



## Scheduling Operations with Heterogeneous Parallel Machines to Minimize Energy Consumption and Total Tardiness Using the Multi-Objective Evolutionary Algorithm

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### Abstract

In recent years, the significant increase in energy consumption and global warming have raised international concerns. Given the interconnectedness of economics, energy, and environmental concerns, energy consumption is critical in planning various systems. Optimizing production operations in various industries is a significant and complex challenge. Given the increasing global market competition and the importance of cost reduction, production process optimization has become increasingly important. One critical issue in this area is job scheduling in production systems with parallel machines. These systems' machine performance and energy consumption differences can significantly impact operating costs and job delivery times. These differences lead to machine heterogeneity, observed in many modern industries. Considering the challenges in managing energy consumption and the negative impacts of delays in product delivery, optimizing production processes to increase system efficiency and reduce energy consumption has become increasingly important. This research investigated the job scheduling problem in production systems with a heterogeneous parallel machine environment to minimize energy consumption and total job tardiness. In this research, a two-objective mathematical model for job scheduling was first designed, and a multi-objective meta-heuristic algorithm based on decomposition was used to solve this model. It was simulated in MATLAB software on several small, medium, and large sample examples. Comparing the results of the proposed method with those of previous methods shows the efficiency and superiority of the proposed method.

**Keywords:** Scheduling, Heterogeneous Parallel Machines, Energy Consumption, Tardiness

**Paper Type:** Original Research

### 1. Introduction

Energy is now a critical necessity for sustaining life, and the need for its efficient supply and use is growing daily due to dwindling energy resources. Consequently, considering energy consumption and associated costs during product planning is valuable for enhancing energy efficiency (Li et al., 2017). In today's competitive manufacturing landscape, proper job scheduling and optimal sequencing of operations are essential. Furthermore, integrating energy metrics into scheduling can lead to lower product costs, reduced production expenses, and increased profitability. In most manufacturing systems, scheduling serves as a critical decision-making process. Determining the sequence of tasks and scheduling involves allocating resources to the required activities; therefore, it can be considered a type of decision-making process to optimize one or more objectives (Bagheri et al., 2015). In modern manufacturing systems, energy consumption and total tardiness are two fundamental factors that directly impact operational costs, efficiency, and environmental sustainability. Given the increasing environmental concerns and the need to reduce the negative impacts of industrial production on the environment, minimizing energy consumption has become a strategic goal in many industries. Total tardiness is a significant issue in production management, referring to delays in delivering products and services to customers. Due to its impact on customer satisfaction and logistics costs, it has become a key performance indicator in manufacturing systems (Wang et al., 2020). Energy consumption in manufacturing systems constitutes a significant portion of total industrial energy consumption. Optimizing energy consumption reduces operational costs and contributes to reducing greenhouse gas emissions and preserving the environment. One of the effective ways to reduce energy consumption is to optimize the scheduling of production operations. This issue is crucial in manufacturing systems with heterogeneous parallel machines where machines differ in processing capability and energy consumption (Garg & Gupta, 2019). In recent years, the significant increase in energy consumption and global warming have raised global concerns

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(Abikarram et al., 2019). The importance of efficient and effective energy use is increasing daily due to the depletion of non-renewable resources. Concurrently with the production and consumption of energy, many greenhouse gases are released into the Earth's atmosphere. Manufacturing industries, due to their extensive energy use, produce a more significant amount of greenhouse gases. Therefore, improving energy efficiency and controlling greenhouse gas emissions in the manufacturing sector seems essential (Wu & Che, 2019). One of the most important tasks that do not require significant investment and can control and minimize energy consumption simply by changing the production process is scheduling as a tool to improve energy efficiency (Liang et al., 2015). Therefore, to achieve energy-saving goals and reduce financial costs, production managers may need the help of scheduling strategies (Pinedo et al., 2015). Considering the setup times is another practical consideration issue in scheduling. Considering the economic ability of the customers in today's world, the desire to satisfy customers requires all production and service units to meet their demands on time and plan in such a way as to minimize the costs of late delivery in meeting customer demand and be able to deliver the customer's order on time (Li et al., 2016). The main issue of this research is to develop a mathematical model and provide a solution that can simultaneously optimize energy consumption and total tardiness of jobs in parallel heterogeneous machine systems. The processing time of jobs and the amount of energy required to process jobs on machines with different speeds are different, and a specific due date is considered for each job. So, it should be determined which job has to be assigned to which machine, along with the sequence of processing of the jobs assigned to each machine. Scheduling in manufacturing environments with heterogeneous parallel machines is inherently complex due to factors such as varying processing speeds and energy consumption among machines, as well as the multi-objective nature of the problem, requiring a balance between conflicting goals. These types of problems fall into the category of NP-hard problems, for which no efficient, exact algorithms have been found (Garey & Johnson, 1979). Therefore, this research proposes a novel multi-objective decomposition-based heuristic algorithm to address this challenge. The obtained results are compared with those of previous studies. The rest of the paper is organized as follows: The second section deals with the literature review on the subject and previous work. The third section discusses the proposed mathematical model, the research methodology, and data analysis. The fourth section examines the findings and simulation results. The conclusions are discussed in the fifth section.

## 2. Literature Review, Challenges, and Previous Work

Multi-objective optimization problems face significant challenges when balancing various and often conflicting objectives. It is often impossible to attain an optimal solution that fully satisfies all objectives. Consequently, solving such problems typically involves finding a Pareto-optimal solution where no objective can be improved without degrading at least one other. The degree to which each objective is satisfied is determined by its assigned weight or importance, a managerial challenge influenced by numerous factors. Some important challenges are discussed in the sections below.

### 2.1 Productivity and Operating Costs

Achieving high efficiency and reducing operational costs are paramount for organizations across all sectors. Energy consumption represents a substantial component of operational expenses within manufacturing industries. Given the escalating energy costs and dwindling natural resources, optimizing energy consumption has become a strategic approach to curtail production costs significantly. By streamlining production processes, enhancing machine scheduling, and employing advanced algorithms, organizations can effectively reduce energy consumption and, consequently, improve the overall efficiency of their production systems. Existing research in scheduling has demonstrated that the integration of optimization models for energy management, coupled with strategies such as allocating lightweight tasks to less energy-intensive machines and heavier tasks to more capable machines, as well as leveraging off-peak electricity rates, can yield energy cost reductions of up to 20% (Zhang et al., 2021). Assigning lightweight tasks, characterized by lower energy requirements and shorter processing times, to less energy-intensive machines contributes to energy cost reduction and overall system performance enhancement. Moreover, this allocation strategy mitigates machine wear and tear, extending its operational lifespan. Conversely, allocating heavy tasks to high-capacity machines capable of handling demanding workloads optimizes machine utilization and expedites product and service delivery (Zhang et al., 2021).

### 2.2 Environmental Sustainability

Considering the environmental crises, climate change, and the consequences of excessive greenhouse gas emissions, industrial machinery and equipment within manufacturing industries have emerged as major energy consumers and significant contributors to environmental pollution (Dornfeld et al., 2006). Optimizing the sequencing of processes and minimizing delays can reduce the demand for energy generated from finite natural resources

such as oil, gas, and coal. Consequently, improving energy efficiency in manufacturing industries helps preserve these resources for future generations and alleviates pressure on natural ecosystems. Conversely, product or service delivery delays can lead to customer dissatisfaction, unnecessary activities, idle machinery, and increased energy consumption. Enhancing energy utilization in these industries can mitigate greenhouse gas emissions and their negative environmental impacts (Weinert et al., 2011).

### 2.3. Improving Competitiveness and Customer Satisfaction

In today's highly competitive marketplaces, timely delivery of products and services has emerged as a critical factor in customer retention and acquisition. Organizations can promptly fulfill customer commitments by optimizing production schedules and minimizing delays. This issue enhances customer satisfaction and loyalty. Moreover, reducing production costs through energy efficiency improvements enables companies to allocate more resources toward innovation and new product development. Such investments can lead to higher-quality products and services, strengthening a company's competitive position in global markets. By emphasizing environmental sustainability, organizations can promote the adoption of green technologies and renewable energy sources. This technique reduces energy consumption and enhances a company's corporate social responsibility image (Gong et al., 2018).

### 2.4. Managerial and Technical Challenges

Managing the production systems with heterogeneous parallel machines, particularly in advanced industries, is fraught with numerous technical challenges and managerial complexities. A primary challenge stems from different machines' varying operational capacities and energy consumption. These disparities necessitate sophisticated scheduling approaches and advanced mathematical models capable of simultaneously optimizing energy consumption and minimizing overall job tardiness. In such contexts, production managers must make strategic and operational decisions that rely on accurate data and sophisticated analytical tools. Multi-objective optimization algorithms are among these tools, aiding managers in balancing various, often conflicting, objectives. Developing optimization models and algorithms to reduce energy consumption and job tardiness can significantly improve the management of these challenges. Moreover, responding to rapid market fluctuations and customer demands is another complex management aspect. Organizations must possess high levels of flexibility to quickly adjust schedules and resource allocations based on varying consumer demands. These changes may be driven by economic fluctuations, seasonal variations, or emerging market trends, requiring swift and effective responses from managers. Effective interaction with internal and external stakeholders is essential for creating a coordinated and efficient schedule, demanding employee training, and implementing new technologies (Lee & Chen, 2020).

### 2.5. Research Methodology

The research problem was modeled using a mathematical programming tool with two primary objectives. The first objective was to minimize the overall energy consumption of production machines, and the second was to minimize total tardiness. Various methods and tools exist for solving scheduling problems, aiding in process optimization. These include mathematical models, optimization algorithms, and application software. Multi-objective optimization is a key approach in optimization problems, seeking to simultaneously optimize two or more objectives. This issue necessitates balancing multiple conflicting or non-aligned goals. It allows decision-makers to explore a set of optimal solutions rather than a single solution, enabling them to make the best choice based on specific needs and priorities. Multi-objective optimization also facilitates more accurate analysis and evaluation of solutions, improving decision-making, especially in contexts requiring a balance between various factors and their interactions. Numerous methods have been proposed for solving multi-objective optimization problems, categorized into classical and metaheuristic methods. Classical methods often attempt to solve the problem by transforming it into a single-objective problem. Evolutionary optimization-based methods, inspired by natural algorithms, have gained significant attention due to their ability to search the solution space effectively and generate efficient Pareto fronts (Coello, 2007). This research uses a multi-objective evolutionary algorithm based on decomposition (MOEA/D) to solve the proposed mathematical model, which will be discussed in detail in the following sections.

### 2.6. Previous Work

Yalaoui and Chen (2002) investigated parallel machine scheduling to minimize total tardiness. Anghinolfi and Paolucci (2007) proposed a hybrid approach for parallel machine scheduling to minimize total tardiness. Langer et al. (2014) contributed to the field of energy-efficient optimization by proposing a model-based approach to energy saving that incorporates integrated and related production processes. Li et al. (2015) investigated the scheduling of unrelated parallel machines to minimize energy costs and tardiness. They considered energy consumption

for various machine states, including idle, processing, and startup, and developed a bi-objective mathematical model integrated into a single objective. Sharma et al. (2015) extended simulated annealing to minimize electricity costs in flexible-speed parallel machine shops. These studies highlight the growing interest in developing efficient algorithms to address complex optimization problems in manufacturing and computing environments. Jia et al. (2017) employed the ant colony optimization algorithm to solve the batch processing problem on parallel machines, aiming to minimize energy consumption and makespan while considering time-varying energy pricing. Their experimental results, obtained from random data, were compared with those of the NSGA-II algorithm. Yin and Liu (2017) proposed a multi-objective algorithm for parallel machine scheduling. Tan et al. (2018) provided an optimal solution to the energy-efficient scheduling problem for batch processing on parallel machines under time-varying energy pricing policies. Wu and Che (2019) delved into the energy-efficient scheduling problem for unrelated parallel machines. By investigating the Memetic Differential Evolution Algorithm (MDEA), they explored instances involving up to 7000 tasks and 200 machines. Johnson & Wang (2019) proposed a model to minimize total completion time and energy consumption on parallel machines. Ayough and Khorshidvand (2019) proposed a new model for solving the cell formation problem and developed an algorithm based on Simulated Annealing(SA) and Particle Swarm Optimization(PSO) to solve the proposed model. Pan et al. (2020) used artificial intelligence techniques to schedule parallel machines to minimize energy consumption. Öztöp et al. (2020) investigated a two-objective energy-efficient scheduling problem in permutation flow shops. Smith and Doe (2020) studied heterogeneous parallel machine scheduling to minimize energy consumption. Zhou and Guo (2021) studied the scheduling problem for unrelated parallel machines, considering multiple resource constraints to minimize makespan and total energy consumption. To address this problem, they proposed a multi-objective artificial immune system algorithm. Modos et al. (2021) focused on dedicated parallel machines while incorporating energy consumption constraints. The researchers examined the problem under four scenarios and developed a heuristic algorithm to solve the general case. Asadpour et al. (2022) investigated the parallel machine scheduling problem, considering workload division to reduce tardiness and energy usage. They employed an epsilon-constraint method for minor instances and simulated annealing for larger ones. Smith and Doe (2022) proposed an algorithm for parallel heterogeneous machine scheduling considering energy consumption. In their research, Ayough and Khorshidvand (2023) considered the assignment problem of the heterogeneous workforce to a U-shaped assembly line with uncertain processing time and proposed a non-linear programming model for solving the problem. Tian and Zheng (2024) explored the single-machine scheduling problem under time-varying electricity prices and introduced a novel model to minimize electricity costs. Recent research has focused on optimizing production processes to balance energy efficiency and performance. Zhang and Chen (2024) developed a novel algorithm for batch production systems to minimize energy consumption while maximizing productivity. Table 1 briefly overviews existing research on energy-efficient scheduling. It details the key characteristics of each study, including the scheduling environment, setup time considerations, energy consumption models, and the corresponding mathematical formulations.

**Table 1.** The Summary of Previous Studies

Reference	Contribution					
	Heterogeneous Parallel Machines	Proposing Mathematical Model	Proposing Heuristic Algorithm	Setup Type	Periodic Energy Consumption	Flowshop Environment
Liu et al. (2013)	-	-	✓	-	-	Permutation Flowshop
Avalos et al. (2015)	✓	✓	✓	Sequence Dependent	✓	-
Dai et al. (2013)	-	-	✓	-	-	Flexible Flowshop
Che et al. (2017)	✓	✓	✓	-	✓	-
Cheraghalikhani et al. (2019)	-	✓	✓	Sequence Dependent	-	-
Saberi et al. (2020)	✓	✓	✓	-	✓	-
Abedi et al. (2020)	-	-	✓	-	-	-
Zhang et al. (2021)	-	-	✓	Sequence Dependent	-	-
Meng & Pan (2021)	✓	-	-	Sequence Dependent	-	✓
Proposed Method	✓	✓	✓	-	-	-

The novelty of this research lies in considering the energy scale in scheduling, minimizing the total task tardiness, and proposing a novel heuristic algorithm that has not been previously employed in this context.

### 3. The Proposed Method

This proposed mathematical model is described in detail in the following section

#### 3.1 Indices, Parameters, and Variables

i	Index of jobs
j	Index of machines
n	Total number of jobs
m	Total number of machines
ri	Release time of the ith job
di	Due date of the ith job
OP <sub>j</sub>	Energy consumption of jth machine (Watt) while working
IP <sub>j</sub>	Energy consumption of jth machine (Watt) while idle
P <sub>ij</sub>	Processing time of jth job by ith machine
T <sub>i</sub>	Tardiness of ith job
C <sub>i</sub>	Completion time of ith job
C <sub>Max</sub>	Total completion time (Make span)
X <sub>ijt</sub>	The binary decisions variable is true if the jth machine processes the ith job at time t; otherwise, it is false.
Y <sub>jt</sub>	The binary decision variable is true if the jth machine is idle at time t; otherwise, it is false.
M	A big number.

#### 3.2 The Objective Functions

The first objective is minimizing the total tardiness, as defined by Equation (1).

$$\text{Min } f_1 = \sum_{i=1}^n T_i \quad (1)$$

The second objective function is minimizing the total energy consumption of all machines. This model assumes time is discrete, starting from zero and progressing in increments of one-time units until C<sub>max</sub>. Therefore, the total energy consumption is equal to the sum of the energy consumption of all machines during both idle and busy times, as expressed in Equation (2).

$$\text{Min } f_2 = \sum_{t=0}^{C_{max}} \sum_{j=1}^m (Y_{jt} \cdot IP_j + (1 - Y_{jt}) \cdot OP_j) \quad (2)$$

#### 3.3 The Mathematical Model

The complete mathematical model is proposed below.

$$\text{Min } f_1 = \sum_{i=1}^n T_i$$

$$\text{Min } f_2 = \sum_{t=0}^{C_{max}} \sum_{j=1}^m (Y_{jt} \cdot IP_j + (1 - Y_{jt}) \cdot OP_j)$$

s.t.

$$T_t = \max(0, C_i - d_i) \quad \forall i \quad (3)$$

$$C_i = X_{ijt} \cdot (t + P_{ij}) \quad \forall i, j, t \quad (4)$$

$$C_{max} \geq C_i \quad \forall i \quad (5)$$

$$t \leq r_i + M(1 - \delta) \quad \forall i, t \quad (6)$$

$$-M(1 - \delta) \leq X_{ijt} \leq M(1 - \delta) \quad \forall i, j \quad (7)$$

$$\sum_{t=t'}^{t'+p_i} Y_j = (1 - X_{ijt'}) + X_{ijt'} \cdot P_{ij} \quad \forall i, j, t' \quad (8)$$

$$\sum_{t=0}^{C_{max}} \sum_{j=1}^m X_{ijt} = 1 \quad \forall i \quad (9)$$

$$\sum_{i=1}^n X_{ijt} \leq 1 \quad \forall j, t \quad (10)$$

$$C_i, r_i, d_i, P_i, P_{ij}, C_{max} \geq 0 \quad (11)$$

$$X_{ijt}, Y_{jt}, \delta \in \{0, 1\} \quad (12)$$

Constraint (3) is used to calculate the tardiness of each job. Constraint (4) calculates the completion time of each job. Specifically, if job  $i$  starts at time  $t$  on machine  $j$ , its completion time will be  $t + p_{ij}$  time units after the start of processing. Constraint (5) is added to the model to minimize the makespan. Constraints (6) and (7) ensure that no job is scheduled before its release date. Equation (8) stipulates that if a job is assigned to a machine at time  $t'$ , the machine must remain busy until the task is completed at time  $t' + p_{ij}$ . This condition is enforced by setting the binary variable  $Y_{jt}$  to 1 for the machine's busy period. Consequently, the summation of  $Y_{jt}$  over this interval must equal  $P_{ij}$ . Constraint (9) ensures that every job in the problem is assigned to a machine and scheduled within the overall planning horizon, defined as the time interval from 0 to  $C_{max}$ . It prevents situations where a job remains unallocated or is not scheduled. Constraint (10) prevents the overloading of any machine. It ensures that a machine is only working on one job at a time, preventing conflicts or overlaps in the schedule. Constraints (12) typically define the nature of the variables and parameters used in the model.

### 3.4. MOEA/D Approach

The Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D), proposed by Zhang and Li (2007), is a highly effective approach for solving multi-objective optimization problems. This evolutionary algorithm addresses multi-objective problems by decomposing them into single-objective subproblems. A distinctive feature of MOEA/D is its focus on iteratively improving a set of weights associated with different objectives rather than directly optimizing the entire objective space. By effectively leveraging information from neighboring subproblems, MOEA/D enhances the quality of the Pareto front during the evolutionary process. Compared to other multi-objective methods such as NSGA-II, MOEA/D often exhibits lower computational complexity and is well-suited for problems with many objectives or constraints. Ultimately, MOEA/D generates a Pareto optimal set, also known as the Pareto front, which comprises a collection of non-dominated solutions. Improving convergence and maintaining diversity are critical considerations in multi-objective algorithms, as highlighted by Wang et al. (2020). This algorithm is based on the Chebyshev weighted decomposition method, which considers various weight vectors for different objectives. By employing these vectors, it endeavors to approximate the results towards the Pareto

optimal front. A salient feature of this algorithm is its ability to reduce solution time and enhance the quality of the obtained outputs.

## 4. The Computational Results

### 4.1. Simulation Examples

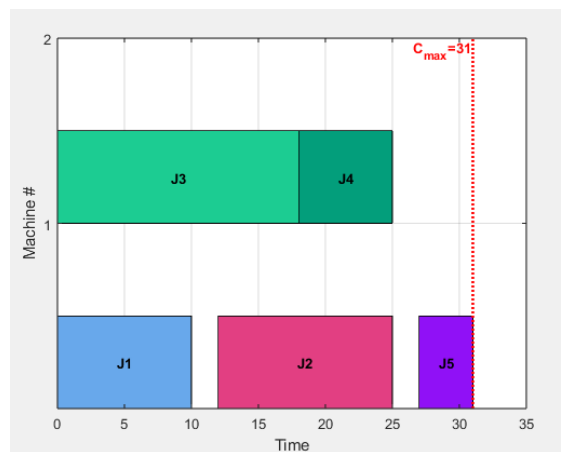
The proposed method was implemented in MATLAB 2023 on a personal computer and tested on several benchmark instances. The characteristics of the simulated instances are summarized in Table (2). For comparative purposes, the same instances were solved using the NSGA-II algorithm, and the computational results are tabulated in Table (3). The release time of jobs, processing time, and due dates were generated randomly following uniform and normal distributions. The number of iterations was determined empirically, increasing until no further improvement in the final solution was observed. It is worth noting that each instance was executed 10 times, and the best results were reported.

**Table 2.** The specification of simulated examples

Example #	(n)	(m)	ri	Di	Pij	No. of Iterations
1	5	2	U[0..50]	ri+U[10..40]	N(20,36)	35
2	10	3	U[0..100]	ri+U[10..50]	N(30,49)	50
3	20	5	U[0..150]	ri+U[10..60]	N(40,64)	150
4	35	7	U[0..200]	ri+U[10..70]	N(50,81)	250

#### 4.1.1 Example 1 Results

In this example, five jobs are scheduled on two machines, as illustrated in the Gantt chart in Figure 1. The horizontal axis represents time, while the vertical axis indicates the machine number. The idle time observed between consecutive jobs on a machine is attributed to the unavailability of jobs at that specific moment. Consequently, the machine remains idle, awaiting the arrival of the next job. Job J5 could have been assigned to machine 2, but this allocation was not executed due to higher associated costs.



**Figure 1.** The Gantt chart of the final scheduling of jobs in example 1

### 4.1.2 Example 2 Results

This example investigates the scheduling of 10 tasks on three parallel machines. Each machine has a unique processing speed and energy consumption when idle or busy. Furthermore, each task has its own ready time, due date, and processing time. The arrival times of tasks and their due dates are randomly generated following uniform distributions between  $a$  and  $b$  ( $U(a,b)$ ). The processing time of each task on each machine is determined by a normal distribution  $N(\mu,\delta^2)$ . The corresponding Gantt chart, presented in Figure (2), illustrates the machine status (busy or idle) at any given time. The value of  $C_{max}$  is 55, representing tasks 9 and 10 completion time, which finish simultaneously.

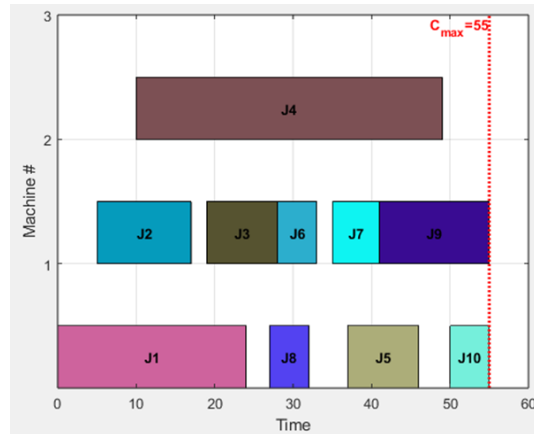


Figure 2. The Gantt chart of the final scheduling of jobs in example 2

### 4.1.3 Examples 3 & 4 Results

Figures (3) and (4) present the final Gantt charts of job scheduling on machines in an environment with more jobs and complex machines. These charts illustrate how the machines have been utilized concurrently and optimally to execute the jobs. Moreover, they depict idle times or machine utilization due to the absence of ready jobs for processing. This analysis aids in scheduling optimization and reveals instances where resources were underutilized. Figure (5) displays the Pareto frontier of the best final solution for Example 4. The horizontal axis represents the first objective function, while the vertical axis corresponds to the second objective function. Since the units and numerical scales of these two objective functions differ, the values of both objective functions were normalized (scaled) before plotting the graph. Red points indicate the optimal solutions located on the Pareto efficient frontier. Other points, considered dominated solutions, are represented by small hollow circles.

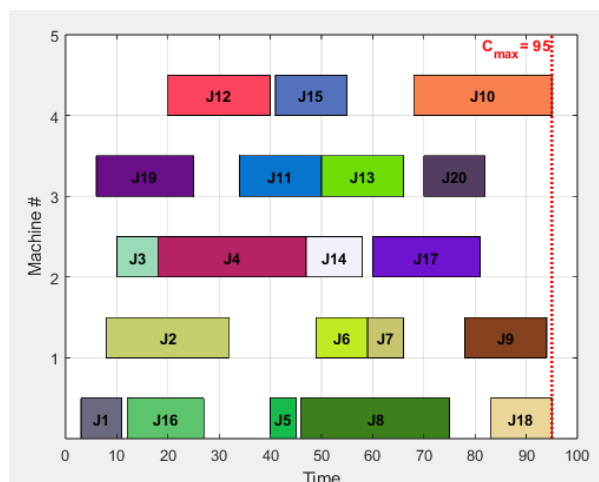


Figure 3. The Gantt chart of the final scheduling of jobs in example 3

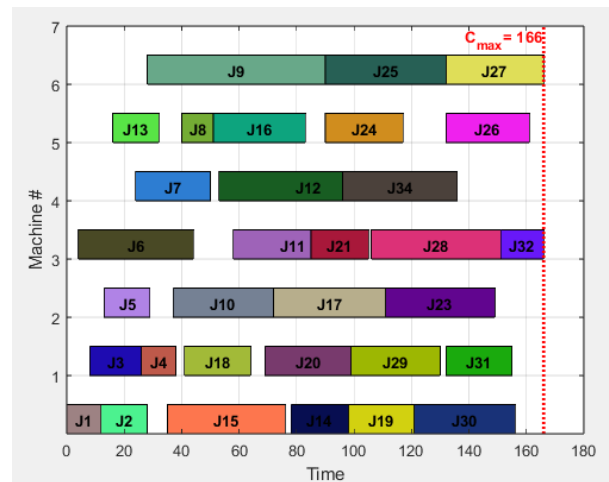


Figure 4. The Gantt chart of the final scheduling of jobs in example 4

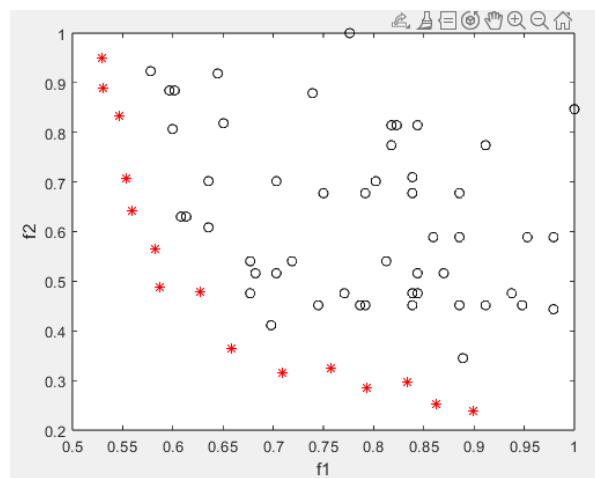


Figure 5. The Pareto frontier of the best solution obtained in example 4

## 4.2 Performance Evaluation

Given the metaheuristic nature and inherent stochasticity of the employed solution method, the obtained solution may exhibit slight variations across different runs. Moreover, metaheuristic algorithms typically involve a primary iterative loop, which must be executed multiple times in each algorithmic run to enhance the solution quality progressively, i.e., to achieve convergence. This study adopted a trial-and-error approach to determine the optimal number of iterations. Precisely, the algorithm was executed for various iteration counts, and the outcomes were evaluated. The algorithm gradually generates superior solutions in each iteration, improving solution quality over time. However, the improvements achieved through additional iterations may diminish or even cease beyond a specific iteration count. Determining the iteration count using a trial-and-error method is an important parameter directly influencing the results' quality. This parameter dictates the frequency of algorithm updates required to attain an optimal or near-optimal solution. Empirical evidence from repeated algorithm executions indicates that incremental improvements in results diminish beyond a certain iteration threshold. The algorithm converges to a steady state, and subsequent iterations produce negligible solution variations. This threshold is denoted as the optimal number of iterations. Numerical results for the best values obtained from repeated simulations of the proposed method and NSGA-II applied to the examples in Table (2) are summarized in Table (3). The results reveal that both methods performed identical in Examples 1 and 2. In Example 3, the proposed method demonstrated superior performance in terms of  $C_{max}$  and the primary objective function, although it marginally underperformed concerning the secondary objective function. However, in Example 4, the proposed method consistently outperformed NSGA-II across all evaluation criteria.

**Table 3.** Numerical results of the proposed method and NSGA-II

Example #	Proposed Method			NSGA-II		
	Cmax	f1	f2	Cmax	f1	f2
1	31	14	489053	31	14	489035
2	55	22	671361	55	22	671361
3	95	29	925105	98	33	912477
4	166	35	1048598	178	42	1129366

## 5. Conclusions

This research investigated the critical role of energy consumption optimization and effective scheduling management in modern production systems with heterogeneous parallel machines. Given the increasing importance of issues such as energy sustainability and system efficiency, the findings of this study can provide practical solutions for improving the performance of parallel processing systems. Since examining energy consumption in conjunction with optimizing scheduling-related criteria is highly efficient, this research focused on optimizing the job scheduling problem to manage energy consumption and reduce job latencies using a heuristic algorithm. A two-objective mathematical model was proposed to optimize the research problem. The proposed model was coded in MATLAB using a decomposition-based multi-objective evolutionary metaheuristic algorithm and simulated on a personal computer using four hypothetical examples. The examples were solved using the genetic algorithm to compare the performance of the proposed method. It was observed that in examples 1 and 2, both methods performed similarly. However, in example 3, the proposed method performed better regarding the first objective function (Cmax) but slightly worse in the second objective function. In example 4, the proposed method outperformed the NSGA-II algorithm in all three aspects. Using the decomposition-based multi-objective evolutionary algorithm, the results obtained showed a significant improvement in performance metrics. The simulations conducted in the MATLAB environment and comparing the results with previous methods, such as the genetic algorithm, confirmed the efficiency and superiority of the proposed algorithm in optimizing scheduling and energy consumption. In particular, for larger-scale examples, the proposed algorithm demonstrated the ability to optimize task completion time, which led to significant energy consumption reductions. These findings underscore the importance of strategic decision-making and the application of advanced algorithms in addressing contemporary challenges within industrial settings. Ultimately, this research empowers managers and researchers to make more informed decisions regarding time and energy resource management through a deeper understanding of optimization algorithms and methodologies. Furthermore, this study can serve as a scientific reference for future researchers in scheduling optimization and energy management in manufacturing industries, paving the way for further research and advancements in this domain. This research has limitations, as the proposed model is confined to specific conditions. Consequently, it may exhibit reduced efficacy in more complex industrial settings. Therefore, conducting empirical and field studies to evaluate and refine the models under diverse real-world circumstances is imperative. To further enhance this research, it is recommended to incorporate assumptions that better align with real-world scenarios. For instance, the model could account for unforeseen events such as processing disruptions, variable machine speeds, and repair times. Developing a similar model for other production environments, such as job shop and flow shop systems, would broaden its applicability. Furthermore, integrating advanced machine learning and artificial intelligence techniques can significantly improve the efficiency and accuracy of scheduling algorithms, thereby paving the way for optimized production processes. Investigating hybrid systems comprising a combination of homogeneous and heterogeneous parallel machines can enrich the research and enhance the model's practical utility. Developing dynamic scheduling models capable of adapting to fluctuating conditions is another promising avenue for future research.

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