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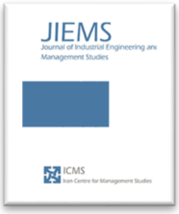

Joint Production and Maintenance Optimization for a Single-Machine Deteriorating System in a Finite Planning Horizon

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Abstract

This paper examines the joint optimization of production and maintenance planning for a single-machine deteriorating system. To achieve optimal performance and meet customer demand at the lowest cost, manufacturing companies need to carefully plan production and maintenance, considering various factors such as time, cost, output levels in any period and its impact on machine deterioration. In this research, we attempt to plan the production and maintenance process for a single-machine single-product system over multi-period. The machine has two operational states during production and gradually deteriorates as it ages. Maintenance operations restore the machine to healthy state and reduce the probability of producing defective products. We model the problem using the Markov decision process and employ the value iteration algorithm to determine the optimal policy, i.e., the best actions to take at each decision epoch. We evaluate the model's effectiveness by solving a numerical example and analyzing how changes in different parameters affect the results. The findings reveal the relationship between various parameters and the average cost rate. Changes in the mentioned rate due to changes in setup cost and the probability of producing conforming products are almost uniform without any drastic fluctuations. If the production cost of each item exceeds a certain threshold, the company's obligations are not enforceable.

Keywords: Markov decision process, Maintenance planning, Production scheduling, Value iteration algorithm

Paper Type: Original Research

1. Introduction

Various factors and their changes affect the performance of manufacturing systems. Considering these effects and providing integrated and comprehensive plans to manage them properly has always been a great challenge for managers of these systems. To face this challenge, researchers have proposed different models and solutions to achieve an optimal policy in the specific conditions of each problem, which is often not simply possible to generalize to other different problems and situations. Production planning is the process of deciding on the resources an organization needs for its future production operations. Production planning is usually done at three levels: strategic level (long-term), tactical level (aggregate production), and operational level (production schedule) (A. Farahani & H. Tohidi, 2020). Production planning determines what, when, and how much to produce to meet the customers' needs, without excessive inventory or back-order costs. Scheduling, conversely, determines how to achieve the goals set in production planning when the resources are limited (D. Sule, 2008). In real production environments, the machine breaks down and may not be available at some times. It is not optimal to carry out production planning, regardless of deterioration and breakdown. Maintenance includes all technical and managerial activities during the life of the equipment, the purpose of which is to maintain or restore the equipment so that it can provide the expected task with an acceptable quality level. In maintenance planning, machine maintenance scheduling is done to prevent sudden equipment failure. When the machine is not in good condition and needs maintenance, the products are not favorable in quality, and the production rate of non-conforming products increases. When an unplanned stop occurs due to a sudden machine breakdown, the current production schedule is not performed, customer orders are delayed, and modifying the production schedule in an emergency usually imposes a high cost on the system. The maintenance will reduce process variation and help increase product quality. The additional maintenance will increase costs, and delayed maintenance will increase the process variability (A. Farahani & H. Tohidi, 2020). Many studies on production scheduling and maintenance planning solely

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concentrate on independent optimization or simple integrated optimization (e.g., the maintenance planning is predetermined, and then the production scheduling is optimized). Therefore, the research of deeply integrated optimization has gradually become a new direction, but a complex coupling relationship exists between production scheduling and maintenance planning, and it will become more complicated when other factors (e.g., processing speed and due date) are incorporated (J. Jiang et al, 2023). Increasing emphasis on sustainable production requires preserving the efficiency of degrading resources over time. Additionally, maintenance planning must consider the interactions with quality control and production planning, since they are fundamental functions for economic success in the manufacturing industry. However, production systems also face the negative effects of deterioration processes. Current approaches result in sub-performing unbalanced systemic solutions that tend to privilege one or two aspects, focusing on the production-maintenance or production-quality interactions, reducing the overall manufacturing system efficiency. Thus, integrated models are needed to determine the right balance between production quality and maintenance functions and improve long-term system performance (H. Rivera-Gomez et al, 2021). Qinming et al. (2019) presented an integrated model for predictive maintenance planning and production scheduling within a single-machine system. This model incorporates not only preventive maintenance but also utilizes diagnostic information and predictions regarding system degradation and its age. Liu et al. (2020) proposed an integrated model to determine the optimal timing for preventive maintenance and the ideal production capacity for each period. Additionally, the model incorporates a limitation on service levels to enhance the likelihood of meeting customer demand in a timely manner. Examining the policy of conditions-based production and maintenance in a single-unit system, focusing on the balance between production and maintenance costs is a study conducted by Broek et al. (2021). The production rate is adjustable, and the equipment failure rate is contingent on the production rate. The degradation process of the system is modeled as a stochastic continuous-time continuous-state process, leading to the formulation of the problem as an MDP (M.A.J. Uit het Broek et al, 2021). Ogunfowora and Najjaran (2023) presented a literature review on the applications of reinforcement and deep reinforcement learning for maintenance planning and optimization problems. Based on the various categories identified in this research, we highlight the key contributions of our model. In the field of maintenance strategy, Condition-Based Maintenance (CBM) has been recognized as one of the most promising strategies for production systems. The overall maintenance strategy in our model is also considered condition-based maintenance. In the realm of solution approaches, the aforementioned review centers on the Markov Decision Process (MDP) framework. Due to the dynamic and stochastic characteristics of our problem, as well as the necessity for sequential decision-making under uncertainty, we also employ the MDP framework to model the issue at hand. The state space in our MDP model is not sufficiently large to warrant the use of deep learning algorithms; therefore, we utilize a well-established dynamic programming algorithm, specifically the value iteration algorithm, to derive the optimal policy. As with some publications, we employ the Markov discrete state degradation model to represent the degradation levels of machines or assets. Finally, one of the proposed areas for future work outlined in the review is to focus on joint or integrated optimization, encompassing production scheduling, inventory management, material handling, shift scheduling, and quality assurance. Therefore, we present a joint production and maintenance optimization model that also considers product quality as a basis for determining the production volume during each period of the planning horizon. Under this background, and based on the conducted literature review we are committed to develop an integrated optimization method for achieving a balance between production scheduling and maintenance planning to fulfill the company's obligation while minimizing the average cost rate. Therefore, we need to balance the production and maintenance costs by determining the appropriate production volume or selecting maintenance operations in each period of the planning horizon. The main contributions of this work are as follows:

- We consider an integrated approach to production and maintenance planning for a single-machine system, taking into account the probabilistic transitions of system operational states, and production quality
- The production machine consists of two operational states, healthy and unhealthy. The quality of the manufactured products is assumed to be influenced by the current state of the machine. Each state exhibits a different probability of producing conforming products
- The problem is modeled in the MDP framework, and we solve it utilizing the value iteration algorithm
- The effects of varying different parameters on derived policy are examined and analyzed

2. Literature Review

In the following, a comprehensive review of previous articles based on the three main categories has been presented:

2.1 Production

Li et al (2009) examined a manufacturing system and formulated a stochastic dynamic programming-based model for production planning in uncertain conditions. The optimal production plan is obtained by the policy iteration algorithm. Moreno and Montagna (2009) presented a public model for designing and planning a multiproduct batch plant under a multi-period scenario. The initial formulation of the problem is nonlinear, and then by transforming the initial formulation into a mixed integer linear planning model, it is possible to solve the problem in a global optimization mode. Determining the optimal production strategy in a production system was the subject of Khaledi and Nafchi in 2013. The problem's objective is to prepare customer orders by considering machine setup costs, delay, and product shortage costs. Cheng et al (2013) used a dynamic programming approach to determine the optimal shutdown policies for deterministic serial production lines. Hilger et al (2016) have considered a stochastic dynamic multi-product capacitated lot sizing problem with remanufacturing. Two approximate approaches are used to provide robust (re)manufacturing plans. Teksan and Geunes (2016) have studied a production planning problem in which the manufacturer procures and buys one of the input components for production by offering a price to the suppliers. In another research, El Ashhab (2016) uses a mixed integer linear programming model to solve the production planning problem in a multi-product system with a multi-level chain and multi-cycle and multi-objective mode. Polotski et al (2017) have considered the problem of optimal production control in a system that uses one facility for both manufacturing and remanufacturing processes with the setup required to switch from one process to another. By solving the problem, the setup times and production and reproduction rates will be determined. Awasthi et al (2019) have proposed a different mathematical programming approach to improve the development, optimization, and production planning decisions in the oil and gas industries and a simultaneous multi-period nonlinear programming model to determine the optimal solution to the oil production planning problem. Liu et al (2019) have investigated a two-product multi-period production-inventory problem under the assumption of uncertainty in demand. The problem is formulated as a multidimensional dynamic plan and lead times are not included in the basic model. Maafa et al (2020) have investigated multi-period production planning for a production system including several parallel workstations. Guillaume et al (2020) investigated the production planning problem of a single product system under demand uncertainty. The robust optimization approach with the min-max criteria has been used to solve the problem. Gomez et al (2021) have modeled a production planning problem with stochastic demand and capacity constraints using a stochastic programming approach. In another study Luo et al (2022) while referring to the emergence of new technologies in the digitalization of production systems, investigated the impact of Industry 4.0 on production planning approaches and expressed the three main challenges in using new technologies in production planning. The use of flexible production systems to minimize costs related to production, maintenance, and outsourcing operations is the subject of Elyasi et al (2023) research. Fornier et al (2024) investigated the joint optimization of daily production planning and energy supply management in an industrial complex. Rahimi et al (2024) have presented a model, based on the Markov decision process for production planning of a multi-period single-product system. The problem is solved by the stochastic dynamic programming method.

2.2 Maintenance

Bohlin and Warja (2012) have presented an integer linear programming model to optimize maintenance operations in single and multi-unit systems. Shafiee and Finkelstein (2015) have proposed a group maintenance policy for a multi-unit series system. The components of the system are subject to different gradual degradation phenomena and a two-stage stochastic model is proposed for modeling degradation of the system. Verbert et al (2016) have presented a two-stage optimization approach for maintenance planning in heterogeneous systems. Matyas et al (2017) research is about Prescriptive maintenance planning in manufacturing industries. Hu et al (2017) have investigated the issue of preventive maintenance planning in a single-machine system. A hybrid model has been presented for modeling the Maintenance operations. Aizpurua et al (2017) proposed a method for maintaining complex dynamic systems, including a new algorithm for the cost-effective grouping of assets. Poppe et al (2018) presented A hybrid condition-based maintenance policy for continuously monitored components with two degradation thresholds. Guiras et al (2018) have presented the optimal maintenance plan for a two-level assembly system on the condition that the failure of the only machine in the system is considered random. Yahyatabar and Najafi (2018) have investigated finding the optimal condition-based maintenance policy based on the proportional hazard model. Rasay et al (2018) have studied the integration of the decisions associated with Maintenance Management (MM) and Statistical Process Control (SPC) in a series production system. Srivastava et al (2019) sought to create a reliable and exhaustive database of agile maintenance attributes to select an effective maintenance strategy in a sugar factory. Wenbin et al (2019) proposed a reliability-dependent cost model for machine tools. Corrective maintenance in combination with overhaul has been investigated as the selected strategy. Dinh et al (2020)

Optimize the maintenance of a multi-component system assuming the existence of structural and economic dependency. Liu et al (2021) have investigated a condition-based maintenance policy for a two-component system, assuming interdependent degradation processes. Zheng et al (2021) have proposed optimizing maintenance operations in a degrading system using reinforcement learning approaches. Taji et al (2022) have presented a new approach for preventive maintenance planning in a multi-component system. It is assumed that failures or inspections are not only causes of stopping devices but also some other activities (non-failure stops) that may interrupt the production process. Using machine learning concepts, Yeardley et al (2022) have presented a new method for predicting machine faults and repair times and using the obtained data to plan maintenance activities. Rasay et al (2022) have presented a stochastic dynamic programming model for maintenance planning on a deteriorating multi-state production system. They use a reinforcement learning (Q-learning) algorithm to solve the presented model . Yilmaz et al (2023) have used the fuzzy DEMATEL method to improve the maintenance performance of the manufacturing system. Considering the issue of manufactured product warranty, Zhao et al (2023) have presented a model for planning imperfect maintenance in a manufacturing system. Based on the concept of turnaround maintenance (TAM), Duffuaa et al (2024) have presented a mathematical model for planning and scheduling for a network of manufacturing plants in the process industry supply chain. Santos et al (2024) have integrated several and not alike fields of study including process mining, multi-criteria decision-making, and data fusion to rank industrial machines to plan maintenance tasks on them.

2.3 Joint production and maintenance planning

Yildirim and Nezami (2014) have presented a mathematical model for planning production and maintenance in a single machine multi-product multi-period system. Bajestani et al (2014) have formulated a Markov decision process model to determine the maintenance plan of each machine in a multi-period multi-machine system, and then, a mixed integer programming model, to find the maintenance and the production schedule in each period of the planning horizon. Joint production and maintenance planning in a failure-prone manufacturing system is the subject of research by Aghezzaf et al (2015). Desforges et al (2017) using inference rules, have proposed a generic function to assess the future availability of a complex system, whose results can be used as decision indicators in production and maintenance planning. Hajej et al (2018) have investigated capacity design, production, and preventive maintenance planning of a production system made of parallel leased machines. Alimian et al (2019) have presented a robust approach for production planning and maintenance of a multi-state production system with demand fluctuation. Qinming et al and Xu et al (2019) have studied the integrated optimization of production planning and maintenance scheduling on a single-machine system . Liu et al (2020) presented an integrated model to determine the optimal time for preventive maintenance and optimal production capacity in a manufacturing company. Broek et al (2021) examined the policy of conditions-based production and maintenance in a single-unit system in line with the balance between production and maintenance costs. In their research, Gomes et al (2021) use a stochastic mathematical model to find the optimal policy of inspection, production, and maintenance in a deteriorating system under quality constraints. Sharifi and Taghipour (2021) have presented a mathematical model for joint production and maintenance scheduling in a single-machine system assuming multiple failure modes. Jiang et al (2023) have investigated the integrated optimization of flow shop scheduling and maintenance planning in conditions where the processing speeds of products are variable. With the aim of simultaneous production planning and maintenance optimization in a system with continuous production, Leo and Engell (2023) have used a stochastic programming approach to handle endogenous uncertainties (type I and II). Li et al (2023) proposed a novel CBM strategy model for manufacturing systems that incorporates working schedules to determine the optimal balance between maintenance and production while considering product quality. They utilize a dual-value iteration algorithm to solve the Bellman equations while an MDP is employed for optimal maintenance strategy. The paper by Hu et al. (2023) addresses the joint optimization problem for a multi-machine system. The authors propose a novel approach known as Knowledge Enhanced Reinforcement Learning (KERL), which employs a centralized multi-agent actor-critic architecture. Bahou et al (2024) have studied the integrated preventive maintenance scheduling and production planning in a system with several parallel components . Zhang et al (2024) have used the Markov decision process framework to formulate the problem of integrated condition-based production and maintenance planning. Random yield and maintenance delay are considered in the presented model. Ouahabi et al (2025) address the joint flexible job shop scheduling and maintenance planning problem, taking into account new job insertions and multi-component machines with economic dependencies among components. The authors employ deep reinforcement learning (DRL) algorithms to tackle this problem. Lv et al (2025) proposed a joint-control model that considers production, inspection, and maintenance within a finite-time scenario for smart manufacturing systems. To enhance the effectiveness of the model, a discrete iterative algorithm with multiple loops is developed. They systematically compare the impact of finite and infinite time strategies on production and maintenance decisions.

Table 1 highlights the contribution of the papers reviewed in this section. In this research, we attempt to present a joint dynamic plan for the production and maintenance of a single-product single-machine manufacturing system. The goal is to meet the customer demand with the optimal cost in a specific number of time periods. The customer demand is certain but it is assumed that there is a probability of producing defective products and all products are not conforming. Consequently, the number of conforming products in any period is uncertain. At the start of each period, based on the machine state and total quantity of conforming items, it can be decided (I) whether to carry out maintenance or not and (II) determine the production volume. We use the Markov decision process framework to model the problem. Then we solve the problem with the value iteration algorithm. A numerical example is presented and the effect of changing different parameters on the average cost rate has been analyzed. The rest of this paper is organized as follows. Section 3, introduces the notations and the problem under consideration. Section 4 presents the details of the MDP* model and the problem solution. Section 5 presents a numerical example and determination of optimal policy using a value iteration algorithm. A comprehensive analysis is done and the results are shown with some tables and graphs. Section 6 concludes the paper and presents some directions for future research.

Table 1. Summary of the contribution of the reviewed papers

	System type	**General maintenance policy**	**Specific conditions of the problem**	**General Modeling and solving approach**
I. production				
Li et al	General	-	Considering remanufacturing issue	Stochastic dynamic programming
Moreno & Montagna	Multi-product	-	Synchronization of design and production decisions	Non-linear model/ Mixed integer programming
Khaledi & Reisi-Nafchi	General	-	Considering the cost of delay and shortage and the randomness of the sales value	Stochastic dynamic programming/ Linear programming
Cheng et al	Serial production line	-	Planning system shutdowns to balance production goals against the need to perform non-production activities	Dynamic programming/ Event-triggered re-optimization
Hilger et al	Multi-product	-	considering remanufacturing and robustness	Stochastic dynamic programming/ Approximate approaches
Teksan et al	General	-	Buying one of the production components by offering the price to the supplier/ considering maintenance cost linearly	Discrete-time planning/ Dynamic programming-based heuristics
El Ashhab	Multi-product	-	Considering the multi-level chain in a multi-period and multi-objective mode, as well as inventory management	Mixed integer programming
Polotski et al	General	-	Considering a hybrid production and reproduction system/ dependence of the production rate on the production mode	Stochastic dynamic programming
Awasthi et al	Oil and gas industries	-	Providing a different approach to mathematical planning, optimization, and production planning in oil and gas industries	Multiperiod nonlinear programming
Liu et al	Two product system	-	Considering inventory control and demand uncertainty	Multi-dimensional dynamic programming
Maafa et al	A single product system including several parallel workstations	-	Production of similar products and capacity limitation in workstations/ Examining a real-world example	Scenario-based linear programming model
Guillaume et al	Single product	-	Assumption of demand uncertainty/ two discrete and continuous mode	Robust optimization with min-max criteria
Gomez-Rocha et al	General	-	Determining the optimal number of personnel, products to be stored/ Integrated production planning approach	Multistage stochastic programming model using scenario tree
Luo et al	General	-	Investigating the effects of industrial 4.0 on planning approaches	-
Elyasi et al	General	-	Considering flexible production systems	Scenario-based approach/ Heuristic algorithms

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* Markov decision process

Fornier et al	General system in industrial complex	-	Improvement of daily production schedule and energy supply management	Mixed integer linear programming in stochastic multistage mode
Rahimi Kakehjoob et al	Single product	-	Taking into account the multi-period opportunity to meet customer demand/ Randomness of the quality of manufactured products/ limitation of production capacity in each period	Markov decision approach/ Stochastic dynamic programming
II. Maintenance				
Bohlin & Warja	Parallel multi-unit systems	Preventive maintenance	Assuming parallel maintenance on a limited number of machines	Integer linear programming model
Shafiee & Finkelstein	Multi-unit series system	Replacement/ Preventive/ Group maintenance	Considering gradual degradation phenomena for the components of the system	A two-stage stochastic model
Verbert et al	Multi-component systems	Condition-based maintenance	Considering heterogeneous systems & economic dependency	A two-stage optimization approach
Matyas et al	General (automotive manufacturing industry)	Prescriptive maintenance	Using a real environment to collect the necessary data and develop the proposed approach	A novel procedural approach/ Using data Analysis and simulation tools
Hu et al	Single machine	Preventive/ Imperfect	Considering a machine working under piecewise constant operating conditions (PCOC)	Combining an age-based imperfect maintenance and an accelerated failure time model/ Using the Maximum likelihood method for estimating the model parameters
Aizpurua et al	Complex dynamic systems	Condition-based/ Predictive	Dividing equipment into critical and non-critical categories/ using group maintenance for non-critical components and condition-based maintenance for critical components	Stochastic Activity Networks (SAN)
Poppe et al	Multi-component	Corrective/ Periodic/ Condition-based	proposing an opportunistic threshold to avoid setup costs and future failures/ an intervention threshold to avoid imminent failures	an enumeration procedure
Guiras et al	Single machine	Preventive/ Corrective	Considering inventory management of production components and products in a two-level assembly system	Mathematical model based on risk assessment
Yahyatabar & Najafi	Series-Parallel systems	Condition-based	Determining different dynamic control limits for each piece of equipment/ proportional hazard model	Multistage stochastic programming model/ Hybrid metaheuristic algorithm
Rasay et al	Series systems	Corrective/ Reactive	Integration of the decisions associated with Maintenance Management (MM) and Statistical Process Control (SPC)	integrated mathematical model/ Sensitivity analyses Using factorial design
Srivastava et al	General system (sugar factory)	Various maintenance and repair strategies	Create and use a reliable database of agile maintenance features	A fuzzy integrated multi-criteria decision-making approach
Wenbin et al	Single machine	Corrective/ Overhaul	Considering Reliability	Markovian reward approach/ Genetic algorithm
Dinh et al	Multi component	Opportunistic	The assumption of economic and structural dependence between component maintenance	Mathematical model based on failure rate and effects of disassembly
Liu et al	Two components	Condition-based	The assumption of component degradation process dependency	Modeling based on Markov decision process/ Dynamic programming
Zheng et al	Single component	Condition-based	Assuming known and unknown equipment degradation process	Reinforcement learning (Q-learning algorithm)

Taji et al	Multi-component	Preventive	Assuming the system stops due to factors other than inspection and failure	Adjustment of parameters by Taguchi method/ Genetic algorithm
Yeardley et al	Multi-component	Predictive	Using machine learning concepts to predict failure and repair time	Three-step approach (prediction, estimation optimization)
Rasay et al	Single machine	Condition-based	Employing The quality of the produced items in each stage as a condition monitoring for condition-based maintenance	Reinforcement learning/ Q-learning
Yilmaz et al	General	-	-	Fuzzy DEMATEL method
Zhao et al	General	replacement and imperfect repair	exploring imperfect maintenance policies for warranted products subject to stochastic performance degradation	Markov decision process/ renewal theory/ Dynamic programming
Duffuaa et al	A network of manufacturing plants/ Process industries	Restore (including full stop, inspection, and different types of maintenance)	Dividing components (factories) into different levels and assuming the existence of sequential flow between components	Mixed integer linear programming model/ Exact and heuristic algorithms
III. Joint production and maintenance				
Santos et al	General system (industrial machines)	-	Integration of several dissimilar fields of study	Integration of process mining, multi-criteria decision making and data integration/ Risk assessment and critical analysis
Yildirim & Nezami	Single machine	Preventive	Considering maintenance time and energy consumption	Mixed integer nonlinear mathematical model
Bajestani et al	Multiple machines (flow shop)	Condition-based	Considering customer demand/ Assumption of determining demand at the beginning of each period	Markovian decision model/ Mixed integer programming model
Aghezzaf et al	General	Imperfect preventive	Predicting the operating mode based on the equipment age	Hybrid failure rate model/ Mixed-integer nonlinear optimization
Desforges et al	Complex	Prognostic/ Condition-based	Using objective-oriented Bayesian networks to estimate system failure probabilities	Using a generic function to evaluate availability by inference rules
Hajej et al	Parallel multi-component	Preventive/ Corrective	Considering the rental machines/ Randomness of customer demand	Multi-stage approach
Alimian et al	General multi-mode	Preventive	Considering demand fluctuations	Robust decision-making
Xu et al	Single machine	Preventive	Considering fixed and flexible time intervals between two successive repairs	Providing six different models/ Solve with exact and heuristic approaches
Qinming et al	Single machine	Prognostic/ Preventive	Predicting degradation and system age	Mathematical model/ genetic algorithm
Liu et al	Single machine	Preventive/ Corrective	Applying service level constraints to increase the probability of timely satisfying customer demand	Mixed integer programming model

Broek et al	Single component	Condition-based	The adjustability of the production rate/ Dependence of the failure rate on the production rate	Markovian decision process framework
Gomes et al	Single product	Minor and major repairs based on age and imperfect inspection	Considering the effect of system degradation on system reliability and quality control issues	Mathematical model with stochastic optimal control approach
Sharifi & Taghipour	Single machine	Replacement/ Corrective /Imperfect and perfect preventive maintenance	Assuming the existence of multiple failure modes with different effects on product quality	Mathematical model/ Genetic algorithm/ Simulated annealing/ Teaching-learning-based optimization
Jiang et al	Multiple machines (flow shop)	Replacement/ Preventive	Considering production speed, variable processing speed of products	Improved genetic algorithm
Leo and Engell	Continuous production/ multiple inputs and products	Condition-based	Handling Type-I and Type-II endogenous uncertainties/ Considering the cost of energy consumption	Mixed integer nonlinear programming model
Li et al	Horizontal machining center	Condition-based	Determining the optimal balance between maintenance and production while considering product quality	MDP/ Dual-value iteration algorithm
Hu et al	Multi machine system	Preventive maintenance	Capturing complex interdependencies between production and maintenance operations under challenging conditions	MDP/ Knowledge Enhanced Reinforcement Learning (KERL)
Bahou et al	Parallel multi-component	Preventive	Effect of the interdependence of components on their degradation process	Integer programming
Zhang et al	Single machine	Condition-based/ Corrective/ Preventive	Considering the random Yield and maintenance delay	Markovian decision process framework
Ouahabi et al	Job-shop production system	Predictive/ Corrective	Considering economic dependencies among components/ Considering opportunistic grouping of maintenance activities	Constraint programming (CP)/ Deep Reinforcement Learning (DRL)
Lv et al	Single-machine intelligent manufacturing system.	Preventive/ Corrective	Compare the impact of finite and infinite time strategies on production and maintenance decisions	Discrete iterative algorithm with multiple loops

3. Notation and Problem Statement

3.1 Notation

The list of parameters, variables, and indices is presented in Table 2:

Table 2. Notation

Symbol	description	symbol	description
N	Total demand	k	Shortage penalty per unit
n	Number of production time periods	M	Maximum production capacity allowed in each period
s	The number of conforming products at the beginning of each period/epoch	P_0	The probability of producing a conforming product when the machine is in state 0
$\boldsymbol{i}$	Machine state in any system state	P_1	The probability of producing conforming products when the machine is in state 1
t	Period number	β	The probability of switching the state of the machine from 0 to 1
E	Setup cost	c_m	Maintenance cost
c	Production cost	V(S)	State value function
$\boldsymbol{r_1}$	Production revenue per unit of conforming product (when the production amount is less than or equal to N unit)	$V^*(S)$	Optimal value of state S
$\boldsymbol{r_2}$	Production revenue per unit of conforming product (when the production amount is greater than N units)	-	-

3.2 Problem statement

Consider a production machine consisting of two operational states, healthy and unhealthy, denoted as 0 and 1 respectively. When the machine is in healthy state, any item produced is independently conforming with a probability p_0, and this value is p_1 for the unhealthy state so that $p_0 \geq p_1$. At the start of the current period, if the machine is in state 0, it transits to state 1 at the next period with probability β. If the machine transits to state 1, it remains in this state until a maintenance action is conducted. Maintenance operations return the machine from state 1 to 0. Performing the maintenance actions takes one period and the cost is c_m. The company is committed to delivering N units of conforming products to the customer. It is assumed there are at most T periods of production opportunity. In each decision epoch, at the start of each period, the company can decide to take the following two actions:

- Setting up the system and produce x units of product, $x \geq 0$
- Conducting Maintenance operation (if the machine is in state 1)

In each period, if the company decides to produce, it incurs a fixed cost E for setting up, and the production of each item has a variable cost c. Two general situations may occur in the last period (At the end of the planning horizon). In the first case, the total number of conforming products is less than N. In this case, each unit of conforming products has an income equal to r_1 ($r_1 > c$) and for each unit of difference between the committed quantity (N) and total number of conforming products, a penalty equal to k must be paid. In the second case, the number of conforming products is greater than or equal to N. In this case, each unit of conforming products has an income equal to r_1 ($r_1 > c$), but each item produced over N has an income equal to r_2 so that r_2 ($r_2 \leq c$). The objective is to find the optimal policy for production and maintenance so that the expected value of the system costs is minimized in the finite planning horizon of T periods. Also, the production capacity in each period is assumed to be limited number denoted by M.

4. MDP model and problem solution

In this section first, the general description of the MDP framework is presented. Second, based on the problem under investigation, the MDP elements and transition probability between defined states are introduced. Third, Bellman's recursive equations are presented as the basis of the problem solution and finally, the problem is solved by the value iteration algorithm

4.1 Markov decision process and MDP formulation

An MDP consists of a finite state space S, a finite state-dependent action space A, state- and action-dependent immediate rewards and transition probabilities (M. Koopmans & B. De Jonge, 2023). In this framework, the decision-making agent will experience another state of the system by performing any action in any state of the system and depending on the transition possibilities, while receiving an instant reward. In other words, the system will be moved to a new state. Modeling problems in the Markov decision process framework requires defining states, actions, and appropriate rewards for performing each action in each state. Most of the problems related to production scheduling and maintenance planning are periodic; in other words, there is a need to make sequential decisions at some epochs. In the problem under investigation, several consecutive epochs have been considered for making decisions about performing maintenance operations and the amount of production in each period. The basis for defining these epochs is the beginning of each time period. The first decision epoch coincides with the first launch of the system, and in this particular case, the end of production in the previous period is irrelevant. Some parameters will inevitably be uncertain when including real-world conditions. To solve this category of problems, it is necessary to use modeling and solving approaches that effectively cover the aspects mentioned above.

4.2 MDP model (Introduction and description of MDP elements)

We formulate the problem as a stochastic dynamic programming model with a finite planning horizon.

State space:

In the MDP model, the basis for decision-making at different epochs is the problem state. In the problem under consideration, each state is represented by a three-element vector:(s, i, t). the first element (s) is the total number of conforming products produced until the start of the period (t), the second (i) is the machine state at the beginning of that period, i.e., 0 for healthy state and 1 for unhealthy state and the third is the period number, t.

Actions:

Let A denote the Action space: $A = \{0,1,2,\dots,"m"\}$, so that j's in the $\{j \in \mathbb{Z}^{+} | 0 \leq j < m\}$ represents setting up the machine and producing j items. In the case where $j = 0$, the number of produced items is zero, and the setup will not be conducted. Also, m represents conducting maintenance action.

Transition probability:

In the following, how to calculate the probability of going from each state to another possible state or in other words how to calculate the transition probability among different system states, focusing on the machine state (i), is described. the system state (s, i, t) is the basis for deciding on appropriate action at the period t. When the machine state (i) is 0, the system state is (s, 0, t):

In this set of states, no maintenance will be performed and the agent's decision is only about the production volume in that period. According to the assumption of the existence positive probability of producing defective products and also the possibility of changing the machine state from 0 to 1 with probability β, when an agent's decision or action is setting up the machine and producing some products, the transition among different states of the problem is stochastic and after selecting a specific action in the current state, the next state of is not known in advance. When the machine state (i) is 1, the system state is (s, 1, t):

In these states, the agent's decision may be performing maintenance, repairing the machine, or producing "a" unit of products. when the selected action is performing maintenance, then the next state is already known and certain.

In both of the above-defined sets, if the selected action by the agent is producing "a" unit of products and the machine is in state (i), the probability of changing the number of conforming products from s in period t to s' in period t+1 is as follows:

$$p_{ss'}(a,i) = \binom{a}{s'-s} {p_i}^{s'-s}(1-p_i)^{a-s'+s} \tag{1}$$

Notice that S and S' are the system states and s and s' are respectively the first element of the mentioned states and in other words, are respectively the number of conforming products in the state S and S'.

4.3 The problem solution

Bellman's recursive equations are represented in the following. Consider that the agent is at the start of the period t ($0 \leq t \leq T-1$), the total number of conforming items at this point is s, and the machine is in state i (i=0 or 1). It means that the state of the MDP is (s, i, t) and the optimal value function is $V^{*}(s,i,t)$. Hence, given Bellman's optimality equation the following equation is provided:

$$\begin{aligned} V^{*}(s,i,t) = {} & \\ \min_a \Bigg\{ & E \times sign(a) + ca + \beta \sum_{s'} V^{*}(s',1,t+1) \times p_{ss'}(a,i) + (1-\beta) \sum_{s'} V^{*}(s',0,t \\ & + 1) \times p_{ss'}(a,i), V^{*}(s,0,t+1) \Bigg\} \end{aligned} \tag{2}$$

$$\forall i = 0, t = 0,1,2,\dots,T, sign(a) = \begin{cases} +1 & if\ a > 0 \\ 0 & if\ a = 0 \\ -1 & if\ a < 0 \end{cases}$$

$$V^{*}(s,i,t) = \min_a \left\{ E \times sign(a) + ca + \sum_{s'} V^{*}(s',1,t+1) \times p_{ss'}(a,i), c_m + V^{*}(s,0,t+1), V^{*}(s,1,t+1) \right\} \tag{3}$$

$$\forall i = 1, t = 0,1,2,\dots,T-1, sign(a) = \begin{cases} +1 & if\ a > 0 \\ 0 & if\ a = 0 \\ -1 & if\ a < 0 \end{cases}$$

While $p_{ss'}(a,i)$ is computed using equation 1.

At the end of the planning horizon, the value function is computed as follows:

$$V(s,i,T) = \begin{cases} -r_1 s + k(N-s); & if\ \ s < N \\ -r_1 N - r_2(s-N); & if\ \ s \geq N \end{cases} \tag{4}$$

4.4 Value iteration algorithm

A way of understanding value iteration is by referencing the Bellman optimality equation. Note that value iteration is obtained by turning the Bellman optimality equation into an update rule. Also, note how the value iteration update is identical to the policy evaluation update except that it requires the maximum to be taken over all actions. Like policy evaluation, value iteration formally requires an infinite number of iterations to converge exactly to V^*. In practice, we stop once the value function changes by only a small amount in a sweep. The box below shows a complete algorithm with this kind of termination condition. in each of its sweeps, Value iteration effectively combines one sweep of policy evaluation and one sweep of policy improvement (R.S. Sutton & A.G. Barto, 2018). The pseudocode of the Value iteration algorithm is presented in Figure 1.

```
Value Iteration, for estimating $\boldsymbol{\pi} \approx \boldsymbol{\pi}^*$

Algorithm parameter: a small threshold, $\theta > 0$ determining accuracy of estimation Initialize V (s), for all $s \in S^+$,
arbitrarily except that V (terminal) = 0

Loop:
    $\Delta \leftarrow 0$
Loop for each $s \in S$:
    $v \leftarrow V(s)$
    $V(s) \leftarrow max_a \sum_{s',r} p(s',r|s,a)[r + \gamma V(s')]$
    $\Delta \leftarrow \max(\Delta, |v - V(s)|$
until: $\Delta < \theta$
output a deterministic policy, $\pi \approx \pi^*$, such that
    $\pi(s) = argmax_a \sum_{s',r} p(s',r|s,a)[r + \gamma V(s')]$
```

Figure1. Value iteration algorithm (R.S. Sutton & A.G. Barto, 2018)

5. Numerical example and Results analysis

In this section first, a numerical example is solved by using the value iteration algorithm to investigate the application and validity of the proposed method. In other words, by setting the parameters according to Table 3 the optimal production and maintenance policy in the assumed time horizon is obtained. Second, Given the base example sensitivity analysis is conducted by changing the main parameters of the problem.

5.1 Solving a hypothetical example

We solve an example of the problem under investigation based on the parameters listed in Table 3. The optimal policy or in other words the best action for each state of the problem is presented in Table 4.

Table 3. The values of the parameters in the numerical example

Parameter	value	Parameter	value	Parameter	value	Parameter	value
N	5	r_1	3	k	5	p_1	0.5
n	3	r_2	1	E	3	c	2
M	10	p_0	0.9	β	0.8	c_m	0.75

Table 4. The optimal policy (best action for each state)

State = (s, i, t) = (number of conforming products, machine state, period number)							
state	Best action	state	Best action	state	Best action	state	Best action
(0,0,0)	8	(5,0,2)	0	(1,1,1)	m	(7,1,2)	0
(0,0,1)	8	(6,0,2)	0	(2,1,1)	m	(8,1,2)	0
(1,0,1)	7	(7,0,2)	0	(3,1,1)	4	(9,1,2)	0
(2,0,1)	5	(8,0,2)	0	(4,1,1)	0	(10,1,2)	0
(3,0,1)	3	(9,0,2)	0	(5,1,1)	0	(11,1,2)	0
(4,0,1)	0	(10,0,2)	0	(6,1,1)	0	(12,1,2)	0
(5,0,1)	0	(11,0,2)	0	(7,1,1)	0	(13,1,2)	0
(6,0,1)	0	(12,0,2)	0	(8,1,1)	0	(14,1,2)	0
(7,0,1)	0	(13,0,2)	0	(9,1,1)	0	(15,1,2)	0
(8,0,1)	0	(14,0,2)	0	(10,1,1)	0	(16,1,2)	0
(9,0,1)	0	(15,0,2)	0	(0,1,2)	10	(17,1,2)	0
(10,0,1)	0	(16,0,2)	0	(1,1,2)	8	(18,1,2)	0
(0,0,2)	0	(17,0,2)	0	(2,1,2)	6	(19,1,2)	0
(1,0,2)	7	(18,0,2)	0	(3,1,2)	4	(20,1,2)	0
(2,0,2)	5	(19,0,2)	0	(4,1,2)	0		
(3,0,2)	4	(20,0,2)	0	(5,1,2)	0		
(4,0,2)	0	(0,1,1)	m	(6,1,2)	0		

5.2 Result analysis

To best understand the analysis and reported results, the three following tips should be considered:

First, the average cost rate is one of the main criteria we use to compare the effect of changing parameters on the optimal policy. different from parameter c (Direct production cost per unit of product) the basis for calculating the average cost rate is dividing the sum of all costs (with a positive sign) and revenues (with a negative sign) by the number of conforming products delivered to the customer at the end of the planning horizon. To investigate the effect of changing each parameter on the average cost rate, in each case, only the same specific parameter is changed in a certain interval and other parameters and values will be by the table provided for solving the numerical example Table 3. **Second**, considering that the main objective function of the problem is defined in the form of cost minimization, in each equation and calculation, costs, have been applied with a positive sign and revenues with a negative sign. Therefore, if implementing the company's obligations leads to profitability, the final cost will be negative. **Third**, after applying changes to each parameter, the optimal policy, or the best action in any probable state, is calculated using the value iteration algorithm. The algorithm provides optimal actions for all possible states, but for calculating the average cost rate we need to know the number of conforming products delivered to the customer at the end of the planning horizon. Accordingly, to solve this challenge, by using the concepts and principles of simulation, the following approach has been adopted to make it possible to analyze and compare the results based on the mentioned criterion. Firstly, the desired change is applied to the parameter under investigation and the optimal action for the first period is determined by the value iteration algorithm. According to problem assumptions and as mentioned earlier, only two types of actions are possible in the first period. Therefore, the best action can be setting up the machine and producing some products or doing nothing. On the one hand, the number of products the algorithm has proposed for producing in the first period is determined. On the other hand, based on the machine state, we can determine the probability of producing conforming products. The probable state for the second period is determined by multiplying the number of products by the probability of producing conforming products. In the other periods, unlike period one, "maintenance" can also be selected as the best action. Maintenance operation only changes the machine state, and if it is the selected action, the next state of the problem is specified and certain. After determining the probable state for the second period, by repeating similar operations for this period and adding the produced units in the first and second periods, the probable state in the third period is also determined. by repeating the mentioned operation for the third period, the average number of conforming products delivered to the customer is estimated. It is important to note that multiplying the probability of being conforming by the number of produced units will not produce integers. For this purpose, if the decimal digits of the obtained number are less than 0.5, the number is rounded to a smaller integer, otherwise, it is rounded to a larger one. In the following, based on the approach mentioned above, the effects of changing different parameters on the trend of average cost rate changes have been investigated:

5.2.1 The effect of changes in setup cost on the average cost rate

Setup cost is considered in calculations, only when the selected action for doing in a specific state produces a non-zero amount of product. The effect of changes in this cost on the average cost rate has been investigated and the results of the relevant calculations are presented in Table 5. In the conditions of the example presented in this research and assuming no change in other parameters, in general, with the increase in the setup cost, the average cost rate increases, and the hope that the implementation of the company's obligations will lead to profitability decreases. The trend of changes in the average cost rate for changes in the setup cost is shown in Figure 2.

Table 5. The effect of changes in setup cost on the average cost rate

Row number	Parameter under investigation	First period	Planning horizon (three time periods)				
	Setup cost	Optimal action	Average of produced units	Average of setup number	Average of repairs	Average of conforming products	Average cost rate
1	0	6	6	1	0	5	-0.65
2	1	7	7	1	0	6	-0.20
3	2	8	8	1	0	7	+0.10
4	3	8	8	1	0	7	+0.25
5	4	9	9	1	0	8	+0.47
6	5	9	9	1	0	8	+0.59
7	6	9	9	1	0	8	+0.72
8	7	9	9	1	0	8	+0.84
9	8	9	9	1	0	8	+0.97
10	9	9	9	1	0	8	+1.09
11	10	9	9	1	0	8	+1.22

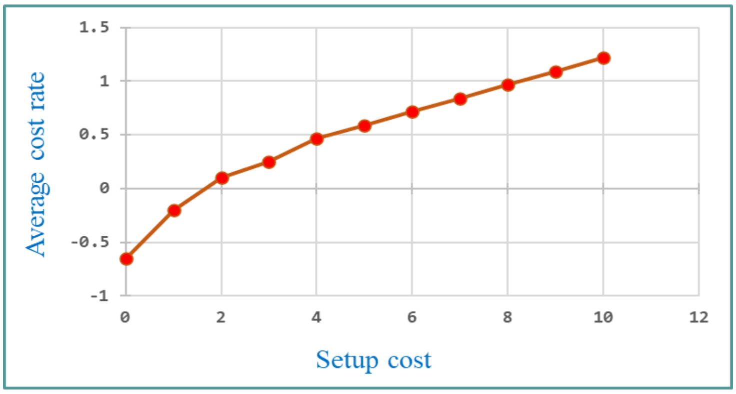

Figure 2. The effect of changes in setup cost on the average cost rate

5.2.2 The effect of changes in maintenance cost on the selecting the maintenance as the optimal policy

Considering the results in Table 6, In the first period, because the machine is new, the model never suggests maintenance operation as an optimal action. In other periods, increasing the maintenance cost decreases the frequency of selecting the maintenance operation as the optimal policy. In other words, the number of states where the optimal policy is doing maintenance will decrease. Both of the above results are consistent with our intuition. Changes in the number of maintenance proposals (as the optimal policy) due to changes in maintenance costs are shown in Figure 3.

Table 6. The effect of changes in maintenance cost on the frequency of selecting the maintenance as the optimal policy

Row number	Parameter under investigation	First period	Planning horizon (three time periods)		
			Number of states where the optimal policy is “Doing maintenance”		
	Maintenance cost	Optimal action	Period one	Period two	Period three
1	0	8	0	11	17
2	0.25	8	0	4	0
3	0.5	8	0	4	0
4	0.75	8	0	3	0
5	1	8	0	0	0

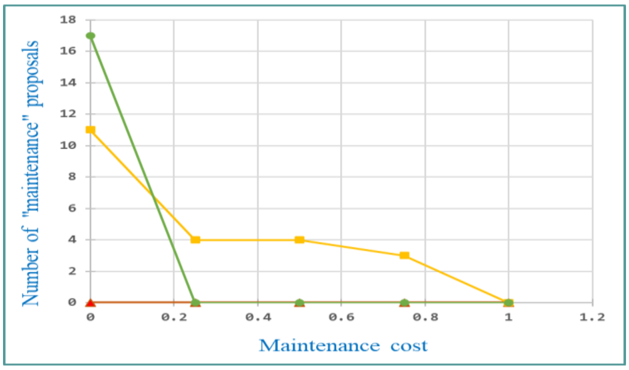

Figure 3. The effect of changes in maintenance cost on the number of maintenance proposals as the optimal policy in each period of planning horizon (Red: First period, Orange: Second period, green: Third period)

5.2.3 Changes in the average cost rate due to changes in the probability of producing conforming products

According to the results in Table 7, with the increase in the probability of producing conforming products (decreasing the probability of producing defective products), the average cost rate has decreased and the company can focus on other influencing parameters on this rate and making the necessary adjustments, hope to create the possibility of fulfilling its commitment (delivering N conforming products to the customer). The positive effect of increasing the probability of producing conforming products on reducing production costs seems obvious. This issue confirms the validity of the model and the presented results. Figure4 shows the Changes in the average cost rate due to changes in the probability of producing conforming products. To produce and fulfill the company's obligation, setting up the system at least once during the planning horizon is inevitable. In the condition of the numerical example (by adjusting parameters based on Table 3, and in the various probabilities listed in Table 7, by performing the system setup only once during the planning horizon, fulfilling the company's obligations is possible.

Table 7. Changes in the average cost rate due to changes in the probability of producing conforming products

Row number	**Parameter under investigation**	**First period**	**Planning horizon (three time periods)**				
	Probabilities of producing conforming products	Optimal action	Average of Produced units	Average of Setup number	Average of repairs	Average of conforming products	Average cost rate
1	0.5	9	9	1	0	5	+0.40
2	0.6	9	9	1	0	5	+0.40
3	0.7	9	9	1	0	6	+0.33
4	0.8	8	8	1	0	6	+0.16
5	0.9	8	8	1	0	7	+0.14
6	0.95	8	8	1	0	8	+0.12
7	1	8	8	1	0	8	+0.12

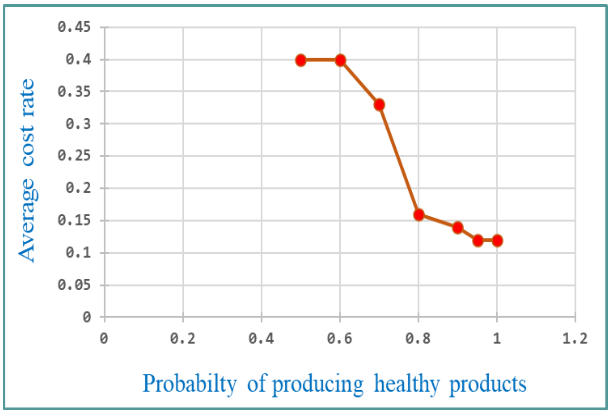

Figure 4. Changes in the average cost rate due to changes in the probability of producing conforming products

5.2.4 Changes in the average cost rate due to changes in the production cost of each unit

Based on the assumptions of the current research, the changes in the average number of produced units and the changes in the average cost rate due to the changes in the production cost are presented in Table 8. Also, the trend of these changes is shown graphically in Figure 5. The results indicate that with the increase in the production cost of each unit, the average cost rate also increases and the hope for the profitability of fulfilling obligations decreases. It is reminded that the production cost of each unit is only related to the direct cost assumed for the production of each unit which is introduced in Table 3 with the symbol c and is different from the average cost rate. As mentioned earlier, the average cost rate is obtained by dividing the sum of all costs (with a positive sign) and all revenues (with a negative sign) by the number of conforming products delivered to the customer.

Table 8. Changes in the average cost rate due to changes in the production cost of each unit

Row number	Parameter under investigation	First period	Planning horizon (Three time periods)				
	Production cost of each unit	Optimal action	Average of Produced units	Average of Setup number	Average of repairs	Average of conforming products	Average cost rate
1	0.5	10	10	1	0	9	-1.33
2	1	10	10	1	0	9	-0.77
3	1.5	9	9	1	0	8	-0.31
4	2	8	8	1	0	7	+0.14
5	2.5	7	7	1	0	6	+0.58
6	3	6	6	1	0	5	+1.00
7	3.5	6	6	1	0	5	+1.60
8	4	5	5	1	0	5	+1.60
9	4.5	0	0	0	0	0	-

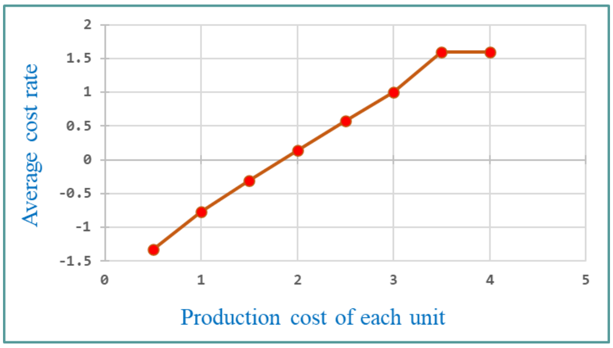

Figure 5. Changes in the average cost rate due to changes in the production cost of each unit

5.2.5 Average cost rate changes due to shortage penalty changes

As mentioned earlier in the problem statement, if the company fails to deliver the Committed quantity to the customer at the end of the planning horizon, it should pay a fine of k units for each unit of product shortage to the customer. Based on the results in Table 9, the risk of the company suffering is low when the shortage penalty is low. By increasing the shortage penalty, this risk also increases. In the conditions of the numerical example, and based on Table 9, the average number of products is often more than N, and increasing the shortage penalty by more than 3, will not have a major impact on the average cost rate. These results and the general trend of changes can be seen from Figure 6.

Table 9. Average cost rate changes due to shortage penalty changes

Row number	Parameter under investigation	First period	Planning horizon (Three time periods)				
	Shortage penalty	Optimal action	Average of Produced units	Average of Setup number	Average of repairs	Average of conforming products	Average cost rate
1	1	0	0	0	0	0	0
2	2	7	7	1	0	6	0
3	3	8	8	1	0	7	+0.14
4	4	8	8	1	0	7	+0.14
5	5	8	8	1	0	7	+0.14

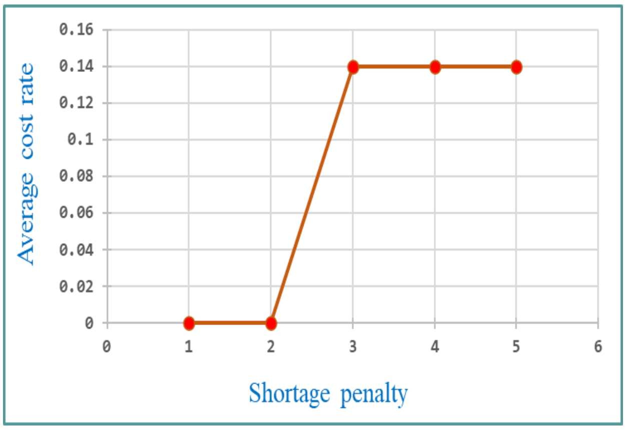

Figure 6. Average cost rate changes due to shortage penalty changes

6. Conclusions

In this paper, an MDP model for joint production and maintenance planning of a single-machine, single-product deteriorating system has been studied. It has been considered that the machine has two operational states, healthy and unhealthy. As the machine deteriorates, its state changes from 0 to 1. The quality of the produced items depends on the machine's state. when the machine is in healthy state, the probability of producing conforming items is greater than the corresponding value when the machine is in unhealthy state. The machine in unhealthy state, will remain in this state until repair is done. Repair operations restore the machine from to healthy state. The system should deliver N units of conforming products to the customer within a finite planning horizon. It is assumed that there are at most T periods of production opportunity and the production capacity is limited in each period. In each period, some actions have been considered and the system manager can decide to take the best actions so that the expected value of the system costs is minimized at the end of the planning horizon. Most of the issues related to production scheduling and maintenance planning are periodic; in other words, there is a need to make sequential decisions at some epochs. To solve this category of problems, it is necessary to use modeling and solving approaches that effectively cover the aspects mentioned above. After modeling the problem in the framework of an MDP model, Bellman's optimality equations for calculating the value of the problem states have been presented. We then use the value iteration algorithm to solve a numerical example and determine the optimal action for each problem state. Result analysis has been done based on changes in different parameters of the problem. The obtained results are compatible with mental logic and confirm the validity of the presented model. Some main results based on the example provided are as follows:

- Average cost rate changes due to setup cost changes and changes in the probability of producing conforming products are almost uniform and we don't see any shocking changes in average cost rate. With the increase in the setup cost, the average cost rate increases, and the hope that the implementation of the company's obligations will lead to profitability decreases.
- By increasing the production cost of each item more than a certain amount (4), The company's obligations are not enforceable.
- If the shortage penalty values are less than 2 or more than 3, the change in the penalty amount has no effect on the average cost rate. Only by changing the shortage penalty from 2 to 3, a major and sudden (shocking) change in the average cost rate is observed.

Considering that one of the basic parameters in determining the average cost rate is the income from the sale of products to the customer, it is possible to use the approach presented in this research to determine the level of income that leads to a negative cost rate and, in other words, making the fulfillment of obligations profitable.While addressing some limitations of previous research, the current study operates under certain assumptions that restrict its ability to capture real-life situations. First, the analysis focuses on a single-machine system, without accounting for dependencies on other system components. Therefore, a potential direction for extending this research is to apply the proposed approach to multi-machine systems with specific sequential, parallel, or hybrid configurations. Second, we have assumed two operational states for the proposed system. This can be enhanced by incorporating additional levels of system deterioration and utilizing specific degradation processes, such as gamma and Weibull, as well as defining more maintenance actions corresponding to various deterioration levels. Third, producing items over several time periods and delivering them at the end of the planning horizon imposes significant holding costs on the system. Thus, another avenue for extending this research could involve integrating inventory management into the proposed model.

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