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An intelligent hybrid model for forecasting the stock price index volatility: The case of Tehran stock exchange

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Abstract

Forecasting the stock price index volatility is considered a strategic and challenging issue in the stock markets, and it is momentous for traders and investors in the decision-making process. Hence, the presentation of an efficient model for forecasting the stock price index volatility is a crucial and hard task because stock market data and price fluctuations have high volatility and nonlinearity characteristics. To beat this challenge, this paper proposes a new hybrid model by applying artificial intelligence algorithms to forecast the stock price index. It incorporates four phases to provide a dynamic and exact model: (1) Select popular and key technical indicators as input variables (2) Apply Adaptive Neuro-Fuzzy Inference System (ANFIS) for designing a substructure to provide a high-quality and quick solution (3) Use Modified Particle Swarm Optimization (MPSO) to enhance predictive accuracy by simultaneously and adjusting the length of each interval in the discourse universe and the appropriate degree of membership (4) Employ Parallel Genetic Algorithm (PGA) to solve complex issues with computational weight optimization and adjusting the decision vectors employing genetic operators. The stock market data of "Tehran Stock Exchange (TSE)" from 01/01/2011 to 31/12/2021 are utilized to examine the functionality of the proposed model. In comparative assessments, the overall performance of the ANFIS-MPSO-PGA model based on 5 criteria achieved 81.45%, which was significantly better than other methods.

Keywords: Artificial intelligence; Technical Indicator; ANFIS; MPSO; PGA

Paper Type: Original Research

1. Introduction

The stock price index reflects the fluctuations of price indices in the stock markets, and it represents the pulse of the stock market. The stock price index has high volatility, and it shows features such as non-stationarity and uncertainty. Hence, forecasting the volatility of the stock price index is a strategic and challenging task in the modern financial world. Stock price indexes are affected by a wide range of factors and parameters in the financial markets, and evidently, each parameter can affect the others. The existence of these factors and parameters makes it difficult to forecast fluctuations in the indexes of stock prices. Due to these reasons, many researchers have persistently sought to propose new models and techniques for accurate forecasting of stock prices. Stock analysts, investors, and financial managers can therefore use these novel models and techniques to analyze the market and make investments more profitable and secure. Accuracy and reliability of forecasts are of great importance because when forecasts are more accurate, more profits will be made. In the financial markets, scientific researchers and financial experts are developing more accurate models and methods to predict the stock market index and targeting greater returns through different investment methods. An analysis of past price patterns, known as technical analysis, is one method used to predict stock price fluctuations (Pring 2002). In order to maximize returns on investments, many studies have focused on technical analysis since financial markets, especially stock markets, are nonlinear and nonstationary (Azoff 1994, Bessembinder and Chan 1995, Neely, Weller et al. 1997, Allen and Karjalainen 1999, Leigh, Purvis et al. 2002). Technical analysis is a tool that forecasts the price index volatility in the financial markets by employing historical price charts and data (Murphy 1999). This tool is regarded as a subset of securities analysis alongside the fundamental analysis. Technical analysis routinely entails the use of technical

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indicators and oscillators. Technical indicators are robust tools that assist traders and investors in forecasting future changes in the stock price index. They are utilized either as standalone instruments or in combination either with each other or with other chart data to display trends, events, and so on (Lo, Mamaysky et al. 2000). In light of the uncertainty and complexity of stock market data, the development of effective approaches for predicting stock prices is crucial. Soft computing is a collection of techniques that are resistant to uncertainty, complexity and unpredictability. One of these techniques is the genetic algorithm, which is used to optimize the objective functions of various problems. The best Artificial intelligence (AI) programs in the 21st century are instances of soft computing. GA as a branch of evolutionary algorithms is a rapidly growing field of AI. Among the most important intelligent optimization algorithms, PSO uses swarm intelligence that is employed in AI. ANFIS is a type of AI that incorporates both neural networks and fuzzy logic. AI is a branch of computer science that builds intelligent systems. AI is widely used in a variety of fields, including finance and investment. The use of AI is able to increase the speed of processes, perform tasks accurately and reduce errors, analyze data and help to make decisions, recognize people's behavior and emotions and generally work intelligently. The effective role of artificial intelligence in financial markets is inevitable (Russell and Norvig 2002). The objective of this study is to present a new intelligent system that integrates ANFIS, MPSO, and PGA to forecast the volatility of the Tehran Stock Exchange's stock index. ANFIS is the first stage in the model, MPSO is the second stage, and PGA is the third stage. ANFIS method learns the principles and membership functions from data. It is adaptive network nodes and directional links with connected learning principles. ANFIS is the adaptive network of selection to be examined systematically and employed for high-frequency predictions (Jang and Sun 1995). ANFIS serves as a substructure for designing a series of fuzzy if-then rules to produce the stipulated input-output pairs with the desirable selection of membership functions. It has been employed to provide a high-quality and quick solution. PSO is a metaheuristic worldwide optimization paradigm that utilizes the notion of public interplay to dealing with troubles. It has been utilized effectively for many different search and optimization troubles. PSO is a simple but strong search approach. PSO executes towards developing the vector. As part of the model, the PSO method enhances predictive accuracy by simultaneously adjusting interval lengths and membership levels in the discourse universe. An optimization method called Modified Particle Swarm Optimization (MPSO) was applied in this model. MPSO is employed to increase the performance of the original PSO via adjusting inertia weight as a new parameter. This new parameter is explained in detail in the methodology section. GA has been utilized to adjusting the decision vectors employing genetic operators. The goal of combining PSO-GA is to incorporate the benefit of the GA and PSO simultaneously (Davis 1991, Shi and Eberhart 1999). GA has been employed to solve complex issues with computational weight optimization. In order to improve the functionality of the model, a parallel genetic algorithm (PGA) was employed. PGA is one type of GA that intends to improve algorithm performance. Based on the above-mentioned items, ANFIS, PSO, and GA are the most prominent methods relative to other evolutionary computing approaches. The presented approach creates a hybrid model that produces more beneficial results relative to those achieved with the implementation of every single technique. In this study, a model named ANFIS-MPSO-PGA has been suggested to deal with stock price index prediction challenge employing stock market data of "TSE". All methods have been implemented in "Mathworks MATLAB R2020b". According to the mentioned points, forecasting the stock price index is necessary for financial specialists, stock traders, and investors. That's why the authors decided to design a powerful and efficient model to forecast the stock price index accurately. After evaluating the model, it becomes clear that the presented model is effective in enhancing predictive accuracy. Based on the evidence, portfolio managers, stock analysts, traders, and investors can apply the presented model in this paper to strengthen their predictive instruments. The main motivations of the current study are the following: (1) Presenting an innovative hybrid model for stock price forecasting; (2) Increasing speed and accuracy in predicting stock price indices using a combination of three techniques; (3) Validation of the presented model employing real data from the TSE; (4) Assessment of the model by five criteria and its comparative analysis with related models. Organizing this paper is as follows: Section 2 reviews related works. Section 3 describes the methodology and framework of the proposed model. Section 4 provides details of the investigation's results. Section 5 presents the conclusions.

2. Literature review

Volatility in the stock price index would be inevitable. Accordingly, many researchers have tried to develop models in order to overcome the problem of predicting the stock price index because of the importance of the topic. In this section, we review the related studies of stock price index forecasting. One of the initial studies in this field is the paper of Kai and Wenhua (1997) that introduced a method based on the GA and NN for forecasting the stock price index on the Shanghai Stock Exchange. The five indices, including Industry Share, Commerce Share, Real Estate Share, Utility Share, and Comprehensive Share, have been used from March 1994 to August 1994. The outcomes demonstrated that their method is appropriate for short-term forecasts, and it has better performance and accuracy in comparison with the ARMA model.

Dutta, Jha et al. (2006) employed ANN for the modeling of the stock price index to predict the BSE stock price index. To achieve this goal, they improved two networks named ANN1 and ANN2, with three hidden layers. In the ANN1, ten-week fluctuations of the weekly closing values are considered, but there are five-week fluctuations in the ANN2. If the moving average reflects a kind of direction in the price for the given period, moving averages are added as inputs to the ANNs. They employed these models to forecast the weekly closing index values of

SENSEX for a two-year period. For the ANN1, RMSE was 4.82%, and MAE was 3.93%, but for the ANN2 RMSE was 6.87%, and MAE was 5.52%. The results indicated that the ANN1 is better compared to the ANN2.

In previous years many scientific researchers proposed new models and methods to overcome the problem of predicting the stock price index. Xiong, Bao et al. (2014) offered an interval forecasting method incorporating the FA and MSVR for short-term and long-term stock price index series. MSVR parameters are adjusted with the FA. Three indexes of the international markets were identified as research databases, including the S&P 500 (from 2010 to 2012), FTSE 100 (from 2007 to 2012), and Nikkei 225 (from 2004 to 2012). Observations for these indices respectively, are 523, 1218, and 2165. The sample data are three indexes with various horizons evaluated on a daily basis, and they used one-step and multi-step-ahead predictions. Their research assessed by various statistical criteria and economic values of the predictors. Firstly, they evaluated the forecast accuracy by calculating the ARV. Secondly, they applied various tests of hypotheses, including ANOVA and HSD tests. According to the findings, this method is more accurate and economically advantageous than other well-established methods. In addition, they demonstrated that the FA-MSVR approach can be used to predict interval-valued financial time series.

Majhi, Rout et al. (2014) developed the functionality of a hybrid forecasting model employing RBFNN and NSGA-II for several stock exchanges. The model functionality was examined by several metrics utilizing S&P500 and DJIA stock data. The outcomes illustrated that the offered model works better than the conventional RBF in terms of MAPE, DA, and ARV. Singh and Borah (2014) presented a novel Type-2 fuzzy time series model, which improved by applying the PSO method. The daily stock index price dataset of SBI and daily stock index prices of Google are utilized to examine functionality of their model. Findings revealed that the offered model performed better than existing fuzzy and conventional time series models. Patel, Shah et al. (2015) attempted to forecast trends and index changes in Indian stock markets. Their research evaluated four forecasting methods including ANN, SVM, RF, and Naive-Baye, and used ten technical parameters applying stock trading information as input variables. All models examined by the historical information from 2003 to 2012 for two stock price indexes. The outcomes of their study revealed that the RF model works better than other methods. Additionally, they found that the functionality of all models is enhanced when ten technical parameters are considered as trend deterministic information.

Inthachot, Boonjing et al. (2016) designed a model incorporating ANN and GA for forecasting movements in the SET50 stock index. Eleven technical indicators reflected the information on past stock trading data. To evaluate the model, the data set of SET50 from 2009 to 2014 has been utilized. Final outcomes demonstrated that their method obtained an average improvement of 12.4011 percent. It indicates that the percentage of average forecasting accuracy is 63.6%. Pan, Xiao et al. (2017) offered a new method named MSVM-UMIDAS that could simultaneously obtain multiple outcomes for sequential points by employing independent variables of mixed frequency. They selected four important technical indicators as independent variables. Additionally, several macroeconomic indicators, including CPI, BOT, ISM Manufacturing Index, and Interest rate, have been utilized. They employed the closing index of the S&P 500 to assess the functionality of the MSVM-UMIDAS approach, which evaluated by four different metrics, including PER, ME, RMSE, and MAE. Final outcomes revealed that their model works better than other approaches. Furthermore, they observed that their approach is an efficient instrument for multi-output and mixed frequency problems. Nelson, Pereira et al. (2017) evaluated the performance of LSTM networks for forecasting the future patterns of stock prices based on the historical prices employing technical indicators. The stock price data of the Brazilian stock market for the year 2014 are used to examine the model functionality. Besides this information, it produced a large number of technical indicators to feed the network as functions. The goal of this study is to test the applicability of recurring neural networks, especially the LSTM networks, to forecast the stock market price fluctuations. The final results showed that the proposed model achieved an average accuracy of 55.9%, and it also specifies the increase or decrease in stock prices. In addition, it can be concluded that the model based on LSTM provides fewer risks compared to the other methods. Chung and Shin (2018) offered a hybrid method by integrating the LSTM network and GA for stock market forecasting. They have selected daily KOSPI data to examine the offered method. The outcomes revealed that the offered model works better compared to the benchmark model. Zhang, Cui et al. (2018) offered a new method that could forecast stock price trends and their interval of growth rate. In order to categorize stocks based on the outlines of their closed prices, they used a heuristic algorithm to cut raw transaction information into clips of predefined length. The clips could be graded into various levels that represent the degree of their growth rates, and their characteristics include prices and technical indicators. They investigated the information of 495 stocks in China from January 2010 to October 2016. The final results demonstrated that the offered method works better than other existing approaches, and it is powerful and efficient for the market fluctuations. Long, Lu et al. (2019) offered the new model called multi-filters neural network (MFNN) to predict the price changes. To build the multi-filter structure, both convolutional and recurrent neurons are combined so that data can be collected from various feature spaces and market views. They employed their model on the CSI 300 Index, aiming to market forecasting and signal-based trading simulation. Their findings indicated that in terms of accuracy, efficiency, and reliability, their offered model performed better than other models.

Seo, Lee et al. (2019) introduced combined models using the ANNs that have multiple hidden layers to enhance the accuracy of market fluctuations. These models are created utilizing approximate input variables from GARCH family patterns and Google domestic trends. To examine the accuracy of models, they have been used to predict weekly and monthly S&P 500 index fluctuations. Final outcomes demonstrated that the hybrid models with Google

domestic trends work better than GARCH family patterns. Chung and Shin (2020) suggested a technique to optimize the parameters for the CNN model by employing GA. To forecast the volatility of the stock index, they used multi-channel CNNs, which is a representative deep learning technique. They also enhanced the model's performance by optimizing the topology of the CNN network. The results indicated that the GA-CNN hybrid model performs better than the other models. Lin, Yan et al. (2021) presented a novel hybrid model called CEEMDAN-LSTM for predicting stock index price. The model was applied to historical data of the S&P500 and the CSI300 indexes for the past ten years. The finding revealed that their model provides optimal forecasting results in comparison with other models. Li, Han et al. (2022) offered the model to predict the stock price index applying an intelligent optimization approach based on frequency component analysis. Data was collected from three stock closing price series of different Chinese industrial companies for their study. The finding confirms the statistical validity of the offerd model. Analyzing horizontal comparisons showed that the multiscale strategy significantly improved forecasting performance.

In this paper, the authors intend to resolve the problems of past models and present a new and efficient model for stock price index prediction that can perform significantly better than other related methods.

3. Methodology

In this section, a new intelligent model to forecast the stock price index volatility is proposed by combining ANFIS, MPSO, and PGA to generate a faster and more accurate model compared to related models. Three parts of the model are explained in the following subsections.

Adaptive Neuro-Fuzzy Inference System (ANFIS)

The ANFIS combines fuzzy logic and neural networks. To predict future values, learning algorithms use historical data to monitor neural networks (Jang 1993). The use of ANFIS makes it easier to reconcile rule base choices with status. A reverse propagation algorithm is used to select the control base in this approach. Using a fuzzy logic approach, for example to approximate a nonlinear system by modifying IF-THEN regulations, such a model is more capable and functional (Jang, Sun et al. 1997).

Two equations were employed in the If-Then technique for the Takagi-Sugeno method:

E1 = If k is R1 and j is S1 Then f1 = c1k + d1k + h1

E2 = If k is R2 and j is S2 Then f2 = c2j + d2j + h2

Wherein:

 R_1, S_1, R_2, S_2 : Membership functions of each input k and j

 c_1 , c_2 , d_1 , d_2 , h_1 , h_2 : Linear parameters in consequent part

According to Figure 1, the ANFIS structure has five layers.

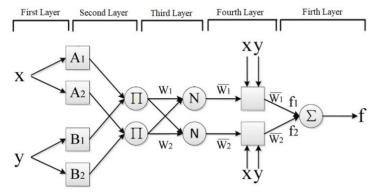


Figure 1. ANFIS Structure

Layer 1:

$$\mu \operatorname{Ai}\left(\mathbf{k}\right) = \exp\left[-\left(\frac{k-c_{i}}{2a_{i}}\right)^{2}\right]$$
(1)

$$\mu \text{Ai}(k) = \frac{1}{1 + \left|\frac{k - c_i}{a_i}\right|^{2b}}$$
(2)

(1)

(5)

(6)

(7)

$$O_{1,i} = \mu_{Ai}(k), \qquad i = 1, 2$$
 (3)

$$O_{1,i} = \mu_{Bi-2}(j), \quad i = 3,4 \tag{4}$$

Layer 2:

$$O_{2,i} = \mu_{Ai}(k) * \mu_{Bi}(j)$$
 $i = 1, 2$

Layer 3:

$$O_{3,i} = \overline{w}_i = \frac{w_i}{\sum_i w_i} \mathbf{V}$$

Layer 4:

$$O_{4,i} = \overline{w}_i f_i = \overline{w}_i (p_i k + q_i j + r_i)$$
⁽⁷⁾

 \overline{w}_i : Normalized firing power from the previous layer

 $(p_i k + q_i j + r_i)$: The parameter in the node.

Layer 5:

$$\mathcal{O}_{5,i} = \sum_{i} \overline{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i}$$
(8)

Particle Swarm Optimization (PSO)

PSO was developed by Kennedy and Eberhart, which is a population-based search method (Kennedy and Eberhart 1995). To begin, the initial particles are generated and velocities are assigned to them. The objective function is measured at each particle position, and the best value and location are specified. In selecting new velocities, the algorithm considers the present velocity, the best places of the particles, and their neighboring best places (Eberhart and Kennedy 1995). Every particle realizes its best value so far (Pbest) and the best among the group (Gbest). Every particle changes its state utilizing its present velocity and its interval from Pbest and Gbest. The velocity of every particle calculated as follows:

$$V_i^{k+1} = W^k V_i^k + C_1 rand \left(P_{best} - X_i^k \right) + C_2 rand \left(G_{best} - X_i^k \right)$$
(9)

Wherein:

 V_i^k : Velocity of ith particle in kth iteration.

 X_i^k : Position of the particle in kth iteration.

W^k: Inertia Weight.

$$W^{k} = W_{max} - \left(\frac{W_{max} - W_{min}}{itermax}\right) \text{ iter}$$
(10)

 $X_i^{k+1} = X_i^k + V_i^{k+1}$ adjusts the position of the particle.

In the present PSO approach, the lower than inequality limitation is supplemented to the objective function to create new objective function as:

$$F = \sum_{i=1}^{NG} a_i P_i^2 + b_i P_i + c_i + \left| \left(\sum_{i=1}^{NG} d_i P_i^2 + e_i P_i + f_i \right) - E_{limit} \right|$$
(11)

Equality limitation of real strength is operating in the following procedure.

The loading on the last generator (NGth generator) selected as a dependent, and the level of NGth generator is as follows:

$$\boldsymbol{P}_{NG} = \boldsymbol{P}_D - \boldsymbol{P}_L - \sum_{i=1}^{NG-1} \boldsymbol{P}_i$$

P_L: A function of all the generators comprising the dependent generator. It calculates as follows:

$$P_{L} = \sum_{i=1}^{NG-1} \sum_{j=1}^{NG-1} P_{i} B_{ij} P_{j} + 2P_{NG} \sum_{i=1}^{NG-1} [B_{NG,i} P_{i}] + B_{NG,NG} P_{NG}^{2}$$
(13)

After replacement the Equation 13 in Equation 12, we achieve Equation 14:

$$B_{NG,NG}P_{NG}^{2} + \left[2\sum_{i=1}^{NG-1}B_{NG,i}P_{i}-1\right]P_{NG} + \left[P_{D}+\sum_{i=1}^{NG-1}\sum_{j=1}^{NG-1}P_{i}B_{ij}P_{j}-\sum_{i=1}^{NG-1}P_{i}\right] = 0$$
⁽¹⁴⁾

Equation 14 is quadratic in PNG that can be solved to achieve PNG.

In the presented model, the approach of Shi and Eberhart (1998) was utilized. They proposed the MPSO to improve the functionality of the PSO via adjusting inertia weight as a new parameter. Their approach indicated that this parameter had a significant and effective impact on the particle swarm optimizer and increased the functionality of the PSO. So an inertia weight (w) is brought into the main equation of the original PSO.

$$V_{id} = w V_{id} + c_1 . rand1() . (P_{id} - X_{id}) + c_2 . rand2() \times (P_{gd} - X_{id})$$
$$X_{id} = X_{id} + V_{id}$$

Wherein:

w: Inertia Weight

 c_2, c_2 : Positive constants

g: Index of the best particle in the population

rand1, *rand2*: Two random functions in the range [0,1]

 $\mathbf{X}_{\mathbf{I}} = (\mathbf{x}_{\mathbf{i1}}, \mathbf{x}_{\mathbf{i2}}, \mathbf{x}_{\mathbf{i3}} \dots, \mathbf{x}_{\mathbf{iD}})$: The ith particle

 $\mathbf{V}_{\mathbf{I}} = (\mathbf{v}_{i1}, \mathbf{v}_{i2}, \mathbf{v}_{i3} \dots, \mathbf{v}_{iD})$: The velocity for particle *i*

 $P_I = (p_{i1}, p_{i2}, p_{i3} \dots, p_{iD})$: The best previous position of any particle

The Parameter settings of PSO are detailed in Table 1.

Table 1 . Parameter settings of PSO										
Maximum iterations	1000									
Particles	100									
Acceleration constant	3.2									
Inertia weight	0.8									
Maximum velocity	200									
Swarm size	40									
Constriction coefficient	0.729									
The multiplier for random numbers	4.1									

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Genetic Algorithm

GA is an algorithm for dealing with both constrained and unconstrained optimization troubles. For the first time, it was released by Holland (1992), and its improvement was made by Goldberg (1989). GA is certainly one of the most-widely applied algorithms utilized by investigators in various areas for the optimization of complex issues. The GA framework, similar to the other evolution algorithms, is comprised of a population, wherein every individual in it is regarded as an answer to the trouble (Goldberg and Holland 1988). An individual is named chromosome, and it consists of various trouble variables that function like genes in the algorithm. The search process is produced by improving a random population of chromosomes and generating the next generation of the population is performed via three operators:

(1) Selection, (2) Crossover, (3) Mutation.

In the proposed model, the Parallel genetic algorithm (PGA) has been used (Pettey, Leuze et al. 1987). PGA employs two independent algorithms to boost its functionality. The difference between these two algorithms is the selection of individuals for mutation and crossover. Generally, PGA is faster, less susceptible to discover only sub-optimal solutions, and able to operate with other search methods in parallel. The Parameter settings of PGA are detailed in Table 2.

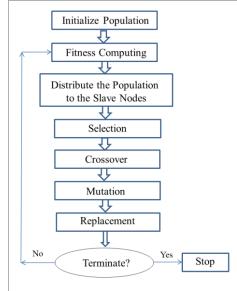


Figure 2. PGA Structure

	0
Population size	3200
Number of generations	300
Initialization	Integer-encoding
Percentage of elite	0.3
Selection	Tournament selection
Crossover	Uniform
Crossover rate	0.7
Mutation	Single point
Mutation rate	0.05

Table 2. Parameter settings of PGA

Technical Indicators

Technical indicators specify the primary element of technical analysis. They are tools that are utilized by traders and investors and help them to forecast the stock price indices and trends. Technical traders make trading decisions according to the price charts. The selected technical indicators provided in Table 3.

Table 3. Selected Technical Indicators										
No.	Indicator									
1	Absolute Price Oscillator (APO)									
2	Ichimoku (ICH)									

3	Average Directional Movement (ADX)
4	Average True Range (ATR)
5	Momentum (MOM)
6	Commodity Channel Index (CCI)
7	Directional Movement Indicators (DMI)
8	Moving Average Convergence Divergence (MACD)
9	On Balance Volume (OBV)
10	Weighted Moving Average (WMA)
11	Price Volume Trend (PVT)
12	Relative Strength Indicator (RSI)
13	Money Flow Index (MFI)
14	Accumulation Distribution Line (ADL)

In this paper, four parameters including "(1) Open (2) High (3) Low (4) Close" have been employed to forecasting the stock price index of the TSE. These parameters used for 12 major indices of the TSE.

Proposed Model

Figure 3 shows the overall model framework, which incorporates five phases to forecast stock prices. These phases as follows:

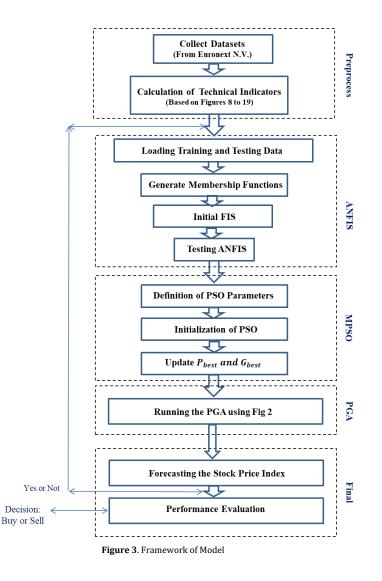
(I) Preprocess. The first phase entails following operations: (1) Collect datasets from TSE; (2) Calculation of technical indicators.

(II) Implementation of ANFIS prediction model. The second phase entails following operations: (1) Loading training and testing data; (2) Generate membership functions; (3) Initial FIS; (4) Testing ANFIS

(III) Implementation of the MPSO. The third phase entails following operations: (1) Definition of PSO parameters; (2) Initialization of PSO; (3) Update P_{best} and G_{best}

(IV) Running the PGA. The fourth phase entails following operations: (1) Initialize population; (2) Fitness computing; (3) Distribute the population to the slave nodes; (4) Selection; (5) Crossover; (6) Mutation; (7) Generate new offspring; (8) Replacement

(V) Final forecasting and Performance Evaluation. The fifth phase entails following operations: (1) Forecasting the stock price index; (2) Performance evaluation by five criteria



4. Results and Discussion

Data Set and Observations

We implement the model utilizing the stock price indices of top companies of TSE. All data gathered from "www.tsetmc.com". The research period of our study is satisfactory for the examination of the model since the significant economic and political events such as different Business cycles, including periods of expansion and contraction, have occurred in this period. The outbreak of the COVID-19 pandemic has also occurred during this period. To improve the model efficiency, the period of investigation divided into several years. As shown in Table 4, training and testing data are available.

Table 4.	Training	and Testing	Data
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Training		Observations	Testing	Observations	
From	То	-	From	То	•
01/01/2011	31/05/2011	425	01/06/2011	31/12/2011	205
01/01/2012	31/05/2012	572	01/06/2012	31/12/2012	281
01/01/2013	31/05/2013	721	01/06/2013	31/12/2013	313
01/01/2014	31/05/2014	843	01/06/2014	31/12/2014	408
01/01/2015	31/05/2015	973	01/06/2015	31/12/2015	491

01/01/2016	31/05/2016	1014	01/06/2016	31/12/2016	523
01/01/2017	31/05/2017	1072	01/06/2017	31/12/2017	588
01/01/2018	31/05/2018	1145	01/06/2018	31/12/2018	657
01/01/2019	31/05/2019	1279	01/06/2019	31/12/2019	723
01/01/2020	31/05/2020	1425	01/06/2020	31/12/2020	793
01/01/2021	31/05/2021	1508	01/06/2021	31/12/2021	824

Forecasting Performance Evaluation

The experimental data has been utilized for analyzing the functionality of the model. The five measures were employed to make a comparison between the actual and predicted prices. The equations of these criteria are as follows:

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^{N} (A_t - P_t)^2}$$
(15)

$$MAE = \frac{1}{N} \sum_{t=1}^{N} |A_t - P_t|$$
(16)

$$MAPE = \frac{1}{N} \sum_{t=1}^{N} \left| \frac{A_t - P_t}{A_t} \right|$$
(17)

Theil's
$$U_1 = \frac{\left[\sum_{t=1}^{N} (A_t - P_t)^2\right]^{\frac{1}{2}}}{\left[\sum_{t=1}^{N} P_t^2\right]^{\frac{1}{2}}}$$
 (18)

Theil's
$$U_2 = \frac{\left[\frac{1}{N} \sum_{t=1}^{N} (A_t - P_t)^2\right]^{\frac{1}{2}}}{\left[\frac{1}{N} \sum_{t=1}^{N} A_t^2\right]^{\frac{1}{2}} + \left[\frac{1}{N} \sum_{t=1}^{N} P_t^2\right]^{\frac{1}{2}}}$$
 (19)

Wherein:

 A_t : Actual price and P_t : Predicted price at time t

The comparison results of key indexes in the TSE have been presented in Tables 5 to 10 which confirmed the functionality of the model.

Table 5. The obtained outcomes of seven key indexes (TSE) based on RMSE

					RMSE						
	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
TEPIX	0.0088	0.0079	0.0065	0.0072	0.0086	0.0101	0.0094	0.0083	0.0078	0.0099	0.0114
Overall Index	0.0110	0.0059	0.0063	0.0141	0.0076	0.0125	0.0136	0.0079	0.0108	0.0087	0.0069
Industry Index	0.0097	0.0073	0.0153	0.0058	0.0132	0.0079	0.0047	0.0105	0.0175	0.0078	0.0066
Top 30 Index	0.0118	0.0068	0.0079	0.0055	0.0201	0.0178	0.0068	0.0114	0.0201	0.0093	0.0127
Main Board Index	0.0065	0.0136	0.0120	0.0083	0.0058	0.0137	0.0211	0.0193	0.0064	0.0076	0.0113
Secondary Index	0.0076	0.0207	0.0143	0.0067	0.0094	0.0118	0.0099	0.0063	0.0126	0.0045	0.0086
Top 50 Index	0.0211	0.0145	0.0096	0.0119	0.0231	0.0072	0.0094	0.0122	0.0140	0.0203	0.0091

Table 6. The obtained outcomes of seven key indexes (TSE) based on MAE

					MAE						
	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
TEPIX	0.0196	0.0137	0.0046	0.0108	0.0211	0.0193	0.0064	0.0090	0.0114	0.0138	0.0212
Overall Index	0.0111	0.0085	0.0057	0.0135	0.0079	0.0100	0.0115	0.0078	0.0221	0.0048	0.0103
Industry Index	0.0125	0.0136	0.0079	0.0108	0.0087	0.0069	0.0047	0.0103	0.0175	0.0001	0.0067
Top 30 Index	0.0065	0.1022	0.0120	0.0083	0.0058	0.0059	0.0063	0.0141	0.0077	0.0060	0.0074
Main Board Index	0.1076	0.0207	0.0143	0.0067	0.0094	0.0119	0.0099	0.0063	0.0126	0.0047	0.0087
Secondary Index	0.0214	0.0142	0.0096	0.0117	0.0231	0.0072	0.0094	0.0122	0.0140	0.0204	0.0092
Top 50 Index	0.1194	0.0039	0.0158	0.0049	0.0076	0.0212	0.0088	0.0079	0.0065	0.0138	0.0089

Table 7. The obtained outcomes of seven key indexes (TSE) based on MAPE

TEPIX 0.0082 0.0063 0.0071 0.0073 0.0045 0.0033 0.0049 0.0098 0.0058 0.0011 0 Overall Index 0.0036 0.0072 0.0035 0.0012 0.0145 0.0236 0.0072 0.0062 0 Industry Index 0.0088 0.0064 0.0071 0.0020 0.0037 0.0049 0.0072 0.0058 0.0011 0.0062 0 Industry Index 0.0088 0.0064 0.0071 0.0020 0.0037 0.0044 0.0057 0.0011 0.0069 0.0179 0 Top 30 Index 0.0135 0.0127 0.0046 0.0072 0.0087 0.0011 0.0069 0.0076 0 Main Board Index 0.0069 0.0056 0.0074 0.0047 0.0035 0.0075 0.0018 0 Secondary Index 0.0028 0.0014 0.0065 0.0022 0.0043 0.0068 0.0050 0.0064 0.0059 0						MAPE						
Overall Index 0.0036 0.0072 0.0035 0.0012 0.0145 0.0236 0.0072 0.0023 0.011 0.0062 0 Industry Index 0.0088 0.0064 0.0071 0.0020 0.0037 0.0044 0.0057 0.0011 0.0069 0.0179 0 Top 30 Index 0.0135 0.0127 0.0046 0.0032 0.0072 0.0086 0.0092 0.0027 0.0010 0.0076 0 Main Board Index 0.0069 0.0056 0.0087 0.0047 0.0035 0.0075 0.0084 0.0095 0.0018 0 Secondary Index 0.0028 0.0014 0.0065 0.0036 0.0022 0.0043 0.0068 0.0050 0.0064 0.0059 0		2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Industry Index 0.0088 0.0064 0.0071 0.0020 0.0037 0.0044 0.0057 0.0011 0.0069 0.0179 0 Top 30 Index 0.0135 0.0127 0.0046 0.0032 0.0072 0.0086 0.0092 0.0027 0.0010 0.0076 0 Main Board Index 0.0069 0.0056 0.0087 0.0074 0.0047 0.0035 0.0075 0.0084 0.0095 0.0018 0 Secondary Index 0.0028 0.0014 0.0065 0.0036 0.0022 0.0043 0.0068 0.0050 0.0064 0.0059 0	TEPIX	0.0082	0.0063	0.0071	0.0073	0.0045	0.0033	0.0049	0.0098	0.0058	0.0011	0.0074
Top 30 Index 0.0135 0.0127 0.0046 0.0032 0.0072 0.0086 0.0092 0.0027 0.0010 0.0076 0 Main Board Index 0.0069 0.0056 0.0087 0.0074 0.0047 0.0035 0.0075 0.0084 0.0095 0.0018 0 Secondary Index 0.0028 0.0014 0.0065 0.0036 0.0022 0.0043 0.0068 0.0050 0.0064 0.0059 0	Overall Index	0.0036	0.0072	0.0035	0.0012	0.0145	0.0236	0.0072	0.0023	0.0101	0.0062	0.0055
Main Board Index 0.0069 0.0056 0.0087 0.0074 0.0047 0.0035 0.0075 0.0084 0.0095 0.0018 0 Secondary Index 0.0028 0.0014 0.0065 0.0036 0.0022 0.0043 0.0068 0.0050 0.0059 0	Industry Index	0.0088	0.0064	0.0071	0.0020	0.0037	0.0044	0.0057	0.0011	0.0069	0.0179	0.0037
Secondary Index 0.0028 0.0014 0.0065 0.0036 0.0022 0.0043 0.0068 0.0050 0.0064 0.0059 0	Top 30 Index	0.0135	0.0127	0.0046	0.0032	0.0072	0.0086	0.0092	0.0027	0.0010	0.0076	0.0041
······································	Main Board Index	0.0069	0.0056	0.0087	0.0074	0.0047	0.0035	0.0075	0.0084	0.0095	0.0018	0.0023
Top 50 Index 0.0049 0.0092 0.0032 0.0005 0.0019 0.0033 0.0065 0.0024 0.0082 0.009 0	Secondary Index	0.0028	0.0014	0.0065	0.0036	0.0022	0.0043	0.0068	0.0050	0.0064	0.0059	0.0047
	Top 50 Index	0.0049	0.0092	0.0032	0.0005	0.0019	0.0033	0.0065	0.0024	0.0082	0.009	0.0085

Table 8. The obtained outcomes of seven key indexes (TSE) based on U1

					U1						
	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
TEPIX	0.0082	0.0063	0.0071	0.0073	0.0045	0.0033	0.0049	0.0098	0.0058	0.0011	0.0074
Overall Index	0.0036	0.0072	0.0035	0.0012	0.0145	0.0236	0.0072	0.0023	0.0101	0.0062	0.0055
Industry Index	0.0088	0.0064	0.0071	0.0020	0.0037	0.0044	0.0057	0.0011	0.0069	0.0179	0.0037
Top 30 Index	0.0135	0.0127	0.0046	0.0032	0.0072	0.0086	0.0092	0.0027	0.0010	0.0076	0.0041
Main Board Index	0.0069	0.0056	0.0087	0.0074	0.0047	0.0035	0.0075	0.0084	0.0095	0.0018	0.0023
Secondary Index	0.0028	0.0014	0.0065	0.0036	0.0022	0.0043	0.0068	0.0050	0.0064	0.0059	0.0047
Top 50 Index	0.0049	0.0092	0.0032	0.0005	0.0019	0.0033	0.0065	0.0024	0.0082	0.009	0.0085

Table 9. The obtained outcomes of seven key indexes (TSE) based on U2

					U2						
	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
TEPIX	0.0103	0.0124	0.0119	0.0042	0.0082	0.0095	0.0082	0.0047	0.0023	0.0086	0.0051
Overall Index	0.0112	0.0099	0.0106	0.0005	0.0019	0.0021	0.0075	0.0034	0.0049	0.0019	0.0095
Industry Index	0.0136	0.0167	0.0145	0.0020	0.0054	0.0034	0.0067	0.0021	0.0079	0.0056	0.0047
Top 30 Index	0.0147	0.0162	0.0137	0.0036	0.0041	0.0043	0.0058	0.0060	0.0054	0.0069	0.0055
Main Board Index	0.0181	0.0144	0.0162	0.0073	0.0045	0.0033	0.0039	0.0088	0.0068	0.0031	0.0084
Secondary Index	0.0107	0.0173	0.0190	0.0012	0.0056	0.0127	0.0062	0.0013	0.0124	0.0073	0.0064
Top 50 Index	0.0125	0.0151	0.0107	0.0138	0.0045	0.0032	0.0074	0.0091	0.0054	0.0022	0.0025

Table 10. The comparison results of models

Model	RMSE	MAE	MAPE	U1	U2	Overall Performance
ANFIS-MPSO-PGA	1.1e-4	0.0007	0.0027	0.0010	0.0007	81.45%
ANFIS-PSO-GA	1.4e-4	0.0009	0.0031	0.0012	0.0009	72.62%
ANFIS-PSO	1.8e-4	0.0142	0.0044	0.0489	0.0324	69.26%
PSO-GA	2.7e-4	0.0098	0.0065	0.0272	0.0256	63.45%
ANFIS-GA	2.8e-4	0.0075	0.0081	0.0311	0.0242	64.88%
ANFIS	3.5e-4	0.0065	0.0072	0.0195	0.0143	58.23%
PSO	4.4e-4	0.0018	0.0086	0.0121	0.0088	53.74%
GA	5.2e-4	0.0182	0.0095	0.1025	0.0963	50.42%
MPSO-PGA	2.5e-4	0.0067	0.0054	0.0234	0.0241	67.14%
ANFIS-MPSO	1.7e-4	0.0125	0.0037	0.0072	0.0224	71.26%
ANFIS-PGA	3.2-4	0.0081	0.0093	0.0376	0.0347	68.53%

Table 10 illustrates that the three criteria for the ANFIS-MPSO-PGA model are lower than the other related models. Furthermore, Theil's U criteria of the model are more than other techniques (Theil 1958, Theil 1971). We utilized the procedure of Sedighi, Jahangirnia et al. (2019) for executing ANFIS that outlined in table 10. Based on the results in Tables 5 to 10, it could be realized that the ANFIS-MPSO-PGA model presents the lowest error and the best precision. The model has been compared to other techniques that are verified by top TSE indices. The actual prices and predicted prices for the ANFIS-MPSO-PGA model shown in Figures 4 to 7. They demonstrate that the model confirms the best performance.

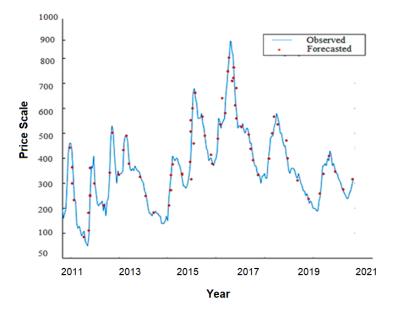


Figure 4. Model Performance Evaluation

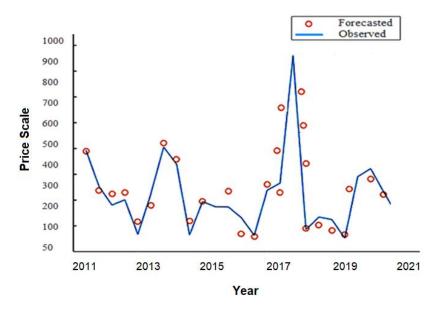


Figure 5. Model Fit (Training)

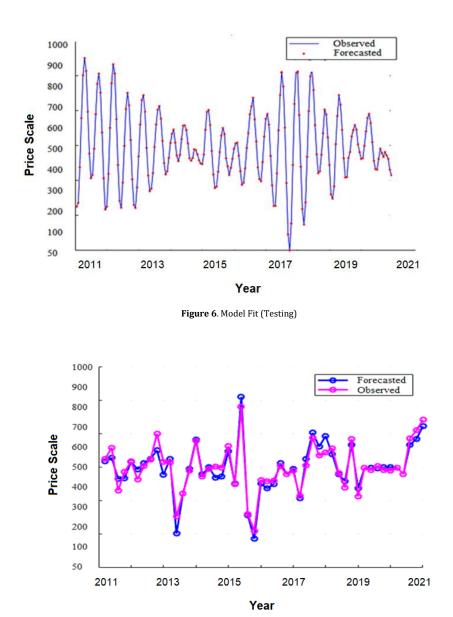


Figure 7. Results of Model Testing

After examining the experimental results; which can be seen in Tables 5 to 10 and Figures 4 to 7; the following cases are evident:

- 1) The ANFIS-MPSO-PGA model is appropriate for parameterized optimization issues, and the proposed method can obtain the minimum RMSE.
- 2) The presented model considers the causality of technical indicators with ANFIS learning and integrates an adaptive equation for stock price prediction.
- 3) The use of PGA raises the functionality and speed of the system that uses evolutionary algorithms.
- 4) The experiments approve that the MPSO is an efficient way to solve specific kinds of search and optimization issues.
- 5) The integration of MPSO and PGA in prediction procedures has a positive impact on the functionality of the model.
- 6) Combined and multi-stage models perform better than single-stage models

In this research, the authors designed a new robust hybrid model named ANFIS-MPSO-PGA to forecast the stock price index volatility, with the purpose to enhance the accuracy of forecasting. After the calculations of technical indicators, they employed as input variables of the presented model. For the aim of model evaluation, the datasets of the stock price index of the TSE from 2011 to 2021 were used. Subsequently, five criteria applied to examine the functionality of the model. Empirical experimentations revealed that the presented model functioned better than other related methods in terms of predicting the direction of the stock price index volatility. Further, the prediction accuracy is enhanced by the implementation of the model. The findings validated that the presented model is an effective tool to predict the volatility of the stock price index. The final results demonstrated that the average functionality of the model (81.45%) is considerably better than other tools. The proposed model is the first combination method that integrates three techniques, including ANFIS, MPSO, and PGA, to predict the volatility of the stock price index in the TSE. The significant contributions of the current research are mentioned as follows:

- I. The difficulty of prediction of the stock price index has been solved by applying the ANFIS-MPSO-PGA model.
- II. The fifteen technical indicators employed in the model are the most applicable indicators of technical analysis.
- III. By hybridizing the ANFIS with the PSO, the accuracy level of the stock price index prediction significantly raised.
- IV. By hybridizing the PSO with the PGA, the processing speed of the model effectively increased.
- V. The prediction accuracy of the ANFIS-MPSO-PGA model is more accurate than the existing related methods.
- VI. The ANFIS-MPSO-PGA model can be used in other financial markets to forecast the price index volatility of types of securities.

The most important restrictions of this study are as follows: The lack of control of many of the conditions affecting the outcomes of the study, such as economic variables, political conditions, the situation of the global economy, trade wars, sanctions, coronavirus, etc. For future research, the following items can be considered to extend the presented model:

1) Using other technical indicators and checking their compatibility with the proposed model.

2) Applying other classifications of genetic algorithms based on the book authored by Sivanandam and Deepa (2008) and evaluating comparative results.

3) Employing different types of PSO including "QPSO, BBPSO, CPSO, FPSO, OPSO, SPSO" that they have been described in paper by Zhang, Balochian et al. (2016) and comparison of experimental results.

Abbreviations

In this article, the following abbreviations have been used:

TSE	Tehran Stock Exchange
TOPIX	Tokyo Stock Price Index
KOSPI	Korea Stock Price Index
SHSE	Shanghai Stock Exchange
SBI	State Bank of India
BPNN	Back Propagation Neural Network
IBCO	Improved Bacterial Chemotaxis Optimization
BPAN	Back Propagation Artificial Neural
NN	Neural Network
ANN	Artificial Neural Networks
DA	Directional Accuracy
ARV	Average Relative Variance
ARMA	Autoregressive-Moving-Average Model
LSTM	Long Short-Term Memory
SVR	Support Vector Regression
FLANN	Functional Link Artificial Neural Network

FA	Firefly Algorithm		
MSVR	Multi-Output Support Vector Regi		
ARV	Average Relative Variance		
ANOVA	Analysis Of Variance		
HSD	Honestly Significant Difference		
CPI	Consumer Price Index		
BOT	Balance Of Trade		
ISM	Institute For Supply Management		
PER	Predictive Error Rate		
RBF	Radial Basis Function		
NB	Naïve Bayes		
DT	Decision Tree		
KNN	K-Nearest Neighbor		
ABC	Artificial Bee Colony		

Autoregressive SVM Support Vector Machine FTSE Financial Times Stock Exchange DJIA Dow Jones Industrial Average European Union

References

AR

EU

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