



A multi-objective fuzzy goal programming model for portfolio selection in Tehran stock exchange

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Abstract

In this research, a new emerging model for the Tehran stock exchange market is considered, and a model with realistic constraints for the mentioned market is provided. Realistic constraints are incorporated in this model for applicable purposes, one of which is the transaction cost. The limited maximum number of stocks that should be invested is also considered. Additionally, constraints have been added to the classic portfolio selection model to prevent stocks from being bought in tiny quantities and avoid over-buying some stocks. The mentioned constraint can improve diversity in the selected portfolio. One of the factors considered by many financial market investors is the amount of liquidity of the stocks they have purchased. This study also considers the amount of portfolio liquidity as one of the essential objective functions affecting the selection of the portfolio. Finally, in the model presented in the present study, the investors can have a different stock portfolio according to their preferences. The proposed model is multi-objective fuzzy goal programming, which can simulate uncertainty in the Tehran stock exchange market and provide a rational framework for investors who invest in the financial markets. As the numerical instances show, the solutions when additional constraints are added to the mathematical model are close to exact results. This difference became significant when the maximum number of stocks increased. According to the results, when the number of stocks increases, GAMS software loses its functionality, and the utilization of meta-heuristics as an option is inescapable. Finally, the harmony search algorithm with added realistic constraints has provided better portfolios in such a situation. To compare the results, a genetic algorithm was used as the competing algorithm. After solving different instances and comparing the results of the algorithms, the superiority of the proposed harmony search algorithm has been proved.

Keywords: Portfolio selection, Harmony search algorithm, Fuzzy goal programming, Liquidity

Paper Type: Original Research

1. Introduction

Investment means buying a number of stocks with expected risk and return. Return in this definition is defined as the ratio of outcomes to incomes, while the risk is considered as a probability of which initial deposit can be reduced. In financial markets, risk and return play an important role, and investors try to reduce risk at a predetermined return level or maximize their investment return at specific risk levels. The selection of various stocks in financial markets was proposed by Markowitz's first time, and the utilization of mathematical programming raised the accuracy of portfolio selection. Markowitz first used a mathematical formulation whose objective functions were to minimize the risk and maximize the return. In this mathematical model, a constraint was considered that limited investment budget (Davies et al., 2016). The problem of portfolio selection is significant in the financial markets. There are several exact and stochastic models for allocating capital in terms of reliability and uncertainty. If the nature of the problem is the same as purchasing limits, then the usual linear programming techniques can be used. However, in some cases, there are more constraints when making decisions. Although several models have been proposed for decision in these circumstances, most mathematical models have sought to minimize risk and maximize return rate. (Yue & Wang, 2017). In the portfolio selection problem, mathematical models have tried to connect a simple relation between risk and return, which has led such models to have unrealistic results. For this reason, the utilization of a fuzzy concept can approximate reality with maximum accuracy. Researchers try to define their models with some simplistic hypotheses in some portfolio selection models, which can have unrealistic outcomes (Saborido et al., 2016). Another constraint in the portfolio selection problem is limited to the number of stocks that should be selected. The constraint of the limited number of stocks in the portfolio will make the model closer to real-world investment decisions. Hence, the proposed model can be widely used in investment decisions.

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Therefore, the investor can determine the number of stocks he or she wants to create efficient portfolios at different risk levels and returns concerning the model's efficiency. The mentioned constraint could dramatically reduce investment risks by lowering transaction costs. In actual financial decision-making, the investor often requires, for various reasons, the exact determination of the number of stocks in his or her investment portfolio. In such a situation, adding the constraint of the number of stocks into a model brings it closer to the real-world condition, and solving it, will bring more efficient and practical decisions to the investor (Liagkouras & Metaxiotis, 2018). In essence, the portfolio selection problem attempts to minimize the portfolio's risk and maximize the rate of return at the same time. Markowitz's model assumes that investors are logically going to maximize returns at a particular risk level or minimize the risk at a certain level of return. For the convenience of the decision-maker, the usual margin of expected returns is considered constant, and then under this constraint, the portfolio risk is minimized (Macedo et al., 2017). There are many multi-objective problems in nowadays environments, and humankind must make decisions in such a situation. One of the most critical conditions is when there is more than one objective that must be considered. The portfolio selection problem is when multi-objective decision models are utilized for such problems (Mashayekhi & Omrani, 2016), (Speranza, 1996) defined a mixed-integer programming model with some realistic constraints such as transaction cost and minimum investment units and examined it in the Milan stock market. Since when the size of the problem increased, the proposed model could not be solved with exact methods, a novel heuristic method was designed (Konno & Yamazaki, 1991). proposed a linear programming formulation for the portfolio selection problem. (Mansini & Speranza, 1999) wrote an article named "Heuristic algorithm for the portfolio selection optimization model with minimum transaction lot" which used a mixed-integer programming method in their research, (Mansini et al., 2007) defined an optimization procedure with linear programming under environmental risks for the portfolio selection problem. (Mercangöz & Eroglu, 2019) designed a novel method for the portfolio selection problem and solved the model by the use of a genetic algorithm that could optimize the final results in each iteration. Yaman and Dalkılıç utilized neural networks in the portfolio selection problem. Yue et al suggested a genetic algorithm for the portfolio selection problem, and semi-variance was used as a risk criterion. Mashayekhi and Omrani presented a multi-objective fuzzy model for the portfolio selection problem, which combined the data envelopment analysis approach with the Markowitz model. In the mentioned model, stocks returns were modeled with fuzzy returns. According to news and social media data, Lamon et al. have done novel research about predicting cryptocurrency prices. A significant area of interest for many researchers is the use of meta-heuristics to tackle complex problems. In their review paper, Soler-Dominguez et al explored the application of meta-heuristic algorithms in financial markets, offering recommendations for future research in this area. Strumberger et al developed a hybrid algorithm combining the ant colony and bat algorithms to address the portfolio problem, comparing it with genetic and particle swarm algorithms and demonstrating its superiority. In his paper, Hosseini et al examined the portfolio problem and used the semi-variance index to identify the risk. Then, a hybrid metaheuristic algorithm was used, and the results show the superiority of the proposed algorithm over the genetic algorithm. Ismail and Pham considered the problem of portfolio selection. In this model, stock returns were considered randomly, and a dynamic programming method was used to model and solve the proposed model. Nystrup et al studied portfolio selection in multi-period situations using a predictive control model for optimization and the Markov concept for prediction, also considering transaction and holding costs. Kar et al proposed a fuzzy multi-objective model for portfolio selection, using the Sharp ratio and value at risk as objectives and solving the model with genetic and scatter search algorithms. Mohammadi and Nazemi focused on portfolio selection under uncertainty, employing the value-at-risk criterion and solving their linear programming model with a neural network method, analyzing the results across various examples. Galankashi et al applied the fuzzy analytic network method to portfolio selection, using a questionnaire to identify and rank essential criteria for this process. Asgari and Behnamian tackled the essential issue of stock portfolio selection in financial markets, aiming to optimize capital allocation among various stocks to maximize returns while minimizing risk. They noted that many existing models rely heavily on historical data and neglect future changes. To address this, they proposed a multi-objective, multi-stage stochastic model that integrates scenario generation with multi-stage stochastic programming, considering both past data and potential future scenarios. The model was solved using a harmony search-based meta-heuristic algorithm, validated with real Iranian market stock data. In a subsequent study, Asgari and Behnamian presented a self-adjusting algorithm for portfolio selection, designed to optimize capital allocation to maximize returns and minimize risk. This research builds on traditional models by introducing new constraints and a unique objective function. Using the Sharp ratio for portfolio comparison, they developed a genetic algorithm-based self-adjusting algorithm, eliminating the need for manual parameter adjustments. The effectiveness of this methodology was demonstrated under various market conditions, with improvements including a new objective function prioritizing the price-to-earnings ratio and the addition of technical analysis constraints, enhancing profitability and risk management. The new research introduces a model tailored for the Tehran stock exchange, incorporating realistic constraints like transaction costs and liquidity, which were not considered in the

previous studies. Unlike Asgari and Behnamian's earlier works, this study employs multi-objective fuzzy goal programming to simulate market uncertainty and allows for investor-specific portfolio customization. Liquidity plays a crucial role in financial markets, ensuring that investors can quickly execute trades without significantly affecting stock prices. This factor is particularly relevant in emerging markets such as the Tehran Stock Exchange (TSE), where market inefficiencies and lower trading volumes compared to developed markets can lead to higher transaction costs and difficulties in executing large orders. Unlike traditional portfolio selection models that focus primarily on risk and return, incorporating liquidity as an objective function provides a more realistic framework for investment decisions in the TSE. A liquid asset is one that can be traded with minimal transaction costs and time delays, enabling investors to reposition their portfolios efficiently in response to market changes. Empirical studies have underscored the relationship between market liquidity and expected returns. For instance, Nasraoui et al found that liquidity is a crucial determinant of asset returns, with lower liquidity often associated with higher expected returns due to the liquidity premium investors demand for holding such assets. Liquidity is a fundamental element in investment decisions, affecting investors' preferences and portfolios' overall performance. According to a recent study, liquidity refers to the ease of asset conversion into cash, which is crucial for achieving optimal returns. The classification of liquidity based on factors such as bid-ask spreads has been explored, suggesting that recognizing different liquidity levels within portfolios can enhance selection processes. Recent studies have demonstrated the effectiveness of fuzzy goal programming in financial markets, particularly in handling uncertainty in investment decisions. For instance, Yadav et al applied fuzzy multi-objective models to optimize portfolio selection under volatile market conditions, showing improved risk-adjusted returns. Our study builds on these advancements by incorporating liquidity considerations, which have been largely overlooked in previous fuzzy goal programming applications. Additionally, the rise of hybrid metaheuristics and machine learning-based methods has expanded the landscape of portfolio optimization. Techniques such as hybrid genetic algorithms with ensemble machine learning (Faridi et al., 2023) and deep learning-assisted portfolio selection (de Moraes et al., 2025) have gained attention for their adaptability to dynamic markets. While these approaches offer promising results, they often require extensive computational resources and large historical datasets. In contrast, our proposed fuzzy goal programming model provides a structured yet flexible decision-making framework that can be implemented with limited data and computational effort, making it particularly suitable for emerging markets like the Tehran Stock Exchange. In this research, a fuzzy goal programming mathematical model is designed to select a portfolio in the Tehran stock exchange. In this model, adding some constraints has been attempted to bring the results closer to a real-world situation. Initially, the cost of trading as one of the critical factors in the financial market has been introduced into the mathematical model. Other constraints have been added to the classic portfolio selection model to prevent the purchase of a small portion of stocks and to avoid over-buying a particular stock. Additionally, liquidity is an essential objective function in buying stocks in financial markets in the presented model. According to the study conducted in this research, for the first time, this objective function is placed simultaneously with the rate of return and portfolio risk in the mathematical model. The model also allows the investor to have a portfolio of stocks that suit his / her constraints based on his / her preferences. Also, by determining the goal values of each objective function and the acceptable tolerance limit for each objective, the results presented in the present model essentially satisfy the needs of investors. Due to the complexity of the problem presented in this research, exact mathematical methods in large-size problems cannot find optimal solutions. In this case, the use of meta-heuristic algorithms would be appropriate. Another innovation of this research is presenting the harmony search algorithm for the introduced problem and comparing the results with the competing genetic algorithm. Finally, due to the uncertainty in the financial markets and the need to model uncertainty in this study, a multi-objective fuzzy goal programming model has been used that can consider the uncertainty properly and bring the final results closer to the real-world condition. The following sections in this research are as follows: in section 2 methodology of this research is discussed. The harmony search optimization algorithm that is designed according to the mathematical formulation is presented in section 3. The evaluation criterion used in this study is presented in section 4. Computational outcomes are illustrated in section 5, and finally, conclusions and ideas for future research are discussed in section 6.

2. Literature Review

The methodology used in this research is described below. In the following, after introducing the concept of liquidity, the mathematical model of the present research is described.

2.1. Liquidity

Liquidity refers to how fast an asset or stock can be sold at a market price. The higher the market value of a stock in a financial market, and the more buyers and sellers that stock is traded, the greater its liquidity. In other words,

if a stock could be sold more quickly in the market, its' liquidity is higher (Lee & Chung, 2018) In this study, the formula presented in research (Chordia et al., 2000) was used to calculate liquidity.

Q_i : Liquidity of the stock i

P_{Ai} : Ask the price of the stock i (the price a seller states they will accept)

P_{Bi} : Bid price of the stock i (the highest price that a buyer is willing to pay for stock)

P_{Mi} : The average price of the stock i

The liquidity value of each stock in the financial market in this research is obtained from equation (1).

$$(1) \quad Q_i = (P_{Ai} - P_{Bi}) / P_{Mi}$$

2.2. Mathematical model

In this section, a multi-objective fuzzy goal programming model for portfolio selection is presented. Also, constraints have been added to bring the model results closer to the real-world results. Since one of the critical factors in profitable investment is transaction cost, this factor has been considered in this modeling. The model also restricts the minimum purchase amount per stock. Also, to diversify the portfolio, a constraint has been added to the model. The mentioned constraint limits the maximum amount of purchases of each particular stock. In addition to maximizing returns and minimizing risk in the proposed model, liquidity maximization has also been considered. This objective function allows the model to select the portfolio of stocks with the highest liquidity available to investors. The model also allows the investor to incorporate his or her preferences into the portfolio in two ways, which is another innovation of the proposed model.

The following notations are utilized in this research:

w_0	Initial investor deposit
r_i	The previous rate of return <i>ith</i> stock
N	The maximum number of stocks in the portfolio
x_i	Amount of money that should be invested in <i>ith</i> stock
c_i	Price of <i>ith</i> stock
λ	The minimum level of the objective function
Z_i	A binary variable that is one if <i>ith</i> stock is bought
e_p	The rate of return of the portfolio
δ_p^2	The variance of the portfolio
δ_p	The standard deviation of the portfolio
Q_p	The liquidity of the portfolio
Q_i	The liquidity of each stock
w_r, w_p, w_Q	The weight of each objective function
b_r^*, δ^*, Q^*	Goal values of rate of return and standard deviation, and liquidity in the portfolio
b_r^-, δ^-, Q^-	Objectives tolerance limit
β_{upper}	The maximum percentage of investment in each stock
β_{lower}	The minimum percentage of investment in each stock
α	Percentage of transactional cost
M	A big positive number
δ_{ij}	Covariance between two stocks
w_i	The weight of stock i in the portfolio
w_j	The weight of stock j in the portfolio

The fuzzy goal programming approach has been utilized for numerous multi-objective decision-making models and has provided a valuable solution for such environments. Jiménez et al. (2018) used a Max-Min operator for the portfolio selection problem. In this situation, a decision-maker determines a membership function and a satisfaction level for each function, and fuzzy programming maximizes minimum satisfaction levels between functions (Jiménez et al., 2018). Gupta et al utilized the fuzzy theory with goal programming to solve the portfolio selection problem.

Fuzzy goal programming can be formalized as follows:

$$\text{Find } X \quad (2)$$

$$\text{To satisfy : } f_j(X) \stackrel{\sim}{\geq} g_j \quad j = 1, 2, \dots, m \quad (3)$$

$$\text{Subject to } BX \leq b \quad (4)$$

$$X \geq 0 \quad (5)$$

In the mentioned representation, $f_j(X)$ shows j^{th} objective function and notation " $\stackrel{\sim}{\geq}$ " illustrates the fuzzy ratio and g_j defines every satisfaction level in each objective function. Since all objective functions cannot be reached simultaneously, the decision-maker must assess a membership function and a low tolerance limit for each objective function, which is shown by l_j . A membership function for j^{th} function is determined as follows bellow:

$$\mu_j(f_j) \begin{cases} 1 & g_j \leq f_j(x) \\ \frac{f_j(x) - l_j}{g_j - l_j} & l_j \leq f_j(x) \leq g_j \\ 0 & f_j(x) \leq l_j \end{cases} \quad (6)$$

The membership functions for return, risk, and liquidity in this research are defined as follows:

$\mu_r(e_p)$	The membership function for return
$\mu_\sigma(\sigma_p)$	The membership function for risk
$\mu_Q(Q_p)$	The membership function for liquidity
br^*	The satisfaction level for return
σ^*	The satisfaction level for risk
Q^*	The satisfaction level for liquidity
br^-	The low tolerance limit for return
σ^-	The low tolerance limit for risk
Q^-	The low tolerance limit for liquidity
e_p	Rate of return objective function
σ_p	Risks objective function
Q_p	Liquidity objective function

$$\mu_r(e_p) \begin{cases} 1 & br^* \leq e_p \\ \frac{e_p - br^-}{br^* - br^-} & br^- \leq e_p \leq br^* \\ 0 & e_p \leq br^- \\ 1 & \sigma_p \leq \sigma^* \end{cases} \quad (7)$$

$$\mu_{\sigma}(\sigma_p) \left\{ \begin{array}{ll} \frac{\sigma^- - \sigma_p}{\sigma^- - \sigma^*} & \sigma^* \leq \sigma_p \leq \sigma^- \\ 0 & \sigma^- \leq \sigma_p \end{array} \right. \quad (8)$$

$$\mu_Q(Q_p) \left\{ \begin{array}{ll} 1 & Q^* \leq Q_p \\ \frac{Q_p - Q^-}{Q^* - Q^-} & Q^- \leq Q_p \leq Q^* \\ 0 & Q_p \leq Q^- \end{array} \right. \quad (9)$$

In this model, the decision-maker can model his or her preferences to select a portfolio in two ways. The first solution is to measure the goal values of each objective function and the tolerance limit for each objective function. The second solution is to assign different weight values to each objective function. Using the two solutions outlined above will allow investors to select the portfolio as much as possible according to their preferences. For example, to maximize portfolio returns, the decision-maker can make many choices by specifying values of br^* , br^- , and w_r , which are the satisfaction level for return, low tolerance limit for return, and return weight, respectively.

According to the mentioned notations and following (Asgari & Behnamian, 2022), the fuzzy goal programming model in this study is presented in the following equations.

$$\max \lambda \quad (10)$$

$$w_r \lambda \leq \frac{e_p - br^-}{br^* - br^-} \quad (11)$$

$$w_{\delta} \lambda \leq \frac{\delta^- - \delta_p}{\delta^- - \delta^*} \quad (12)$$

$$w_Q \lambda \leq \frac{Q_p - Q^-}{Q^* - Q^-} \quad (13)$$

$$\delta_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \delta_{ij} \quad (14)$$

$$e_p = \sum x_i c_i r_i (1 + \alpha) \quad (15)$$

$$Q_p = \sum Q_i \quad (16)$$

$$w_i = \frac{c_i x_i}{\sum_{i=1}^n x_i c_i} \quad (17)$$

$$\sum_{i=1}^n Z_i \leq N \quad (18)$$

$$\beta_{lower} - M(1 - Z_i) \leq x_i c_i (1 + \alpha) \leq \beta_{upper} + M(1 - Z_i) \quad (19)$$

$$\frac{x_i}{M} \leq Z_i \leq x_i \quad (20)$$

$$\sum x_i c_i (1 + \alpha) \leq w_0 \quad (21)$$

$$Q_i = (P_{Ai} - P_{Bi}) / P_{Mi} \quad (22)$$

$$x_i \in Z, Z_i \in \{0,1\} \quad (23)$$

In the proposed mathematical model, equation (10) maximizes minimum levels for objective functions. Equations (11, 12 and 13) are fuzzy goal programming equations. Equations (14 and 15) calculate the rate of return and risk

of the selected portfolio. Equation (16) calculates portfolio liquidity. Equation (16) quantifies the portfolio's liquidity by aggregating the liquidity values of individual stocks, weighted by their respective proportions in the portfolio. Unlike traditional portfolio models that primarily focus on risk-return trade-offs, our approach ensures that selected stocks maintain a high level of liquidity, reducing execution delays and transaction costs. In classical models, constraints typically limit portfolio composition based on capital allocation, risk, and return expectations, without explicitly considering liquidity. However, in emerging markets like the Tehran Stock Exchange, liquidity fluctuations can significantly impact portfolio performance. By incorporating Equation (16), the model prioritizes liquid assets, making it more adaptable to real-world trading conditions. Equation (17) checks that the summation of selected stocks' weights must be one. In equation (18), the maximum number of stocks must be equal to or lower than a predefined value. Equation (19) illustrates buying and selling constraints in the presented portfolio, and consequently, equation (20) relates decision variables and integer variables in the model. Sequentially, equation (21) considers that the maximum amount of investment must be equal to or lower than the initial investor's deposit. Equation (22) calculates the liquidity value of each stock.

3. Harmony search algorithm

Meta-heuristics are common algorithms, frequently nature-inspired, that are used to solve complicated optimization problems. They have been an increasing research region for a few decades. Additionally, for solving optimization problems that encompass their mathematical formulation of uncertain, stochastic, and dynamic information, these algorithms are utilized (Soler-Dominguez et al., 2017). Optimization methods are generally divided into two categories: exact algorithms and approximate algorithms. Exact algorithms are capable of finding the precise optimal solution, but they become inefficient for complex optimization problems, as their runtime increases exponentially with the problem size. On the other hand, approximate algorithms can produce good (near-optimal) solutions within a short time frame, making them more suitable for complex optimization problems. Approximate algorithms are further divided into three types: heuristic algorithms, meta-heuristic algorithms, and hyper-heuristic algorithms (He et al., 2016). Meta-heuristic algorithms are approximate optimization algorithms with solutions that outsource local optimal points and can be used in various problems. Various categories of this type of algorithm have been developed in recent decades, all of which are a subset of the meta-heuristic algorithm (Pai & Michel, 2017). The harmony search algorithm (HSA) is a stochastic exploration algorithm replicating the melodic invention procedure (Pai & Michel, 2017). Like other algorithms, the harmony search algorithm works on a population of iteratively multiplied solutions over several generations. The harmony search algorithm generates a novel solution at each generation using three measures: harmony memory consideration rate, arbitrary consideration, and pitch adjustment rate (Seyedhosseini et al., 2016).

The pseudo-code used in the present study is presented below.

```

for (j=1 to MNI) do
    if ( r_1<HMCR )then
        X_new (j)=X_a (j)  where a∈(1,2,3,...,HMS)
        if ( r_2<PAR )then
            X_new (j)=X_new (j)±r_3*BW  where (r_1,r_2,r_3 ∈(0,1))
        end if
    else
        X_new (j)= [(LB) _j+r*( [(UB) _j- [(LB) _j)] where r∈(0,1)
    end if
end for

```

3.1. Harmony search steps

The following are the steps of the harmony search algorithm.

3.1.1. Step 1: Initialize Parameters

Define the harmony memory size (HMS), pitch adjustment rate (PAR), harmony memory consideration rate (HMCR), and maximum iterations.

3.1.2. Step 2: Generate Initial Harmony Memory

Harmony memory (HM) holds a set of solutions, and its size is equivalent to harmony memory size (HMS). In this step, the harmony search algorithm generates solutions using either an arbitrary or heuristic process and then transfers them to harmony memory (HM) (Pai & Michel, 2017). In this research, to deal with the portfolio selection problem, each solution is signified by a matrix with two rows and whose columns are equal to the number of the stocks available that are chosen to be in the portfolio. In this research, the first 100 stocks in the Tehran stock exchange market are being considered. The first row indicates which stock must be selected, which is a binary variable. Consequently, the second row shows the weight of the selected stock in the portfolio. Figure (1) presents a solution representation in this algorithm. As can be seen, in this representation, three stocks are selected, and their weights are shown in the second row. As mentioned before, the second row in this matrix shows the percentage of initial deposits which must be invested in each stock and randomly filled between $\beta_{lower}(1-\alpha), \beta_{upper}(1-\alpha)$

$$\begin{bmatrix} 1 & 0 & 1 & \dots & 1 \\ 0.3 & 0 & 0.2 & \dots & 0.5 \end{bmatrix}$$

Figure 1: Solution representation

Randomly generate an initial set of solutions (portfolios) while ensuring feasibility.

3.1.3. Step 3: Improvise a New Harmony

Generate a new solution using three mechanisms:

- 1- Memory consideration: Select variables from existing harmonies with probability HMCR.
- 2- Pitch adjustment: Fine-tune selected variables using a small random adjustment with probability PAR.
- 3- Randomization: Introduce new solutions to maintain diversity.

3.1.4. Step 4: Update Harmony Memory

If the new solution is better than the worst solution in the memory, replace it.

3.1.5. Step 5: Repeat Until Convergence

Continue until the stopping criterion (maximum iterations or convergence) is met.

3.2. Fitness function evaluation

For evaluating each solution, equation (24) is utilized as a fitness function.

$$fitnessfunction = \min \left\{ \frac{e_p - br^-}{br^* - br^-} * \left(\frac{1}{w_r} \right), \frac{\delta^- - \delta_p}{\delta^- - \delta^*} * \left(\frac{1}{w_\delta} \right), \frac{e_p - Q^-}{Q^* - Q^-} * \left(\frac{1}{w_p} \right) \right\} \quad (24)$$

3.3. Setting harmony search algorithm parameters

This subsection aims to define the parameters of the proposed algorithm, which are detailed below:

- **Harmony Memory Size (HMS):** This represents the population size or the variety of solutions stored in the Harmony Memory (HM).
- **Harmony Memory Consideration Rate (HMCR):** This factor determines whether the variables for the new solution should be chosen from the existing ones in the HM or generated randomly during some steps of the solution generation process. It takes a value between zero and one.
- **Pitch Adjustment Rate (PAR):** This factor also takes a value between zero and one and is used to decide whether to modify the elements selected from the HM.
- **Maximum Number of Iterations (MNI):** This parameter sets the stopping condition.

In this study, the Taguchi parameter setting method was used to determine the optimal values of each parameter. Table (1) presents the best values for the mentioned parameters.

Table 1. Harmony search algorithm parameters

Parameter's name	parameters	Problem size	
		30 stock	200 stock
Harmony memory size	HMS	300	250
Harmony memory consideration rate	HMCR	0.5	0.7
Maximum number of iterations	MNI	2000	1500
Pitch adjustment rate	PAR	0.5	0.7

4. Algorithms evaluation criterion

In multi-objective problems, how to evaluate the values of the solutions is based on criterion and cannot be the sole purpose of evaluating the final solution. Accordingly, we define the following criterion for evaluating and comparing the algorithms. In this criterion, an ideal solution to the problem is first considered, and then the mean deviations of the Pareto set of ideal solutions are calculated. The ideal solution is a situation where both solutions are at the same time. In the following, the mean ideal deviation criterion (MID) criterion equation is presented.

$$MID = \frac{\sum_{i=1}^n C_i}{n} \quad (25)$$

In the above equation, n is the number of Pareto solution and the value of C_i is $C_i = \sqrt{(f_{1i} - f_1^*)^2 + (\frac{1}{f_{2i}} - \frac{1}{f_2^*})^2 + (\frac{1}{f_{3i}} - \frac{1}{f_3^*})^2}$. Given that the first objective function is the minimization type and the other objective functions are the maximization type, the maximization of the objective function has been converted to the minimization to determine the ideal point. We consider the point (0,0) as an ideal point. The lower the MID criterion, the better the algorithm's performance.

5. Computational results

In this section, the proposed fuzzy goal programming model has been solved two times. Firstly, it has been solved without considering realistic constraints that are added in this research. Secondly, the proposed mathematical formulation is coded according to the harmony search optimization procedure, and finally, the proposed model has been solved by GAMS software. In this research, the first 100 stocks, according to their market capital, are selected, and numerical computations are executed. In this study, a genetic algorithm was used as a competing algorithm. Then, algorithms have investigated the final model presented in this study, and numerical results are evaluated. In this section, we first consider the problem without considering the added realistic constraints and then consider the constraints added to the model. The results of solving the models presented with and without realistic constraints are presented by the harmony search algorithm and the GAMS software. In this instance, it is assumed that there are 30 stocks to choose from. Then, for the different combinations of objective function weights in this study, the problem is solved once by GAMS software. Then, by keeping the problem's parameters constant, the same problem is solved by the harmony search algorithm concerning the added constraints and without the added constraints. The value of r represents the return of the proposed portfolio, the value of δ is equal to the standard deviation of the proposed portfolio, and finally Q represents the portfolio's liquidity. The calculation results are presented in Table (2).

Table 2. Computational results when 30 stocks are available

	W_r	W_δ	W_Q	With realistic constraints			GAMS result			Without realistic constraints		
				r	δ	Q	r	δ	Q	r	δ	Q
I=30	1	0	0	1.52	2.06	0.19	1.66	2.15	0.23	1.11	2.56	0.46
	0.9	0.5	0.5	1.83	2.19	0.16	1.97	2.36	0.13	1.32	2.86	0.34
	0.8	0.1	0.1	2.39	2.56	0.19	2.49	2.52	0.16	1.85	3.24	0.45
	0.7	0.2	0.1	2.92	4.88	0.36	2.84	3.3	0.25	2.14	4.3	0.36
	0.6	0.2	0.2	3.48	5.58	0.45	3.57	3.99	0.36	2.44	4.96	0.66
	0.5	0.3	0.2	4.1	6.49	0.18	4.08	5.56	0.13	3	6	0.36
	0.4	0.3	0.3	4.27	6.49	0.44	4.27	6.49	0.42	3.37	6.99	0.64
	0.3	0.35	0.35	4.27	6.49	0.55	4.27	6.49	0.56	3.67	7.52	0.36
	0.2	0.4	0.4	4.27	6.49	0.26	4.27	6.49	0.36	3.77	7.81	0.46
	0.1	0.45	0.45	4.27	6.49	0.38	4.27	6.49	0.46	3.77	7.88	0.13
	0	0.5	0.5	4.27	6.49	0.77	4.27	6.49	0.61	3.77	7.88	0.55

The following is shown in Figures (2-4) to better compare the results in Table (2).

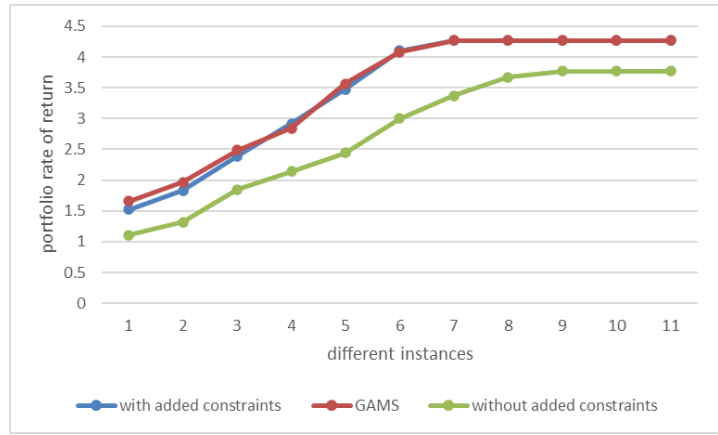


Figure 2. Comparison according to portfolio rate of return

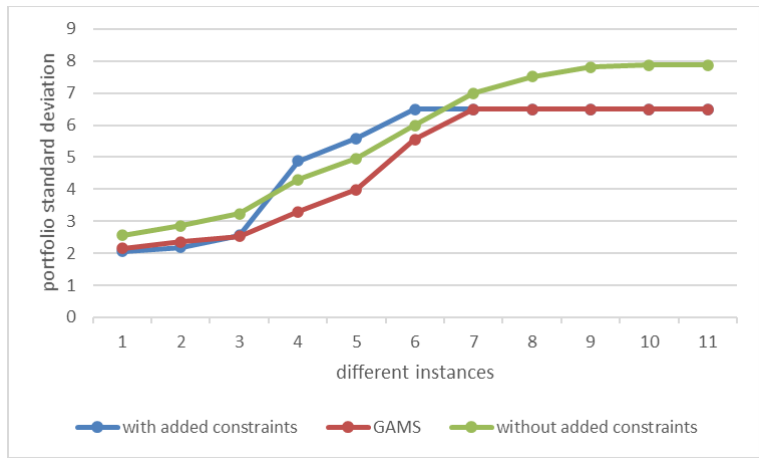


Figure 3. Comparison according to portfolio standard deviation

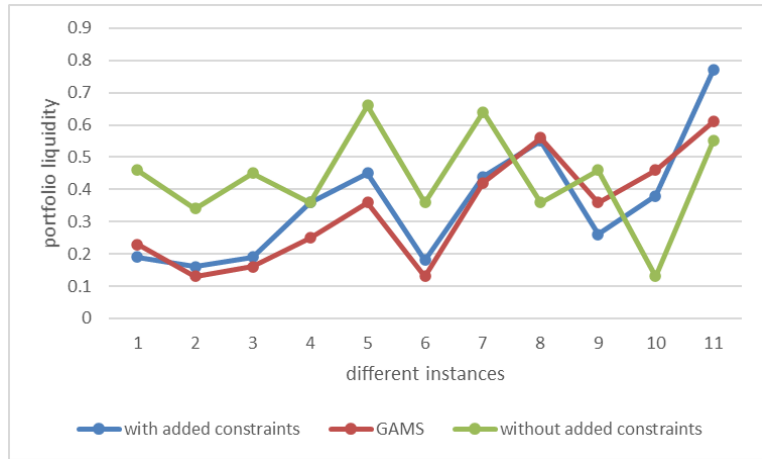


Figure 4. Comparison according to portfolio liquidity

As shown in Figures (2) to (4), by adding the realistic constraints mentioned in the present study to the classical portfolio selection problem, the results of the harmony search algorithm were closer to the results obtained using GAMS software. In addition, it is clear that by adding such constraints to the portfolio selection problem, the results will be closer to the real-world situation in financial markets. While this study demonstrates the effectiveness of the harmony search algorithm in portfolio selection, further robustness tests such as sensitivity analysis and out-of-sample validation could provide deeper insights into the model's generalizability. These analyses are beyond the scope of this paper but are valuable directions for future research. Additionally, the computational complexity of the harmony search algorithm is influenced by the number of iterations and the size of the solution space.

Compared to traditional methods such as genetic algorithms, the harmony search algorithm provides competitive efficiency while maintaining high-quality solutions. A more detailed theoretical complexity analysis can be explored in subsequent studies. The empirical validation in this study is based on a subset of stocks from the Tehran Stock Exchange (TSE), focusing on the stocks to ensure reliable computational analysis. While this approach helps maintain data quality and consistency, it may introduce a selection bias, as less liquid stocks are excluded. Future studies could explore the model's performance across a more diverse dataset to examine its robustness in varying market conditions. Additionally, the proposed multi-objective fuzzy goal programming model is designed to be adaptable to other stock markets with appropriate modifications. While liquidity constraints and transaction costs are critical factors in emerging markets like the TSE, developed markets may require adjustments to account for high-frequency trading effects and regulatory differences. The model's flexibility in handling uncertainty through fuzzy goal programming makes it a valuable tool for application in global financial markets with proper calibration of constraints and objectives.

5.1 Algorithm results

Then, to validate the harmony search algorithm presented in this research, different instances are solved in different sizes. A genetic algorithm has been used as a competing algorithm in this research. The genetic algorithm presented in the research [38] has been investigated as a competing algorithm. As shown in the results table, the harmony search algorithm performs better on the MID criterion. The MID criterion is not significantly different for genetic and harmony search algorithms at small-size problems, but in the larger size problems, the greater the difference. As can be seen, the harmony search has a lower MID than the genetic algorithm. Since using the meta-heuristic algorithm is to solve large-size problems, it is better to have an algorithm with smaller MID in large-size problems. Furthermore, in the following, the observed difference is also proved that be significant. It should be noted that the size of each instance is shown in Table (3) by the number of stocks in the portfolio and the number of available stocks in the market. For example, (4*20) means selecting only four stocks out of 20 available stocks in the market. In terms of the runtime of the algorithms, as can be seen from the results in Table (3), there is no significant difference between the algorithms, and both algorithms are not significantly different in terms of runtime.

Table 3. Results of solving different instances by harmony search and genetic algorithm

Problem's size (<i>N * I</i>)	Algorithms results			
	MID		Average runtime (s)	
	HS	GA	HS	GA
3*8	180.4	210.6	26	23
4*8	250.26	138012	33	43
5*8	150.29	340.92	67	56
3*10	140.07	230.78	73	80
4*10	160.20	520.26	95	102
5*10	330.81	32.26	173	112
3*100	350.29	714.19	860	970
4*100	250.06	481.18	1100	1230
5*100	512.12	752.16	1630	1560
3*200	390.6	650.15	2460	2930
4*200	280.49	590.46	2866	2600
5*200	400.05	714.19	4200	3860

To evaluate the performance of the proposed algorithm, 12 instances were used. In the following, the significance of the difference between the values of the algorithms is examined. For this purpose, each algorithm was run ten times under the same conditions for a specific problem. The factors here are algorithms that are examined at two levels. We have to judge by the value of the statistic obtained about the null hypothesis. If $\alpha \geq p$ -value is true, then the hypothesis H_0 (no significant difference between algorithmic averages) is rejected, but if $\alpha \leq p$ -value has no reason to reject H_0 . The α value is 0.05 in this test.

Figure 5. Equality of variance of data in genetic algorithm and harmony search

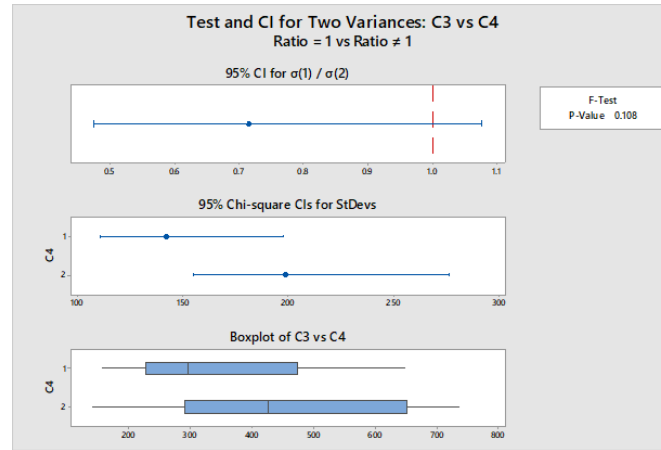


Table 4. Analysis of variance of the MID criterion

Source of Changes	SS	Degrees of freedom	Variance of MS	F	P-value
Between groups	43650	1	44236	1.45	0.231
Within groups	920364	30	25641		
Total	964014	31			

In the analysis of variance, the null hypothesis to be considered is that all mean groups are equal, assuming that at least one group's mean is different from the others. Given the statistical value (1.45) and the significant probability (0.231), the null hypothesis must be rejected, and therefore, the difference between the two algorithms is significant.

6. Conclusion

In this research, a new fuzzy goal programming approach is presented. The number of realistic constraints is added to the classic portfolio selection model, facilitating investment in such volatile markets. The use of fuzzy goal programming allows the investor to minimize investment risk and maximize the rate of return. Additionally, portfolio liquidity is considered in this study as an objective function that must be maximized. Multi-objective function simultaneously allows the final results to be an optimal combination of these objective functions. The model also adds realistic constraints to bring the final results closer to the real-world situation in the financial markets. In this regard, the transactional cost also was added to the mathematical model. Additional constraints are also considered, one of which is the maximum number of stocks that must be selected. Another one is the constraint in the amount of money that should be invested in the specified stock. The model also has additional constraints to the classic portfolio selection model to avoid buying too much of a particular stock and buying too little from a stock. Then, by allocating different weights to each of the objective functions by the investor and using the fuzzy goal programming method, the investor can make choices according to their preferences. Finally, the proposed model was solved using a harmony search algorithm. After comparing the results with the competing genetic algorithm, the superiority of the harmony search algorithm was proved in different instances. As can be seen from the numerical results, adding realistic constraints has yielded better results and is closer to the real-world situation. Also, considering liquidity as an objective function causes stocks to be selected with high liquidity and occurs when buying and selling transactions at current prices immediately. Also, the use of fuzzy goal programming provides a variety of selections based on investor priorities. In this model, uncertain conditions of financial markets are also considered in the results. Finally, the algorithm presented in this research can provide appropriate solutions in a reasonable runtime that many people can use. The introduced model and the algorithm designed in the present study can be used by financial market investors, investment company managers, and financial software developers. For further research, other meta-heuristic algorithms such as particle swarm optimization algorithms can be used. Using methods such as stochastic programming and dynamic programming can also yield good results. Also, combining machine learning methods with meta-heuristic algorithms can increase the power of these algorithms to solve various problems.

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