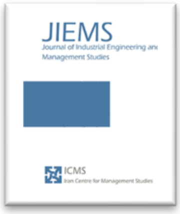
Vol. 12, No. 1, 20254, Pages 53-70

JIEMS
Journal of Industrial Engineering and Management Studies

journal homepage: www.jiems.icms.ac.ir



Coordination strategies in two-stage supply chains under partial stochastic demand information: A focus on corporate social responsibility and marketing efforts

Yalda Sedighi Sereshke [1], Mohammad Reza Gholamian [*1], Seyyed-Mahdi Hosseini-Motlagh [1], Somayeh Fathollahi Arani[1], Marzieh Raei[1]

[1] Department of Logistics and Supply Chain Engineering, School of Industrial Engineering, Iran University of Science and Technology

Received: Jan 2025-27/ Revised: Feb 2025-26 / Accepted: Apr 2025-14

Abstract

Recently, there has been a notable increase in research efforts focused on achieving supply chain (SC) coordination by considering corporate social responsibility (CSR) as a crucial factor in decision-making. This study explores the coordination of a two-stage SC, including a manufacturer with CSR investment and a retailer with marketing efforts, under deterministic and stochastic with partial information demand scenarios. Furthermore, it analyzes decisions related to pricing, order quantity, CSR investment, and marketing efforts by presenting decentralized, centralized, and coordinated strategies. A comprehensive numerical analysis, involving a numerical example and a set of sensitivity analyses, is conducted to evaluate the efficacy of the proposed models. Results indicate that a two-part tariff (TPT) contract can effectively coordinate the SC and motivate the manufacturer to enhance its CSR investment, ultimately resulting in profitability equivalent to that of the centralized strategy. Moreover, the manufacturer's CSR investment and the retailer's marketing efforts contribute to increased market demand and profitability.

Keywords: Supply chain coordination, Corporate social responsibility, Marketing efforts, Stochastic demand, Distribution-free approach

Paper Type: Original Research

1. Introduction

Bowen and Johnson (1953) were the first to introduce CSR. However, various definitions of its true meaning have existed over the past half-century. According to Carroll (1991), “CSR encompasses the economic, legal, ethical, and philanthropic expectations that society has of organizations at any given time.” This definition is widely accepted in CSR research due to its sustainable application and positive connotation. In today's global competitive environment, businesses increasingly adopt CSR as a valuable strategy to create goodwill and social value for their organizations (Bowen and Johnson (1953)). Cruz and Wakolbinger (2008) state that many companies engage in CSR activities to improve their corporate image, mitigate risks, foster customer loyalty, and more. In reality, allocating resources to CSR initiatives can boost customer demand and ultimately increase business profitability. As a result, many international brands such as Nike, Walmart, and Adidas have integrated CSR into their SC networks (Gimenez et al. (2012), Amaeshi et al. (2008)). The SC primarily consists of suppliers, manufacturers, and distribution intermediaries such as wholesalers and retailers. In practice, CSR extends beyond a company's boundaries due to the interactions among SC participants. Therefore, the impact of CSR issues expands to all SC players (Ashby et al. (2012). For example, a manufacturer's investment in CSR initiatives can increase the product market size, ultimately benefiting all SC players. Hence, coordinating CSR decisions within the SC is crucial for achieving an enduring competitive edge for all involved players (Nematollahi et al. (2017). This coordination is typically facilitated through contracts and information sharing. Various types of contracts, such as profit-sharing contracts, flexible quantity contracts, buyback contracts, wholesale price contracts, and TPT contracts enable interactions among SC participants. However, there is no one-size-fits-all contract for SCs due to substantial variations in their structure and performance metrics. This study makes a valuable contribution by coordinating the SC through the TPT contract under stochastic market demand with an unknown distribution. In addition, it explores the interactions between CSR and marketing and their impact on SC performance. The main objectives of this study are as follows: (1) To compare the decisions of SC players regarding pricing, order quantity, CSR investment, and marketing

*Corresponding Author: Gholamian@iust.ac.ir

efforts derived through the distribution-free approach under decentralized, centralized, and coordinated strategies. (2) To examine how CSR and marketing influence the decisions and profits of SC players. (3) To coordinate the SC under stochastic demand with an unknown distribution through the TPT contract. The remainder of this study is structured as follows: Section 2 reviews the related literature. Section 3 formulates the decentralized, centralized, and coordinated strategies under deterministic and stochastic with partial information demand scenarios. Section 4 presents the numerical analysis of the proposed models developed through the distribution-free approach. Finally, Section 5 concludes the study and outlines directions for future research.

2. Related literature

The focus on social and environmental concerns has given rise to the concept of CSR, which has become a significant and widely debated topic in recent years. Consequently, researchers have conducted valuable studies on CSR and its incorporation into SC coordination models. Notably, Goering (2012) analyzed the impact of CSR in a two-stage SC using a TPT scheme within a bilateral monopoly situation. Barcos et al. (2013) investigated the effect of CSR implementation on a company's inventory policies. Hsueh (2014) coordinated a manufacturer-retailer SC through a novel contract that combined revenue-sharing and CSR. Modak et al. (2016a) studied a distribution system involving a manufacturer, multiple distributors, and retailers. The manufacturer's primary objective was to improve the welfare of all stakeholders through CSR practices. In addition, a novel revenue-sharing contract was introduced to mitigate potential channel conflicts. Modak et al. (2016b) also explored channel coordination through TPT contract and profit distribution in a two-stage SC consisting of a socially responsible manufacturer and two competing retailers engaged in Cournot and Collusion games. Nematollahi et al. (2017) designed a novel collaborative model for a two-stage SC consisting of a supplier and a retailer, where the supplier could potentially boost the popularity of products by investing in CSR activities. Wu et al. (2017) proposed flexible quantity and wholesale price incentive contracts to improve the performance of a socially responsible two-stage SC, in which the supplier’s socially irresponsible behavior could significantly reduce the manufacturer’s sales. Raza (2018) presented quantitative models for a two-stage SC consisting of a socially responsible manufacturer and a retailer under different demand scenarios. Liu et al. (2019b) proposed a three-stage Stackelberg game model to analyze the impact of government subsidies in promoting CSR on the profitability of SC members, the level of CSR effort, and overall social welfare. Hosseini-Motlagh et al. (2019) suggested an adjustable bi-level wholesale price contract to coordinate a three-stage SC consisting of a socially responsible manufacturer, a distributor, and a retailer under a scenario-based stochastic demand. Cheng and Ding (2021) shifted the focus from a static to a dynamic perspective to investigate the implementation of CSR in competitive SCs. Sharma and Singh (2022) considered a two-stage SC involving a socially responsible manufacturer and a fairness-concerned retailer. They investigated how the retailer’s fairness concern impacts channel coordination through a wholesale price contract. Moraux et al. (2023) explored reverse factoring and cost-sharing contracts to help SCs address CSR challenges, focusing on small and medium-sized suppliers with limited working capital access. Chen et al. (2024) suggested a Nash-revenue-sharing contract to coordinate a two-stage SC consisting of a green manufacturer and a socially responsible retailer under yield uncertainty. Zhang et al. (2025) examined the impact of CSR within an e-commerce SC including an E-platform and a manufacturer, where the E-platform owned a store brand product and supported online sales of the manufacturer’s product through agency selling or reselling. Additionally, CSR has gained significant importance in more complex SCs such as dual-channel and closed-loop systems. For example, Modak et al. (2014) proposed an all-unit quantity discount contract to coordinate a dual-channel SC. They also discussed how the manufacturer's social responsibility influences product compatibility and operational success. Modak et al. (2019) suggested a TPT contract for coordinating a two-stage closed-loop SC, which considered social work donation as a CSR activity. Hosseini-Motlagh et al. (2020) examined a socially responsible closed-loop SC with two stages, where the manufacturer invested in green research and development, while two retailers competed to enhance their CSR efforts to increase market demand and collection rates. Additionally, they proposed a profit-sharing contract to resolve channel conflicts. Raza (2020) developed mathematical models to optimize pricing, CSR investment, and inventory decisions in dual-channel SC under different demand scenarios. Modak and Kelle (2021) incorporated CSR into a closed-loop SC through social work donations and recycling investments under stochastic market demand. Moreover, several researchers have extended their studies by considering information asymmetry in CSR costs among the SC members. For instance, Ma et al. (2017) coordinated a two-stage SC under CSR cost information asymmetry, where a contract manufacturer and a brand-name retailer invested in CSR activities and marketing efforts, respectively. They also analyzed TPT and wholesale price contracts under information symmetry and asymmetry. Liu et al. (2019a) designed a transfer payment mechanism for coordinating a two-stage SC involving a socially responsible contract supplier and a brand retailer under CSR cost information asymmetry. The growing emphasis on competitive differentiation and social communication has heightened interest in integrating marketing and advertising into SC coordination. In this context, Ma et al. (2013) designed an innovative coordination contract that combined the manufacturer's quality costs and the retailer's marketing costs. Khorshidvand et al. (2021b) introduced a two-stage approach for modeling and solving a sustainable closed-loop SC. In the first stage, the optimal decisions for pricing, greening, and advertising were determined. In the second stage, a fuzzy multi-objective Mixed Integer Linear Programming model was employed to maximize total profit, minimize CO2 emissions, and improve social impacts. In their subsequent work, Khorshidvand et al. (2021a) proposed a two-stage model for a sustainable closed-loop SC. In the first stage, the optimal pricing decision was determined by considering the optimal levels of advertising and greening. In the second stage, multi-objective Mixed-Integer Linear Programming was applied to maximize the total profit, minimize CO2 emissions, and improve employee safety. Li et al. (2021) investigated the effects of government subsidies in a two-stage SC involving a manufacturer with green technology investment and a retailer with green marketing efforts under a cap-and-trade mechanism. Khorshidvand et al. (2021c) developed a novel hybrid method to simultaneously address SC coordination decisions and SC network design objectives under demand uncertainty. This method first determined pricing, greening, and advertising decisions, and subsequently focused on maximizing profit and minimizing CO2 emissions. Huang et al. (2021) explored a two-stage green SC, where a manufacturer invested in green technology and a retailer engaged in green marketing efforts, to analyze the impact of CSR initiatives undertaken by either party and the influence of different power structures. Taleizadeh et al. (2021) proposed a cooperative advertising strategy to coordinate a manufacturer-retailer SC while accounting for the noise effect of new products. Khorshidvand et al. (2021d) presented a modified centralized model for a three-echelon multi-channel SC by considering pricing, greening, and advertising decisions. Fadavi et al. (2022) suggested a green marketing cost-sharing contract to coordinate a manufacturer-retailer SC under different scenarios for producing green products. Hosseini-Motlagh et al. (2022) coordinated a two-stage SC including a retailer with sales efforts and a population of manufacturers with innovation efforts through a profit surplus distribution mechanism based on evolutionary game theory. Yang et al. (2022) proposed a revenue-sharing contract to coordinate a manufacturer-retailer SC under environmental research and development uncertainty, wherein the manufacturer invested in green manufacturing while the retailer focused on green marketing. Khorshidvand et al. (2023) designed a cost-sharing collaborative model for a dual-channel green closed-loop SC under an incentive-based recycling program by addressing pricing, greening, and advertising decisions. Xia et al. (2024) analyzed how precision marketing and contract types, namely consignment and reselling, influence SC members' decision-making and consumer surplus. Khorshidvand et al. (2025) developed a green-cost-sharing collaborative model for an omnichannel closed-loop SC. They also explored the role of government subsidies in promoting sustainability by focusing on pricing, greening, marketing, and recycling decisions. A summary of the related literature is presented in Table 1. All the studies discussed in the related literature section have made valuable contributions to the field of CSR. However, no comprehensive research has explored the relationship between a manufacturer's social responsibility and a retailer's marketing efforts in presenting the manufacturer's CSR initiatives to consumers and increasing sales, particularly under stochastic demand with an unknown distribution.

Table 1. Summary of the related literature

		SC structure		CSR issue				Demand type			Demand sensitivity				Information		Game theory			
Row	Authors (Year)	Two-level	Three-level	Consumer surplus	CSR investment	CSR performance level	Others	Deterministic	Stochastic	Stochastic with partial information	Price	CSR	Marketing/Advertising	Others	Symmetry	Asymmetry	Stackelberg	Nash	Others	Coordination model
1	Goering (2012)	✓		✓				✓			✓				✓		✓			Two-part tariff contract
2	Barcos et al. (2013)						✓													_
3	Hsueh (2014)	✓				✓			✓			✓			✓				✓	New revenue-sharing contract with CSR
4	Modak et al. (2014)	✓		✓				✓			✓				✓		✓			All-unit quantity discount contract
5	Modak et al. (2016a)		✓	✓				✓				✓			✓		✓			Revenue-sharing contract
6	Modak et al. (2016b)	✓		✓				✓			✓				✓		✓		✓	Two-part tariff contract
7	Nematollahi et al. (2017)	✓			✓				✓			✓			✓		✓			Collaborative decision-making
8	Ma et al. (2017)	✓			✓			✓			✓	✓	✓		✓	✓	✓			Wholesale price & two-part tariff contracts
9	Wu et al. (2017)	✓			✓			✓			✓				✓		✓			Flexible quantity & wholesale price incentive contracts
10	Raza (2018)	✓			✓			✓	✓	✓	✓	✓			✓		✓			Revenue-sharing contract
11	Modak et al. (2019)	✓			✓			✓			✓	✓			✓		✓			Two-part tariff contract
12	Liu et al. (2019b)		✓		✓			✓			✓	✓			✓		✓			_
13	Liu et al. (2019a)	✓			✓			✓			✓	✓			✓	✓	✓			Transfer payment mechanism
14	Hosseini-Motlagh et al. (2019)		✓			✓			✓			✓			✓		✓			Adjustable bi-level wholesale price contract
15	Hosseini-Motlagh et al. (2020)	✓			✓			✓				✓		✓	✓		✓			Profit-sharing contract
16	Raza (2020)	✓			✓			✓	✓	✓	✓	✓			✓		✓			_
17	Modak and Kelle (2021)	✓			✓				✓	✓	✓	✓			✓					_
18	Cheng and Ding (2021)	✓				✓		✓			✓	✓		✓	✓		✓	✓		Combined decision-making
19	Khorshidvand et al. (2021d)		✓					✓			✓		✓	✓	✓		✓			Modified centralized structure
20	Huang et al. (2021)	✓		✓				✓			✓		✓	✓	✓		✓	✓		_
21	Sharma and Singh (2022)	✓		✓				✓			✓				✓		✓			Wholesale price contract
22	Hosseini-Motlagh et al. (2022)	✓						✓					✓	✓	✓				✓	Profit surplus distribution mechanism
23	Moraux et al. (2023)	✓			✓				✓			✓			✓				✓	Reverse factoring & cost-sharing contracts
24	Khorshidvand et al. (2023)		✓					✓			✓		✓	✓	✓		✓			Collaborative decision-making
25	Chen et al. (2024)	✓		✓				✓			✓			✓	✓		✓	✓		Nash-revenue-sharing contract
26	Zhang et al. (2025)	✓		✓				✓			✓			✓	✓		✓			_
27	Khorshidvand et al. (2025)	✓						✓			✓		✓	✓	✓		✓			Collaborative decision-making
28	This study	✓			✓			✓	✓	✓	✓	✓	✓		✓		✓			Two-part tariff contract

3. Problem description

This study examines a SC consisting of a manufacturer and a retailer in a single selling period. The manufacturer invests in CSR activities, while the retailer focuses on marketing efforts. The manufacturer incurs a cost of c_m for producing each product unit and determines the wholesale price, w, and CSR investment, γ. The retailer incurs a cost of c_r for retailing each product unit, on top of the wholesale price, and sets the retail price, p, marketing efforts, e, and order quantity, q, before the selling period begins. This problem is formulated for decentralized, centralized, and coordinated strategies under deterministic and stochastic with partial information demand scenarios. The clear goal is to maximize the average profit for each player in these three strategies. Before constructing the model, this study establishes the following assumptions.

Assumption 1. The SC operates under information symmetry, meaning that both the manufacturer and the retailer have identical and comprehensive information at the start of the selling period. Moreover, all players are risk-neutral and rational.

Assumption 2. The effects of overstocking and understocking are not taken into consideration.

Assumption 3. The decision-making process is structured as a Stackelberg game where the manufacturer is a leader and the retailer is a follower.

Assumption 4. The retailer experiences stochastic demand, $D(p, e, \gamma, \xi) = y(p, e, \gamma) + \xi$. $y(p, e, \gamma) = \alpha - \beta p + k\sqrt{\gamma} + \lambda\sqrt{e}$ is a deterministic part of market demand that is influenced by price, CSR investment, and marketing efforts. ξ is a stochastic part of market demand that is both additive and independent. The stochastic demand factor is defined in the range $[\underline{\xi}, \overline{\xi}]$ with a mean μ and a standard deviation σ. Additionally, it follows a probability distribution function $f(\xi)$ and a cumulative distribution function $F(\xi)$. All the notations used in this study are shown in Table 2.

Table 2. Notations

	Parameters:
c_r	Retailer's cost per unit product, $c_r \geq 0$, r = retailer
c_m	Manufacturer's cost per unit product, $c_m \geq 0$, m = manufacturer
α	Market potential
β	Consumer sensitivity to price
k	Consumer sensitivity to CSR investment, k ∈ $[0, \overline{k}]$
λ	Consumer sensitivity to marketing efforts, $\lambda \in [0, \overline{\lambda}]$
z	Marketing efforts cost coefficient, $z \in [0. \overline{z}]$
$y(p, e, \gamma)$	Deterministic demand, $y = \alpha - \beta p + k\sqrt{\gamma} + \lambda\sqrt{e}, y \geq 0$
$D(p, e, \gamma, \xi)$	Stochastic demand
μ	Mean of stochastic demand factor, $\xi \in [\underline{\xi}, \overline{\xi}]$
σ	Standard deviation of stochastic demand factor, ξ, σ > 0
π_r	Retailer's profit function
π_m	Manufacturer's profit function
	Accents:
$\overline{x}$	Maximum value of a parameter, $x \in \mathbb{R}$
$\underline{x}$	Minimum value of a parameter, $x \in \mathbb{R}$
$\hat{x}$	Parameter, $x \in \mathbb{R}$ when the demand is deterministic
$\check{x}$	Parameter, $x \in \mathbb{R}$ when the demand is stochastic with unknown distribution
$\tilde{x}$	Parameter, $x \in \mathbb{R}$ when a worst-case analysis is used for a distribution-free approach
	Superscripts:
x^d	Decentralized decisions
x^c	Centralized decisions
x^{TPT}	Coordinated decisions under a TPT contract
	Others:
$E(x)$	Expected value of a parameter, $x \in \mathbb{R}$ $[x]^+ = max\{0, x\}$. where $x \in \mathbb{R}$ $min\{a, b\} = a - E[a - b]^+$, where $a, b \in \mathbb{R}$
	Decision variables:
w	Wholesale price, $w \geq c_m$
γ	CSR investment, γ ≥ 0
p	Retail price, p ≥ (w+c_r)
e	Marketing efforts, e ≥ 0
q	Order quantity, q ≥ 0
F	Lump sum fee, $F \geq 0$

3.1. Deterministic demand

In a scenario where the retailer faces deterministic demand influenced by price, CSR investment, and marketing efforts, there is no risk of excess inventory or stock shortages. Once the retailer decides on the retail price, p, and marketing efforts, e, the order quantity, q, is automatically achieved as $q = y$, due to the deterministic nature of demand and the lack of need for safety stock. Consequently, the retailer does not need to make a separate decision regarding the order quantity, q. This concept has been discussed in various studies, including Raza (2018), Raza et al. (2018), Smith et al. (2007), and Yao et al. (2006).

3.1.1. Decentralized strategy

In the decentralized strategy, each player aims to maximize their individual profits. Specifically, the manufacturer, as a leader, first determines the wholesale price, w, and CSR investment, γ. Then, the retailer, as a follower, establishes the retail price, p, and marketing efforts, e. The profit functions for the manufacturer and the retailer can be defined as follows:

$$\pi_m(w.\gamma) = (w - c_m)y - \gamma \tag{1}$$

$$\pi_r(p, e) = (p - w)y - c_r y - ze \tag{2}$$

The optimal retail price, p, and marketing efforts, e, can be determined by solving for $\frac{\partial \pi_r}{\partial p} = 0$, and $\frac{\partial \pi_r}{\partial e} = 0$.

$$p(w, \gamma) = c_r + w + \frac{2z(\alpha - (c_r + w)\beta + k\sqrt{\gamma})}{4z\beta - \lambda^2} \tag{3}$$

$$e(w, \gamma) = \frac{(\alpha - (c_r + w)\beta + k\sqrt{\gamma})^2\lambda^2}{(-4z\beta + \lambda^2)^2} \tag{4}$$

The optimal conditions for the concavity of π_r are given in Appendix A.
In the decentralized strategy based on the Stackelberg game, the manufacturer (leader) anticipates the best response of the retailer (follower) using backward induction, as shown by Song and Gao (2018). By substituting equations (3) and (4) into equation (1), π_m can be derived as follows:

$$\pi_m(w, \gamma) = \frac{2(c_m - w)(-\alpha + (c_r + w)\beta - k\sqrt{\gamma})(z\beta - \lambda^2)}{4z\beta - \lambda^2} - \gamma \tag{5}$$

Next, the manufacturer determines the optimal values for the wholesale price, w, and CSR investment, γ, by solving for $\frac{\partial \pi_m}{\partial \gamma} = 0$, and $\frac{\partial \pi_m}{\partial w} = 0$. The accent "^" denotes the deterministic demand scenario, while the superscript "d" indicates the decentralized strategy.

$$\widehat{w}^d = \frac{z\beta(-4\alpha + c_m(k^2 - 4\beta) + 4c_r\beta) + (\alpha - c_r\beta + c_m(-k^2 + \beta))\lambda^2}{z(k^2 - 8\beta)\beta - (k^2 - 2\beta)\lambda^2} \tag{6}$$

$$\hat{\gamma}^d = \frac{k^2(\alpha - (c_m + c_r)\beta)^2(-z\beta + \lambda^2)^2}{(z(k^2 - 8\beta)\beta - (k^2 - 2\beta)\lambda^2)^2} \tag{7}$$

The optimal conditions for the concavity of π_m are given in Appendix B.
Then, the optimal values of w and γ, as derived in equations (6) and (7), can be substituted into equations (3) and (4). So, the revised expressions for the retail price and marketing efforts can be obtained as follows:

$$\hat{p}^d = \frac{z(-6\alpha + (c_m + c_r)(k^2 - 2\beta))\beta + (\alpha - (c_m + c_r)(k^2 - \beta))\lambda^2}{z(k^2 - 8\beta)\beta - (k^2 - 2\beta)\lambda^2} \tag{8}$$

$$\hat{e}^d = \frac{\beta^2(\alpha - (c_m + c_r)\beta)^2\lambda^2}{(z(k^2 - 8\beta)\beta - (k^2 - 2\beta)\lambda^2)^2} \tag{9}$$

Finally, the optimal values of $\hat{\pi}_m^d$ and $\hat{\pi}_r^d$ can be calculated by substituting $\widehat{w}^d$, $\hat{\gamma}^d$, $\hat{p}^d$, and $\hat{e}^d$ into equations (1) and (2).

3.1.2. Centralized strategy

In the centralized strategy, the primary goal is to maximize the overall profit of the SC, while the decision variables are the retail price, p, marketing efforts, e, and CSR investment, γ. The profit function for the SC can be defined as follows:

$$\pi_{sc}(p, \mathrm{e}, \gamma) = (p - (c_m + c_r))y - \gamma - ze \tag{10}$$

The optimal retail price, p, marketing efforts, e, and CSR investment, γ, can be determined by solving for $\frac{\partial \pi_{sc}}{\partial p} = 0$, $\frac{\partial \pi_{sc}}{\partial e} = 0$, and $\frac{\partial \pi_{sc}}{\partial \gamma} = 0$. The superscript "c" indicates the centralized strategy.

$$\hat{p}^c = c_m + c_r + \frac{-2z\alpha + 2(c_m + c_r)z\beta}{k^2 z - 4z\beta + \lambda^2} \tag{11}$$

$$\hat{e}^c = \frac{(\alpha - (c_m + c_r)\beta)^2 \lambda^2}{(k^2 z - 4z\beta + \lambda^2)^2} \tag{12}$$

$$\hat{\gamma}^c = \frac{k^2 z^2 (\alpha - (c_m + c_r)\beta)^2}{(k^2 z - 4z\beta + \lambda^2)^2} \tag{13}$$

The optimal conditions for the concavity of π_{sc} are given in Appendix C.
Finally, the optimal value of $\hat{\pi}_{sc}^c$ can be calculated by substituting $\hat{p}^c$, $\hat{e}^c$, and $\hat{\gamma}^c$ into equation (10).

3.1.3. Coordinated strategy through a TPT contract

Under the TPT contract, the retailer incentivizes the manufacturer to lower the wholesale price and invest more in CSR activities by offering a lump sum fee, F. The superscript "TPT" indicates the coordinated strategy through the TPT contract. The profit functions for the manufacturer and the retailer can be defined as follows:

$$\pi_m^{TPT} = \pi_m^{TPT*} + F = (w - c_m)y - \gamma + F \tag{14}$$

$$\pi_r^{TPT} = \pi_r^{TPT*} - F = (p - w)y - c_r y - ze - F \tag{15}$$

The optimal retail price, p, can be determined by solving for $\frac{\partial \pi_r^{TPT}}{\partial p} = 0$.

$$p(w, \gamma, e) = \frac{\alpha + (c_r + w)\beta + k\sqrt{\gamma} + \sqrt{e}\lambda}{2\beta} \tag{16}$$

Substituting $\hat{e}^c$ and $\hat{\gamma}^c$, as derived in equations (12) and (13), into equation (17) yields:

$$p(w) = \frac{1}{2}\left(-c_m + w + \frac{2\alpha}{\beta} + \frac{4z(\alpha - (c_m + c_r)\beta)}{k^2 z - 4z\beta + \lambda^2}\right) \tag{17}$$

Comparing equations (11) and (17), the optimal wholesale price, w, can be determined as follows:

$$\hat{w}^{TPT} = 3c_m + 2c_r - \frac{2\alpha}{\beta} + \frac{-8z\alpha + 8(c_m + c_r)z\beta}{k^2 z - 4z\beta + \lambda^2} \tag{18}$$

It should be noted that the SC players will only agree to the TPT contract if:

$$\begin{aligned} &\pi_m^{TPT*} + F \geq \pi_m^d \\ &\pi_r^{TPT*} - F \geq \pi_r^d \end{aligned} \tag{19}$$

So, the lower and upper bounds of the lump sum fee, F, can be determined as follows:

$$F^L \geq \pi_m^d - \pi_m^{TPT*}$$

$$F^U \leq \pi_r^{TPT*} - \pi_r^d \tag{20}$$

Finally, the optimal values of $\hat{\pi}_m^{TPT}$ and $\hat{\pi}_r^{TPT}$ can be calculated by substituting $\hat{w}^{TPT}$, $\hat{p}^{TPT} = \hat{p}^c$, $\hat{e}^{TPT} = \hat{e}^c$, and $\hat{\gamma}^{TPT} = \hat{\gamma}^c$ into equations (14) and (15).

3.2. Stochastic demand with partial information

This section extends the models to a scenario where the retailer faces stochastic demand influenced by price, CSR investment, and marketing efforts. The stochastic demand, represented as $D = y + \xi$, consists of a deterministic component, $y = \alpha - \beta p + k\sqrt{\gamma} + \lambda\sqrt{e}$, and an additive random factor, ξ, which is independent of price, CSR investment, and marketing efforts. Unlike deterministic demand, the retailer must decide on the order quantity, q, in this scenario.

3.2.1. Decentralized strategy

The profit functions for the manufacturer and the retailer can be defined as follows:

$$\pi_m(w, \gamma) = (w - c_m)\, q - \gamma \tag{21}$$

$$\pi_r(p, \text{e}, q) = p\, min\{q,D\} - (w + c_r)\, q - z\, e \tag{22}$$

Such that,

$$min\,\{q,D\} = q - E[q - D]^+ \tag{23}$$

Based on equation (23), π_r can be rewritten as follows:

$$\pi_r(p, \text{e}, q) = p\, q - p\, E[q - D]^+ - (w + c_r)\, q\, - z\, e \tag{24}$$

The lack of information about the distribution of stochastic demand necessitates using the distribution-free approach based on Scarf (1958)'s rule. Considering $\mu = 0$, an upper limit for the expected surplus inventory, $E[q - D]^+$, can be determined as follows (Raza (2018); Raza et al. (2018); Raza (2014); Gallego and Moon (1993)):

$$E[q - D]^+ \leq \frac{\sqrt{\sigma^2 + (q - y)^2} + (q - y)}{2} \tag{25}$$

The accent "ˇ" denotes the stochastic demand scenario with an unknown distribution.

Therefore, π_r can be reformulated as follows:

$$\check{\pi}_r(\check{p}, \check{\text{e}}, \check{q}) = \check{p}\, \check{q} - \check{p}\, \frac{\sqrt{\sigma^2 + (\check{q} - \check{y})^2} + (\check{q} - \check{y})}{2} - (\check{w} + c_r)\, \check{q}\, - z\, \check{e} \tag{26}$$

Based on equation (26), the retailer's profit maximization problem is initially addressed, and the optimal order quantity, $\check{q}$, can be determined by solving for $\frac{\partial \check{\pi}_r}{\partial q} = 0$.

$$\check{q} = \check{y} + \frac{\sigma(2\check{\rho} - 1)}{2\sqrt{\check{\rho}(1 - \check{\rho})}} \tag{27}$$

Where $\check{\rho} = \frac{\check{p} - (\check{w} + c_r)}{\check{p}}$. Substituting equation (27) into equation (26), $\check{\pi}_r$ is reformulated as follows:

$$\check{\pi}_r(\check{\text{p}}, \check{\text{e}}, \check{\text{w}}) = \left(\check{p} - (\check{w} + c_r)\right)\check{y} - \sigma\sqrt{(\check{w} + c_r)\left(\check{p} - (\check{w} + c_r)\right)} - z\, \check{e} \tag{28}$$

The expression $\sqrt{(\check{w} + c_r)\left(\check{p} - (\check{w} + c_r)\right)}$ can be simplified to $\check{p}\sqrt{\check{\rho}(1 - \check{\rho})}$. This provides a modified expression for $\check{\pi}_r$ as follows:

$$\check{\pi}_r(\check{\text{p}}, \check{\text{e}}, \check{\text{w}}) = \left(\check{p} - (\check{w} + c_r)\right)\check{y} - \sigma\, \check{p}\sqrt{\check{\rho}(1 - \check{\rho})} - z\, \check{e} \tag{29}$$

The term $\sqrt{\breve{p}(1-\breve{p})}$ is expressed as $s(\breve{p}) = \sqrt{\breve{p}(1-\breve{p})}$. Solving the partial derivatives $\frac{\partial s}{\partial \breve{p}} = 0$, $\frac{\partial s}{\partial \breve{e}} = 0$, and $\frac{\partial s}{\partial \breve{w}} = 0$ using the Chain Rule, it becomes clear that only $\frac{\partial s}{\partial \breve{p}} = 0$ is valid, and $\breve{p} = \frac{1}{2}$ maximizes the function s. This allows for a worst-case analysis of $\breve{\pi}_r$. The accent "~" denotes the distribution-free approach based on the worst possible demand result. Therefore, π_r can be reformulated as follows:

$$\tilde{\pi}_r(\tilde{p}, \tilde{e}, \tilde{w}) = \left(\tilde{p} - (\tilde{w} + c_r)\right)\tilde{y} - \frac{\tilde{p}\,\sigma}{2} - z\,\tilde{e} \tag{30}$$

The optimal retail price, $\tilde{p}$, and marketing efforts, $\tilde{e}$, can be determined by solving for $\frac{\partial \tilde{\pi}_r}{\partial \tilde{p}} = 0$, and $\frac{\partial \tilde{\pi}_r}{\partial \tilde{e}} = 0$.

$$\tilde{p}(\tilde{w}, \tilde{\gamma}) = \frac{(-c_r - \tilde{w})\lambda^2 - z(-2\left(\alpha + (c_r + \tilde{w})\beta + k\sqrt{\tilde{\gamma}}\right) + \sigma)}{4z\beta - \lambda^2} \tag{31}$$

$$\tilde{e}(\tilde{w}, \tilde{\gamma}) = \frac{\lambda^2(-2\alpha + 2(c_r + \tilde{w})\beta - 2k\sqrt{\tilde{\gamma}} + \sigma)^2}{4(-4z\beta + \lambda^2)^2} \tag{32}$$

The optimal conditions for the concavity of π_r are given in Appendix D.
Given that $\breve{p} = \frac{1}{2}$, the optimal order quantity would be $\tilde{q} = \tilde{y}$. Therefore, π_m can be reformulated as follows:

$$\tilde{\pi}_m(\tilde{w}, \tilde{\gamma}) = (\tilde{w} - c_m)\,\tilde{y} - \tilde{\gamma} \tag{33}$$

Substituting equations (31) and (32) into $\tilde{\pi}_m$, yields:

$$\tilde{\pi}_m(\tilde{w}, \tilde{\gamma}) = \frac{(c_m - \tilde{w})(\lambda^2\sigma - 2z\beta(2\alpha - 2(c_r + \tilde{w})\beta + 2k\sqrt{\tilde{\gamma}} + \sigma))}{8z\beta - 2\lambda^2} - \gamma \tag{34}$$

Next, the manufacturer determines the optimal values for the wholesale price, $\tilde{w}$, and CSR investment, $\tilde{\gamma}$, by solving for $\frac{\partial \tilde{\pi}_m}{\partial \tilde{w}} = 0$, and $\frac{\partial \tilde{\pi}_m}{\partial \tilde{\gamma}} = 0$.

$$\tilde{w}^d = \frac{4c_m z\beta^2(k^2 z - 4z\beta + \lambda^2) - (4z\beta - \lambda^2)(-\lambda^2\sigma + 2z\beta(2\alpha - 2c_r\beta + \sigma))}{4z\beta^2(k^2 z - 8z\beta + 2\lambda^2)} \tag{35}$$

$$\tilde{\gamma}^d = \frac{k^2(2z\beta(-2\alpha + 2(c_m + c_r)\beta - \sigma) + \lambda^2\sigma)^2}{16\beta^2(k^2 z - 8z\beta + 2\lambda^2)^2} \tag{36}$$

The optimal conditions for the concavity of π_m are given in Appendix E.
Then, the optimal values of $\tilde{w}^d$ and $\tilde{\gamma}^d$, as derived in equations (35) and (36), can be substituted into equations (31) and (32). So, the revised expressions for the retail price and marketing efforts can be determined as follows:

$$\tilde{p}^d = \frac{4z\beta\left(-z\left(6\alpha - (c_m + c_r)(k^2 - 2\beta)\right)\beta + (\alpha + (c_m + c_r)\beta)\lambda^2\right) - (2z^2(k^2 - 2\beta)\beta - 4z\beta\lambda^2 + \lambda^4)\sigma}{4z\beta^2(k^2 z - 8z\beta + 2\lambda^2)} \tag{37}$$

$$\tilde{e}^d = \frac{(\lambda^3\sigma + z\lambda(4\alpha\beta - 4(c_m + c_r)\beta^2 + (k^2 - 6\beta)\sigma))^2}{16z^2\beta^2(k^2 z - 8z\beta + 2\lambda^2)^2} \tag{38}$$

Finally, the optimal values of $\tilde{\pi}_r^d$ and $\tilde{\pi}_m^d$ can be calculated by substituting $\tilde{w}^d$, $\tilde{\gamma}^d$, $\tilde{p}^d$, and $\tilde{e}^d$ into equations (30) and (33).

3.2.2. Centralized strategy

The profit function for the SC, using the distribution-free approach, can be defined as follows:

$$\tilde{\pi}_{sc}(\tilde{p}, \tilde{e}, \tilde{\gamma}) = \left(\tilde{p} - (c_m + c_r)\right)\tilde{y} - \frac{\tilde{p}\,\sigma}{2} - \tilde{\gamma} - z\tilde{e} \tag{39}$$

The optimal retail price, $\tilde{p}$, marketing efforts, $\tilde{e}$, and CSR investment, $\tilde{\gamma}$, can be determined by solving for $\frac{\partial \tilde{\pi}_{sc}}{\partial \tilde{p}} = 0$, $\frac{\partial \tilde{\pi}_{sc}}{\partial \tilde{\gamma}} = 0$, and $\frac{\partial \tilde{\pi}_{sc}}{\partial \tilde{e}} = 0$.

$$\tilde{p}^c = \frac{c_m(k^2z - 2z\beta + \lambda^2) + c_r(k^2z - 2z\beta + \lambda^2) + z(-2\alpha + \sigma)}{k^2z - 4z\beta + \lambda^2} \quad (40)$$

$$\tilde{e}^c = \frac{\lambda^2(-2\alpha + 2(c_m + c_r)\beta + \sigma)^2}{4(k^2z - 4z\beta + \lambda^2)^2} \quad (41)$$

$$\tilde{\gamma}^c = \frac{k^2z^2(-2\alpha + 2(c_m + c_r)\beta + \sigma)^2}{4(k^2z - 4z\beta + \lambda^2)^2} \quad (42)$$

The optimal conditions for the concavity of π_{sc} are given in Appendix F.

Finally, the optimal value of $\tilde{\pi}_{sc}$ can be calculated by substituting $\tilde{p}^c$, $\tilde{e}^c$, and $\tilde{\gamma}^c$ into equation (39).

3.2.3. Coordinated strategy through a TPT contract

The profit functions for the manufacturer and the retailer, using the distribution-free approach, can be defined as follows:

$$\tilde{\pi}_m^{TPT} = \tilde{\pi}_m^{TPT*} + \tilde{F} = (\tilde{w} - c_m)\tilde{y} - \tilde{\gamma} + \tilde{F} \quad (43)$$

$$\tilde{\pi}_r^{TPT} = \tilde{\pi}_r^{TPT*} - \tilde{F} = (\tilde{p} - (\tilde{w} + c_r))\tilde{y} - \frac{\tilde{p}\sigma}{2} - z\tilde{e} - \tilde{F} \quad (44)$$

The optimal retail price, $\tilde{p}$, can be determined by solving for $\frac{\partial \tilde{\pi}_r^{TPT}}{\partial \tilde{p}} = 0$.

$$\tilde{p}(\tilde{w}, \tilde{\gamma}, \tilde{e}) = \frac{2\left(\alpha + (c_r + \tilde{w})\beta + k\sqrt{\tilde{\gamma}} + \sqrt{\tilde{e}}\lambda\right) - \sigma}{4\beta} \quad (45)$$

Substituting $\tilde{e}^c$ and $\tilde{\gamma}^c$, as derived in equations (41) and (42), into equation (45) yields:

$$\tilde{p}(\tilde{w}) = \frac{k^2\tilde{w}z - 4z\alpha - 4\tilde{w}z\beta + \tilde{w}\lambda^2 + c_m(k^2z + \lambda^2) + 2c_r(k^2z - 2z\beta + \lambda^2) + 2z\sigma}{2(k^2z - 4z\beta + \lambda^2)} \quad (46)$$

Comparing equations (40) and (46), the optimal wholesale price, $\tilde{w}$, can be determined as follows:

$$\tilde{w}^{TPT} = c_m \quad (47)$$

It should be noted that the SC players will only agree to the TPT contract if:

$$\tilde{\pi}_m^{TPT*} + \tilde{F} \geq \tilde{\pi}_m^d$$
$$\tilde{\pi}_r^{TPT*} - \tilde{F} \geq \tilde{\pi}_r^d \quad (48)$$

So, the lower and upper bounds of the lump sum fee, $\tilde{F}$, can be determined as follows:

$$\tilde{F}^L \geq \tilde{\pi}_m^d - \tilde{\pi}_m^{TPT*}$$
$$\tilde{F}^U \leq \tilde{\pi}_r^{TPT*} - \tilde{\pi}_r^d \quad (49)$$

Finally, the optimal values of $\tilde{\pi}_m^{TPT}$ and $\tilde{\pi}_r^{TPT}$ can be calculated by substituting $\tilde{w}^{TPT}$, $\tilde{p}^{TPT} = \tilde{p}^c$, $\tilde{e}^{TPT} = \tilde{e}^c$, and $\tilde{\gamma}^{TPT} = \tilde{\gamma}^c$ into equations (43) and (44).

4. Numerical analysis

In this section, a numerical analysis is performed to confirm the correctness and validity of the proposed models developed under stochastic demand with partial information. The input parameters are taken from Raza and Govindaluri (2019) including: $\alpha = 10000$, $\beta = 800$, $c_m = 5$, $c_r = 1$, $z = 1$, $\sigma = 100$, $\lambda = 10$, and $k = 15$. The optimal values of decision variables achieved through the distribution-free approach are presented in Table 3.

Table 3. Optimal values under decentralized, centralized, and coordinated strategies

Optimal values	Decentralized		Centralized	Coordinated	
	Manufacturer	Retailer		Manufacturer	Retailer
$\boldsymbol{p}$	-	11.0966	9.58261	-	9.58261
$\boldsymbol{w}$	8.40279	-	-	5	-
$\boldsymbol{\gamma}$	173.503	-	721.974	721.974	-
$\boldsymbol{e}$	-	71.7219	320.877	-	320.877
$\boldsymbol{F}$	-	-	-	(5329.45, 7893.95)	
$\boldsymbol{\pi}$	4607.48	1753.24	8925.22	(4607.48, 7171.98)	(1753.24, 4317.74)

Results in Table 3 show that the decentralized strategy results in a profit of 6360.72 for the SC, which is 40.32% lower than the profit achieved through the centralized strategy. The TPT coordination contract allows the SC members to benefit from a win-win situation when $F \in (5329.45, 7893.95)$. The profit ranges of the manufacturer and the retailer under the proposed contract are obtained (4607.48, 7171.98) and (1753.24, 4317.74), respectively.
In addition, a set of sensitivity analyses are conducted to examine the impact of key parameters on optimal decision variables and profits. The results are presented in Figures 1 to 5.
Figure1. compares SC profits under decentralized, centralized, and coordinated strategies while analyzing the impacts of consumer sensitivity to CSR investment, k, and marketing efforts, λ. The parameter k is changed in the range [0, 15] in increments of 5, while λ is varied in the range [0, 10] in increments of 2.5. Two main points can be derived from Figure. 1: (1) The TPT contract achieves profitability equivalent to that of the centralized strategy. (2) Increasing both k and λ leads to higher profitability for the SC. Fig. 2 graphically indicates that as both k and λ increase, the order quantity also increases.

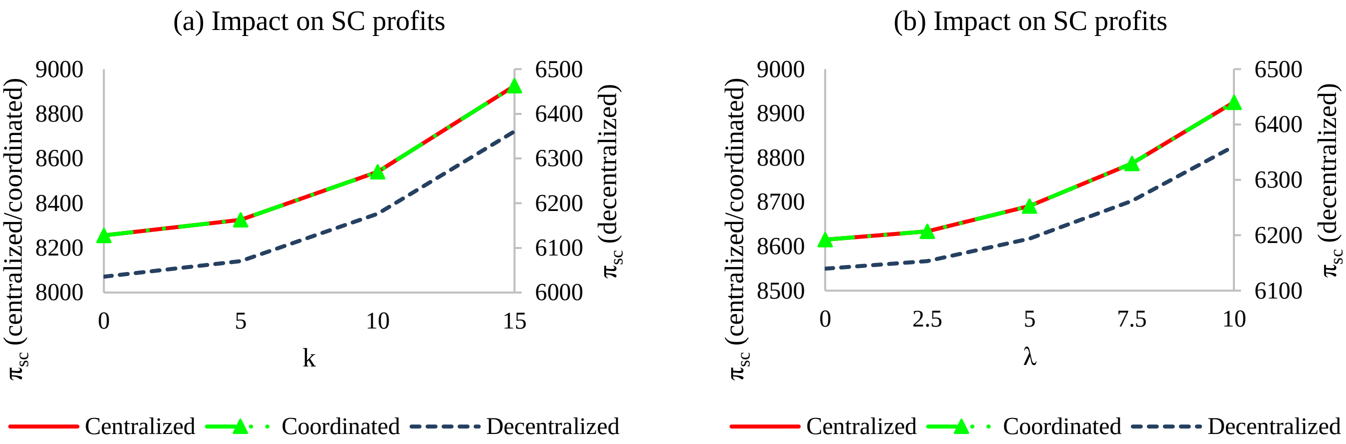


Figure 1. Impacts of k and λ on π_{sc}

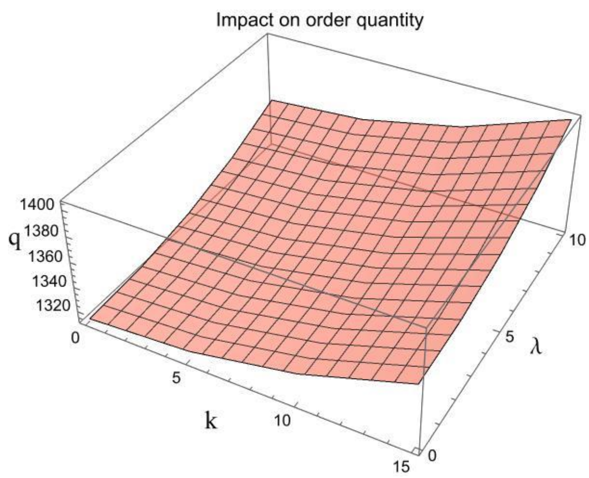


Figure 2 Impacts of k and λ on $\boldsymbol{q}$

In the coordinated strategy under the TPT contract, the impacts of k and λ on the lower and upper bounds of the lump sum fee are illustrated in Figure. 3. Two key conclusions can be drawn from Figure. 3: (1) As k increases, the manufacturer is incentivized to enhance its CSR investment. To sustain profitability alongside this increased investment, the manufacturer demands a higher lump sum fee from the retailer. (2) As λ increases, the retailer is encouraged to increase its marketing efforts. Since the retailer's marketing efforts are directly linked to the manufacturer's CSR investment, higher marketing efforts necessitate greater CSR investment by the manufacturer. To maintain profitability with this increased CSR investment, the manufacturer demands a larger lump sum fee from the retailer.

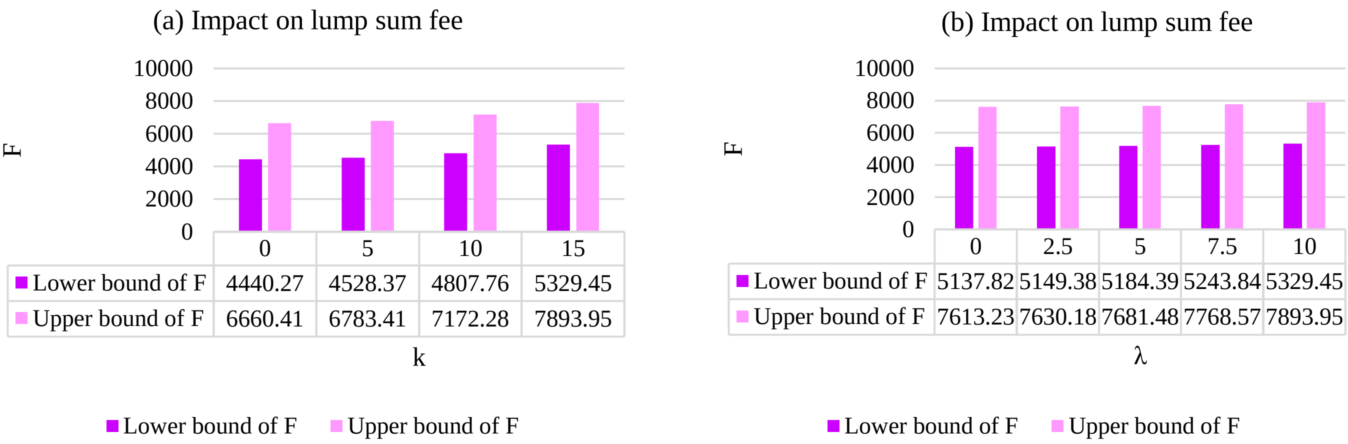
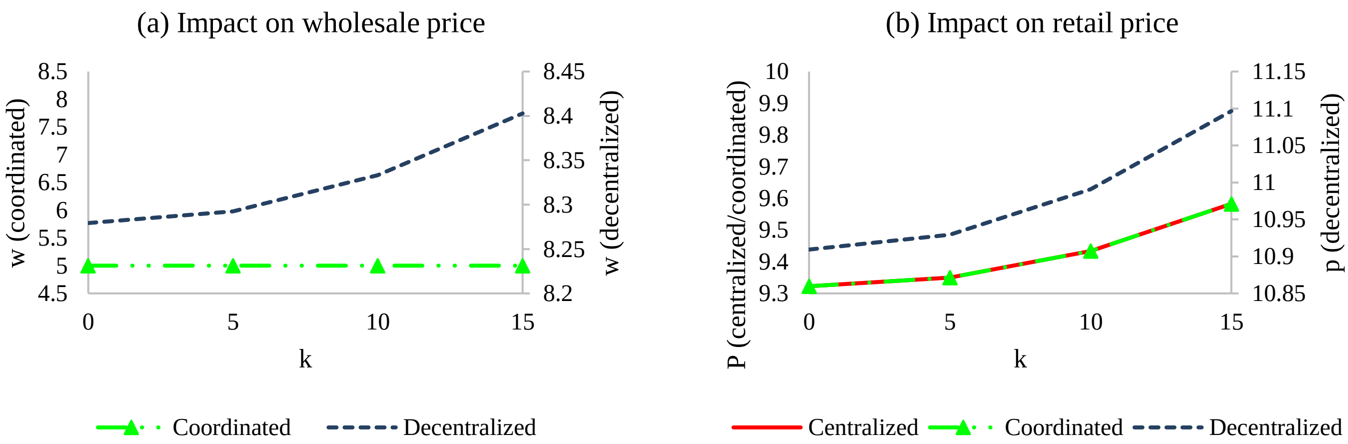


Figure 3. Impacts of k and λ on ***F***

Figure. 4 illustrates how the optimal decision variables are influenced by k under decentralized, centralized, and coordinated strategies. Three key conclusions can be drawn from Figure 4: (1) In the decentralized strategy, when k increases, the wholesale price, retail price, marketing efforts, and CSR investment all increase. This trend is driven by the manufacturer's incentive to enhance CSR investment to attract more consumers. Higher CSR investment leads to increased marketing efforts, prompting higher wholesale and retail prices to sustain profitability. (2) In the coordinated strategy through the TPT contract, as k increases, the wholesale price remains unchanged, while the retail price, marketing efforts, and CSR investment increase. This occurs because greater CSR investment encourages higher marketing efforts, leading the retailer to set a higher retail price to maintain profitability. (3) The TPT contract results in lower wholesale and retail prices while enhancing marketing efforts and CSR investment.



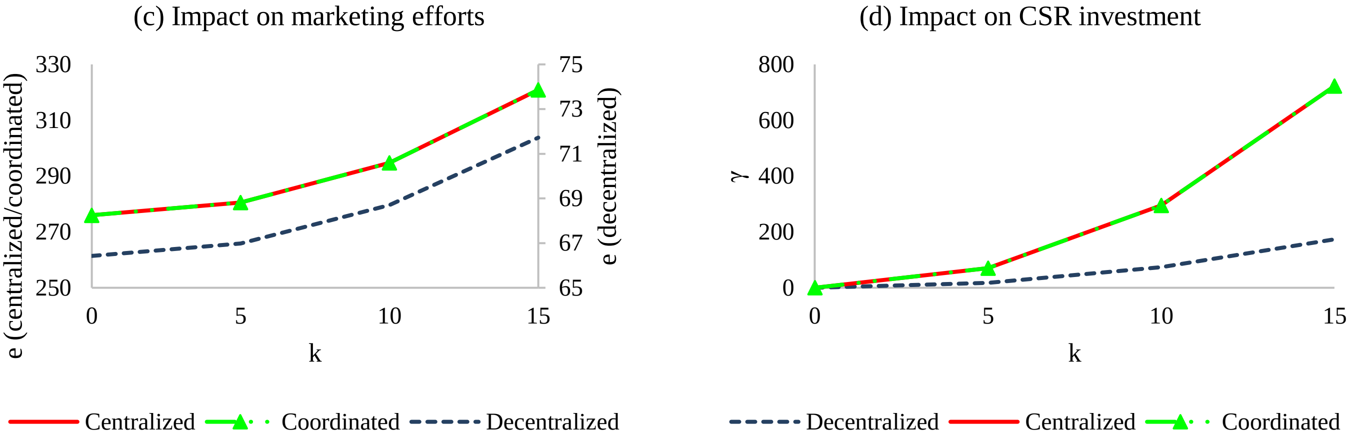
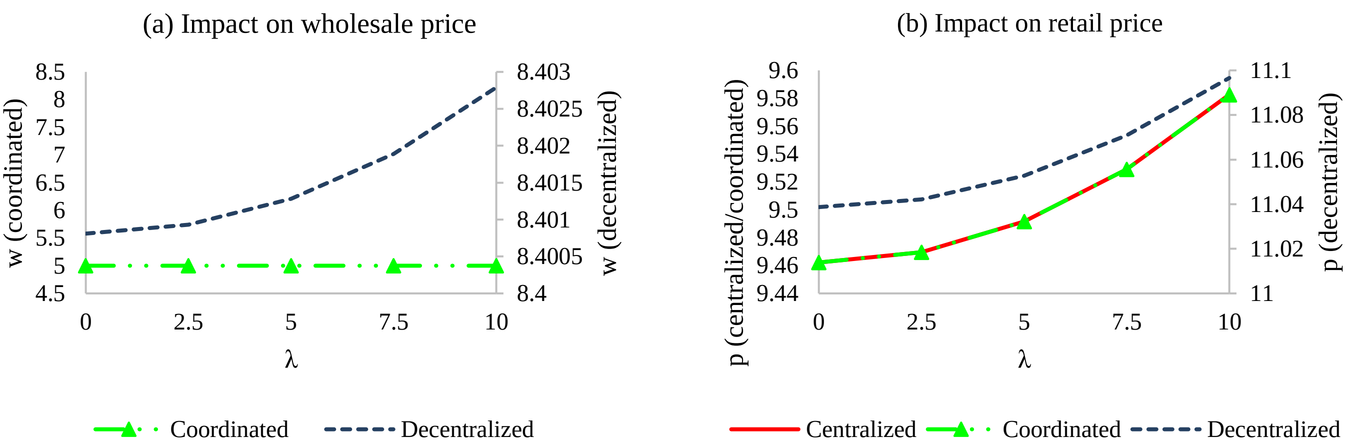


Figure 4. Impacts of k on optimal decision variables

Figure 5. illustrates the impacts of λ on the optimal decision variables under decentralized, centralized, and coordinated strategies. Two key main points can be derived from Figure 5: (1) In the decentralized strategy, as λ increases, the wholesale price, retail price, marketing efforts, and CSR investment all rise. This trend is driven by the retailer's incentive to enhance marketing efforts to attract more consumers. Greater marketing efforts require higher CSR investment from the manufacturer, leading to increased wholesale and retail prices to sustain profitability. (2) In the coordinated strategy through the TPT contract, as λ increases, the wholesale price remains unchanged, while the retail price, marketing efforts, and CSR investment increase. This occurs because greater marketing efforts lead to a higher retail price and necessitate a greater CSR investment from the manufacturer.



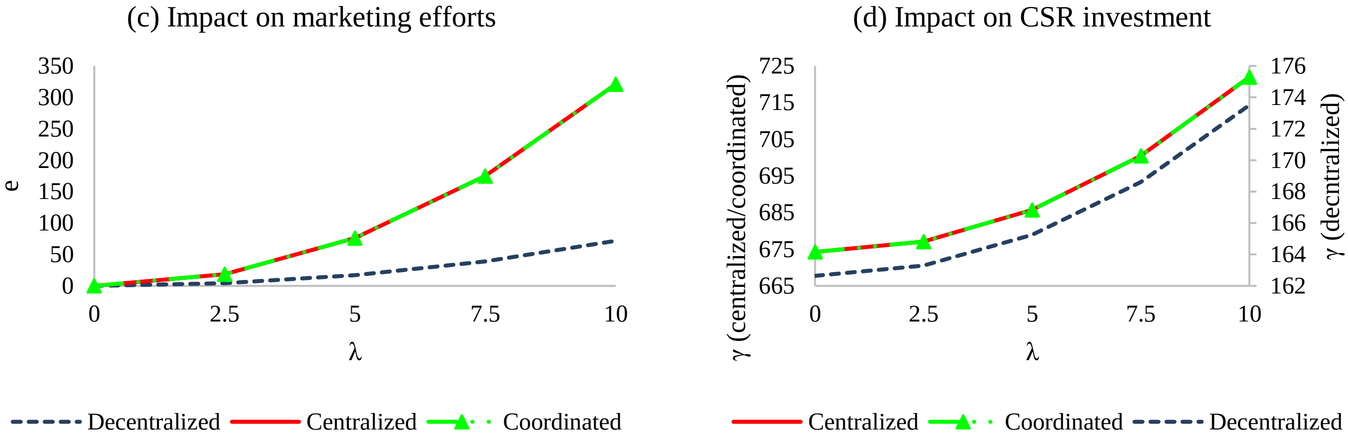


Figure 5. Impacts of λ on optimal decision variables

The results of this section offer several managerial insights:
(1) With the growing public awareness of social issues, manufacturers should recognize that engaging in CSR initiatives provides a competitive advantage. Despite the initial costs, it has significant long-term effects on profitability.
(2) Retailers play a crucial role in presenting the manufacturers' CSR initiatives to consumers through various marketing channels. This strategic alignment boosts brand image, strengthens market positioning, and increases profitability.
(3) The TPT coordination contract, by providing one-time funding, serves as an effective strategy for incentivizing manufacturers to increase their investments in CSR while also controlling product prices.

5. Conclusions

This study examines the coordination of a two-stage SC involving a manufacturer and a retailer within a single sales period. The manufacturer invests in CSR, while the retailer manages marketing efforts. Quantitative models have been developed for decentralized, centralized, and coordinated strategies under deterministic and stochastic with partial information demand scenarios. The distribution-free approach, rooted in Scarf (1958)'s rule, enables SC players to coordinate effectively without needing precise knowledge of the stochastic demand distribution. Additionally, it takes into account the possibility of the demand distribution being at its worst, allowing SC players to achieve lower bounds on their profits that are close to the optimal results. A comprehensive numerical study has shown that the TPT contract can coordinate the SC and stimulate the manufacturer to improve its CSR investment, ultimately resulting in profitability equivalent to that of the centralized strategy. Moreover, the manufacturer's CSR investment and the retailer's marketing efforts increase market demand and profitability. Besides, there is a direct correlation between the manufacturer's CSR investment and the retailer's marketing efforts. This work demonstrates several exciting paths for future research. By reducing the assumptions made in this study, a closer alignment with real-world scenarios can be achieved. For example, contrary to the assumption of this study, the market demand can be non-linear, the SC players might be risk-averse, the information between SC players can be asymmetric, and the effects of overstocking and understocking can be considered. Furthermore, the SC can be developed into a dual-channel or a closed-loop system.

Appendices

Appendix A

To prove that π_r is concave in both the retail price, $\hat{p}^d$, and marketing efforts, $\hat{e}^d$, its Hessian matrix is computed as follows:

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r}{\partial p^2} & \frac{\partial^2 \pi_r}{\partial p \partial e} \\ \frac{\partial^2 \pi_r}{\partial e \partial p} & \frac{\partial^2 \pi_r}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -2\beta & \frac{\lambda}{2\sqrt{e}} \\ \frac{\lambda}{2\sqrt{e}} & \frac{((c_r + w) - p)\lambda}{4e^{3/2}} \end{bmatrix} \tag{A.1}$$

The Hessian matrix indicates $\frac{\partial^2 \pi_r}{\partial p^2} = -2\beta \leq 0$ and $\frac{\partial^2 \pi_r}{\partial e^2} = \frac{((c_r+w)-p)\lambda}{4e^{3/2}} \leq 0$. Additionally, $|H| = \frac{\lambda\,(2\,\beta\,(p-(w+c_r))-\lambda\,\sqrt{e})}{4e^{3/2}}$ is positive if $2\,\beta\,(p-(w+c_r)) \geq \lambda\,\sqrt{e}$. This establishes an upper limit on marketing efforts, $0 \leq e \leq \overline{e}$, where $\overline{e} = (\frac{(2\,\beta\,(p-(w+c_r))}{\lambda})^2$.

Appendix B

To prove that π_m is concave in both the wholesale price, $\hat{w}^d$, and CSR investment, $\hat{\gamma}^d$, its Hessian matrix is computed as follows:

$$H = \begin{bmatrix} \frac{\partial^2 \pi_m}{\partial w^2} & \frac{\partial^2 \pi_m}{\partial w \partial \gamma} \\ \frac{\partial^2 \pi_m}{\partial \gamma \partial w} & \frac{\partial^2 \pi_m}{\partial \gamma^2} \end{bmatrix} = \begin{bmatrix} 4\beta(-1 - \frac{3z\beta}{-4z\beta + \lambda^2}) & \frac{k(z\beta - \lambda^2)}{\sqrt{\gamma}(4z\beta - \lambda^2)} \\ \frac{k(z\beta - \lambda^2)}{\sqrt{\gamma}(4z\beta - \lambda^2)} & \frac{k(c_m - w)(z\beta - \lambda^2)}{2\gamma^{3/2}(4z\beta - \lambda^2)} \end{bmatrix} \tag{B.1}$$

The Hessian matrix indicates $\frac{\partial^2 \pi_m}{\partial w^2} = 4\beta(-1 - \frac{3z\beta}{-4z\beta+\lambda^2}) \leq 0$ and $\frac{\partial^2 \pi_m}{\partial \gamma^2} = \frac{k(c_m-w)(z\beta-\lambda^2)}{2\gamma^{3/2}(4z\beta-\lambda^2)} \leq 0$ if $\lambda^2 < 4z\beta$ and $\lambda^2 < z\beta$. Also, $|H| = \frac{k(2\beta(w-c_m)-k\sqrt{\gamma})(-z\beta+\lambda^2)^2}{\gamma^{3/2}(-4z\beta+\lambda^2)^2}$ is positive if $(2\beta(w-c_m)) \geq k\sqrt{\gamma}$. This establishes an upper limit on CSR investment, $0 \leq \gamma \leq \overline{\gamma}$, where $\overline{\gamma} = (\frac{2\beta(w-c_m)}{k})^2$.

Appendix C

To prove that π_{sc} is concave in the retail price, $\hat{p}^c$, CSR investment, $\hat{\gamma}^c$, and marketing efforts, $\hat{e}^c$, its Hessian matrix is computed as follows:

$$H = \begin{bmatrix} \frac{\partial^2 \pi_{sc}}{\partial p^2} & \frac{\partial^2 \pi_{sc}}{\partial p \partial \gamma} & \frac{\partial^2 \pi_{sc}}{\partial p \partial e} \\ \frac{\partial^2 \pi_{sc}}{\partial \gamma \partial p} & \frac{\partial^2 \pi_{sc}}{\partial \gamma^2} & \frac{\partial^2 \pi_{sc}}{\partial \gamma \partial e} \\ \frac{\partial^2 \pi_{sc}}{\partial e \partial p} & \frac{\partial^2 \pi_{sc}}{\partial e \partial \gamma} & \frac{\partial^2 \pi_{sc}}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -2\beta & \frac{k}{2\sqrt{\gamma}} & \frac{\lambda}{2\sqrt{e}} \\ \frac{k}{2\sqrt{\gamma}} & \frac{k((c_m + c_r) - p)}{4\gamma^{3/2}} & 0 \\ \frac{\lambda}{2\sqrt{e}} & 0 & \frac{((c_m + c_r) - p)\lambda}{4e^{3/2}} \end{bmatrix} \tag{C.1}$$

The Hessian matrix indicates $\frac{\partial^2 \pi_{sc}}{\partial p^2} = -2\beta \leq 0$, $\frac{\partial^2 \pi_{sc}}{\partial \gamma^2} = \frac{k((c_m+c_r)-p)}{4\gamma^{3/2}} \leq 0$, and $\frac{\partial^2 \pi_{sc}}{\partial e^2} = \frac{((c_m+c_r)-p)\lambda}{4e^{3/2}} \leq 0$. In addition, $|H_2| = \frac{k(2\beta(p-(c_m+c_r))-k\sqrt{\gamma})}{4\gamma^{3/2}}$ is positive if $2\beta(p-(c_m+c_r)) \geq k\sqrt{\gamma}$. Furthermore, $|H_3| = \frac{k(p-(c_m+c_r))\lambda(2\beta((c_m+c_r)-p)+k\sqrt{\gamma}+\lambda\sqrt{e})}{16e^{3/2}\gamma^{3/2}}$ is negative if $2\beta((c_m+c_r)-p) + k\sqrt{\gamma} + \lambda\sqrt{e} \leq 0$.

Appendix D

To prove that $\tilde{\pi}_r$ is concave in both the retail price, $\tilde{p}^d$, and marketing efforts, $\tilde{e}^d$, its Hessian matrix is computed as follows:

$$H = \begin{bmatrix} \frac{\partial^2 \tilde{\pi}_r}{\partial \tilde{p}^2} & \frac{\partial^2 \tilde{\pi}_r}{\partial \tilde{p} \partial \tilde{e}} \\ \frac{\partial^2 \tilde{\pi}_r}{\partial \tilde{e} \partial \tilde{p}} & \frac{\partial^2 \tilde{\pi}_r}{\partial \tilde{e}^2} \end{bmatrix} = \begin{bmatrix} -2\beta & \frac{\lambda}{2\sqrt{\tilde{e}}} \\ \frac{\lambda}{2\sqrt{\tilde{e}}} & \frac{((c_r + \tilde{w}) - \tilde{p})\lambda}{4\tilde{e}^{3/2}} \end{bmatrix} \tag{D.1}$$

The Hessian matrix indicates $\frac{\partial^2 \tilde{\pi}_r}{\partial \tilde{p}^2} = -2\beta \leq 0$ and $\frac{\partial^2 \tilde{\pi}_r}{\partial \tilde{e}^2} = \frac{((c_r+\tilde{w})-\tilde{p})\lambda}{4\tilde{e}^{3/2}} \leq 0$. Moreover, $|H| = \frac{\lambda\,(2\,\beta\,(\tilde{p}-(\tilde{w}+c_r))-\lambda\,\sqrt{\tilde{e}})}{4\tilde{e}^{3/2}}$ is positive if $2\,\beta\,(\tilde{p}-(\tilde{w}+c_r)) \geq \lambda\,\sqrt{\tilde{e}}$. This establishes an upper limit on marketing efforts, $0 \leq \tilde{e} \leq \overline{e}$, where $\overline{e} = (\frac{(2\,\beta\,(\tilde{p}-(\tilde{w}+c_r))}{\lambda})^2$.

Appendix E

To prove that $\tilde{\pi}_m$ is concave in both the wholesale price, $\tilde{w}^d$, and CSR investment, $\tilde{\gamma}^d$, its Hessian matrix is computed as follows:

$$H = \begin{bmatrix} \frac{\partial^2 \tilde{\pi}_m}{\partial \tilde{w}^2} & \frac{\partial^2 \tilde{\pi}_m}{\partial \tilde{w} \partial \tilde{\gamma}} \\ \frac{\partial^2 \tilde{\pi}_m}{\partial \tilde{\gamma} \partial \tilde{w}} & \frac{\partial^2 \tilde{\pi}_m}{\partial \tilde{\gamma}^2} \end{bmatrix} = \begin{bmatrix} \frac{4z\beta^2}{-4z\beta + \lambda^2} & \frac{kz\beta}{\sqrt{\tilde{\gamma}}(4z\beta - \lambda^2)} \\ \frac{kz\beta}{\sqrt{\tilde{\gamma}}(4z\beta - \lambda^2)} & -\frac{k(c_m - \tilde{w})z\beta}{2\tilde{\gamma}^{3/2}(-4z\beta + \lambda^2)} \end{bmatrix} \tag{E.1}$$

The Hessian matrix indicates $\frac{\partial^2 \tilde{\pi}_m}{\partial \tilde{w}^2} = \frac{4z\beta^2}{-4z\beta+\lambda^2} \leq 0$ and $\frac{\partial^2 \tilde{\pi}_m}{\partial \tilde{\gamma}^2} = -\frac{k(c_m - \tilde{w})z\beta}{2\tilde{\gamma}^{3/2}(-4z\beta+\lambda^2)} \leq 0$ if $\lambda^2 < 4z\beta$. Besides, $|H| = \frac{kz^2\beta^2(2\beta(\tilde{w}-c_m)-k\sqrt{\tilde{\gamma}})}{\tilde{\gamma}^{3/2}(-4z\beta+\lambda^2)^2}$ is positive if $2\beta(\tilde{w} - c_m) \geq k\sqrt{\tilde{\gamma}}$. This establishes an upper limit on CSR investment, $0 \leq \tilde{\gamma} \leq \overline{\gamma}$, where $\overline{\gamma} = (\frac{2\beta(\tilde{w}-c_m)}{k})^2$.

Appendix F

To prove that $\tilde{\pi}_{sc}$ is concave in the retail price, $\tilde{p}^c$, CSR investment, $\tilde{\gamma}^c$, and marketing efforts, $\tilde{e}^c$, its Hessian matrix is computed as follows:

$$H = \begin{bmatrix} \frac{\partial^2 \tilde{\pi}_{sc}}{\partial \tilde{p}^2} & \frac{\partial^2 \tilde{\pi}_{sc}}{\partial \tilde{p} \partial \tilde{\gamma}} & \frac{\partial^2 \tilde{\pi}_{sc}}{\partial \tilde{p} \partial \tilde{e}} \\ \frac{\partial^2 \tilde{\pi}_{sc}}{\partial \tilde{\gamma} \partial \tilde{p}} & \frac{\partial^2 \tilde{\pi}_{sc}}{\partial \tilde{\gamma}^2} & \frac{\partial^2 \tilde{\pi}_{sc}}{\partial \tilde{\gamma} \partial \tilde{e}} \\ \frac{\partial^2 \tilde{\pi}_{sc}}{\partial \tilde{e} \partial \tilde{p}} & \frac{\partial^2 \tilde{\pi}_{sc}}{\partial \tilde{e} \partial \tilde{\gamma}} & \frac{\partial^2 \tilde{\pi}_{sc}}{\partial \tilde{e}^2} \end{bmatrix} = \begin{bmatrix} -2\beta & \frac{k}{2\sqrt{\tilde{\gamma}}} & \frac{\lambda}{2\sqrt{\tilde{e}}} \\ \frac{k}{2\sqrt{\tilde{\gamma}}} & \frac{k((c_m + c_r) - \tilde{p})}{4\tilde{\gamma}^{3/2}} & 0 \\ \frac{\lambda}{2\sqrt{\tilde{e}}} & 0 & \frac{((c_m + c_r) - \tilde{p})\lambda}{4\tilde{e}^{3/2}} \end{bmatrix} \tag{F.1}$$

The Hessian matrix indicates $\frac{\partial^2 \tilde{\pi}_{sc}}{\partial \tilde{p}^2} = -2\beta \leq 0$, $\frac{\partial^2 \tilde{\pi}_{sc}}{\partial \tilde{\gamma}^2} = \frac{k((c_m+c_r)-\tilde{p})}{4\tilde{\gamma}^{3/2}} \leq 0$, and $\frac{\partial^2 \tilde{\pi}_{sc}}{\partial \tilde{e}^2} = \frac{((c_m+c_r)-\tilde{p})\lambda}{4\tilde{e}^{3/2}} \leq 0$. Also, $|H_2| = \frac{k(2\beta(\tilde{p}-(c_m+c_r))-k\sqrt{\tilde{\gamma}})}{4\tilde{\gamma}^{3/2}}$ is positive if $2\beta(\tilde{p} - (c_m + c_r)) \geq k\sqrt{\tilde{\gamma}}$. Furthermore, $|H_3| = \frac{k(\tilde{p}-(c_m+c_r))\lambda(2\beta((c_m+c_r)-\tilde{p})+k\sqrt{\tilde{\gamma}}+\lambda\sqrt{\tilde{e}})}{16\tilde{e}^{3/2}\tilde{\gamma}^{3/2}}$ is negative if $2\beta((c_m + c_r) - \tilde{p}) + k\sqrt{\tilde{\gamma}} + \lambda\sqrt{\tilde{e}} \leq 0$.

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