) and (2) of the equation denote the total of fixed and variable machine costs. The fixed cost enforces the model to maximize the utilization of the machines. The variable cost includes the total of the workload allocated to each machine in each cell and also balances the workload. Expression (3) is related to the cost of inter-cellular movements; it indicates that if two consecutive operations are done for a part in two separate cells, the intercellular movement is required. Expression (4) indicates that two consecutive operations will need inter-cellular movements if both operations are done within a cell but on different machines. It should be noted that since each part movement transmission unit is done twice, a 1.2 coefficient is applied. Expression (5) is related to the total production planning costs which include inventory maintenance costs, shortage costs, and part outsourcing costs. Expression (6) refers to redeployment cost which is obtained from the total machine installation costs added to one period and the cost of removing deducted machines from a period.

3.7. The constraints

Equations (7) and (9) ensure that if a fraction of a demand for a part is to be produced at a specific period, each operation is allocated to one machine and one cell. Expression (8) guarantees that the demand is supplied and machine capacity is not exceeded. Expression (10) indicates the upper limit of cell size. Limitation (11) denotes the number of machines in all periods. Expression (12) refers to inventory balance during the period; it indicates that the inventory level (additional inventory or inventory shortage) of each part at the end of each period is equal to the inventory level of the part at the end of the previous period as well as the number of outsourced parts in the previous period minus part demand rate at the current period.
Expression (13) indicates that the net inventory level at the final period should be zero and the demand for all parts should be supplied and fulfilled throughout the planning horizon. According to expression (14), the value of variable Sph is declared and the upper and the lower outsourcing limits are defined as demand coefficients. Constraint (14) is a complement of equation (7) which indicates that if the required operation for manufacturing parts is performable in equation (8), a fraction of its demand can be internally produced at a specific period. Equation (15) reveals that inventory maintenance and inventory shortage cannot occur simultaneously. That is, they are two dependent variables. Finally, expressions in (16) determine the type and range of decision variables.

3.8 The major problems and weaknesses of the base model

The base model proposed by Safaei and Tavakkoli-Moghaddam (2009a) is studied and some examples are implemented both in Lingo and GAMS software, but the obtained execution results were completely unexpected. A precise assessment of the values obtained for decision variables is also performed in Excel and confirmed the Lingo and GAMS output. The investigation and deep analysis of the model revealed two major problems of the base model proposed in the mentioned reference which are distinguished and fixed in this paper as follows:

1) There is no guarantee to perform all necessary operations for manufacturing a specific part in every period. That is, some operations of producing a part do NOT take place in production planning and the base model suffers the lack of such a constraint. So, the following constraint should be added to this model to ensure that none of the part production operations will be ignored.

$$\sum_{c=1}^{C} \sum_{j=1}^{OP} \sum_{m=1}^{M} x_{jpmch} = OP_p \qquad \forall\, p, h \tag{17}$$

Where OPp is the number of operations that should be performed on the part p.

2) In all of the sample test cases, the model prefers to produce most of the parts by outsourcing method; rather than internal producing, which leads to increasing the idle time of the machines, the number of voids, and extra machine investment costs. To solve this issue, the following constraints as the lower and upper bound of outsourcing amount are also added to the base model to avoid excessive and unnecessary outsourcing of the parts:

$$S_{ph} \geq L_o * D_{ph} \qquad \forall\, p, h \tag{18}$$

$$S_{ph} \leq U_o * D_{ph} \qquad \forall\, p, h \tag{19}$$

Where L0 and U0 are the lower and upper bound coefficients of outsourcing which are considered equal to 0.5 and 2 in the proposed model consequently. In a real environment, these values along with other parameters are set according to the managerial insight.

In this way, the weak points and disadvantages of the base model are covered and fixed in this paper. In the next section, the proposed GWO-based algorithm for solving the revised model is described in detail.

4. The GWO algorithm

The GWO algorithm was first introduced by Mirjalili and Mirjalili (2014) is a population-based algorithm inspired by the social behavior of gray wolves. Gray wolves live in groups of populations 5 through 12 and are divided into four types: α, β, δ, and ω wolves hierarchically. The α wolf is the single leader of the group and responsible for the decision about hunting, resting place, etc. At the second level of the hierarchy, the β wolves, are the followers of α wolf and help him in decision making. The δ wolves are at the lowest level of the hierarchy and should obey other dominant wolves. The rest of the group are ω wolves.
In designing GWO, in line with mathematical modeling of the social hierarchy of the wolves, the most suitable solution (the least objective function value) is regarded as α. Hence, given the best solutions, the second best and the third-best solutions are labeled as β and δ wolves. The remaining candidate solutions are labeled as ω. In GWO, the hunting (optimization) process is led by α, β, δ wolves. The ω wolves follow these three groups as follows (Mirjalili and Mirjalili, 2014):

$$\vec{D} = |\vec{C}\vec{X}_p(t) - \vec{X}| \tag{20}$$

$$\vec{X}_{(t+1)} = \vec{X}_P(t) - \vec{A} \tag{21}$$

In equations (20) and (21), t refers to the current iteration number; $\vec{D}$ is the distance vector of the wolves from the hunt; $\vec{A}$ and $\vec{C}$ denote coefficient vectors; $\vec{X}_P$ indicates hunt position vector and $\vec{X}$ refers to the position vector of a grey wolf. Vectors $\vec{A}$ and $\vec{C}$ are computed in the following way:

$$\vec{A} = 2\vec{a}\vec{r}_1 - \vec{a} \tag{22}$$

$$\vec{C} = 2\,\vec{r}_2 \tag{23}$$

In equations (22) and (23), $\vec{a}$ is linearly reduced from 2 to 0. $\vec{r}_1$ and $\vec{r}_2$ indicate random vectors within the [0,1] interval. The α, β, and δ wolves change their position randomly within the space which includes the bait by using equations (24), (25), and (26) consequently. The same concept can be generalized to the n-dimension search space. For a mathematical simulation of the hunting behavior of grey wolves, it was assumed that alpha (the best available solution), beta, and delta have better awareness about the potential location of the bait. Thus, three of the best-obtained solutions are saved and other search factors (omega wolves) are forced to update their positions concerning the best search factors. Equations (27), (28), (29), and (30) are presented in this regard.

$$\vec{D}_\alpha = |\vec{C}_1\vec{X}_\alpha - \vec{X}| \tag{24}$$

$$\vec{D}_\beta = |\vec{C}_2\vec{X}_\beta - \vec{X}| \tag{25}$$

$$\vec{D}_\delta = |\vec{C}_3\vec{X}_\delta - \vec{X}| \tag{26}$$

$$\vec{X}_1 = \vec{X}_\alpha - \vec{A}_1(\vec{D}_\alpha) \tag{27}$$

$$\vec{X}_2 = \vec{X}_\beta - \vec{A}_2(\vec{D}_\beta) \tag{28}$$

$$\vec{X}_3 = \vec{X}_\delta - \vec{A}_3(\vec{D}_\delta) \tag{29}$$

$$\vec{X}_{(t+1)} = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \tag{30}$$

When the hunt stops moving, grey wolves start attacking it and terminate the hunting operation. For mathematical modeling of the wolves' approaching to the bait, we reduce the value of $\vec{a}$. It should be noted that the fluctuation range $\vec{A}$ is also reduced by $\vec{a}$. In other words, $\vec{A}$ is a random value within the interval [-2a, 2a]. However, during the iteration, its value is reduced from 2 to 0. When the random values of $\vec{A}$ are within the interval [-1, 1], the next position of a search agent can be at any position between its current position and the bait position (Mirjalili and Mirjalili, 2014).

4.1. The proposed GWO for solving the CMS model

Some fundamental tools are needed for designing GWO and this algorithm should be designed about the nature of the problem. The pseudo-code of the algorithm can be summarized in the following way:

1. Begin
2. Randomly generate the initial population of the wolves (random initial solutions).
3. Calculate the finesses of the solutions using objective function equation, and select three superior solutions as alpha, beta, and delta wolves.
4. In each iteration, three superior solutions are capable of estimating the position of the hunt; calculate the distance of the wolves form the hunt using equations (24), (25) and (26).
5. Update vector $\vec{a}$ (and consequently vector $\vec{A}$) and vector $\vec{C}$.
6. Repeat from step 3 until the termination condition is met.
7. At the end of the iterations, the position of alpha wolf is introduced as the optimal point.
8. End

Figure 1. The pseudo-code of the WGO algorithm

In theory, the hunt is the best (optimal or near-to-optimal) solution which the model is supposed to find. In each iteration, the wolves are trying to decrease their distance from the hunt; this issue in the solution space is equal to decreasing the difference between the objective function value and the optimal value. Unfortunately, in an abstract search space, there no idea about the position of the bait (the value of the optimal solution is not known), and since the objective function is in minimization form, the optimal value can be assumed equal to zero, or a positive lower bound should be calculated. Since the value of the objective function will never be equal to zero; in this paper, the lower bound is considered as the sum of the fixed costs of the machines. The stopping condition is considered as no improvement of the objective function value during the last 15 iterations which is tuned up using the try-and-error method.

The main limitation of the meta-heuristic approaches (as well as GWO) is that there is no guarantee to obtain the optimal solution. Besides, they usually use many parameters that require tuning and the performance of the algorithm highly depends on the fine-tuning process. The WGO approach uses some random vectors and numbers during the iterations and the convergence speed depends on the performance of the uniform random number generator.

4.2. Demonstration of alpha, beta, and gamma wolves

The demonstration manner of alpha wolves refers to the demonstration of a feasible solution that corresponds with the CMS model and its components should meet the following issues:

1. Each operation related to one part should be allocated to only one machine in one cell. Hence, one feasible solution should demonstrate both operation allocation to the machine, and operation allocation to the cell at the same time. That is, one specific operation may be processed by several machines but it should be assigned to only one of the machines.
2. In each period, the number of the available m-type machines in the cell, Nmch, should be known. Also, the number of machines that have been added to the cell c (K+mch) or have been deducted from this cell (K-mch) in this period should be determined.
3. In each period, the value of the optimal internal production of the part p, Qph, and also, the number of outsourcings of the part p, Sph, should be estimated. Then, by capitalizing on these values, the amount of the inventory and the shortage of the part p (Iph) should be estimated.
4. The structure of a single wolf (Figure 2) consists of many fields which represent a feasible solution for the problem. It contains information about the allocation of machines, operations, cells, internal and outsourcing production volume, etc. Hence, in each solution, the following four matrices and three vectors should be involved and valued:

   - The matrix X for allocating operation to the machines.
   - The matrix Y for allocating operation to the cells.
   - The matrix N for determining the number of available m-type machines in cell c.
   - The matrix K determines the number of m-type machines that have been added to/deducted from cell c.
   - The vector Q estimates the amount of internal production of the part p.
   - The vector S estimates the amount of outsourcing of part p.

- The vector I estimates and measures the amount of inventory or the shortage of the part p.

Given the constraints (11) and (12) of the model, matrix K is updated according to matrix N of the current period and matrix N of the previous period. Also, vector I is updated according to vector Q of the current period and vector S of the previous period, and vector I of the previous period. Hence, in each solution, a block structure including four matrices and three vectors are required for each period which indicates the structure of the grey wolves in the GWO algorithm. The matrix and vector components of the grey wolves are as follows:

X: is a P×r matrix in which:

$$r = \max_{i=1}^{P}\{op_i\} \tag{31}$$

Where OPi refers to the number of operations of the ith part and Xij denotes the number of the machine to which the jth operation of the ith part is assigned.
Y: is a P×r matrix in which Yij indicates the number of the cell to which the jth operation of part i is assigned.
N: is a M×c matrix whose members indicate the number of machine types that have been added to or deducted from cell c.
Q: is a vector of length P where Qp indicates the estimated amount of the internal production of the part p in the current period.
S: is a vector of length P in which Sp denotes the amount of outsourcing of the part p in the current period.
I: is also a vector of length P in which Ip indicates the amount of inventory or the shortage of the part p in the current period. If Ip>0 in the current period, it represents the current inventory; and if Ip<0, it demonstrates the shortage.

X matrix	Y Matrix	N Matrix	K Matrix	Qph	Sph	Fixed Machine Cost	Variable Machine Cost	Inter-cell Movement Cost	Intra-cell Movement Cost	Prod. Planning Cost	Rede-ployment Cost	Total Cost

Figure. 2 The structure of a wolf (solution) in the proposed GWO algorithm

5. Examining the numerical example

In this section, example (1) given in Safaei and Tavakkoli-Moghaddam (2009a) is investigated for evaluating and validating the performance of the proposed model and the capability of GWO in solving the optimization of the proposed model. This example includes 5 parts, 2 operations, 5 machines, 3 cells, and 3 periods. The data of this example is identical to the one given in the reference.
In GWO, the search operation begins by creating a random population of grey wolves for the candidate solution. Throughout the iteration, alpha, beta, and delta wolves estimate the probable position of the hunt. Each candidate solution updates its distance from the bait. The parameter a is reduced from the value 2 to 0 to support the identification and attack process to the bait. When $|A| > 1$, the candidate solutions diverge from each other. In contrast, when $|A| < 1$, they converge to the bait. GWO algorithm was coded in the MATLAB software and was executed 50 times on the proposed model. The cellular structure proposed obtained in Safaei and Tavakkoli-Moghaddam (2009a) and by the proposed method is according to tables 1 and 2 consequently. The cells are demonstrated in shaded rectangles. The integer value in the left column of each machine denotes the number of the needed machine should be assigned to the cell. The bolded values out of the cell blocks show the inter-cellular movements which are considerably less in the proposed method. Safaei and Tavakkoli-Moghaddam (2009a) also propose an optimal cell configuration for a typical test problem that suffers another mistake. In the third period, operations 2 and 3 of part P1 are performed both by machines M3 and M5, which is not applicable; besides, according to the problem parameters and specifications, the machine M5 is NOT capable of performing these operations.

Table 1. The cell structure proposed by Safaei and Tavakkoli-Moghaddam (2009a) for a typical test problem

Period 1

			C1		C2	
			P2	P1	P3	P4
C1	4	M1	1,2,3			
	1	M2		2	3	1
C2	1	M3		1	2	3
	1	M4		3	1	2

Period 2

				C1		C2
			P1	P2	P4	P3
C1	3	M1		1,2,3		
	1	M5	1		2,3	
	1	M2				3
C2	2	M3	**2,3**			2
	1	M4				1

Period 3

			C1		C2	
			P1	P5	P3	P4
C1	3	M5	1	1		**2,3**
	1	M2	**2**		3	
C2	2	M3		**3**	2	1
	1	M4	**3**	**2**	1	

Table 2. The cell structure obtained by the proposed method for the same typical test problem

Period 1

			C1	C2			
			P2	P1	P3	P4	P5
C1	3	M1	1,2,3			**1**	
	1	M2		2	3		
C2	1	M4		3	1		2
	2	M5		1	2	2,3	1,3

Period 2

			C1	C2			
			P2	P1	P3	P4	P5
C1	1	M1	1,2,3		**2**		
	1	M3		2,3		1	3
C2	1	M4			1		2
	1	M5		1	3	2,3	1

Period 3

			C1			C2
			P1	P3	P4	P2
C1	1	M2	2	3	3	
	1	M3	1,3	2	1	
C2	1	M4		1		
	1	M1			**2**	1,2,3

The details of the solution obtained by the proposed GWO method are given in table 3.

Table 3. The optimal production plan obtained by the proposed GWO algorithm

	h = 1					h = 2					h = 3				
	P1	P2	P3	P4	P5	P1	P2	P3	P4	P5	P1	P2	P3	P4	P5
Q_{ph}	363	830*	521	39	150	139	6	52	195	62	37	33	182	108	0
S_{ph}	198	567	390	0	387	366	114	205	808	0	0	0	0	0	0
I_{ph}	63	-20	321	39	-100	0	-147	62	34	350	0	0	0	0	0
D_{ph}	300	850	200	0	250	400	700	700	200	0	400	0	450	950	350

* The corresponding value reported in the reference is 1112; but the correct value should be 1111

Table 4. The comparison of the cost result for the sample example

Reference	Objective function value	Machine constant cost	Machine variable cost	Inter-cell movements cost	Intra-cell movements cost	Planning cost*	Reconfiguration cost
Safaei and Tavakkoli-Moghaddam (2009)	144612**	26100	46114	521	645	63854	7375
The proposed GWO	140145	24600	31532	1735	570	60458	21250

* Planning cost = Inventory carrying cost + Backorder incurring cost + Subcontracting cost.
** The sum of the costs is equal to 144609; not 144612. This is another mistake in the values reported in the reference.

Comparing the results given in table (4) indicates that the value of the objective function in this research is about 3% less than obtained in the reference. Whereas the cost of planning production and the fixed (constant) and variable costs of the machine have been reduced, inter-cellular and intra-cellular movement costs and reconfiguration costs have been increased. In this study, considering the advantage of adding the constraint for controlling the subcontracting and outsourcing parts and the reduction of the inventory carrying and shortage costs, the cost of planning production has been reduced in comparison with the base model proposed by Safaei and Tavakkoli-Moghaddam (2009a). Since an attempt was made to use fewer machines for reducing costs, the fixed machine cost decreased; however, the inter-cellular movement had to be used for better and more exploitation of the machine capacity. Consequently, the inter-cellular movement increased in this solution. Also, in selecting machinery for performing operational stages, the processing time of the operations was selected as a significant factor for reducing costs. Accordingly, machines with shorter processing times were used for manufacturing the parts. In this way, the variable cost of the machinery was reduced.

Figure 3 indicates that the values of the solutions in the GWO were notably reduced during 30 iterations. Hence, in a descending trend with a relatively steady slope, the responses reached the near-optimal value.

For evaluating the performance of the proposed GWO algorithm, 5 problems adopted from references Safaei and Tavakkoli-Moghaddam (2009a) and Ardakani et al. (2012) with different dimensions are also solved. The specifications of the problems are given in table 5. Since LINGO 8.0 is not able to solve problems with large dimensions, the sample problems were considered in a small size so that they can be comparable with the proposed model. Figure 4 depicts the comparative illustration of the final solutions (local or near to optimal) for all three methods. As given in table 5, the results of the comparisons indicate that GWO outperformed the other method for solving all these 5 problems.

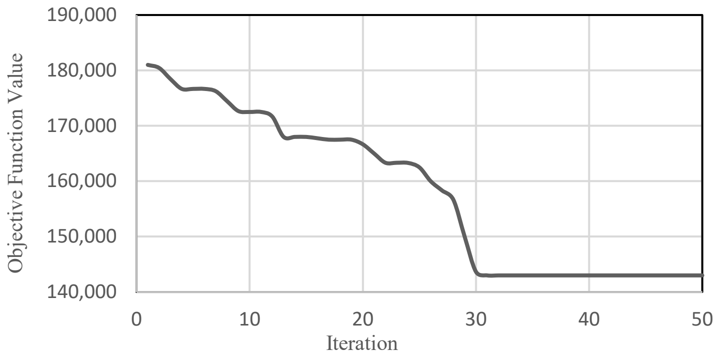

Figure 3. The convergence diagram of the proposed GWO algorithm

Table 5. The specifications and simulation results of other sample test cases

Problem No.	Dimension	Objective function value obtained by		
	P(OP)×M×C×H	LINGO	PSO*	Proposed GWO
1	2×2×2) ×2(2	42547	42985	41480
2	3×2×2)× 3(2	77425	79884	75489
3	3×2×3) ×2(3	99823	104425	96819
4	3×2×3) ×2(4	109825	121524	107129
5	3×3×5) ×3(6	159698	175450	155514

* Adopted from Ardakani et al. (2012)

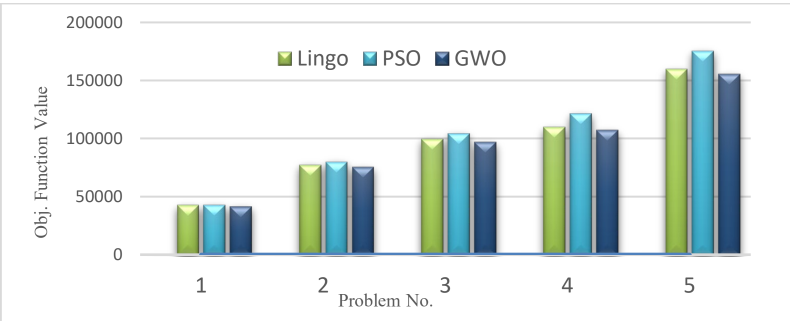

Figure 4. The histogram of the objective function values of table 5

To evaluate the efficiency and stability of the proposed GWO algorithm, two sensitivity analyses are also performed on problem #5. The first analysis tests the impact of the number of the initial wolf population on convergence speed (in terms of the number of iterations needed to reach the final best value). The simulation results show that the convergence speed increases as the number of wolves increase from 10 up to 60, but no more meaningful improvement happens by increasing the number of wolves to higher values. Figure 5 demonstrates the test result.

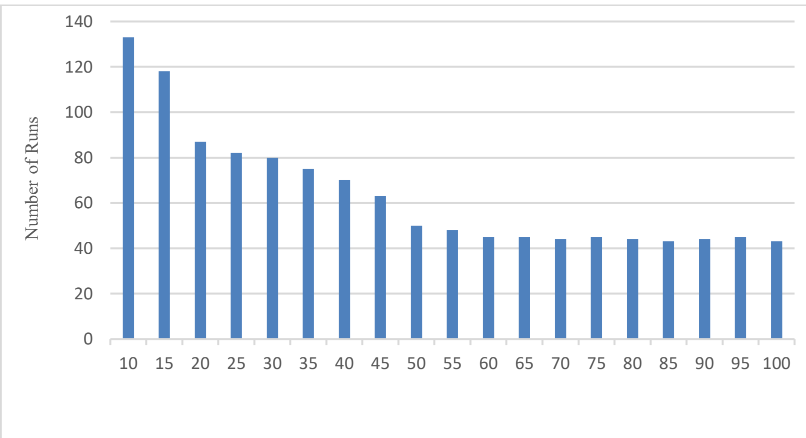

Fig. 5. The impact of increasing the initial wolf population size on convergence speed (# of Runs)

Meta-heuristic approaches have a random nature and the results may change at different runs. In the second test, problem #5 is executed for 50 different runs and the best obtained solutions are plotted in Figure 6. According to the results, the proposed GWO algorithm was capable of reaching the same solution in 47 out of 50 runs. The small deviation shows the stability of the proposed algorithm.

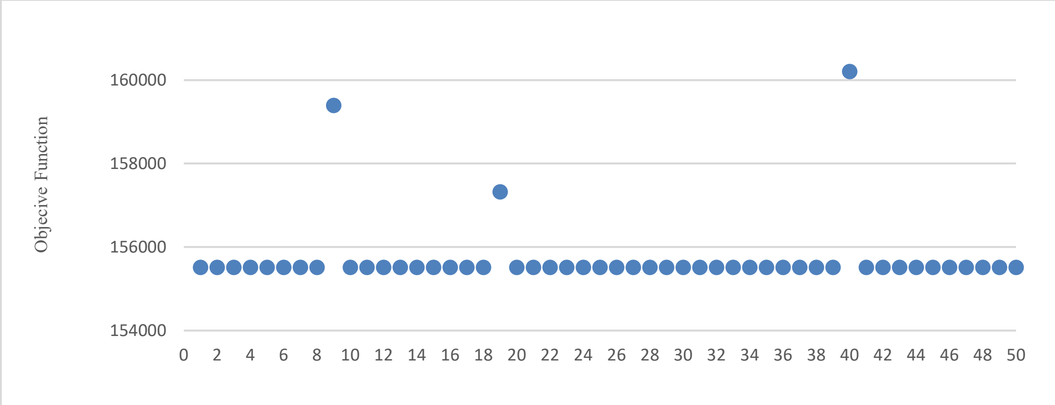

Fig. 6. The stability diagram of the proposed GWO algorithm

6. Conclusions

This study investigated the basic non-linear integer programming model for solving the dynamic cell formation problem and production planning to minimize the total sum of machine costs; inert/intra cellular movements cost; reconfiguration cost; inventory carrying, shortage, and outsourcing cost originally proposed by Safaei and Tavakkoli-Moghaddam (2009a). The proposed model in the mentioned reference suffers lacking some critical equations such as constraints to limit amoun of outsourcing, and a constraint to guarantee all the operations needed to produce a part will be processed. Besides, the machine investment cost is calculated at every period; instead of considering it just once at beginning of the first period. Finally, some reported solutions for solved test cases in the reference are distinguished to be wrong and completely inapplicable.
In this paper, this model is revised and modified by adding required constraints on the amount of part outsourcing and guaranteeing the completion of all the operational processes related to the parts. Adding these constraints, led to obtaining better solutions that are all applicable (feasible) and considerably better in objective function value. Considering the NP-Hard nature of the problem, a meta-heuristic algorithm based on the GWO method was also developed and simulated in Matlab and the sample example was presented in detail and solved by the proposed model which led to the significant reduction of total costs by 3% in comparison with the base model. For a fair comparison, the machine investment costs are considered at all the periods in the proposed model. Furthermore, five more sample problems adopted from the literature are also simulated and the results were compared with that of Lingo and PSO. The comparison of the computational results obtained by the proposed GWO algorithm revealed that the proposed method is more efficient in optimizing the objective function and reaching a better local optimum.
As a direction for further research, a mathematical model can be developed for considering other effective costs that have impacts on the production costs (labor, maintenance and repair, depreciation of machinery, etc.). Also, it is suggested that the optimization problem be solved using other meta-heuristic algorithms for finding the best algorithm and sorting out such issues.

References

Aghajani, A., Kazemi, M., Rezaeian, J. (2010).” Designing a Dynamic Cellular Manufacturing Model with Considering Alternative Routing”, 3rd International Conference of Iranian Operations Research Society, Amirkabir University, Tehran, Iran.

Ahi, A., Aryanezhad, M.B., Ashtiani, B., Makui, A. (2009). “A novel approach to determine cell formation, intracellular machine layout and cell layout in the CMS problem based on TOPSIS method”, Computers & Operations research, 36(5), 1478-1496.

Almonacid, B. (2019). Resolve the cell formation problem in a set of three manufacturing cells (No. e27692v1). PeerJ Preprints.

Ardakani, A., Barzinpour, F., Tavakkoli-Moghaddam, R. (2012). “Developing the PSO algorithm for solving the integrated production planning and dynamic cellular manufacturing model”, Industrial Engineering Journal (in Persian), 46, 77-89.

Ayough, A., Khorshidvand, B. (2019). “Designing a manufacturing cell system by assigning workforce”. Journal of Industrial Engineering and Management, 12(1), 13-26.

Bajestani, M.A., Rabbani, M., Rahimi-Vahed, A.R., Baharian-Khoshkhou, G. (2009). “A multi-objective scatter search for a dynamic cell formation problem”, Computers & Operations research, 36(3), 777-794.

Ballakur A., Steudel, H.J. (1987). “A within Cell Utilization Based Heuristic for Designing Cellular Manufacturing Systems”, International Journal of Production Research, 25(5), 639-655.

Dehnavi-Arani, S., Sadegheih, A., Mehrjerdi, Y.Z. and Honarvar, M. (2020). “A new bi-objective integrated dynamic cell formation and AGVs’ dwell point location problem on the inter-cell unidirectional single loop”, Soft Computing, 24(21), 6021-16042.

Deljoo, V., Mirzapour Al-e-hashem, S.M.J., Deljoo, F., Aryanezhad, M.B. (2010). “Using genetic algorithm to solve dynamic cell formation problem”, Applied Mathematical Modelling, 34, 1078-1092.

Dmytryshyn, T., Ismail, M., & Rashwan, O. (2018, November). A novel modeling approach for solving the cell formation problem. In ASME International Mechanical Engineering Congress and Exposition (Vol. 52019, p. V002T02A074). American Society of Mechanical Engineers.

Doulabi, S.H., Hojabri, H., Davoudpour, H., Seyed-Alagheband, S.A., Jaafari, A. (2009). “Two-phase approach for solving cell-formation problem in cell manufacturing”, In Proceedings of the world congress on engineering and computer science, Vol II, October 20–22, San Francisco.

Golmohammadi, A. M., Honarvar, M., Hosseini-Nasab, H., & Tavakkoli-Moghaddam, R. (2020). A bi-objective optimization model for a dynamic cell formation integrated with machine and cell layouts in a fuzzy environment. Fuzzy Information and Engineering, 12(2), 204-222.

Hafezalkotob, A., Tehranizadeh, M., Sarani Rad, F., Sayadi, M. (2015). “An Improved DPSO Algorithm for Cell Formation Problem”, Journal of Industrial and Systems Engineering, 8(2), 30-53.

Hamza, S., & Jehad, A. (2019). “Heuristic Method for Solving Cell Formation Problem in Cellular Manufacturing System Based on Hamming Distance”, The Iraqi Journal for Mechanical and Materials Engineering, 19(1), 75-90.

Karim, R., Biswas, S.K. (2015). “A literature review on cell formation problem in a batch-oriented production system”, International Journal of scientific research and management, 3(2), 2187-2192.

Kheirkhah, A., Ghajari, A. (2018). “A three-phase heuristic approach to solve an integrated cell formation and production planning problem”, Uncertain Supply Chain Management, 6(2), 213-228.

Kiaa, F., Khaksar-Haghanib, A., Javadian, N., Tavakkoli-Moghaddam, R. (2014). “Solving a multi-floor layout design model of a dynamic cellular manufacturing system by an efficient genetic algorithm”, Journal of Manufacturing Systems, 33, 218–232.

Kia, R., Baboli, A., Javadian, N., Tavakkoli-Moghaddam, R., Kazemi, M., Khorrami, J. (2012). “Solving a group layout design model of a dynamic cellular manufacturing system with alternative process routings, lot splitting and flexible reconfiguration by simulated annealing”, Computers & Operations research, 39(11), 2642-2658.

Kioon, S.A., Bulgak, A. A., Bektas, T. (2009). “Integrated cellular manufacturing systems design with production planning and dynamic system reconfiguration”, European Journal of Operations Research, 192(2), 414-428.

Mahdavi, I., Aalaei, A., Paydar, M.M., Solimanpur, M. (2010). “Designing a mathematical model for dynamic cellular manufacturing systems considering production planning and worker assignment”, Computers and Mathematics with Applications, 60(4), 1010-1025.

Mirjalili, S., Mirjalili S.M., Lewis, A. (2014). “Grey Wolf Optimizer”, Advances in Engineering Software, 69, 46-61.

Nagaraj, G., Arunachalam, M., Vinayagar, K. (2020). “Enhancing performance of cell formation problem using hybrid efficient swarm optimization”, Soft Computing, 24, 16679–16690.

Noktehdan, A., Karimi, B., Husseinzadeh Kashan, A. (2009). “A differential evolution algorithm for the manufacturing cell formation problem using group-based operators”, Expert System with Applications, 37(7), 4822-4829.

Rafiee, M. Rabbani, H. Rafiei, H., Rahimi-Vahed, A. (2011). “A new approach towards integrated cell formation and inventory lot sizing in an unreliable cellular manufacturing system”, Applied Mathematical Modelling, 32, 1810-1819.

Rafiee, H, Ghodsi, R. (2012). “A bi-objective mathematical model toward dynamic cell formation considering labor utilization”, Applied Mathematical Modelling, APM 8939.

Rajesh, K.V.D, Abidali, M.D., Chalapathi, P.V. (2018). “Voids Based Approach for Solving Cell Formation Problems”, MaterialsToday: Proceedings, Vol. 5, Issue 13, part 3, pp. 27185-27192.

Saeedi, S., Solimanpur, M., Mahdavi, I., Javadian, N. (2010). “Heuristic Approaches for Cell Formation in Cellular Manufacturing”, Journal of Software Engineering & Applications, 3, 674-682.

Saeedi, S., Solimanpur, M., Mahdavi, I., Javadian, N. (2014). “A multi-objective genetic algorithm for solving cell formation problem using a fuzzy goal programming approach”, International Journal of Advanced Manufacturing Technology, 70, 1635-1652.

Safaei, N., Tavakkoli-Moghaddam, R. (2009a). “Integrated multi-period cell formation and subcontracting production planning in dynamic cellular manufacturing systems”, International Journal of Production Economics, 120(2), 301-314.

Safaei, N., Tavakkoli-Moghaddam, R. (2009b). “Integrated multi-period cell formation and subcontracting production planning in dynamic cellular manufacturing”, International Journal of Production Economics, 120(2), 301-314.

Solimanpur, M, Saeedi, S., Mahdavi, I. (2010). "Solving cell formation problem in cellular manufacturing using ant-colony-based optimization", International Journal of Advanced Manufacturing Technology, 50(9-12), 1135-1144.

Zhu, Y., & Li, S. (2018). Simultaneous Hierarchical Clustering for Cell Formation Problems with Production Information. Mathematical Problems in Engineering, 2018.

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