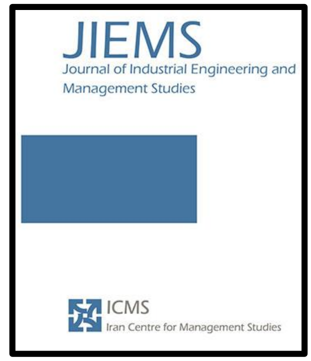
JIEMS

Journal of Industrial Engineering and Management Studies

Vol. 2, No. 2, pp. 55-82

www.jiems.icms.ac.ir



A mixed integer linear programming formulation for a multi-stage, multi-Product, multi-vehicle aggregate production-distribution planning problem

H. Mokhtari[1,*]

Abstract

In today's competitive market place, companies seek an efficient structure of supply chain so as to provide customers with highest value and achieve competitive advantage. This requires a broader perspective than just the borders of an individual company during a supply chain. This paper investigates an aggregate production planning problem integrated with distribution issues in a supply chain so as to simultaneously optimize characteristics of these supply chain drivers. The main contribution of this paper is to consider the aggregate production-distribution planning (APDP) problem jointly with multiple stage, multiple product, and multiple vehicle. Moreover, we considered both routing and direct shipment as transportation system which is not considered in APDP literature so far. A mixed-integer linear programming formulation is suggested for two distinct Scenarios: (i) when we have direct shipment in which all shipments are transported directly from manufacturer to customers, and (ii) when we have routing option in which the vehicles can move through routes to deliver products to more than one customer at a trip. A numerical analysis is performed to compare performance of problem in two above Scenarios. Moreover, to assess applicability of problem, some computational experiments are implemented on small, medium and large sized problems.

Keywords: Mixed-Integer Programming; Production Planning; Production-Distribution, Transportation; Vehicle Routing; Setup Times.

Received: June 2015-29

Revised: July 2015-25

Accepted: November 2015 -22

[*] Corresponding Author.

[1] Department of Industrial Engineering, University of Kashan, Kashan, Iran.

1. Introduction

The Aggregate Production Planning (APP) which is a class of mid-term planning can be defined as delineation of production quantity, inventory size and workforce level during a finite planning horizon. The APP can be carried out without need to get detailed material and capacity resource requirements for individual products. An APP can be categorized as a decision-making problem in tactical level of supply chain. Since minimum amount of detailed data is required in APP, it enables planners to update plan more frequently, and compensate disruptions occurring in product demand, costs, capacity and material supply. Because APP is one of the most critical areas of supply chain planning, it has attracted many attentions.

One of the first studies on APP was presented by Holt et al. (1955). In order to establish a production plan with actual operational costs of a real paint factory, he investigated APP models. A pharmaceutical case was evaluated by Ashayeri and Selen (2003) considering an APP with strategic planning. Moreover, Wang and Liang (2004) investigated a multi-product APP in a fuzzy environment with a fuzzy multi-objective linear programming approach. Additionally, Wang and Liang (2005) presented a multi-objective APP with imprecise demand, and suggested an interactive possibilistic programming approach. Jain and Palekar (2005) introduced a configuration-based formulation with dissimilar machines and production lines. Jamalnia and Soukhakian (2009) considered an APP in a fuzzy environment and developed a hybrid fuzzy multi-objective nonlinear model. Moreover, Zhang et al. (2012) developed a hybrid heuristic for solving a mixed-integer linear model with capacity expansion and multiple centers. Ghasemi Yaghin et al. (2012) devised a hybrid fuzzy multiple-objective model for solving an integrated pricing and APP in a multi-period, multi-product environment. Karmarkar and Rajaram (2012) proposed a competitive extension of APP problem for process industries.

To the best of our knowledge, as can be seen, the majority of research has studied the optimization of production plans inside a manufacturing company, and ignored distribution of products to customers. In a vast amount of researches, APP problem has been investigated in detail but without taking distribution into account. A new approach is to analyze production and distribution problems based on integration of decisions in production and distribution contexts in a two-echelon supply chain. However, the literature dealing with coordination problems between production and distribution is still scarce (Boudia et al. 2007). Fahimnia et al. (2013) presented a systematic literature review on distribution problems of produced items. Lee and Kim (2002) developed a linear model for production-distribution problem minimizing production, distribution, inventory, and shortage costs. Chan et al. (2005) considered a multi-factory production-distribution problem with a hybrid genetic algorithm and analytic hierarchy process. Demirli and Yimer (2006) suggested a mixed-integer fuzzy programming for a build-to-order environment with uncertain inventory and shortage costs. Besides, Rizk et al. (2006) investigated a mixed-integer model for planning problem of a manufacturing location and a distribution center with parallel machines and transportation costs. Nishi et al. (2007) introduced Lagrangian decomposition for a distributed production and distribution. Coronado (2008) studied a model with uncertainty at suppliers’ capacity with regular-time and overtime production. In another study, Hamedi et al. (2009) constructed a mixed-integer model to analyze a real-world gas industry with multi-period and single-product.

Recently, Raa et al. (2013) developed a matheuristic for solving an aggregate production–distribution problem for a producer of plastic products.

Table 1 shows a comparison among previous literature and present work at a glance. As can be seen, one of the major shortcomings in all of the above studies is that they just consider direct shipment of products to customers in generating distribution plan, and then discard vehicle routing aspects. The author could find very few papers such as Boudia et al. (2007) and Bard and Nananukul (2009) that considered production and routing decisions simultaneously. Not only the literature dealing with joint production planning and routing is very scarce, but also they just considered simplified models with single-product, single-vehicle and single-stage. Hence, to overcome these shortcomings, we will present a generalized production and distribution in current paper and formulate it as mixed integer linear programming problems. There are many successful applications of mathematical programming formulation for addressing scheduling problems in literature (e.g., Wong et al., 2012; Karimi-Nasab and Fatemi Ghomi, 2012; Ma et al. 2013; Wong et al., 2014).

The reminder of this paper is organized as follows. Section 2 defines suggested APDP problem. The mathematical formulations are presented in Section 3. Section 4 describes a numerical analysis. Finally Section 5 concludes paper.

Table 1. Characteristics of existing models

Paper	products		period		stage		vehicles		customers		setup times		transportation	
	single	multiple	single	multiple	single	multiple	single	multiple	single	multiple	machine	vehicle	direct	routing
Lee and Kim (2002)	✓			✓	✓		✓			✓			✓	
Chan et al. (2005)	✓			✓	✓		✓			✓			✓	
Demirli and Yimer (2006)	✓			✓	✓		✓			✓			✓	
Rizk et al. (2006)	✓			✓	✓		✓		✓				✓	
Nishi et al. (2007)	✓			✓	✓			✓		✓			✓	
Coronado (2008)	✓			✓	✓		✓		✓				✓	
Hamedi et al. (2009)	✓			✓	✓		✓			✓			✓	
Raa et al. (2013)		✓	✓		✓		✓			✓			✓	
Boudia et al. (2007)	✓			✓	✓		✓			✓				✓
Bard and Nananukul (2009)	✓			✓	✓			✓		✓				✓
Aliev, Rafik A., et al. (2007)	✓			✓	✓		✓		✓				✓	
Niknamfar, et al., (2015)		✓		✓	✓		✓			✓			✓	
Perumal et al., (2013)	✓			✓	✓		✓		✓				✓	
Pathak and Sarkar (2012)		✓		✓	✓		✓		✓				✓	
Moghaddam et al., (2012)	✓		✓		✓		✓		✓				✓	
Moattar Husseini et al. (2015)		✓		✓	✓		✓			✓	✓		✓	
Torabi and Moghaddam (2012)		✓		✓	✓		✓		✓		✓		✓	
Proposed research		✓		✓		✓		✓		✓	✓	✓	✓	✓

2. Scope of APDP problem

In this section, the Aggregate Production-Distribution Planning (APDP) problem suggested in current paper will be described. This problem is relevant to integration between production planning and vehicle transportation with multi-stage, multi-product and multi-vehicle systems in a two-echelon supply chain. The production part of our APDP is an extended version of classical APP problem with several machines and setup decisions. A manufacturer seeks a cost effective production level, inventory level, backorder level, and overtime and subcontract productions. It is also interested to find best workforce level including number of hired and laid-off workers. In addition, each product has a setup on machines. At each stage of production system, a number of products received from last stage and some other purchased parts are assembled to establish one unit of a product in current stage. The production system in our APDP is depicted in Figure 1.

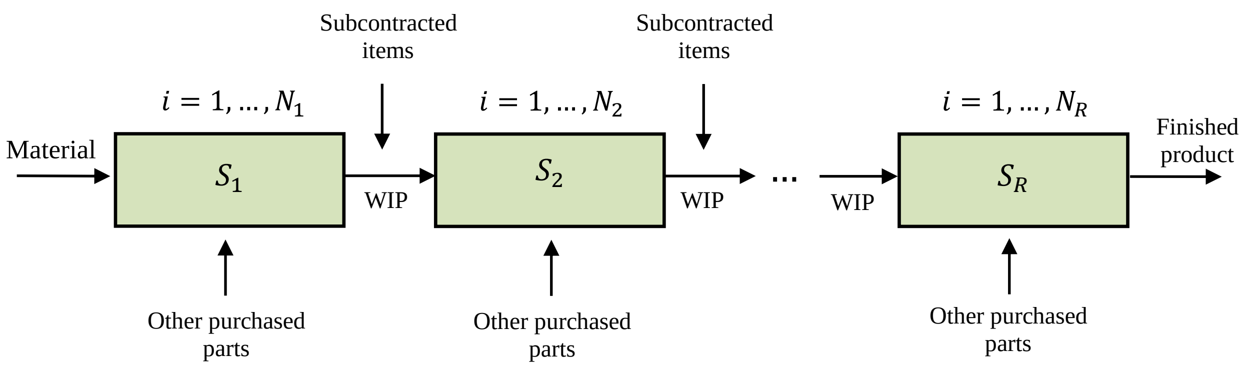


Figure 1. Production stages within a specific time period

After last stage is completed, finished product is transported to customers by vehicles. A maximum number of vehicles, with limited capacities, are available to transport products from manufacturer to customers. In addition to direct shipment, vehicles can moved through a route in order to deliver products to more than one customer during a trip. It is also assumed that each vehicle is responsible for satisfying demand of all customers on route. A route is a sequence of customers in which a vehicle delivers product from a single manufacturer to multiple customers. A customer should be visited before its due date time. The route time of a vehicle is the sum of total traveling time and service time taken to visit on-route customers. The main characteristics of proposed problem are given in Table 2.

Table 2. Main characteristics of proposed APDP problem

Feature	Proposed APDP Model
Number of products	Multiple
Number of periods	Multiple
Number of stages	Multiple
Number of vehicles	Multiple
Number of customers	Multiple
Backorder	Allowed
Routing delivery	Allowed
Subcontracting option	Considered
Overtime option	Considered
Labor hiring	Considered
Labor lay off	Considered
Machine setup times	Considered
Vehicle setup times	Considered
Machine capacity	Considered
Workforce legal limitations	Considered
Objective components (Costs):	
Production	Considered
Overtime	Considered
Subcontract	Considered
Inventory	Considered
Setup time	Considered
Backorder	Considered
Workforce	Considered
Transportation	Considered
Vehicles	Considered

3. MILP formulation

In this section, suggested APDP problem will be formulated. In order to cope with complexity of problem, we formulate suggested APDP via two Scenarios. At first, a situation is considered in which manufacturer makes direct shipments to each customer and routing decision is not incorporated. Next, we investigate second Scenario in which vehicles move through a route to deliver products to more than one customer. The parameters and variables are summarized below.

Problem parameters

i, j	Index of product type $i, j = 1, \dots, N_k$
k	Index of production stage $k = 1, \dots, R$
t	Index of time period $t = 1, \dots, T$
m	Index of machine $m = 1, \dots, M$
q	Index of part type required for producing products $q = 1, \dots, Q$
n, c, d, e	Index of customers $n, c, d, e = 1, \dots, N$
ξ	Index of routs and associated vehicles $\xi = 1, \dots, E$
CV	Maximum capacity of each vehicle
ϖ	Vehicle setup time factor
θ_n	Service time of customer n
du_n^t	Preferred due date of customer n at period t
t_{cd}	Travel time between customers c and d
$\rho^{\xi t}$	Vehicle setup time associated with route ξ at period t
D_{it}^n	Demand of customer n for product type i at period t
TC_v	Transportation cost of vehicle v (Scenario I)
TC_ξ	Transportation cost of vehicle associated with route ξ (Scenario II)
FC_ξ	Fixed cost of vehicle associated with route ξ
MTT	Maximum travel time allowed for each vehicle (units of time at each period)
f	Working hours of a worker at each stage of each period
p_{it}^k	Regular time production cost of product type i in stage k of period t

o_{it}^{k}	Over time production cost of product type i in stage k of period t
c_{it}^{k}	Subcontracting cost of product type i in stage k of period t
h_{it}^{k}	Inventory cost of product type i at end of stage k in period t
b_{it}^{k}	Backorder cost of product type i in stage k of period t
ws_{t}^{k}	Salary cost of a worker in stage k of period t
wt_{t}^{k}	Cost to hire one worker in stage k of period t
wl_{t}^{k}	Cost to lay off one worker in stage k of period t
H_{tk}^{max}	The maximum workforce level available for hiring in stage k of period t
W_{tk}^{max}	The maximum overall workforce level in stage k of period t
W_{t}^{max}	The maximum overall workforce level in period t
L^{max}	The maximum workforce level can be laid off during μ periods
r_{it}^{mk}	The setup cost of product type i on machine m in stage k of period t
A_{ij}^{k}	Number of product type i in stage k that is needed for producing one unite of product type j in stage $k+1$
σ_{ik}^{q}	Number of other purchased part type q that is needed for producing one unite of product type i in stage k
U_{tk}^{q}	Maximum available number of part type q in stage k of period t
d_{im}^{k}	Capacity of machine m that is needed for producing one unit of product type i in stage k
f_{im}^{k}	Capacity of machine m that is used for setting up in producing one unit of product type i in stage k
Ca_{mt}^{k}	Total available capacity of machine m in regular time of stage k in period t
β_{mt}^{k}	A fraction of capacity of machine m that is available in over time of stage k in period t

C_{tk}^{max}	Maximum number of products which can be subcontracted in stage k of period t
δ_{ik}^{space}	Amount of warehouse space occupied by one unit of product type i at end of stage k
P_{tk}^{space}	Maximum warehouse space that is available after stage k in period t
e_{it}^{k}	Worker hours required per unit of product type i in stage k of period t
γ_{t}^{k}	The ratio of regular time working hours of a worker available for use in over time in stage k of period t
$BigM$	An arbitrary big positive number

Problem variables

P_{it}^{k}	Units of product type i produced in regular time of stage k in period t
O_{it}^{k}	Units of product type i produced in over time of stage k in period t
C_{it}^{k}	Units of product type i subcontracted in stage k of period t
I_{it}^{k}	The inventory level of product type i at end of stage k in period t (WIP)
B_{it}^{k}	The backorder level of product type i in stage k of period t
W_{t}^{k}	The overall workforce level in stage k of period t
H_{t}^{k}	The number of workers hired in stage k of period t
L_{t}^{k}	The number of workers laid off in stage k of period t
Y_{it}^{mk}	A binary variable that indicates setup decision of product type i on machine m in stage k of period t
X_{vc}^{t}	A binary variable that indicates whether vehicle v is allocated to customer c at period t (Scenario I)
t_{c}^{vt}	Start time of service at customer c by vehicle v in period t (Scenario I)
$X_{cd}^{\xi t}$	A binary variable that indicates whether (c, d) is in route ξ at period t (Scenario II)
$Z_{c}^{\xi t}$	A binary variable that indicates whether customer c visited in route ξ at period t (Scenario II)

$t_c^{\xi t}$	Start time of service at customer c in route ξ of period t (Scenario II)
$t_0^{\xi t}$	Start time of route ξ at period t (Scenario II)
$V^{\xi t}$	A binary variable that indicates whether vehicle associated with route ξ is selected at period t (Scenario II)

3.1. Scenario I: APDP with Direct Shipment

The mixed integer linear programming (MILP) model for Scenario I of suggested APDP problem, called $\boldsymbol{P_1}$, is presented in this section. The objective function seeks to minimize total cost of production and transportation. At first we define the parts of production cost. The costs of regular time production, overtime production and subcontracting for all product types is as presented below.

$$\sum_{k=1}^{R}\sum_{i=1}^{N_k}\sum_{t=1}^{T}\left(p_{it}^k P_{it}^k + o_{it}^k O_{it}^k + c_{it}^k C_{it}^k\right) \tag{1}$$

The total inventory cost including work-in-process (WIP) inventory and finished product inventory costs is formulated as follows.

$$\sum_{k=1}^{R}\sum_{i=1}^{N_k}\sum_{t=1}^{T}\left(h_{it}^k I_{it}^k\right) \tag{2}$$

The setup costs for all product types on associated machines are formulated in following way.

$$\sum_{k=1}^{R}\sum_{m=1}^{M}\sum_{i=1}^{N_k}\sum_{t=1}^{T}\left(r_{it}^{mk} Y_{it}^{mk}\right) \tag{3}$$

Since real demand of customers is related to finished products in last stage, backorder quantity is associated to Rth stage of period. Therefore backorder cost is expressed as follows.

$$\sum_{i=1}^{N_R}\sum_{t=1}^{T}\left(b_{it}^R B_{it}^R\right) \tag{4}$$

Due to fluctuation of market demand at each period, it is necessary to determine how many additional workers are required to handle extra production or how many workers are required to be laid off to reduce overheads. Total workforce cost including salary, hiring and laying off cost is as follows.

$$\sum_{t=1}^{T}\sum_{k=1}^{R}\left(ws_t^k W_t^k\right)+\sum_{t=1}^{T}\sum_{k=1}^{R}\left(wt_t^k H_t^k\right)+\sum_{t=1}^{T}\sum_{k=1}^{R}\left(wl_t^k\, L_t^k\right) \tag{5}$$

Regarding production costs presented in (1)-(5), total production cost (TPC) is expressed as follows.

$$\begin{aligned} TPC = &\sum_{k=1}^{R}\sum_{i=1}^{N_k}\sum_{t=1}^{T}\left(p_{it}^k P_{it}^k + o_{it}^k O_{it}^k + c_{it}^k C_{it}^k\right) + \sum_{k=1}^{R}\sum_{i=1}^{N_k}\sum_{t=1}^{T}\left(h_{it}^k I_{it}^k\right) \\ &+ \sum_{k=1}^{R}\sum_{m=1}^{M}\sum_{i=1}^{N_k}\sum_{t=1}^{T}\left(r_{it}^{mk} Y_{it}^{mk}\right) + \sum_{i=1}^{N_R}\sum_{t=1}^{T}\left(b_{it}^R B_{it}^R\right) \\ &+ \sum_{t=1}^{T}\sum_{k=1}^{R}\left(ws_t^k W_t^k\right)+\sum_{t=1}^{T}\sum_{k=1}^{R}\left(wt_t^k H_t^k\right)+\sum_{t=1}^{T}\sum_{k=1}^{R}\left(wl_t^k\, L_t^k\right) \end{aligned} \tag{6}$$

We next define direct shipment transportation costs. The transportation cost which is associated to total transportation among customers and manufacturer can be formulated as follows.

$$\sum_{t=1}^{T}\left(TC_v \sum_{v=1}^{E}\sum_{c=1}^{N}\left(t_{0c}+t_{c0}\right)X_{vc}^t\right) \tag{7}$$

Here t_{0c} and t_{c0} represents transportation time from manufacturer to customer c and vice versa. Another important component of transportation cost is related to fixed cost of vehicles. Since number of vehicles is assumed to be determined in model, variable V^{vt} is defined to delineate whether corresponding vehicle is selected or not. Therefore total cost of vehicles, at all periods, is defined as follows.

$$\sum_{t=1}^{T}\sum_{v=1}^{E}\left(FC_v\, V^{vt}\right) \tag{8}$$

Therefore, total transportation cost (TTC) incurred in Scenario I can be expressed as follows.

$$TTC = \sum_{t=1}^{T}\left(TC_v \sum_{v=1}^{E}\sum_{c=1}^{N}\left(t_{0c}+t_{c0}\right)X_{vc}^t\right)+\sum_{t=1}^{T}\sum_{v=1}^{E}\left(FC_v\, V^{vt}\right) \tag{9}$$

Regarding two components of objective function presented in (6) and (9), total objective function is to minimize $TPC + TTC$.

Next, we aim at defining constraints considered in APDP model as follows.

$$P_{it}^{R} + O_{it}^{R} + C_{it}^{R} + B_{it}^{R} - B_{i(t-1)}^{R} + I_{i(t-1)}^{R} - I_{it}^{R} = \sum_{n=1}^{N} D_{it}^{n} \qquad (10)$$
$$i = 1, \dots, N_R, \;\; t = 1, \dots, T$$

The constraint set (10) ensures that total amount of finished products (last stage R) including regular time and overtime production, subcontracting, and inventory level be equal to sum of total demand of product from all customers at current period and backorders from previous periods.

$$P_{it}^{k} + O_{it}^{k} + C_{it}^{k} + I_{i(t-1)}^{k} - I_{it}^{k} = \sum_{j=1}^{N_{k+1}} A_{ij}^{k} \left(P_{jt}^{k+1} + O_{jt}^{k+1}\right) \qquad (11)$$
$$i = 1, \dots, N_k, \quad k = 1, \dots, R-1, \quad t = 1, \dots, T$$

The constraint set (11) is associated with requirement of production at each stage. It ensures an enough number of product type i in stage k needed for producing all products in stage $k+1$.

$$\sum_{i=1}^{N_k} \sigma_{ik}^{q} (P_{it}^{k} + O_{it}^{k}) \leq U_{tk}^{q} \;, \quad k = 1, \dots, R \;, \;\; t = 1, \dots, T \;, \;\; q = 1, \dots, Q \qquad (12)$$

The constraint set (12) indicates market limitation of parts required at each stage of periods.

$$\sum_{i=1}^{N_k} C_{it}^{k} \leq C_{tk}^{max} \;, \quad k = 1, \dots, R \;, \quad t = 1, \dots, T \qquad (13)$$

$$\sum_{i=1}^{N_k} \left(\delta_{ik}^{space} I_{it}^{k}\right) \leq P_{tk}^{space} \;, \quad k = 1, \dots, R \;, \quad t = 1, \dots, T \qquad (14)$$

The constraint sets (13) and (14) express quantities of subcontracting and required shortage space should not exceed maximum number of products which can be subcontracted and maximum warehouse space available, respectively.

$$P_{it}^{k} + O_{it}^{k} \leq BigM \sum_{m=1}^{M} Y_{it}^{mk} \;, \;\; i = 1, \dots, N_k \;, \;\; k = 1, \dots, R \;, \quad t = 1, \dots, T \qquad (15)$$

The constraint set (15) is relevant to setup decisions on machines. It expresses that it is not possible to produce any amount of a product (regular time and overtime) when none of corresponding machines are setup.

$$\sum_{i=1}^{N_k}\left(d_{im}^{k}P_{it}^{k}+f_{im}^{k}Y_{it}^{k}\right)\leq Ca_{mt}^{k}\,,\quad m=1,\ldots,M\,,\quad k=1,\ldots,R\,,\quad t=1,\ldots,T \tag{16}$$

$$\sum_{i=1}^{N_k}\left(d_{im}^{k}O_{it}^{k}\right)\leq \beta_{mt}^{k}Ca_{mt}^{k}\,,\quad m=1,\ldots,M\,,\quad k=1,\ldots,R\,,\quad t=1,\ldots,T \tag{17}$$

The constraint sets (16) and (17) indicate machine capacity. Constraint (16) ensures that sum of processing times and setup times on a specific machine should not be greater than its regular time capacity, and constraint (17) ensures that processing time of overtime production does not violate machine overtime capacity.

$$W_t^k = W_{t-1}^k + H_t^k - L_t^k\,,\quad k=1,\ldots,R\,,\quad t=1,\ldots,T \tag{18}$$

$$\sum_{i=1}^{N_k}\left(e_{it}^{k}P_{it}^{k}\right)\leq fW_t^k\,,\quad k=1,\ldots,R\,,\quad t=1,\ldots,T \tag{19}$$

$$\sum_{i=1}^{N_k}\left(e_{it}^{k}O_{it}^{k}\right)\leq f\gamma_t^k W_t^k\,,\quad k=1,\ldots,R\,,\quad t=1,\ldots,T \tag{20}$$

$$W_t^k \leq W_{tk}^{max}\,,\quad k=1,\ldots,R\,,\quad t=1,\ldots,T \tag{21}$$

$$H_t^k \leq H_{tk}^{max}\,,\quad k=1,\ldots,R\,,\quad t=1,\ldots,T \tag{22}$$

$$\sum_{k=1}^{R} W_t^k \leq W_t^{max}\,,\quad t=1,\ldots,T \tag{23}$$

$$\sum_{t=\lambda}^{\lambda+\mu}\sum_{k=1}^{R} L_\lambda^k \leq L^{max}\,,\quad \lambda=1,\ldots,T-\mu \tag{24}$$

The constraint sets (18)-(24) state workforce limitations incurred due to technical and legal considerations. The first constraint (18) indicates flow conservation of workers and defines number of workers for each stage at each period. The next constraint sets (19) and (20) express limitation of working hours in regular time and overtime. They guarantee that amount of hours required for production in regular time and overtime should not exceed maximum working hours available, respectively. The constraint set (21) restricts overall workforce level to be less than a maximum threshold, while constraint set (22) ensures that hired workforce for each stage in each period should do not exceed maximum workforce level available. The constraint set (23) presents limitation of total workforce level in each period. The constraint (24) indicates legal limitation to laying off workers during μ periods. The total number of workers laid off should not exceed a legal threshold.

$$\sum_{v=1}^{E} X_{vc}^{t} = 1 , \quad c = 1, \dots, N , \;\; t = 1, \dots, T \tag{25}$$

$$\sum_{c=1}^{N} X_{vc}^{t} = 1 , \quad v = 1, \dots, E , \;\; t = 1, \dots, T \tag{26}$$

The constraint set (25) ensures that demand of each customer should be satisfied at each period, and constraint set (26) enforces that each vehicle is assigned to exactly one customer.

$$\sum_{i=1}^{N} D_{it}^{c} X_{vc}^{t} \leq CV , \quad c = 1, \dots, N , \;\; t = 1, \dots, T , \;\; v = 1, \dots, E \tag{27}$$

The constraint set (27) guarantees that total demand of a customer satisfied by vehicle allocated to that customer should not exceed vehicle capacity.

$$t_{c}^{vt} \leq du_{c}^{t}\, X_{vc}^{t} \;\; , \;\; c = 1, \dots, N , \quad v = 1, \dots, E \;\; , \quad t = 1, \dots, T \tag{28}$$

The constraint set (28) enforces vehicles to start service of customers before due date.

$$t_{c}^{vt} \geq (\rho^{vt} + t_{0c}) X_{vc}^{t} , \quad c = 1, \dots, N , \quad v = 1, \dots, E \;\; , \quad t = 1, \dots, T \tag{29}$$

In order to consider setup operation of vehicles, start time of customer service should be greater than sum of vehicle setup time and transportation time between manufacturer and customer. This constraint is indicated by Eq. (29).

$$\rho^{vt} = \varpi \sum_{c=1}^{N} \theta_{c} X_{vc}^{t} \;\; , \;\; v = 1, \dots, E \;\; , \quad t = 1, \dots, T \tag{30}$$

The constraint set (30) calculates vehicle setup time as a fraction of sum of service times of all customers on route.

$$\sum_{c=1}^{N} X_{vc}^{t} \leq V^{vt}, \quad v = 1, \dots, E, \quad t = 1, \dots, T \tag{31}$$

The constraint (31) indicates whether a vehicle is employed in a period. If one of customers is allocated to vehicle v, vehicle v is employed ($V^{vt} = 1$). Otherwise, we have $V^{vt} = 0$.

$$\sum_{c=1}^{N} \left(X_{vc}^{t}(t_{0c} + t_{c0})\right) \leq MTT, \quad v = 1, \dots, E, \quad t = 1, \dots, T \tag{32}$$

The maximum travel time allowed for each vehicle is ensured by constraint set (32).

$$\begin{aligned} &P_{it}^{k} \geq 0,\ O_{it}^{k} \geq 0,\ C_{it}^{k} \geq 0,\ I_{it}^{k} \geq 0,\ B_{it}^{k} \geq 0, \\ &W_{t}^{k} \geq 0,\ H_{t}^{k} \geq 0,\ L_{t}^{k} \geq 0, t_{c}^{vt} \geq 0, \\ &Y_{it}^{mk} = (0,1),\ X_{cv}^{t} = (0,1),\ V^{vt} = (0,1) \\ \\ &i = 1, \dots, N_k, \quad t = 1, \dots, T,\ k = 1, \dots, R,\ m = 1, \dots, M \\ &v = 1, \dots, E,\ c = 0, \dots, N+1,\ d = 0, \dots, N+1 \end{aligned} \tag{33}$$

Finally, binary and non-negativity natures of variables are indicated by constraints in Eq. (33).

3.2. Scenario II: APDP with Routing

The mixed-integer linear programming (MILP) model for Scenario II of suggested APDP problem, called $\boldsymbol{P_2}$, will be presented in this section. As mentioned before, vehicles can move through a route to deliver products to more than one customer in Scenario II. Therefore, production part of Scenarios I and II is identical, and transportation part should be revised. The transportation issue is appeared in two sections of mathematical formulation, i.e., (i) second term of objective function (TTC), and (ii) transportation related constraints. In sequel, we define routing costs considered in objective function.

The first component of routing cost is total cost of transportation carried out from manufacturer to customers. This transportation cost which is associated to total transportation time and distance among manufacturer and customers on selected routes can be formulated as follows.

$$\sum_{t=1}^{T} \sum_{\xi=1}^{E} \left(TC_{\xi} \sum_{c=0}^{N+1} \sum_{d=0}^{N+1} \left(t_{cd} X_{cd}^{\xi t} \right) \right) \tag{34}$$

Another important component of routing cost is related to fixed cost of vehicles. Since number of vehicles is assumed to be determined in model, variable $V^{\xi t}$ is defined to delineate whether vehicle is selected or not. Therefore total cost of vehicles is presented as follows.

$$\sum_{t=1}^{T}\sum_{\xi=1}^{E}\left(FC_{\xi}\,V^{\xi t}\right) \tag{35}$$

Hence, total transportation cost (TTC) considered in Scenario II is expressed as follows.

$$TTC=\sum_{t=1}^{T}\sum_{\xi=1}^{E}\left(TC_{\xi}\sum_{c=0}^{N+1}\sum_{d=0}^{N+1}\left(t_{cd}X_{cd}^{\xi t}\right)\right)+\sum_{t=1}^{T}\sum_{\xi=1}^{E}\left(FC_{\xi}\,V^{\xi t}\right) \tag{36}$$

Here we formulate transportation related constraints in Scenario II. According to definition of two different Scenarios, the objective function and all the constraints presented in Eq.s (10)-(24) are identical in both Scenarios. The transportation related constraints for Scenario II are as follows.

$$\sum_{d=0}^{N+1}X_{cd}^{\xi t}=Z_{c}^{\xi t}\,,\quad c=1,\dots,N\;,\;\;\xi=1,\dots,E\;,\quad t=1,\dots,T \tag{37}$$

$$\sum_{\xi=1}^{E}Z_{c}^{\xi t}=1\;,\quad c=1,\dots,N\;,\quad t=1,\dots,T \tag{38}$$

$$\sum_{c=0}^{N+1}X_{ce}^{\xi t}-\sum_{d=0}^{N+1}X_{ed}^{\xi t}=0,\;\;e=1,\dots,N\,,\xi=1,\dots,E\,,\;\;t=1,\dots,T \tag{39}$$

$$\sum_{d=1}^{N+1}X_{0d}^{\xi t}=1\,,\quad \xi=1,\dots,E\;,\quad t=1,\dots,T \tag{40}$$

$$\sum_{c=1}^{N}X_{c(N+1)}^{\xi t}=1\,,\quad \xi=1,\dots,E\;,\quad t=1,\dots,T \tag{41}$$

$$\sum_{n=1}^{N}\sum_{i=1}^{N_R}\left(D_{it}^{n}Z_{n}^{\xi t}\right)\le CV\,,\quad \xi=1,\dots,E\;,\quad t=1,\dots,T \tag{42}$$

$$\begin{aligned}&t_{c}^{\xi t}+\theta_{c}+t_{cd}\le t_{d}^{\xi t}+BigM\left(1-X_{cd}^{\xi t}\right)\,,\;\;c=0,\dots,N+1\\&d=0,\dots,N+1\,,\quad \xi=1,\dots,E\;,\quad t=1,\dots,T\end{aligned} \tag{43}$$

$$t_c^{\xi t} \le du_c^t \, Z_c^{\xi t} \ , \ \ c = 1, \dots, N \ , \quad \xi = 1, \dots, E \ , \quad t = 1, \dots, T \tag{44}$$

$$t_0^{\xi t} \ge \rho^{\xi t} \ , \quad \xi = 1, \dots, E \ , \quad t = 1, \dots, T \tag{45}$$

$$\rho^{\xi t} = \varpi \sum_{c=1}^{N} \theta_c Z_c^{\xi t} \ , \ \ \xi = 1, \dots, E \ , \quad t = 1, \dots, T \tag{46}$$

$$\sum_{c=1}^{N} Z_c^{\xi t} \le N \, V^{\xi t} \ , \quad \xi = 1, \dots, E \ , \quad t = 1, \dots, T \tag{47}$$

$$\sum_{c=0}^{N+1} \sum_{d=0}^{N+1} \left(t_{cd} X_{cd}^{\xi t} \right) \le MTT \ , \quad \xi = 1, \dots, E \ , \quad t = 1, \dots, T \tag{48}$$

The constraint sets (37)-(48) are relevant to routing decisions in transportation of products from manufacturer to customers and substitute instead of Eq.s (25)-(32) in Scenario I to establish Scenario II. The constraint (37) defines relation between customer assignment variables $X_{cd}^{\xi t}$ and $Z_c^{\xi t}$. The constraint (38) ensures that each customer should be visited exactly once. Constraint (39) enforces each customer to be visited after and before other unique customers on a specific route. The constraints (40) and (41) define first and last customer on routes. The constraint (42) guarantees that total demand on a route should not violate vehicle capacity. The feasibility of vehicle schedules is ensured by constraints (43). The constraint (44) guarantees that customer demand should be met before due date. The constraint (45) restricts route start time to be greater than vehicle setup time. Constraint (46) calculates vehicle setup time. The constraint (47) indicates whether a vehicle is employed in a period or not. If at least one customer is selected for a specific route ξ in each period t ($\sum_{c=1}^{N} Z_c^{\xi t} \ge 1$), vehicle should be employed in that period ($V^{\xi t} = 1$). The constraint (48) ensures a maximum travel time is allowed for each vehicle. The variables $t_c^{\xi t}$ and $t_0^{\xi t}$ are non-negative, and variables $X_{cd}^{\xi t}$, $Z_c^{\xi t}$ and $V^{\xi t}$ are binary.

4. Numerical analysis

In previous section, proposed APDP problem was formulated as MILP models for two different Scenarios. In sequel, we investigate performance of formulations. For this purpose, a numerical example with following structure has been considered:

- Number of product types: 4
- Number of time periods: 5
- Number of stages at each period: 3
- Number of machines at each stage: 3
- Number of vehicles available: 3
- Number of customers: 3
- Number of routes possible: 3

- Number of required parts: 3

The total demand of customers is given in Table 3. Tables 4-6 show unit cost of production, holding/backorder and workers respectively.

Table 3. Information on customer demand

Customer	Product	Period				
		$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
$c = 1$	$i = 1$	50	200	150	250	400
	$i = 2$	350	750	150	350	550
	$i = 3$	400	850	50	150	750
	$i = 4$	200	300	450	500	300
	$i = 5$	100	350	250	400	200
$c = 2$	$i = 1$	300	350	200	450	250
	$i = 2$	850	600	400	650	900
	$i = 3$	200	50	150	300	200
	$i = 4$	850	400	600	750	450
	$i = 5$	350	400	500	450	350
$c = 3$	$i = 1$	200	150	50	400	200
	$i = 2$	650	700	400	550	250
	$i = 3$	450	400	650	850	950
	$i = 4$	50	250	300	250	400
	$i = 5$	650	150	200	250	350

Table 4. The unit costs of production in regular time, over time and subcontracting

Stage	Product	Period				
		$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
$k = 1$	$i = 1$	(5,10,20)	(10,15,20)	(5,15,25)	(10,20,20)	(5,15,20)
	$i = 2$	(20,30,35)	(25,30,40)	(20,35,45)	(25,25,40)	(20,25,35)
	$i = 3$	(10,15,20)	(10,10,20)	(15,15,25)	(5,15,25)	(10,10,30)
	$i = 4$	(40,50,55)	(45,45,60)	(50,55,55)	(60,60,65)	(40,50,50)
	$i = 5$	(10,15,18)	(10,15,18)	(10,15,18)	(10,15,18)	(10,15,18)
$k = 2$	$i = 1$	(40,40,55)	(35,40,40)	(50,55,60)	(40,50,55)	(30,50,70)
	$i = 2$	(10,30,30)	(5,20,30)	(20,35,40)	(30,30,35)	(10,30,40)
	$i = 3$	(20,25,25)	(35,40,50)	(15,30,45)	(5,20,30)	(25,30,30)
	$i = 4$	(55,55,75)	(50,55,55)	(40,45,55)	(25,35,40)	(15,30,45)

Stage	Product	Period				
		$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
	$i=5$	(30,40,55)	(25,30,45)	(35,45,50)	(30,40,55)	(20,25,35)
$k=3$	$i=1$	(60,70,85)	(30,45,55)	(55,75,80)	(35,50,60)	(40,55,65)
	$i=2$	(10,20,25)	(40,50,55)	(25,30,35)	(15,30,40)	(35,55,65)
	$i=3$	(25,30,40)	(50,55,55)	(20,45,50)	(20,30,45)	(10,15,15)
	$i=4$	(30,40,40)	(25,25,45)	(35,45,60)	(55,65,65)	(45,50,50)
	$i=5$	(45,55,55)	(25,35,50)	(20,30,45)	(35,45,50)	(25,35,50)

Table 5. The unit costs of holding and backorder $(\boldsymbol{h}, \boldsymbol{b})$

Customer	Product	Period				
		$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
$c=1$	$i=1$	(10,15)	(5,20)	(10,5)	(25,15)	(5,5)
	$i=2$	(15,25)	(5,10)	(10,10)	(20,10)	(10,25)
	$i=3$	(30,20)	(20,10)	(15,5)	(25,10)	(10,25)
	$i=4$	(25,30)	(5,20)	(20,30)	(15,5)	(25,20)
	$i=5$	(20,30)	(5,15)	(20,10)	(15,25)	(15,10)
$c=2$	$i=1$	(5,20)	(25,10)	(15,30)	(20,10)	(5,15)
	$i=2$	(10,20)	(25,5)	(15,10)	(15,5)	(5,25)
	$i=3$	(15,10)	(20,10)	(25,5)	(30,10)	(5,5)
	$i=4$	(20,10)	(30,25)	(20,30)	(10,30)	(10,10)
	$i=5$	(25,25)	(10,20)	(30,15)	(20,20)	(15,30)
$c=3$	$i=1$	(25,20)	(15,10)	(25,10)	(15,30)	(5,30)
	$i=2$	(20,5)	(15,15)	(5,10)	(20,15)	(30,30)
	$i=3$	(15,5)	(5,20)	(25,15)	(5,20)	(30,10)
	$i=4$	(5,20)	(30,10)	(15,15)	(30,5)	(10,25)
	$i=5$	(5,10)	(10,30)	(25,10)	(10,5)	(30,20)

Table 6. The unit costs of worker salary (ws_t^k), hiring (wt_t^k) and lay off (wl_t^k)

Parameter	Stage	Period				
		$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
ws_t^k	$k=1$	1070	1210	1450	1390	1480
	$k=2$	1320	1020	1420	1460	1340
	$k=3$	1380	1370	1190	1320	1080
wt_t^k	$k=1$	398	390	430	442	451
	$k=2$	356	436	432	333	324
	$k=3$	400	492	369	418	345
wl_t^k	$k=1$	276	226	251	270	290
	$k=2$	296	255	214	215	226
	$k=3$	285	226	282	225	293

The transportation times among manufacturer and customers are presented in Table 7.

Table 7. Transportation times

	Manufacturer	Customer $k=1$	Customer $k=2$	Customer $k=3$
Manufacturer	0	150	275	225
Customer $k=1$	150	0	225	300
Customer $k=2$	225	250	0	200
Customer $k=3$	275	350	300	0

Table 7 shows capacity of machines needed for producing one unit of products and carrying out setup $\left(d_{im}^k, f_{im}^k\right)$.

Table 8. Required capacity of machines for production and setup

Stage	Product	Machine		
		$m = 1$	$m = 2$	$m = 3$
$k = 1$	$i = 1$	(6,2)	(6,4)	(11,3)
	$i = 2$	(13,3)	(15,2)	(7,4)
	$i = 3$	(11,3)	(10,3)	(6,3)
	$i = 4$	(9,3)	(7,2)	(13,2)
	$i = 5$	(9,2)	(11,2)	(7,2)
$k = 2$	$i = 1$	(12,2)	(8,4)	(12,4)
	$i = 2$	(12,4)	(13,3)	(10,3)
	$i = 3$	(6,3)	(8,4)	(15,3)
	$i = 4$	(7,2)	(14,4)	(11,3)
	$i = 5$	(15,2)	(6,3)	(10,2)
$k = 3$	$i = 1$	(7,2)	(15,4)	(6,4)
	$i = 2$	(13,3)	(14,2)	(14,2)
	$i = 3$	(6,3)	(9,4)	(8,2)
	$i = 4$	(14,2)	(10,2)	(15,3)
	$i = 5$	(7,4)	(8,3)	(7,3)

Table 9 indicates total available capacity of machines in regular time Ca_{mt}^{k} and a ratio of regular time capacity in overtime β_{mt}^{k}.

Table 9. Information on machines' capacity (Ca, β)

Stage	Machine	Period				
		$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
$k = 1$	$i = 1$	$(7920, 0.78)$	$(5760, 0.80)$	$(4450, 0.67)$	$(5040, 0.66)$	$(5640, 0.94)$
	$i = 2$	$(6380, 0.51)$	$(5050, 0.89)$	$(6420, 0.91)$	$(6850, 0.85)$	$(4890, 0.54)$
	$i = 3$	$(4470, 0.61)$	$(5190, 0.65)$	$(5280, 0.80)$	$(5700, 0.56)$	$(6040, 0.82)$
$k = 2$	$i = 1$	$(5230, 0.54)$	$(6040, 0.79)$	$(6050, 0.72)$	$(7280, 0.85)$	$(7180, 0.82)$
	$i = 2$	$(6580, 0.91)$	$(5520, 0.90)$	$(7250, 0.65)$	$(6140, 0.81)$	$(5410, 0.58)$
	$i = 3$	$(7760, 0.51)$	$(7510, 0.83)$	$(6210, 0.72)$	$(6490, 0.71)$	$(6350, 0.90)$
$k = 3$	$i = 1$	$(4840, 0.77)$	$(5210, 0.78)$	$(5890, 0.86)$	$(4930, 0.75)$	$(7380, 0.58)$
	$i = 2$	$(4780, 0.60)$	$(4910, 0.89)$	$(4690, 0.51)$	$(4920, 0.72)$	$(5750, 0.71)$
	$i = 3$	$(5250, 0.51)$	$(7700, 0.80)$	$(5730, 0.52)$	$(4740, 0.53)$	$(7620, 0.73)$

The preferred due date of customers at each period is presented by Table 10.

Table 10. Customer due dates

Customer	Period				
	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
$c = 1$	520	730	860	770	580
$c = 2$	650	690	900	570	850
$c = 3$	760	660	580	680	700

Two MILP models are implemented in a commercial solver LINGO 11.0. Based on the attained results, the main significant difference between two Scenarios is that direct shipping provides the benefit of eliminating intermediate warehouses, whereas routing causes lower transportation cost by shipment to multiple customers on a single vehicle and the use of routing mode results in better utilization of the vehicles. According to the results, the inventory cost of Scenario I is obtained 487652, while that of Scenario II is calculated 623902. Moreover, transportation cost incurred by Scenario I is 329113, while this value is 148294 by Scenario II.

In addition to above analysis, to assess the performance of the proposed MILP models, some computational experiments will be conducted and implemented in the sequel. Different test problems were designed in three classes: small, medium and large sizes. Table 10 shows the structure of test problems in different classes. The MILP models were run on a PC with 2.81 GHz processor and 1 GB of RAM under Microsoft XP operating system. In order to illustrate complexity of the problems, the total number of variables including total number of integer variables and total number of continuous variables, and total number of constraints are recorded. In addition, the CPU times are also recorded to assess the efficiency of models. The computational results are shown in Tables 12 and 13 for Scenarios I and II separately. Additionally, the increasing trend of problems' dimension (variables and constraints) and resulting CPU time are depicted in Figures 2-4.

Table 11. Structure of test problems

Class	Problem	Periods	Stages	Products	Machines	Customers	Vehicles	Parts
Small	P1	5	2	2	1	5	2	2
	P2	5	2	2	1	10	2	2
	P3	5	2	3	2	15	2	4
	P4	5	2	3	2	20	2	4
Medium	P5	8	3	4	4	25	4	5
	P6	8	3	4	4	30	4	5
	P7	8	3	5	6	35	4	7
	P8	8	3	5	6	40	4	7
Large	P9	10	4	8	8	45	6	8
	P10	10	4	8	8	55	6	8
	P11	10	4	10	10	60	6	10
	P12	10	4	10	10	80	6	10

Table 12. The computational results for Scenario I

Class	Problem	Number of Variables		Number of Constraints	CPU(s)
		Integer	Continuous		
Small	P1	180	70	333	32
	P2	230	120	458	48
	P3	330	210	643	59
	P4	380	260	768	106
Medium	P5	1352	1184	2646	331
	P6	1512	1344	3006	456
	P7	1792	1840	3558	538
	P8	1952	2000	3918	603
Large	P9	4420	5260	8048	1205
	P10	5020	5860	9348	1458
	P11	5720	7600	10398	1672
	P12	6920	8800	12998	2192

Table 13. The computational results for Scenario II

Class	Problem	Number of Variables		Number of Constraints	CPU(s)
		Integer	Continuous		
Small	P1	190	330	333	98
	P2	240	1130	458	121
	P3	340	2470	643	155
	P4	390	4270	768	198
Medium	P5	1384	21216	2646	714
	P6	1544	30176	3006	794
	P7	1824	41072	3558	846
	P8	1984	53232	3918	1015
Large	P9	4480	126820	8048	2025
	P10	5080	187420	9348	2463
	P11	5780	223660	10398	2584
	P12	6980	392860	12998	3454

As can be seen, total number of constraints is equal in both Scenarios and varies from 333 to 12998. It can be verify from enumeration of number of constraints in both models. In Scenario I, the total number of integer variables varies from 180 to 6920, while in Scenario II it varies from 190 to 6980. Therefore, it can be concluded that the total number of integer variables for a specific problem in Scenario II is slightly greater than that value of same problem in Scenario I. In addition, it is observed that there is a significant difference in total number of continuous variables between Scenario I and II. The minimum and maximum number of continuous variables in Scenario I is 700 and 8800, while those values of Scenario II is 330 and 392860. The increasing behavior of number of variables and constraints of Scenario I and II is depicted by Figures 2 and 3 respectively. There is obviously a high difference between two Scenarios. As indicated in Figure 3, the total number of continuous variables in Scenario II exceeds the figure limit (12000) in medium and large classes of problems.

Moreover, Tables 12 and 13 reports the CPU time for each Scenario separately. In both Scenarios, the increasingly trend of number of variables and number of constraints causes the solution time to increase. Since the number of variables in Scenario II is much greater than that number of Scenario I, the run process of Scenario II is more time consuming than Scenario I. As the results reveal, LINGO solver could find high quality solutions, but when the problem sizes become larger, the CPU time shows an increasing trend. Figure 4 depicts this trend graphically.

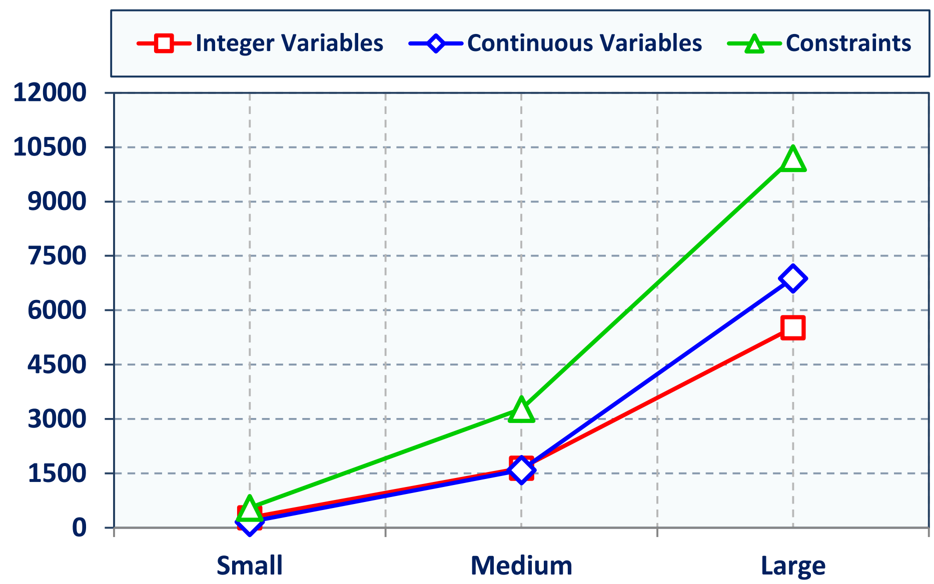


Figure 2. Problem dimension versus problem size in Scenario I

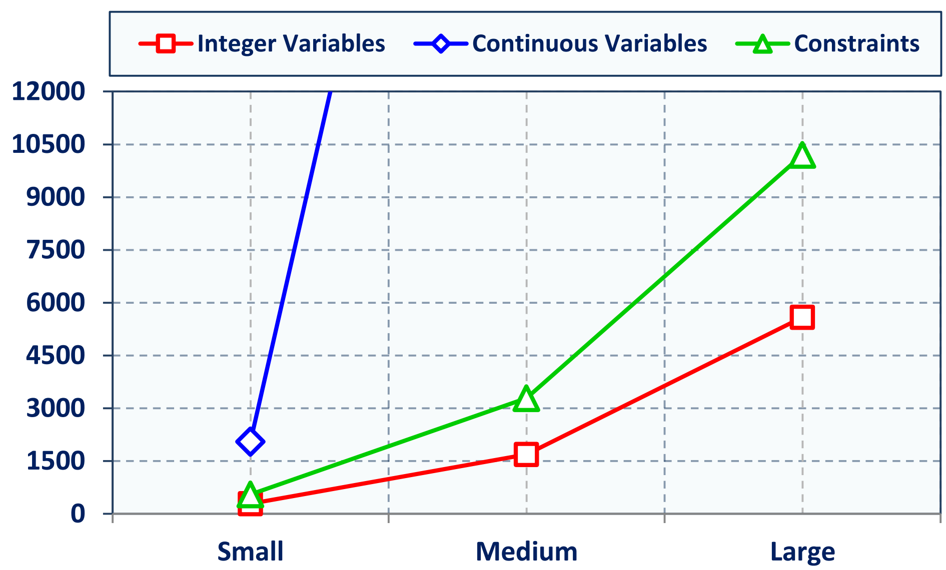


Figure 3. Problem dimension versus problem size in Scenario II

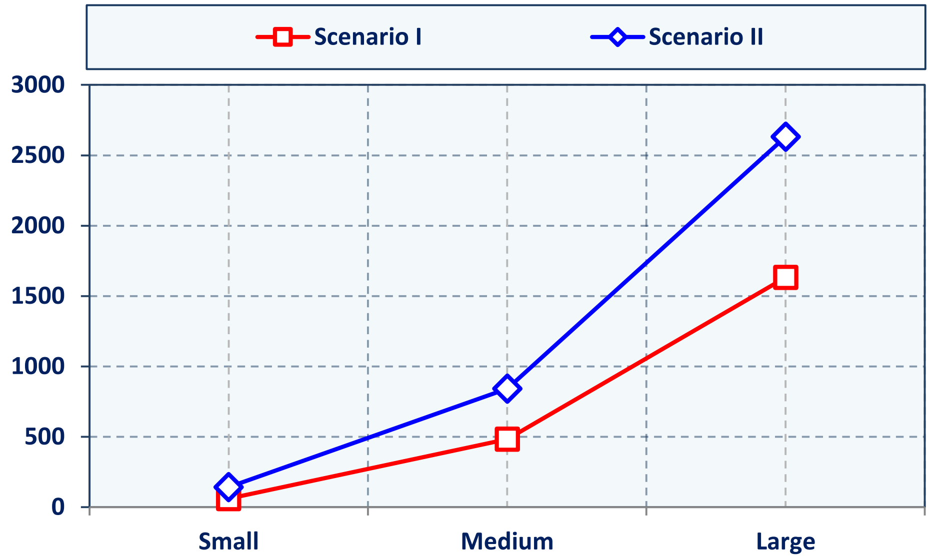


Figure 4. The CPU time versus problem size

5. Conclusion

In this paper we introduced an aggregate production-distribution planning problem, consisting of two sub-problems, aggregate production planning problem and distribution planning problem in a two-echelon supply chain. The suggested APDP aims to jointly optimize production, inventory, subcontracting, and transportation decisions in order to supply and deliver demand of geographically dispersed customers while minimizing total cost of system. Unlike most of past researches, an extended version of sub-problems with multi-stage, multi-product, multi-vehicle including several shop-floor machines and setup decisions was considered. We also introduced two distinct scenarios for distribution decisions, i.e., direct shipment and routing, and formulated each one as MILP models. An illustrative numerical example and some test problems were employed and comparison results of MILP models on number of constraints, number of variables and CPU time were reported.

References

Aliev, Rafik A., et al., 2007, “Fuzzy-genetic approach to aggregate production–distribution planning in supply chain management’, *Information Sciences*, 177, 4241-4255.

Ashayeri J, Selen W., 2003, “A production planning model and a case study for the pharmaceutical industry in the Netherlands”, *Journal of Logistics: Research and Applications*, 6, 37–50.

Bard JF, Nananukul N., 2009, “The integrated production–inventory–distribution–routing problem”, *Journal of Scheduling*, 12, 257–80.

Boudia M, Louly MAO, Prins C., 2007, “A reactive GRASP and path relinking for a combined production–distribution problem”, *Computers & Operations Research*, 34, 3402–3519.

Chan FTS, Chung SH, Wadhwa S., 2005, “A hybrid genetic algorithm for production and distribution”, *Omega*, 33, 345–55.

Coronado JL., 2008, “An optimization model for strategic supply chain design under stochastic capacity disruptions. Texas: College of Engineering”, Texas A&M University; 1–110.

Demirli K, Yimer AD., 2006, “Production–distribution planning with fuzzy costs. In: Proceedings of annual meeting of North American fuzzy information processing society”, (NAFIPS 2006). 702–707.

Fahimnia, B., Farahani, R.Z., Marian, R. Luong, L., 2013, "A review and critique on integrated production–distribution planning models and techniques", *Journal of Manufacturing Systems*, 32, 1–19.

Ghasemy Yaghin, R., Torabi, S.A., and Fatemi Ghomi S.M.T., 2012, "Integrated markdown pricing and aggregate production planning in a two echelon supply chain: A hybrid fuzzy multiple objective approach", *Applied Mathematical Modelling*, 36, 6011–6030.

Haehling LC., 1970, "Production and employment scheduling in multi-stage production systems", *Naval Research Logistics Quarterly*, 17(2), 193 –8.

Hamedi M, Farahani RZ, Husseini MM, Esmaeilian GR., 2009, "A distribution planning model for natural gas supply chain: a case study", *Energy Policy*, 37, 799–812.

Holt CC, Modigliani F, Simmon HA., 1955, "A linear decision rule for production and employment scheduling", *Management Science*, 2(1), 1–30.

Jain, A., and Palekar, U.S., 2005, "Aggregate production planning for a continuous reconfigurable manufacturing process", *Computers & Operations Research*, 32, 1213–1236.

Jamalnia, A., and Soukhakian, M.A., 2009, "A hybrid fuzzy goal programming approach with different goal priorities to aggregate production planning", *Computers & Industrial Engineering*, 56, 1474–1486.

Karimi-Nasab, M., and Fatemi Ghomi S.M.T., 2012, "Multi-objective production scheduling with controllable processing times and sequence-dependent setups for deteriorating items", *International Journal of Production Research*, 50, (24), 7378-7400.

Karmarkar, U.S., and Rajaram, K., 2012, "Aggregate production planning for process industries under oligopolistic competition", *European Journal of Operational Research*, 223, 680–689.

Lee YH, Kim SH., 2002, "Production–distribution planning in supply chain considering capacity constraints", *Computers & Industrial Engineering*; 43, 169–190.

Ma, H.L. Felix T.S. Chan and Chung, S.H., 2013, "Minimizing earliness and tardiness by integrating production scheduling with shipping information", *International Journal of Production Research*, 51, (8), 2253-2267.Moattar Husseini, Zohre, et al., 2015, "Multi-objective integrated production distribution planning concerning manufacturing partners", *International Journal of Computer Integrated Manufacturing*. 28, 1313-1330.

Moghaddam, Mohsen, Masoud Rabbani, and Babak Maleki-Shoja, 2012, "Integrating lateral transshipment to aggregate production–distribution planning considering time value of money and exchange rate" *International Journal of Operational Research*, 13, 439-464.

Niknamfar, Amir Hossein, Seyed Taghi Akhavan Niaki, and Seyed Hamid Reza Pasandideh, 2015, "Robust optimization approach for an aggregate production–distribution planning in a three-level supply chain", The *International Journal of Advanced Manufacturing Technology*, 76, 623-634.

Nishi T, Konishi M, Ago M., 2007, "A distributed decision-making system for integrated optimization of production scheduling and distribution for aluminum production line", *Computers & Chemical Engineering*, 31:1205–1221.

Pathak, Savita, and Seema Sarkar, 2012, "A fuzzy optimization model to the aggregate production/distribution planning decision in a multi-item supply chain network", *International Journal of Management Science and Engineering Management*, 7, 163-173.

Perumal, Ashoka Varthanan, N. Murugan, and G. Mohan Kumar, 2013, "A discrete PSO approach for generating an integrated multi-plant aggregate production-distribution plan", *International Journal of Knowledge-based and Intelligent Engineering Systems*, 17, 195-207.

Raa, B. Dullaert, W., and Aghezzaf, EL-H., 2013, "A matheuristic for aggregate production–distribution planning with mould sharing", *Int. J. Production Economics*, 145, 29–37.

Rizk N, Martel A, D'Amours S., 2006, "Multi-item dynamic production–distribution planning in process industries with divergent finishing stages", *Computers & Operations Research*, 33, 3600–3623.

Torabi, S. A., and M. Moghaddam, 2012, "Multi-site integrated production-distribution planning with trans-shipment: a fuzzy goal programming approach", *International Journal of Production Research*, 50, 1726-1748.

Wang, R-C, and Liang, T-F., 2005, "Applying possibilistic linear programming to aggregate production planning", *Int. J. of Production Economics*, 98, 328–341.

Wang, R-C, and Liang, T-F., 2004, "Application of fuzzy multi-objective linear programming to aggregate production planning", *Computers and Industrial Engineering*, 46, 17–41.

Wong, C.S., Felix T.S. Chan and Chung, S.H., 2012, "A genetic algorithm approach for production scheduling with mould maintenance consideration", *International Journal of Production Research*, 50, (20), 5683-5697.

Wong, C.S., Felix T.S. Chan, and Chung, S.H., 2014, "Decision-making on multi-mould maintenance in production scheduling", *International Journal of Production Research*, 52, (19), 5640-5655.

Zhang, R., Zhang, L., Xiao, Y., and Kaku, I., 2012, "The activity-based aggregate production planning with capacity expansion in manufacturing systems", *Computers & Industrial Engineering*, 62, 491–503.