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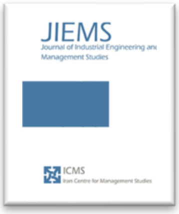

A Mathematical Model of Hub Location for War Equipment under Uncertainty Using Meta-Heuristic Algorithms

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Abstract

providing timely transportation and dispatch of raw materials and finished goods, freight transport plays an essential role in industries, commercial activities, and trade war industries. It also has a significant impact on the overall performance of associated organizations and the ultimate costs of their products. Therefore, freight transport providers are under pressure to decrease costs and increase their service levels and should overcome these pressures by redesigning and improving their logistics processes on strategic, tactical, and operational levels. In this research, a multi-objective model is proposed for hub location in the field of war equipment under uncertainty. The first objective is to minimize costs, the second objective is to maximize the fulfillment of demands, and the third objective is to minimize congestion on the routes. Taking into account the parameters in the state of uncertainty, the mathematical model is modeled in a robust state and a robust counterpart model of the problem is proposed. In order to solve the problem on a small scale, the exact epsilon constraint method is used in GAMS software. Also, meta-heuristic approaches of grey wolf optimizer (GWO) and non-dominated sorting genetic algorithm (NSGA-II) are used to solve the model in medium and large dimensions. Finally, proper performance indicators are used to compare the performance of the used algorithms, and as a result of solving several numerical examples and calculating their performance indicator, it is concluded that the GWO algorithm has a better performance in solving the model.

Keywords: Hub Location, War Equipment, Uncertainty, Meta-heuristic Algorithm.

Paper Type: Original Research

1. Introduction

Hubs are specialized facilities used as exchange, transfer, and classification points in many distribution systems. Instead of providing direct services between each origin and destination, hub facilities concentrate the flows in order to use the resulting economic savings. In this research, hub location for war equipment is performed under uncertainty. Thus, some points are considered as hub points and these points are considered as equipment transfer centers so that the cost of transporting the equipment is minimized and this transfer takes place in the shortest time, and also the minimum number of transport vehicles is used for transportation (Chobar et al. 2022). War equipment is one of the tools that, despite their ineffectiveness in peacetime, are a suitable factor for deterring and defending against enemy attacks in times of war. Along with the strategic importance of these kinds of equipment, their location is also of great importance. Because in case of wrong location, it is not possible to equip the troops during an attack or crisis, and therefore this can cause serious damage during the enemy's attack in such a way that in some cases it is very difficult to compensate and sometimes impossible to compensate for these damages. Accurate and proper location of war equipment can prevent the enemy's attack on time and will lead to maximum reaction at this time. However, despite the high importance of this area, it can be seen that not much research work has been conducted on locating military equipment, which can be considered an important research gap. In fact, the number of research works hub location is very large and the research works have paid attention to different areas in this field, but the lack of attention to the issue of locating war equipment can be a major problem and an important research path for future researches. Furthermore, given the similarity of war and crisis conditions, uncertainty should naturally be taken into consideration in this field, in other words, uncertainty will be an integral part of locating and routing problems, especially in the field of crisis and war. Considering the shortcomings and deficiencies in the field of hub location for war equipment, this research seeks to solve this research gap by proposing a mathematical programming model for hub location and solving it using meta-heuristic algorithms. One of the important points considered in this research is to provide a robust model to consider the uncertainty, while taking into account that the routes leading to the hubs are subject to disruption, costs related to flow in these routes should be considered separately, while the route demand is considered through more than one arch due to the

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critical nature of the problem. While many similar studies have considered a maximum of one arc for each hub, considering several arcs for each hub can bring the problem closer to the real world and its uncertain nature. Another point that is considered as the third objective of the research and also as its innovation is network congestion. Naturally, there is a possibility of congestion in war conditions; hence, choosing a route with less congestion is one of the priorities of the present problem. Thus, the selected algorithm selects the corresponding hub by considering the existing congestion in the network. The meta-heuristic algorithm of gray wolf is used because of its new nature. This algorithm is less used in the research related to hub location, and considering the multi-objective nature of the model, the effort is to evaluate the efficiency of this algorithm in order to solve the desired problem.

In general, the objectives of the research are as follows:

- Proposing a mathematical model of hub location for war equipment under uncertainty using meta-heuristic algorithms

- Proposing a mathematical model of hub location for war equipment considering the flow-dependent costs in disturbed routes

- Proposing a mathematical model of hub location for war equipment considering the route demand through more than one arc hub

- Proposing a mathematical model of hub location for war equipment considering network congestion

2. Literature review

The idea of hub networks was proposed by Goldman (1969). Then, O'Kelly (1987) proposed the first study of hub network in the field of aerial networks. Although O'Kelly provided the first known mathematical formulation of a hub location problem by studying airline passenger networks. His formulation refers to the single-hub allocation problem (Alumur et al., 2008). Nevertheless, the first linear integer mathematical model was presented by Campbell (1994). The first generation of hub location research can be seen as the results of the work of Campbell and O'Kelly (1994), which made great progress in the understanding of intermediary systems and the development of basic models with a major focus on minimizing the flow cost and fixed cost. Taghipourian et al. (2011) provided a fuzzy integer linear programming approach to express a dynamic virtual hub location problem with the aim of minimizing the transportation cost in the network. The authors tested the effectiveness of their model using the famous CAB dataset. Alumur et al. (2012) evaluated the hierarchical multiple hub location problem with certain delivery time. In this article, a mixed integer programming model was presented for the analysis of Turkey's air transport system. They showed that airport hub locations were less sensitive to cost parameters than ground hub locations, and service quality was better in this case. Lindsay et al. (2014) researched regional logistics hubs for freight activity and industrial space demand. In this article, the importance of logistics and supply chain economics for companies with industrial activity was taken into account. In such a way that these companies should consider their warehousing location and operational centers based on availability, quality, and transportation costs. This study performed an econometric analysis of the data set that includes 20 urban markets and identified an identification method for ranking urban markets according to industrial space consumption. Sun (2016) investigated the location problem, which was a combination of the hub location problem and the multi-hub vehicle location problem in a delivery service. To solve these problems, the authors presented a new solution framework dividing the problem into two sub-problems. The two-phase approach created two computational techniques, one of which was the ant optimization algorithm and the other was the genetic algorithm. The authors determined the locations of prescribed hubs in a set of sorting centers and assigned them to each client node. Lopes et al. (2016) investigated the many-to-many hub localization problem, which consisted in partitioning the set of nodes of a graph into routes containing exactly one hub each, and determining an extra route interconnecting all hubs. A variable neighborhood descent with neighborhood structures based on remove/add, swap and exchange move nested with routing and location operations was used as a local search procedure in a multistart algorithm. The authors also considered a model of this local search in the multistart. In addition, a biased random-key genetic algorithm working with a local search routine, which also considered routing and location operations, was applied to the problem. Rahmati and Bashiri (2018) sought to model the multi-allocation hub location problem without capacity in an uncertain environment. First, a certain model of hub location with multiple allocation without capacity was introduced, then a robust optimization approach was used to deal with uncertain parameters. Mathematical formulation of the problem was developed with uncertainty in demand, fixed hub set-up cost and internal hub flow discount factor. The level of conservatism was controlled by the uncertainty budget. Peer models were compared with each other using AP and CAB datasets with different uncertain levels. The results showed that with increasing uncertainty, more hubs were set up despite demand uncertainty, while the number of hubs decreased when the set-up cost was uncertain.

In the article of Bashiri et al. (2018), mathematical modeling for the problem of hub location in a dynamic environment was considered by genetic algorithm. In this study, a new problem in the field of mobile phones in a dynamic environment was proposed. The numerical examples analyzed in the article showed that the mobile hub network was more efficient than the mobile hub network in a dynamic environment. The main goal of this study was to propose a proper solution algorithm by genetic algorithm and annealing simulation method. Li et al. (2020) considered two sources of uncertainty: the flows from origins to destinations and the set-up costs of hubs. A robust optimization formulation was proposed for both single and multiple allocation cases, in which the flow between each pair of nodes was assumed to be uncertain and correlated. In addition, the set-up cost of a hub was related to the total flow through the hub. Nonlinear integer program models were presented for both single and multiple allocation cases, and they were solved using CPLEX. Computational tests using the Civil Aeronautics Board and Australian Post datasets were provided. The numerical results suggested that the robust optimization strategy located more hubs than in the deterministic case with a relatively small cost increase, and the total cost of the robust solution calculated for the multiple allocation case was marginally lower than that for the single allocation case. The robust optimization strategy was proven to be effective for protecting the solution against the worst case for different uncertain parameters. Soltanpour et al. first considered the classical p-median location problem on a network in which the vertex weights and the distances between vertices were uncertain variables. The uncertainty distribution of the optimal objective value of the p-median problem was given and the concepts of the α-p-median, the most p-median and the expected p-median were introduced. Then, it was shown that the uncertain p-median problem was NP-hard on general networks. Finally, the researchers investigated the inverse 1-median problem on a tree with uncertain vertex weights and presented a programming model for the problem. Then, it was shown that the proposed model could be reformulated into a deterministic programming model The article by Golabi et al. (2017) was conducted on the premise that people move to the location of distribution centers in accessible and intact routes to receive the relief. This study aimed to develop a mathematical model minimizing the aggregate traveling time for both people and UAVs over a set of feasible scenarios. Since the network problems were NP-hard, some metaheuristic algorithms were developed to solve the proposed model. In order to demonstrate the applicability of developed model, a case study based on feasible earthquake scenarios in Tehran was presented. Shavarani et al. (2019) sought to develop a bi-objective mathematical model to account for the optimum number and spatial location of facilities among a set of candidate locations such that the total travel distance, costs, and lost demand would be minimized simultaneously. The proposed bi-objective capacitated facility location model was NP-hard, thus non-dominated sorting genetic algorithm II and reference-point based non-dominated sorting genetic algorithm were applied to solve the problem. The performance of the algorithms, quality of solutions, and the results were investigated and discussed. Yılmaz et al. (2018) introduced an optimization problem attempting to optimize location and routing of a homogeneous unmanned aerial vehicle fleet. The problem also allocated the available capacity to the potential locations while it sustained the feasibility defined by synchronization constraints including time windows at visited points, capacity monitoring in the stations and a limited number of multiple sorties. A mixed integer linear programming formulation for the problem was given and a heuristic method based on ant colony optimization approach was suggested. The suggested heuristic was compared to a commercial solver. It was observed that the suggested heuristic provided the best solutions, while the commercial solver was able to produce only poor solutions in longer time periods. The learning component, which was the main difference between the suggested heuristic and its simplified version, made a significant change. The results of the experiments strongly suggested the usage of the metaheuristic method for the introduced problem. The research by Mokhtarzadeh et al. (2021) presented a novel p-mobile hub location–allocation problem. Hub facilities can be transferred to other hubs for the next period. Implementation of mobile hubs can reduce the costs of opening and closing the hubs, particularly in an environment with rapidly changing demands. On the other hand, the movement of facilities reduces lifespan and adds relevant costs. Three objective functions were considered to minimize costs, noise pollutions, and the harassment caused by the establishment of a hub for people. A multi-objective mixed-integer non-linear programming (MINLP) model was developed. To solve the proposed model, four meta-heuristic algorithms, namely multi-objective particle swarm optimization (MOPSO), a non-dominated sorting genetic algorithm (NSGA-II), a hybrid of k-medoids as a famous clustering algorithm and NSGA-II (KNSGA-II), and a hybrid of K-medoids and MOPSO (KMOPSO) were implemented. The results indicated that KNSGA-II was superior to other algorithms. In the research by Soleimani et al. (2021), a multi-objective mathematical programming model was presented in the form of fuzzy figures, taking into account the uncertainty in some parameters, especially the cost. In addition, backup hubs were selected for each primary hub to deal with disruption and natural disasters. Then, a robust possibilistic method was proposed to deal with uncertainty. As the hub location-allocation problem was considered as NP-Hard problems so that exact methods could not solve them in large sizes, two metaheuristic algorithms including a non-dominated sorting genetic algorithm non-dominated sorting genetic algorithm (NSGA-II) and multi-objective particle swarm optimization (MOPSO) were applied. Numerical results showed the proposed model was valid. Also, it was demonstrated that the NSGA-II algorithm outperformed the MOPSO algorithm. The article by Zahiri and Suresh (2021) considered a material transportation network design problem based on hub location under uncertainty. The research variables included decisions regarding hubs and hazmat response teams, along with the routing schemes. The model aimed to minimize the total risk in the network. The measure of risk incorporated average response time to hazmat incidents as well as waiting time at hubs.

To tackle the uncertainty of the problem, an efficient interactive approach using mean-absolute deviation and possibilistic programming was developed. Two heuristic algorithms were presented to solve the model in large scale instances. The advantages of the presented methodologies were demonstrated through numerical experiments based on transportation networks. The researchers investigated different design schemes and discussed optimal decisions under different configurations. Demir et al. (2022) conducted a meta-heuristic experimental evaluation for the problem of multi-allocation hub location with multi-objective capacity. In this study, two meta-heuristic approaches based on the non-dominated sorting genetic algorithm (NSGA-II) and archived multi-objective simulated annealing method (AMOSA) to solve MOCMAHLP. To statistically analyze the behavior of both algorithms, they conducted experiments on two well-known data sets, namely Turkish and Australian Post (AP). After analyzing different operators for both algorithms, a comparison between the NSGA-II based and AMOSA based approaches was performed with the best settings. As a result, it was concluded that both of the algorithms were able to find feasible solutions of the problem. Moreover, NSGA-II performed better for larger, whereas AMOSA performed better for smaller size networks. Li et al. (2023) presented the design of multimodal hub-and-spoke transportation network for emergency relief under COVID-19 pandemic using a meta-heuristic approach. In this research, a multimodal hub-and-spoke transportation network for emergency relief schedules was considered which included a bi-objective model aiming at minimizing both transportation time consumption and transportation costs. This study thus employed and redesigned grey wolf optimizer (GWO) to tackle it. Results indicated that the customized GWO could solve such a problem in a reasonable time with higher accuracy. The research could provide significant practical management insights for related government departments and transportation companies on designing an effective transportation network for emergency relief schedules when faced with the unexpected COVID-19 pandemic. In this research, using a mathematical model, we intend to minimize the transportation cost and determine the best location for war equipment storage in war zones according to the demand of the regions. The demand of the regions is considered dynamically and based on different situations. After the mathematical model is prepared, first we code the model in small dimensions in GAMS software and run the model to make sure the validity of the model. Then, meta-heuristic algorithms are used in MATLAB software to analyze information and data in large dimensions.

3. Problem statement

Locating military equipment has special conditions, if these conditions are not taken into consideration, one can witness serious problems regarding the allocation of a proper location for military equipment warehouses. Regardless of security issues, fulfilling the demand is naturally considered a necessity. Therefore, one of the important objectives in hub location of military equipment is to meet the maximum demand according to the demands in the demand areas or routes. On the other hand, in such matters, as military products must be transported at maximum speed, choosing routes with the least congestion can be another necessity. In other words, both in terms of security and speed of transportation, it is important to choose routes with the least amount of traffic, so that vehicles are less aware of the existence of vehicles carrying military equipment and the equipment can be quickly transferred to the desired location or headquarters. Demand as well as route congestion are both among the components that can be subject to uncertainty, that is, with the occurrence of scenarios, one would witness that the route congestion increases or demand changes, so the issue of uncertainty is naturally important in the relevant issues and becomes meaningful and it can be defined based on different scenarios such as pessimistic, optimistic, and possible scenarios. In the pessimistic scenarios, the route congestion reaches the highest level, while the demand also increases significantly; however, in other scenarios, we see better conditions both in terms of congestion and demand. According to the mentioned cases, the present research seeks to provide a location model for military equipment under uncertainty. The model has three objectives, the first objective is to minimize the cost, the second objective is to maximize the fulfillment of demand, and the third objective is to minimize congestion on the routes.

The assumptions of the problem are as follows:

- It is a single period model.
- The location of the hubs is not known and must be determined.
- The inventory at the beginning of the period is zero.
- There is a capacity limit.
- The model is scenario oriented.
- The number of existing vehicles is considered as congestion.
- The parameters of demand and product transfer cost are uncertain.
- The maintenance cost is considered variable in different periods, but it is the same for all hubs.

Indices

i	Node
j	Hub
k	Location
r	Route
t	Period
l	Demand point
s	Scenario

Parameters

TC_{rs}	Cost of transporting a unit of product on route r under scenario s
DEM_{rst}	Product demand on route r under scenario s in period t
FC_{jk}	Cost of constructing hub j in location k
DIS_{irl}	Distance from node i to demand point l on route r
COG_{rt}	Number of vehicles on route r in period t
CAP_j	Node capacity i
HC_t	Product maintenance cost in time period t
M	A big number

Decision variables

X_{ij}	1 if node i is selected as hub j and zero otherwise
Y_{jk}	1 if hub j is built in location k and zero otherwise
V_{rt}	1 if route r is selected in period t and zero otherwise
Z_{rsjt}	Transfer flow on route r under scenario s to hub j in time period t
U_{jt}	Product inventory in hub j in time period t

3.1. Mathematical model

According to the assumptions of the research, the indices, parameters, and defined variables of the mathematical model of the research are presented as follows:

Objective functions

$$min\,z1 = \sum_i \sum_r \sum_l \sum_s \sum_t DIS_{irl} Z_{jrst}\, TC_{rs} + \sum_j \sum_k FC_{jk}\, Y_{jk} + \sum_j \sum_t HC_t\, U_{jt} \tag{1}$$

The above equation seeks to minimize the costs, including transfer cost, fixed construction cost, and maintenance cost based on inventory.

$$max\,z2 = \sum_r \sum_s \sum_t DEM_{rst}\, V_{rt} \tag{2}$$

The above equation seeks to maximize the fulfillment of the demand according to the appropriate chosen route.

$$min\,z3 = \sum_r \sum_t COG_{rt}\, V_{rt} \tag{3}$$

The above equation seeks to maximize congestion on the route by choosing the appropriate route.

Constraints

$$\sum_{i} X_{ij} = 1 \tag{4}$$

The above equation shows that each node is assigned to one hub only.

$$\sum_{k} Y_{jk} = 1 \tag{5}$$

The above equation shows that each hub is built in one place.

$$\sum_{k} Y_{jk} \leq \sum_{i} X_{ij} \tag{6}$$

The above equation shows that if a node is assigned to a hub, it is possible to build it.

$$\sum_{r} \sum_{t} V_{rt} = 1 \tag{7}$$

The above equation shows that only one route is selected in all periods.

$$Z_{rsj} \leq M\, V_{rt} \tag{8}$$

The above relation states that if a route is selected, there is a flow of product transfer from that route.

$$U_{jt} = U_{jt-1} + \sum_{r} Z_{rsjt} \tag{9}$$

The above equation shows the product inventory in each period.

$$U_{jt} \leq CAP_j \tag{10}$$

The above relation indicates the limitation of hub capacity.

$$U_{j1} = 0 \tag{11}$$

The above relation indicates the assumption that the first period includes zero inventory.

$$Z_{rsj} \leq DEM_{rst} \tag{12}$$

The above relation shows the fulfillment of demand by transfer flow.

$$U_{jt} \leq M\, Y_{jk} \tag{13}$$

The above relation states that there is an inventory for a hub if that hub has been built.

$$X_{ij} \in \{0.1\} \tag{14}$$

$$Y_{jk} \in \{0.1\} \tag{15}$$

$$V_{rt} \in \{0.1\} \tag{16}$$

The above relations indicate the constraint of the binary variables of the problem.

$$Z_{rsj} \geq 0 \tag{17}$$

$$U_{jt} \geq 0 \tag{18}$$

The above relations indicate the constraint of integer variables of the problem.

3.2. Providing a robust uncertain model

Given that the problem is in the state of uncertainty and the previous model is a certain model; therefore, in this section, the uncertain problem and the uncertainty approach regarding the present problem are discussed. Mulvey et al. (1995) provided a framework for robust optimization that includes two important definitions of robust solution and robust model: a solution to an optimization model is called a robust solution when that solution remains close to optimal under all scenarios and also when a model is a robust model that is almost plausible under all scenarios. Accordingly, the general model of robust optimization has been developed. This optimization is related to problems whose data type is of the scenario type, in other words, the data values of the problem are described by a set of scenarios. Mulvey states in his research that operation research in mathematical programming models is faced with fluctuating and uncertain data, and dealing with this type of data through sensitivity analysis or probabilistic planning faces problems. In general, when dealing with optimization models, we are faced with two separate parts: the structural part which is fixed and does not have any fluctuations in its input data, and the control part which is a function subject to uncertain and fluctuating data. The optimization model is as follows:

$$\min C^T X + d^T y \tag{19}$$

$$\text{subject to: } AX = b \tag{20}$$

$$BX + Cy = e \tag{21}$$

$$x, y \geq 0 \tag{22}$$

$$x \in R^{n1}, y \in R^{n2} \tag{23}$$

In the above model:

X represents the decision variables of the certain parameters

Y represents the decision variables of the control part

Constraints are divided into two parts:

Structural constraints whose coefficients are fixed and so-called certain.

Control constraints whose coefficients include the uncertain state.

We assume a limited set of scenarios $\Omega = \{1,2,3, \ldots . S\}$ for the uncertain parameters of the model, and corresponding to each scenario $s \in \Omega$, the set of {ds, Bs, Cs, es} is defined as the realization of the performance in each scenario, on the other hand, Ps shows the probability of the occurrence of each scenario which is $\sum \; P_s$. The general form of Mulvey et al. (1995) robust optimization model is as follows:

$$\min \sigma(X, y1, y2, \ldots, ys) + \text{ῳ} \sum \text{ῥ}(\partial 1, \ldots . \partial s) \tag{24}$$

$$st{:}\, AX = b \tag{25}$$

$$B_s X + C_s y_s + Z_s = e_e \; \forall s \in \Omega \tag{26}$$

$$X \geq 0, y_s \geq 0 \; \forall s \in \Omega \tag{27}$$

In the above robust model, the set $\{y1, y2, y3, \ldots ys\}$ is a set of control variables for each scenario $s \in \Omega$ and also a set of error vectors that measure the implausible value allowed in the control constraints under scenario s. According to multiple scenarios, the objective function $\varepsilon = C^T X + d^T$ is a random variable that takes the value of $\varepsilon_s = C^T X + d_s^T y_s$ with probability of P_s . The exchange between robustness of the solution and robustness of the model is done using the concept of multi-criteria decision making. In fact, the above optimization model is able to measure the amount of this exchange. The expression ∂_0 is considered as a non-linear expression. In fact, this robust model is based on the applied scenario of the probabilistic nonlinear programming model. The second expression in the objective function, i.e. $\{\partial 1, \partial 2, \partial 3, \ldots \partial s\}$, is a plausibility penalty function, which is considered in order to penalize the violation of the control constraints according to some scenarios. With the help of ῳ, the balance and trade-off between robustness of the answer measured by ∂_0 and robustness of the model measured by ῥ_0 can be modeled under the multi-criteria decision process. For example, if ῳ=0, the objective function minimizes the expression ∂_0 and the answer may be implausible, while if a relatively large value is assigned to ῳ, it will bring more cost. The expression $\partial(x, y1, \ldots ys)$ includes the mean value ∂_0 plus the constant value λ multiplied by its variance.

$$\partial(x, y1, \ldots ys) = \sum_{s \in S} P_s \, \mathcal{E}_s + \lambda \sum_{s \in S} P_s \, (\mathcal{E}_s - \sum_{s \in S} P \mathcal{E}_{s_s}) \tag{28}$$

Since the above expression contains a part that has the second power and is a quadratic form in modeling, the above expression is converted into the following form formulated by Yu and Lee (2009):

$$\partial(x, y1, \ldots ys) = \sum_{s \in S} P_s \, \mathcal{E}_s + \lambda \sum_{s \in S} P_s \, |\mathcal{E}_s - \sum_{s \in S} P_s \, \mathcal{E}_s \tag{29}$$

However, this objective function is still non-linear, but it can be converted into a linear function by adding two non-negative deviation variables with the "Yu and Lee approach". In fact, instead of minimizing the reference of the absolute deviations from the average of the above two functions, the deviations are minimized according to the constraints in such a way that:

$$\min \sum_{s \in S} P_s \, \mathcal{E}_s + \lambda \sum_{s \in S} P_s \, [\left(\mathcal{E}_s - \sum_{s \in S} P_s \, \mathcal{E}_s \right) + 2\theta_s] \tag{30}$$

$$st{:}\, \mathcal{E}_s - \sum_{s \in S} P_s \, \mathcal{E}_s + \theta_s \geq 0 \tag{31}$$

$$\theta_s \geq 0 \tag{32}$$

As can be seen, n is considered as a scenario in the above model and P_s is the probability of the scenario occurring.

4. Method of solution

In this section, the used solution methods are provided. Since the problem model is presented in an uncertain and robust mode, in order to solve it on a small scale, the exact method is used in GAMS commercial optimization software. Then, meta-heuristic approaches of GWO and NSGA-II are used to solve the model with medium and large-scale numerical problems. Also, since the problem model is multi-objective, the epsilon constraint approach is used to solve it, so that it can be solved in GAMS software. Additionally, to solve it, the mentioned meta-heuristic approaches are used in the multi-objective optimization mode, in other words, the multi-objective GWO and the multi-objective NSGA-II are used, which will be explained later.

4.1. Multi-Objective Grey Wolf Optimizer (MOGWO)

The GWO algorithm was presented by Mirjalili and Lewis in 2014. Social leadership and hunting of grey wolves was the main inspiration for this algorithm. In order to mathematically model the social hierarchy of wolves when designing the GWO, the best solution is considered as the alpha wolf (α). As a result, the second and third best solutions are named beta (β) and delta (δ) wolves, respectively. The rest of the candidate solutions are assumed to be omega wolves (ω). In the GWO algorithm, hunting (optimization) is guided by α, β, and δ. ω wolves follow these three wolves in search of optimization. In addition to social leadership, the following equations are presented to simulate the encirclement behavior of grey wolves during hunting.

$$\vec{D} = \left|\vec{C}.\vec{X}_P(t) - \vec{X}(t)\right| \tag{33}$$

$$\vec{X}(t+1) = \vec{X}_P(t) - \vec{A}\cdot\vec{D} \tag{34}$$

where t represents the current iteration of A and C is the coefficient vector, X p is the prey position vector, and X represents the grey wolf position vector. Vectors A and C are calculated as follows:

$$\vec{A} = 2\vec{a}\cdot\vec{r_1} - \vec{a} \tag{35}$$

$$\vec{C} = 2\cdot\vec{r_2} \tag{36}$$

It decreases linearly from 2 to 0 during each iteration and r1, r2 are random vectors in [0,1]. Figure 1. The GWO algorithm uses simulation of social leadership and encirclement to find optimal solutions to optimization problems. The algorithm stores the three best solutions found so far and requires other search agents (including Omega) to update their positions about them. The following formulas are run continuously for each search agent during optimization in order to simulate hunting and finding possible regions of the search space.

$$\vec{D}_\alpha = \left|\vec{C}_1\cdot\vec{X}_\alpha - \vec{X}\right| \tag{37}$$

$$\vec{D}_\beta = \left|\vec{C}_2\cdot\vec{X}_\beta - \vec{X}\right| \tag{38}$$

$$\vec{D}_\delta = \left|\vec{C}_3\cdot\vec{X}_\delta - \vec{X}\right| \tag{39}$$

$$\vec{X}_1 = \vec{X}_\alpha - \vec{A}_1\cdot\left(\vec{D}_\alpha\right) \tag{40}$$

$$\vec{X}_2 = \vec{X}_\beta - \vec{A}_2\cdot\left(\vec{D}_\beta\right) \tag{41}$$

$$\vec{X}_3 = \vec{X}_\delta - \vec{A}_3\cdot\left(\vec{D}_\delta\right) \tag{42}$$

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \tag{43}$$

From the prey. Another component of GWO that is seeking exploration is element C. C generates random values in [0, 2], where a random weight is provided for the prey to emphasize the random or the prey effect to define the distance in the first equation. This helps GWO to exhibit random behavior during optimization, favoring exploration and avoiding local optimization. It is worth noting here that C, unlike A, does not decrease linearly. It was noted here that C does not decrease linearly unlike A. To emphasize exploration not only during the initial but also the final iterations, parameter C is intentionally required to provide random values. This component is very useful in case of local stagnation of the optimum, especially in the last iterations. The operation of the GWO algorithm starts when the random values of A are within [-1,1]. The next position of a search agent can be anywhere between the current position and the position of the prey, which helps the search agents move towards the estimated position of the prey provided by the alpha, beta, and delta solutions. The GWO algorithm starts optimization with generating a set of random solutions as the first population. During optimization, the three best obtained solutions so far are saved and considered as alpha, beta, and delta solutions. For every omega wolf (search agents except α, β, and δ), the position updating Formula (3.5) to (3.11) are triggered. Meanwhile, parameters C and A are linearly decreased over the course of iteration. Therefore, search agents tend to diverge from the prey when hunting and converge towards the prey. Finally, the position and score of the alpha solution is returned as the best solutions obtained throughout optimization when an end condition is satisfied. In order to perform multi-objective optimization by GWO, we integrate two new components. The employed components are very similar to those of MOPSO. The first one is an archive, which is responsible for storing non-dominated Pareto optimal solutions obtained so far. The second component is a leader selection strategy that assists to choose alpha, beta, and delta solutions as the leaders of the hunting process from the archive. The archive is a simple storage unit that can save or retrieve non-dominated Pareto optimal solutions. The key module of the archive is an archive controller, which controls the archive when a solution wants to enter the archive or when the archive is full. Note that there is a maximum number of members for the archive. During the course of iteration, non-dominated solutions obtained so far are compared against the archive residents. There would be three different possible cases. The new member is dominated by at least one of the archive residences. In this case the solution should not be allowed to enter the archive. The new solution dominates one or more solutions in the archive. In this case the dominated solution(s) in the archive should be omitted and the new solution will be able to enter the archive. If neither the new solution nor archive members dominate each other, the new solution should be added to the archive. If the archive is full, the grid mechanism should be first run to re-arrange the segmentation of the objective space and find the most crowded segment to omit one of its solutions. Then, the new solution should be inserted to the least crowded segment in order to improve the diversity of the final approximated Pareto optimal front. The probability of deleting a solution is increased proportional to the number of solutions in the hypercube (segment). For removing solutions if the archive was full, the most crowded segments are first selected, and a solution is omitted from one of them randomly in order to provide a space for the new solution. There is a special case where a solution is inserted outside the hypercubes. In this case, all the segments are extended in order to cover the new solutions. So, the segments of other solutions can be changed as well. The second component is the leader selection mechanism. In GWO, three of the best solutions obtained so far are used as alpha, beta, and delta wolves. These leaders guide the other search agents.

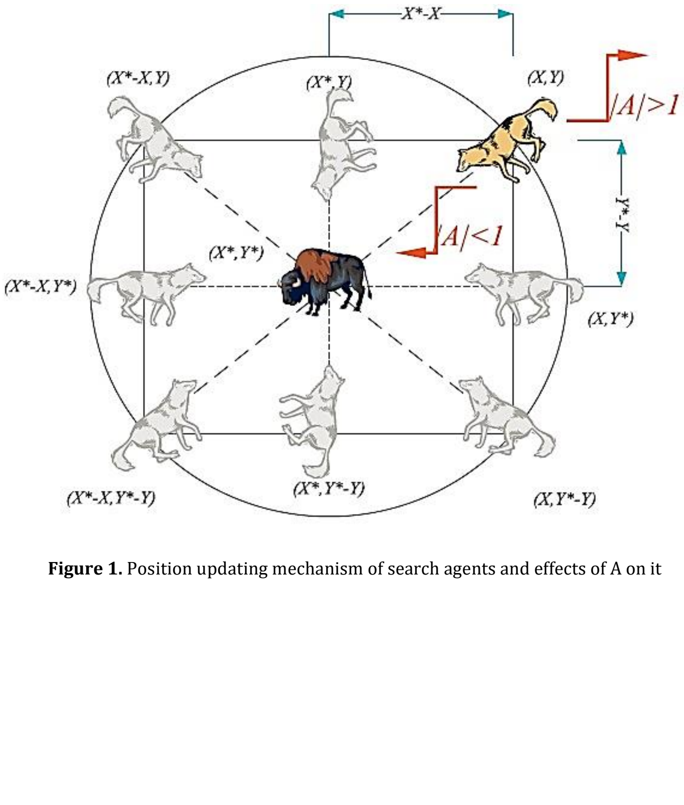

Figure 1. Position updating mechanism of search agents and effects of A on it

toward promising regions of the search space with the hope to find a solution close to the global optimum. In a multi-objective search space, however, the solutions cannot easily be compared due to the Pareto optimality concepts as discussed in the preceding subsection. The leader selection mechanism is designed to handle this issue. As mentioned above there is an archive of the best non-dominated solutions obtained so far. The leader selection component chooses the least crowded segments of the search space and offers one of its non-dominated solutions as alpha, beta, or delta wolves. The selection is done by a roulette-wheel method with the following probability for each hypercube:

$$P_i = \frac{c}{N_i} \quad (44)$$

It should be noted that there might be some special cases since we have to choose three leaders. If there are three solutions in the least crowded segment, three of them are randomly assigned to alpha, beta, and delta solutions. If there are less than three solutions in the least crowded hypercube, the second least crowded hypercube is also found to choose other leaders from. This scenario is the same if the second least crowded hypercube has one solution, so the delta leader should be chosen from the third least crowded hypercube.

Using our method, we prevent MOGWO from picking similar leaders for alpha, beta, or delta. Consequently, the search is always toward the unexplored/unexposed areas of the search space since the leader selection mechanism favors the least crowded hypercubes and offers leaders from different segments if there is not enough number of leaders (less than 3) in the least crowded segment. The computational complexity of MOGWO is of O(MN2) where N is the number of individuals in the population and M is the number of objectives. To see how the proposed MOGWO algorithm can be theoretically effective in solving multi-objective problems some remarks may be noted as follows:

- The external archive saves the best solutions without candidates obtained.
- Since MOGWO inherits the encircling mechanism of GWO, there is a circle-shaped neighborhood around the solutions which can be extended to higher dimensions as a hyper-sphere (in the parameter space).
- The random parameters A and C assist candidate solutions to have hyper-spheres with different random radii.
- Since MOGWO inherits the hunting mechanism of GWO, the search agents are allowed to locate the probable position of the prey.
- Exploration and exploitation are guaranteed by the adaptive values of a and A.
- The adaptive values of parameters a and A allow MOGWO to smoothly transition between exploration and exploitation. Therefore, the convergence of the MOGWO algorithm is guaranteed.
- With decreasing A, half of the iterations are devoted to exploration ($|A| \geq 1$) and the other half is dedicated to exploitation.
- The MOGWO has only two key parameters to be adjusted (a and C).
- The grid mechanism and selection leader component maintain the diversity of the archive during optimization.
- Employed roulette wheel in the leader selection component provides a low probability to choose leaders from most crowded hypercubes as well. This emphasizes local front avoidance of MOGWO.
- Non-adaptive random values for parameter C during optimization also boost exploration and the local front avoidance of the MOGWO algorithm simultaneously.

4.2. Metaheuristic Non-Dominated Sorting Genetic Algorithm II (NSGA-II)

Multi-objective evolutionary algorithms which use non-dominated sorting and sharing have been mainly criticized for three difficulties: (i) computational complexity, (ii) non-elitism approach, and (iii) the need for specifying a sharing parameter. In this section, we explain a non-dominated sorting based multi-objective evolutionary algorithm called NSGA-II which alleviates all the above three difficulties. Genetic algorithm is one of the heuristic algorithms that uses animal population modeling. In this algorithm, animal features are likened to the values of objective functions and improvement in generational features, and the emergence of new generations from the interbreeding of previous generations helps to improve objective functions. Non-dominated sorting genetic algorithm is one of the multi-objective modes of genetic algorithm, whose general procedure is as follows:

- Generating the initial population
- Calculating the fitness criteria
- Sorting the population based on the conditions to overcome
- Calculating the crowding distance
- Selection based on population rank and distance calculation: Populations in lower ranks are selected. Assuming that q and p are two members of the same rank, the member with a greater congestion distance is selected. Selection is prioritized first according to ranking and then according to crowding distance.
- Performing crossover and mutation to produce new offspring
- Combining the initial population and the population obtained from crossover and mutation
- Replacing the parent population with the best members of the population combined in previous steps. First, the members with lower rank replace the previous parents and finally they are sorted according to the crowding distance.
- All the above steps are repeated until the optimality is met.
- The general procedure of NSGA-II can be shown as below.

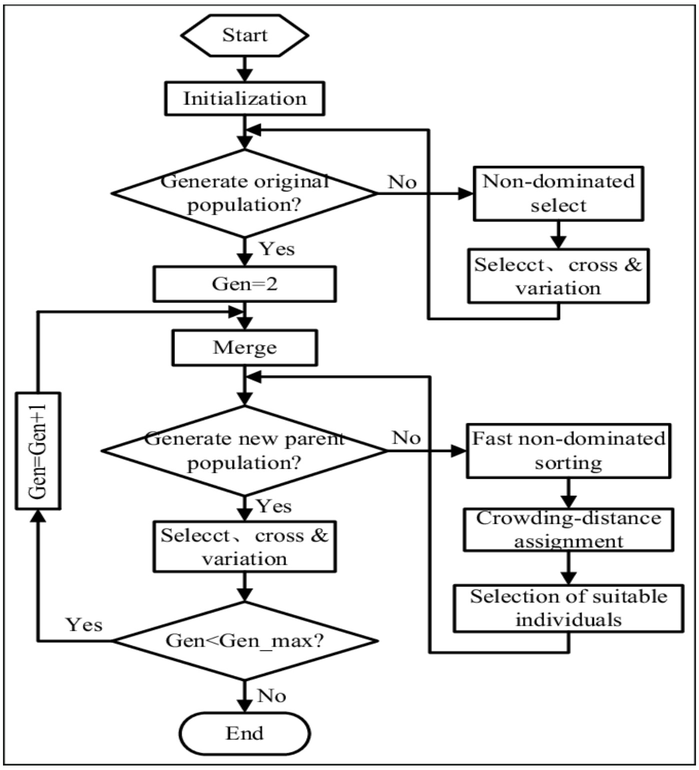

Figure 2. Display of the general approach of NSGA-II in flowchart

A Fast Non-Dominated Sorting Approach

In a simple approach, in order to sort a population of size N according to the level of non-domination, each solution must be compared with every other solution in the population to find if it is dominated. This requires O(mN) comparisons for each solution, where M is the number of objectives. When this process is continued to find the members of the first non-dominated class for all population members, the total complexity is $O(MN^2)$ At this stage, all individuals in the first non-dominated front are found. In the worst case, the task of finding of the second front also requires $O(MN^2)$ computations, especially when O(N) is the number of answers belonging to the second and highest non-dominated levels. Therefore, in the worst case, there are N number of fronts and only one answer in each front, which requires $O(MN^3)$ comparisons. First, for each solution we calculate two entities: 1. Dominance count (n_p), the number of solutions which dominate the solution p, and 2. S_p, a set of solutions which the solution p dominates. This requires $O(MN^2)$ comparisons. The dominance count for all solutions in the first non-dominated front is zero. For each solution P with $n_p = 0$, We monitor each member q in its set S_p and reduce its dominance count by one. In doing so, if for any member q the count becomes zero, we put it in set Q. These members belong to the second non-dominated front. Then continue this process using the newly identified front Has our current front. Then, the above procedure is continued with each member of set Q and the third front is identified. This continues until all fronts are identified.

Density estimation

To get an estimate of the density of solutions surrounding a particular point in the population, we take the average distance of the two points on either side of this point along each of the objectives. i serves as an estimate of the cuboid formed using nearest neighbors to the vertices (crowding distance). The crowding distance of the i-th solution in its front is the average side-length of the cuboid (Figure 3).

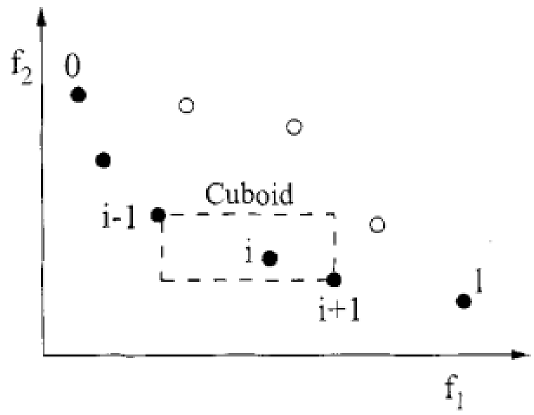

Figure 3. calculation of the crowding distance

Calculating crowding distance requires sorting the population according to each objective function value in an ascending order. Afterwards, for each objective function, the boundary solutions are assigned an infinite distance value. All other intermediate solutions are assigned a distance value equal to the absolute normalized difference in the function values of two adjacent solutions. The overall crowding-distance value is calculated as the sum of individual distance corresponding to each objective. The objective values are normalized before calculating the crowding distance.

Crowded Comparison Operator

The crowded comparison operator guides the selection process at the various stages of the algorithm towards a uniformly spread-out Pareto-optimal front. Let us assume that every individual i in the population has two attributes.

- Non-domination rank
- Local crowding distance

Here, we specify a relative order of the population comparison operator:

$$\begin{aligned} & i \prec_n j \quad \text{if } (i_{\text{rank}} < j_{\text{rank}}) \\ & \text{or } ((i_{\text{rank}} = j_{\text{rank}}) \\ & \text{and } (i_{\text{distance}} > j_{\text{distance}})) \end{aligned} \tag{45}$$

Between two solutions with differing non-domination ranks, we prefer the point with the lower rank. Otherwise, if both the points belong to the same front then we prefer the point which is located in a region with lesser number of points.

Main loop

Initially, a random parent population P_0 is created. The population is sorted based on the non-domination. Each solution is assigned a fitness equal to its non-domination level. Binary tournament selection, recombination, and mutation operators are used to create a child population Q_0 of size N. Since the elitism is compared by comparing the current population with the best non-dominated solutions found, the procedure of the algorithm is different after the first generation. First, a combined population $R_t = P_t \cup Q_t$ is formed. The population R_t will be of size 2N. The population R_t is sorted according to non-domination. Since all members of the current and former population are present in R_t, elitism is guaranteed. Solutions belonging to the best non-dominated set consisting of F_1 the best solutions in the combined population need to be reinforced more than any other answer. If the size of F_1 is less than N, we select all members of set F_1 for the new population P_{t+1}. The remaining members P_{t+1} are selected from consecutive non-dominated fronts. This procedure continues until no more sets can be accommodated. This procedure is shown in Figure 4.

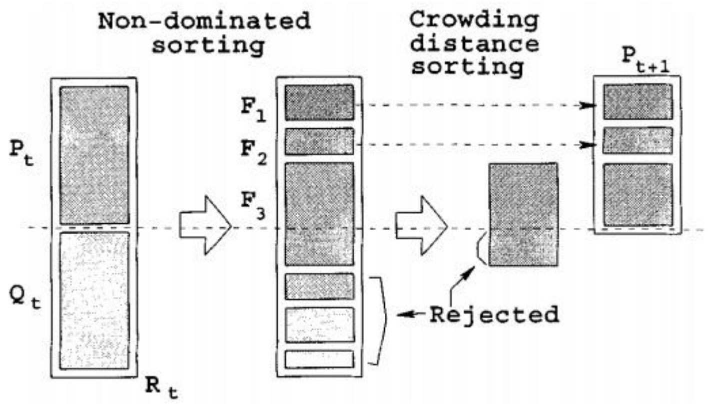

Figure 4. Procedure of non-dominated sorting genetic algorithm

4.3. Exact solution methods for multi-objective optimization

Multi-objective optimization is the process of simultaneously optimizing two or more conflicting objective functions, each of which has its own limitations. In multi-objective optimization, due to conflicting or incomparable objectives, in practice, we cannot reach a solution that simultaneously optimizes all functions and reaches optimal points for all functions. Therefore, it is necessary to use the concept of non-dominated solution or Pareto solution. In such a way that A dominates C if it is not worse than it in any objective function and is better than it in at least one of the functions. In solving multi-objective optimization problems, we seek non-dominated solutions. If we approximate the non-dominated solutions by a continuous curve, we have achieved the Pareto procedure or the Pareto surface or the Pareto set. The goal of multi-objective optimization problem solving methods is to obtain the Pareto set. Methods for solving multi-objective optimization problems can be divided into two categories: constructive methods and preference-based methods. The first category of methods seeks to find the Pareto set without any input or data from the decision maker. The second category of methods uses additional information provided by the decision maker as part of the solution process. Constructive methods can be divided into the following three categories. Non-preference methods do not require prior information about the problem and usually provide a Pareto optimal point, such as the absolute criterion method. Inductive method with scalarization method solves them by turning the problem into one or a set of problems with an objective function, such as the weighting method and the epsilon constraint method. The multi-objective inductive method solves them by ranking the solutions based on the value of the objective function, such as genetic algorithm and ant colony. Preference-based methods are also grouped into two categories: preferential and interactive methods. In the preferential methods, the taste of the decision maker is considered in the initial formulation of the single objective function problem, such as the lexicographic method and the goal programming method. Interactive method requires interaction with decision makers in the solution process, such as the NIMBUS method. Another type of multi-objective problem solving method is the weighted or weighted sum method, which tries to optimize the weighted sum of the objective functions by assigning a non-negative weight value to each objective function.

$$\begin{aligned} &\min\ w_1 f_1 + w_2 f_2 + \ldots + w_n f_n \\ &s.t. \quad g(\mathbf{x}) \le 0\,, \quad h(\mathbf{x}) = 0 \end{aligned} \tag{46}$$

By this method, the multi-objective problem becomes a single-objective problem. Another method used in multi-objective optimization is the normalized weighted sum method, which is the developed state of the weighted sum method, which puts objective functions in the interval [1, 0] by normalizing them.

$$\begin{aligned} &\text{Min}\ \frac{w_1(f_1 - z_1^U)}{z_1^N - z_1^U} + \cdots + \frac{w_n(f_n - z_n^U)}{z_n^N - z_n^U} \\ &\text{s.t.} \quad g(x) \le 0\ ,\ h(x) = 0 \end{aligned} \tag{47}$$

One of the popular methods in multi-objective optimization, which will be used in this research, is the epsilon constraint method, in which one of the objective functions is selected for optimization and other objective functions become constraints with an upper limit of ε. The primary idea of the epsilon-constraint method is that, first, one of the multiple objectives is selected as the main objective function of the optimization problem and the rest of the objective functions are transferred to the constraints of the problem while considering an upper and lower limit for them. Thus, by changing the right side of the constraints of these functions from their upper limit to their lower limit and repeating the problem solving, all possible Pareto solutions for the multi-objective problem are generated. The general form of the epsilon-constraint problem is as follows:

$$\begin{aligned} &Min\ Z_j(x) \\ &\text{s.t.} \\ &Z_k(x) + s_k = \varepsilon_k \quad \forall k \neq j \\ &x \in X, s_k \in R^+ \end{aligned} \tag{48}$$

where the value of Sk is the auxiliary variable related to the constraint of the k-th objective function. Another method is the hybrid method, which is actually a combination of the epsilon constraint method and normalized weighting.

$$\begin{aligned} &min \sum_{i=1}^{k} w_i(f_i(x) - z_i^u) \\ &\text{s.t} \quad f_j(x) \le \varepsilon_j \end{aligned} \tag{49}$$

5. Problem solving

5.1. Parameter adjustment

When working with meta-heuristic algorithms, the operation of parameter adjustment is highly important, because with improper parameter choices, the efficiency of the algorithm may decrease, and as a result, the obtained solutions are far from the optimal solutions. These parameters are performed through numerical experiments and there are different methods for designing numerical experiments. One of the simplest methods is to perform the experiments through complete factors, which can have problems because with a large number of factors, it becomes complicated to perform calculations and there will be a possibility of errors. Another method is to set the parameters through the method provided by Taguchi (1995), which includes a series of experiments with a fractional factor. Thus, by maintaining the information required for display, the number of experiments is greatly reduced. According to Taguchi's presentations, the factors affecting parameter setting are generally divided into two groups: controllable factors and uncontrollable factors. In this method, optimal levels of controllable factors and reducing the effects of uncontrollable factors are desired. In this method, we need to first measure the qualitative characteristics of the experiments, which are calculated as $\frac{S}{N}$. S represents the value of the signal and N represents the noise. This rate indicates the value of deviations in the solution variable, which in this research represents the objective function. In this way, each of the mentioned algorithms will be parameterized based on the objective value of the problem, and the parameter adjustment operation is performed twice. In the following, we will examine adjusting the parameters of each algorithm separately.

5.2. Parameter adjustment for the MOGWO algorithm

In the MOGWO algorithm, we will have five parameters ni, np, (w), C_1, C_2. We use a three-level Taguchi design for the operation of adjusting the said parameters, where three different values are considered for each parameter, as shown in the table below.

Table 1. Display of the level of parameters in the MOGWO algorithm according to Taguchi method

Parameter	ni	np	w	C1	C2
Three-level values	50, 75, 100	20, 30, 40	0.3, 0.6, 0.9	1,1.5,2	1,1.5,2

To perform numerical experiments in order to set the above parameters, we will carry out Taguchi tests from the Stat tab and the DOE section through the MiniTab software. In this way, we run each test five times in the software to reduce the effect of their randomness. Finally, the value obtained from the mean execution of experiments is equivalent to the value of the solution level in the algorithm. Figure 5, which follows this section, is the results of data analysis obtained from the Minitab software.

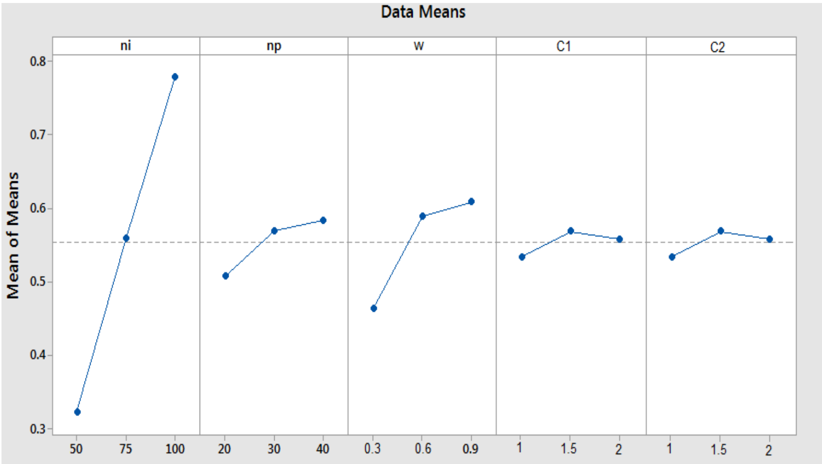

Figure 5. The parameter adjustment analysis diagram for MOGWO in Taguchi method

The best value for each parameter is listed in Table 2. These values are calculated and displayed for each index in the MOGWO algorithm.

Table 2. Adjusted values of MOGWO algorithm parameters

Algorithm	ni	np	w	C1	C2
GWO	100	40	0.9	1.5	1.5

5.3. Parameter adjustment for the NSGA-11

The NSGA-II has four parameters: the maximum number of iterations (mi), population number (np), mutation (pm), and crossover (pc) that need to be adjusted. We use a three-level Taguchi design for the adjustment of the said parameters. Therefore, three different values for each parameter are considered based on the literature and our expertise, which are shown in Table 3.

Table 3. Level of parameters for Taguchi design

Parameter	Values of each level		
	Level 1	Level 2	Level 3
Percentage of Crossover (Pc)	0.7	0.8	0.9
Percentage of Mutation (Pm)	0.05	0.1	0.15
Number of Solutions in the Population (N-pop)	50	100	150
Maximum iteration(Max-iteration)	100	200	300

Taguchi tests for each algorithm are determined using MiniTab software and tests are performed. Each test is run five times to eliminate the effect of randomness. Therefore, the mean objective value of five times of performing each test is considered as its solution level value. Figure 6, which follows this section, is the results of data analysis.

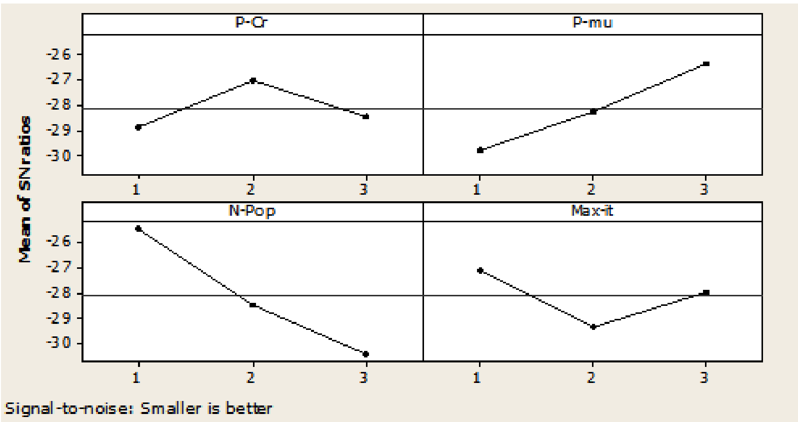

Figure 6. The parameter adjustment analysis diagram for NSGA-II in Taguchi method

The best value for each parameter is listed in Table 4. These values are calculated and displayed for each index in the genetic algorithm.

Table 4. Adjusted values of the NSGA-II parameters

Parameter	Optimal value
Percentage of Crossover (Pc)	0.7
Percentage of Mutation (Pm)	0.05
Number of Solutions in the Population (N-pop)	150
Max iteration	200

5.4. Comparison of the performance of algorithms

In this section, we compare the performance of NSGA-II and MOGWO. There are different methods to do so, we use the method of relative percentage increase in performance, which is defined as the following formula.

$$RPI_s = \frac{f_s - f_b}{f_b} \times 100 \tag{50}$$

In the above equation, f_s means the value of the objective function, which is calculated by the meta-heuristic-method s. s represents the used algorithm. f_b is equal to the most optimal objective value obtained from the algorithms. In the following, each numerical example is solved 10 times in order to ensure more certainty of the efficiency of the algorithms. Table 5 shows the RPI values obtained from the best results of 10 tests.

Table 5. RPI obtained from the most optimal solutions of 10 numerical tests of NSGA-II and MOGWO

Data set	NSGAII	MOGWO
1	5.65	0.6
2	6.85	3.69
3	4.87	3.3
4	6.48	5.48
5	9.14	0.26
6	0.49	0.32
7	7.19	0.98
8	1.66	1.56
9	7.24	11.48
10	5.69	3.65
Mean	5.526	3.132

In the continuation of this section, the results obtained from the best solutions of 10 numerical tests can be seen in Figure 5. The RPI indices for the two proposed algorithms are tested in MiniTab software based on the indices defined in the previous section, and interval graphs for the best value of the objective function with CI=0.95% are drawn and displayed. By looking at the diagram in Figure 3, it can be concluded that the MOGWO algorithm has a better performance than the other algorithm. It should be noted that because the reports in Table 6 and Figure 7 are obtained according to the best results of 10 times of running the algorithms, it is possible that they are not completely valid. Therefore, this conclusion cannot be drawn with certainty.

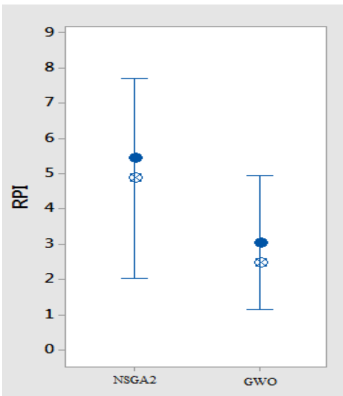

Figure 7. Interval graphs of MOGWO and NSGA-II for the best objective performance values

As with the numerical tests in Table 6, which were implemented according to the best results, in the following, each numerical test is implemented 10 times based on the mean values of the obtained results. Table 6 reports the RPI values obtained from the mean results obtained from 10 tests.

Table 6. RPI obtained from the mean results of 10 iterations of the numerical test of NSGA-II and MOGWO

Data set	NSGAII	MOGWO
1	3.26	9.82
2	5.62	2.1
3	4.16	6.26
4	10.66	6.56
5	6.5	0.69
6	9.8	3.13
7	6.3	0.68
8	9.6	2.98
9	3.72	7.56
10	3.69	5.91
Mean	6.331	4.569

The results obtained from the mean answers of 10 numerical tests are shown in Figure 4. The RPI indexes for two algorithms are analyzed in MiniTab software and the interval graphs are drawn for the mean values of the objective function with CI=0.95% index (while in Figure 8, the graphs were drawn according to the best values of the objective function). According to Figure 4, it can be concluded that the MOGWO algorithm behaves more optimally than the other algorithm. For this reason, it is concluded that it is better to use the MOGWO algorithm to solve the problem.

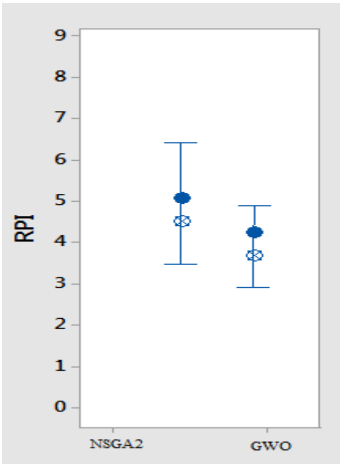

Figure 8. Interval graphs of NSGA-II and MOGWO for the mean values of objective performance

In the following, we compare the solution time of two algorithms. 10 numerical tests with different dimensions were designed and implemented through MOGWO and NSGA-II. Thus, the time to solve the numerical tests is recorded and displayed as follows. According to Figure 9, it is clear that the time to solve the MOGWO problem with the blue graph is less than the other algorithm.

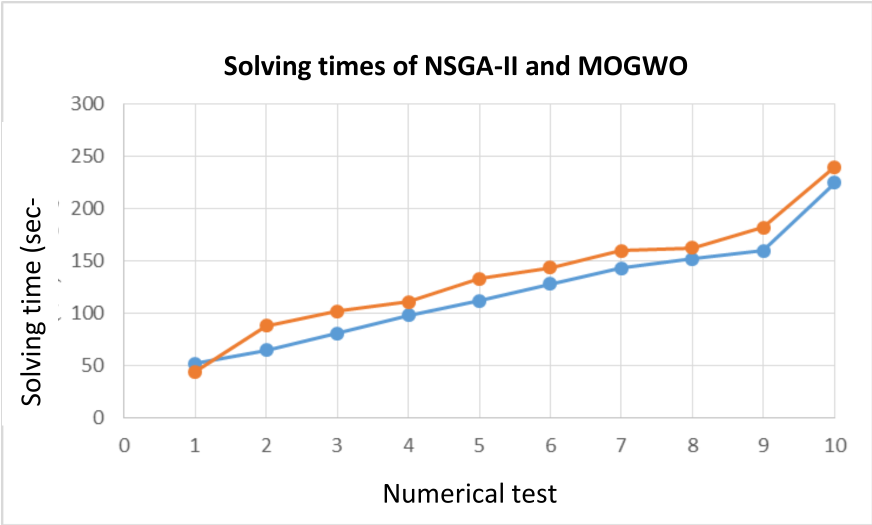

Figure 9. Comparison of the solution times of NSGA-II and MOGWO

6. Conclusion

Locating military equipment has special conditions, if these conditions are not taken into account, one can witness serious problems regarding the allocation of a proper location for military equipment warehouses. Regardless of security issues, fulfilling the demand is naturally considered a necessity. Therefore, one of the important objectives in hub location of military equipment is to meet the maximum demand according to the demands in the demand areas or routes. On the other hand, in such matters, as military products must be transported at maximum speed, choosing routes with the least congestion can be another necessity. In other words, both in terms of security and speed of transportation, it is important to choose routes with the least amount of traffic, so that vehicles are less aware of the existence of vehicles carrying military equipment and the equipment can be quickly transferred to the desired location or headquarters. In this research, using a mathematical model, it was tried to minimize the cost of transportation and determine the best location for war equipment storage in war zones according to the demand of the regions. The demand of the regions was considered dynamically and based on different situations. After designing the certain mathematical model and then the counterpart of the model in uncertain robust conditions, we first coded and executed the model in small dimensions in GAMS software to make sure of the validity of the model. Then, meta-heuristic algorithms were used in MATLAB software to analyze information and data in large dimensions. In order to solve the problem on a small scale, the exact epsilon constraint method was used in GAMS software. Also, meta-heuristic approaches of grey wolf optimizer (GWO) and non-dominated sorting genetic algorithm (NSGA-II) were used to solve the model in medium and large dimensions. Finally, proper performance indicators were used to compare the performance of the used algorithms, and as a result of solving several numerical examples and calculating their performance indicator, it was concluded that the MOGWO algorithm has a better performance in solving the model.

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