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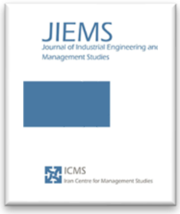

A game theoretic approach for novel pricing mechanism in a duopolistic supply chain considering price shock, market forecasting, customer behavior, and tax deduction policy

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Abstract

Today, due to the instability in the various markets, the modeling of price shock becomes the challenging task due to continuous price changes caused by numerous external factors. Therefore, in this paper, a novel pricing mechanism for manufacturers who produces substitution products in a duopoly is proposed considering price shock, market prediction and customer behavior. Consequently, a game theory approach based on the Cournot model is designed to determine the equilibrium decisions. To extract managerial insights, Nash and Stackelberg approaches investigated in two scenarios before and after occurring price shock, considering the behavior of manufacturers and consumers. Moreover, the government policies are investigated in these two scenarios. The obtained results from parametric analysis indicated that the market forecasting parameter plays a significant role in the profitability and production quantity of manufacturers in both scenarios. Besides, the tax deduction policy provides better conditions for the government only before occurring price shock.

Keywords: Game theory, Price shock, Pricing, Market Forecasting, Customer behavior, Government, duopolistic supply chain

Paper Type: Original Research

1. Introduction

Economic activities have always been affected by various external and unexpected factors such as a sudden increase in exchange rates, economic and political sanctions, global oil prices, various wars, and so on. In the economic literature, this external phenomenon that makes deviance in the macroeconomic variable, especially prices are known as a price shock. In other words, any deviation of prices from the long-term trend of their expected values is interpreted as a price shock. Hence, in today's volatile and uncertain world, one of the critical challenges regarding the advancement of manufacturers and economic analysts is the existence of price shock and its effects. Price shocks are the consequence of the unavoidable external and unexpected economic factors such as sudden increase of the exchange rates, economic and political sanctions, temporary increase in global oil price, various wars, etc., Which can cause a negative or positive effect on the economic variables such as supply levels, costs, and in the particular product price. Therefore, it is very important to pay attention to these random and generally unknown factors in economic analysis. Understanding and processing the price shock is crucial because persistent changes in prices can expose manufacturers and industrial consumers to make risky decisions and affect the quality of their decisions. As an example, the spot price of Brent crude oil decreases from $115.19 per barrel on June 19, 2014 to $45.13 per barrel on January 13, 2015, demonstrating more than 60% decline within seven months. This shock of crude oil prices caused the manufacturers and investors to change future strategies and decisions. As an example, the decrease of the crude oil price leads to the decline of the profits of petroleum enterprises, which forces some petroleum investors to adjust the benchmark achievement for desired and profitable investments (Gao *et al.*, (2017)). Thus, a better understanding of price shock and the market temporary volatility can help the researchers, market participants, and policymakers to make more proper and more profitable decisions. With the substantial importance of the price shock in economic activities, in recent decades, a great number of theoretical researches devoted to determine the effects of different shocks on the economic variables. In numerous amount of studies, the different statistical econometrics models such as Generalized Autoregressive Conditional Heteroscedasticity

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(GARCH), Vector autoregressive (VAR), and Threshold Autoregressive (TAR) are employed to identify and analyze the impact of price shock on various markets. Volkov and Yuhn (2016) investigated the impacts of the oil price shocks on the foreign exchange rate in the five oil-exporting countries. Aloui *et al*. (2013) provided a comprehensive analysis of the impact of the oil price shock on the foreign exchange rate by employing different kinds of Autoregressive Conditional Heteroscedasticity (ARCH) models. Basher et al. (2018) explored the impact of the oil price shock on the stock return rate by the VAR model. The literature of the modeling and analyzing the price shock regarding its impact on the economic activates are summarized in Table 1.

Table 1. The recent literature of the studies modelling price shocks

Ref.	Year	Model	The main contribution
Kilian and Zhou	2023	VAR	This paper develops a vector autoregressive model that quantifies the joint impact of shocks to several energy prices on headline and core CPI inflation.
Kröger et al.	2023	Linear Regression	This paper focuses on the distributional effect of the natural gas2 price shock on heating expenditures in Germany.
Zhang et al	2023	VAR	This paper explores whether the government's control of refined oil prices can slow down this impact and play a “shock absorber” function
Rahman	2022	Nonlinear bivariate structural SVR	The paper studies the relation between the crude oil price and U.S. stock returns
Sohag et.al	2022	Autoregressive Distributed Lags (DSARDL)	The current transition to a greener economy affects world markets dynamics

In today’s market, managers must continually respond to a rapid change occurred in various market segments to keep their competitive advantage (Lang et al. (2016)). Thus, for achieving this decisive goal, they can follow some principal approaches such as marketing (Day (2011)), innovation (Bharadwaj et al. (2015)), strategic management (D'Aveni et al. (2010)), and forecasting (Z. Hajirahimi, M.Khashei, 2019)). Forecasting and estimating the future manner of the market is widely-used method to deal with the uncertainty. One of the most important and challenging issues that manufacturer and economic analyst deal with, is to predict the future market and particularly the price shock. The ability of making effective forecasting about future market is important, especially for managers and analysts to develop appropriate strategies and implement plans, and also forecast probabilistic price shock. Market forecasting defines as some theoretical and numerical approaches that gathering information about current market and using them to forecast future market behavior to improve managerial decisions. Thus, due to the undeniable existence of the price shock, market forecasting and awareness of the future behavior of the market is inevitable task. Many recent studies, especially in economic, focused on market movements in terms of price shock. A substantial challenging problem is to forecast the direction of the market movement. In this way, appropriately analyzing and processing historical data would be an effective solution for forecasting future market direction and can help market analysts. Time series models in which historical observations of the variables such as price, demand, and so on are analyzed to develop a model describing the underlying relationship have been used widely for forecasting market direction. This modeling approach is particularly useful when little knowledge is available on the underlying data generating process or when there is no satisfactory explanatory model that relates the prediction variable to other explanatory variables (Zhang et al. (1998)). Different time series models are proposed to forecast unexpected price shocks. Consequently, manufacturers by analyzing historical data and time series models can forecast the future scheme of the market. Table (2) summarized the brief literature of forecasting price shock employing time series models.

Table 2. The brief literature of forecasting shocks employing time series models.

Ref.	Year	Time series forecasting model	The main contribution
Hirose and Kurozumi	2012	Bayesian model	Identifying news shocks in a dynamic stochastic general equilibrium model estimated with not only actual data but also forecast data.
Lee and Brorsen	2017	kalman filter	New stochastic time-series process is proposed to describe both permanent shocks related to structural breaks and temporary shocks. A permanent break is captured by a Poisson-jump or a Bernoulli-jump process, and a

			temporary shock is represented by a white noise process. the kalman filter model is used for forecasting.
Alessandsri and umtaz	2017	Regime-Switching models	Regime-Switching models employed to capture financial shocks. Such models could have sent a credible advance warning ahead of the great recession.

One of the initial issues manufactures faced is to determine the price of the productions or in other words, pricing in a competitive market environment (Khorshidvand et al. (2021)). The optimal pricing decision leads to obtain the maximum value of the profit for manufacturers. Thus, the growing literature has evolved through pricing problems. Due to the importance of the pricing for manufacturers to maximize their profit and make efficient decisions, several studies have been developed by proposing various pricing techniques in recent years. Soon (2011) and Chan et al. (2004) made a broad overview of pricing approaches and methodologies. Ai et al. (2012) proposed the pricing mechanism in a supply chain for substitution products by considering return policies and uncertain demand. Karakul and Chan (2008) presented a single-period pricing and procurement model concurrently for two substitution products. A literature review indicated that game theory approaches are known as an effective and widely-used tool to exploit strategic analysis and recognizing the best interactions among multiple players. Due to the competitive essence of different markets and the nature of the game theory concept, various game theory approaches are applied to develop and solve the pricing problems in a wide variety of studies. Esmaeli et al. (2016) evaluated different kinds of pricing models using game theory concepts for the closed-loop supply chain. Safarzadeh and Rasti-Barzoki (2019) proposed a pricing mechanism in the energy market considering the energy rebound effect, consumer behavior, and government policies. Khorshidvand et al. (2023) presented pricing strategies in a dual-channel green closed-loop supply chain considering incentivized recycling and circular economy. The results of this study highlighted that targeted advertising can stimulate demand for green products, and incentivizing circular economy practices through collaboration can yield positive environmental and economic outcomes. Recent papers, using game theory approaches to solve pricing problems are summarized in Table (3).

Table 3. The recent literature of the solving pricing problems using game theory approaches

Ref.	Year	Type of game theory model	The pricing problem
Mahdavisharif et al.	2023	Stackelberg	Pricing and inventory policy for non-instantaneous deteriorating items in vendor-managed inventory systems
Cai al.	2023	Stackelberg	How reverse information sharing supports pricing and sales effort decisions
Huag et.al	2022	Stackelberg	Dynamic pricing model of real-time energy management based on the source-load interaction optimization for hybrid energy.
Ling et al. [32]	2022	leader-follower Stackelberg game	Pricing decisions in a duopolistic market

It can be concluded that because of the direct impact of price shocks on the supply level, production quantity, and production price, considering this important factor in the pricing modeling procedure can lead to provide a more desired pricing mechanism and close it to the real-world system. Although several studies have already been conducted in the field of product pricing, no study in the literature considered the price shock in the pricing modeling by the use of the game theory concept. Therefore, given the existing gap in the literature on the non-investigation of the effects of price shocks on the pricing mechanism using the game theory approach, this paper proposed the game theory approach to assess and analyze the effect of price shock for two substitution products considering some important parameters such as market forecasting and customer behavior. Besides, the pricing mechanism will also be analyzed under two scenarios before and after the occurring price shock. Therefore, the following questions are answered in this research study:

1) What will be the equilibrium values of decision variables, including production quantity and price?
2) How will change the amount of profit and production quantity made by the manufacturer in both before and after occurring price shock scenarios?
3) How can it affect the market forecasting parameter before and after occurring price shock on the production quantity and profits made by the manufacturer?

4) How does consumer behavior affect the pricing mechanism before price shock?

5) How may change the government's profit by deducting tax policy in each scenario?

As mentioned before in the current research a new pricing mechanism by considering price shock, market forecasting, and customer behavior is proposed. In this paper, two scenarios are designed, namely before and after occurring price shock and analyzing the profit and production quantity of two manufacturers which are discriminated by the forecasting market characteristic. Consequently, two game approaches are proposed in this paper for solving the pricing problem considering price shock, market forecasting, and customer behavior that has not been discussed previously. Thus, in the first stage, the Nash game is proposed to determine equilibrium decisions. Then in the second phase, the Stackelberg game approach is applied to the defined problem for considering government policies to optimize its decision-making procedure. The rest of the paper is organized as follows: The problem statement is described in section 2. In section 3, the formulation of the proposed model is introduced. The sensitivity analysis is given in section 4 in detail. The discussion and summary of implications are provided in section 5. Section 6, contains the summary and concluding results.

2. The problem statement

Suppose that a market with two manufacturers produced two substitution products. In this study, two manufacturers that are differentiated with the important characteristic namely the forecasting market are considered. Thus, in the first stage with the importance of the market forecasting criteria, the impact of this crucial parameter on the profit and the production quantity of manufacturers is investigated in two scenarios, including before and after occurring price shock as well as other parameters such as customer expectation and intensity of price shock. Therefore, in this stage, there is Nash competition between the manufacturers. In the second phase, a government as a leader, to increase its profit using the tax scheme, is added to the problem structure. In this phase, a two-stage Stackelberg game approach is designed. The general framework of the proposed problem is illustrated in Figure1.

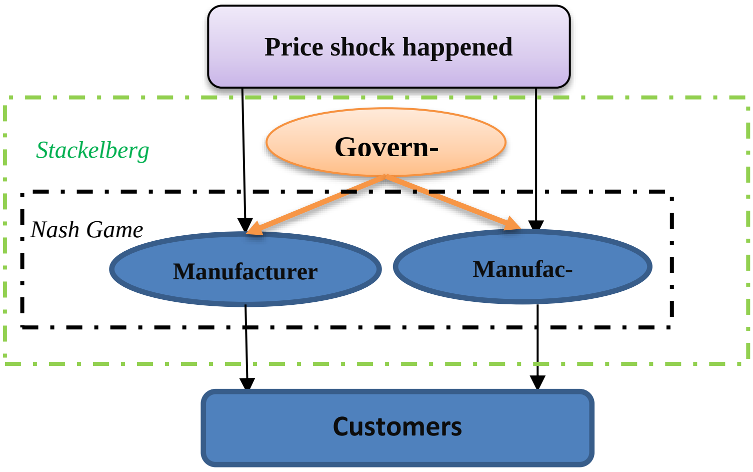

Figure1. The general framework of the proposed model

2.1. Assumptions

Now, in this subsection the required assumptions considered in this study are given as follows:

Assumption 1: The effect of price shock (μ) is positive.

Assumption 2: The expectation of the customers from a temporary increase in prices (r) can get values between zero and one.

Assumption 3: The prices of the manufacturer's product are considered as a linear function of production quantity.

3. Model Formulation

In this section, the proposed game model and its requirements are proposed, as well as two considered scenarios. So, at first, the notations of the mathematical model are given, as follows:

3.1. Notations

The required notations for proposed game model are described as follows:

Indices

i	Index of the manufacturers (i=1, 2)
j	Index of the type of game approach Stackelberg (j=S) or Nash (j=N)

Parameters

α	The products-market base
λ	Degree of substitutability between the substitute products from the two manufacturers
θ	The forecasting degree of the first manufacturer from the probabilistic price shock
μ	The intensity effect of the price shock
t_0	Time of occurring price shock
c	Production cost

Decision variable

r	The expectation of the customers from temporary increase in prices
t	Tax rate on each unit of product
q'_{ij}	The production quantity of product which is made by the manufacture' i applying game approach j in the second scenario, after occurring price shock
q'_{ij}	The production quantity of product which is made by the manufacture' i applying game approach j in the second scenario, after occurring price shock
p_{ij}	Price of the manufacture' i applying game approach j in the first scenario, before occurring price shock
p'_{ij}	Price of the manufacture' i applying game approach j in the second scenario, after occurring price shock
π_{ij}	Profit function of the manufacture' i applying game approach j in the first scenario, before occurring price shock
π'_{ij}	Profit function of the manufacture' i applying game approach j in the second scenario, after occurring price shock
π_g	Government's utility function in the first scenario, before occurring price shock
π'_g	Government's utility function in the second scenario after occurring price shock)

3.3 The players' profits without government intervention: modelling and solution

In this subsection, the players' utility functions are explained, which are formulated in the Nash equilibrium approach and also two designed scenarios. This study aims to research the impact of price shock on the profitability of manufactures considering market forecasting and customer behavior. Thus, two scenarios including before and after happening price shock are designed due to analyze the profit of manufactures differentiated by market forecasting feature. Moreover, we want to determine how may change the profit and price of manufacturers under the influence of price shock defined in second scenario who forecasts or not the future of market before happening price shock. In other words, these scenarios are designed to represent the reaction of manufacturers when faced price shock in the market considered or not market forecasting before price shock occurred. So, two manufacturer prices for the substitution products before happening price shock scenario are given in Eq. (1) and (2) respectively, as follows:

If $t < t_0$

$$p_{1N} = a - \frac{q_{1N}}{\theta} - \lambda\, q_{2N} + \frac{1}{(1-r)^2} \tag{1}$$

$$p_{2N} = a - q_{2N} - \lambda\, q_{1N} \tag{2}$$

Where, according to the market forecasting characteristic of the first manufacturer and based on its prediction rate, it can decrease its production quantity. Also, for the first manufacturer, if customers expect a temporary increase in future prices, it may cause an increase in first manufacturer production's price. Consequently, the customer expectation has an inverse relationship with the production price of the first manufacturer. Accordingly, the player's utility functions can be defined in Eq (3) and (4) for the manufacturer (1) and (2) respectively.

$$\pi_{1N} = \left(p_{1N} - \frac{c}{\theta}\right) q_{1N} \tag{3}$$

$$\pi_{2N} = (p_{2N} - c)\, q_{2N} \tag{4}$$

Where, according to Eq. (3), the first manufacture based on the market forecasting rate and future market estimation can reduce its production cost. In the second scenario, the main aim is to investigate the behavior of manufacturers when price shock has occurred who forecasts or not the future of the market before happening of this phenomenon. Since market forecasting is a highlighted feature of manufacturers that its effectiveness has appeared in an unusual situation like happening price shock, however, manufacturers should suitably analyze the market and also forecast the future scheme of the market before happening such unexpected situations. Accordingly, the second scenario is designed to represent the reaction of manufacturers when faced price shock analyzing the market or not before price shock occurred and evaluate the impact of market forecasting on the price, production quantity and, profit of manufacture. Thus, based on the market forecasting whether is considered or not by manufactures before happening price shock, the player's price functions and their utility functions in the second scenario are given in the following formulas:

If $t \geq t_0$

$$p'_{1N} = a - \frac{q'_{1N}}{(\mu-\theta)^2} - \lambda q'_{2N} \tag{5}$$

$$p'_{2N} = a - \frac{q'_{2N}}{\mu^2} - \lambda\, q'_{1N} \tag{6}$$

In this scenario and the presence of price shock, the first manufacturer based on the difference of the market forecasting rate and the actual effect of the price shock, have a decrease in its production quantity. However, the second manufacture which has no estimation of the future market has observed the whole effect of the price shock on its production quantity. Accordingly, the utility functions of the two manufacturers are given in Eq (7) and (8).

$$\pi'_{1N} = (p'_{1N} - c(\mu-\theta)^2)\, q'_{1N} \tag{7}$$

$$\pi'_{2N} = (p'_{2N} - c\,\mu^2)\, q'_{2N} \tag{8}$$

Similarly, the utility functions are also affected by the market forecasting rate. As it is addressed in Eq. (7), the production costs of the first manufacture increase based on the actual effect of the price shock and the estimated value of this manufacturer. Besides, the production cost of the second manufacturer enhanced based on the actual effect of the price shock. **Calculations of the Nash equilibria (1).** The Nash equilibrium decisions of the manufacturer under the first scenario are presented in Eqs. (9) -(12), as follows.

$$q_{1N} = \frac{2\left(-a - \frac{1}{(1-r)^2} + \frac{c}{\theta}\right) + (a-c)\,\lambda}{-\frac{4}{\theta} + \lambda^2} \tag{9}$$

$$q_{2N} = \frac{2a - 2c - 4ar + 4cr + 2ar^2 - 2cr^2 + c\lambda - 2cr\lambda + cr^2\lambda - \theta\lambda - a\theta\lambda + 2ar\theta\lambda - ar^2\theta\lambda}{(-1+r)^2(4-\theta\lambda^2)} \tag{10}$$

$$p_{1N} = a + \frac{1}{(1-r)^2} + \frac{-2\left(a + \frac{1}{(1-r)^2} - \frac{c}{\theta}\right) + (a-c)\lambda}{\theta\left(\frac{4}{\theta} - \lambda^2\right)} + \frac{\lambda(2a - 2c - 4ar + 4cr + 2ar^2 - 2cr^2 + c\lambda - 2cr\lambda + cr^2\lambda - \theta\lambda - a\theta\lambda + 2ar\theta\lambda - ar^2\theta\lambda)}{(-1+r)^2(-4+\theta\lambda^2)} \tag{11}$$

$$p_{2N} = a + \frac{\lambda(-2(a + \frac{1}{(1-r)^2} - \frac{c}{\theta}) + (a-c)\lambda)}{\frac{4}{\theta} - \lambda^2} + \frac{2a - 2c - 4ar + 4cr + 2ar^2 - 2cr^2 + c\lambda - 2cr\lambda + cr^2\lambda - \theta\lambda - a\theta\lambda + 2ar\theta\lambda - ar^2\theta\lambda}{(-1+r)^2(-4+\theta\lambda^2)} \tag{12}$$

Calculations of the Nash equilibria (2). The Nash equilibrium values for the production quantity of two manufacturers under the second scenario are presented in Eq. (13) and (14).

$$q'_{1N} = \frac{\lambda(a - c\mu^2) - \frac{2(a - c(-\theta+\mu)^2)}{\mu^2}}{\lambda^2 - \frac{4}{\mu^2(-\theta+\mu)^2}} \tag{13}$$

$$q'_{2N} = \frac{2a\mu^2 - a\theta^2\lambda\mu^2 + c\theta^4\lambda\mu^2 + 2a\theta\lambda\mu^3 - 4c\theta^3\lambda\mu^3 - 2c\mu^4 - a\lambda\mu^4 + 6c\theta^2\lambda\mu^4 - 4c\theta\lambda\mu^5 + c\lambda\mu^6}{4 - \theta^{\,2}\lambda^2\mu^2 + 2\theta\lambda^2\mu^3 - \lambda^2\mu^4} \tag{14}$$

The equilibrium values of the price for two manufacturers obtained by Nash equilibrium are given in Eqs. (15) and (16).

$$p'_{1N} = a + \frac{\lambda(2a\mu^2 - a\theta^2\lambda\mu^2 - c\theta^4\lambda\mu^2 + 2a\theta\lambda\mu^3 - 4c\theta^3\lambda\mu^3 - 2c\mu^4 - a\lambda\mu^4 + 6c\theta^2\lambda\mu^4 - 4c\theta\lambda\mu^5 + c\lambda\mu^6)}{\theta^2\mu^2\lambda^2 - 4 - 2\theta\lambda^2\mu^3 + \lambda^2\mu^4} + \frac{\lambda(a - c\mu^2) - \frac{2(a - c(\mu-\theta)^2}{\mu^2}}{(\mu-\theta)^2(\frac{4}{\mu^2(\mu-\theta)^2} - \lambda^2)} \tag{15}$$

$$p'_{2N} = a + \frac{2a\mu^2 - a\theta^2\lambda\mu^2 - c\theta^4\lambda\mu^2 + 2a\theta\lambda\mu^3 - 4c\theta^3\lambda\mu^3 - 2c\mu^4 - a\lambda\mu^4 + 6c\theta^2\lambda\mu^4 - 4c\theta\lambda\mu^5 + c\lambda\mu^6)}{\mu^2(\theta^2\mu^2\lambda^2 - 4 - 2\theta\lambda^2\mu^3 + \lambda^2\mu^4)} + \frac{\lambda(\lambda(a - c\mu^2) - \frac{2(a - c(\mu-\theta)^2}{\mu^2})}{(\frac{4}{\mu^2(\mu-\theta)^2} - \lambda^2)} \tag{16}$$

All the proofs are presented in Appendix A.

3.4 The players' profits with government intervention: modelling and solution

In this subsection, the defined problem is considered by the intervention of the government in a Stackelberg game. A Stackelberg refereed to the two-stage game by defining leader and followers. In pricing models, the government and members of the supply chain considered as a leader and followers respectively. In this game-theoretic approach, the leaders and followers devoted to maximize their profits. The Stackelberg model has been widely adopted solution for solving pricing problems in a supply chain that leads to determine the effective policies can be developed by governmnet. Accordingly, in this modeling phase of the defined problem, for evaluating the effectivness of the tax deduction policy applied by the government as a leader, a Stackelberg game model is proposed. In Stackelberg game approach, a tax deduction system for the government with the same structure and players (two manufacturers) with the first Nash game procedure is proposed. Thus, the utility functions for players and government in both scenarios are given as follows:

$If\, t < t_0$

$$\pi_{1S} = \left(p_{1S} - \frac{c}{\theta} - t\right) q_{1S} \tag{17}$$

$$\pi_{2S} = (p_{2S} - c - t) q_{2S} \tag{18}$$

$$\pi_g = t\,(q_{1S} + q_{2S}) \tag{19}$$

Similar to the previous scenario, the players' profit functions and also government under the second scenario are addressed in the following equations:

$If\, t \geq t_0$

$$\pi'_{1S} = \left(p'_{1S} - (c\ (\mu - \theta)^2) - t\right) q'_{1S} \tag{20}$$

$$\pi'_{2S} = \left(p'_{2S} - \left((c\,\mu^2) - t\right)\right) q'_{2S} \tag{21}$$

$$\pi'_g = t\,(q'_{1S} + q'_{2S}) \tag{22}$$

In addition, in the first scenario, the general mathematical model is presented in Eq. (23).

$$\begin{cases} (\underset{t}{max}\, \pi_g \rightarrow [\underset{q_{1N}}{max}\, \pi_{1S} \leftrightarrow \underset{q_{2N}}{max}\, \pi_{2S}\ ]) \\ \pi_{g,}\ \pi_{1S}\ , \pi_{2S} > 0 \ and \quad t\ , q_{1N}\ ,\ q_{2N} > 0 \end{cases} \tag{23}$$

Similar to the first scenario, the equilibrium decisions of two manufacturers under the second scenario and considering government intervention are obtained by the same formula.

Calculations of the Nash equilibria (3). The equilibrium decision of the government under the first scenario is presented respectively in Eq. (24) as follows:

$$t = \frac{\frac{a\lambda - 2a - c\lambda}{4-\lambda^2} - \frac{2}{(1-r)^2(4-\lambda^2)} + \frac{2\,c}{\theta(4-\lambda^2)} + \frac{2a - 2c - 4ar + 4cr + 2ar^2 - 2cr^2 - \lambda - a\lambda + 2ar\lambda - ar^2\lambda}{(-1+r)^2(-4+\lambda^2)} + \frac{c\lambda + 2cr\lambda + cr^2\lambda}{(-1+r)^2\theta(-4+\lambda^2)}}{\frac{-4}{4-\lambda^2} + \frac{2 - 4r + 2r^2 - \lambda + 2r\lambda - r^2\lambda}{(-1+r)^2(-4+\lambda^2)} + \frac{2\theta - 4r\theta + 2r^2\theta - \theta\lambda + 2r\theta\lambda - r^2\theta\lambda}{(-1+r)^2\theta(-4+\lambda^2)}} \tag{24}$$

Calculations of the Nash equilibria (4). The equilibrium decisions of both manufacturers under the first scenario is described in Eq. (25) and (26).

$$q_{1S} = \frac{2\left(a + \frac{1}{(1-r)^2} - t - \frac{c}{\theta}\right) - (a - c - t)\lambda}{4 - \lambda^2} \tag{25}$$

$$q_{2S} = \frac{\begin{array}{c}2c\theta-2a\theta+4ar\theta-4cr\theta-2ar^2\theta+2cr^2\theta+\\ 2t\theta-4rt\theta+2r^2t\theta-c\lambda+2cr\lambda-cr^2\lambda+\theta\lambda+a\theta\lambda-2ar\theta\lambda+ar^2\theta\lambda\\ -t\theta\lambda+2rt\theta\lambda-r^2t\theta\lambda\end{array}}{(-1+r)^2\theta(-4+\lambda^2)} \quad (26)$$

The equilibrium values of the decision variables for two manufacturers are obtained by replacing the Eq. (24) in Eq. (25) and (26). In a similar fashion, the equilibrium values of the decision variables considering government intervention can be yielded in the second scenario.

4. Sensitivity analysis and managerial implications

In this section, according to the proposed game model, a comprehensive numerical analysis of the main parameters is addressed for both scenarios in the two following subsections. Firstly, the individual effect of the r and θ parameters is investigated on the price and production quantity, and in the next step, the simultaneous effects of these two parameters on the profit of the manufacturers in both scenarios are evaluated and analyzed. Besides, in Stackelberg game approach the effect of μ and θ parameters on the government's profit are investigated simultaneously in both scenarios. Consequently, by these comprehensive analyses, the corresponding managerial insights can be extracted. Then, the discussion about sensitivity analysis of the main parameters is described. Accordingly, three crucial managerial insights that can help the manufactures and government to raise their performance in two scenarios are proposed.

4.1. Sensitivity analysis of the first scenario

Table (4) shows the default parameter values which are considered for both scenarios applying the Nash game approach in this study. Therefore, Table (5) shows the equilibrium values which are obtained by replacing the default parameter values in Eq. (9) and (10) as well as the profit functions. In this scenario, the effects of important parameters such as the manufacturer's market forecasting, the expectation of customers of price increases, the degree impact of the price shock on the profit and production quantity as well as the of manufactures' profits are investigated.

Corollary 1: According to Fig. (2), the first manufacturer which made the forecasting of the future behavior of the market, can increase its production quantity with the increasing value of the market forecasting parameter. In other words, when the ability of the market forecasting of the first manufacturer from the price shock increases, it will try to produce more. Also, with the increasing parameter θ, the manufacturer's production quantity which does not have an estimate of the future market situation decreases.

Corollary 2: The sensitivity analysis result regarding the parameter r is shown in Fig. (3). The results indicated that if the consumer's expectation from a temporary increase in the first manufacture's price becomes higher, the price of the first manufacture increase as well as the production quantity.

Table 4. The default value of the parameters

Parameter	Value
a	100
λ	1
c	20
r	0.6
θ	1.6
μ	2

Thus, the first manufacturer can yield a higher profit. However, the second manufacture encounters a significant decline in its price as well as profit.

Table 5. The equilibrium decisions of the players under numerical study

Variable	Value
q_{1N}	71.66
q_{2N}	4.16
π_{1N}	3210.07
π_{1N}	17.36

In contrast, the amount of production quantity produced by the second manufacturer decreases in the Nash equilibrium.

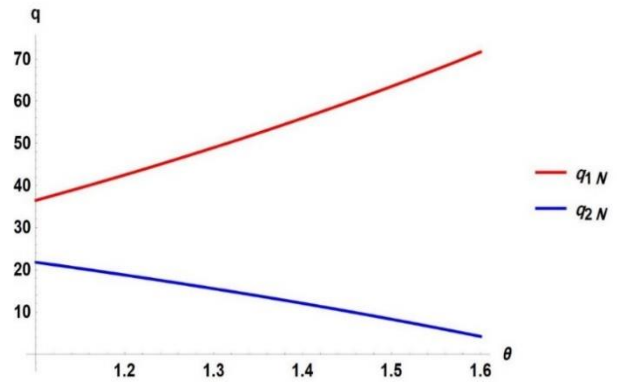

Figure2. Effect of the parameter θ on the production quantity in the first scenario

Corollary 3: The concurrent effect of two parameters θ and r on the profit of the manufacturer (1) and (2) are shown in Fig. (4) and (5), respectively.

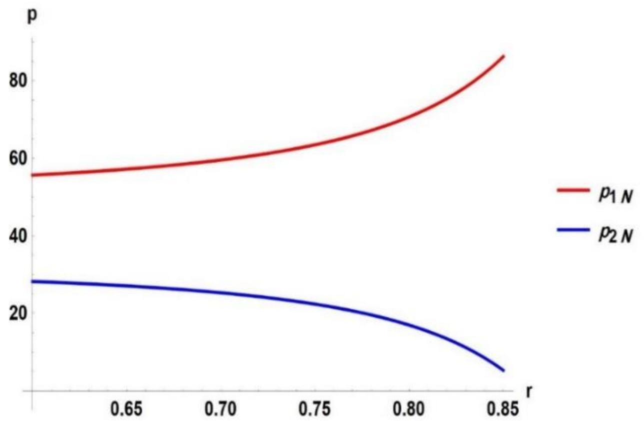

Figure3. Effect of the parameter r on the price of the manufacturers in the first scenario

It is confirmed that with the increase of the two parameters, the profit of the first manufacturer increases and the second manufacture faced decrease in its profit.

Managerial insight 1: According to the obtained results of the sensitivity analysis in this scenario, it can be induced that market forecasting and proper estimation of future market behavior and especially price shock may play a significant role in the amount of production quantity and profits of manufactures before occurring price shock.

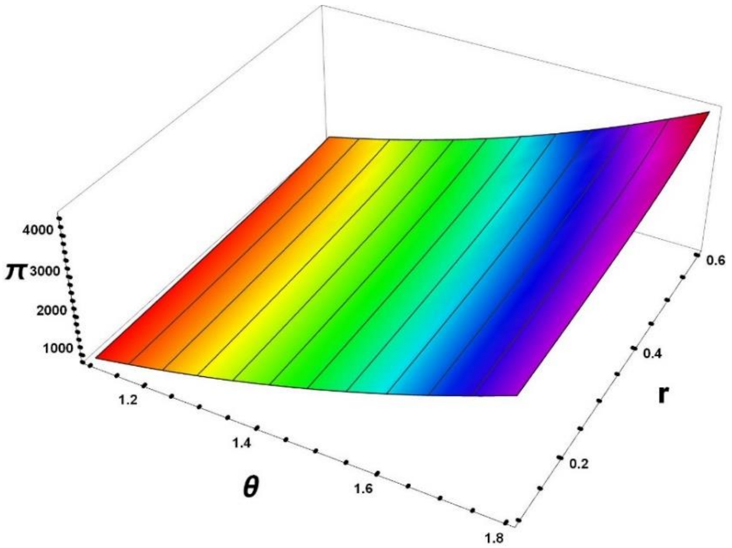

Figure4. Effect of r and θ parameters on the profit of the first manufacturer simultaneously in the first scenario

Whatever the manufacture forecasts and analyzes the probable price shock effect, it can produce more and devote to develop a reduction in production costs. In other words, before happening price shock, as a manufacturer makes continuous market analysis, it can be able to estimate the future price shock more accurately and consequently its profit and production quantity will increase. Also, if consumers expect a temporary increase in the manufacturer's price, this may lead to an increase in the price, production quantity, and profitability of the manufacturer.

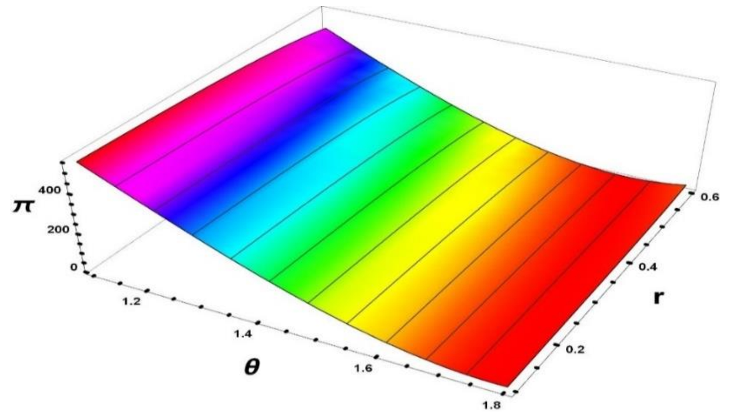

Figure5. Effect of r and θ parameters on the profit of the second manufacturer simultaneously in the first scenario

4.2. sensitivity analysis of the second scenario

Similarly, in this scenario, the following numerical example is given in Table (4) is considered, and the Nash equilibrium decisions of the players under numerical study are presented in Table (6).

Corollary 4: As shown in Fig. (6), with increasing intensity of the price shock the parameter, the amount of the production quantity of both manufactures decreases, which is more significant for the second manufacturer. In fact, because of this issue that the second manufacturer has no estimate of this phenomenon, the effect of a price shock on its amount of production quantity is greater.

Table 6. The equilibrium decisions of the players under numerical study

Variable	Value
${q'}_{1N}$	5.40
${q'}_{2N}$	29.18
${\pi'}_{1N}$	182
${\pi'}_{2N}$	212

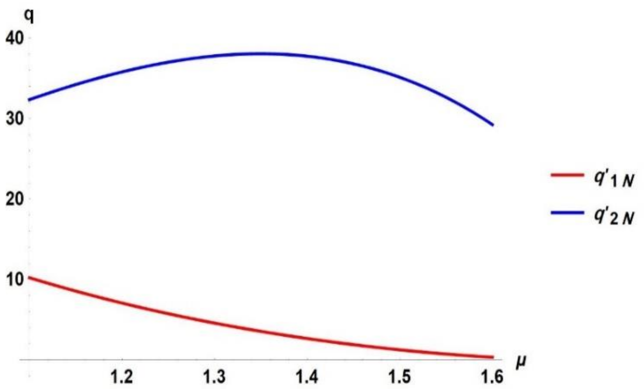

Figure6. Effect of the parameter μ on the production quantity in the second scenario

Corollary 5: The simultaneous effect of the two parameters μ and θ on the profit of the first and second manufacturer are shown in Fig. (7) and (8), respectively. Based on the obtained results illustrated in Fig. (7) and (8), with the increasing impact of the price shock, the manufacturers' profits increase firstly due to increasing the product prices. However, when the intensity of the price shock increases, due to the increase in the production costs, the profitability of the two manufacturers decreases. Remarkably, the amount of profit reduction for the second manufacture is higher than the first one.

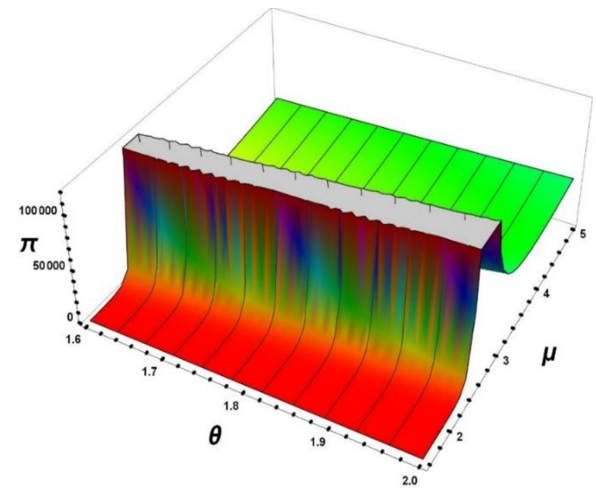

Figure7. Effect of μ and θ parameters on the profit of the first manufacturer simultaneously in the second scenario

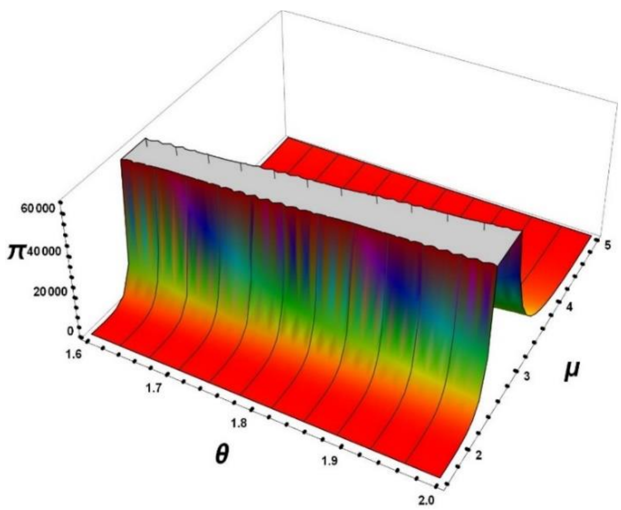

Figure 8. Effect of μ and θ parameters on the profit of the second manufacturer simultaneously in the second scenario

Managerial insight 2: The obtained results indicated that, when the price shock is happening, we faced a recession. In other words, in this scenario, prices are rising and production quantity is decreasing. It is confirmed that the θ parameter plays a significant role in the profitability of manufacturers after happening price shock. At the beginning of happening price shock, the profit of both manufactures increases due to the increasing price however the amount of profitability of the first manufacturer is more remarkable due to forecasting this situation. Then by intensifying price shock the decrease in profit of the first manufacturer happened with a lower slop. This is because that the first manufacture can adapt to this situation due to having planning for this phenomenon. This manufacturer had effective preparation due to market forecasting feature and it made the necessary plans for the manufacturing sector to minimize profit reduction. Thus, profit is decreased in a controlled manner. However, the second manufacture who has not to estimate this situation leads to sudden shock. Consequently, the second manufacturer cannot easily adjust to make effective preparation and may lose its situation in the market. Thus, it can cause significant failure.

4.3. Sensitivity analysis of the Stackelberg game approach

After analyzing the Nash game approach, the effect of the tax policy considering the government as a leader is investigated by designing and implementing a Stackelberg game. Figure 9 indicated that by considering a tax deduction policy before occurring price shock, the government's profit increased with the increasing of the θ and r parameters concurrently. However, when the price shock is happening, the government's profit decreases with the increase of the parameter as it is shown in Figure (10).

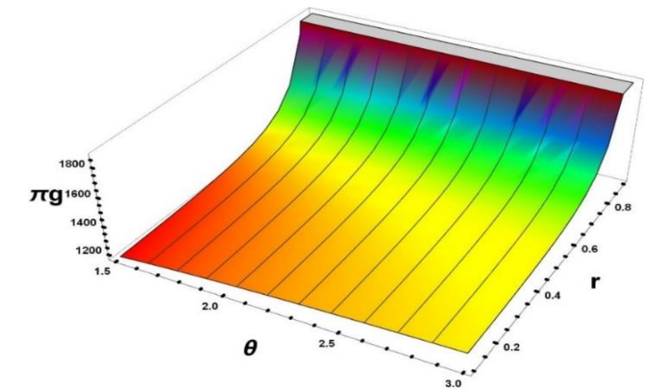

Figure 9. Effect of μ and θ parameters on the government's profit simultaneously in the first scenario

Managerial insight 3: By designing a two-stage Stackelberg game, the obtained results verified that the tax deduction policy is an effective strategy before the occurring price shock. However, the price shock caused the economic recession where the tax deduction policy reduces dramatically government's profit and it is not suggested as an appropriate policy in the second scenario.

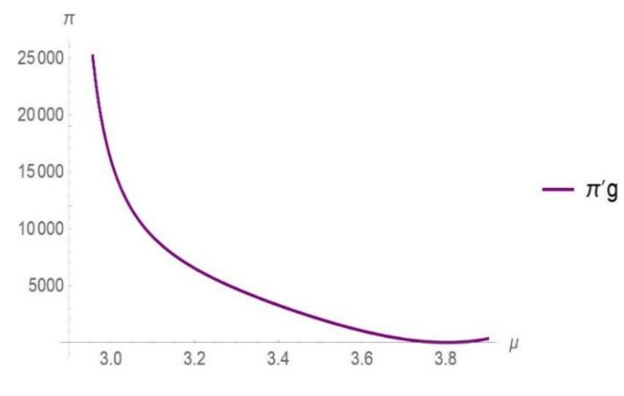

Figure 10. Effect of μ parameter on the government's profit in the second scenario

5. Discussion and obtained implications

In this section, we present a summary of results obtained in two scenarios also in both Nash and Stackelberg game approaches in the following point.

1) As it is discussed in section 4.1, before occurring price shock, the increase in the market forecasting and customer expectation parameters can lead to more production quantity as well as profit for the first manufacturer. In contrast, increasing these two parameters has a negative impact on the production quantity and profit of the second manufacturer.
2) The results showed that after happening price shock we faced two periods. In the beginning of price shock the profits of both manufactures starts to increase. The sensitivity analysis confirmed that this increase is more remarkable for first manufacture by considering market forecasting. By intensifying the price shock in the second period, the manufactures experienced the significant decrease in their profits. However, the decrease in the second manufacturer's profit is more remarkable due to the lack of the market forecasting ability.
3) The market forecasting and customer expectation are introduced as two highlighted parameters that have a significant effect on the manufacturer's profitability that not discussed in previous pricing problems. The proposed pricing models show how the profit, price and production quantity can be affected by market forecasting and customer expectation for two manufactures in the supply chain before and after happening price shock (Equilibrium equations (9-16))
4) By government intervention and analyzing tax deduction policy, it can be inferred that tax deduction policy can be an effective strategy before occurring price shock. In contrast, after price shock is happening, it causes the significant reduction in the government's profit.

6. Conclusion

Price shock and volatility are known as the key factors that affect the variables and economic activities. These critical phenomena result from political and economic crises and such other external factors. One of the consequences of this kind of economic phenomenon is the reduction in the production quantity and also immediate rise in prices. Therefore, it is very important to consider and recognize price shocks in pricing mechanisms. However, in the large amount of economic literature, several studies analyzed and modelled price shock and volatility in different markets, no study is considered pricing modeling procedure considering price shock by applying game theory approach. Therefore, in this paper, for the first time, the pricing mechanism for two substation product is proposed to consider market forecasting, consumer behavior, as well as the government's policies in two scenarios before and after the price shock occurred. Therefore, in the first step, the simultaneous Nash game approach is applied and in the second step, a two-stage game model is proposed to formulate the defined problem using Stackelberg game. Furthermore, comprehensive numerical analysis of the main parameters is provided in this study. As a result, the main objective of this research can be summarized in following points:

- In this paper, a new pricing mechanism for manufacturers who produce substitution products in a duopoly, using a Cournot model considering price shock as a key factor affecting the price, the amount of production and profits of the manufacturers is proposed for the first time.
- The market forecasting and customer behavior are addressed as two effective parameters.
- The Sensitivity analysis is proposed under two scenarios (before and after occurring price shock).
- The effectiveness of the government policy (tax deduction) is investigated for defining the problem by designing a Stackelberg game.

The parametric analysis results of the crucial parameters indicated that before occurring the price shock, the market forecasting parameter plays an important role in the profitability of the first manufacturer. Manufacturers by analyzing and estimating the future of the market can increase the profit and production quantity before occurring price shock. Another important implication yielded in this study is that with the presence of the price shock, we are faced with a recession. In such a way that prices increase immediately, production quantity is decreased as well as manufacturers' profit. However, this reduction in the profit and the amount of production quantity for the first manufacturer who estimates and forecasts this phenomenon is less than the other manufacturer. Even though at the beginning of the price shock, manufacturers' profit may increase as a result of the immediate increase in the prices, but with rising the effect of the price shock, manufacturers' profits will decrease dramatically due to rising production costs. This reduction in profits for the manufacturer who predicted the price shock is less than the other manufacturer. Also, by reformulating this problem and considering the government as a leader, it can be inferred that tax deduction policy can be an effective policy only before happening price shock. As a research direction for future studies, the defined problem in this study can be addressed in the energy market where oil price shocks are a common phenomenon. Besides researchers can examine this problem by considering more than two manufacturers and reformulate the problem, using an evolutionary game approach.

Appendix A

To investigate the concavity of the profit functions in section 3.3, the second derivative of the functions is calculated with respect to decision variables given in Eq. (A.1) and (A.2). The following equations verified that the second derivation is negative and satisfied with the concavity of utility functions.

$$\frac{d{\pi_{1N}}^2}{d{q_{1N}}^2} = -\frac{2}{\theta} \tag{A.1}$$

$$\frac{d{\pi_{2N}}^2}{d{q_{2N}}^2} = -2 \tag{A.2}$$

Similar to the previous scenario, the concavity of utility functions is proven by Eq. (A.3) and (A.4). Where represent the second derivation of the utility functions.

$$\frac{d{\pi'_{1N}}^2}{d{q'_{1N}}^2} = -\frac{2}{(-\theta+\mu)^2} \tag{A.3}$$

$$\frac{d{\pi'_{2N}}^2}{d{q'_{2N}}^2} = -\frac{2}{\mu^2} \tag{A.4}$$

Declaration of interests

The author declares that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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