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Development of two mathematical models for age-based maintenance policies of production systems with different operational states

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Abstract

Maintenance management along with production planning and control are two major components of production systems and operation management. In this paper, a production system that has two operational states plus a failure state is considered. In this system, maintenance actions are carried out based on the age-based policy which means the maintenance is performed after passing a specified time from the age of the system or after a system failure whichever occurs first. According to the random variables of the system failure, i.e., transition among the states of the system, two approaches are proposed to model the age-based policy considering different scenarios and their respective probabilities that may occur in the age-based policy. Both models aim to optimize the expected cost of the system per time unit, while the optimal time to terminate the production cycle and conduct preventive maintenance is determined as the main decision variable of the models. Some numerical examples employing different statistical distributions for the system failure mechanism, e.g., Weibull, gamma, normal, are also provided.

Keywords: maintenance management, age-based maintenance, run-to-failure policy, mathematical models

Paper Type: Original Research

1. Introduction

Maintenance management along with production planning and control are two major components of a production system and operation management. Maintenance is defined as the combination of all technical and administrative actions intended to retain a system in the operational state or restore it to the operational state (Rasay et al,2019, R. Dekker,1996). Regular maintenance is essential to detect potential failures and keep equipment, machines and the work environment safe and reliable and ensures a reliable and smooth production flow throughout the system useful life. The corrective and preventive maintenances are among the most well-known maintenance strategies. Corrective maintenance, also known as run-to-failure or reactive maintenance, is a strategy that is done in order to repair or replace some equipment to its required function after it has failed. This strategy contributes to higher levels of maintenance costs and production loss due to unexpected or sudden failure. An alternative to the corrective maintenance strategy is the preventive maintenance (PM) strategy which is more effective and involves the performance of maintenance activities prior to the failure of the equipment. Preventive maintenance could be further classified into the age-based and condition-based preventive maintenance (CBM). The effectiveness of CBM, which is also known as the predictive maintenance has been widely studied by many researchers (S. Ding &S. Kamaruddin,2015). In comparison with the age-based maintenance, CBM is usually more effective in reducing the probability of catastrophic failures and may avoid unnecessary repairs (S. Ding &S. Kamaruddin,2015 & R. Ahmad & S. Kamaruddin,2012). Condition monitoring process is the heart of CBM, which is continuous monitoring of the signals using certain types of sensors or other appropriate indicators. Therefore, it is more applicable for systems with evident degradation paths and requires advanced condition monitoring techniques to obtain necessary degradation information of the system (J. Wang et al ,2020). On the other hand, for systems with non-measurable degradation information (e.g., those that are subjected to hard failure), age-based preventive maintenance is commonly adopted (J. Wang et al ,2020). Age based preventive maintenance policy is one of the widely studied policies which is used to avoid unnecessary replacement costs. According to a standard age-based policy, an asset/unit is replaced at a specified planned time as T or at a failure, whichever occurs first. In a classical/standard age-based maintenance policy, two main costs affect the optimal time to terminate the life cycle of an assent, i.e., *T*: the cost of corrective maintenance action, denoted by C_{CM}, and the cost of preventive maintenance action, denoted as C_{PM} (J. Wang et al ,2020). However, for a production system under the age-based policy, besides C_{CM} and C_{CM}, the optimal termination time of a production cycle is affected by the operational costs of the system in the different

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operational states (J. Wang et al ,2020). The main objective of a good age-based preventive maintenance program is to determine the best replacement time to optimize some major criteria such as maintenance costs, lifetime, and availability function. Therefore, it is important for managers to find an optimal policy for decision making about the maintenance of the system. In order to achieve this, an appropriate interval for scheduled maintenance must be determined which are usually prescribed based on the reliability theory and statistical analyses of the system failures. One way to do that is to use the mathematical models describing the age-based maintenance policy. A manufacturing system consisting of two operational states, i.e., an in-control state and an out-of-control state, and a failure state is considered in the paper. Transition among the states of the system is assumed to follow random variables with general probability density functions. Maintenance of the system is conducted according to the age-based policy. In this paper, according to the definitions of the variables characterizing the transition among the states of the system, two mathematical models are derived for the age-based maintenance policy. The models determine the optimal time to terminate the production cycle and conduct the preventive maintenance so that expected cost per time unit can be minimized. Some numerical examples and sensitivity analyses are then provided considering different probability density functions for the system failure mechanism including Weibull, gamma and normal distributions. The rest of the paper is organized as follows: in Section 2, a literature review about the subject of the paper is provided. Section 3 presents the problem of the study. In Section 4, two mathematical models are presented for the age-based policy of the system. In Section 5, the run-to-failure maintenance model is discussed. In Section 6, numerical examples and sensitivity analyses are conducted. Finally, Section 7 concludes the paper.

2. Literature review

Age-based maintenance is a well-established maintenance policy and developments of its mathematical models have been extensively studied. Considering the large body of literature exits in this subject, some authors have provided literature review papers about the mathematical models of age-based or essential concepts of this policy. One of the initial works providing a comprehensive literature review of different types of age-based policies for single-unit and multi-unit systems is H. Also, in (R. Ahmad & S. Kamaruddin,2012) an overview about the age-based and condition-based maintenance policies is provided. Reference (X. Zhano,2017) discusses the age-replacement models considering both criteria of maximizing availability and minimizing costs. Optimal age-replacement models are provided for different systems/conditions: parallel systems with random number of units, maintenance done in a discreet working cycle, the policy of "whichever occurs last" are among others. In reference (J. Wang,2019), using a semi-Markov decision process, an age-based opportunistic maintenance model is developed for a two-unit series system. Application of the model is illustrated for a two-unit wind turbine system and comparative studies with other maintenance polices are also conducted. In (M.sha & M. Finkelstein,2015), an optimal age-based group maintenance policy is provided for a complex multi-unit series system whose components are subject to different degradation phenomena. The authors show the application of the model for the bearings of wind turbine and the results are compared with the reactive maintenance policy. Development of an age-based preventive maintenance policy for a k-out-of-n system is discussed in (S. Eryilmaz,2020) while minimizing the expected cost per time unit is considered as the objective function. An integrated model of age-based maintenance and spare parts inventory control is developed for a two-unit series system in (J.Wang et al ,2021). A semi-Markov decision process model is proposed to optimally determine the parameters of the inventory system and the optimal time to conduct maintenance actions. An age-replacement policy is proposed in (F. Safaei et al,2020) for series and parallel systems with dependent components. Two optimal age-based replacement polices are developed for minimizing the cost and maximizing the availability of the system. In addition, applications of the models are discussed for the offshore wind turbine. Reference (K. T. Huynh,2018) develops two age-based maintenance polices with minimal repair for a single unit system which is under the effects of competing failures and shocks. The performance of the proposed models is compared with a classic age-based policy. While some studies have directly developed and discussed mathematical models of age-based repair/replacement polices, in some studies, this policy is employed as a benchmark to evaluate the effect of the other maintenance strategies. Reference (H.Rasay et al,2018) compares the performance of an age-based policy against condition-based maintenance policy for a production system deteriorating according to a general continuous distribution. It is concluded that the condition-based policy has a better performance in decreasing the operational costs of the system. In reference (F. Naderkhani ZG & V. Makis,2015), the age-based and condition-based maintenance polices are compared while a Bayesian control chart is employed to monitor the condition of the system. In many studies and models of the age-based maintenance policy, corrective and preventive maintenance costs, i.e., C_{CM} and C_{PM}, are two main factors affecting the optimal time to terminate the life cycle of the asset and implement the preventive maintenance. In other words, the termination of the life cycle is determined to make a trade-off between C_{PM} and C_{CM}. Maximizing the system availability or minimizing the costs of the system per time unit are two major criteria in the mathematical models of maintenance. However, in these models, it is usually ignored the fact that a system may have different operational costs for different operational states, e.g., the operational costs of a production system in the healthy state or in-control state are usually less than the corresponding value for the unhealthy state. This paper presents mathematical models for the age-based maintenance while taking into consideration the corrective/preventive maintenance costs and the differences of the operational costs of the system in the healthy and unhealthy states. To make the model more practical, degradation of the system and transition among the states of the system are considered as a general continuous random variable.

3. Problem statement

Consider a production system which has three states: an in-control state, an out-of-control state and a failure state. The in-control state, out-of-control state and the failure state are denoted by 0,1 and 2 respectively. Each production cycle starts at the zero-age time, while the system is in the in-control state. After passing a random time, the system shifts from state 0 to 1 or may directly shift from state 0 to state 2. In the case that the system enters state 1, if no maintenance action is conducted, the system eventually shifts to state 2. Once the system shifts to state 2, it is immediately observable as the system operation in this state stops. On the other hand, shift of the system to state 1 is not directly observable unless an inspection is performed. Operation of the system in State 1, in comparison with the operation in State 0, is undesirable. It is due to the increase of the operational costs in state 1, increase of the failure rate of the system or increase of the quality costs. Figure 1 illustrates the transition among the different states of the system. In developing the model, no restrictive assumptions are made about the random variables describing the transition among the states, except that they are continuous random variable. The following maintenance policy is applied for this system. Maximum duration of a production cycle is considered t_m. If the system reaches t_m, it is stopped and an age-based maintenance action denoted as PM is preventively conducted. The PM renews the system to the as-good-as-new condition. On the other hand, during a production cycle, once the system enters State 2, a corrective maintenance (CM) action is implemented. After that the system returns to the as-good-as new state and a new production cycle starts. In this paper, according to the definitions of the random variables describing the system deterioration, two models are developed for the age-based policy. The aim of the models is to optimally determine t_m so that the expected total cost of the system per time unit be minimized.

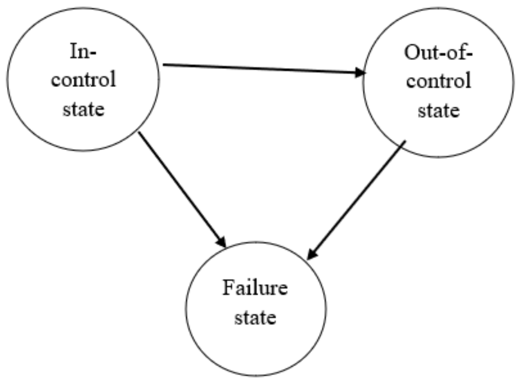


Figure 1. Transition among the states of the system

4. Development of the models of the Age-based Policy

Notations

ECT	Expected cost per time unit
$E[C]$	Expected cost during a production cycle
E[T]	Expected duration of a production cycle
C_0	Operational cost of the system per time unit while the system is in the in-control state
C_1	Operational cost of the system per time unit given that the system is in the out-of-control state ($C_1 > C_0$)
C_{CM}	Cost of performing a corrective maintenance action

C_{PM}	Cost of performing a preventive maintenance action
T_{CM}	Duration of time required for conducting a corrective maintenance action
T_{PM}	Duration of time required for conducting a preventive maintenance action
X_1	A random variable denoting the time from the start of a production cycle until transition to the out-of-control state
X_2	A random variable denoting the time from the beginning of the out-of-control state until transition to the failure state
X_3	A random variable denoting the time from the start of the production cycle until the time that the system directly shifts from the in-control state to the failure state
$f_i(x)$	Probability density functions (p.d.f) of $X_i, (i = 1,2,3)$ at x
$F_i(x)$	Cumulative distribution functions (c.d.f) of $X_i (i = 1,2,3)$ at x
$\bar{F}_i(x)$	Complement of c.d.f of variable X_i , i.e., $\bar{F}_i(x) = 1 - F_i(x), i = 1,2,3$ at x
t_m	Maximum duration of a production cycle (decision variable)

In the field of reliability and maintenance engineering, the variables X_1 , X_2 and X_3 describing the system deterioration are commonly known as the system failure mechanism (SFM). (H. Rasay et al,2023 & J. Taji et al,2023). In the following two subsections, according to the definition of the SFM, two models will be developed for the age-based policy which are explained in the following subsections.

4.1. The first model of the age-based policy

In this model, shifts among the states of the system, i.e., SFM, are described according to the variables X_1, X_2 and X_3 . Definitions of them are provided in the introduction of the notations. During a production cycle, evolution of the system may occur according to one of the following four scenarios. These scenarios are shown in Table 1 and elaborated as follows. Scenario 1: In this scnario, the system remains in the in-control state throughout the production cycle. It means that $X_1 > t_m$ and $X_3 > t_m$. Hence, at t_m, the system is stopped and the PM action is conducted. The probability of occurrence of this scenario can be computed as follows:

$$P(S_1) = \bar{F}_1(t_m)\bar{F}_3(t_m) \tag{1}$$

Scenario 2: In this scenario, at a random time as t , $(0 < X_1 = t < t_m)$, the system shifts from state 0 to state 1 and remains in this state until t_m. Hence, at t_m, the system is stopped and the PM action is conducted. The following equation is held for the occurrence probability of this scenario:

$$P(S_2) = \int_0^{t_m} f_1(t)\, \bar{F}_3(t)\bar{F}_2(t_m - t)dt \tag{2}$$

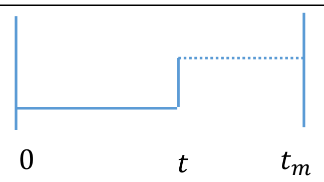
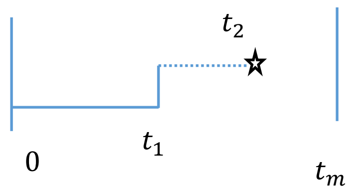
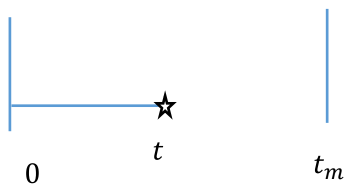
Scenario 3: In this scenario, at a random time as t_1, $(0 < X_1 = t_1 < t_m)$, the system shifts from state 0 to state 1, then after spending a random time at state 1, at time point t_2, $(t_1 < X_2 = t_2 - t_1 < t_m)$, the system enters the failure state and the production cycle terminates. To make clearer the derivation of the equation computing the probability of this scenario, first, consider the definition of random variable X_2 provided as follows: A random variable denoting the time from the beginning of the out-of-control state until transition to the failure state. It means that the system spends $t_2 - t_1$ time units in the out-of-control state (State 1) and then shifts to the failure state. Hence, the following equation holds in this scenario:

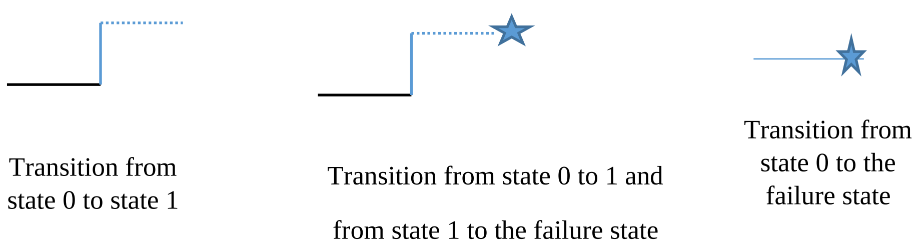
$$P(S_3) = \int_0^{t_m} \int_{t_1}^{t_m} f_1(t_1)\,\bar{F}_3(t_1) f_2(t_2 - t_1) dt_2 dt_1 \qquad (3)$$

Scenario 4: In this case, at a random time as t, $(0 < X_3 = t < t_m)$, the system directly shifts from state 0 to the failure state. Thus, the probability of occurrence of this scenario is as follows:

$$P(S_4) = \int_0^{t_m} f_3(t)\bar{F}_1(t)dt \qquad (4)$$

Table 1. Different scenarios that may occur in the age-based policy

Scenario	Figure	In-control time	Out-of-control time	Type of maintenance action conducted
1		t_m	0	Preventive maintenance
2		t	$t_m - t$	Preventive maintenance
3		t_1	$t_2 - t_1$	Corrective maintenance
4		t	0	Corrective maintenance



According to the renewal reward process, ECT for this policy can be computed as follows:

$$ECT = \frac{E[C]}{E[T]} \tag{5}$$

While E[C] and E[T] are the expected cost and expected duration of a production cycle, respectively, which can be computed as in the following:

$$E[T] = \sum_{i=1}^{4} P(S_i)E[T|S_i] = (t_m + T_{PM})\bar{F}_1(t_m)\bar{F}_3(t_m) + \tag{6}$$

$$(t_m + T_{PM})\int_0^{t_m} f_1(t)\,\bar{F}_3(t)\bar{F}_2(t_m - t)dt$$

$$+\int_0^{t_m}\int_{t_1}^{t_m}(t_2 + T_{CM})f_1(t_1)\,\bar{F}_3(t_1)f_2(t_2 - t_1)dt_2dt_1$$

$$+\int_0^{t_m}(t + T_{CM})f_3(t)\bar{F}_1(t)dt$$

In Equation 6, the first term computes the expected duration of a production cycle given the occurrence of Scenario 1. In the same manner, the second, third, and fourth terms compute the expected duration of a production cycle assuming the occurrence of the scenarios 2, 3 and 4 respectively. For the E[C], the following equations holds:

$$E[C] = \sum_{i=1}^{4} P(S_i)E[C|S_i] = \tag{7}$$

$$(C_0t_m + C_{PM})\bar{F}_1(t_m)\bar{F}_3(t_m)$$

$$+\int_0^{t_m}(tC_0 + (t_m - t)C_1 + C_{PM})f_1(t)\,\bar{F}_3(t)\bar{F}_2(t_m - t)dt$$

$$+\int_0^{t_m}\int_{t_1}^{t_m}(t_1C_0 + (t_2 - t_1)C_1 + C_{CM})f_1(t_1)\,\bar{F}_3(t_1)f_2(t_2 - t_1)dt_2dt_1$$

$$+\int_0^{t_m}(tC_0 + C_{CM})f_3(t)\bar{F}_1(t)dt$$

The first term of Equation 7 computes the expected cost of a production cycle given the occurrence of Scenario 1. Similarly, the second, third, and fourth terms of this equation compute the expected cost of a production cycle provided that Scenarios 2, 3 and 4 occur, respectively.

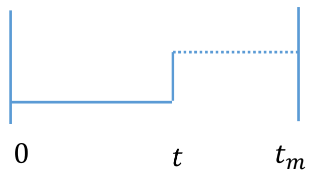
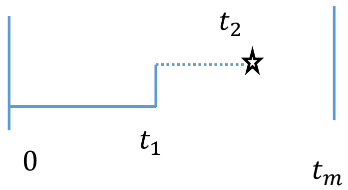
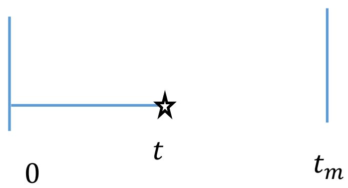
4.2. The second model of the age-based policy

In this model, variable X_2 is defined as follows:

X_2 A random variable denoting the time from the start of a production cycle until the time point that the system shifts from the out-of-control state to the failure state

The change of the definition of X_2 affects the formulation of the age-based maintenance model as elaborated in the following. Firstly, in this model, the probability of occurrence of each scenario changes. Table 2 shows the four scenarios along with their respective probabilities.

Table 2. Different scenarios that may occur in the age-based policy and their respective probabilities considering the second model

Scenario	Figure	Probability of occurrence
1		$\bar{F}_1(t_m)\bar{F}_3(t_m)$
2		$\int_0^{t_m} f_1(t)\frac{\bar{F}_3(t)\bar{F}_2(t_m)dt}{\bar{F}_2(t)}$
3		$\int_0^{t_m}\int_{t_1}^{t_m} f_1(t_1)\frac{\bar{F}_3(t_1)f_2(t_2)dt_2}{\bar{F}_2(t_1)}dt_1$
4		$\int_0^{t_m} f_3(t)\bar{F}_1(t)dt$

Expected duration of a production cycle, $E[T]$, and the expected cost of each production cycle, $E[C]$, are computed as in the following:

$$E[T] = \sum_{i=1}^{4} P(S_i)E[T|S_i] = (t_m + T_{PM})\,\bar{F}_1(t_m)\bar{F}_3(t_m) \tag{8}$$

$$+(t_m + T_{PM})\int_0^{t_m} f_1(t)\frac{\bar{F}_3(t)\bar{F}_2(t_m)dt}{\bar{F}_2(t)} + \int_0^{t_m}\int_{t_1}^{t_m}(t_2 + T_{CM})f_1(t_1)\frac{\bar{F}_3(t_1)f_2(t_2)dt_2}{\bar{F}_2(t_1)}dt_1$$

$$+\int_0^{t_m}(t + T_{CM})f_3(t)\bar{F}_1(t)dt$$

In Equation 8, the first term computes the expected duration of a production cycle given the occurrence of Scenario 1. In the same manner, the second, third and fourth terms compute the expected duration of a production cycle assuming the occurrence of scenario 2, 3 and 4 respectively. Also, E[C] is computed as follow.

$$E[C]=\sum_{i=1}^{4}P(S_i)E[C|S_i] \tag{9}$$

$$=(C_0t_m+C_{PM})\bar{F}_1(t_m)\bar{F}_3(t_m)$$

$$+\int_0^{t_m}(tC_0+(t_m-t)C_1+C_{PM})f_1(t)\frac{\bar{F}_3(t)\bar{F}_2(t_m)dt}{\bar{F}_2(t)}$$

$$+\int_0^{t_m}\int_{t_1}^{t_m}(t_1C_0+(t_2-t_1)C_1+C_{CM})f_1(t_1)\frac{\bar{F}_3(t_1)f_2(t_2)dt_2}{\bar{F}_2(t_1)}dt_1$$

$$+\int_0^{t_m}(tC_0+C_{CM})f_3(t)\bar{F}_1(t)dt$$

The first term of Equation 9 computes the expected cost of a production cycle given the occurrence of Scenario 1. Similarly, the second, third and fourth terms of this equation compute the expected cost of a production cycle provided that Scenario 2, 3 and 4 occur, respectively. Finally, the expected cost per time unit can be computed using Equation 5.

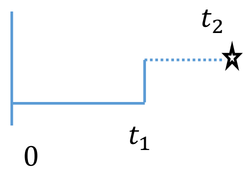
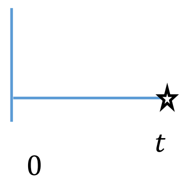
4.3. Discussion about the models

In the subsections 4.1. and 4.2, two models are developed for the age-based maintenance policy of the deteriorating three states system. Both models optimally determine the maximum duration of the production cycle (t_m) so that ECT can be minimized. As discussed before, no restrictive assumptions are made regarding the distributions of X_1, X_2 and X_3, thus the models can be applied for different statistical distributions, e.g., Weibull, gamma, normal or lognormal. For the both proposed models of the age-based policy, there is no difference between the definitions of variables X_1 (the time that the system shifts from state 0 to 1) and variable X_3 (the time that the system shifts directly from state 0 to the failure state). The factor that leads to the two different models is change in the definition of variable X_2 which specifies the time that the system shifts from the out-of-control state to the failure state. By comparing the definitions of X_2 for the both models, it can be inferred that variable X_2 in the second model is in fact sum of the variables X_1 and X_2 of the first model. Let denote the variable X_2 of the second model by X_2'. It means that $X_2'=X_1+X_2$. Considering the independence of $X_1$and X_2, for some special statistical distributions, distribution of variable X_2' can be easily specified without conducting mathematical computations. For example, if X_1 follows a normal distribution with mean and variance μ_1 and σ_1^2 , and X_2 follows is a normal distribution with mean and variance μ_2 and σ_2^2, it means that X_2' follows a normal distribution with $\mu_1+\mu_2$ and $\sigma_1^2+\sigma_2^2$. As another example, if X_1 follows a gamma distribution with shape parameter α_1 and scale parameter β and X_2 follows a gamma distribution with shape parameter α_2 and scale parameter β, then X_2' follows a gamma distribution with shape parameter $\alpha_1+\alpha_2$, and scale parameter β.

5. Corrective maintenance policy

In this policy, the system continues the production until it transits to the failure state, after that the CM action is conducted and the system renews. Under this policy, two scenarios may occur during a production cycle. These scenarios and their corresponding probabilities are shown in Table 3. In the first scenario, at time point $\boldsymbol{t_1}$, the system shifts from State 0 to State 1, and at time point $\boldsymbol{t_2}$ the system enters the failure state. For the second scenario, at time point $\boldsymbol{t}$, the system directly shifts from State 0 to the failure state. Instead of computing ECT employing the laws of probability theory, i.e., as it computed in the previous section, we use the fact that this policy can be considered as an extreme case of the age-based policy while $\boldsymbol{t_m}\to\infty$, and the value of $\boldsymbol{C_{PM}}$ and $\boldsymbol{T_{PM}}$ are equal to $\boldsymbol{C_{CM}}$ and $\boldsymbol{T_{CM}}$., respectively. Thus, the models of the previous section can also be employed for this corrective policy.

Table 3. Different scenarios of the runt-to-failure policy and their respective probabilities

Scenario	Figure	Probability of occurrence (variable X_2 is defined based on the definition provided in the second model of the age-based policy)	Probability of occurrence (variable X_2 is defined based on the definition provided in the first model of the age-based policy)
1		$\int_0^\infty \int_{t_1}^\infty f_1(t_1)\frac{\bar{F}_3(t_1)f_2(t_2)dt_2}{\bar{F}_2(t_1)}dt_1$	$\int_0^\infty \int_{t_1}^\infty f_1(t_1)\,\bar{F}_3(t_1)f_2(t_2 - t_1)dt_2dt_1$
2		$\int_0^\infty f_3(t)\bar{F}_1(t)dt$	$\int_0^\infty f_3(t)\bar{F}_1(t)dt$

6. Numerical Examples and sensitivity analyses

In this section, some numerical examples and sensitivity analyses are provided considering different probability density functions for the SFM including Weibull, gamma and normal distributions. The Weibull distribution has been broadly employed to describe the SFM ([19],[1],[20]). Hence, at the first step, we use Weibull distribution for our model analyses. More precisely, the following cumulative distribution function (c.d.f) is considered:

$$F(x) = 1 - \exp\{-(\lambda x)^{-v}\} \qquad (10)$$

In Eq. 10, λ and v are commonly known as the scale and shape parameters of the Weibull distribution. Thus, it is assumed that variable $X_i (i = 1,2,3)$ follows Weibull distribution with shape parameter v_i and scale parameter λ_i. The parameters of the example are shown in Table 4. It should be noted that the values of v_i and λ_i provided in Table 4 is based on the definition of X_2 in subsection 4.1. Figure 2 shows the changes of ECT and $t_m$versus the changes of the shape parameter of the distribution. Accordingly, as the shape parameter (v) increases from 1, the value of ECT decreases while the value of t_m increases. Figure 3 compares the value of ECT for the age based and run-to-failure policies. As the figure shows, the values of ECT for the run-to-failure policy are much larger than the ECT for the age-based policy. According to Figure 2, for $v = 2$, as an example, the optimal value of t_m is 22 which leads to $ECT = 11.85$. It means that maximum duration of each production cycle is 22. For the same value of v, if the maintenance is conducted according to the run-to-failure policy, the value of ECT is 22.83 which is much larger that the corresponding value of the age-based policy. It is noteworthy that analyses of Figures 2, 3, and 4 are conducted assuming $v_1 = v_2 = v_3$. Figure 4 shows the effect of the out-of-control operational costs, C_1, on the optimal values of ECT and t_m. As the value of C_1 is increasing from 35 to 100, the values of t_m are decreasing while the values of ECT are increasing. Decrease of t_m due to the increasing $\frac{C_1}{C_0}$ is intuitive to some extent because it means that the maximum duration of the production cycle decreases as the operational cost of the system in State 1 increases in comparison with the operational cost in State 0.

Table 4. The parameters of the example

C_0	C_1	C_{PM}	C_{CM}	T_{PM}	T_{MJR}	λ_1	λ_2	λ_3
10	50	50	100	2	2	0.01	0.02	0.005

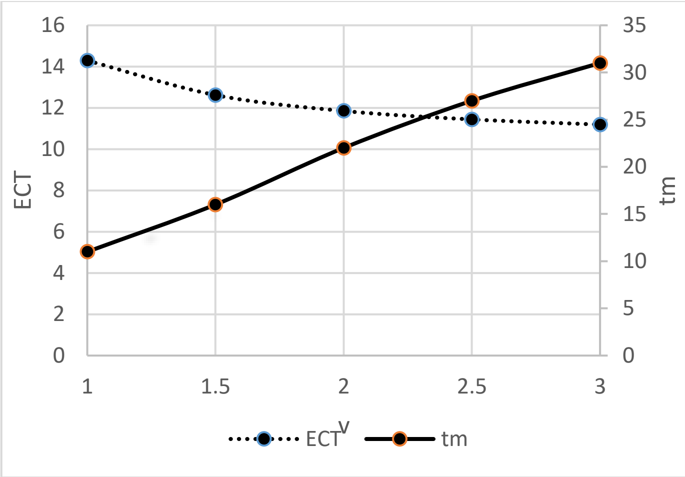


Figure 2. Changes of ECT and t_m versus the shape parameter of the Weibull distribution

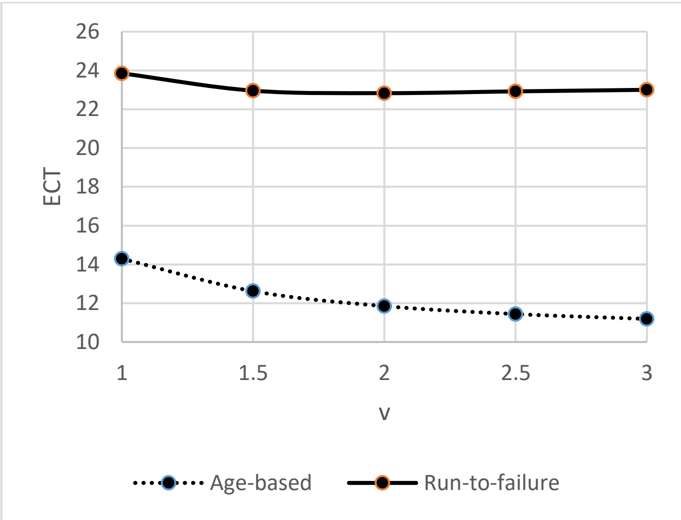


Figure 3. Comparison of ECT for the age-based and run-to-failure policy

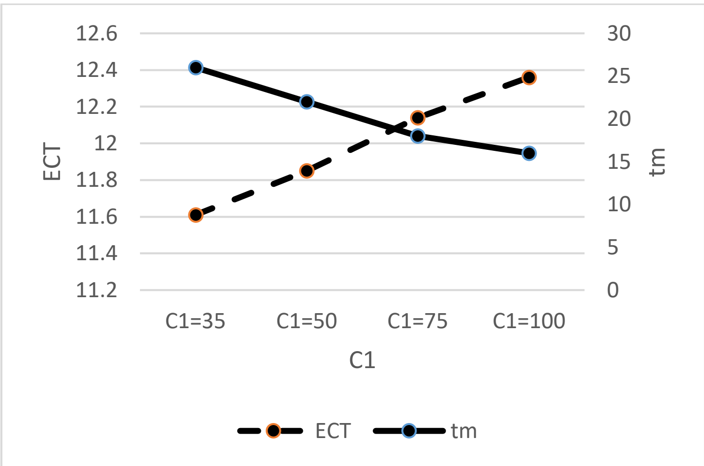


Figure 4. The effects of the out-of-control operational costs on the values of ECT and t_m

In the following, more examples are presented considering normal distribution for the SFM). More precisely, it is assumed that $X_i(i = 1,2,3)$ follows a normal distribution with mean μ_i and variance σ_i^2. Table 5 provides the results for different values of mean and variance of SFM. In case 2 compared to case 1, the mean of X_i $(i = 1,2,3)$ increases while the standard deviations do not change. This change increases t_m from 77 to 125 and decreases the value of

ECT. Decrease of ECT and increase of t_m due to the increase of μ_i are not far from expectation as an increase of mean of X_i means a system with less failure rate. On the other hand, from case 1 to case 3, μ_i does not change while σ_i increases. This change increases ECT and decreases t_m. This change is also intuitive to some extent as the increase of variance of X_i means that the failure of the system is less predictable which necessitates the decrease of t_m . As the figures of Table 5 show, for all cases the ECT of run-to-failure policy is significantly larger than the corresponding values of the age-based policy. It should be noted that analyses of Table 5 are based on the values of $C_0, C_1, C_{PM}, C_{CM}, T_{PM}$ and T_{CM} of Table 4. Also, the parameters of μ_i and σ_i are provided considering the definitions of X_2 for the first model. According to the discussion provided in subsection 4.3, the parameters of X_2 for the second model can be easily obtained.

Table 5. the results of the examples while SFM follows normal distribution

Case	Parameters of the normal distributions	Age based policy		Run to failure policy
		t_m	ECT	ECT
Case 1	$\mu_1 = 100, \sigma_1 = 10, \mu_2 = 50, \sigma_2 = 10, \mu_3 = 200, \sigma_3$=20	77	10.40	23.68
Case 2	$\mu_1 = 150, \sigma_1 = 10, \mu_2 = 100, \sigma_2 = 10, \mu_3 = 300, \sigma_3$=20	125	10.24	26.19
Case 3	$\mu_1 = 100, \sigma_1 = 20, \mu_2 = 50, \sigma_2 = 15, \mu_3 = 200, \sigma_3 = 30$	56	10.56	23.66

In the rest of this section, the results of the models optimization are provided while the SFM follows gamma distribution. More specifically, it is supposed that X_i $(i = 1,2,3)$ follows a gamma distribution with the following p.d.f:

$$f(x) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{-x/b} \tag{11}$$

In Equation 11, a and b are commonly referred as the shape and scale parameters of the gamma distribution. Hence, in the following, it is assumed that X_i follows a gamma distribution with shape parameter a_i and scale parameter b_i. Table 6 shows the results for different SFMs of gamma distribution. From case 1 to case 2, the values of a_i do not change while the values of b_i increase. Increase of b_i leads to the increase of the mean of the gamma distribution and consequently the mean life of the system. Therefore, as expected, increase of b_i leads to an increase of t_m from 29 to 38 and decrease of ECT from 11.71 to 11.31

Table 6. The results of the examples while the SFM follows gamma distribution

Case	Parameters of the gamma distributions	Age based policy		Run to failure policy
		t_m	ECT	ECT
Case 1	$a_1 = 2, b_1 = 100, a_2 = 2, b_2 = 50, a_3 = 1, b_3 = 200$	29	11.71	21.79
Case 2	$a_1 = 2, b_1 = 150, a_2 = 2, b_2 = 100, a_3 = 1, b_3 = 300$	38	11.31	24.12

7. Conclusion

According to a standard age-based maintenance policy, an asset/unit is replaced at a specified planned time as T or at a failure, whichever occurs first. In the classical/standard age-based models, two main costs affect the optimal time to terminate the life cycle of an assent: the cost of corrective maintenance action, and the cost of preventive maintenance action. However, a production system/process may have different operational costs in each operational state. This phenomenon is commonly overlooked in the development of the age-based maintenance models. In the proposed models in this study, besides the corrective and preventive maintenance costs, differences among the operational costs of the system in different operational states are also taken into consideration. The proposed models have widespread applications as in development of them no restrictive assumptions about the system failure mechanism (SFM) are placed. In this paper, according to the definitions of the random variables describing the SFM, two mathematical models are developed for the age-based maintenance policy. The models minimize the expected cost per time unit by optimally determining the maximum duration of each production cycle. It was discussed that the model of the run-to-failure maintenance policy can be considered as a special case of the models of the age-based maintenance. Numerical examples and comparative studies are conducted according to the different distributions for the SFM including Weibull, gamma and normal distributions. Also, considering the definitions of the random variables describing the SFM, one of the proposed models can be employed which better represent that condition. Using the idea of the current study for the multi-component series/parallel systems and investigating the impact of economic, physical and structural dependencies among the system units can be promising directions for the future developments of this study.

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