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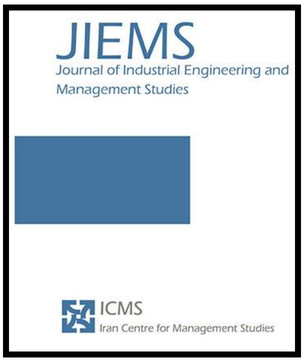

Designing a new multi-objective fuzzy stochastic DEA model in a dynamic environment to estimate efficiency of decision making units (Case Study: An Iranian Petroleum Company)

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Abstract

This paper presents a new multi-objective fuzzy stochastic data envelopment analysis model (MOFS-DEA) under mean chance constraints and common weights to estimate the efficiency of decision making units for future financial periods of them. In the initial MOFS-DEA model, the outputs and inputs are characterized by random triangular fuzzy variables with normal distribution, in which data are changing sequentially. Since the initial MOFS-DEA model is a complex model, we convert it to its equivalent one-objective stochastic programming by using infinite-norm approach. To solve it, we design a new hybrid meta-heuristic algorithm by integrating Imperialist Competitive Algorithm and Monte Carlo simulation. Finally, this paper presents a real application of the proposed model and the designed hybrid algorithm for predicting the efficiency of five gas stations for the next two periods of them, with using real information which gathered from credible sources. The results will be compared with the Qin's hybrid algorithm in terms of solution quality and runtime.

Keywords: Data envelopment analysis; Random fuzzy variable; Dynamic stochastic programming; Monte Carlo simulation; Imperialist competitive algorithm.

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1. Introduction

Evaluating and comparing performance of similar decision making units is an important part of the responsibilities of each organization management. Data envelopment analysis (DEA) is a managerial tool for evaluating and improving the efficiency of decision making units)DMUs). DEA which was initially proposed by Charnes, Cooper and Rhodes (1978), has been widely applied to evaluate the relative efficiency of DMUs. Since then hundreds of papers have been published in this field. However, this method has some major shortcomings such as inability to predict the performance of DMUs and also impossibility to consider stochastic variations in the data. The first stochastic DEA model was developed by Cooper, Huang and Li (1996). Also, we

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usually obtain fuzzy data from DMUs; Sengupta (1992) is the first one who considered fuzziness both in constraints and objective function in fuzzy DEA models. Since in real world problems, decision makers may encounter an uncertain environment where fuzziness and randomness coexist in a DMU, they represent the outputs and inputs in these DMUs by random fuzzy variables to characterize the hybrid uncertainty. The random fuzzy variable (Kwakernaak 1978), possibility theory (Z.Q. Liu and Y.K. Liu 2010), credibility and mean chance theories (Liu 2014) have been presented to treat fuzzy phenomena existing in real world problems. Wang and Watada (2009) discussed the analytical properties of critical value functions of random fuzzy variables and mean chance distribution functions in CCR model. A new fuzzy DEA approach based on parametric programming was developed by Razavi, Amoozad and Zavadskas (2013).

The basic idea of the proposed method is applying the notion of α-cuts. Based on these studies, all papers focused on analyzing current performance of DMUs, without considering the need of predicting efficiency for future planning of them. So, this paper proposes a new fuzzy multi-objective stochastic DEA model to predict the performance of decision making units.

The rest of this paper is organized as follows: Section 2 surveys the literature review on DEA models with common weights in dynamic and static environments. Section 3 presents some basic fuzzy theories, which are useful for modeling the proposed model in the next section. Section 4 presents a new multi-objective fuzzy stochastic DEA model with common weights in dynamic environment (MOFS-DEA). In Section 5, we first convert the expected values of objective functions and mean chance constraints of initial MOFS-DEA model to their equivalents stochastic representations, and we then convert the initial MOFS-DEA model to one objective stochastic model with using infinite-norm approach. In section 6 we integrate Monte Carlo (MC) simulation and Imperialist Competitive Algorithm (ICA) to design a new hybrid meta-heuristic algorithm (ICA-MC) for solving the proposed model. Section 7 provides a real example to express the idea and effectiveness of the designed approach to predict the efficiency of DMUs. Finally in this section, we compare our results with the results of hybrid algorithm which was proposed by Qin and Liu (2010). Section 8 draws the concluding remarks and suggestions.

2. Literature review

The application of multi-objective linear programming (MOLP) procedure to select the preferred outputs and inputs was proposed by Golany (1988). He attempted to present a new model to improve the discriminating power in classical DEA models. Thanassoulis and Dyson (1992) presented a new data envelopment analysis model which incorporated the preferences over input–output improvements to reach the preferred input–output target. The first classic multi-objective stochastic DEA models in dynamic environment were proposed by Sengupta (1995) and the framework of dynamic DEA by Nemoto and Goto (2003). Sueyoshi and Sekitani (2005) presented a new type of dynamic DEA with common weighs set. They used concept of returns to scale (RTS) in the dynamic DEA of Nemoto and Goto (2003). Teimoori (2006) proposed a new multi-objective DEA model in dynamic environment which produced aggregate performance of the total planning horizon. An equivalence model between multi objective linear programming and dynamic DEA models was presents by Yang, Wong, Xu and Stewart (2008). Omrani (2013) proposed a new method to find common weights set in DEA with imprecise data. Ramezani and Khodabakhshi (2013) proposed a new approach to rank decision making units in dynamic DEA with using common weights set. Wang, Lu and Liu (2014) proposed a new two stage multi-objective fuzzy DEA model in dynamic environment for evaluating the performance of US bank holding companies. Yaghoubi, Amiri and Safi- Samghabadi (2015) presented a new Stochastic DEA model to predict performance of DMUs. They designed a new hybrid algorithm by integrating genetic algorithm and MC simulation to solve the proposed model.

The crisp outputs and inputs in traditional DEA models become random fuzzy variables in fuzzy stochastic environment, and modeling with such data is meaningless directly because the meanings of the constraints and the objective function are not clear at all. So, in order to obtain a meaningful model in such environments, we employ the expected value to objective function and the mean chance theory to constraints to propose a new multi-objective fuzzy stochastic DEA model (MOFS-DEA) in dynamic environment. In general, the mean chance functions in the constraints and expected value in objective function are difficult to compute, so we convert them to their equivalent stochastic representations. As a consequence, the initial MOFS-DEA model can be converted to its equivalent stochastic programming one. A summary development of the above literature and the characteristics of the proposed model in this paper are listed in Table 1.

Table 1. A summary development of the review literature

Authors	Method		Inputs & Outputs		Environment		Objective	
	DEA	SDEA	Precise	Imprecise	Static	Dynamic	Single	Multiple
Golany (1988)	×		×		×			×
Sengupta (1992)		×		×	×		×	
Thanasolis (1992)	×			×	×		×	
Sengupta (1995)		×	×			×		×
Cooper (1996)		×		×	×		×	
Sueyoshi (2005)	×		×			×		×
Teimoori (2006)	×		×			×		×
Yang et al. (2008)	×			×		×		×
Qin et al. (2010)		×		×	×		×	
Omrani (2013)	×			×	×			×
Ramezani (2013)	×		×			×		×
Wang et al. (2014)	×			×		×		×
Yaghoubi (2015)		×		×		×	×	
proposed model		×		×		×		×

As it can be observed in Table 1, the previous researches have not proposed any model based on multi-objective SDEA model with using imprecise inputs and outputs in a dynamic environment to predict the performance of DMUs, however, in this paper provided.

3. Preliminaries

3.1. Credibility approach

Let ξ be a fuzzy variable with a possibility distribution function μ. The credibility of a fuzzy event {ξ ≥r} for $r \in R$ is defined as (Qin and Liu 2010):

$$Cr\{\xi \geq r\} = \frac{1}{2}(1 + \sup_{t \geq r} \mu(t) - \sup_{t \prec r} \mu(t)) \qquad (1)$$

The mean chance of random fuzzy event {ξ ∈B} when ξ be n-dimensional random fuzzy vector, and B be a subset of R, is defined as:

$$Ch\{\xi \in \beta\} = \int Cr\{\xi \in \beta\} P\{\xi \in \beta\} \qquad (2)$$

3.2. Mean chance distributions for random triangular fuzzy variables

Theorem 1. Let $\xi=(X-a,X,X+b)$ be a continuous random triangular fuzzy variable, in which X is a random variable, and a,b being positive numbers. If $X\sim N(\mu,\sigma^2)$ then we have:

$$Ch\{\xi\ge r\}=\frac{\sigma(b-a)}{2ab\sqrt{2\pi}}\exp(-\frac{(r-\mu)^2}{2\sigma^2})+\frac{\sigma}{2b\sqrt{2\pi}}\exp(-\frac{(b+\mu-r)^2}{2\sigma^2})-\frac{\sigma}{2a\sqrt{2\pi}}\exp(-\frac{(r+a-\mu)^2}{2\sigma^2})$$
$$+\frac{\mu-a-r}{2a}\phi(\frac{r+a-\mu}{\sigma})-\frac{(\mu-r)(b-a)}{2ab}\phi(\frac{r-\mu}{\sigma})+\frac{\mu+b-r}{2b}\phi(\frac{b+\mu-r}{\sigma})+\frac{r+b-\mu}{2b} \tag{3}$$

where $\phi(0)$ is the probability distribution of standard normal distribution function (Qin and Liu 2010).

Theorem 2. Let $\xi_i=(X_i-a_i,X_i,X_i+b_i)$ be mutually independent triangular fuzzy variables. If $X_i\sim N(\mu_i,\sigma_i^{\,2})$ with $a_i,b_i\succ 0$, then we have:

$$Ch\left\{\sum_{i=1}^n x_i\xi_i\ge r\right\}=\frac{\sqrt{\sum_{i=1}^n x_i^2\sigma_i^2}\sum_{i=1}^n x_i(b_i-a_i)}{2\sqrt{2\pi}\sum_{i=1}^n x_ia_i\sum_{i=1}^n x_ib_i}\exp(-\frac{(r-\sum_{i=1}^n x_i\mu_i)^2}{2\sum_{i=1}^n x_i^2\sigma_i^2})+\frac{\sqrt{\sum_{i=1}^n x_i^2\sigma_i^2}}{2\sqrt{2\pi}\sum_{i=1}^n x_ib_i}\exp(-\frac{(\sum_{i=1}^n x_i(b_i+\mu_i)-r)^2}{2\sum_{i=1}^n x_i^2\sigma_i^2})$$
$$-\frac{\sqrt{\sum_{i=1}^n x_i^2\sigma_i^2}}{2\sqrt{2\pi}\sum_{i=1}^n x_ia_i}\exp(-\frac{(r+\sum_{i=1}^n x_i(a_i-\mu_i))^2}{2\sum_{i=1}^n x_i^2\sigma_i^2})-\frac{(\sum_{i=1}^n x_i\mu_i-r)\sum_{i=1}^n x_i(b_i-a_i)}{2\sum_{i=1}^n x_ia_i\sum_{i=1}^n x_ib_i}\phi(\frac{r-\sum_{i=1}^n x_i\mu_i}{\sqrt{\sum_{i=1}^n x_i^2\sigma_i^2}}) \tag{4}$$
$$+\frac{\sum_{i=1}^n x_i(b_i+\mu_i)-r}{2\sum_{i=1}^n x_ib_i}\phi(\frac{\sum_{i=1}^n x_i(b_i+\mu_i)-r}{\sqrt{\sum_{i=1}^n x_i^2\sigma_i^2}})+\frac{\sum_{i=1}^n x_i(\mu_i-a_i)-r}{2\sum_{i=1}^n x_ia_i}\phi(\frac{r+\sum_{i=1}^n x_i(a_i-\mu_i)}{\sqrt{\sum_{i=1}^n x_i^2\sigma_i^2}})+\frac{r+\sum_{i=1}^n x_i(b_i-\mu_i)}{2\sum_{i=1}^n x_ib_i}$$

where x_i, (i=1,2,...,n) are nonnegative real numbers (Qin and Liu 2010).

Theorem 3. Let $\xi=(X-a,X,X+b)$ and $\eta=(Y-c,Y,Y+d)$ be two mutually independent random triangular fuzzy variables. If $X\sim N(\mu_1,\sigma_1^2)$, $Y\sim N(\mu_2,\sigma_2^2)$ and a,b,c,d being positive numbers, then we have (Qin and Liu 2010):

$$Ch\{x_1\xi-x_2\eta\ge r\}=\frac{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}\,(x_1(b-a)+x_2(c-d))}{2(x_1a+x_2d)(x_1b+x_2c)\sqrt{2\pi}}\exp(-\frac{(r-(x_1\mu_1-x_2\mu_2))^2}{2(x_1^2\sigma_1^2+x_2^2\sigma_2^2)}) \tag{5}$$
$$+\frac{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}}{2\sqrt{2\pi}(x_1b+x_2c)}\exp(-\frac{(x_1(b+\mu_1)+x_2(c-\mu_2)-r)^2}{2(x_1^2\sigma_1^2+x_2^2\sigma_2^2)})-\frac{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}}{2\sqrt{2\pi}(x_1a+x_2d)}\exp(-\frac{(r+x_1(a-\mu_1)+x_2(d+\mu_2))^2}{2(x_1^2\sigma_1^2+x_2^2\sigma_2^2)})$$
$$+\frac{x_1(\mu_1-a)-x_2(\mu_2+d)-r}{2(x_1a+x_2d)}\phi(\frac{r+x_1(a-\mu_1)+x_2(d+\mu_2)}{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}})+\frac{((x_1\mu_1-x_2\mu_2)-r)(x_1(b-a)+x_2(c-d))}{2(x_1a+x_2d)(x_1b+x_2c)}$$
$$*\phi(\frac{r-(x_1\mu_1-x_2\mu_2)}{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}})+\frac{x_1(b+\mu_1)-x_2(\mu_2-c)-r}{2(x_1b+x_2c)}\phi(\frac{x_1(b+\mu_1)+x_2(c-\mu_2)-r}{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}})+\frac{r+x_1(b-\mu_1)+x_2(c+\mu_2)}{2(x_1b+x_2c)}$$

where x_1 and x_2 are nonnegative real numbers and at least one of them is nonzero.

Theorem 4. Suppose $\xi = (X - a, X, X + b)$ and $\eta = (Y - c, Y, Y + d)$ are two mutually independent triangular fuzzy variables, in which X,Y∈R and a, b, c, d being positive numbers, then we have (Qin and Liu 2010):

$$E\left[\frac{\eta}{\xi}\right] = -\frac{c}{2b} - \frac{d}{2a} + \frac{1}{2b}(Y + \frac{c}{b}X)\ln(1 + \frac{b}{X}) + \frac{1}{2a}(Y + \frac{d}{a}X)\ln\frac{X}{X - a} \tag{6}$$

4. Multi-objective fuzzy stochastic DEA model (MOFS-DEA) formulation

In a dynamic environment, it is assumed that there are "n" DMUs and these are surveyed in T periods. In the *t*-th period, each DMUj uses two kind of inputs: K^{t-1} (an *l* dimensional vector of quasi-fixed inputs) and X^t (an *m* dimensional vector of inputs) in order to produce two kind of outputs: Y^t (an *r* dimensional vector of inputs) and K^t (an *l* dimensional vector of quasi-fixed outputs). In Fig. 1, the vertical axis indicates the order of DMUs and the horizontal axis denotes the order of periods. As shown in this figure, the output vector (K^t) in the *t*-th period is used as the quasi-fixed or feedback input vector (link data) at the next (t+1) period. The conventional dynamic CCR model is built as:

$$\begin{aligned}
&Max: \quad Z_0^t = \frac{U^t\, Y_0^t + \rho^t\, K_0^t}{V^t\, X_0^t + \beta^{t-1}\, K_0^{t-1}} \\
&st: \\
&\frac{U^t\, Y_j^t + \rho^t\, K_j^t}{V^t\, X_j^t + \beta^{t-1}\, K_j^{t-1}} \le 1 \qquad j = 1,2,\ldots,n \\
&\quad U^t, V^t, \rho^t, \beta^{t-1} \ge \varepsilon \qquad j = 1,..,n \quad t = 1,..,T
\end{aligned} \tag{7}$$

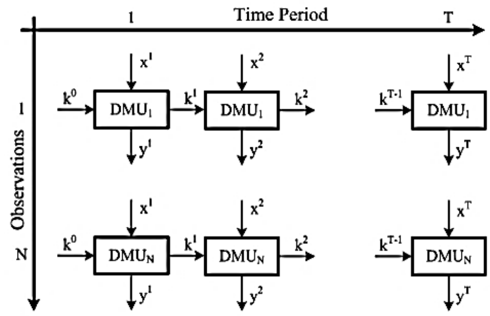

Fig. 1. Execution of the DEA model in dynamic framework

where X_j^t , Y_j^t are the column vectors of random fuzzy inputs (outputs) of DMUj (j=1,..,n) in period t, respectively. K_j^t is the quasi–fix random fuzzy outputs column vector of DMUj at period t and the quasi–fix random fuzzy inputs column vector of DMUj in period t+1 (link data). K_j^{t-1} is the quasi–fix random fuzzy inputs column vector of DMUj at period t and the quasi–fix random fuzzy outputs column vector of DMUj in period t-1 (link data). Also ρ^t and β^{t-1} are the weights vectors of K_j^t and K_j^{t-1}, respectively. Model (7) is used to evaluate the relative efficiency (Z_0^t) of DMU_0 with precise inputs and outputs at period t. However, in many cases, the data cannot be known with certainty, and are always derived by statistic or given by experts according to their experience, so randomness and fuzziness may exist simultaneously in these data. In many cases, we can only obtain the possibility distributions of the inputs and outputs. Thus in this paper, we assume that the inputs and outputs are random triangular fuzzy variables with normal distributions, following as:

$$
\begin{aligned}
X_j^t &= (X_{ij}^t - a_{ij}^t, X_{ij}^t, X_{ij}^t + b_{ij}^t) \\
Y_j^t &= (Y_{rj}^t - c_{rj}^t, Y_{rj}^t, Y_{rj}^t + d_{rj}^t) \\
K_j^t &= (K_{lj}^t - e_{lj}^t, K_{lj}^t, K_{lj}^t + f_{lj}^t) \\
K_j^{t-1} &= (K_{lj}^{t-1} - e_{lj}^{t-1}, K_{lj}^{t-1}, K_{lj}^{t-1} + f_{lj}^{t-1})
\end{aligned} \tag{8}
$$

Where $X_{ij}^t \sim N(\mu_{ij}^t, \sigma_{ij}^{2^t})$, $Y_{rj}^t \sim N(\overline{\mu}_{rj}^t, \overline{\sigma}_{rj}^{2^t})$, $K_{lj}^t \sim N(\overline{\overline{\mu}}_{lj}^t, \overline{\overline{\sigma}}_{lj}^{2^t})$, $K_{lj}^{t-1} \sim N(\overline{\overline{\mu}}_{lj}^{t-1}, \overline{\overline{\sigma}}_{lj}^{2^{t-1}})$. Also $a_{ij}^t, b_{ij}^t, c_{rj}^t, d_{rj}^t, e_{lj}^t, f_{lj}^t, e_{lj}^{t-1}, f_{lj}^{t-1}$ being positive numbers for each i (i=1,..,m), r (r=1,..,s) and l (l=1,..,L) which are predicted by decision maker for the next financial period to predict the efficiency of each DMU. In this case, the objective function of model (7) is also a random fuzzy variable, but the meaning of the model (7) is not clear. If we consider the efficiency ratio for all DMUs, we can then establish the multiple objectives programming which uses common weights set to efficiency measurement (Lozano and Villa 2007). To build a meaningful model, we utilize the expectation value in objective functions and the mean chance theory in the constraints to formulate the initial proposed MOFS-DEA model with common weights:

$$
\begin{aligned}
&Max: \quad Z_1^t = E\left[\frac{U^t Y_1^t + \rho^t K_1^t}{V^t X_1^t + \beta^{t-1} Y_1^{t-1}}\right] \\
&\vdots \\
&Max: \quad Z_n^t = E\left[\frac{U^t Y_n^t + \rho^t K_n^t}{V^t X_n^t + \beta^{t-1} Y_n^{t-1}}\right] \\
&st: \; Ch\left\{(V^t X_j^t + \beta^{t-1} K_j^{t-1}) - (U^t Y_j^t + \rho^t K_j^t) \geq 0\right\} \geq 1 - \alpha_j^t \qquad j = 1,2,\ldots,n \\
&\qquad U^t, V^t, \rho^t, \beta^{t-1} \geq \varepsilon \quad t = 1,..,T
\end{aligned} \tag{9}
$$

where α_j^t is considered as a risk criterion of failing to satisfy the *j*-th constraint at period t $\left(\alpha_j^t \in [0,1)\right)$. This model contains "n" objective functions and the purpose is to seek a common weights set $(U^t, V^t, \rho^t, \beta^{t-1})$ with the maximum value of each objective function at period t (t=1,..,T), while the fuzzy events $\left\{(V^t X_j^t + \beta^{t-1} K_j^{t-1}) - (U^t Y_j^t + \rho^t K_j^t) \geq 0\right\}$ are satisfied with the least confidence level $(1 - \alpha_j^t)$ for j=1,..,n.

5. Equivalent stochastic programming representation of MOFS-DEA model

5.1. Equivalent stochastic representation of the constraints

According to Theorems (2) and (3) and with considering the defined inputs and outputs (8), the *j*-th constraint of initial proposed MOFS-DEA model can be transformed to the following equivalent stochastic one:

$$g_j^t(U^t,V^t,\rho_l^t,\beta_l^{t-1}) = Ch\left\{(V^t X_j^t + \beta^{t-1}K_j^{t-1}) - (U^t Y_j^t + \rho^t K_j^t) \geq 0\right\}$$
$$= \frac{\sqrt{V^{2^t}\sigma_j^{2^t} + \beta_l^{2^{t-1}}\overline{\overline{\sigma}}_j^{2^{t-1}} + U^{2^t}\overline{\sigma}_j^{2^t} + \rho^{2^t}\overline{\overline{\sigma}}_j^{2^t}}\,(V^t\ (b_j^t - a_j^t)^2 + \beta^{t-1}\ (f_j^{t-1} - e_j^{t-1}) - U^t(d_j^t - c_j^t) - \rho^t(f_j^t - e_j^t))}{2\sqrt{2\pi}(V^t a_j^t + \beta^{t-1}e_j^{t-1} + U^t d_j^t + \rho^t f_j^t)(v^t b_j^t + \beta^{t-1}\ f_j^{t-1} + U^t c_j^t + \rho^t e_j^t)}$$
$$*\exp(-\frac{(V^t\mu_j^t + \beta^{t-1}\ \overline{\overline{\mu}}_j^{t-1} - U^t\overline{\mu}_j^t - \rho^t\ \overline{\overline{\mu}}_j^t)^2}{2(V^{2^t}\sigma_j^{2^t} + \beta_l^{2^{t-1}}\overline{\overline{\sigma}}_j^{2^{t-1}} + U^{2^t}\overline{\sigma}_j^{2^t} + \rho^{2^t}\overline{\overline{\sigma}}_j^{2^t})}) + \frac{\sqrt{V^{2^t}\sigma_j^{2^t} + \beta_l^{2^{t-1}}\overline{\overline{\sigma}}_j^{2^{t-1}} + U^{2^t}\overline{\sigma}_j^{2^t} + \rho^{2^t}\overline{\overline{\sigma}}_j^{2^t}}}{2\sqrt{2\pi}(V^t b_j^t + \beta^{t-1}\ f_j^{t-1} + U^t c_j^t + \rho^t e_j^t)}$$
$$*\exp(-\frac{(V^t(b_j^t + \mu_j^t) + \beta^{t-1}(f_j^{t-1} + \overline{\overline{\mu}}_j^{t-1}) + U^t(c_j^t - \overline{\mu}_j^t) + \rho^t(e_j^t - \overline{\overline{\mu}}_{lj}^t))^2}{2(V^{2^t}\sigma_j^{2^t} + \beta_l^{2^{t-1}}\overline{\overline{\sigma}}_j^{2^{t-1}} + U^{2^t}\overline{\sigma}_j^{2^t} + \rho^{2^t}\overline{\overline{\sigma}}_j^{2^t})}) - \frac{\sqrt{V^{2^t}\sigma_j^{2^t} + \beta_l^{2^{t-1}}\overline{\overline{\sigma}}_j^{2^{t-1}} + U^{2^t}\overline{\sigma}_j^{2^t} + \rho^{2^t}\overline{\overline{\sigma}}_j^{2^t}}}{2\sqrt{2\pi}(V^t a_j^t + \beta^{t-1}e_j^{t-1} + U^t d_j^t + \rho^t f_j^t)}$$
$$*\exp(-\frac{(V^t(a_j^t - \mu_j^t) + \beta^{t-1}(e_j^{t-1} - \overline{\overline{\mu}}_j^{t-1}) + U^t(d_j^t + \overline{\mu}_j^t) + \rho^t(f_j^t + \overline{\overline{\mu}}_{lj}^t))^2}{2(V^{2^t}\sigma_j^{2^t} + \beta_l^{2^{t-1}}\overline{\overline{\sigma}}_j^{2^{t-1}} + U^{2^t}\overline{\sigma}_j^{2^t} + \rho^{2^t}\overline{\overline{\sigma}}_j^{2^t})})$$
$$+\frac{V^t(\mu_j^t - a_j^t) + \beta^{t-1}(\overline{\overline{\mu}}_j^{t-1} - e_j^{t-1}) - U^t(d_j^t + \overline{\mu}_j^t) - \rho^t(f_j^t + \overline{\overline{\mu}}_{lj}^t)}{2(V^t a_j^t + \beta^{t-1}e_j^{t-1} + U^t d_j^t + \rho^t f_j^t)}$$
$$*\phi(\frac{V^t(a_j^t - \mu_j^t) + \beta^{t-1}(e_j^{t-1} - \overline{\overline{\mu}}_j^{t-1}) + U^t(d_j^t + \overline{\mu}_j^t) + \rho^t(f_j^t + \overline{\overline{\mu}}_{lj}^t)}{\sqrt{V^{2^t}\sigma_j^{2^t} + \beta_l^{2^{t-1}}\overline{\overline{\sigma}}_j^{2^{t-1}} + U^{2^t}\overline{\sigma}_j^{2^t} + \rho^{2^t}\overline{\overline{\sigma}}_j^{2^t}}}$$
$$-\frac{(V^t\mu_j^t + \beta^{t-1}\ \overline{\overline{\mu}}_j^{t-1} - U^t\overline{\mu}_j^t - \rho^t\ \overline{\overline{\mu}}_j^t)(V^t\ (b_j^t - a_j^t)^2 + \beta^{t-1}\ (f_j^{t-1} - e_j^{t-1}) - U^t(d_j^t - c_j^t) - \rho^t(f_j^t - e_j^t))}{2(V^t a_j^t + \beta^{t-1}e_j^{t-1} + U^t d_j^t + \rho^t f_j^t)(v^t b_j^t + \beta^{t-1}\ f_j^{t-1} + U^t c_j^t + \rho^t e_j^t)}$$
$$*\phi(\frac{-V^t\mu_j^t - \beta^{t-1}\ \overline{\overline{\mu}}_j^{t-1} + U^t\overline{\mu}_j^t + \rho^t\ \overline{\overline{\mu}}_j^t}{\sqrt{V^{2^t}\sigma_j^{2^t} + \beta_l^{2^{t-1}}\overline{\overline{\sigma}}_j^{2^{t-1}} + U^{2^t}\overline{\sigma}_j^{2^t} + \rho^{2^t}\overline{\overline{\sigma}}_j^{2^t}}}) + \frac{V^t(b_j^t + \mu_j^t) + \beta^{t-1}(f_j^{t-1} + \overline{\overline{\mu}}_j^{t-1}) - U^t(c_j^t - \overline{\mu}_j^t) - \rho^t(e_j^t - \overline{\overline{\mu}}_{lj}^t)}{2(V^t b_j^t + \beta^{t-1}\ f_j^{t-1} + U^t c_j^t + \rho^t e_j^t)}$$
$$*\phi(\frac{V^t(b_j^t + \mu_j^t) + \beta^{t-1}(f_j^{t-1} + \overline{\overline{\mu}}_j^{t-1}) + U^t(c_j^t - \overline{\mu}_j^t) + \rho^t(e_j^t - \overline{\overline{\mu}}_{lj}^t)}{\sqrt{V^{2^t}\sigma_j^{2^t} + \beta_l^{2^{t-1}}\overline{\overline{\sigma}}_j^{2^{t-1}} + U^{2^t}\overline{\sigma}_j^{2^t} + \rho^{2^t}\overline{\overline{\sigma}}_j^{2^t}}})$$
$$+\frac{V^t(b_j^t - \mu_j^t) + \beta^{t-1}(f_j^{t-1} - \overline{\overline{\mu}}_j^{t-1}) + U^t(c_j^t + \overline{\mu}_j^t) + \rho^t(e_j^t + \overline{\overline{\mu}}_{lj}^t)}{2(V^t b_j^t + \beta^{t-1}\ f_j^{t-1} + U^t c_j^t + \rho^t e_j^t)} \tag{10}$$

5.2. Equivalent stochastic representation of objective functions

Suppose X_j^t, Y_j^t, K_j^t and K_j^{t-1} are random triangular fuzzy vectors of DMUj at period t as (8), then we have:

$$\mu_j^t = \sum_{i=1}^m v_i^t\mu_{ij}^t \qquad \overline{\mu}_j^t = \sum_{r=1}^s u_r^t\overline{\mu}_{rj}^t \qquad \overline{\overline{\mu}}_j^t = \sum_{l=1}^L \rho_l^t\overline{\overline{\mu}}_{lj}^t \qquad \overline{\overline{\mu}}_j^{t-1} = \sum_{l=1}^L \beta_l^{t-1}\overline{\overline{\mu}}_{lj}^{t-1}$$
$$\sigma_j^{2^t} = \sum_{i=1}^m v_i^t\delta_{ij}^{2^t} \qquad \overline{\sigma}_j^{2^t} = \sum_{r=1}^s u_r^t\overline{\delta}_{rj}^{2^t} \qquad \overline{\overline{\sigma}}_j^{2^t} = \sum_{l=1}^L \rho_l^t\overline{\overline{\delta}}_{lj}^{2^t} \qquad \overline{\overline{\sigma}}_j^{2^{t-1}} = \sum_{l=1}^L \beta_l^{t-1}\overline{\overline{\delta}}_{lj}^{2^{t-1}}$$
$$a_j^t = \sum_{i=1}^m v_i^t a_{ij}^t \qquad b_j^t = \sum_{i=1}^m v_i^t b_{ij}^t \qquad c_j^t = \sum_{r=1}^s u_r^t c_{rj}^t \qquad d_j^t = \sum_{r=1}^s u_r^t d_{rj}^t$$
$$e_j^t = \sum_{l=1}^L \rho_l^t e_{lj}^t \qquad f_j^t = \sum_{l=1}^L \rho_l^t f_{lj}^t \qquad e_j^{t-1} = \sum_{l=1}^L B_l^{t-1}e_{lj}^{t-1} \qquad f_j^{t-1} = \sum_{l=1}^L \beta_l^{t-1}f_{lj}^{t-1}$$

According to Theorem (4), the j-th objective function of initial MOFS-DEA model (9) has the following equivalent representation:

$$Z_j^t = E(\frac{U^t Y_j^t + \rho^t K_j^t}{V^t X_j^t + \beta_j^{t-1} K_j^{t-1}}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \{-\frac{1}{2}(\frac{c_j^t + e_j^t}{b_j^t + f_j^{t-1}}) - \frac{1}{2}(\frac{d_j^t + f_j^t}{a_j^t + e_j^{t-1}}) + \frac{1}{2(b_j^t + f_j^{t-1})}[y_j^t + k_j^t + c_j^t + e_j^t + x_j^t(\frac{c_j^t + e_j^t}{b_j^t + f_j^{t-1}})$$

$$+ k_j^{t-1}(\frac{c_j^t + e_j^t}{b_j^t + f_j^{t-1}}) + b_j^t(\frac{c_j^t + e_j^t}{b_j^t + f_j^{t-1}}) + f_j^{t-1}(\frac{c_j^t + e_j^t}{b_j^t + f_j^{t-1}})] * Ln(\frac{x_j^t + k_j^{t-1}}{x_j^t + k_j^{t-1} + b_j^t + f_j^{t-1}}) + \frac{1}{2(a_j^t + e_j^{t-1})}[y_j^t + d_j^t + k_j^t + f_j^t$$

$$+ x_j^t(\frac{d_j^t + f_j^t}{a_j^t + e_j^{t-1}}) + k_j^{t-1}(\frac{d_j^t + f_j^t}{a_j^t + e_j^{t-1}}) - a_j^t(\frac{d_j^t + f_j^t}{a_j^t + e_j^{t-1}}) - e_j^{t-1}(\frac{d_j^t + f_j^t}{a_j^t + e_j^{t-1}})] * Ln(\frac{x_j^t + k_j^{t-1}}{x_j^t + k_j^{t-1} - a_j^t - e_j^{t-1}})\} \tag{11}$$

$$* \frac{1}{4\pi^2 \sigma_j^t \bar{\sigma}_j^t \bar{\bar{\sigma}}_j^t \bar{\bar{\sigma}}_j^{t-1}} \exp(-\frac{(x_j^t - \mu_j^t)^2}{2\sigma_j^{2^t}} - \frac{(y_j^t - \bar{\mu}_j^t)^2}{2\bar{\sigma}_j^{2^t}} - \frac{(k_j^t - \bar{\bar{\mu}}_j^t)^2}{2\bar{\bar{\sigma}}_j^{2^t}} - \frac{(k_j^{t-1} - \bar{\bar{\mu}}_j^{t-1})^2}{2\bar{\bar{\sigma}}_j^{2^{t-1}}}) dk_j^{t-1} dk_j^t dy_j^t dx_j^t$$

So, the initial proposed MOFS-DEA model (9) was transformed to its equivalent multi-objective stochastic programming model as follows:

$$\begin{aligned} & Max: Z_1^t \\ & \vdots \\ & Max: Z_n^t \\ & \quad st: \\ & \quad g_j^t(U^t, V^t, \rho^t, \beta^{t-1}) \geq 1 - \alpha_j^t \quad j = 1,2,\dots,n \\ & \quad U^t, V^t, \beta^{t-1}, \rho^t \geq \varepsilon \quad j = 1,2,\dots,n \quad t = 1,..,T \quad i = 1,..,m \quad r = 1,..,s \quad l = 1,..,L \end{aligned} \tag{12}$$

where $g_j^t(u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1})$ and Z_j^t are determined by (10) and (11), respectively.

5.3. Final representation of MOFS-DEA model

The proposed equivalent multi-objective stochastic model (12) has "n" objective function in each period and is established by individually maximizing the efficiency of each DMU which consumes much computational time. It can be converted to equivalent one objective stochastic model by using infinite-norm approach, so model (12) can be rewritten as following:

$$
\begin{aligned}
& Max: Z^t = w + A \\
& \quad st: \\
& \quad g_j^t(U^t, V^t, \rho^t, \beta^{t-1}) \geq 1-\alpha_j^t \quad j=1,2,...,n \\
& \quad Z_j^t \geq w \quad j=1,2,...,n \\
& \quad \left.\begin{aligned}
& u_r^t \geq w_r A \\
& u_r^t \leq 1-(1-w_r)A \\
& v_i^t \geq w_i' A \\
& v_i^t \leq 1-(1-w_i')A \\
& \rho_l^t \geq w_l'' A \\
& \rho_l^t \leq 1-(1-w_l'')A \\
& \beta_l^{t-1} \geq w_l''' A \\
& \beta_l^{t-1} \leq 1-(1-w_l''')A
\end{aligned}\right\} \quad (\mathrm{I}) \\
& \quad 0 \prec w \leq 1 \\
& \quad 0 \prec A \leq 1 \\
& \quad U^t, V^t, \beta^{t-1}, \rho^t \geq \varepsilon \quad j=1,2,...,n \quad t=1,..,T \quad i=1,..,m \quad r=1,..,s \quad l=1,..,L
\end{aligned}
\tag{13}
$$

Where w is the percent which all DMUs reach to optimum efficiency level. Also (I) shows the weights control constraints and A is the real number as $0 \prec A \prec 1$. On the other hand, w_r, w_i', w_l'' and w_l''' are preferred weight for inputs and outputs which are determined by the decision makers. Model (13) is the equivalent one objective stochastic model or the final representation of proposed MOFS-DEA model. Since the stochastic constraints (10) in this model are still in form of the integral, we cannot solve the proposed model (13) via the exact optimization algorithms. In order to overcome the difficulty, in the next section we design a new hybrid algorithm to solve it by incorporating ICA and MC simulation, in which MC simulation will employ to compute the integrals involved in the constraints and ICA will use to find the optimal of problem.

6. Solution methodology

According to Atashpaz-Gargari and Lucas (2007), ICA is a new socio-politically motivated global search strategy that has recently been introduced for dealing with different optimization tasks. Hence, in addition to develop mathematical model, comparing the results to other well-known techniques in terms of solution quality and running time is also considered in this paper. So, ICA can be considered as a very practical tool to solve and exloring the solution space of complex problems such as the MOFS-DEA model, successfully. The procedure of proposed hybrid algorithm for solving the final MOFS-DEA model (13) is summarized as follows.

6.1. Solution representation

Each solution in the ICA is in a form of an array. Each array consists of variables which should be optimized. This array is called country. Suppose there are pop-size countries in the population, representing the solutions of the final proposed MOFS-DEA model (13), in which the decision variables include $u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1} \in (0,1)$ for each i (i=1,..,m), r (r=1,..,s) and l (l=1,..,L) and $R^t = [u_1^t,...,u_s^t, v_1^t,...,v_m^t, \rho_1^t,...,\rho_l^t, \beta_1^{t-1},...,\beta_l^{t-1}] \in (0,1)$ is characterized as a country to show a decision array at period t.

6.2. Initialization process

Generate randomly $u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1}$ from the interval (0,1). Compute $g_j^t(u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1})$ and Z_j^t via formulas (10) and (11) respectively, where the integrals which applied in Z_j^t are approximated by MC simulation that will discribe in next section. If R^t satisfies the constraints, then it is feasible and take it as an initial country. Repeat this process until pop-size initial feasible countries $R_1^t, R_2^t, \ldots R_{pop-size}^t$ are produced. After generating countries, the best solutions are selected from population as the imperialists and the remaining countries are colonies. For calculating the cost value of each imperialist, the value of objective function is obtained for each imperialist. Then, the cost value for objective function is computed by:

$$C_n = \frac{W_n^t - W_{best}^t}{W_{\max}^t - W_{\min}^t} \tag{14}$$

where C_n is the normalized value of objective function for imperialist n. $W_{best}^t, W_{\max}^t$ and $W_{\min}^t$ are the best, maximum and minimum values of the objective function in each iteration, respectively. The power of each imperialist is calculated after obtaining the normalized cost as shown below and the colonies distributed among the imperialist according to power of each imperialist country.

$$P_n = \left| C_n / \sum_{i=1}^{N_{imp}} C_i \right| \tag{15}$$

Then, the initial number of colonies of an empire will be as follows:

$$NC_n = round\{P_n . N_{col}\} \tag{16}$$

where NC_n is the initial number of colonies of the *n*-th imperialist, and N_{col} is the number of all colonies. We randomly select NC_n of colonies and give them for each imperialist.

6.3. Moving the colonies of an empire toward the imperialist

After dividing colonies between imperialists, colonies are moved toward their related imperialist. This movement is shown in Fig. 2, in which X is the distance between colony and imperialist.

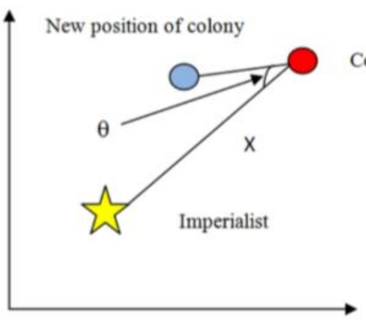

Fig. 2. Moving colonies toward the imperialist with a random angle θ

α is a random variable with a uniform distribution between 0 and β × X , in which β is a number greater than 1. The direction of movement is shown by θ, which is a uniform distribution between - γ and γ.

6.4. Exchanging positions of the imperialist and a colony

While moving toward the imperialist, a colony might reach to a place with lower cost than the imperialist. In this case, the imperialist and the colony change their positions.

6.5. Total power of an empire

The total power of an empire is calculated as below:

$$TC_n = \cos t(imperialist_n) + \zeta \times mean(colonies \quad of \quad empire_n) \tag{17}$$

where TC_n denotes the total cost of the *n*-th empire and zeta (ζ) is positive number which is less than 1. The cost of imperialist and colonies are calculated by Equations (14) and (15).

6.6. Imperialistic competition

All empires competition is to take the possession of the weakest colony of the weakest empire. This competition is modeled by just selecting one of the weakest colonies of the weakest empires and then for calculating the possession probability of each empire first the normalized total cost is obtained as follows:

$$NTC_n = \max\{TC_i\} - TC_n \tag{18}$$

where NTC_n is the normalized total cost of *n*-th empire and TC_n is the total cost of *n*-th empire. Having the normalized total cost, the possession probability of each empire is calculated by:

$$P_{pn} = \left| NTC_n / \sum_{i=1}^{N_{imp}} NTC_i \right| \tag{19}$$

6.7. Eliminating the powerless empires

Powerless empires will collapse and their colonies are distributed among other empires in the imperialistic competition. In this paper, when an empire loses its colonies, we consider it is collapsed.

6.8. Stopping criteria

In the proposed model, the state in which there is only one empire between all countries is considered as stopping criterion.

6.9. Monte Carelo simulation

MC simulation is a method to deal with the stochastic behavior in complex systems (Chuen , Kuan and Wai 2012). In order to solve the final proposed MOFS-DEA model (13), for any given solution (R^t), we need to check its feasibility. Since the some constraints (11) include integrals which cannot solve via the conventional optimization algorithm, so we should approximate their values by MC simulation. This method, firstly changes variable in the integral to convert infinite interval to finite interval as following (Gander and Gautschi 2000):

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-1}^{+1} f\left(\frac{h}{1-h^2}\right)\frac{1+h^2}{(1-h^2)^2}dh \tag{20}$$

So, the process of MC simulation is described as follows:

Procedure: MC simulation for approximating (20)
begin

n←number-simulation
for i =1 to n do
hi←Generate a uniform distributed random point in the interval [a,b]=[-1,1]
Determine the average value of the function: $\hat{f} = \frac{1}{n} * \sum_{i=1}^{n} f(h_i)$
Compute the approximation to the integral: $\int_{-\infty}^{+\infty} f(x)dx \approx (b-a)\hat{f}$

end

By integrating ICA and MC simulation, we design a new hybrid algorithm (ICA-MC) for solving the proposed model (13) which summarized as follows:

- Initialize the empires and their colonies positions randomly whose feasibility must be checked by MC simulation in the constraints of model (13). .
- Moving the colonies towards the imperialist's position.
- Compute the total cost of all empires. Pick the weakest colony (colonies) from the weakest empire and give it (them) to the empire that has the most likelihood to possess it (Imperialistic competition).
- Eliminate the powerless empires.
- If there is just one empire, then stop else continue.
- Check the termination conditions.
- Select the best country as an optimal solution.

7. Practical example

This section compares computational efforts of the proposed model and ICA-MC algorithm with the results of hybrid GA algorithm which was proposed by Qin and Liu (2010) in terms of solution quality and runtime. We used the real example which was proposed by Yaghoubi, Amiri and Safi (2015) in their paper as a benchmark. In this real example, the outputs and inputs are characterized by random triangular fuzzy variables; like the outputs and inputs in the proposed model in this paper. The problem consists of five Gas stations (DMUs) with two input parameters, one output parameter, and one quasi-fixed input parameter. It is assumed that there are two prediction periods for efficiency of DMUs (t=1, 2): Autumn (first period) and Winter (second period) of 2013.

To present the effectiveness of the solution methodolgy and the modeling idea, the final proposed MOFS-DEA model (13) and ICA-MC algorithm are used to predict the efficiency of five DMUs. Table 2 documents the predicted efficiency values under $\alpha = 0.5$ for the next two periods. This practical experiments are performed on a personal computer, using the Microsoft Windows 7 operating system, and ICA-MC algorithm is written by C++ programming language. In this paper, appropriate tuning of the parameters has been carried out for optimizing the behavior of the proposed algorithm. For this purpose, response surface methodology (RSM) is employed. RSM is defined as a collection of mathematical and statistical method-based experiential, which can be used to optimize processes (Azizmohammadi et al. 2013). So the tuned parameters of the ICA-MC algorithm are obtained as follows: pop-size=30, B=0.65, γ=0.5, N_{imp}=5. From the solution results for each period, we can see the information about each DMU. For example in the first period, DMU2 has the biggest α-expected efficient value 0.991, followed by DMU4, DMU1, DMU3 and DMU5.

Table 2. The predicted efficiency scores for both periods under $\alpha = 0.5$ in June 2013

Period	DMU	Optimal solution ($u_1^t, v_1^t, v_2^t, \rho_1^t, \beta_1^{t-1}$)	α-expected efficient value
t=1	1	(0.281 , 0.102 , 0.874 , 0.341 , 0.178)	0.964
	2	(0.478 , 0.162 , 0.741 , 0.821, 0.103)	0.991
	3	(0.251 , 0.625 , 0.881 , 0.373 , 0.425)	0.962
	4	(0.654 , 0.431 , 0.879 , 0.332 , 0.834)	0.988
	5	(0.439 , 0.286 , 0.8113 , 0.956 , 0.161)	0.9
t=2	1	(0.352 , 0.135 , 0.828 , 0.199 , 0.171)	0.96
	2	(0.521 , 0.207 , 0.606 , 0.824 , 0.154)	0.997
	3	(0.198 , 0.725 , 0.902 , 0.474 , 0.524)	0.912
	4	(0.534 , 0.469 , 0.812 , 0.238 , 0.756)	0.922
	5	(0.544 , 0.072 , 0.518 , 0.806 , 0.105)	0.978

Table 3 presnts the total amount of real inputs and outputs for the Autumn and Winter of 2013 which was gathered in January 2014 in order to survey whether the predicted efficiencies are different from actual efficiency values. So, the results of actual efficiencies that have been achieved by conventional dynamic CCR model (7) and real data which are shown in Table 3 for DMUs. Generally there are three types of classification: (a) $\alpha \prec 0.5$ is conservative, (b) $\alpha = 0.5$ is risk-natural and (c) $\alpha \succ 0.5$ is risk-taking in DEA. It is easily thought that the conventional use of DEA belongs to the risk-natural (Nemoto and Goto 2003). This finding can be easily confirmed by comparing the actual efficiencies with the predicted efficiencies under $\alpha = 0.5$. The two approaches exhibit very similar results on ranks scores and efficiencies.

Table 3. The real inputs and outputs of DMUs with their actual and predicted efficiencies for both periods

Period	DMU_j	Inputs			Outputs		Actual efficiencies	Predicted efficiencies	Actual ranks	Predicted ranks
		Employeis salaries	Operation costs	Net profits	Gasoline	Net profit				
t=1	1	4.01	2.15	7.85	3.95	8.16	0.98	0.964	2	3
	2	3.02	1.51	7.11	3.41	7.41	1	0.991	1	1
	3	4.6	2.34	9.07	4.91	9.52	1	0.962	1	4
	4	4.11	2.11	8.91	4.55	9.24	1	0.988	1	2
	5	5.12	3.03	9.89	5.19	10.2	0.96	0.9	3	5
t=2	1	4.12	2.21	8.16	4.01	8.61	1	0.96	1	3
	2	3.11	1.42	7.41	3.75	7.79	1	0.997	1	1
	3	4.78	2.48	9.52	4.61	9.68	0.97	0.912	3	5
	4	4.15	2.15	9.24	5.3	9.39	0.98	0.922	2	4
	5	5.28	3.22	10.2	5.92	10.6	1	0.978	1	2

For example in second period from table 3, three DMUs (the 1th, 2th, 5th gas stations) are efficient based on real efficiencies and have been in the first place, while they are in the first to third place and separated based on the predicted efficiency scores. Also, the high Pearson correlation rates have obtained (0.91 and 0.92) for both periods between predicted and real efficiencies under $\alpha = 0.5$.

In order to show the validation of the results, we compare these results with the results of the similar hybrid GA algorithm which was proposed by Qin and Liu (2010). The computational results of the predicted efficiency scores for DMU1 are reported in Table 4, in which parameter "CPU(s)" is the computational time consumed by the two hybrid algorithms to get (near) optimal predicted efficiency score (Z_1^*). It can be from Table 4 that the proposed hybrid algorithm solves all instances optimally in average of less than 61s of CPU time requirement for both periods. As a result, we conclude that the designed hybrid algorithm outperforms the Qin's hybrid GA algorithm in terms of CPU time. In addition, the predicted efficiencies are closer than the results of the Qin's hybrid algorithm to the actual efficiency scores.

Table 4. Comparison between the results of ICA-MC and Qin's hybrid GA algorithms

Period	α	**ICA-MC algorithm**		**Qin's hybrid GA algorithm**	
		Z_1^*	CPU(s)	Z_1^*	CPU(s)
t=1	0.05	0.823	51.301	0.784	107.412
	0.1	0.841	50.147	0.816	108.521
	0.2	0.868	52.332	0.847	107.217
	0.5	0.964	50.124	0.881	109.698
	0.8	0.962	53.418	0.901	110.665
Average		0.891	51.464	0.846	108.701
t=2	0.05	0.921	59.124	0.775	108.701
	0.1	0.942	58.412	0.796	107.942
	0.2	0.956	60.663	0.824	109.432
	0.5	0.960	62.127	0.897	110.121
	0.8	0.979	61.202	0.932	110.789
Average		0.951	60.305	0.845	109.397

Fig. 3 shows the comparison between the actual and predicted efficiencies under $\alpha = 0.5$ for DMU1 in the first period.

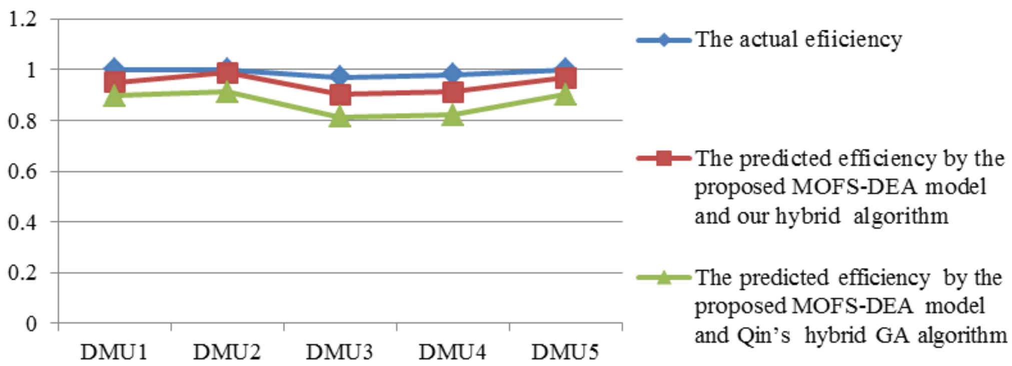

Fig. 3. Comparison between real efficiencies with predicted efficiencies

According to study which was proposed by Qin and Liu (2010), to further test the effectiveness of the proposed hybrid algorithm, a careful variations about the parameters B and γ in ICA-MC is made in view of the identification influence on the solution quality for DMU1 under $\alpha = 0.5$ at the first period (t=1). The computational results are collected in Table 5. To compare these results, we give the relative error as follows:

$$\frac{\text{optimal } \alpha\text{-expexted efficient value - actual } \alpha\text{-expexted efficient value}}{\text{optimal}\,\alpha\text{-expexted efficient value}}\times 100\% \tag{21}$$

Table 5. Comparison the solutions for DMU1 under different ICA's parameters in first period under $\alpha = 0.5$

β	γ	α-expected efficient value	Realative error (%)
0.2	0.3	0.959	0.5
0.3	0.4	0.948	1.7
0.5	0.2	0.915	5
0.6	0.1	0.930	3.5
0.7	0.6	0.964	0

where the optimal α-expected efficient value is the maximum one of the five α-expected efficient values in Table 5. Generally, findings from the above tables can be summarized as follows:

Finding 1: Table 3 shows the high correlation rates have obtained for both periods (0.91 and 0.92) between predicted and real efficiencies; it can represents the validity of the proposed MOFS-DEA model.

Finding 2: The comparsion between predicted and real efficienies in Table 3 reveals significant improvement in discriminating power between efficient DMUs.

Finding 3: Table 4 shows that the designed ICA-MC algorithm outperforms the Qin's hybrid GA algorithm in terms of solution quality and CPU time.

Finding 4: It can be shown from Table 5 that the relative errors do not exceed 5%, which implies that the designed ICA-MC algorithm is robust for parameters selection.

8. Conclusions

This paper attempted to present a new multi-objective fuzzy stochastic DEA model (MOFS-DEA) to predict efficiency of DMUs for the next financial periods. The previous researches have not proposed any model based on multi-objective SDEA model with using imprecise data in a dynamic environment to predict performance of DMUs, however, in this paper provided to get closer to real-world problems. The major results of the paper include the following several aspects: (i) A new multi-objective DEA model was built in uncertain and dynamic environment, in which the outputs and inputs are characterized by random traingular fuzzy variables. Under this assumption, the initial proposed MOFS-DEA model transformed to its equivalent multi-objecyive stochastic model. (ii) In order to simplify during the solution process, the equivalent multi objecyive stochastic model converted to one objective stochastic model by infinite-norm approach. (iii) To solve the final MOFS-DEA model, this paper designed a new hybrid meta-hueristic algorithm by integrating ICA and Monte Carlo simulation. Finally, the MOFS-DEA model was used to predict efficiencies for five gas stations in an Iranian petroleum company for the next two periods. The results showed that the designed ICA-MC algorithm outperforms the hybrid GA algorithm which was proposed by Qin and Liu (2010) in terms of solution quality and CPU time. It is suggested to be used another continuous distribution (other than normal) for the inputs (outputs) of DMUs in modeling. Also, we used ICA algorithm as an evolutionary algorithm to optimize the proposed model, but it is possible to use another evolutionary algorithm such as GA, SA, TS, ACO, that it can future research task.

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