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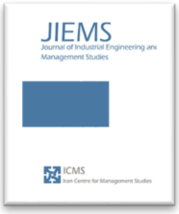

Effect of different types of failure and repair policies in determining warranty policy parameters

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Abstract

Nowadays offering attractive warranty offers along with selling products become a common tool in marketing. Warranty offers make sure the customer about receiving some services in the case of product failure during its useful lifetime. On the other hand, they help manufacturers to protect their reputation in the case of product failure by providing repair services for their customers. In this study, a repairable product after initial performance test (which is known as the burn-in test) sends to the market with a non-renewable linear pro-rata warranty offer. If during burn-in test or warranty periods minor or major failure occurs, depend on the failure type, the product is repaired minimally, generally, or replaced. In this study, the cost model for these two periods is extracted and optimum values for the length of burn-in test and warranty period with the aim of minimizing average total cost from manufacturer perspective is obtained. In order to show the applicability of the proposed model, a numerical example is presented.

Keywords: Warranty, Minor and major failure, Minimal and general repair, Burn-in test.

Paper Type: Original Research

1. Introduction

Nowadays warranty policy as a modern strategy has been used widely along with selling a product. Based on the vital role of warranty policies in marketing, researchers pay attention to this area during recent years. An attractive warranty offer can assure the buyer about receiving repair/replacement services in the case of product failure. On the other hand, warranty services can protect producer's reputation in the case of product failure during warranty period. Since selling warranty offers along with the selling of products, increase the product price for the buyers, hence, calculating this cost is very important. If a producer evaluates this cost below the real value, it will result in decreasing producer's profit and if it is evaluated more than its real cost, the producer will lose its market share. As the manufactured products are different, the offered warranty policies for them will also be different. For example the warranty offers for electrical devices are different from those for mechanical devices. Warranty policies can be classified from different perspectives. For instance, warranty policy can be renewable or non-renewable. In the renewable warranty, after any failure in the product, and then replacement or repair of the defective part, the warranty period starts from the beginning. But in the non-renewable warranty, after any repair or replacement, the previous warranty period continues. Khorshidvand et. al. (2014) analyzed two different systems in their research subject to shocks occurring based on a non-homogeneous Poisson process. The first type of the system consists of only a single unit and the second one consists of two units in parallel that operate identically and simultaneously. In the first type system, the system stops at shock occurrence and receives minimal repair after that. For this system, preventive maintenance policy is applied. However, in the second type of the system, if any of the units experiences a shock, it is repaired minimally and the other unit receives preventive maintenance service without stopping the system. The goal of the model was to minimize the total average cost. Shang et.al. (2018) with the aim of maximizing the manufacturer's profit by optimizing the warranty period, sale price, and replacement threshold, proposed a condition-based renewable replacement warranty. They showed that in a monopoly market it is more profitable to let the replacement threshold equal the failure threshold. But, the optimal replacement threshold should not be more than failure threshold in competitive market. Alem Tabriz and his team of researchers (2016) determined the optimal replacement cycle for a system subject to shocks and failure rate with the aim of minimizing the system maintenance costs by obtaining the optimal parameters of the proposed age-based replacement model. There exist many researches that non-renewable warranty policy is considered. For example,

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Alqahtani and Gupta (2018) proposed an optimization model with the aim of maximizing the manufacturer's profit for remanufactured items sold with "one-dimensional non-renewing money-back guarantee" warranty policy. Since in their model an End-Of-Life product is subjected to upgrade action at the end of its past life, during warranty period, when the remaining life of the product reaches a pre-specified value, preventive maintenance actions are carried out. Warranty policies can be free repair/replacement warranty (FRW) or pro-rata warranty (PRW). In free repair/replacement warranty, if during warranty period a failure occurs, manufacturer will be responsible for repair/replacement of the failed product and consumer should not pay the cost of this service. But in the pro-rata warranty, both consumer and manufacturer are responsible for the repair cost. Based on the cost sharing function, consumer and manufacturer should pay their share of the repair cost. Luo and Wu (2018) in their paper, optimized warranty policy for a set of different products which produced by one manufacturer, whose failures are statistically dependent. They proved the existence of the optimal solutions for different scenarios. Liu et.al. (2021) derived the expected life cycle cost rate of the product from the customer view. In their study, they investigated an optimal replacement problem by considering different type of failures (repairable failure and catastrophic failures), different type of repairs (minimal repair and replacement), and also different type of warranties (pro-rata refund warranty and free replacement warranty). It is worth mentioning that under both warranty policies, during warranty period the repair cost is free for the customer while within the post warranty period all repair cost should be borne by the customer. MoghimiHadji (2021) constructed the cost model for a non-repairable system and obtained optimal values for the length of burn-in test and warranty periods by minimizing the average total cost during burn-in test and warranty periods. In his proposed model, if a component fails during burn-in test period, it will replace with a new component, while if the component fails during warranty period, the consumer will receive a pro-rata warranty service. Generally, warranty policies are one dimensional, but there are some products that their warranty offers are two-dimensional. For example the warranty offer for automobiles is two-dimensional, namely age (time) and usage (traveled distance). Although problem formulation and model solution in two-dimensional problems are much more complicated than one-dimensional problems, research in this field is also progressing. Du et.al. (2022) proposed a new warranty policy which is two-dimensional renewing free-replacement. They investigated two kinds of post warranty maintenance model (uniform and customized model) by extending the warranty model to the post warranty maintenance model and for each proposed model; they provided an algorithm to seek the optimum solution. Using numerical examples, they showed that the traditional renewing free replacement warranty (RFRW) cost is higher than the proposed warranty policy and the customized one is inferior to the uniform model. Zheng and Su (2020) with the objective of determining the optimal boundary between the two regions from the perspective of customers, proposed a flexible two-dimensional basic warranty policy with two continuous rectangular regions. Then, they developed a warranty pricing model based on the polynomial failure intensity model. After that, the customer's expected repair cost over the warranty period is analyzed and a numerical method is proposed for the optimization problem. Mitra (2021) considered a two-dimensional policy for extended warranty. In his research, decision variables are the length of warranty period and usage in the initial policy, the length of warranty period and usage in the extended policy, the product price, and the premium to be charged by the manufacturer for extending the warranty. Warranty policies can be classified from one-stage, two-stage, or multi-stage point of view. In two stage or multi-stage warranty policies, the warranty policies or the type of repairs that can be done in any phase of the warranty, can be different. In such warranty policies, several different types of warranty can be combined together. Yousefi and her collogues (2019) examined a warranty model where after the end of the two- phase warranty period, an extra warranty period is provided to the customer. The first part of the two-phase warranty period, is free replacement/repair warranty and the second part is pro-rata warranty. During extended warranty period only minimal repair is done. Since the customers consumption rates are different, they advised to classify customers based on this rate, which results in reducing the warranty cost. Zheng and Su (2018) in their paper, proposed a warranty policy for repairable products with bathtub-shape failure rate function. They employed a two-fold Weibull competing risk model to describe the bathtub-shape cure. In their study, they presented a numerical optimization model to achieve the optimal preventive maintenance strategy. MoghimiHadji (2020) investigated the total cost incurred during the burn-in and warranty periods from manufacturer perspective. In the proposed model, he considered different types of failure and different types of repair services, and obtained the average total cost in each period. He presented an optimization example to calculate optimum values of burn-in test and warranty periods using proposed model. MoghimiHadji and EslamiParsa (2022) considered a three-stage non-renewing warranty for a repairable system, in their study. By separating whole warranty period into three different stages and extracting cost model during these phases, they calculated optimum values for the length of each phases with the aim of minimizing the total cost. In new researches, the combination of warranty policies with product maintenance policies has also been considered. For example, choosing the optimal policy for preventive maintenance of a product during the warranty period or after this period has been the focus of some researchers. Liu and his team of researchers (2021) derived the expected life cycle cost rate of the product during warranty time from customer's point of view. By minimizing the average cost rate, they theoretically proved the existence and uniqueness of the optimal maintenance strategy for the warranty product. Ullah et.al. (2021) with the aim of developing a model for determining whether how much effort of preventive maintenance action is worthwhile for the consumer over the post-sale product life cycle of a repairable complex product, they designed expected life cycle cost for a warranted product from the consumer perspective. Those interested can refer to the article by Taravat Sarabi and MoghimiHadji (2023) which provides a complete classification of different types of warranty policies. In this study, a repairable product is considered. Before selling the product in the market, in order to be sure about preventing early failure (infant mortality) in the consumer place, manufacturer arranges a burn-in test with the length of b for all the manufactured products in the factory. During this period, if the product faces a minimal failure with the probability P1, the manufacturer repairs this damaged product minimally. If a major failure take happens during this test with the probability 1-P1, manufacturers replace this item with a new one and again burn-in test starts from the beginning for this new item. After completing burn-in test successfully, these products will send to the market with a non-renewable linear pro-rata warranty offer of length W. If during this warranty period a minor failure occurs (with the probability P2), minimal repair will be done to repair this failed item. But, if a catastrophic failure occurs (with the probability 1-P2), the damaged item will repair generally with the rejuvenation factor $\boldsymbol{\delta}$ (generally, $0\leq\boldsymbol{\delta}\leq1$). If the system at age t, repairs generally, after this general repair the age of the system will decrease to $\boldsymbol{\delta}$t. Since in the real world all the failures are not covered by the warranty services, it is supposed that, based on the experience of the manufacturer, averagely, P percent of the warranty claims are acceptable. Of course, this value can be different from one manufacturer to another one. In this problem only P percent of warranty claims are acceptable. Because some damages such as physical damages are not covered by warranty services. Such a model, which considers different failure types along with different types of failures with a linear pro-rata warranty offer, has not been studied by any of the researchers so far. The aim of this study is to minimize the average total cost during burn-in and warranty periods by calculating optimum values for the length of burn-in test (b) and warranty (W) periods. The organization of the rest of the article will be as follows. In the next section, the notations used in the article are listed. After that, in section 3, the model is presented and the costs during these two periods are derived. Then in section 4, the numerical optimization example is explained. Concluding remarks are made in the last section.

2. Notations used

f(t): Probability density function of the product life time

F(t): Probability distribution function of the product life time

b: Burn-in test period length

ti: Time of the ith failure

h(t): Failure rate function which, $h(t) = \frac{f(t)}{\bar{F}(t)}$

$\bar{F}(t)$: Survival function which, $\bar{F}(t) = 1 - F(t)$

P1: Probability of minor failure occurrence during burn-in test period

1-P1: Probability of major failure occurrence during burn-in test period

P2: Probability of minor failure occurrence during warranty period

1-P2: Probability of major failure occurrence during warranty period

W: Length of the warranty period

P: The possibility of a failure being covered by the warranty

C1: Product unit price for manufacturer

C2: Installation cost for a new product

C3: Operation cost of a product per unit time

C4: Minimal repair cost in the manufacturer place

C5: Replacement cost in the manufacturer place

Cm: Minimal repair cost during warranty period

Cg: General repair cost during warranty period

Cr: Replacement cost during warranty period

C(t): Pro-rata warranty cost function

N(b)-1: Number of replacement during burn-in period until for the first time one product pass this period without major failure

$\boldsymbol{\delta}$: rejuvenation factor

Ng(W): Number of major failure during warranty period when general repair is done

Mg(W): Average number of Ng(.)

3. The model

During burn-in period the main costs relating to the product are; installation and setup cost, C2, operating cost of a product per unit time, C3, cost of minimal repair, C4 and cost of damaged part replacement, C5. If during this period a product suffers minor failure, the related costs for the manufacturer will be consist of the product installation and setup cost during burn-in period, operating cost, and minimal repair cost depends on the number of failures. Thus, the total cost during this period if the product faces only minor failures, can be shown as:

$$C_{b1} = c_2 + c_3 \mathrm{b} + c_4 \int_0^b h(t)dt \quad (1)$$

where $h(t)$ is the failure rate function when a product repairs minimally after any minor failure.

On the other hand, if the product faces major failure during this period, manufacturer will replace it with a new product. Hence, the related costs will be consist of installation and setup cost depends on the number of major failure, operation cost, and replacement cost based on the number of major failures which can be shown as the following:

$$C_{b2} = c_2\, N(b) + c_3 \left(\sum_{i=1}^{N(b)-1} t_i + b\right) + c_5(N(b) - 1) \quad (2)$$

where $N(b) - 1$ is the number of damaged part replacement until for the first time one product can pass this period successfully. Since the probability of having a minor failure is P1 and the probability of having major failure is 1-P1 during burn-in period, the total cost during this period can be calculated as:

$$C_b = P_1 \left[c_2 + c_3 \mathrm{b} + c_4 \int_0^b h(t)dt\right] + (1 - P_1)\left[N(b)c_2 + c_3 \left(\sum_{i=1}^{N(b)-1} t_i + b\right) + c_5(N(b) - 1)\right] \quad (3)$$

Based on the definition of N(b), it can be seen that N(b) has geometric distribution, thus the probability density function of N(b) can be defined as:

$$P(N(b) = m) = \mathrm{F}(\mathrm{b})^{m-1}.\bar{F}(b) \quad ,k > 0 \quad (4)$$

Based on the properties of geometric distribution, the expected value for number of failures is:

$$E[N(b)] = \frac{1}{\bar{F}(b)} \quad (5)$$

By considering Wald's identity,

$$E\left[\sum_{i=1}^{N(b)-1} t_i\right] = E[N(b)].E(t_i) - E\left[t_{N(b)}\right] = \frac{\int_0^b \bar{F}(t)dt}{\bar{F}(b)} - b \quad (6)$$

wherein $E\left[t_{N(b)}\right]$ is the average operation time of the last product was replaces which was able to pass the burn-in period successfully. Thus, the average total cost for the manufacturer during burn-in period will be:

$$C_b = P_1 \left[c_2 + c_3 \mathrm{b} + c_4 \int_0^b \frac{f(t)}{\bar{F}(t)} dt\right] + (1 - P_1)\left[\frac{c_2}{\bar{F}(b)} + c_3 \left(\frac{\int_0^b \bar{F}(t)dt}{\bar{F}(b)}\right) + c_5 \frac{F(b)}{\bar{F}(b)}\right] \quad (7)$$

Until the end of burn-in period (b), all the repair/replacement costs will be borne by the manufacturer. At the end of this period, a product with age b, will send to the market for sale with a non-renewable linear pro-rata warranty offer of length W. During warranty period, repair cost (minimal or general repair) will be share between manufacturer and consumer based on a linear pro-rata function which can be defined as the follows:

$$C(t) = \begin{cases} (1 - \frac{t-b}{W}).c & b < t \leq b + W \\ 0, & b + W < t \end{cases} \quad (8)$$

Although there are other forms of this function (even nonlinear forms) can be found in the literature, in this study the above linear form is considered. If the failure time is equal to b, all the costs should be borne by the manufacturer. Depends on the type of failure, it can be minimal repair cost (Cm) or cost of general repair (Cg). If the failure time is equal to b+W, all the costs should be paid by the consumer. Hence, based on this function, if the failure time during warranty period is between (b, b+W], the repair costs (based on the failure type) will be shared between manufacturer and consumer. Here, the function of C(t) shows the manufacturer share. During the second period, only P percent of the warranty claims can be accepted by manufacturer, since some types of failures, such as physical damages or electrical connections due to spilled liquids on electrical parts, generally are not covered by warranty services. During this period, the probability of minor failure occurrence is P2 which is solved by a minor repair action and the probability of facing a major failure is 1-P2 which is solved by doing a general repair action. Similar to the Nasrollahi and Asgharizadeh (2016) reasoning, if the product faces only minor failures, the repair cost for the manufacturer during warranty period can be calculated using the following relation.

$$C_{W1} = \int_b^{b+W} C(t)f(t)dt.P \int_b^{b+W} h(t)dt \quad (9)$$

If the product faces major failure, it repairs generally and the number of failures during warranty period can be shown by Ng(W). The total repair cost during warranty period if the product faces major failure can be calculated using the following relation.

$$C_{W2=}\int_{b}^{b+W} C(t)f(t)dt.P\,N_g(W) \tag{10}$$

If a product at age t, repairs generally, the age of the product will be δt ($0 \leq \delta \leq 1$) after general repair. If δ is equal to 0, the age of the product will be zero. It means that the product will be new after this repair. On the other side, if δ is equal to 1, the age of the product will be again t, after this repair. It means that the repair action was minimal repair. Parameter δ is called rejuvenation factor. In this study δ will be something between 0 and 1. It means that the repaired product is not as good as the same before failure and is not same as new product, but something between them. The average value of Ng(W) is shown by Mg(W), which for the first time was introduced by Kijima (1989) and called g-renewal function.

$$M_g(t) = Q(t|0) + \int_0^t Q(t-x|x).m(x)dx \tag{11}$$

where in

$$Q(t|x) = \int_0^t q(y|x)dy = \int_0^t \frac{f(y+\delta x)}{\bar{F}(\delta x)}dy \tag{12}$$

It is worth mentioning that the number of failures when a product repairs generally, will be depended on rejuvenation factor. Also it should be considered that, only when the lifetime distribution function follows gamma distribution, it is possible to calculate Mg(t), analytically . In other cases researchers try to calculate Mg(t), numerically. In this research a technique which was introduced by Rangan and MoghimiHadji (2011), is employed.

If the replacement cost for a damaged product during warranty period is shown by Cr, the general repair cost can be shown by:

$$C_g = (1-\delta)C_r \tag{13}$$

Thus, the total cost during warranty period can be shown using the following relation:

$$C_W = P_2.\int_b^{b+W}(1-\frac{t-b}{W})\,C_m\,f(t)\,dt\,.P\int_b^{b+W}h(t)dt \;+\; (1-P_2)\int_b^{b+W}(1-\frac{t-b}{w}).(1-\;\delta)C_r f(t)dt.p.M_g(\mathrm{W})$$
(14)

The average total cost during burn-in and warranty periods, can be calculated using the below equation:

$$E(TC) = \frac{C_b+C_W}{W} \tag{15}$$

The aim of solving this model is to minimize the average total cost by finding optimum values for b (Length of burn-in period) and W (Length of warranty period).

4. Numerical example

In order to show the performance and simplicity of implementation of the proposed model, a product with a bathtub failure rate curve is considered. In this example a five-parameter failure rate function which was introduced by Dhillon (1979), is considered.

$$h(t) = kC\lambda t^{C-1} + (1-k)Bt^{B-1}.\beta e^{\beta t^B} \qquad ,0 \leq k \leq 1,\; t \geq 0 \tag{16}$$

In this five-parameter hazard function, β and λ are scale parameters, B and C are shape parameters and one can generate different shapes of failure curve by making changes in their values. In this example the five parameters are chosen as follows: λ=1, β=1, k=0.5, B=2.5 and C=0.3. Such a choice results in a failure rate curve with a steep slope of decreasing failure rate over a short period. Typical examples of products with such a failure rate curve are electronic items. The shape of this curve is shown in the figure below (Figure 1).

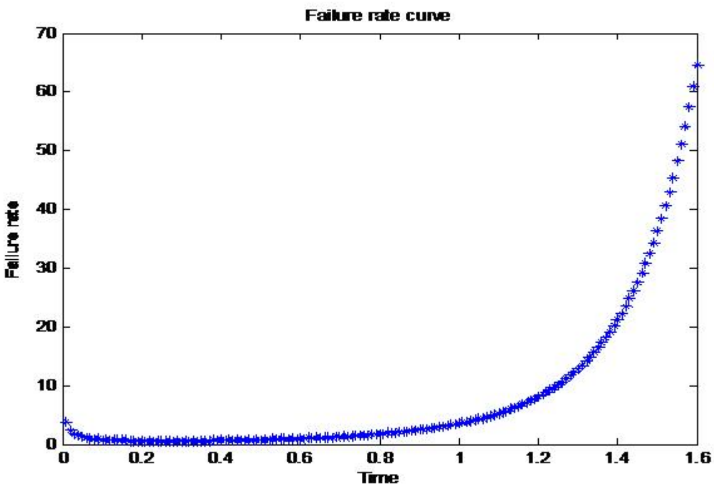

Figure 1. Curve of bathtub failure rate with the above parameters

Since failure rate cure of many deteriorating systems shows a bathtub shaped curve wherein three distinguished phase can be seen, in this example such a form has chosen. As it can be seen in Figure 1, this bathtub shape curve shows three distinguishable areas. In the first part of this curve, it decreases sharply in a short time. During the initial period of operation of a product, the main reason for failures is related to defective materials or poor quality control. This period is called the infant mortality period. In the second phase of this curve, it continues without sharp increasing or decreasing for a long period of time. During the middle period of a product's life, the amount of failure rate is often small and almost constant. The main reason for failures at this stage is human errors, improper use, and accidental overload on the system. This period is also known as the useful life period of the system, and the warranty period is also a part of this period. In the last phase of this cure, the failure rate increases rapidly in a short period of time. The last period of the process depicts product aging, which results in increased failure rates. The system cannot work continuously for a long time and it needs more break times for maintenance. This period is known as the period of wear and tear of the product, and breakdowns occur during this period due to erosion, wear and tear, and excessive operation of the product. In this phase, the condition of the system becomes worse and worse with the passage of time. Although in this numerical example a bathtub shaped failure rate function is employed, it is possible to use any other failure rate function in this model. It is worth mentioning that if the failure rate curve has another shape, based on the shape of this curve; one should revise the total cost function. For instance, if the failure rate curve has exponential form (see Figure 2), since there is not a decreasing failure rate at the beginning of this curve, it is not necessary to consider a short period time as the burn-in period.

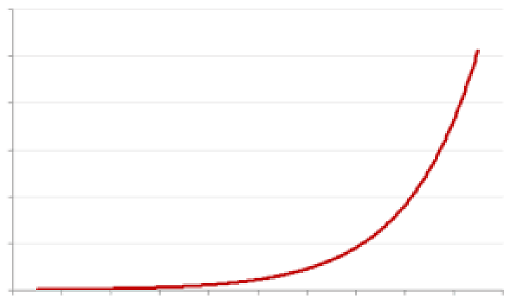

Figure 2. Exponential form of failure rate curve

In this example, the probability of having a minor failure during burn-in test (P1) is equal to 0.99, the probability of facing minor failure during warranty period (P2) is equal to 0.8, and the probability of warranty coverage for a failure during warranty period (P) is considered as 0.9. The cost parameters are chosen to be C2=3, C3=1, C4=6, C5=105, Cm=10 and Cr=120. These values for C1 to C5 are arbitrary. One can choose any other values for these cost parameters. In this case the value for rejuvenation factor (δ) is considered as 0.6 (although any other value between 0 and 1 is acceptable). As the value of δ decreases, the costs of the warranty period increase accordingly. Considering that performing burn-in test is a costly process for the manufacturer, the length of this period should be logically very short. For example, the duration of this period was only 24 hours for old television sets with a lifespan of about 15 years. Therefore, the length of this period is considered to be between 0.001 and 0.01 of the average lifetime of the product. The length of the warranty period is also considered between 0.5 and 1.5 times the average length of the product life time. Since the calculation of the average total cost is complicated, the necessary calculation is done with the help of programming in MATLAB software. Table 1 shows the average total cost for different values of b and W.

Table 1. Average total cost per unit time from manufacturer point of view

Warranty / Burn-in	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
0.001	13.8430	12.0545	10.8396	10.0007	9.5046	9.1475	8.9645	8.9372	9.0589	9.3320	9.7664
0.002	13.8519	12.0362	10.8006	9.9446	9.4298	9.0586	8.8617	8.8205	8.9278	9.1856	9.6037
0.003	13.8704	12.0342	10.7832	9.9146	9.3865	9.0052	8.7986	8.7476	8.8451	9.0927	9.4997
0.004	13.8921	12.0389	10.7751	9.8962	9.3574	8.9678	8.7535	8.6949	8.7847	9.0243	9.4230
0.005	13.9153	12.0471	10.7721	9.8842	9.3363	8.9399	8.7189	8.6540	8.7374	8.9705	9.3624
0.006	13.9390	12.0574	10.7725	9.8765	9.3206	8.9180	8.6914	8.6210	8.6989	8.9265	9.3127
0.007	13.9630	12.0690	10.7750	9.8718	9.3086	8.9006	8.6689	8.5936	8.6667	8.8894	9.2707
0.008	13.9870	12.0816	10.7791	9.8692	9.2995	8.8864	8.6501	8.5704	8.6393	8.8577	9.2346
0.009	14.0109	12.0948	10.7845	9.8683	9.2925	8.8748	8.6343	8.5506	8.6156	8.8302	9.2032
0.010	14.0347	12.1085	10.7907	9.8687	9.2872	8.8653	8.6208	8.5335	8.5949	8.8061	9.1755

Also, the 3D chart of the average total cost according to the duration of the burn-in test and warranty periods can be seen in Figure 3, below:

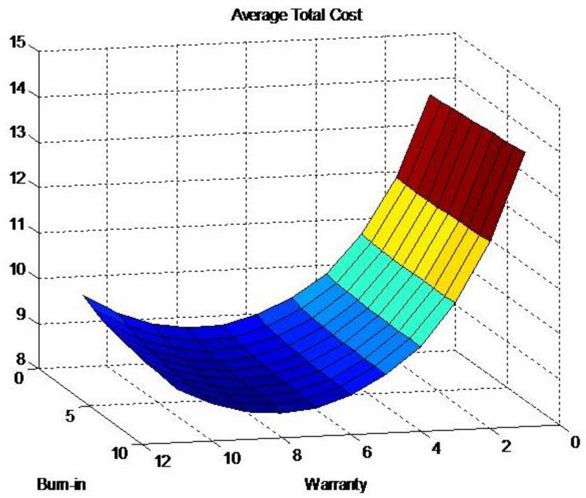

Figure 3. The graph of the average total cost per unit of time according to the length of the burn-in test and warranty periods

As it can be seen from the values in Table 1, when the length of the warranty period is short (for example, half of the average lifetime of the product), the model tends to choose the minimum possible value for the length of the burn-in test period (here, 0.001 of the average lifetime length). As the length of the warranty period increases, the model tends to choose larger values for the length of the burn-in test period. If the length of the burn-in test period, b is considered fixed, as the length of the warranty period increases, the average total cost will decrease up to a certain point and then this cost will start to increase. In this example, according on the ranges defined for b and W values, the optimum value of W is 1.2 and the optimum value of b is 0.01 and the average total cost is 8.5335. Obviously, by changing the values of the cost coefficients and other parameters (for example, the parameters of the shape in the product failure rate function), the answers (the optimal value of the cost and the duration of the burn-in test and warranty periods of the product) will be different. utilizing sensitivity and fuzzy analysis, with a specific focus on road and belt projects. Nunes and Abreu (2020)consider open innovation project risks through the lens of social network analysis. Marle (2020) contributes to project risk management by applying complex systems and agile project management techniques. Nunes, Abreu et al. (2021) develop a model for managing risks in collaborative projects aimed at fostering knowledge creation and promoting sustainable business practices. Antoniou (2021) presents several models dedicated to evaluating delay risks in road projects. Alvand, Mirhosseini et al. (2023) identify and assess risks in construction projects using the SWARA, FMEA, and WASPAS methodologies within a fuzzy context, specifically targeting construction projects in Iran. Bepari, Narkhede et al. (2024) perform a comparative study on project risk management focusing on failure structures. Erlita, Amin et al. (2023) explore multi-step project risk management for construction projects in the food manufacturing industry. Senova, Tobisova et al. (2023) propose innovative approaches for project risk evaluation leveraging Monte Carlo methodologies. Nikolaenko and Sidorov (2023) analyze risks associated with information technology projects. Alawneh, Jannoud et al. (2024) introduce a novel method for evaluating project risk in sustainable construction projects within developing countries. He, Wang et al. (2024) assess advancements and systematic risk analysis concerning investment safety in water transfer projects and operational failures. Nge'tich and Munene (2024) investigate the impacts of project risk management on road projects in Kenya, while Gichohi, Iravo et al. (2024) examine its influence on the performance of road construction projects in the same region. Lastly, Khalilzadeh, Banihashemi et al. (2024) propose a hybrid stepwise approach that integrates multi-criteria methodologies with multi-objective optimization models for effective project risk management. Upon reviewing the literature, we observe that some studies, such as those conducted by Andrić, Wang et al. (2019) and Alvand, Mirhosseini et al. (2023), primarily concentrate on risk identification. In contrast, other research efforts, including those by Dandage, Mantha et al. (2018), Antoniou (2021), and Senova, Tobisova et al. (2023), focus on risk evaluation. A considerable number of studies address risk management, with notable contributions from Nunes, Abreu et al. (2021), and Marle (2020). However, there is a scarcity of research that concurrently examines all three dimensions: identification, evaluation, and management. Moreover, Alvand, Mirhosseini et al. (2023) and Alawneh, Jannoud et al. (2024) integrate uncertainty into their risk evaluation processes. Methodologically, the studies by Erlita, Amin et al. (2023) and Alvand, Mirhosseini et al. (2023) employ multi-criteria decision-making (MCDM) techniques. Heydarpour et al(2023) provide a DEA and AHP hybrid model to evaluate contractors performance. Rasouli et al(2023) evaluate risk factors in solar energy investment. They have a strategic approach for iran market. Ayough et al(2022) provide a new interactive method based on multi criteria preference degree functions for solar power plant site selection. Overall, the literature review reveals a significant gap in addressing the evaluation and management of risks specifically associated with startup projects. In particular, there is a lack of research utilizing hybrid techniques that combine statistical analysis, MCDM, and multi-objective mathematical models for effective project risk management. Given the growing importance of startup projects in the contemporary business landscape, it is essential to focus more intently on the risks they face. To address this identified gap, this research introduces several key innovations:

- Defining a hybrid approach that integrates risk identification, evaluation, and management
- Focusing on the specific risks associated with startup projects
- Considering the uncertainty in the risk assessment process
- Optimizing the risk response strategies
- Developing a model along with proposed solution

5. Concluding remarks

In this research, a repairable product with a bathtub-shaped failure rate function was considered, which is offered to the consumer after conducting burn-in test at the manufacturer's place and with a nonrenewable warranty offer. During the burn-in test period, if a minor failure occurs, the product is minimally repaired and the test continues, and if a major failure occurs, the product is abandoned and a new product is replaced. During the warranty period, only some failures are covered by the warranty service. If the failure is covered by the warranty service, then according to the type of failure, the manufacturer will carry out minor repairs or general repairs. The suggested type of warranty is linear pro-rata type. That is, the longer the warranty period is passed, the share of the producer's costs will be lower and the share of the consumer's costs will be higher. The costs related to this problem were mathematically modeled and with the aim of minimizing the average total costs of the producer's share per unit of time, the optimal length of the burn-in test and warranty periods was calculated. The advantage of this model is its simplicity in use. Using the same logic, one can develop his customized model for a specific system. MATLAB software was used to solve the numerical example. Solving the numerical example showed that with the increase of the warranty period, the model tends to increase the length of the burn-in test period. For future research, other warranty models or other types of cost sharing (such as non-linear cost sharing) can be used. It is also possible to determine the maximum cost for the warranty period in advance and then determine the duration of the periods.

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