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Strategies for service, price, and lot size with a capacitated manufacturer and multi retailers in a VMI system

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Abstract

Vendor-managed inventory (VMI) is a popular inventory management system that allows a vendor to access sales data and manage inventory levels for his retailers. The formulation of service level and pricing decisions are finite in the VMI model literature. The study examines how a manufacturer and its retailer communicate with one another to optimize their net profits through modifying service level, pricing, and inventory policy in a VMI system employing a Stackelberg game. The manufacturer produces a product and distributes it to several retailers at a similar wholesale price. The retailers subsequently offer the product at retail pricing in independent marketplaces. The Cobb-Douglas demand function could characterize the demand rate in every market, which is an enhancing function of the service level, however, a reducing function of retail prices. The manufacturer selects its wholesale pricing, replenishment cycles, backorder amount, and binary variable for production capacity to optimize profit. Retailers determine retail pricing and service levels so that they may optimize their profitability. Solution procedures are evolved for finding the Stackelberg game equilibrium from which no firm is inclined to deviate from maximizing its profit. The balance benefits the manufacturer while increasing the revenues of the retailers. If the retailers are prepared to engage with the manufacturer via a cooperative contract, the equilibrium could be enhanced to the advantage of both the manufacturer and his retailers. Ultimately, a numerical example is shown, along with the appropriate sensitivity analysis, to demonstrate that. 1) In certain circumstances, the manufacturer might benefit from his leadership and monopolize the additional profit generated by the VMI system. 2) The manufacturer's profit, and later the retailers' profit, could be increased more by the cooperative contract, in comparison to the Stackelberg equilibrium; 3) Market-related parameters have a substantial impact on the net profit of any enterprise.

Keywords: vendor managed inventory; service; pricing; Stackelberg game; multi retailers.

Paper Type: Original Research

1. Introduction

Vendor-managed inventory (VMI) mainly may assist a decentralized supply chain in optimizing its inventory management performance. The vendor supervises the retailer's inventory by making replenishment decisions for the retailer under the VMI approach. The vendor seems to have the option of organizing the manufacturing and distribution processes based on actual demand. In contrast to a traditional supply chain, the VMI system vendor will prefer to accept more responsibility for the system's operating expenses than the customers to maximize the system's overall profit. As a result, rather than centralized decision-making as in an ordinary supply chain, determining the optimal operating strategy for the VMI system is based on the Stackelberg game, in which the vendor seems to be the leader and the customers are the followers (Deng et al. (2020)).

Stackelberg games are often employed in the literature on VMI systems. Wu et al. (2005) analyze supply chain coordination in a one-vendor, one-retailer environment using a VMI contract. The contract states that the vendor, as a Stackelberg leader, maintains the retailer's inventory and covers inventory-holding expenses; it is the retailer's responsibility to sell products and set their retail pricing. The findings indicate that the contract allows the vendor and the retailer to interact while achieving optimal profit coordination. Viswanathan (2009) utilizes the Stackelberg game to represent discount pricing decisions in a vendor–buyer supply chain. The findings reveal that the leader's optimal conditional approach improves VMI system coordination. Wang et al. (2010) investigated two alternative models of the Stackelberg game in the VMI system, where retailers selling prices are supposed to be exogenously set, with the cooperative advertisement issues of a monopolistic manufacturer with competitive duopolistic retailers. Seyed Esfahani, Biazaran, and Gharakhani (2011) utilized an identical Stackelberg- manufacturer or Stackelberg-retailer game to depict the interaction between the manufacturer and the retailer. Braide, Cao, and Zeng (2013) employed a discount pricing strategy for coordinating VMI supply chains with several retailers, and the Stackelberg game was utilized in the supply chain modeling. Taleizadeh, Noori-daryan, and Cárdenas-Barrón (2015) used

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the VMI model in a two-level supply chain with deterministic and price-sensitive demand, including a singular vendor and numerous retailers. They established the optimal retailer pricing and the optimal replenishment value for raw materials and finished goods to optimize the profit function using the Stackelberg method. De Giovanni, Karray and Martín-Herrán (2019) employed a differential Stackelberg paradigm to assess the effects of collaborative advertising inside bilateral monopolies. Wei et al. (2020) used Stackelberg's game theory to compare VMI and retailer-managed inventory (RMI) techniques and the contributions of stochastic learning. Karimi et al. (2021a) examined a Stackelberg game model in a VMI system with one manufacturer as the leader and numerous retailers as followers to choose the optimal retailers. Karimi et al. (2021b) suggested a vendor managed inventory model with a single manufacturer and a single retailer. The demand rate for the product is considered a declining function of price and a rising function of service level. Mahdavisharif et al. (2022) developed a complete model depending on the Stackelberg game to address the problem of a two-echelon supply chain with unpredictable demand, which is price and time-dependent. Poursoltan, Seyedhosseini, and Jabbarzadeh (2021) examined a closed-loop VMI-type supply chain with a retailer, a manufacturer, and a remanufacturer. In this study, the impact of the learning effect is considered in both manufacturing and remanufacturing. They developed a new hybrid approach to solve the MINLP model. Cai et al. (2021) developed a bi-level supply chain model under the VMI strategy with a single retailer and a single supplier. They examined the customer's strategic behavior on system performance. The VMI model was studied by Kusuma and Kallista (2022) at three levels: vendor, distributor, and retailer. The goal of this concept is for retailers to choose the finest vendors. Each vendor has a scheduler that calculates the number of products that should be produced on the customer’s inventory and preferences.

In addition, the distributor has three levels of control over the products delivered: vendor inventory, customers, and customer preferences. Modares, Farimani, and Emroozi (2022) studied a two-level SC model with several retailers and vendors under the VMI strategy. This study considers three objectives: minimization of total supply chain costs, maximization of the retailer weight value and maximization of production reliability. The model was solved using PSO and GA metaheuristic algorithms. Karampour et al. (2022) presented a bi-objective Green VMI-type SC model. This study aims to minimize carbon emissions and maximize inventory profit. They solved the NLP model by MORDA, NSGA-II, and MOKA algorithms.

It is critical to develop and assign demands in the VMI system. Various demand functions are suggested in the literature on VMI-type supply chains. Several investigations have shown that demand is based on quality (Chen et al. (2021)), quality and marketing effort (Huang, He, and Wang (2019)), and stock (Zanoni and Jaber, (2015)). Whereas some have argued that demand is a function of pricing (– for example, Yu, Huang, and Liang (2009); Almehdawe and Mantin (2010); Rasay, Zare Mehrjerdi and Fallah Nezhad (2013); Yu et al. (2013); Zare Mehrjerdi, Fallah Nezhad and Rasai (2014); Niknamfar and Pasandideh (2014); Rasay, Zare Mehrjerdi and Fallah Nezhad (2015); Taleizadeh, Noori-daryan, and Cárdenas-Barrón (2015); Haji, Afzalabadi, and Haji (2018); Bahrami and Pasandideh (2019); Batarfi, Jaber and Glock (2019)). Various economists have claimed that consumer behavior cannot be adequately described by referring to demands merely depending on sales prices. As a result, research started to focus on other factors, such as price and stock level (Hemmati, Fatemi Ghomi, and Sajadieh (2017)), price and advertising (Yu and Huang (2010) and Deng et al. (2020)), price and time (Mahdavisharif et al. (2022)), as well as price and service level (Karimi et al. (2021b)). As previously stated, Karimi et al. (2021b) is the only research considering demand function as a function of price and service level in the VMI system. While the service level influences the consumer's buying decision, a greater degree of service encourages people to purchase the products. Nevertheless, raising the service level could increase service costs. Therefore, the retailer and the manufacturer should collaborate on pricing and service level decisions to optimize profit. As a result, the emphasis of this study is on how to coordinate pricing and service level decisions for a decentralized supply chain under VMI and the impact of the interplay between retail price and service level strategies.

Karimi et al. (2021b) proposed a VMI system with a single retailer and manufacturer. Demand is a linear function of price and service level, and the model is solved using a combination of analytical and numerical approaches. However, in this research, in addition to the presence of several retailers, the Cobb-Douglas demand function is used to define every retailer's demand (which is reliant on price and service level). Additionally, to reduce expense, a binary decision variable is employed to ensure that the manufacturing process continues uninterrupted by production setup when production capacity is used up. Furthermore, depending on the Stackelberg equilibrium, the manufacturer realizes that his profit could be higher by altering his wholesale price to obtain an extra system profit. The retailers benefit from the increased profit by signing a profit-sharing contract with the manufacturer. Ultimately, only the analytical technique is used to solve the models. Table 1 shows the novelty of the paper in comparison to other relevant VMI model studies.

Table 1. The novelty of this research

Article	Number of retailers	Demand dependent to			Demand function type			Production line setup variable	cooperative contract	solution methods/Tools
		Service level	price	others	Linear	Cobb-Douglas	others			
Almahda & Mantin (2010)	n		*			*		*		Analytical and KKT
Braide, Cao, and Zeng (2013)	n		*			*		*		Heuristic
Niknamfar and Pasandideh (2014)	n		*			*				GA
Rasay, Zare Mehrjerdi and Fallah Nezhad (2015)	n		*			*			*	KKT
Hemmati, Fatemi Ghomi and Sajadieh (2017)	1		*	*	*					Heuristic
Haji, Afzalabadi and Haji (2018)	1		*			*				Analytical
Huang, He, and Wang (2019)	1			*	*					Analytical
Batarfi, Jaber and Glock (2019)	1		*	*	*					Analytical
Deng et al. (2020)	n		*	*			*			Heuristic
Chen et al. (2021)	1			*	*					Analytical
Karampour et al. (2022)	n		*		*					MORDA, NSGA-II, and MOKA
Mahdavisharif et al. (2022)	1		*	*			*			Analytical and iterative
Karimi et al. (2021b)	1	*	*		*					Analytical and KKT
Current study	n	*	*			*		*	*	Analytical

The remaining research is structured as follows: The next section outlines the issue; Then the profit functions of the participants discuss in section 3; In Section 4, the Stackelberg game model is presented, and the model' solution mechanism is presented in section 5. Section 6 explores a cooperative contract enhancement; Section 7 provides a numerical example and sensitivity assessments, and Section 8 concludes the article.

2. A description of the problem

This article investigates a VMI supply chain comprising one manufacturer and numerous retailers. The manufacturer fabricates a single product with limited production capacity and sells it via various retailers. The manufacturer and retailer have a leader-follower relationship, as depicted in Fig. 1. Instead of focusing on pricing, retailers provide service levels to attract more consumers. As a result, in this research, the product demands of retailers i, $D_i(p_i, s_i)$, i =1, 2, ..., n are influenced by their retail price (p_i) and the service level of the retailer i (s_i). The Cobb–Douglas demand function accurately depicts the link between $D_i(p_i, s_i)$ and p_i and s_i:

$$D_i(p_i, s_i) = K_i \frac{s_i^{\alpha_i}}{p_i^{a_i}} \quad i = 1,2, \dots, n \tag{1}$$

Where Ki is a positive constant, it indicates the retailer's market scale α_i and a_i represent the elasticity parameters s_i and p_i, respectively.

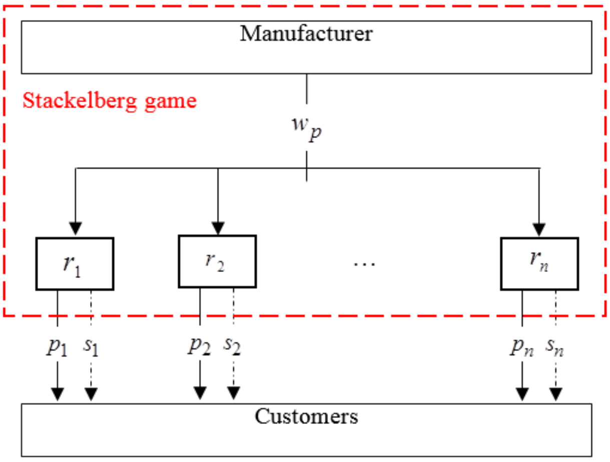

Figure. 1. Problem schematic and Stackelberg game

The annotations included the following:

Parameters:

$D_i(p_i, s_i)$	The demand of i -th retailer
K_i	Basic market demand of i -th retailer
a_i	The price elasticity of demand for retailer i
α_i	The elasticity of retailer i 's demand in terms of service
Cap	Production capacity of the product for the manufacture
Cm	The manufacturer's product manufacturing cost
Hr_i	Holding costs at retailer i
Hm	manufacturer's holding cost
Br_i	Costs associated with backorders paid by the manufacturer to the retailer i
Sr_i	The product's fixed order cost for the retailer i
Sm	The product's fixed order cost for the manufacturer
ξ_i	The cost of inventory for retailer i
φ_i	The product's transportation cost from the manufacturer to the retailer i
η_i	The service cost factor for retailer i
γ	an adequately large positive number $\gamma > Cap$

Variables influencing manufacturer decisions

C	product's common replenishment cycle
y_i	a percentage of retailer i 's backlog time
w_p	wholesale price set by the manufacturer
z	A binary variable that equals one if the production capacity of manufacture is upper

Variables influencing retailer decisions

p_i	Retailer i 's selling price
s_i	i 's retailer service level

3. Player profit function

3.1. The net profit of the manufacturer

The net profit of a manufacturer is calculated by deducting the revenue from the total cost. Furthermore, the retailer i pays $(w_p + \xi_i)$ to the manufacturer for each unit of product. As a result, the manufacturer's revenue is determined as follows:

$$TR_m = \sum_{i=1}^{n} D_i\,(p_i, s_i)(w_p + \xi_i) \tag{2}$$

The transportation and manufacturing expenses are included in the direct manufacturing cost.

$$TDC_m = \sum_{i=1}^{n} D_i\,(p_i, s_i)(Cm + \phi_i) \tag{3}$$

Moreover, indirect costs include those associated with the supply chain inventory system separated into retailer and manufacturer inventory system costs. The retailer warehouse inventory level is demonstrated in Fig. 2(a). As noticed, retailer i's inventory costs, including holding and ordering expenses, can be calculated from Eq. (4).

$$TIC_{r_i} = \frac{1}{C}\left[\frac{D_i(p_i, s_i)C^2}{2}(Hr_i(1-y_i)^2 + Br_i y_i{}^2) + Sr_i\right] \tag{4}$$

The manufacturer warehouse inventory level is displayed in Fig. 2(b). A redundant production capacity ($\sum_{i=1}^{n} D_i\,(p_i, s_i) < Cap$) means that a manufacturer cannot regularly make its product, increasing its product invtory costs. The production setup cost (Sm), equivalent to the economic production quantity (EPQ), is necessary. Thus, the manufacturer's inventory cost for the product equals

$$TIC_m^1 = \frac{1}{C}\left[Hm\sum_{i=1}^{n}\frac{D_i(p_i, s_i)^2C^2}{2Cap} + Sm\right] \tag{5}$$

Nevertheless, $\sum_{i=1}^{n} D_i(p_i, s_i) = Cap$, the manufacturing process continues uninterrupted after the production capacity is used up. The manufacturer's inventory cost is:

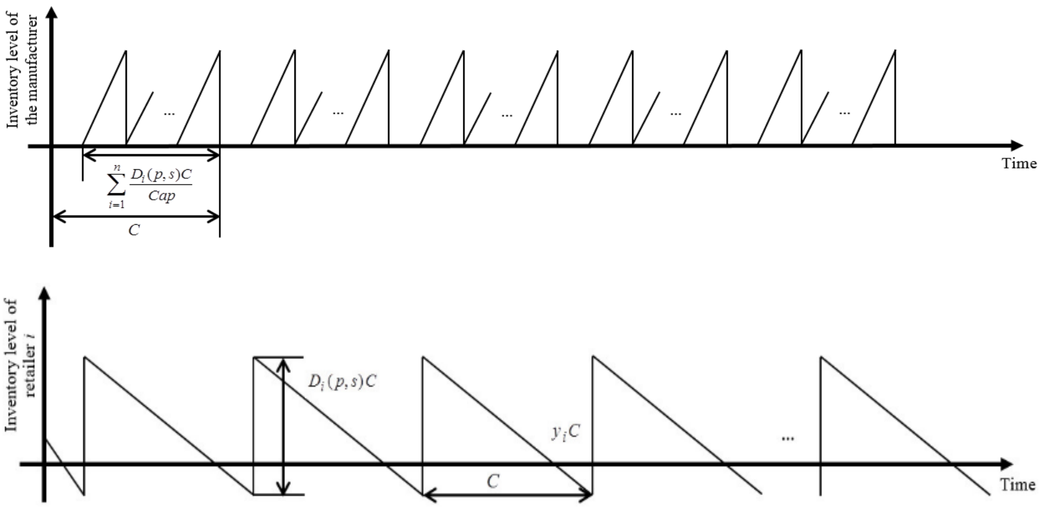

Figure. 2. (a) Retailer i's inventory level and (b) manufacturer's inventory level

$$TIC_m^2 = \frac{1}{C}\left[Hm\sum_{i=1}^{n}\frac{D_i(p_i, s_i)^2C^2}{2Cap}\right] \tag{6}$$

Based on the research above, the overall inventory cost of the VMI system could be stated as

$$TIC = TIC_{r_i} + zTIC_m^1 + (1-z)TIC_m^2 \tag{7}$$

Where $Z = 1$ denotes the presence of a production setup cost and z =0 otherwise. Therefore, the net manufacturer profit may be calculated as follows:

$$NP_m(y_i, C, w_p, z) = TR_m - TIC - TDC_m = \sum_{i=1}^{n} D_i\,(p_i, s_i)(w_p + \xi_i) - \frac{1}{C}\left[\sum_{i=1}^{n} Sr_i + zSm\right]$$
$$-\frac{C}{2}\left[\sum_{i=1}^{n} D_i(p_i, s_i)(Hr_i(1-y_i)^2 + Br_i y_i{}^2) + Hm\sum_{i=1}^{n}\frac{D_i(p_i, s_i)^2}{Cap}\right] - \sum_{i=1}^{n} D_i\,(p_i, s_i)(Cm + \phi_i) \tag{8}$$

3.2. Net profits of every retailer

The retailer is expected to have a service cost of $1/2\eta_i s_i^2$ for the service level s_i of the retailer i. This provides easy analysis control and assures that the profit function on s_i is concave. Improving service level could result in a lower return on service expenditure (Tsay and Agrawal (2000), Xiao and Yang (2008), Giri and Maiti (2014), Ali et al. (2018), and Pi, Fang, and Zhang (2019)). The net retailer profit could be calculated as follows:

$$NP_{r_i}(p_i, s_i) = (p_i - w_p - \xi_i)D_i(p_i, s_i) - \frac{\eta_i s_i^2}{2} \tag{9}$$

4. Stackelberg game model

In this study, the Stackelberg game model was developed.

SG Model:

$$\begin{aligned} NP_m(y_i, C, w_p, z) &= \sum_{i=1}^{n} D_i\,(p_i, s_i)(w_p + \xi_i - Cm - \phi_i) - \frac{1}{C}\left[\sum_{i=1}^{n} Sr_i + zSm\right] \\ &- \frac{C}{2}\left[\sum_{i=1}^{n} D_i(p_i, s_i)(Hr_i(1 - y_i)^2 + Br_i y_i^2) + Hm\sum_{i=1}^{n}\frac{D_i(p_i, s_i)^2}{Cap}\right] \end{aligned} \tag{10}$$

S.t:

$$\sum_{i=1}^{n} D_i\,(p_i, s_i) \le Cap \tag{11}$$

$$Cap - \sum_{i=1}^{n} D_i\,(p_i, s_i) \le \gamma z \tag{12}$$

$$0 \le y_i \le 1 \quad i = 1,2, \dots, n \tag{13}$$

$$z\epsilon\{0,1\}, \qquad C > 0, \qquad w_p \ge 0 \tag{14}$$

$$\text{Max } NP_{r_i}(p_i, s_i) = (p_i - w_p - \xi_i)D_i(p_i, s_i) - \frac{\eta_i s_i^2}{2} \tag{15}$$

$$(p_i - w_p - \xi_i) > 0 \tag{16}$$

$$p_i, s_i \ge 0 \tag{17}$$

Where Eq. (10) is the objective function of the manufacturer, Eq. (15) denotes the objective function of the retailer. Eq. (11) ensures that the manufacturer's total demand does not surpass his production capacity; constraint (12) is utilized for setting z =1 when the manufacturer's capacity is redundant. Eq. (13) is used to set the limits for the proportion of backlog that cannot exceed 100% of the demand. The present basic retailing condition is expressed by Eq. (13), and Eqs show the range of variables. (14, 17).

5. Solution procedure

The optimal retailer response function is employed to calculate equilibrium. The optimal manufacturing decision is then examined depending on the optimal retailer reactions.

5.1. Retailers' optimal approach

The optimal service level and retail pricing are expressed as follows for the maximizing $NP_{r_i}(p_i, s_i)$ as the concave function of s_i and p_i:

Theorem 1 states that the optimal retail pricing for $a_i > 1$ and $0 < \alpha_i < 1$ would be as follows:

$$p_i^*(w_p) = \frac{a_i(w_p + \xi_i)}{a_i - 1} \qquad i = 1,2, \dots, n. \tag{18}$$

In addition, the service level is as follows:

$$s_i^*(w_p) = \left[\frac{K_i\alpha_i}{\eta_i}\frac{(w_p+\xi_i)}{a_i-1}\right]^{\frac{1}{2-\alpha_i}}\left(\frac{a_i(w_p+\xi_i)}{a_i-1}\right)^{\frac{-a_i}{2-\alpha_i}} \quad i=1,2,\dots,n \tag{19}$$

Proof: Appendix A contains the proof.

The retailer profit is calculated by inserting Eqs. (18), (19) into Eq. (15) as follows:

$$NP_{r_i}^*(w_p) = \frac{\eta_i(2-\alpha_i)}{2\alpha_i}\left[\frac{K_i\alpha_i}{\eta_i}\frac{(w_p+\xi_i)}{a_i-1}\right]^{\frac{2}{2-\alpha_i}}\left(\frac{a_i(w_p+\xi_i)}{a_i-1}\right)^{\frac{-2a_i}{2-\alpha_i}} \quad i=1,2,\dots,n \tag{20}$$

As a result, the optimal retailer demand rate may be expressed as:

$$D_i^*(w_p) = K_i\left[\frac{K_i\alpha_i}{\eta_i}\frac{(w_p+\xi_i)}{a_i-1}\right]^{\frac{\alpha_i}{2-\alpha_i}}\left(\frac{a_i(w_p+\xi_i)}{a_i-1}\right)^{\frac{-2a_i}{2-\alpha_i}} \quad i=1,2,\dots,n \tag{21}$$

Property1. If the manufacturer raises w_p, every retailer raises (reduces) p_i^* (s_i^*) to maximize $NP_{r_i}(p_i, s_i)$, as well as the manufacturer's demand rate decreases.

Proof. $\frac{\partial p_i^*}{\partial w_p} = \frac{a_i}{a_i-1} > 0$ ($\frac{\partial s_i^*}{\partial w_p} = -\frac{(a_i-1)}{(2-\alpha_i)(w_p+\xi_i)}\left[\frac{K_i\alpha_i}{\eta_i}\frac{(w_p+\xi_i)}{a_i-1}\right]^{\frac{1}{2-\alpha_i}}\left(\frac{a_i(w_p+\xi_i)}{a_i-1}\right)^{\frac{-a_i}{2-\alpha_i}}$) < 0 for any $a_i > 1$ ($0 < \alpha_i < 1$). As a result, whenw_p rises, the retailer i raises (reduces), p_i^* (s_i^*) to mazimize$NP_{r_i}(p_i, s_i)$. Furthermore, we have $\frac{\partial D_i(p_i^*(w_p), s_i^*(w_p))}{\partial w_p} = -\frac{K_i(2a_i-\alpha_i)}{(2-\alpha_i)(w_p+\xi_i)}\left[\frac{K_i\alpha_i}{\eta_i}\frac{(w_p+\xi_i)}{a_i-1}\right]^{\frac{\alpha_i}{2-\alpha_i}}\left(\frac{a_i(w_p+\xi_i)}{a_i-1}\right)^{\frac{-2a_i}{2-\alpha_i}} < 0$ since $a_i > 1$and$0 < \alpha_i < 1$. Consequently, when a manufacturer increases its wholesale price, the demand rate of each retailer decreases, and therefore the manufacturer's demand rate decreases. Property 1 has been proved.

5.2. The manufacturer's optimal approach

The SG model may be reformulated by replacing the retailers' optimal reaction functions from Eqs. (19) and (20) into Eq. (10) and constraints (11) and (12).

$$NP_m(y_i, C, w_p, z) = \sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right)(w_p+\xi_i-Cm-\phi_i) - \frac{1}{C}\left[\sum_{i=1}^{n} Sr_i + zSm\right]$$
$$-\frac{C}{2}\left[\sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right)(Hr_i(1-y_i)^2 + Br_i{y_i}^2) + Hm\sum_{i=1}^{n}\frac{D_i\left(p_i^*(w_p)s_i^*(w_p)\right)^2}{Cap}\right] \tag{22}$$

S.t:

$$\sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right) \le Cap \tag{23}$$

$$Cap - \sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right) \le \gamma z \tag{24}$$

$$0 \le y_i \le 1 \quad i=1,2,\dots,n \tag{25}$$

$$z\epsilon\{0,1\}, \quad C > 0, \quad w_p \ge 0 \tag{26}$$

To calculate net profit NP_m analytically, we proceed by simplifying restrictions (23 and 24). According to property (1), the manufacturer demand rate $\sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right)$ is a declining function with regard to w_p. There is just one solution w_p to the equation $\sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right) = Cap$, which is the lowest wholesale price (denoted by w_p^{min}). In two cases, constraints (23 and 24) could be simplified. $z = 1$ is one case, and $\sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right) < Cap$ could be used to replace the two constraints. SG model could subsequently be reformulated as the following model (designated as the SG1 model):

SG1 Model:

$$NP_m(y_i, C, w_p) = \sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right)(w_p + \xi_i - Cm - \phi_i) - \frac{1}{C}\left[\sum_{i=1}^{n} Sr_i + Sm\right]$$
$$-\frac{C}{2}\left[\sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right)(Hr_i(1-y_i)^2 + Br_i y_i^2) + Hm\sum_{i=1}^{n}\frac{D_i\left(p_i^*(w_p)s_i^*(w_p)\right)^2}{Cap}\right] \tag{27}$$

S.t:

$$0 \le y_i \le 1 \quad i = 1,2,\dots,n \tag{28}$$

$$\sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right) < Cap \tag{29}$$

$$C > 0, \quad w_p \ge 0 \tag{30}$$

The alternative case is $z = 0$, and the two limitations (23 and 24) could be replaced by $w_p = w_p^{min}$. Afterward, the SG model may be rearranged as the following model (designated as the SG2 model):

SG2 Model:

$$NP_m(y_i, C, w_p) = \sum_{i=1}^{n} D_i\left(p_i^*(w_p^{min})s_i^*(w_p^{min})\right)(w_p^{min} + \xi_i - Cm - \phi_i) - \frac{1}{C}\left[\sum_{i=1}^{n} Sr_i\right]$$
$$-\frac{C}{2}\left[\sum_{i=1}^{n} D_i\left(p_i^*(w_p^{min})s_i^*(w_p^{min})\right)(Hr_i(1-y_i)^2 + Br_i y_i^2) + Hm\sum_{i=1}^{n}\frac{D_i\left(p_i^*(w_p^{min})s_i^*(w_p^{min})\right)^2}{Cap}\right] \tag{31}$$

S.t:

$$0 \le y_i \le 1 \quad i = 1,2,\dots,n \text{ and } C > 0 \tag{32}$$

Let us proceed with the case $z = 1$. With respect to y_i, the second derivative of Eq. (27) is:

$$\frac{\partial^2 NP_m(y_i, C, w_p)}{\partial y_i^2} = -C\, D_i\left(p_i^*(w_p)s_i^*(w_p)\right)(Hr_i + Br_i) < 0 \tag{33}$$

As a result, $NP_m(y_i, C, w_p)$ is a concave function of y_i, regardless of C and w_p. The optimal y_i is determined using the first derivative $\frac{\partial NP_m(y_i,C,w_p)}{\partial y_i} = 0$ of Eq. (27) with respect to y_i as follows:

$$y_i^* = \frac{Hr_i}{Hr_i + Br_i} \tag{34}$$

Inserting Eq. (34) into Eq. (27) yields:

$$NP_m(C, w_p) = \sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right)(w_p + \xi_i - Cm - \phi_i) - \frac{1}{C}\left[\sum_{i=1}^{n} Sr_i + Sm\right]$$
$$-\frac{C}{2}\left[\sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right)\left(\frac{Hr_i Br_i}{Hr_i + Br_i}\right) + Hm\sum_{i=1}^{n}\frac{D_i\left(p_i^*(w_p)s_i^*(w_p)\right)^2}{Cap}\right] \tag{35}$$

With respect to C, the second derivative of Eq. (35) is

$$\frac{\partial^2 NP_m(C, w_p)}{\partial C^2} = -\frac{2}{C^3}\left(\sum_{i=1}^{n} Sr_i + Sm\right) < 0 \tag{36}$$

As a result, regardless of w_p, $NP_m(C, w_p)$ is a concave function of C.

Since $\frac{\partial NP_m(C,w_p)}{\partial C} = 0$ the optimal C (represented by C_1^*) is as follows:

$$C_1^* = \sqrt{\frac{2(\sum_{i=1}^{n} Sr_i + Sm)}{\sum_{i=1}^{n} D_i\big(p_i^*(w_p)s_i^*(w_p)\big)\left(\frac{Hr_i Br_i}{Hr_i + Br_i}\right) + Hm\sum_{i=1}^{n}\frac{D_i\big(p_i^*(w_p)s_i^*(w_p)\big)^2}{Cap}}} \tag{37}$$

By inserting Eq. (37) in (35), the net manufacturer profit becomes a function of w_p as follows:

$$NP_m(w_p) = \sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right)(w_p + \xi_i - Cm - \phi_i) - \sqrt{2(\sum_{i=1}^{n} Sr_i + Sm)\left(\sum_{i=1}^{n} D_i\big(p_i^*(w_p)s_i^*(w_p)\big)\left(\frac{Hr_i Br_i}{Hr_i + Br_i}\right) + Hm\sum_{i=1}^{n}\frac{D_i\big(p_i^*(w_p)s_i^*(w_p)\big)^2}{Cap}\right)} \tag{38}$$

Equation (38) is a continuous function w_p.Given that the manufacturer's capacity is redundant ($z = 1$) and that the maximum of Eq. (38) exists, the optimal w_p (referred to as w_{p1}^*) for maximizing (38) is to satisfy

$$\left.\frac{\partial NP_m(w_p)}{\partial w_p}\right|_{w_p = w_{p1}^*} = 0 \tag{39}$$

Concerning Eq. (39), one question remains unanswered: how to handle a case $\sum_{i=1}^{n} D_i\left(p_i^*(w_{p1}^*)s_i^*(w_{p1}^*)\right) \geq Cap$. If $\sum_{i=1}^{n} D_i\left(p_i^*(w_{p1}^*)s_i^*(w_{p1}^*)\right) \geq Cap$, Eq. (38) is a monotone function with respect to $w_p^* > w_p^{min}$, and the case $z = 1$ may be ignored since w_p is w_p^{min} (belong to the case $z = 0$). When $z = 0$ $\sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right) = Cap$ or $w_p = w_p^{min}$.

A similar analysis could be utilized to obtain the optimal y_i^* that is the same as Eq. (34) and the optimal C.

$$C_2^* = \sqrt{\frac{2\sum_{i=1}^{n} Sr_i}{\sum_{i=1}^{n} D_i\big(p_i^*(w_p^{min})s_i^*(w_p^{min})\big)\left(\frac{Hr_i Br_i}{Hr_i + Br_i}\right) + Hm\sum_{i=1}^{n}\frac{D_i\big(p_i^*(w_p^{min})s_i^*(w_p^{min})\big)^2}{Cap}}} \tag{40}$$

The manufacturer's net profit is then calculated.

$$NP_m(w_p^{min}) = \sum_{i=1}^{n} D_i\left(p_i^*(w_p^{min})s_i^*(w_p^{min})\right)(w_p^{min} + \xi_i - Cm - \phi_i) - \sqrt{2\sum_{i=1}^{n} Sr_i\left(\sum_{i=1}^{n} D_i\big(p_i^*(w_p^{min})s_i^*(w_p^{min})\big)\left(\frac{Hr_i Br_i}{Hr_i + Br_i}\right) + Hm\sum_{i=1}^{n}\frac{D_i\big(p_i^*(w_p^{min})s_i^*(w_p^{min})\big)^2}{Cap}\right)} \tag{41}$$

The above analyses demonstrate that the Stackelberg game equilibrium exists and could be identified by one of the two conditions that result in the manufacturer's net profit being greater than the other.
Condition 1: Eqs. (18), (19), (34), (37) and (39) and $z^* = 1$
Condition 2: Eqs. (18), (19), (34), (40), $w_p^* = w_p^{min}$, $z^* = 0$.
Theorem 2: In the Stackelberg game offered by the SG model, there is only one Stackelberg equilibrium point.
Proof. Consider Appendix B.

5.3. Algorithm procedure

In a series of phases, a solution algorithm is suggested for driving the unique Stackelberg game equilibrium, including:
Step 1: By solving the equation$\sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right) = Cap$, we can calculate the minimum wholesale price w_p^{min}.
Step 2: Calculating $NP_m(w_{p1}^*)$ to correspond to $z = 1$: calculate the candidate solution for$\frac{\partial NP_m(C,w_p)}{\partial C} = 0$using equation (39). If $w_{p1}^* > w_p^{min}$ the optimal w_{p1}^* is obtained, and $NP_m(w_{p1}^*)$ is obtained by substituting into Eq (38), otherwise, set $NP_m(w_{p1}^*) = 0$.
Step3: $NP_m(w_p^{min})$ corresponding to $z = 0$ is calculated as follows: $NP_m(w_p^{min})$ is obtained by substituting w_p^{min} into Eq. (41).

Step 4: if $\max\{NP_m(w_{p1}^*), NP_m(w_p^{min})\} \leq 0$ stop. If $NP_m(w_{p1}^*) > NP_m(w_p^{min})$ and $\max\{NP_m(w_{p1}^*), NP_m(w_p^{min})\} > 0$ proceed to Step 5. If not, proceeds to Step 6.

Step 5: The following equations provide the Stackelberg game equilibrium: $z^* = 1$, $w_p^* = w_{p1}^*$, p_i^* $i = 1,2,\dots,n$ (from Eq. (18)), s_i^* $i = 1,2,\dots,n$ (from Eq. (19)), y_i^* $i = 1,2,\dots,n$ (from Eq. (34)), and C^* (from Eq (37)).
Step 6: The following equations provide the Stackelberg game equilibrium: $z^* = 0$, $w_p^* = w_{p1}^*$, p_i^* $i = 1,2,\dots,n$, (from Eq. 18), s_i^* $i = 1,2,\dots,n$, (from Eq. (19)), y_i^* $i = 1,2,\dots,n$ (from Eq. (34)), and C^* (from Eq (40)).

6. Enhancement via a cooperative contract

The Stackelberg game's equilibrium is the outcome of a non-cooperative game in which the manufacturer uses his leadership to maximize his profits by considering the optimal reactions of his retailers. Any deviation from the Stackelberg equilibrium by the manufacturer results in a loss for him. Nevertheless, these deviations may not result in a loss for his retailers, as the manufacturer can perceive. As a result, the manufacturer is aware of which deviations might increase the overall profit of the VMI system. Therefore, if the retailers are eager to collaborate with him, the manufacturer could be able to increase profits for all enterprises. The section will examine how the manufacturer leverages his informational advantages to increase his profit, which benefits both the manufacturer and his retailers. The enhancement is only possible by adjusting the wholesale price w_p since, according to Eq. (20), the retailer's profit is impacted by a single manufacturer decision variable (w_p). The section assumes that:

i. The manufacturer is eager to deviate from the Stackelberg equilibrium to generate a higher system profit than the Stackelberg equilibrium.
ii. The manufacturer and his retailers may establish a cooperative agreement to divide the increased profit associated with a given weight δ_i $i = 0,1,\dots,n$ and $\sum_{i=0}^{n} \delta_i = 1$, where $i = 0$ represents the manufacturer.

As a result, the manufacturer and his retailers might boost their revenues in comparison to the Stackelberg game equilibrium by following the instructions below:
Step 1: The manufacturer modifies w_p and computes the maximum additional benefit. (The additional profit is equal to the difference between the system profit in the two modes w_p current and w_p optimal).
Step 2: If the maximum additional profit is less than zero, the process is terminated, and the Stackelberg equilibrium is maintained in its present state. If not, go to Step 3.
Step 3: Sign a cooperative arrangement outlining how the enterprises will split the additional profit.

7. Numerical example

In this part, we analyze 1) the advantages of the Stackelberg game; and 2) the effect of the cooperative contract on the manufacturer's and retailers' profits compared to the outcomes at the Stackelberg game equilibrium. 3); the impact of specific market characteristics on the manufacturer's and his retailers' decisions and profitability. According to Almehdawe and Mantin (2010), the inputs are derived from the suggested assumptions and previous research. The retailers received identical inputs as in (Deng et al. (2020)) for the aim of simplicity. The inputs encompass $n = 3, K_i = 3 \times 10^6, a_i = 1.8, \alpha_i = 0.2, \eta_i = 10, \xi_i = 9, Br_i = 200, Hr_i = 4, Sr_i = 40, \phi_i = 3, Cap = 200, Hm = 3, Sm = 100, Cm = 160, \delta_i = 0.25\ (i = 0,1,2,3)$.

7.1. The manufacturer's benefit and future improvement

We modify the wholesale price from \$229 (w_p^{min}) to \$430 to demonstrate the benefit of the Stackelberg to the manufacturer, as well as potential additional benefits to the manufacturer and his retailers via a cooperative agreement and yield the corresponding outcomes demonstrated in Fig. 3, where the series $n * NP_{r_i}\_current$ $TNP\_current$, $NP_m\_current$, and NP_m_base represent the sum of three-retailer net profits, the total net profit of the VMI system, the manufacturer's net profit at the current modifying wholesale price (w_p), and the manufacturer's net profit at the base instance's Stackelberg equilibrium. Because of Figure. 3, we deduce the following:

[1] The manufacturer benefits from the Stackelberg game's equilibrium in a non-cooperative context under the assumptions specified in section 3.2. If the initial w_p is \$229, we could observe from series $NP_m\_current$ and NP_m_base that if the manufacturer decides to adjusted w_p from \$229 to \$342 (the Stackelberg equilibrium), the manufacturer's profit could rise (17548-14306)/14306=22.66 percent. [2] If the initial w_p is more than the Stackelberg equilibrium price w_p^*, the price shift from the initial w_p to the Stackelberg game equilibrium price w_p^*, benefits both the manufacturer and its retailers. If the initial w_p=430 is changed to w_p, the retailer's profit will rise by (38037-31178)/ 31178=22 percent, while the manufacturer's profit would rise by (17622-17037)/ 17037=3.43 percent.

[3] Depending on the assumptions in Section 6, if the manufacturer and his retailers are inclined to collaborate, the manufacturer must deviate w_p from \$342 (the Stackelberg equilibrium price) to \$229, bringing the VMI system the maximum extra profit of \$68032-\$55659=\$12373. The manufacturer and his retailers subsequently benefit from them by applying a specified profit-sharing weight of $\delta_0 = \delta_1 = \delta_2 = \delta_3$ =0.25.

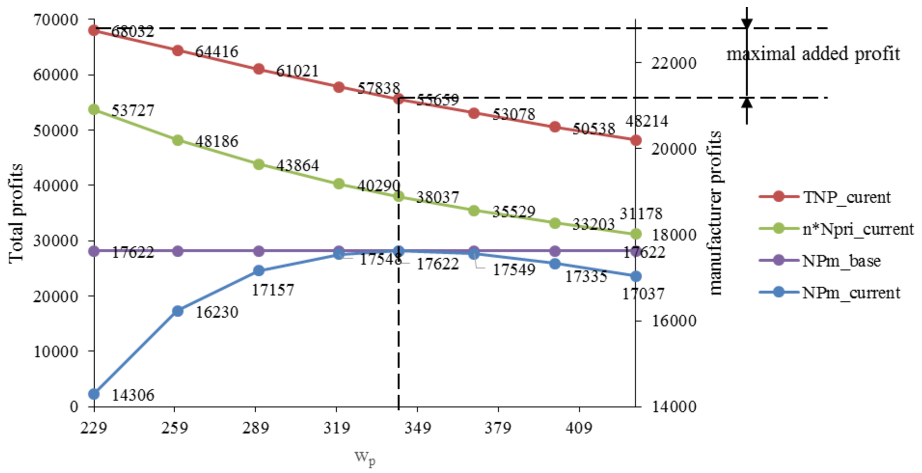

Figure. 3. The Stackelberg equilibrium and its enhancement

In comparison to the Stackelberg equilibrium, the manufacturer and his retailer improve their profits by 0.25*12373/17548=17.62 percent and 0.25*12373/ (38037/3)=24.39 percent, correspondingly.

7.2. Sensitivity analysis

Table 2 shows the ideal values of decision variables, as well as a sensitivity analysis of a few parameters.

Table 2: A sensitivity analysis.

Parameters	value	s_i^*	p_i^*	y_i^*	w_p^*	D_i^*	C^*	z^*	$NP_{r_i}^*$	NP_m^*
Basic example		16.77	791.25	0.020	342.67	31.99	1.15	1	12657.65	17677.55
K_i	2000000	13.37	793.51	0.020	343.67	20.28	1.48	1	8046.30	11203.41
	4000000	19.69	790.02	0.020	342.12	44.17	0.96	1	17449.24	24413.09
a_i	1.7	23.69	916.60	0.020	368.42	52.04	0.87	1	25252.14	32906.45
	1.9	11.93	700.98	0.020	323.05	19.30	1.52	1	6409.66	9461.96
α_i	0.16	13.90	801.32	0.020	347.14	27.11	1.26	1	11103.41	15314.84
	0.24	19.95	781.24	0.020	338.22	38.20	1.04	1	14590.86	20636.40
n	1	16.73	795.86	0.020	344.72	31.64	1.70	1	12592.44	5837.83
	6	16.79	789.80	0.020	342.02	33.33	0.74	0	13217.21	35571.46
Cap	100	16.76	792.89	0.020	343.40	33.33	0.70	0	13279.33	17767.80
	300	16.78	790.67	0.020	342.41	32.04	1.17	1	12665.88	17685.47
Cm	100	19.93	536.44	0.020	229.42	66.67	0.49	0	17881.17	26597.07
	220	14.60	1081.18	0.020	471.52	17.74	1.75	1	9590.37	13392.72
Hr_i	2	16.79	788.92	0.010	341.63	32.17	1.53	1	12690.86	17786.50
	6	16.75	793.20	0.029	343.53	31.85	0.96	1	12630.02	17592.76
Hm	1	16.78	790.08	0.020	342.15	32.08	1.19	1	12674.40	17693.55
	5	16.76	792.36	0.020	343.16	31.91	1.11	1	12641.93	17662.19
Br_i	100	16.77	791.17	0.038	342.63	32.00	1.16	1	12658.83	17681.26
	300	16.77	791.28	0.013	342.68	31.99	1.15	1	12657.24	17676.29
Sr_i	20	16.79	789.48	0.020	341.88	32.13	1.02	1	12682.85	17741.09
	60	16.76	792.80	0.020	343.35	31.88	1.27	1	12635.73	17622.36
Sm	50	16.79	789.80	0.020	342.02	32.11	1.15	1	12678.35	17729.75
	150	16.76	792.55	0.020	343.25	31.89	1.15	1	12639.21	17631.11
ϕ_i	1	19.93	536.44	0.020	229.42	66.67	0.49	0	17881.17	17871.76
	5	16.68	800.91	0.020	346.96	31.27	1.17	1	12521.84	17487.78
ξ_i	7	16.77	791.25	0.020	344.67	31.99	1.15	1	12657.65	17677.55
	11	16.77	791.25	0.020	340.67	31.99	1.15	1	12657.65	17677.55
η_i	5	28.77	558.76	0.020	239.34	66.67	0.75	1	18625.31	19108.46
	15	13.39	791.45	0.020	342.76	30.57	1.18	1	12097.38	16890.45

A few observations are chosen from Table 2 and summarized as follows.

7.2.1. Market parameters

Let us examine the retailer's service level elasticity (α_i). According to Fig. 4b, when α_i increases, so do the net profits of all enterprises. For instance, by increasing α_i from 0.28 to 0.32, the manufacturer's production capacity is adequate, and its net profit grows from 24363.6 to 29053.9 by 19.25 percent. The retailer's net profit rises by 17.89 percent from 17007.33 to 20050.12. The explanation for this is because, with such alteration α_i, each retailer's demand rises by 22.26 percent, from 46.15 to 56.43, with a moderate decrease in wholesale and retail prices. The wholesale price drops by 1.31 percent from 333.79 to 329.40, while every retailer's retail price drops by 1.28 percent from 771.29 to 761.40.

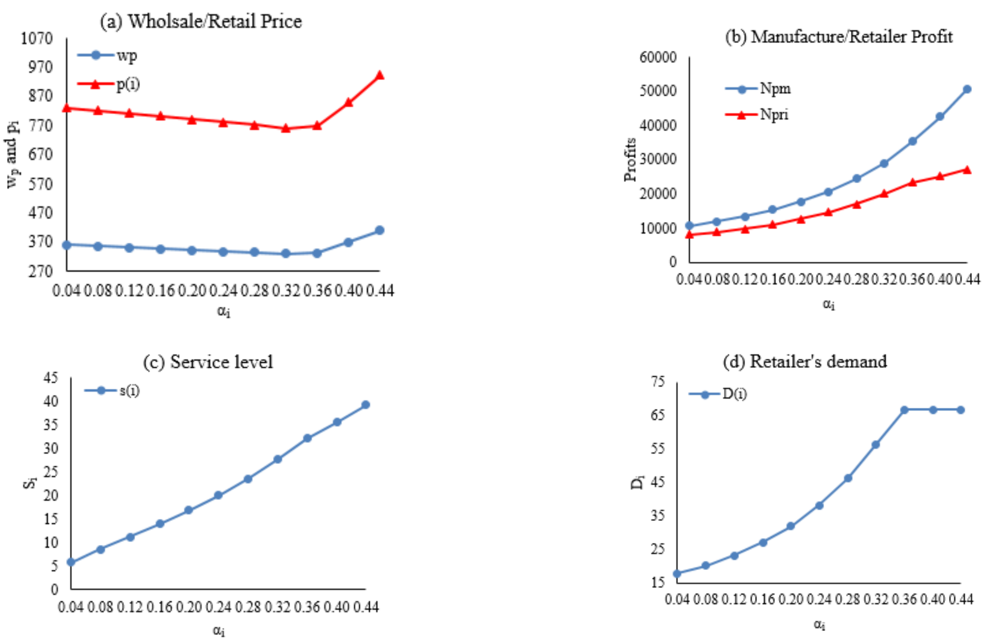

Figure. 4. The impact of α_i, Take note that when $\alpha_i \geq 0.36$, the manufacturer's manufacturing capacity is used up

In a VMI system where the production capacity is redundant, a higher profit rise may be achieved for a similar boost in α_i than in a system where the capacity remains utilized up. For instance, in the case of redundant manufacturing capacity, a rise in α_i from 0.28 to 0.32 enhances the profits of the manufacturer and each retailer by 19.25 percent, from 24363.6 to 29053.9, and by 17.89 percent, from 17007.33 to 20050.12, correspondingly. Their profits rise from 35318.19 to 42500.13 by 20.33 percent and from 23371.44 to 25195.39 by 7.80 percent, likewise, when α_i is increased from 0.36 to 0.40 (due to inadequate manufacturing capacity). There is significant differentiation in improvement on the retailer profit between such two situations. As long as demand remains within total capacity, retailers may raise their profitability by offering services to boost sales. When capacity is achieved, increasing service levels will not increase sales; however, it may rise in pricing. Let us now consider another market parameter – the price elasticity a_i. From Fig. 5 and Table 2, with the decrease a_i, net profit, and prices of both the manufacturer and retailers and service level of retailers are significantly increased, no matter whether the manufacturer's production capacity is sufficient or not (the manufacturer reaches its capacity when a_i is about 1.6).

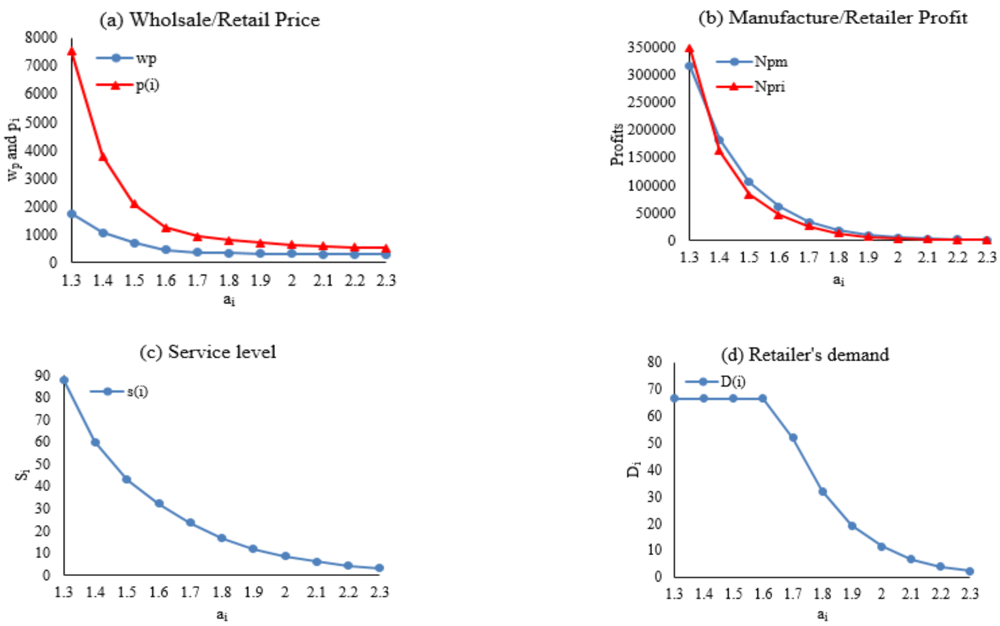

Figure.5. The impact of a_i, note. When $a_i < 1.6$, the manufacturer's manufacturing capacity is used up.

For example, when a_i decreases from 1.8 in the base example to 1.7, the manufacturers and each retailer's profits increase dramatically by (32906.45-17677.55)/ 17677.55=86.45% and (25252.14-12657.65)/ 12657.65=99.50%. For instance, when a_i is decreased from 1.8 to 1.7, the manufacturer's (retailer's) net profit increases by (32906.45-17677.55)/ 17677.55=86.45% ((25252.14-12657.65)/ 12657.65=99.50. The reason for the increase in retailer's profit is that, when a_i decreases, each retailer's demand and retail price increase from 31.99 units and $791.25 to 52.04 units and $916.60. Similarly, due to the increasing product demand and w_p^* rising, the manufacturer's profit can sky-rocket. The effects K_i on the manufacturer's and all retailer's profits are comparable to those ofα_i; thus, they are deleted.

7.2.2. Inventory parameters

Inventory-related parameters have a minor impact on manufacturers' and retailers' profits; however, they might have a significant effect on their inventory policies on occasion. When the holding cost per unit completed product Hr_i is reduced from $4 to $2, the manufacturer's profit increases only by (17786.50-17677.55)/ 17677.55=0.61 percent, while the common replenishment cycle C boosts by (1.53-1.15)/ 1.15=33.04 percent, as illustrated in Table 2.

8. Conclusion

Using the Stackelberg model, the article describes a VMI system with one manufacturer and multiple retailers, and when it comes to price, service levels, and inventory rules, the manufacturer takes the lead, and all retailers follow. In the investigated VMI system, the manufacturer produces a single product at a wholesale price due to manufacturing capacity constraints and provides the product to retailers, who subsequently sell the product in the dispread market at retail price. The Cobb-Douglas demand function describes the demand rate for a product in the retail market, a declining function of price, and a rising service level function. The Stackelberg game equilibrium is proved to be unique and could achieve using the algorithm steps provided. We demonstrate that, compared to the Stackelberg equilibrium, the manufacturer's and their retailers' profits might be significantly higher, and we prove the conditions for raising the profit of any enterprise depending on certain additional assumptions in Section 6. Ultimately, a numerical analysis is carried out to fully understand the suggested models and assess the influence of market and inventory parameters, resulting in the following management implications:
1) The manufacturer and its retailers must recognize market-related parameters since they substantially influence their profits. By increasing α_i and K_i, When the manufacturer's production capacity is sufficient, a minor drop in retail and wholesale pricing, as well as an improvement in retail service level, can boost the manufacturer's net profit. When a manufacturer's manufacturing capacity is depleted, they might boost their net profits by raising retail service levels and charge higher prices while demand is constrained by continuous supply.
2) The profits of the manufacturer and his retailers in the VMI system always increase as a_i is reduced. For example, in our analysis, a 10-percent drop in price elasticity (from 1.8 to 1.7) may improve the manufacturer's and every retailer's profit by 86.45 percent and 99.50 percent, respectively.
3) The manufacturer might utilize the retailer's information in the VMI system to maximize its profit. In our example, the Stackelberg equilibrium may enhance the manufacturer's profit by 22.66 percent at the starting wholesale price w_p=229 (before utilizing the Stackelberg game).
4) When the manufacturer's capacity is used up, he may monopolize all of the system's additional profit owing to lower manufacturing costs. However, in most situations, it will be more logical for the manufacturer to share the additional profit generated by the VMI system with his retailers because of higher system revenue or lower system costs.
5) Retailers benefit from the Stackelberg equilibrium if the wholesale price is initially set higher than that at the equilibrium Stackelberg pricing. Assuming the starting wholesale price w_p=430, the Stackelberg equilibrium may boost the retailer's profit by 22%.
6) Only when it is smaller than that of the Stackelberg equilibrium can the system profit, and then the profit of every enterprise is enhanced compared to the Stackelberg equilibrium. In our case, the manufacturer's and retailers' profits might rise roughly 20% compared to the Stackelberg equilibrium.
This study has some limitations that can be considered in future studies. On the one hand, while our theoretical conclusions are potentially helpful and generic for VMI practices, collecting data from real-world VMI systems to conduct a specific case study is desirable. Analyzing an actual VMI situation would be fascinating to produce more precise recommendations for supply chain participants to acquire their optimal prices and service level investments. It is proposed that a centralized model of the investigated VMI system is evolved and that our findings be compared to those of the centralized model. In addition, the suggested model included deterministic demand. In the future, stochastic demand may be used. In addition, it is suggested that future research take competition among retailers into account. Ultimately, as an extension of our Stackelberg game, advertising promos (Khorshidvand et al. 2021(a), 2021(b) and 2021(c)) may be integrated with price and service level. This study used only the traditional retail channel to sell the manufacturer's products. Due to the changes in the buying process caused by modern technology, considering the online channel is also recommended for the manufacturer. Considering a discount strategy can also be an exciting topic for future research.

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Appendixes

Appendix A. The proof of Theorem 1

The Hessian matrix is a negative definite.

$$\frac{\partial^2 NP_{r_i}(p_i, s_i)}{\partial p_i^2} = K_i a_i s_i^{\alpha_i} p_i^{-a_i-2}[(a_i-1)p_i - (a_i+1)(w_p+\xi_i)]$$

$$\frac{\partial^2 NP_{r_i}(p_i, s_i)}{\partial p_i \partial s_i} = \frac{\partial^2 NP_{r_i}(p_i, s_i)}{\partial s_i \partial p_i} = K_i \alpha_i s_i^{\alpha_i-1} p_i^{-a_i-1}[(1-a_i)p_i + a_i(w_p+\xi_i)]$$

$$\frac{\partial^2 NP_{r_i}(p_i, s_i)}{\partial s_i^2} = K_i \alpha_i(\alpha_i-1) s_i^{\alpha_i-2} p_i^{-a_i}(p_i - w_p - \xi_i)$$

The Hessian matrix is thus equal to

$$\begin{bmatrix} \frac{\partial^2 NP_{r_i}(p_i, s_i)}{\partial p_i^2} & \frac{\partial^2 NP_{r_i}(p_i, s_i)}{\partial p_i \partial s_i} \\ \frac{\partial^2 NP_{r_i}(p_i, s_i)}{\partial s_i \partial p_i} & \frac{\partial^2 NP_{r_i}(p_i, s_i)}{\partial s_i^2} \end{bmatrix}$$

The Hessian matrix's determinant is positive and $\left.\frac{\partial^2 NP_{r_i}(p_i, s_i)}{\partial p_i^2}\right|_{p_i=p_i^*} = -K_i s_i^{\alpha_i}\left(\frac{a_i(w_p+\xi_i)}{a_i-1}\right)^{-a_i-2}(w_p+\xi_i) < 0$ the Hessian matrix is negative definite. Therefore, solving the first-order conditions $\frac{\partial NP_{r_i}(p_i,s_i,w_p)}{\partial p_i} = 0$ and $\frac{\partial NP_{r_i}(p_i,s_i,w_p)}{\partial s_i} = 0$ for (p_i, s_i) yields the optimal retailer reactions in (16) and (17).

$$\frac{\partial NP_{r_i}(p_i, s_i)}{\partial p_i} = K_i s_i^{\alpha_i} p_i^{-a_i-1}[(1-a_i)p_i + a_i(w_p+\xi_i)] = 0 \Rightarrow p_i^*(w_p) = \frac{a_i(w_p+\xi_i)}{a_i-1}$$

$$\frac{\partial NP_{r_i}(p_i, s_i)}{\partial s_i} = K_i \alpha_i s_i^{\alpha_i-1} p_i^{-a_i}(p_i - w_p - \xi_i) - \eta_i s_i = 0 \quad \Rightarrow$$

$$s_i^*(w_p) = \left[\frac{K_i\alpha_i}{\eta_i}\frac{(w_p+\xi_i)}{a_i-1}\right]^{\frac{1}{2-\alpha_i}}\left(\frac{a_i(w_p+\xi_i)}{a_i-1}\right)^{\frac{-a_i}{2-\alpha_i}}$$

Theorem 1 is proved.

Appendix B. The proof of Theorem 2

Proof. When the manufacturer's capacity is used up ($z^* = 0$), $\sum_{i=1}^{n} D_i\left(p_i^*(w_p)s_i^*(w_p)\right) = Cap$ may be used to determine the unique $w_p^* = w_p^{min}$. Afterward, we may derive unique p_i^*,s_i^*, y_i^*, and C^* by substituting w_p^{min} into Esq. (18), (19), (34), and (40). When a manufacturer's capacity is redundant (z^*=1), we could use Eq. to identify the critical point (w_p^*) at which the manufacturer's profit is maximized (38). Without losing generality, we assume there are two key point points w_p^* designated by $w_{pc}^* > w_{pd}^*$ if there is more than one w_p^*. Eq. (20) states that $NP_{r_i}^*$ is a diminishing function of w_p^* because $\frac{\partial NP_{r_i}^*}{\partial w_p^*} < 0$. As a result, the manufacturer, as the leader, knows $NP_{r_i}^*(w_{pc}^*) < NP_{r_i}^*(w_{pd}^*)$, and w_{pd}^* will be chosen as the ultimate optimal solution if the manufacturer attempts to satisfy his retailers. In such a case, we derive the unique values of p_i^*, s_i^*, y_i^*, and C^* by inserting w_{pd}^* into Eqs. (18), (19), (34), (37), and (39). Ultimately, at both z^*=1 and 0, the manufacturer cannot achieve his maximum profit. We are assuming that the manufacturer can maximize his profit at both z^*=1 and 0. Therefore, we have two w_p^* in this circumstance. We deduce from the analysis in case z^*=1 that only the smaller w_p^* of them will be chosen. As a result, we achieve the unique Stackelberg equilibrium. Theorem 2 is proved.