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Integrated production-inventory-routing problem incorporating greenness consideration: A mathematical model and heuristic solver

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Abstract

This research focuses on the integrated production-inventory-routing planning (PIRP) problem, which persuades necessary decisions to study the supply chains (SCs). Previous research studies confirm that corporations coping with production, inventory, and routing problems, can remarkably decrease the total costs and meet the customers' demands efficaciously. Currently, because of severe obligations, corporations must consider environmental factors and cost optimization in their activities. Accordingly, in this article, a green PIRP (GPIRP) is addressed using mixed-integer linear programming (MILP), which simultaneously takes into account the economic and social decisions of the SCs. Furthermore, because the SCs routing-oriented problems belong to the NP-hard categories, we propose a two-phase heuristic solution method; in the first phase, the inventory and production decisions are determined using MILP formulation. The second phase seeks to find optimal vehicle routing and transportation decisions using a genetic algorithm (GA). Two main deals leading to this paper's unique position are to develop a bi-objective MILP model for the GPIRP and present a novel hybrid two-phase heuristic solution method that sequentially utilizes the CPLEX solver and the proposed GA. To validate the computational performance of the proposed solution method, we conduct a case study from the Ahvaz Sugar Refinery Company in Iran to demonstrate the advantages of the formulated model. Moreover, we handle sensitivity analyses to study the effectiveness of the suggested method for the large-sized examples.

Keywords: integrated production and inventory; vehicle routing; supply chain planning; mathematical optimization; genetic algorithm.

Paper Type: Original Research

1. Introduction

Nowadays, because of increasing competition, all supply chain (SC) activities, such as extraction of immature materials, production, and distribution of products in the customer zones, need effective decision-making. Besides, SCs are preferred to integrate vital decisions to maximize the potential benefit and coordinate various parts of the network more efficiently.

One of the various kinds of coordination among SC parts is coordinating both inventory and routing problems. In this system, both decisions of inventory and routing are optimized concurrently. One of the applications of this problem is in the vendor-managed inventory (VMI) systems, which can result in a win-win solution for both supplier and customer. In the VMI systems, both parties can benefit by reducing inventory investment, transportation costs, and inventory management strategies' complexity.

The connection between separate parts in each SC is realized through the transportation system. As a result, it is vital to design an efficient transportation system to improve the efficiency of SC. The correct choice of vehicle type and routes is one of the significant challenges in designing transportation systems. This concept was introduced as a vehicle routing problem (VRP) by Dantzig and Ramser (Dantzig and Ramser, 1959).

Motivated by an essential and critical issue in the SC planning, optimal integration of operational decisions such as lot-sizing and cost-efficient product routing to the customers, we aim to develop an integrated model for the production-inventory-routing problem. The model can be computationally efficient and solved by a heuristic solver based on mathematical programming and meta-heuristic search. Although integration can always reduce supply chain costs and increase competitiveness, related integrated models often are intractable. They cannot be applied to a real-world scale of the SCs problems. Consequently, in addition to integration, we should present a treatable solution method, which has an acceptable level of adaption to the model and is computationally efficient. Therefore, the primary motivation of this paper is to develop an integrated model for which a heuristic solver is suggested to solve the problem in large-scale instances.

Literature has developed inventory-routing models by integrating both inventory control and routing problems. The classical inventory-routing problem (IRP) assumes that several vehicles with a limited capacity transport

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products from the producer to the customers, and optimal routes are selected based on the inventory level in the producer and customer. In these problems, the supplier must control the inventory level to minimize the probability of shortage. Thus, the concept of inventory adds a time dimension to the routing problem, making the problem complex to solve.

We can summarize the properties of the developed IRP models in the literature as follows:

- Limited or unlimited planning horizon;
- Deterministic or probabilistic production and consumption rates;
- Discrete or continuous production and consumption time intervals.

The general purpose of these problems is to find the best distribution strategy to optimize distribution and inventory costs for a long time. Today, most industries employ these models using the VMI approach, such as aerospace, airplane manufacturing, clothing, automotive, chemical products, assembly, home appliances, metal, petroleum, and printing and publishing industries. One of the challenging issues related to the distribution problem in the petroleum industry is the dangerous pollution that can harmfully threaten global health (Brown and Graves, 1981).

Recently, the problem of global warming has become more dangerous; hence, reducing carbon production is the priority of all governments (Mirzapour Al-e-hashem and Rekik, 2014). Green transportation is one of the suitable strategies to decrease carbon emissions (Mirzapour Al-e-hashem and Rekik, 2014). Accordingly, researching the discussed matters in green SC has become crucial.

Concisely, the main research questions are as follows: I- What is the formulation of an efficient mathematical optimization model for the PIRP problem in which environmental objectives are also considered? II- Regarding the complexity of the problem, are conventional mathematical programming models and classical solvers able to solve the problem? If not, how do we utilize a hybrid solution method with an acceptable level of computational performance? Finally, III- What is the best trade-off between the economic and environmental objective function, and how can we discover the best Pareto solution? Considering the mentioned research questions, the main aims of this paper are to develop an integrated PIRP model in a bi-level green SC and propose a hybrid two-phase solution method. The developed model and solution approach are respectively based on a mixed-integer program and a metaheuristic search to cope with the NP-hard complexity of the problem (Adulyasak, Cordeau, and Jans, 2015). Furthermore, we analyze the impact of maximum possible environmental effects on the cost minimization objective and propose the global Pareto front on which the decision-makers can select the best efficient solution.

The rest of the paper is structured as follows: In Section 2, we present a review of significant studies in the field of PIRP. Section 3 explains the developed GPIRP model, and Section 4 proposes the structure of the developed solution algorithm to solve the model. In Section 5, numerical experiments, including a case study, are carried out to evaluate the model and the solution approach. Finally, Section 6 includes the main conclusions.

2. Literature review

Studies on PIRP have quickly expanded in recent years due to the importance of the problem. The integration of production, inventory, routing, and distribution processes has been considered extensively in the SCs to achieve competitive advantages. In particular, this is very important for companies with limited capacity utilization and high distribution costs (Fumero and Vercellis, 1999; Brown et al., 2001; Gupta et al., 2002; Jabbarzadeh et al., 2016; Saeedi Mehrabad et al., 2017). Sindhuchao et al. (2005) proposed an integrated IRP for a multi-product replenishment with a limited vehicle capacity. They introduced a mathematical programming approach for integrating inventory and transportation decisions in a collection and distribution system. Lei et al. (2006) were among the first researchers who formulated the PIRP using a MILP approach. Their innovative method managed to determine the size of demands and routes simultaneously. Then, Boudia et al. (2006) solved the previous model by proposing a memetic algorithm, including a greedy randomized adaptive search procedure. Boudia et al. (2007) investigated the production routing problem (PRP) in a multi-period planning horizon in another study. Their objective was to minimize total production, inventory, and distribution costs. Yu et al. (2008) examined an IRP with a size constraint for a public transport fleet. Due to the complexity of their problem, they utilized a Lagrangian relaxation approach. Bard & Nananukul (2009) answered the critical question of the best replenishment strategy by analyzing a group of innovative methods. Their objective function was to maximize the net income of delivering the shipment in a specific period. Bard & Nananukul (2010) proposed a MIP model for the PIRP to minimize production, inventory, and delivery costs in different phases of a production system. Their model included production facilitation and a group of customers with varying demands over time.

Moin et al. (2011) proposed an IRP by considering the many-one distribution network of an assembly factory and some specific suppliers. They utilized a hybrid GA to solve the problem under a two-phase strategy. Coelho et al. (2012) introduced an IRP with transshipment, where vehicle routing and inventory decisions had to be simultaneously made. Coelho et al. (2014) presented a combination of production, inventory, distribution, and routing problems in another study. They prioritized production planning to determine the optimal quantity of production, considering inventory and vehicle-routing constraints.

The problem of collecting animals is a VRP considering the inventory control constraints used mainly in the animal slaughter industry. The objective is to determine the vehicle routes to collect animals from slaughterhouses. A group of fundamental limitations must be considered, such as production and inventory in the slaughterhouses.

By taking into account these constraints for the problem, Oppen et al. (2010) employed exact solution methods based on the column-generation technique to solve the large-sized instances. In a study, Bertazzi et al. (2015) proposed an IRP where the objective function was minimizing the costs of shortage, holding, and transportation over a period. Adulyasak et al. (2015) presented a review of PIRP. They explained the problem and its basic modeling without considering the shortage and examined various solution methods. They implemented a two-phase approach in which they first estimated daily delivery quantities and then solved the routing problem.

Yantong et al. (2016) formulated PIRP as linear programming for food products. They conducted the modeling based on the maximization of profit, considering product perishability. Agra et al. (2016) utilized a Lagrangian relaxation algorithm to solve a single product PIRP. Hasni et al. (2017) formulated a multi-product IRP and used a variable neighborhood search (VNS) to solve their complex model. Malladi & Sowlati (2018) presented a complete content analysis-based review of articles on IRP incorporating sustainability-related aspects. They selected and reviewed 40 papers by categorizing them into single-objective and multiple-objective models. They also proposed studies with single-objective models, including waste management, returnable transport item management, waste prevention and reduction, and emission reduction.

Qiu et al. (2018) developed a MILP model for PRP with reverse logistics and remanufacturing for the first time in a closed-loop SC. They proposed a solution approach based on a branch-and-cut algorithm. Fakhrzad & Alidoosti (2018) presented a practical perishability inventory model. Their proposed model added spoilage of products and variable prices within a period to the location-inventory-routing models to make it more realistic. Chitsaz et al. (2019) formulated a general model for the assembly PIRP as a MILP. They expanded a three-phase decomposition metaheuristic based on the iterative solution of different sub-problems. Zhang et al. (2019) presented an electric vehicle battery swap station LRP with stochastic demands. They attended to determine a minimum cost scheme, including the optimal number and location of stations with an optimum route plan, and proposed a hybrid VNS. Adeli et al. (2019). investigated the integrated sourcing and inventory policy problem in a pharmaceutical distribution company. They considered the number of shortages as a separate objective.

Bertazzi et al. (2020) studied an inbound IRP concerned with the minimal-cost collection of distinct components from a network of suppliers and subsequent delivery to a manufacturing plant. They assumed known and constant production of end products at the plant and developed a branch-and-cut algorithm. Avci and Yildiz (2020) developed the classical PRP by considering transshipments, either from supplier to retailers or between retailers, the total cost further. They also proposed a mathematical programming-based heuristic to solve the problem. Chan et al. (2020) extended a model to build a unified planning problem for efficient food logistics operations. They considered four critical objectives, including minimizing the total expense of the system, maximizing the average food quality, minimizing the amount of CO2 emissions in transportation along with production, and total weighted delivery lead time minimization. Recently, Schenekemberg et al. (2021) introduced the two-echelon PRP to the literature. Their studied problem is motivated by the petrochemical industry, enlarging the supply chain integration by considering production, inventory, and routing decisions in the two-echelon vendor-managed inventory systems. Aazami and Saidi-Mehrabad (2021) developed a new multi-period production-distribution planning for perishable products with a fixed lifetime in a seller-buyer system. They proposed a hierarchical heuristic approach based on Benders decomposition and GA. Peng et al. (2021) assessed the effectiveness of the physical internet for dealing with various disruptions in an integrated production-inventory-distribution system. They developed a two-stage stochastic programming model which incorporates pre-event and post-event mitigation strategies in an integrated way.

In the green inventory-routing field, Mirzapour Al-e-hashem and Rekik (2014) defined a multi-period GIRP in a bi-echelon green SC in which products were transferred from multiple suppliers to a single plant. They assumed the demand as a deterministic case and did not consider any production constraints. Their objective function was the trade-off between transportation costs and greenhouse gas emissions. Also, Qiu et al. (2017) proposed a model for the PIRP, including carbon emissions minimization. They decomposed the problem into two main problems and then used an innovative algorithm based on Dantzig–Wolfe decomposition to solve each main problem. Abad et al. (2018) developed an integrated model for coordination between decisions related to picking up the cargo, routing, allocation of vehicles, consolidation of cargo, and allocation of sorted cargo. They expanded some metaheuristics based on GA and PSO. Darvish et al. (2019) compared the effect of operational decisions on costs and emissions. They formulated some logistic problems considering new objectives. Besides, they studied two integrated systems dealing with PIRP decisions in which a product is shipped to the customers over a finite time. Yong et al. (2019) focused on freight distribution, introduced a transportation resource sharing strategy to consider the multi-depot green VRP, and incorporated the time-dependency of speed and piecewise penalty costs for earliness and tardiness of deliveries, as well. They developed a bi-objective model to minimize operating costs and total carbon emission.

Karakostas, Sifaleras, and Georgiadis (2020) introduced the fleet-size and mix pollution IRP with a just-in-time replenishment policy and capacity planning considering capacity selection decisions and heterogeneous fleet composition. They adopted a general VNS-based framework to solve more realistic-sized problem instances. In recent studies on the green inventory-routing field, Khorshidvand et al. (2021) addressed a novel two-stage model for a sustainable closed-loop SC. They provided a balance among economic aims, environmental concerns, and social responsibilities based on price, green quality, and advertising level. Their objectives were maximizing the profit of the whole SC, minimizing the environmental impacts of CO2 emissions, and maximizing employee safety. Also, Khorshidvand, Soleimani, Sibdari, et al. (2021) offered a two-stage approach to model and solved a sustainable closed-loop SC, considering pricing, green quality, and advertising in another research. They introduced suitable solution methods according to the scale of the problem, including the augmented ϵ-constraint and a Lagrangian relaxation algorithm. Additionally, Khorshidvand, Soleimani, Sibdari, et al. (2021a) proposed a hybrid method in which SC coordination decisions and closed-loop SC objectives are simultaneously involved. They developed their model based on the sensitivity of the return rate to green quality and the customers' maximum tolerance, while the demands are uncertain. They used a robust optimization model to overcome the uncertain demands. Liu et al. (2021) proposed an integrated multi-objective model of IRP for perishable products considering the factors of carbon emissions and product freshness. They analyzed the economic cost, carbon emission levels, and freshness of the perishable products.

In this study, we use the formulation approaches proposed in (Mirzapour Al-e-hashem and Rekik, 2014), (Adulyasak, Cordeau and Jans, 2015), and (Qiu, Qiao and Pardalos, 2017) as the base models to develop our new GPIRP. Table 1 lists the most relevant studies in this field, along with the present study, to show the contributions of our paper. In summary, the main contributions of the present study include: paying attention to the environmental consideration, developing an integrated PIRP model in a bi-echelon green SC, proposing a two-phase GA by decomposing the main problem, and using the actual data of a manufacturing company.

Table 1. Comparison of the most relevant studies

Researchers and Year	Decisions			Production Level				Inventory Level		Routing Level		Environmental Considerations (Green Gas Emissions)	Solution Method			Case Study
	Production	Inventory	Routing	Multi-site	Multi-product	Multi-period	Capacity Constraints	Capacity of Warehouse	Multi-period	Capacity of Vehicle	Multi-period		Exact/CPLEX	Heuristic	Metaheuristic	
Brown et al. (2001)	✓	✓	✓	✓	✓	✓	✓		✓		✓			✓		✓
Sindhuchao et al. (2005)		✓	✓					✓		✓			✓	✓		
Lei et al. (2006)	✓	✓	✓			✓	✓	✓	✓	✓	✓			✓		✓
Boudia et al. (2007)	✓		✓			✓	✓		✓	✓	✓				✓	
Yu et al. (2008)		✓	✓					✓	✓	✓	✓		✓			
Bard and Nananukul (2009)	✓	✓	✓			✓	✓	✓	✓	✓	✓				✓	
Bard and Nananukul (2010)	✓	✓	✓			✓	✓	✓	✓	✓	✓		✓	✓		
Moin et al. (2011)		✓	✓		✓				✓	✓	✓		✓		✓	
Coelho et al. (2012)		✓	✓					✓	✓	✓	✓			✓		
Coelho et al. (2014)		✓	✓		✓			✓	✓	✓	✓			✓		
Mirzapour Al-e-hashem and Rekik (2014)		✓	✓		✓				✓	✓	✓	✓	✓			✓
Bertazzi et al. (2015)		✓	✓						✓	✓	✓		✓			
Yantong et al. (2016)	✓	✓	✓			✓	✓	✓	✓	✓	✓		✓			
Agra et al. (2016)	✓	✓	✓			✓	✓		✓	✓	✓		✓			
Hasni et al. (2017)		✓	✓		✓				✓	✓	✓			✓		
Qiu et al. (2017)	✓	✓	✓			✓	✓	✓	✓	✓	✓	✓		✓		
Abad et al. (2018)			✓		✓		✓			✓		✓			✓	
Qiu et al. (2018)	✓	✓	✓	✓		✓	✓	✓		✓	✓			✓		
Darvish et al. (2019)	✓	✓	✓					✓	✓	✓	✓	✓	✓			
Chitsaz et al. (2019)	✓	✓	✓			✓	✓	✓	✓	✓	✓			✓		
Avci and Yildiz (2020)	✓		✓			✓	✓			✓				✓		
Chan et al. (2020)	✓	✓	✓			✓	✓	✓	✓	✓	✓				✓	✓
Liu et al. (2021)		✓	✓		✓			✓	✓	✓	✓	✓		✓		✓
Schenekemberg et al. (2021)	✓		✓	✓		✓				✓	✓			✓		
This Research	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

3. Problem definition

In this section, firstly, the problem is defined, and then the proposed formulation of the problem as a MILP model is presented

3.1. Problem description

We propose a bi-echelon SC problem, including a manufacturing factory in the first echelon and multiple DCs in the second echelon. The factory can produce different products. There are constraints on each product's diversity and manufacturing quantity, considering the available production equipment. Products are stored in the factory warehouse and transferred to DCs via a fleet of heterogeneous vehicles.

In each period, the factory starts producing products to satisfy customer demands through DCs. Some parts of the manufactured products may be stored in the warehouse to be supplied in the next period. Vehicles are heterogeneous with different capacities. Some properties of the vehicles, such as the load-carrying capacity, the type of fuel, and the volume of pollution per traveled distance unit, differ among them.

It is assumed that vehicles are rented in each period. The company intends to reduce its costs in each period (e.g., a year) to minimize its total cost by the end of a limited planning horizon. These costs include:

(1) Production costs (PC), i.e., variable PC (VPC) resulting from the production of one unit of each product and fixed PC (FPC) resulting from the activity of the factory in each period;

(2) Transportation costs (TC), i.e., variable TC (VTC), which is proportionate to the traveled distance by different vehicles, and fixed TC (FTC) resulting from renting each vehicle;

(3) Holding cost (HC) of storing products in factory warehouses and DCs; and

(4) Backorder cost (BC), resulting from failing to fully satisfy customers' demands in each period.

Nowadays, considering the tremendous growth of different industries, environmental pollution is increasing. Greenhouse gas emission is a type of pollution directly related to the trend of global warming. Thus, limitations and restrictions are applied to companies to prevent the excessive emission of these gases. Transportation has the highest share in emitting these gases (Mirzapour Al-e-hashem and Rekik, 2014).

Therefore, companies must control routing decisions to consider not only their economic activities but also environmental aspects for society. It must be noted that an optimal routing decrease traveling distance for vehicles and, thus, decreases transportation costs and greenhouse gas emissions. However, renting vehicles that conform to environmental purposes may violate economic objectives. Therefore, a proper trade-off between economic and environmental objectives must be considered.

The company's objective is to minimize its total cost at the end of the planning horizon. Attaining this objective requires an optimal solution for PIRP. The operational constraints of GPIRP can be partitioned into the following categories:

- Production capacity of the factory: Product diversity is limited, and production quantity for each product cannot exceed a certain level in each period.
- Factory warehouse capacity: Inventory level must not exceed factory warehouse capacity.
- DCs capacity: DCs inventory must not exceed their capacity.
- Transportation capacity: In each period, a specific number of each vehicle type is available, and each vehicle type has a specific load-carrying capacity.
- Greenhouse gas emissions limitation: In each period, the greenhouse gas emissions of the transportation process must not exceed its allowable limit.

3.2. MILP formulation of the problem

Notions used in the proposed MILP model are as follows:

Table 2. Nomenclatures

Indices	
$F = \{1,2,3,\dots,\lvert F\rvert\}$	Set of manufacturing factories
$DC = \{1,2,3,\dots,\lvert DC\rvert\}$	Set of DCs
$\{0\}$	Transportation fleet
$N = D \cup F$	Set of DCs and factories
$E = N \times N$	Set of total edges between DCs and factories
$T = \{1,2,3,\dots \lvert T\rvert\}$	Set of time periods
$K = \{1,2,3,\dots,\lvert K\rvert\}$	Set of vehicles
$P = \{1,2,3,\dots,\lvert P\rvert\}$	Set of products
Parameters	

vcp_p	The variable production cost of each unit of product type
fcp_p	The fixed production cost of product type p (resulting from the factory activities in each period)
v_k	The variable cost of the k^{th} vehicle per unit of traveled distance
$d_{(i,j)}$	Distance between factories and DCs per $(i, j) \in E$
f_k	Fixed cost of renting a vehicle k
$hc^t_{p,F}$	Holding cost for a unit of product type p in a factory F in period t
$hc^t_{p,d}$	Holding cost for a unit of product type p in DC d in period t
bc_p	Backorder cost per unit of product pin DCs
$capF_p$	Production capacity of product type p in factory
$CapV_k$	The load-carrying capacity of vehicle type k
S_P	Space occupied per unit of product type p
CapSF	Factory warehouse capacity
$CapSD_d$	Warehouse capacity of DC d
$dem^t_{p,d}$	The demand for product P from DC d in period t
GH_k	The volume of greenhouse gases emitted per unit of transportation of vehicle K
$MaxGH^t$	Maximum greenhouse gas emission in period t
Decision variables	
x^t_p	The amount of product type p produced in the period t
$sup^t_{p,d}$	The amount of product type p supplied from DC d in period t
$supF^t_p$	The amount of product type p supplied from the factory in period t
$supF^t$	The total amount of product supplied from the factory in period t
$rec^t_{p,d}$	The amount of product type p received from DC d in period t
$I^t_{p,d}$	Inventory level of product p in DC d in period t
$I^t_{p,F}$	Inventory level of product type p in a factory in period t
$y^t_{(O,F),k}$	A binary variable representing renting (1) or not renting (0) of the vehicle k in period t (Binary variable representing moving (1) or not moving (0) of the vehicle k from the fleet to the factory in period t
$y^t_{(i,j),k}$	A binary variable representing passing (1) or not passing (0) vehicle k from the edge $(i, j) \in E$ in period t
$openF^t$	A binary variable represents a working (1) or not working (0) factory in period t
Total Cost	Total cost at the end of the planning horizon
PC_t	Production cost in period t
VPC_t	Variable production cost in period t
FPC_t	Fixed production cost in period t
TC_t	Transportation cost in period t
VTC_t	Variable transportation cost in period t
FTC_t	Fixed transportation cost in period t
HC_t	Holding cost in period t
BC_t	Backorder cost in period t

Objective function:

$$\text{Total Cost} = \sum_{t \in T} PC_t + TC_t + HC_t + BC_t \quad (1)$$

$$PC_t = VPC_t + FPC_t = \sum_{p \in P} vcp_p .\ x^t_p + fcp_p .\ openF^t \qquad \forall t \quad (2)$$

$$HC_t = \sum_{p \in P} hc^t_{p,F} .\ I^t_{p,F} + \sum_{p \in P} \sum_{d \in DC} hc^t_{p,d} .\ I^t_{p,d} \qquad \forall t \quad (3)$$

$$BC_t = \sum_{p \in P} \sum_{d \in DC} bc_p .\ (dem^t_{p,d} - sup^t_{p,d}) \qquad \forall t \quad (4)$$

$$TC_t = VTC_t + FTC_t = \sum_{(i,j) \in E} \sum_{k \in K} v_k .\ d_{(i,j)} .\ y^t_{(i,j),k} + \sum_{k \in K} f_k .\ y^t_{(o,F),k} \qquad \forall t \quad (5)$$

Eq. (1) shows the objective function of GPIRP, including four parts: 1- fixed and variable production costs (Eq. (2)), 2- holding cost in the factory and the warehouses of DCs (Eq. (3)), 3- backorder cost (Eq. (4)), and 4- fixed and variable transportation costs (Eq. (5)). The total cost at the end of the planning horizon will be achieved by summing these costs in each period.

Constraints:

The constraints of the developed GPIRP include production capacity, transportation capacity, warehouse capacity, environmental constraints (maximum greenhouse gas emission). Also, we have other constraints considering the specification of the network, e.g., mass balance constraint in each node, dynamic relationship of inventory level in different periods, and allowable transportation route for vehicles. These constraints are explained in the following equations:

$$I_{p,F}^{t} = I_{p,F}^{t-1} + x_{p}^{t} - supF_{p}^{t} \qquad \forall t, p \qquad (6)$$

$$I_{p,d}^{t} = I_{p,d}^{t-1} + rec_{p,d}^{t} - sup_{p,d}^{t} \qquad \forall t, p, \qquad (7)$$

$$x_{p}^{t} \leq capF_{p}. openF^{t} \qquad \forall t, p \qquad (8)$$

$$\sum_{d} rec_{p,d}^{t} = supF_{p}^{t} \qquad \forall t, p \qquad (9)$$

$$supF_{p}^{t} \leq I_{p,F}^{t-1} + x_{p}^{t} \qquad \forall t, p \qquad (10)$$

$$sup_{p,d}^{t} \leq I_{p,d}^{t-1} + rec_{p,d}^{t} \qquad \forall t, p, d \qquad (11)$$

$$sup_{p,d}^{t} \leq dem_{p,d}^{t} \qquad \forall t, p, d \qquad (12)$$

$$supF^{t} = \sum_{p} supF_{p}^{t} \qquad \forall t \qquad (13)$$

$$\sum_{k} capV_{k}. y_{(O,F),k}^{t} \geq supF^{t} \qquad \forall t \qquad (14)$$

$$\sum_{p} s_{p} I_{p,F}^{t} \leq CapSF \qquad \forall t \qquad (15)$$

$$\sum_{p} s_{p} I_{p,d}^{t} \leq CapSD_{d} \qquad \forall t, d \qquad (16)$$

$$\sum_{(i,j)\in E^{*}} \sum_{k\in VehType} GH_{k} . d_{(i,j)} . y_{(i,j),k}^{t} \leq MaxGH^{t} \qquad \forall t \qquad (17)$$

$$\sum_{(i,j)\in E} y_{(i,j),k}^{t} \leq 1 \qquad \forall t, k \qquad (18)$$

$$\sum_{(i,n)\in E} y_{(i,n),k}^{t} = \sum_{(n,j)\in E} y_{(n,j),k}^{t} \qquad \forall t, k, n \qquad (19)$$

$$\sum_{(i,j)\in E} y_{(i,j),k}^{t} \leq y_{(O,F),k}^{t} \qquad \forall t, k \qquad (20)$$

$$u_{i,k}^{t} - u_{j,k}^{t} + |N| y_{(i,j),k}^{t} \leq |N| - 1 \qquad \forall t, k, (i, j) \qquad (21)$$

$$\begin{cases} x_{p}^{t}, sup_{p,d}^{t}, supF_{p}^{t}, supF^{t}, rec_{p,d}^{t}, I_{p,d}^{t}, I_{p,F}^{t} \geq 0 \\ \qquad \forall\, p, d, F, t \\ y_{(O,F),k}^{t}, y_{(i,j),k}^{t}, openF^{t} = \{0,1\} \\ \qquad \forall\, i, j, o, k, F, t \end{cases} \qquad (22)$$

4.Solution approach

Because the routing problem is NP-hard and belongs to the GPIRP, standard solvers such as CPLEX can be applied to only small-sized examples. The large-sized examples require an efficient solution procedure. In this research, we concentrate on combined exact linear programming (LP) solver (CPLEX) and metaheuristics solver (GA).

We utilize a two-phase method in GA to solve the GPIRP because the problem is extremely complicated to be solved in a single stage. We should note that the two-phase procedure has been broadly applied in the literature (Alvarenga, Mateus and De Tomi, 2007; Shen et al., 2010; He and Tan, 2012; Mjirda et al., 2014; Azad et al., 2019). In phase I, production planning and inventory control are taken into consideration. Here, the optimal amount of each produced product in factories and inventory in the factory and DC are obtained during each period. In phase II, considering the first phase results, transportation planning is performed. In conclusion, "Phase I is production and inventory planning," and "Phase II is vehicle routing to transport the products."

Phase I is solvable by the CPLEX LP solver since all variables in this phase are continuous, and the extended model is linear. In Phase II, a VRP should be addressed. In the previous twenty years, metaheuristic algorithms have appeared as the most hopeful way of research for all types of VRP problems (Golden, Raghavan and Wasil, 2008;

Ayough et al., 2020; Goli et al., 2018). Because of the special solution representation structure of the VRP, the GA is an ordinary solver for VRP and Green VRP (Lin et al., 2014; Karakatič and Podgorelec, 2015). Figure 1 displays a flow diagram of the suggested two-phase solution procedure, further described in the following sub-sections.

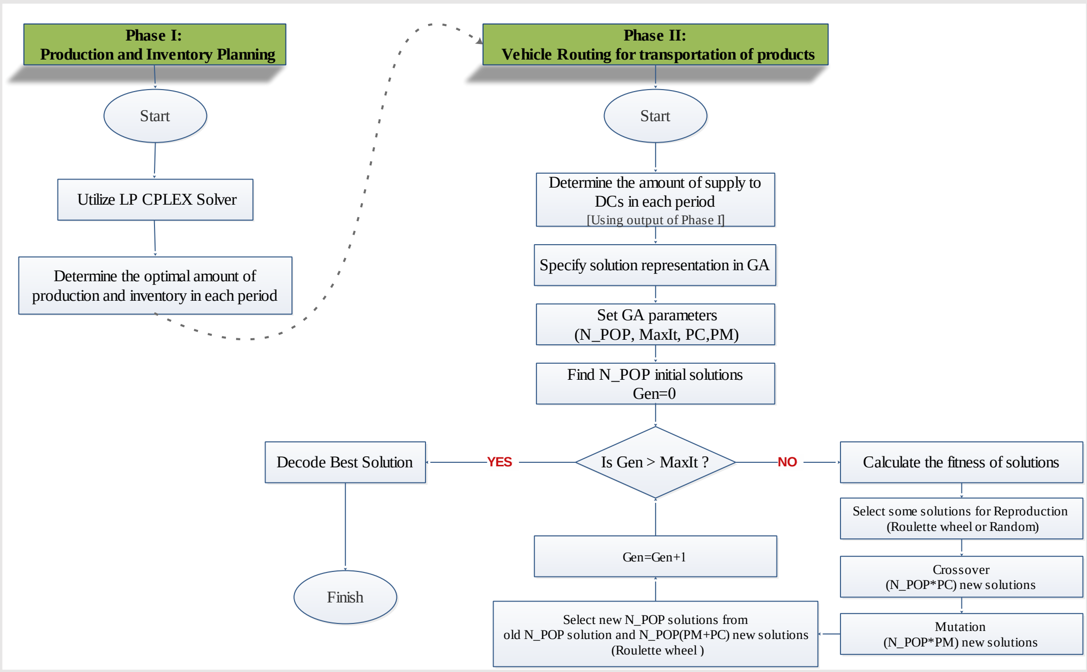

Figure 1. Flow diagram of the suggested two-phase solution procedure

Phase I: production and inventory planning

We determine the quantity of production and inventory variables for each period, such as the optimal production amount for each product, the number of products stored in the warehouse, and the optimal volume for each product supplied to DCs. Therefore, the following model (23) as a sub-problem of the GPIRP model is obtained, in which transportation cost, vehicle routing, and the environmental constraint are relaxed. This model is easily solved using commercial solvers, e.g., CPLEX, and the optimal solution of the problem is transferred to Phase II. Note that this phase of the problem is formulated as an LP model, and despite MILP models, a large-sized LP model can be solved by CPLEX quickly(Crowder, Johnson and Padberg, 1983; Bixby, 1994). At the end of Phase I (by solving model 23), we determine the optimal amount of physical flow in the SC network, which is an input for the routing decision in phase II.

$$\begin{cases} \text{Min} \quad \text{Total Cost} = \sum_{t\in T} PC_t + HC_t + BC_t \\ \qquad\qquad \text{s.t.} \\ \qquad \text{Eqs } (6-17) \end{cases} \tag{23}$$

Routing and environmental transportation

GA is used for solving phase II. In the following, solutions representation, initial solutions generation, neighborhood finding (based on mutation and crossover operators), selection mechanism, repairing strategy, fitness function, and finally, stop criteria of the proposed GA are explained.

Solutions representation

We assume that there are m DCs, K different vehicle types with specific capacities and different pollution emission levels. In PIRP, product supply to each DC in each period is performed using one vehicle. Therefore, we can place the customers in a k-member partition, and the order of each partition member is shipped to DC using one vehicle. Figure 2(a) shows the solution representation. The first vehicle starts from factory F, delivers products of DC1 to DCr, and then returns to fleet O. Similarly, the second vehicle starts from factory F, delivers products of DCr+1 to DCr+t, and then returns to fleet O. Finally, the last vehicle starts from factory F, delivers products of DCm-j to DCm, and then returns to fleet O. For example, Figure 2(b) depicts a route, and Figure 2(c) presents its encoding.

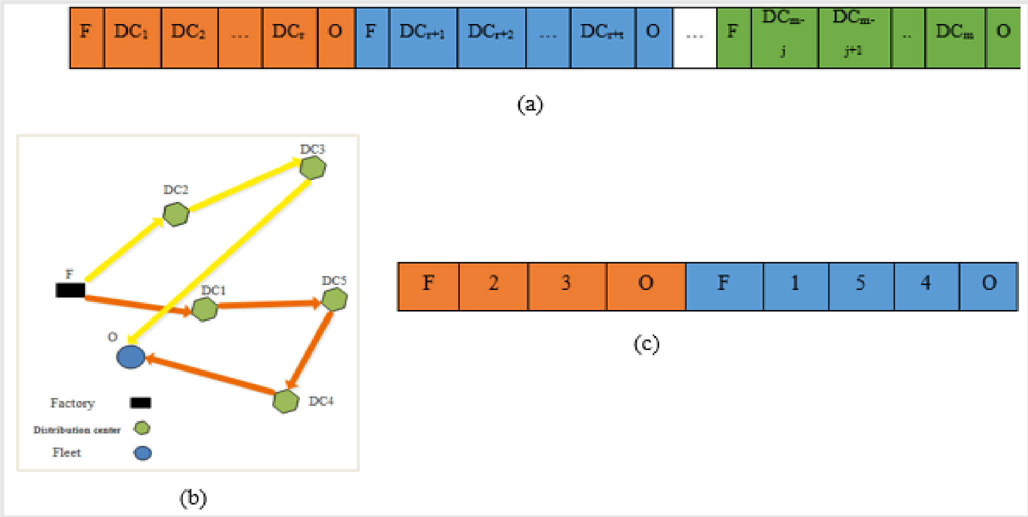

Figure 2. Solution representation and an example

Initial solutions

We employ random generation and a greedy search to find an initial solution. We first ensure that the routing of vehicles decreases total transportation cost and satisfies the vehicle capacity and environmental constraints.

To do so, we first consider the environmental constraint. Therefore, a vehicle with a minimum greenhouse gas emission per distance unit is selected. This vehicle is placed in factory F. That DC near the factory is chosen in which the selected vehicle has a load-carrying capacity of its products. The number of the chosen DC is placed in the first cell after the first F. This process is repeated for the remaining DCs, but this time the distance from the previous selected DC is considered. This process is continued until the selected vehicle does not have the load-carrying capacity for the products of any remaining DC. In this case, we place the sign of fleet O after the final DC, suggesting that the vehicle has returned to the fleet from that DC. The sign of the factory is also placed after the fleet, and a similar vehicle is selected among the remaining ones.

This vehicle performs a similar procedure for the remaining DCs. This process continues until all the products are supplied to all DCs.

Neighborhood's search (mutation and crossover operators)

As mentioned, DCs are turned into k-member partitions where each member is covered by one vehicle. Let us assume that one of the partition members is $A = X_1, X_2, \dots, X_n$ with n members. Therefore, with n! possible routes, the products are shipped from the factory to DCs using one vehicle. We can find new solutions (offspring) for each initial solution (parent) by defining the following two operators.

Crossovers

Consider couples solutions A and B, selected with the Roulette wheel method from the initial answers. Two new answers (children) are generated from each parent called C1 and C2, according to Figure 3.

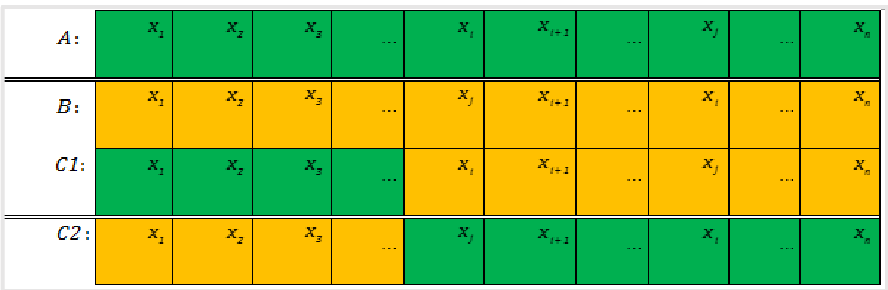

Figure 3. Crossover operator

Mutation (Swap / Reversion / Insertion)

For solution A, according to the swap operator, the new neighboring solution is shown in Figure 4(a); also $X^{new} = Rev(X, i, j)$ in Figures 4(b) and 4(b) is two neighborhoods generated by Reversion and Insertion operators, respectively.

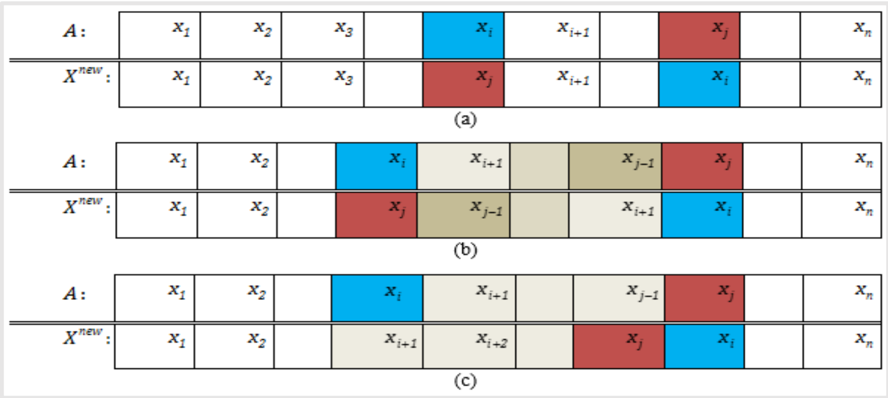

Figure 4. Mutation operators

Fitness function

We select the new generation among the solutions after generating an initial population (different initial solutions) and creating their offspring (new solutions) using the defined mutation and crossover operators. We assume that the number of the initial population is N_POP, and the percent of mutation and crossover is equal to pc and pm, respectively. Therefore, at the end of each repetition, there are solutions. We evaluate each solution by determining the fitness function. For this purpose, the transportation part, removed in phase I, is used in this section, and the fitness function for each solution is defined as follows:

$$\text{Fitness}(Y) = \frac{1}{\sum_{(i,j)\in E} \sum_{k\in K} v_k \cdot d_{(i,j)} \cdot y^{t}_{(i,j),k}} \qquad \forall t \qquad (24)$$

In fact, the denominator in Eq. (24) represents the transportation cost. It is clear that the smaller value of the transportation cost results in a higher value for the solution. The main loop of the proposed GA continues with a certain number of iterations until the optimal solution or near one concludes. In other words, the stop condition of the algorithm is achieving the defined maximum iteration (MaxIt).

5. Computational results

In this section, some numerical experiments (one case study, 20 small-sized and large-sized random test problems) are performed to evaluate the performance of the developed model and solution method. CPLEX can solve the proposed MILP model in small-sized instances. To solve the large-sized instances, we use the proposed two-phase method in the previous section.

Experiment on the case study

In this sub-section, we explain a case study based on the real data gathered from Ahvaz Sugar Refinery Company. The company network is a bi-echelon SC, including one factory and five DCs.
The factory produces three types of products: Caster Sugar, Sugarcane, and Sugar Lump, such that we named these products for simplicity p1, p2, and p3, respectively. The company's objective is to supply products from the factory to DCs at a minimum cost. Also, transportation is performed using vehicles provided by a transportation fleet.
Tables 3 to 7 explain the main data of this case. These data are gathered for the annual planning horizon with six two-month periods. Besides, the initial inventory of the factory and DCs is equal to 1000 units. We assume that each period's maximum greenhouse gas emission equals 500 units.
The optimal solution of the case study is presented as the economic production and inventory quantity, as well as vehicle routing decisions in a six-period.
Table 8 shows the factory's open status and the optimal production amount in each period.
Table 9 presents the amount of each product received by each DC in each period.

Finally, holding and backorder costs for each DC in each period are reported in Table 10. For the vehicle routing decision, vehicles K2 and K4 are rented for delivering products from the factory to DCs. These two vehicles are chosen in the optimal solution considering environmental and other constraints.

Table 11 indicates DCs met by each vehicle, transportation cost, and emission volume at the end of the planning horizon.

Table 3. Data of GPIRP problem (Products data)

Products	Production capacity of the factory	Variable production cost	Fixed production cost	Holding cost per unit	Backorder cost per unit	Space occupied per unit
P1	3000	1.8	600	1.8	2.9	0.6
P2	2000	2.05	600	1.6	3.1	0.8
P3	2500	3.65	600	2.3	3.7	0.55

Table 4. Data of GPIRP problem (Vehicle's data)

Vehicle	Variable cost per unit	Fixed cost of renting	Capacity	The volume of pollutant emission per unit
K1	7	700	1200	0.8
K2	6	900	1100	0.65
K3	9	1050	1500	0.9
K4	11	1300	1000	0.5

Table 5. Data of GPIRP problem (Facilities capacity data)

Facility	F	DC1	DC2	DC3	DC4	DC5
Capacity	400	500	600	320	260	400

Table 6. Data of GPIRP problem (Customers/demand data)

Product for DC	Planning horizon					
	T1	T2	T3	T4	T5	T6
P1.DC1	59	249	480	170	293	112
P1.DC2	376	128	253	350	445	480
P1.DC3	274	69	75	129	420	127
P1.DC4	407	122	465	175	98	126
P1.DC5	308	237	176	415	293	275
P2.DC1	459	143	379	377	190	284
P2.DC2	38	27	265	390	467	65
P2.DC3	284	235	6	169	81	397
P2.DC4	156	264	83	301	131	327
P2.DC5	345	374	225	42	114	457
P3.DC1	76	413	269	498	39	221
P3.DC2	53	481	2	387	409	434
P3.DC3	42	200	130	400	216	455
P3.DC4	91	132	73	68	435	290
P3.DC5	**275**	**72**	**427**	**311**	**175**	**257**

Table 7. Data of GPIRP problem (Distance matrix)

	O	F	DC1	DC2	DC3	DC4	DC5
O	0	100	80	250	480	310	285
F	100	0	125	140	385	305	280
DC1	80	125	0	100	240	185	133
DC2	250	140	100	0	142	448	318
DC3	480	385	240	142	0	400	210
DC4	310	305	185	448	400	0	165
DC5	285	280	133	318	210	165	0

Table 8. Economic production in each period

period	Factory activity	Economic Production product type P1 (*10^3)	Economic Production product type P2 (*10^3)	Economic Production product type P3 (*10^3)
T1	1	865	744	865
T2	1	764	1016	1160
T3	1	1449	923	901
T4	1	1239	1279	1664
T5	1	1549	983	1274
T6	1	1120	1530	1657

Table 9. Optimal amount of each product received by DCs in each period

Time. Product	DC1	DC2	DC3	DC4	DC5
T1.P1	0	276	174	307	208
T1.P2	359	0	184	56	245
T1.P3	0	0	0	0	175
T2.P1	208	128	69	122	237
T2.P2	143	0	235	264	374
T2.P3	389	434	142	123	72
T3.P1	480	253	75	465	176
T3.P2	379	230	6	83	225
T3.P3	269	2	130	73	427
T4.P1	170	350	129	175	415
T4.P2	377	390	169	301	42

Time. Product	DC1	DC2	DC3	DC4	DC5
T4.P3	498	387	400	68	311
T5.P1	293	445	420	98	293
T5.P2	190	467	81	131	114
T5.P3	39	409	216	435	175
T6.P1	112	480	127	126	275
T6.P2	284	65	397	327	457
T6.P3	221	434	455	290	257

Table 10. PC, BC, and HC in each period and at the end of the planning horizon

Costs	Time periods					
	T1	T2	T3	T4	T5	T6
Production cost (PC)	3955.95	8292	8389	11525.75	10053.45	11800.55
Backorder cost (BC)	0	0	0	0	0	0
Holding cost (HC)	490.4	56	0	0	0	0
Total	4446.35	8358	8389	11525.75	10053.45	11800.55

Table 11. Transportation planning, transportation cost, and the volume of greenhouse gas emission

Vehicle	DCs				
	DC1	DC2	DC3	DC4	DC5
K1	×	×	×	×	×
K2	✓	✓	✓	×	✓
K3	×	×	×	×	×
K4	×	×	×	✓	×
Transportation costs at the end of the planning horizon				60690	
The volume of greenhouse gas emissions at the end of the planning horizon				2880	

Evaluation of the proposed model and solution method

In this sub-section, we seek to evaluate the model performance and solution method in various sizes of 20 random test problems. For this goal, to assess a broad domain of problem dimensions, we apply the sizes offered in (Mirzapour Al-e-hashem and Rekik, 2014). For each instance, we generate the parameters randomly explained in Table 7. It should be noted that the Taguchi method is used for parameter tuning of the proposed two-phase GA. The proposed MILP (by CPLEX solver in GAMS) and two-phase heuristic method (based on GA) are implemented for all test problems.

Table 12 shows the result of solving experimental problems using the proposed model and solution method. We consider 500 minutes as the maximum time for solving the problem using the proposed MILP model. It is worth noting that the first ten small-sized test problems are solved within an acceptable time. But solving the problem requires much more time in test problems 11 to 15, which increases the size of the problems. Finally, solving the instances in test problems 16 to 20 is impossible using the CPLEX-based MILP model. While all experimental problems are solved by the proposed two-phase method within the desired time, and the optimality gap of the best solution is 0 or is closed to zero.

Therefore, the proposed MILP can be solved to determine the optimal solution only for the experimental problems, which are not large. In contrast, the proposed GA-based two-phase method has excellent performance for large-sized problems.

Finally, the effect of the environmental considerations is analyzed. As we mentioned in Table 13, the value of maximum greenhouse gas emission ($\mathtt{MAXGH}$) is randomly generated by $\mathtt{Uniform(1200,2500)}$. In the final test problem, $\mathtt{MAXGH} = 1923.12$is generated. Now, we fix all the parameters of this test problem and increase the value of MAXGH and report the best solutions. Figure 5 shows the percentage of the increase in $\mathtt{MAXGH}$ and the total cost decreases. In fact, the model has more flexibility by increasing the value of $\mathtt{MAXGH}$ and, therefore, will result in a lower total cost

As a managerial insight driven by numerical results, the total cost is in most of its value when the environmental constraint is applied with the most stringency (flexibility 0). However, applying small flexibility to the environmental constraint (10 to 20 percent) significantly decreases the total cost (about 20 percent). Finally, the environmental impacts dramatically increase by applying more flexibility, but the total cost does not considerably decrease.

Table 12. Random generation of GPIRP test problem

Parameters	Values in ten first instances	Values in ten second instances
vcp_p	Uniform(1,3)	Uniform(1.3,3.5)
fcp_p	Uniform(500,1500)	Uniform(500,1500)
v_k	Uniform(1.5,5.5)	Uniform(1,7)
$d_{(i,j)}$	Uniform(10,1000)	Uniform(10,1000)
f_k	Uniform(400,700)	Uniform(500,1000)
$hc_{p,F}^t$	Uniform(0.5,3.5)	Uniform(0.25,2.5)
$hc_{p,d}^t$	Uniform(0.3,2.5)	Uniform(0.20,2.10)
bc_p	Uniform(1,5)	Uniform(0.5,6.5)
$capF_p$	Uniform(2000,4000)	Uniform(2000,6000)
$capV_k$	Uniform(1500,3500)	Uniform(1800,3900)
s_p	Uniform(0.2,0.8)	Uniform(0.2,0.8)
CapSF	Uniform(1000,4000)	Uniform(1000,4000)
$CapSD_d$	Uniform(3000,6000)	Uniform(3000,6000)
GH_k	Uniform(0.05,0.60)	Uniform(0.05,0.80)
$MaxGH^t$	Uniform(500,1000)	Uniform(1200,2500)

Table 13. Test problems (performance evaluation of the proposed MILP model and two-phase solution method)

Test problem	Problem size				Proposed MILP (CPLEX)		The Proposed Hybrid Heuristic Solver		Gap (%)
	|T|	|P|	|DC|	|K|	Cost	CPU time (m)	Cost	CPU time (m)	
1	3	1	2	2	60919.11	3.39	60919.11	4.01	0
2	4	2	2	2	65083.15	5.67	65083.15	4.12	0
3	4	2	3	2	73914.60	5.83	73914.60	5.47	0
4	5	2	2	2	76715.38	8.34	86652.90	6.16	12.95
5	5	3	3	2	76719.66	9.13	80628.31	8.32	5.09
6	5	3	5	3	79664.22	10.42	85111.73	9.50	6.84
7	5	4	5	3	98975.58	11.83	103676.40	10.37	4.58
8	6	3	5	4	102541.49	20.22	112428.95	12.35	6.65
9	6	5	5	5	115472.47	25.95	115472.47	12.54	0
10	6	4	8	4	118811.81	32.38	125202.10	13.30	5.38
11	10	5	10	5	320200.13	107.56	320200.13	51.54	0
12	12	5	10	6	332843.08	321.63	340129.38	53.06	2.19
13	13	6	12	7	357489.55	432.19	381201.12	56.63	6.63
14	14	6	12	6	388735.23	409.21	400213.31	55.69	2.95
15	15	6	14	7	422274.87	487.19	445691.05	60.37	5.55
16	15	7	15	7	NA	+500	461852.97	62.37	-
17	15	8	15	8	NA	+500	587093.09	72.00	-
18	16	8	17	8	NA	+500	616215.85	75.35	-
19	16	9	20	10	NA	+500	620580.78	80.50	-
20	18	10	20	10	NA	+500	762599.27	93.11	-

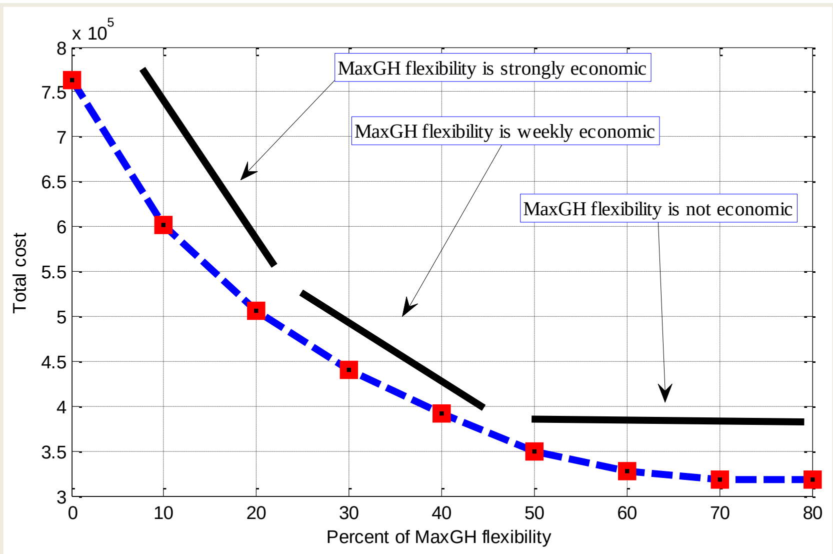

Figure 5. Sensitivity analysis on maximum greenhouse (MaxGH) and total cost

Although Table 13 shows that the proposed hybrid two-phase heuristic solver is able to obtain optimal or near to optimal solutions for various scales of the PIRP problem, in the following, we also illustrate a new performance evaluation analysis during which the quality and stability of the proposed solution method is compared with a metaheuristic solver base on common GA. For this purpose, we randomly generate ten large-scale instances of the PIRP problem and solve each instance ten times by the proposed hybrid solution method and commonly-used GA, respectively. Table 14 shows the instances and solution obtained by solvers after 3600 seconds. As illustrated in Figure 6, the proposed solution method frequently dominates the classic GA. Furthermore, to validate the stability of the proposed solution method in comparison with GA, Figure 7 shows the boxplot of the two methods for each problem instance. The boxplot visualization presents that the stability of the proposed heuristic solution method is acceptable and better than classic GA.

Table 14. Compression of the proposed Hybrid Heuristic Solver and Metaheuristics (GA) Solver in solving large-scale instances

Instances	Problem size				Proposed Hybrid Heuristic Solver		Metaheuristics (GA) Solver	
	|T|	|P|	|DC|	|K|	Mean	Deviation	Mean	Deviation
1	10	5	25	10	102343.43	2235.54	107460.60	2459.09
2	10	5	30	10	114338.54	3035.14	125772.39	3490.41
3	10	10	35	10	125435.58	3554.65	130453.00	4016.75
4	15	10	40	10	134659.75	3454.87	150818.92	4042.20
5	15	15	50	10	145835.09	5485.61	148751.79	6857.01
6	15	15	60	20	167546.65	6985.31	184301.32	8452.23
7	20	20	70	20	168033.04	6904.54	181475.68	9113.99
8	20	20	80	20	179954.74	8454.03	201549.31	10567.54
9	20	25	90	20	189534.94	9855.61	204697.74	11728.18
10	30	25	100	20	195479.03	11544.95	201343.40	16393.83

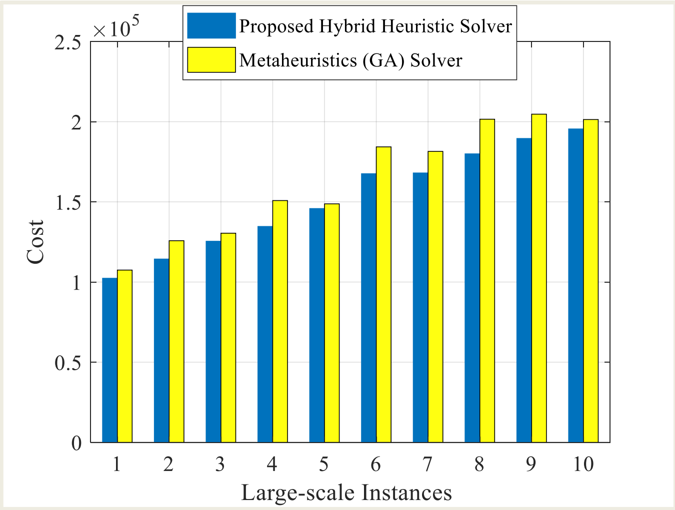

Figure 6. Compression of the proposed Hybrid Heuristic Solver and Metaheuristics (GA) Solver in solving large-scale instances with respect to the objective function

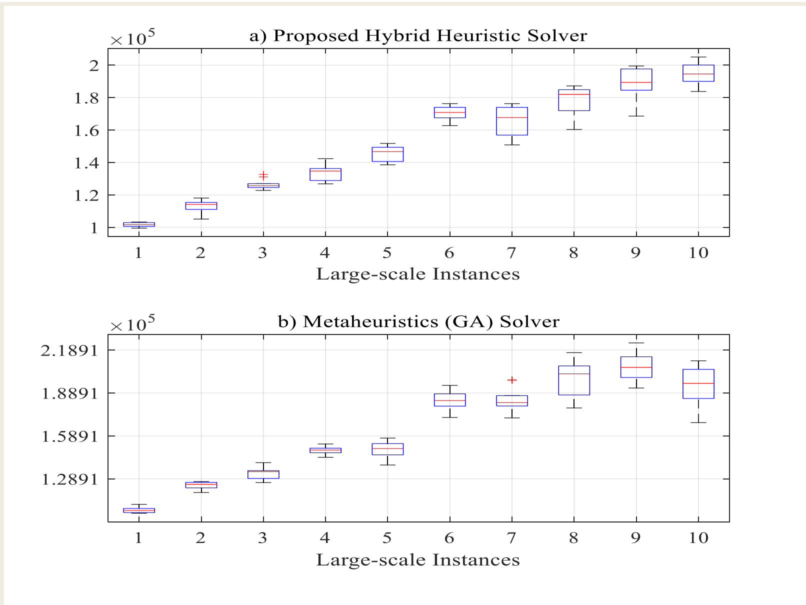

Figure 7. Boxplot to illustrate the stability of the proposed Hybrid Heuristic Solver in comparison with the GA Solver

Practical results and managerial insights

In this sub-section, some practical and managerial insights have been derived from the computational results and briefly explained. The managers of the production and inventory sections of the case study company found the results of this research quite valuable for solving their problem, as it offered solutions in a very short time and with a tiny error. Before this study, they could not find an excellent solution to the problem in the short decision-making time available. Therefore, by comparing the costs before and after implementing this research, and based on the short time of achieving a good solution, they introduced this study as an important step to progress their systems. The results of production planning and inventory control show that the three parameters of production capacity, holding cost, and holding capacity are among the effective factors in the production and inventory of products in each period. The changes in these three parameters are discussed in the following:

1) Assuming the high product holding capacity, the opportunity for greater product storage for high demand periods is increased by increasing the production capacity. Therefore, in these periods, without the cost of budget deficits, the re-activation costs of the factory can be reduced, and the inventory of previous periods can be used. In this strategy, it is necessary that "the cost related to increasing the factory production capacity" should not be higher than "reducing the total costs with increasing the production capacity". Assume that CapF is the factory production capacity and $\mathrm{Cost}_{\mathrm{CapF}}$ is the total cost for this level of capacity. Now, if the production capacity is increased by a unit size X, and the new production capacity is $\mathrm{CapF}_{\mathrm{new}} = \mathrm{CapF} + \mathrm{X}$, then the total cost for this level of production is equal to $\mathrm{Cost}_{\mathrm{CapF}_{\mathrm{new}}}$, which is not certainly higher than $\mathrm{Cost}_{\mathrm{CapF}}$, $(\mathrm{Cost}_{\mathrm{CapF}_{\mathrm{new}}} \leq \mathrm{Cost}_{\mathrm{CapF}})$. Now, if the cost relevant to increasing the production capacity is Y, then the real/net new cost is equal to $\mathrm{Cost}^{\mathrm{R}}_{\mathrm{CapF}_{\mathrm{new}}} = \mathrm{Cost}_{\mathrm{CapF}_{\mathrm{new}}} + \mathrm{Y}$, which might be higher compared to $\mathrm{Cost}_{\mathrm{CapF}}$. Thus, in increasing the production capacity, the total cost per new production level should be seen along with the cost related to increasing the production capacity. Figures 8(a) and 6(b) are shown for a better explanation. In Figure 8(a), reducing the total cost and increasing the capacity cost are presented at different levels relevant to increasing the production capacity. In the beginning, the total reduced cost is often higher than the increased costs of the capacity. Then, they are equal at a point such as A, and the increased cost of the capacity is raised, while the total reduced cost becomes 0 from point B. Figure 8(b) clearly shows that point A is the best level related to increasing the production capacity because the new net cost is at its lowest. With sensitive analysis of production capacity parameter, the decision-making managers can have a strategy to increase the production capacity as described and reach the optimum point A.

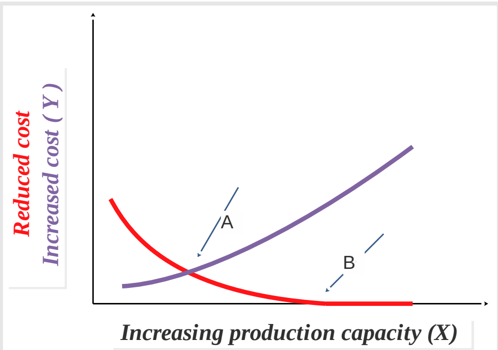

Figure 8(a). Reduced and increased cost with respect to increasing the production capacity

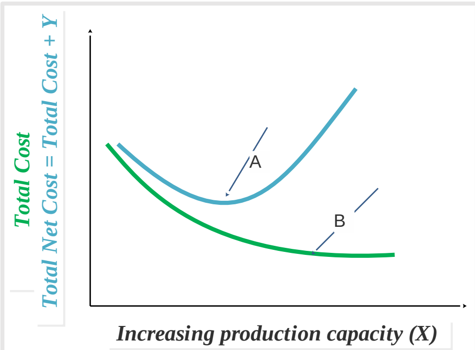

Figure 8(b). Total cost and total net cost with respect to increasing the production capacity (Y = the cost related to increasing the production capacity)

2) Assuming that the production capacity is high, increasing related to the holding capacity of the products has a very similar analysis to the previous mode (increasing the production capacity). It should be noted that if the holding cost of each product unit reduces instead of increasing the holding capacity, the same result is obtained. Also, if the warehouse inventory is zero in different periods or less than the warehouse capacity in the optimum solution for the problem, the strategy related to increasing the holding capacity is wrong because only the net cost is increased.
3) Increasing the production capacity and the holding capacity simultaneous are similar to the previous two modes, with the difference that at the time of net cost calculation, two components of increasing the cost resulting from increasing the production and holding capacities must be added to the total cost.

Obviously, without considering the environmental constraints, the total cost will be the lowest. Considering the environmental constraints would increase the total cost, but how the environmental constraints should be considered is a question for the managers, the answer to which is explained. The results show that although the total cost is reduced with the flexibility in the environmental constraints, no significant reduction can be seen in the cost after a level. In other words, as shown in Figure 9(a), the flexibility in the greenhouse gas emissions would reduce the total transportation cost and, as a result, the total cost. While it can be seen in Figure 9(b) that from a point such as A onwards, no significant reduction can be seen in the total cost. In fact, point A can show a reasonable decision to minimize the costs with respect to the environmental constraints in the green SC. Another proposed point is a point such as B, where the percent of changes in the cost and greenhouse gas is equal. Point B is suggested in the problems where environmental constraints are more sensitive.

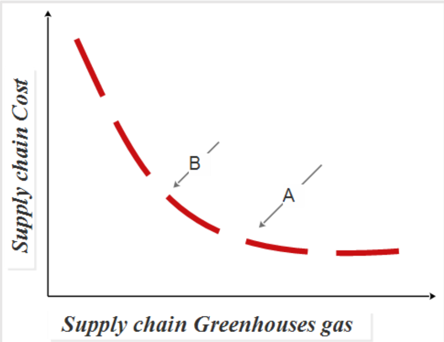

Figure 9(a). Pareto front of the SC cost and greenhouses gas

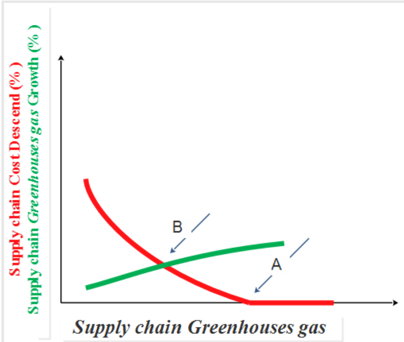

Figure 9(b). Two proposed levels of the SC greenhouses gas (A and B)

6. Conclusion

In this research, a model is extended in the field of the PIRP. This model seeks to minimize the entire costs of production, transportation, and inventory. Corporations now must consider environmental factors and the economic aspects of the SCs in all activities, from manufacturing to supply. Regarding effective transportation plans, corporations can simultaneously reduce costs and environmental contamination. This paper investigated a green SC problem named GPIRP by adding environmental constraints to the PIRP. This problem simultaneously controls economic decisions as well as the social ones related to production and supply in the corporations. The problem was formulated as a MILP model and solved using a proposed two-phase GA.

Based on the literature, although metaheuristic algorithms do not ensure a global optimal solution, they can present satisfactory solutions to complex problems in a short period. Subsequently, the initial problem is decomposed into two sub-problems through the two-phase GA. The outcomes revealed that the suggested algorithm returns a solution with an allowable error in a short period.

Further, we conducted a case study to show the proposed model's applicability and environmental feasibility in a real case. Besides, numerical small- and large-sized instances and their analyses were tested to show the efficiency and effectiveness of the algorithm and developed model.

As two main quantitative findings, results driven by numerical analysis show that 1) we can obtain a Pareto optimal solution in which the greenness is near 90% while total cost can be near-optimal. In other words, using the proposed model, we can do a proper trade-off between the economic and environmental objective functions so that the two objectives have a near to ideal value. 2) The proposed hybrid two-phase heuristic solver can solve the small and medium sizes of the PIRP problems with less than a 5% gap on average. Furthermore, the stability test verifies that the proposed method had an acceptable level of robustness/stability in different runs for a specific instance.

While we have shown the efficiency of the proposed MILP model and the two-phase solution method, our work is not without limitations. In this paper, customer demand, the number of available vehicles, the production capacity of factories, and some other important parameters were assumed deterministic. While these parameters sometimes include uncertainty in real life. Therefore, future studies can use optimization approaches under uncertainty, especially robust optimization methods, to capture the uncertain parameters. Also, it may be of interest to develop the proposed model into a multi-objective formulation, such as minimizing the delivery time and maximizing the production stability over time.

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