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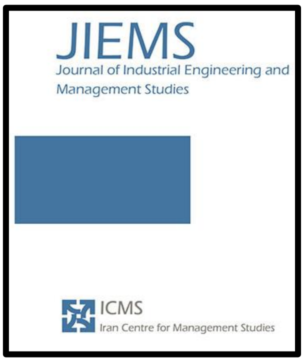

Electromagnetism-like Algorithms for The Fuzzy Fixed Charge Transportation Problem

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Abstract

In this paper, we consider the fuzzy fixed-charge transportation problem (FFCTP). Both of fixed and transportation cost are fuzzy numbers. Contrary to previous works, Electromagnetism-like Algorithms (EM) is firstly proposed in this research area to solve the problem. Three types of EM; original EM, revised EM, and hybrid EM are firstly employed for the given problem. The latter is being firstly developed and proposed in this paper. Another contribution is to present a novel, simple and cost-efficient representation method, named string representation. It is employed for the problem and can be used in any extended transportation problems. It is also adaptable for both discrete and continues combinatorial optimization problems. The employed operators and parameters are calibrated, according to the full factorial and Taguchi experimental design. Besides, different problem sizes are considered at random to study the impacts of the rise in the problem size on the performance of the algorithms.

Keywords: Fuzzy Fixed Charge Transportation Problem, Electromagnetism-like Algorithms, String representation, Fuzzy numbers, Taguchi.

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1. Introduction

In recent decades, there have been many researchers who reported new models or methods to determine the transportation or the logistics activities that can give the least cost (Gen & Cheng, 2000). Thomas and Griffin (1996) provided an extensive review and discussion of the supply chain literature. They pointed out that for many products, logistics expenditures can constitute as much as 30% of the net production cost. There is no doubt that logistics is an important function of business and is evolving into strategic supply chain management (New & Payne, 1995).The transportation problem (TP) is a well-known and basic network problem. It is also a basic model in the logistic networks. In the traditional transportation problem (TP), two kinds of constraints, source constraint and destination constraint, and one kind of cost are considered. There are many efficient algorithms to solve the traditional TP.

The fixed cost linear programming problem was introduced by Hirsch & Dantzig (1968). It has been widely applied in many decision making and optimization problems and utilized both in academia and industry. In a Fixed Charge Transportation Problem (FCTP), a single merchandise

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is shipped from supplier locations to customer locations. The fixed cost occurs for every route that is used for shipping, while the variable cost is proportional to the amount which is shipped. The objective minimizes the summation of both costs in accompany with meeting the supply and demand requirements of each location. Since the existence of fixed costs leads to discontinuities in the objective function, the FCTP is much more difficult to solve in comparison with the TP. It is unsolvable by the direct application of the transportation algorithm (Clover *et al*. 1992).

It has been shown that the FCTP is *NP*-hard problem (Hirsch & Dantzig 1968, Klose, 2008). Adlakha and Kowalski (2003) developed a simple heuristic algorithm for solving small FCTP and stated that the proposed method is more time consuming than the algorithms for solving a regular TP. Several heuristic algorithms were presented for solving the FCTP (Gottlieb & Paulmann, 1998; Sun *et al.* 1998, Gen & Cheng, 2000, Adlakha and Kowalski, 2003, Ida *et al*. 2004, Liu et al. 2008, Yang & Liu, 2007). Also, some exact methods such as mixed integer programming, the branch-and-bound, and the cutting plane method have been reported to solve the problem. But these methods are generally inefficient and computationally expensive (Steinberg, 1970).

Gen et al. (2005) surveyed evolutionary algorithms for solving various network design problems such as FCTP. Jo et. al (2007), utilized the Prüfer number representation and developed a criterion to check the feasibility of the generated chromosomes. They also used a repairing procedure for infeasible chromosomes. Their proposed repairing procedure may take long time to repair and make a feasible Prüfer number.

Hajiaghaei-Keshteli *et al*. (2010) studied the FCTP and proposed a novel method to design a chromosome that does not need a repairing procedure for feasibility, i.e. all the produced chromosomes are feasible. Besides, they corrected the procedure provided by Jo *et al*. (2007), which designs transportation tree with feasible chromosomes. They illustrated that the previous decoding procedure, introduced by Jo *et al*. (2007), does not produce any transportation tree in some situations. In addition, some new crossover and mutation operators are developed by them. Othman et al. (2011) continued their work and present two Genetic Algorithms (GAs) for this problem. Using Prüfer number representation, they developed two fuzzy logic controllers to automatically tune the parameters.

In another related work, Molla-Alizadeh-Zavardehi *et al.* (2011) developed a mathematical model for a capacitated FCTP in a two-stage supply chain network. They proposed and compared an artificial immune algorithm (AIA) and a GA based on the Prüfer number representation. Besides, they introduced a new method to calculate the affinity value by using an adjustment rate in the proposed AIA.

Later, Lotfi and Tavakkoli-Moghaddam (2012) solved the FCTP via GA using priority-based encoding (pb-GA). They used a new approach in encoding scheme. Besides, El-Sherbiny and Alhamali (2012) solved the same problem by a hybrid particle swarm algorithm with artificial immune learning in which a flexible particle is used instead of Prüfer number. Xie and Jia (2012) solved a nonlinear problem via a hybrid minimum cost flow-based HGA, which reduces the time and the memory space of computer to achieve the global optimum. Furthermore, Molla-Alizadeh-Zavardehi *et al.* (2013) presented a fuzzy fixed charge solid transportation problem by Metaheuristics. They solved the problem under a fuzzy environment via VNS and a hybrid algorithm of VNS and SA. And the recent paper, Altassan *et al.* (2014) developed a new encoding scheme instead of Prüfer number and developed an algorithm for decoding the problem. They used Artificial Immune Algorithm to solve the problem.

Generally, it is often difficult to estimate the actual penalties (e.g., transportation cost, quantity of goods delivered, demands, availabilities, the capacities of different modes of transport between origins and destinations). Therefore, the typical models which uses crisp numbers for their parameters, fail in many practical applications. The purpose of introducing fluctuation in transportation problem is to get better customer service. Bit (2005), Bit *et al.* (1993a, 1993b), Li and Lai (2000) presented the fuzzy compromise programming approach to multi-objective transportation problem. Samanta and Roy (2005) proposed an algorithm for solving multi-objective entropy transportation problem under fuzzy environment. Omar and Samir (2003), and Chanas and Kuchta (1996) discussed the solution algorithm for solving the transportation problem in fuzzy environment. The entropy optimization in transportation models as well as other models also is discussed in the book of Kapur and Kesavan (1992). Ojha *et. al.* (2009) discussed a solid transportation problem with entropy in fuzzy environment. Problems involving uncertainty has become the subject of extensive research in the last decade.

For most of the real-world processes, some parameters (e.g. costs) are not precisely known a priori. There are several approaches to modeling the uncertainty in optimization. The natural one is to apply the theory of Fuzzy. Therefore, because of all factors mentioned above, we consider fuzzy cost numbers for the FCTP in this paper. In the fuzzy FCTP (FFCTP) both mentioned costs are supposed to be fuzzy number instead of crisp number.

The literature of incorporating fuzzy on FCTP is rather limited and relatively recent. Liu et al. (2008) modeled fuzzy fixed charge transportation problem as chance-constrained programming by using the possibility measure and credibility measure. They used Prüfer number representation and employed a GA. Also, Yang and Liu (2007) studied the fixed charge solid transportation problem under fuzzy environment. They employed hybrid intelligent algorithm which is based on the fuzzy simulation technique and tabu search algorithm.

Birbil and Fang (2003) first introduced the EM as a new stochastic population-based heuristic optimization tool. Solutions are considered as charged particles in EM. The performance of each solution is measured by its own charge. All particles magnetize each other in which this force leads to a global movement of all particles towards the particles with higher charges or solutions with better objective function value. This system provides an iterative method which simulates particle interactions, and movement in search space under the impact of electromagnetic force.

In the electromagnetic space, every particle attracts or repels every other particles according to its charge. The direction of particles to move in subsequent iterations is determined by the resultant force determined with all the forces exerted by on the particle by other particles. In this system, the candidate solutions with a better objective function values attract others while those with worse values repel. The amount of attraction or repulsion between two particles in the population is directly proportional to the product of their charges and inversely proportional to the distance between them. The philosophy behind the algorithm is that the force leads to a global movement of all particles into the solutions with higher quality.

The EM approach has been recently employed to solve several combinatorial optimization problems such as set covering problem (Naji-Azimi *et al.* 2010), project scheduling (Debels *et al.* 2006), nurse scheduling (Maenhout and Vanhoucke, 2007), inventory control (Tsou and Kao, 2008), etc.

Here, in order to find the optimal solution for the given problem, FFCTP, we attempt to use both efficient algorithms and efficient solution representation way. Some deficiencies of using Prüfer number representation, motivates us to develop a new way of representation. This type of representation is a new, simple and cost-efficient representation method, named string representation. It is being firstly proposed in this paper. The generated string by this type of representation, does not need to check or repairing procedure for feasibility. All produced strings with our proposed method are feasible, and also if an operator in an algorithm, like mutation or crossover operator in GA, operates on a generated string in order to exploit and explore the solution space, the operated string will be certainly feasible. In addition, there is an unique relation between the generated string and its related transportation network. It means that for each string there is only one transportation network and vice versa.

Another contribution of this paper is to utilize three types of EM in order to solve the given problem. The EM is known as an efficient metaheuristic algorithm to tackle *NP*-hard problems.

The motivation behind this algorithm has risen from an attraction-repulsion mechanism to move a population of points gradually toward optimality. We utilize two existing well-known EM proposed by Birbil and Fang (2003) and *Birbil et al.* (2004), and moreover, we attempts to present a novel high-performing hybrid EM algorithm for solving the given problem, which improves the original EM in two aspects. Intensification and diversification are two major issues for designing a global search method. Diversification generally refers to the ability to visit many and different regions of the search space, whereas intensification refers to the ability to obtain high quality solutions within those regions (Lozanoa García-Martínezb (2010)). Besides, the full factorial and Taguchi experimental design are employed as parameter tuning methods to calibrate the used operators and parameters.
Five sections follow this Introduction. The next section briefly introduced some knowledge of fuzzy costs. Section 3 describes the problem's details and elaborates the mathematical formulation of our model. The proposed algorithms are detailed in Section 4. Section 5, describes the Taguchi experimental design and compares the computational results. Finally, in Section 6, conclusions are provided and some areas of further research are then presented.

2. Preliminaries

The theory of the fuzzy set was firstly introduced by Zadeh (1965) with the membership function and then it has been well developed in an extensive range of real problems. In order to measure a fuzzy event, the term fuzzy variable was introduced by Kaufman (1975).
Here, we briefly introduce some primary concepts and results of fuzzy measure theory initiated by Bellman and Zadeh (1970). Here, we give definitions and notations taken from Bezdek (1993).

Definition 2.1 If X is a collection of objects denoted generically by x, then a fuzzy set in X is a set of ordered pairs:
$\tilde{A} = \{x, \tilde{A}(x) | x\epsilon X\}$, where $\tilde{A}(x)$ is called the membership function which associates with each $x \in X$ a number in [0,1] indicating to what degree x is a number.

Definition 2.2 The α-level set of $\tilde{A}$ is the set $\tilde{A}_\alpha = \{x | \tilde{A}(x) \geq \alpha\}$ where $\alpha \in [0, 1]$. The lower and upper bounds of any α -level set $\tilde{A}_\alpha$ are represented by finite number $inf_{x\in\tilde{A}_\alpha}$ and $sup_{x\in\tilde{A}_\alpha}$.

Definition 2.3 A fuzzy set A is convex if

$$\tilde{A}(\lambda x + (1-\lambda)y) \geq \min\{\tilde{A}(x), \tilde{A}(y)\} \qquad \forall x, y \in X, \lambda \in [0,1]$$

Definition 2.4 A convex fuzzy set $\tilde{A}$ on $\mathbb{R}$ is a fuzzy number if the following conditions hold:

(a) Its membership function is piecewise continuous function.
(b) There exist three intervals [a,b], [b,c] and [c,d] such that A is increasing on [a,b], equal to 1 on [b,c], decreasing on [c,d] and equal to 0 elsewhere.

Definition 2.5 The support of a fuzzy set $\tilde{A}$ is a set $\tilde{A}$ is a set of elements in X for which $\tilde{A}(x)$ is positive, that is,

$$supp\ \tilde{A} = \{x \in X | \tilde{A}(x) > 0\}$$

Definition 2.6 Let $A = (a^l, a^u, \alpha, \beta)$ denote the trapezoidal fuzzy number, where $[a^l - \alpha, a^u + \beta]$ is the support of $\tilde{A}$ and $[a^l, a^u]$ its core.

Remark 2.1 In this paper we denote the set of all fuzzy numbers by F ($\mathbb{R}$).

We next define arithmetic on trapezoidal fuzzy numbers.
Let $\tilde{a} = (a^l, a^u, \alpha, \beta)$ and $\tilde{b} = (b^l, b^u, \gamma, \theta)$ be two trapezoidal fuzzy numbers. Define,

$$x > 0 \quad , \quad x \in \mathbb{R} : x\tilde{a} = (xa^l, xa^u, x\alpha, x\beta),$$
$$x < 0 \quad , \quad x \in \mathbb{R} : x\tilde{a} = (xa^u, xa^l, -x\beta, -x\alpha),$$
$$\tilde{a} + \tilde{b} = (a^l + b^l, a^u + b^u, \alpha + \gamma, \beta + \theta),$$
$$\tilde{a} - \tilde{b} = (a^l - b^u, a^u - b^l, \alpha + \theta, \beta + \gamma).$$

2.1. Ranking function

A convenient method for comparing of the fuzzy numbers is by use of ranking function (Maleki, 2002). We define a ranking function$\Re : F(\mathbb{R}) \rightarrow \mathbb{R}$, which maps every fuzzy number into the real line. Here, suppose that $\tilde{a}$ and $\tilde{b}$ be two trapezoidal fuzzy numbers. So, we define,

$$\tilde{a} \underset{\Re}{\geq} \tilde{b} \quad \text{If and only if} \quad \Re(\tilde{a}) \geq \Re(\tilde{b}) \tag{1}$$
$$\tilde{a} \underset{\Re}{>} \tilde{b} \quad \text{If and only if} \quad \Re(\tilde{a}) > R(\tilde{b}) \tag{2}$$
$$\tilde{a} \underset{\Re}{=} \tilde{b} \quad \text{If and only if} \quad \Re(\tilde{a}) = \Re(\tilde{b}) \tag{3}$$

Besides we write

$$\tilde{a} \underset{\Re}{\leq} \tilde{b} \quad \text{If and only if} \quad \tilde{b} \underset{\Re}{\geq} \tilde{a} \ .$$

Because there are many ranking function for comparing fuzzy numbers, we only utilize linear

$$\Re(k\tilde{a} + \tilde{b}) = k\Re(\tilde{a}) + \Re(\tilde{b}),$$

for any $\tilde{a}$ and $\tilde{b}$ belonging to $F(\mathbb{R})$ and any $k \in F(\mathbb{R})$.
Here, we introduce a linear ranking function developed by Maleki (2002). For a trapezoidal fuzzy number$\tilde{a} = (a^l, a^u, \alpha, \beta)$, we employ ranking function as follows:

$$\Re(\tilde{a}) = \int_0^1 (\inf \tilde{a}_\alpha + \ sup\, \tilde{a}_\alpha) d\alpha,$$

This reduces to

$$\Re(\tilde{a}) = a^l + a^u \ + \frac{1}{2}\ (\beta - \alpha).$$

Hence, for trapezoidal fuzzy numbers $\tilde{a} = (a^l, a^u, \alpha, \beta)$ and $\tilde{b} = (b^l, b^u, \gamma, \theta)$ we have

$$\tilde{a} \ \geq \ \tilde{b} \ \text{ if and only if } a^l + \ a^u + \frac{1}{2}\ (\beta - \alpha) \geq \ b^l + b^u + \frac{1}{2}(\theta - \gamma).$$

3. Mathematical model and descriptions

The FFCTP can be stated as a transportation problem in which there are *m* potential distribution centers (DCs) and *n* customers (destinations or demand points). Each of the *m* potential DCs can ship to any of the *n* customers at a fuzzy shipping cost per unit $\tilde{c}_{ij}$ (unit cost for shipping from potential DC *i* to customer *j*) plus a fuzzy fixed cost $\tilde{f}_{ij}$, assumed for opening this route and an opening cost $\tilde{f}_i$, assumed for opening potential DC *i*. Each potential DC $i=1, 2,\ldots, m$ has a_i units of supply, and each customer $j=1, 2,\ldots, n$ has a demand of b_j units. The objective is to find that (1) which candidate places are to be opened as distribution centers, (2) which routes are to be opened and (3) the size of the shipment on those routes, so that the total cost of meeting demand, given the supply constraints, is minimized. The FFCTP is formulated as follows:

Min $\tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \times x_{ij} + \tilde{f}_{ij} \times y_{ij}) + \sum_{i=1}^{m} \tilde{f}_i \times y_i$

S.t.

$\sum_{j=1}^{n} x_{ij} \le a_i$ *i=1,2,…,m,*

$\sum_{i=1}^{m} x_{ij} \ge b_j$ *j=1,2,…,n,*

$\forall\, i,j \;\; x_{ij} \ge 0$

$y_{ij} = 0$ if $x_{ij} = 0,$

$y_{ij} = 1$ if $x_{ij} > 0,$

$y_i = 0$ if $\sum_{j=1}^{n} x_{ij} = 0$ *i=1,2,…,m,*

$y_i = 1$ if $\sum_{j=1}^{n} x_{ij} > 0$ *i=1,2,…,m.*

The crisp equivalent of objective function can be written as follows:

$$\text{Min} \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}^{l} \times x_{ij} + f_{ij}^{l} \times y_{ij}) + \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}^{u} \times x_{ij} + f_{ij}^{u} \times y_{ij}) + \frac{1}{2}\Big(\sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}^{\beta} \times x_{ij} + f_{ij}^{\beta} \times y_{ij}) - (\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{\alpha} \times x_{ij} + f_{ij}^{\alpha} \times y_{ij})\Big) + \sum_{i=1}^{m} f_i^{l} \times y_i + \sum_{i=1}^{m} f_i^{u} \times y_i + \frac{1}{2}\Big(\sum_{i=1}^{m} f_i^{\beta} \times y_i - \sum_{i=1}^{m} f_i^{\alpha} \times y_i\Big)$$

Where x_{ij} is the unknown quantity to be transported on the route *(i,j)* that from plant *i* to consumer *j*, $\tilde{c}_{ij}$is the shipping fuzzy cost per unit from plant *i* to consumer *j*. a_i is the number of units available at plant *i*, and b_j is the number of units demanded at costumer *j*. The transportation cost for shipping per unit from plant *i* to consumer *j* is $\tilde{c}_{ij} \times x_{ij}$. Also $\tilde{f}_{ij}$ is the fuzzy fixed cost associated with route *(i,j)* while $\tilde{f}_i$ is the fuzzy opening cost assumed for opening potential DC *i*. For an illustration more about of the above model, we solve an example here. Suppose there are four potential DCs and six costumers. The transportation costs, fixed cost and opening cost are given in Table 1.

Table 1: The example parameters and costs

Customers		1	2	3	4	5	6
b_j		40	20	10	30	50	20
Suppliers	a_i			$\tilde{c}_{ij}$			
1	50	(2,3,1,5)	(2,7,1,4)	(2,5,1,4)	(3,8,1,5)	(5,6,2,4)	(3,18,1,5)
2	100	(4,6,2,3)	(6,8,5,1)	(4,10,1,5)	(8,9,5,1)	(3,7,1,5)	(6,10,1,3)
3	150	(3,7,2,2)	(3,9,1,1)	(6,11,1,3)	(5,7,3,2)	(3,7,2,5)	(3,15,2,2)
4	120	(3,5,1,1)	(5,7,3,6)	(8,13,2,4)	(4,5,1,1)	(5,9,3,1)	(4,5,3,4)
	$\tilde{f}_i$			$\tilde{f}_{ij}$			
1	(200,400,50,100)	(50,60,10,10)	(20,30,10,20)	(30,50,20,10)	(30,50,10,10)	(40,60,10,30)	(30,50,10,20)
2	(700,800,100,100)	(50,60,10,30)	(60,90,20,10)	(40,100,10,50)	(80,90,20,40)	(20,50,20,30)	(60,80,10,20)
3	(400,700,50,100)	(40,80,20,10)	(20,70,10,20)	(50,90,10,20)	(30,60,30,20)	(30,40,20,10)	(30,60,40,20)
4	(100,200,50,50)	(40,50,10,10)	(30,50,20,10)	(50,70,20,30)	(50,70,30,60)	(20,30,10,20)	(30,60,20,10)

As shown in Fig. 1, we suppose that customers 1, 3, 5, and 6 are served their demands from DC 4, and customer 4 is served from DC 1 and also customer 2 receives its demands from both DCs 1 and 4. In this occasion, we select potential DCs 1 and 4 to be opened and serve the customers' demands. The total cost occurred in this allocation is calculated as follows:

- The transportation cost from DCs to customers is equal to:
  $\tilde{Z}_1 = \tilde{c}_{12} \times x_{12} + \tilde{c}_{14} \times x_{14} + \tilde{c}_{41} \times x_{41} + \tilde{c}_{42} \times x_{42} + \tilde{c}_{43} \times x_{43} + \tilde{c}_{45} \times x_{45} + \tilde{c}_{46} \times x_{46}$
  $= (2,7,1,4) \times 20 + (3,8,1,5) \times 30 + (3,5,1,1) \times 40 + (5,7,3,6) \times 10 + (8,13,2,4) \times 10 + (5,9,3,1) \times 40 + (4,5,3,4) \times 20$
  = (40,140,20,80) + (90,240,30,150) + (120,200,40,40) + (50,70,30,60) + (80,130,20,40) + (200, 360, 120, 40) + (80, 100, 60, 80) = (660,1240,320,490),
  $\Re(\tilde{Z}_1) = \Re(660,1240,320,490) = 1985.$

- The fixed cost of routs $\tilde{Z}_2 = \tilde{f}_{12} + \tilde{f}_{14} + \tilde{f}_{41} + \tilde{f}_{42} + \tilde{f}_{43} + \tilde{f}_{45} + \tilde{f}_{46}$ = (20,30,10,20) + (30,50,10,10) + (40,50,10,10) + (30,50,20,10) + (50,70,20,30) + (20,30,10,20) + (30,60,20,10) = (220,340,100,110),
  $\Re(\tilde{Z}_2) = \Re(220,340,100,110) = 565.$

- The opening cost of DCs $\tilde{Z}_3 = \tilde{f}_1 + \tilde{f}_4$ = (200,400,50,100) + (100,200,50,50) = (300,600,100,150),
  $\Re(\tilde{Z}_3) = \Re(300,600,100,150) = 925.$

- The objective function value is:
  $\tilde{Z} = (660,1240,320,490) + (220,340,100,110) + (300,600,100,150) = (1180,2180,520,750),$
  $\Re(\tilde{Z}) = \Re(1180,2180,520,750) = 3475.$

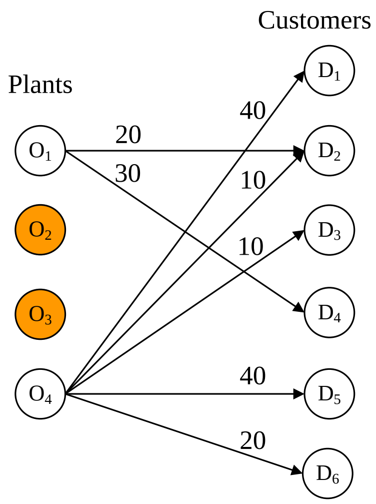


Fig. 1: Illustration of designing a transportation network

4. The electromagnetism-like algorithm

4.1. The original EM

The EM motivated by the real electromagnetism theory, to solve the problems with bounded variables in the form of:

$$\text{Min } f(x) \quad (4)$$
$$\text{s:t: } x \in [L,U] \quad (5)$$

where [L,U] = $\{x \in R^n \mid L_k \leq x_k \leq U_k, k = 1,...,n\}$ and $x_1, \ldots, x_n$ stand for the decision variables. U_k, L_k and $f(x)$ represent, upper and lower bounds on the k^{th} variable ($k = 1, \ldots, n$) and the objective function value, respectively.

The general structure of the EM algorithm is illustrated in Algorithm 1. The process begins with an initial set of solutions to sample population size (*popsize*) of solutions from the feasible region. This step can be performed by a random procedure which generates a pool of *popsize* uniformly. The next steps are local search (*LSITER*), computation of total force exerted on each particle, and movement along the direction of the force.

The local search procedure is utilized for each solution and tries to improve the quality of the current population solutions. Then, according to their charges, all the particles or solutions forces each other and the total force which exerted on each of them is calculated in the force computation procedure. The total force is computed by using the charge of each solution X^i or objective function value $f(X^i)$, and the best objective value $f(X^{best})$ in the current population (Birbil and Fang, 2003).

ALGORITHM 1.EM (*popsize, Maximum Iteration (MAXITER), Local Search Iteration (LSITER)*)

1. Initialize (*popsize*)
2. iteration ← 1
3. **while** termination criterion is not satisfied **do**
4. Local search
5. Calculate Forces
6. Move
7. iteration ← iteration + 1
8. **end while**

At last, in the movement procedure, by considering a random step length λ which is uniformly distributed between 0 and 1, each solution is moved along the direction of the total force exerted on it to its new place in the feasible space. In this work, the specific formulas to compute the charges, forces and the movement action of each solution will be explained in Sections 4.1.3 and 4.1.4.

4.1.1. Encoding scheme and initialization

Despite the EM approach has been designed for continuous optimization problems, here we adapt it to utilize in the discrete space. As mentioned in the most works which used metaheuristic algorithms to solve optimization problem, the impact of the types of solution representation on obtaining optimal solution is inevitable. This is one of the most important decisions in designing a metaheuristic. It deals with deciding how to represent solutions and relate them in an efficient way in the searching space. It also affects on the time to reach the optimal solution. Besides, because of this, some works in different research areas studied and evaluated different types of representation. Representation should be easy to decode in order to reduce the cost of the algorithm. Several principles have been proposed to evaluate an encoding by (Schwefel, 1995, Gen & Cheng, 1997). Required amounts of memory for representation, time complexity for executing evaluation and operation on representation, feasibility, legality, completeness, uniqueness, heritability, and locality are the main principles that would be considered when one wants to develop a representation or evaluate the efficiency of a representation method.

In this paper, we introduce a new type of representation, named string representation for the problem. The string representation is an encoding way which has simple procedure and easy to

code. Hence, it does not take an excessive amount of memory. In comparison with the similar encoding methods, types of operation, like recombination or fitness evaluation take shorter time in this type of representation. Because it uses just random numbers and sort function in a vector, it may take a short time to operate on solution or evaluate the fitness value, in comparison to previous ways with more probability, especially in the matrix representation. Because in matrix representation n^2 digits are required to represent a solution while n digits are needed in string representation. By increasing the solution dimension (n), the difference between these two types of representation will be highly increased. In addition, every generated solution by string representation is certainly feasible. It does not need any repairing procedure, rejecting procedure or penalty strategy. This property is very important for a representation.
The mapping from solution representation to solutions (decoding) may belong to one of the following three cases: 1-to-1 mapping, n-to-1 mapping and 1-to-n mapping (Gen & Cheng, 1997). For more detail we suggest the reader to see the mentioned reference (Gen & Cheng, 1997) but in a nutshell we explain 1-to-n mapping. The 1-to-n mapping means that with one string as a solution representation we may reach to several and different solutions in a real space by decoding. The 1-to-1 mapping is the best among three cases and 1-to-n mapping is the most undesired one. The string representation method employed 1-to-1 mapping. Also, by using this method, a small change in represented solution imply a small change in its corresponding solution, which is a good property for a representation method.

As mentioned earlier in declaring our problem, the network has *m* potential DCs (warehouses or factories) and *n* customers (destinations or demand points). Each of the *m* potential DCs can ship to any of the *n* customers. Most of the evolutionary algorithms use a random procedure to generate an initial set of solutions. Here, we number potential DCs from 1 to *m* and put them to set *S* and number the customers from 1 to *n* and put them to set *C*.
The proposed string representation is a two-part vector in which the first part (in the right side) is related to the set *S* and the second corresponds to the set *C*. The initialization of a string is performed from randomly generated *m*+*n* digits in range [0,1] for each supplier and customer. By ascending sorting of the value corresponding to each supplier and customer, separately in each part, the sequence of potential DCs and customers in each part is obtained. This simple operation is illustrated in Fig. 2 for a network with four potential DCs and six customers.

	Potential DCs				Customers					
A string for 4 potential DCs and 6 customers	1	2	3	4	1	2	3	4	5	6
Initialization (randomly generated)	0.23	0.83	0.68	0.07	0.23	0.68	0.05	0.91	0.42	0.19
Sequence order for each part	4	1	3	2	3	6	1	5	2	4
Sorted digits in each part, separately	0.07	0.23	0.68	0.83	0.05	0.19	0.23	0.42	0.68	0.91

Figure 2: Illustration of designing a sequence by a string

After having a sequence of potential DCs and customers in two parts, we can use it to compute the objective function value of this solution and design its corresponding transportation network by the following procedure:

Procedure: Convert string representation to its corresponding transportation network
Input: A two-part vector
Output: A transportation network

Repeat the following process (1 to 7) until all customers' demands be satisfied:

1. Let *i* be the leftmost digit in supplier part, and *j* be the leftmost digit in customer part, add the edge (*i*, *j*) in transportation network.
4. Assign the available amount of units $x_{ij} = \min\{a_i, b_j\}$ to the edge (*i*, *j*) where $i \in S$ and $j, k \in C$.
5. Update availability $a_i = a_i - x_{ij}$ and $b_j = b_j - x_{ij}$.
6. Remove *i* from vector if the capacity of i^{th} suppler reaches to zero, i.e. all capacity of i^{th} supplier is consumed.
7. Remove *j* from vector if all demand of j^{th} customer reaches to zero, i.e. all demand of i^{th} customer is satisfied.

This procedure is illustrated by an example in Fig. 3.

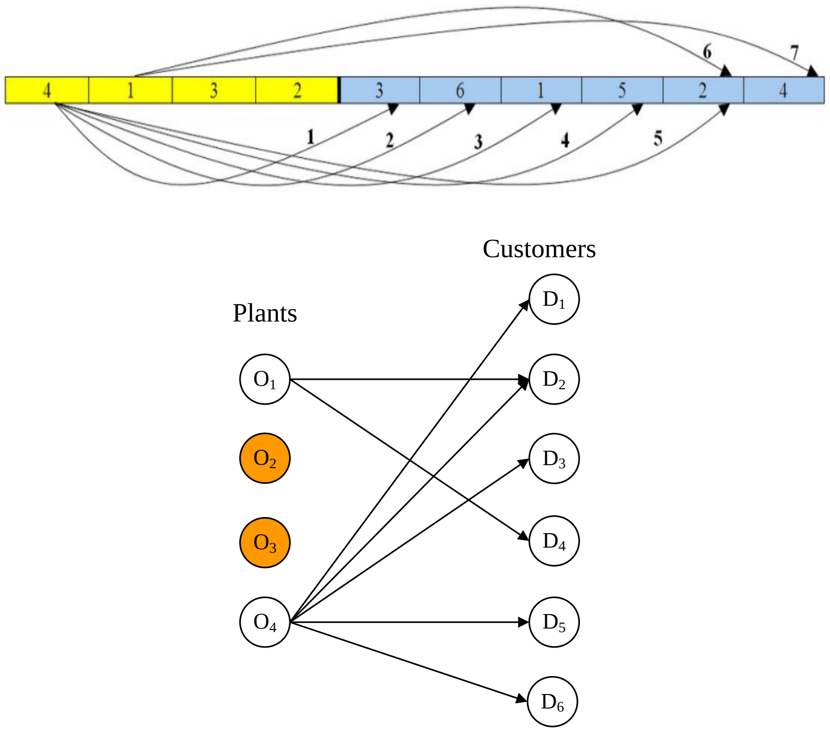


Figure 3: The allocation of transportation network according to the procedure and string representation

4.1.2. Local search

The local search procedure agitates each coordinate of a solution and then finds its related sequence and evaluates its objective value. The new designed temporary solution will be replaced instead of the current solution when its objective value is better than the current solution. This procedure is shown in Algorithm 2.

ALGORITHM 2. Local (*LSITER*)

counter $\leftarrow 1$
for $i = 1$ to *popsize* **do**
    for $k = 1$ to $m+n$ **do**
        $\lambda_1 \leftarrow U(0, 1)$
        while counter $< LSITER$ **do**
            $Y \leftarrow X^i$
            $\lambda_2 \leftarrow U(0, 1)$
            if $\lambda_1 > 0.5$ **then**
              $Y_k \leftarrow Y_k + \lambda_2$

```
            else
                Yk ← Yk − λ2
            end if
            if f (Y) < f (X^i) then
               X^i ← Y
               counter ← LSITER − 1
            end if
            counter ← counter + 1
        end while
    end for
end for
X^best ← argmin{f (X^i), i}
```

4.1.3. Total forces Computation

As mentioned before, by using the main structure of EM, the best solutions encourage other ones to converge to attractive valleys while the inferior solutions discourage the others to move toward this region. We define the charge q^i, and the components $F_j^i \, (j \in J)$ of the total force exerted on each solution X^i and the direction of movement are obtained by adapting the equations.

$$F_j^i = \sum_{\substack{k=1 \\ k \neq i}}^{popsize} \left\{ \begin{array}{ll} \left(x_j^k - x_j^i\right) \dfrac{q^i q^k}{\left\|x^k - x^i\right\|^2} & if \quad f\left(X^k\right) < f\left(X^i\right) \\ \left(x_j^i - x_j^k\right) \dfrac{q^i q^k}{\left\|x^k - x^i\right\|^2} & if \quad f\left(X^k\right) \geq f\left(X^i\right) \end{array} \right\}, \quad i = 1, \ldots, popsize, \; j \in J \tag{6}$$

where

$$q^i = \exp\left(-(m+n) \frac{f\left(X^i\right) - f\left(X^{best}\right)}{\sum_{k=1}^{popsize} \left(f\left(X^i\right) - f\left(X^{best}\right) \right)} \right), \quad i = 1, \ldots, popsize, \; j \in J \tag{7}$$

$$\left\|x^k - x^i\right\| = \left(\sum_{j \in J} \left(x_j^k - x_j^i\right)^2 \right)^{1/2} \tag{8}$$

and X^{best} is the current best solution in the population.

4.1.4. Movement procedure

After evaluating the effects of all the other solutions, each solution is moved in the direction of the force by a random step length λ, uniformly distributed between [0,1]. The formulation proposed to calculate the new position of X^i is as follows:

$$x_j^i = x_j^i + \lambda \frac{F_j^i}{\left\|F^i\right\|} (RNG_j) \quad i = 1, \ldots, popsize, \; j \in J \tag{9}$$

where RNG_j (Range) denotes the amount of feasible movement toward the zero or one. Since RKs (Random Keys) are real numbers between zero and one, the adaptation of Eq. (9) for the *RK*s gives the following formula:

$$x_j^i = \begin{cases} x_j^i + \lambda \dfrac{F_j^i}{\left\|F^i\right\|}\left(1 - x_j^i\right) & if \quad F_j^i > 0 \\ x_j^i + \lambda \dfrac{F_j^i}{\left\|F^i\right\|}\left(x_j^i\right) & if \quad F_j^i \le 0 \end{cases} \quad i = 1, \ldots, popsize, \; j \in J \tag{10}$$

where

$$\left\|F^i\right\| = \left(\sum\nolimits_{j \in J} {F_j^i}^2\right)^{1/2}.$$
(11)

It is important to notice that we do not move the best solution X^{best} in the current population and apply this procedure only to the others.

4.2. The revised EM

The original EM may converge prematurely when the total force exerted on the particles neglect some parts of the solution space, so the original EM an attraction–repulsion mechanism was modified to be more convergent (Birbil et al. 2004). In the revised EM, the current population perturbed so that a perturbed point denoted by X^P, is considered as the farthest point from the current best point, X^{best}. The calculation of the total force vector remains the same for all points except X^P. The components of force exerted to the farthest point are calculated in which they are perturbed by a random number λ which is uniformly distributed between 0 and 1.

$$F_j^p = \sum_{\substack{k=1 \\ k \ne p}}^{popsize} \begin{cases} \left(x_j^k - x_j^p\right) \dfrac{\lambda q^p q^k}{\left\|x^k - x^p\right\|^2} & if \quad f\left(X^k\right) < f\left(X^p\right) \\ \left(x_j^p - x_j^k\right) \dfrac{\lambda q^p q^k}{\left\|x^k - x^p\right\|^2} & if \quad f\left(X^k\right) \ge f\left(X^p\right) \end{cases}, \qquad j \in J \tag{12}$$

Also in the revised EM, the direction of the total forces exerted to X^P is perturbed, i. e., if parameter λ is less than parameter $v \in (0,1)$, then the direction of the component force is reversed. After these modifications, Birbil et al. (2004) mentioned that their revised EM is so convergent.

4.3. The Proposed Hybrid EM

One of the important issues in designing the hybrid meta-heuristic algorithm is to keep the diversity to explore new unvisited regions of the solution space. The right balance between intensification and diversification makes metaheuristic algorithms naturally effective to solve the complex problems.

The original EM, explained in 4.1, often suffers from loss of diversity through premature convergence of the population, which causes the search to be trapped in a local optimum. Diversification refers to the process of replacing inferior solutions of the current population by replacing new randomly generated solutions.

Here, we propose a new diversification procedure that is utilized when the similarity rate of a generation is more than the pre-specified threshold, θ. For each iteration, we calculate the similarity rate of the iteration, $\bar{s}$. This rate is calculated by the following formula:

$$\bar{s} = \frac{\sum_{i=1, i\neq best}^{popsize}\left(\frac{\sum_{j=1}^{m+n}\left(1-\left|x_j^i - x_j^{best}\right|\right)}{m+n}\right)}{popsize-1} = \frac{\sum_{i=1, i\neq best}^{popsize}\left(\frac{(m+n)-\sum_{j=1}^{m+n}\left(\left|x_j^i - x_j^{best}\right|\right)}{m+n}\right)}{popsize-1}$$

$$= \frac{\sum_{i=1, i\neq best}^{popsize}\left(1-\frac{\sum_{j=1}^{m+n}\left(\left|x_j^i - x_j^{best}\right|\right)}{m+n}\right)}{popsize-1} = 1 - \frac{\sum_{i=1, i\neq best}^{popsize}\sum_{j=1}^{m+n}\left(\left|x_j^i - x_j^{best}\right|\right)}{(m+n)(popsize-1)} \qquad (13)$$

If $\bar{s} \geq \theta$, then ω percent of individuals in that iteration should be regenerated randomly from the solution space to join the population such that the population size remains *popsize*.

We propose a mechanism in order to select ω percent of individuals in each iteration. The fact behind this mechanism is to regenerate the solutions that are similar (near) to the best solution or have greater inferior rate (greater objective functions in minimization problem). If two solutions have the same similarity, i.e., the same distance to the best solution, we assign more probability to select the solution with greater inferior rate. We denote the similarity of solution i by s_i, and the inferior rate of each solution by I_i, and calculate them by the following formulas:

$$s_i = \frac{\sum_{j=1}^{m+n}\left(1-\left|x_j^i - x_j^{best}\right|\right)}{\sum_{i=1, i\neq best}^{popsize}\sum_{j=1}^{m+n}\left(1-\left|x_j^i - x_j^{best}\right|\right)} = \frac{m+n-\sum_{j=1}^{m+n}\left(\left|x_j^i - x_j^{best}\right|\right)}{\left(m+n(popsize-1)\right)-\sum_{i=1, i\neq best}^{popsize}\sum_{j=1}^{m+n}\left(\left|x_j^i - x_j^{best}\right|\right)} \qquad (14)$$

$$I_i = \frac{f(x^i)}{\sum_{i=1, i\neq best}^{popsize} f(x^i)} \qquad (15)$$

As mentioned earlier, in order to use both properties of s_i and I_i, we use P_i, the selection probability, to give a selection chance for each solution. This probability is a linear combination of s_i and I_i, and is evaluated by the following equation:

$$P_i = \alpha(s_i) + (1-\alpha)I_i \qquad (16)$$

The P_i, is the probability of selection and the selection mechanism acts like roulette wheel. The greater P_i a solution has, the more chance it has to be selected.

In order to explain the s_i, an example is explained here. Suppose that we have two solutions and the length of digits in each solution, $m+n$, is equal to 2+3=5. Here, we show how the s_i for each solution is calculated and depict the schematic presentation in Fig. 4.

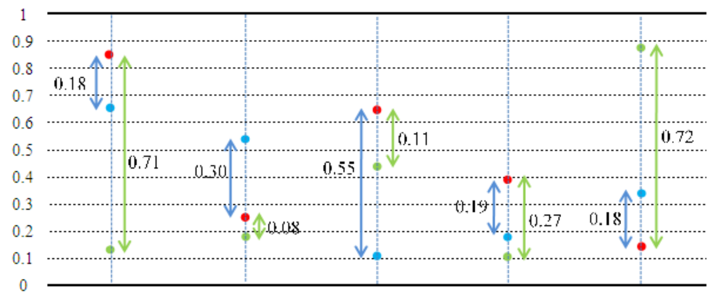
$popsize = 3$

$$s_1 = \frac{5-(|0.67-0.58|+|0.55-0.25|+|0.11-0.66|+|0.19-0.38|+|0.34-0.16|)}{\left(5\times(3-1)\right)-\left((|0.67-0.58|+|0.55-0.25|+|0.11-0.66|+|0.19-0.38|+|0.34-0.16|)+(|0.14-0.58|+|0.17-0.25|+|0.45-0.66|+|0.11-0.38|+|0.88-0.16|)\right)}$$

$$s_1 = \frac{5-1.4}{10-(1.4+1.89)} = 0.53$$

$$s_2 = \frac{5-(|0.14-0.58|+|0.17-0.25|+|0.45-0.66|+|0.11-0.38|+|0.88-0.16|)}{\left(5\times(3-1)\right)-\left((|0.67-0.58|+|0.55-0.25|+|0.11-0.66|+|0.19-0.38|+|0.34-0.16|)+(|0.14-0.58|+|0.17-0.25|+|0.45-0.66|+|0.11-0.38|+|0.88-0.16|)\right)}$$

$$s_2 = \frac{5-1.89}{10-(1.4+1.89)} = 0.46$$



Best	0.85	0.25	0.66	0.38	0.16
x_1	0.67	0.55	0.11	0.19	0.34
x_2	0.14	0.17	0.45	0.11	0.88

Figure 4: The graphic scheme of similarity between two solutions and the best one

5. Experimental design

5.1. Taguchi Parameter Design

The robustness of a product or a process greatly depends on the correct choice of the related parameters and operators. Hence, the different parameters and operators of the proposed algorithm should be studied here. There are some methods in design of experiments. The full factorial design, which tests all possible combinations of factors, is a widely used method in the most researches but when the number of factors significantly increases, it does not seem to be an effective way.

As it would be explained clearly later, for the EM there are 28 test problems, two 3 level factors in our case that each of which should be run three times. Therefore, the total number of running the problem in GA is $28\times3^2\times3$, which is equal to 756. In the revised EM, there are 28 test problems, three 3 level factors in our case that each of which should be run three times. Hence, the total number of running the problem in revised EM is $28\times3^3\times3$, which is equal to 2268. In the hybrid EM, there are 28 test problems, five 3 level factors in our case that each of which should be run three times. Hence, the total number of running the problem in hybrid EM is $28\times3^5\times3$, which is equal to 20’412.

So, to be economic, we can use one of the several experimental designs which have been suggested to reduce the number of experiments. Among the proposed experimental design techniques, Taguchi method is a systematic and efficient approach which uses many ideas for experimental design in order to evaluate and implement improvements in operating conditions in complex systems and it has been applied successfully in various engineering problems. This method uses an orthogonal array to organize the experimental results.

Dr.Taguchi postulated that there are two types of factors which operate on a process: control factors and noise factors. He tends to both minimize the impact of noise and also find the best level of the influential controllable factors on the basis of robustness. In addition, Taguchi determines the relative importance of each factor with respect to its main impacts on the performance of the algorithm. A transformation of the repetition data to another value which is the measure of variation is developed by Taguchi. It is the signal-to-noise (S/N) ratio, which explains why this type of parameter design is called a robust design (Phadke 1989). Here, the term ‘signal’ denotes the desirable value (response variable) and ‘noise’ denotes the undesirable value (standard deviation). So the S/N ratio indicates the amount of variation present in the response variable. The objective is to maximize the signal-to-noise ratio. The S/N ratio of the minimization objectives is:

$$\text{S/N ratio} = -10\log_{10}(objective\ function)^2$$

The control factors of EM are as follows: Population size, $LSITER$, for the revised EM, the factors are: Population size, $LSITER$, ν, and for the hybrid EM, the factors are: Population size, $LSITER$, θ, α and ω. Levels of these factors are illustrated in Table 2.

Table 2: Factors and their levels

Factors	**original EM symbols**	**revised EM symbols**	**hybrid EM symbols**	**original EM Levels**	**revised EM Levels**	**hybrid EM Levels**
Population size	A	A	A	A(1)- 80	A(1)- 70	A(1)- 60
				A(2)-85	A(2)-75	A(2)-65
				A(3)- 90	A(3)- 80	A(3)- 70
$LSITER$	B	B	B	B (1)- 30	B (1)- 40	B (1)- 50
				B (2)- 35	B (2)- 45	B (2)- 55
				B (3)- 40	B (3)- 50	B (3)- 60
ν	-	C	-	-	C (1)-0.4	-
				-	C (2)-0.5	-
				-	C (3)-0.6	-
θ	-	-	C	-	-	C(1)- 0.8
				-	-	C(2)- 0.85
				-	-	C(3)- 0.9
α	-	-	D	-	-	D (1)- 0.5
				-	-	D (2)- 0.6
				-	-	D (3)- 0.7
ω	-	-	E	-	-	E (1)- 60%
				-	-	E (2)- 70%
				-	-	E (3)- 80%

In EM and revised EM, the number of factors are just two and three, respectively. Hence, we employ the full factorial design in these algorithms. But, As we can see, we have five factors in the hybrid EM. So, in order to reduce the number of experiments, we use the Taguchi method for the algorithm.

To select the appropriate orthogonal array for the hybrid EM, it is necessary to calculate the total degree of freedom. The proper array should contain a degree of freedom for the total mean, two degrees of freedom for each factor with three levels (2×5=10). Thus, the sum of the required degrees of freedom is 1+2×5=11. Therefore, the appropriate array must have at least 11 rows.
The selected orthogonal array should be able to accommodate the factor level combinations in the experiment. Considering this, L16 (4^5) is an appropriate array that satisfies these conditions. Since there are five factors with three levels and this scheme offers the factor with four levels, we should adjust this array to the problem by means of adjustment techniques (Park 1995).

Using the dummy level technique we convert the four-level columns into the three-level columns. To assign the three-level factor to the four-level column from the orthogonal array L16 (4^5), one of these levels is required to be replicated twice. In this research first level is chosen to be replicated twice. It is essential to notice that, after applying these techniques, the obtained array remains orthogonal. Furthermore, the accuracy of this level that is replicated twice is twice the accuracy of the other levels. Table 3 shows the orthogonal array L16, where control factors are assigned to the columns of the orthogonal array and the corresponding integers in these columns indicate the actual levels of these factors.

Table 3: The modified orthogonal array L16 for hybrid EM

Trial	A	B	C	D	E
1	1	1	1	1	1
2	1	2	2	2	2
3	1	3	3	3	3
4	1	3	3	3	3
5	2	1	2	3	3
6	2	2	1	3	3
7	2	3	3	1	2
8	2	3	3	2	1
9	3	1	3	3	2
10	3	2	3	3	1
11	3	3	1	2	3
12	3	3	2	1	3
13	3	1	3	2	3
14	3	2	3	1	3
15	3	3	2	3	1
16	3	3	1	3	2

5.2. Data generation

In order to present the efficiency of the proposed algorithms for solving the problem, a plan is utilized to generate test data. Following Hajiaghaei-Keshteli (2011), the data required for a problem consists of the number of DCs and customers, total demand, and range of variable costs, route fixed costs and opening fixed costs. For running the algorithms, 28 problem sets were generated at random in which seven size of problem are implemented for experimental study. The problem size is determined by the number of DCs and customers. The lower and upper bounds of variable costs are 2 and 9, such that a^l, $a^u - a^l$, α and β are made from a uniform distribution of U(3, 7), U(0, 1), U(0.25, 1) and U(0.25, 1). Within each problem size, four problem types A, B, C, and D are considered. For each problem size, problem types are different in range of fuzzy route fixed cost and fuzzy opening fixed cost numbers, which increases according to the alphabetic order of the problem types. Variable costs, are uniformly generated in the small interval, while the lower and upper bounds of fixed costs are generated in larger interval. The problem sizes, types, DCs/customers, and fixed costs ranges are shown in Table 4.

Table 4: Test problems characteristics

			Range of variable costs			Range of route fixed costs			Range of opening fixed costs		
Problem size	Total Demand	Problem type	a^l	a^l-b^l	α and β	a^l	a^l-b^l	α and β	a^l	a^l-b^l	α and β
10×10	10,000	A	U(3, 7)	U(0, 1)	U(0.25, 1)	U(50, 200)	U(0, 25)	U(5, 25)	U(1000, 4000)	U(0, 500)	U(100, 500)
10×20	15,000	B	U(3, 7)	U(0, 1)	U(0.25, 1)	U(100, 400)	U(0, 50)	U(10, 50)	U(2000, 8000)	U(0, 1000)	U(200, 1000)
15×15	15,000	C	U(3, 7)	U(0, 1)	U(0.25, 1)	U(200, 800)	U(0, 100)	U(20, 100)	U(4000, 16,000)	U(0, 2000)	U(400, 2000)
10×30	15,000	D	U(3, 7)	U(0, 1)	U(0.25, 1)	U(400, 1,600)	U(0, 200)	U(40, 200)	U(8000, 32,000)	U(0, 4000)	U(800, 4000)
50×50	50,000										
30×100	30,000										
50×200	50,000										

5.3. Parameter Tuning

To evaluate the performance of employed algorithms, twenty eight test problems, with different sizes, are developed. As we discussed in the previous sections, the experiments were designed according to the full factorial design for the original EM and revised EM, because of the few number of factors, and Taguchi experimental design for the hybrid EM. We used the L16 orthogonal array for the hybrid EM, so 16 different combinations of control factors were considered. Because of the stochastic nature of EMs, three replications were performed for every trial in order to achieve the more reliable results. Because the scale of objective functions in each instance is different, they could not be employed directly. To solve this dilemma, the relative percentage deviation (RPD) is used for every instance.

$$\text{RPD} = \frac{\text{Alg}_{sol} - Min_{sol}}{Min_{sol}} \times 100$$

Where Alg_{sol} and Min_{sol} are the obtained objective value for each replication of trial in a given instance and the obtained best solution respectively. After converting the objective values to RPDs, the mean RPD is calculated for every trial. As shown in Figure 5, in original EM, best parameters of factors A, and B, are obviously 2, and 2 respectively, according to the RPD results. Similar to original EM, in revised EM, the best parameters for the revised EM are 2,2, and 2 respectively, according to their alphabetical order in Figure 6.

To do according Taguchi parameter design instructions for the hybrid EM, the objective values, are transformed to S/N ratios. The S/N ratios of trials are averaged in each level and the value is shown in Figure 7. But, in determining best parameters of other factors more investigations are needed. Thus, in addition to S/N ratios, another measurement, the RPD, is used. The results of RPD for each parameter level are demonstrated in Figure 8. As can be seen in Figure 8, the RPD illustrates the best parameters of factors A, B, C, D and E as 1, 2, 1, 2 and 2 respectively which confirms the same results as S/N ratios.

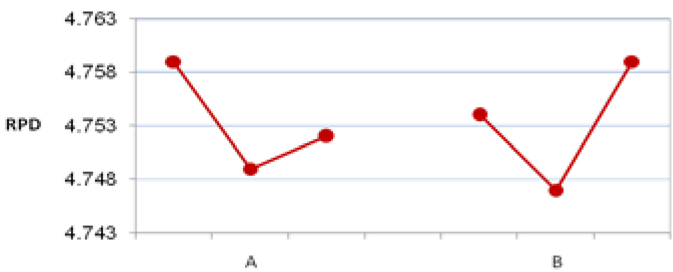


Figure 5: RPD for original EM factors

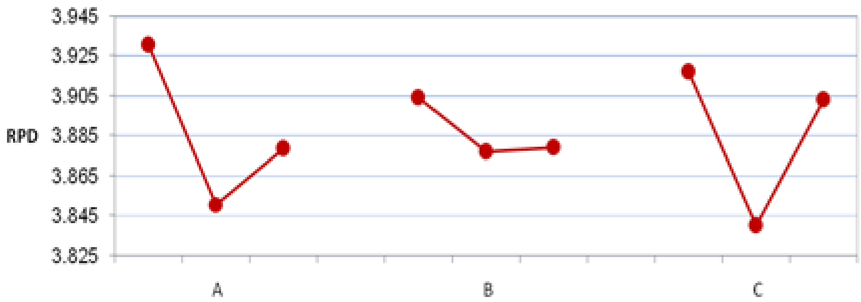


Figure 6: RPD for the revised EM factors

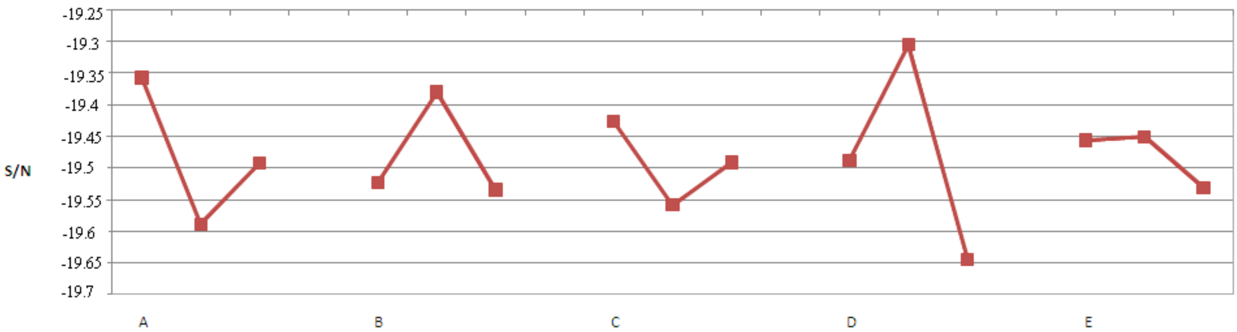


Figure 7: S/N ratio for hybrid EM factors

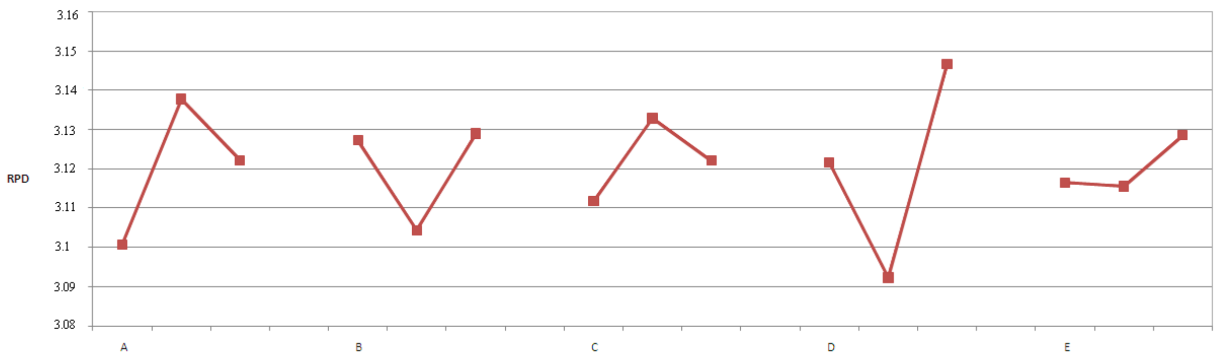


Figure 8: RPD ratio for hybrid EM factors

5.4. Experimental results

For the three algorithms the searching time is set identical and equal to 2×m×n milliseconds. By using this criterion, we consider both sizes, m and n, and searching time increases according to the rise of either number of potential DCs or customers. Twenty instances are generated for each of the seven problem sizes i.e. totally 140 instances, different from the ones used for calibration, to avoid bias in the results.

Each instance is solved three times. We use RPD measure to compare the algorithms. In order to verify the statistical validity of the results, we have performed an analysis of variance (ANOVA) to accurately analyze the results. The results demonstrate that there is a clear statistically significant difference between performances of the algorithms. The means plot and LSD intervals (at the 95% confidence level) for the three algorithms are shown in Figure 9. As can be seen, between original and revised EM, there is not as significant difference as between hybrid EM and both original and revised EM.

In order to evaluate the robustness of the algorithms in different situations, we analyzed the effects of the problem size on the performance of the algorithms. Figure 10 shows the results of the experiments for each problem size, 60 data per average, due to twenty instance for each problem size and running three times. It also shows the interaction between the quality of the algorithms and the size of problems. As one can conclude, hybrid EM demonstrates a robust performance, when the problems size rises. It also shows remarkable performance improvements of hybrid EM in large size problems versus other algorithms. The results obtained from revised EM are better than original EM, except in size 50×50, but for all problem sizes, the hybrid EM could find the better results.

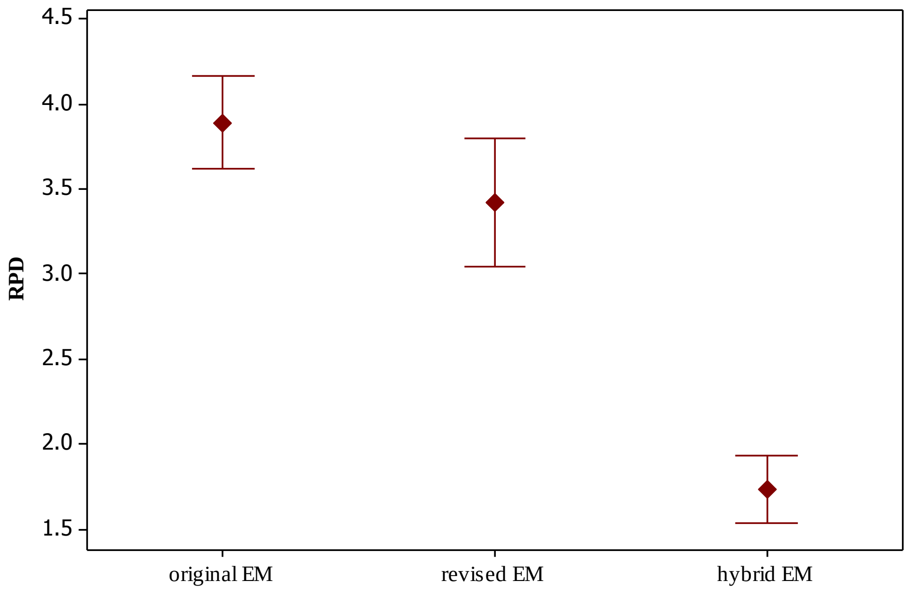


Figure 9: Means plot and LSD intervals for the algorithms

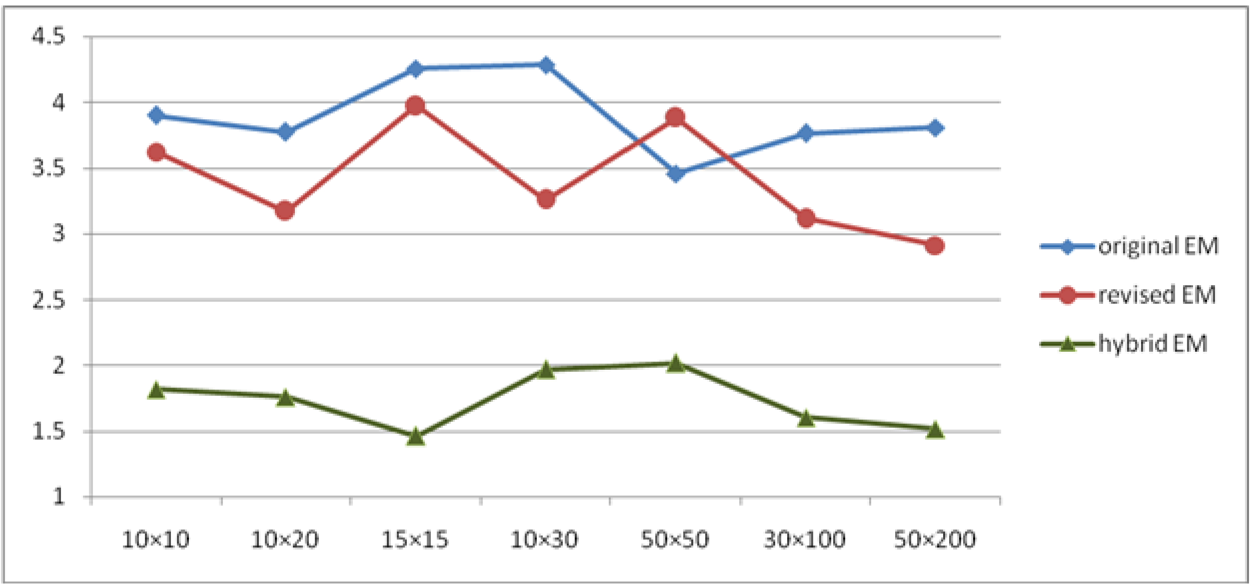


Figure 10: Means plot for the interaction between each algorithm and problem size

6. Conclusion and future works

In this paper several algorithms presented to handle the fuzzy fixed-charge transportation problem. One of our innovation lies in presenting a new solution representation in this research area. We also focused on the intensification and diversification phases of the EM and develop a new version of EM. In order to tune the parameters of the proposed algorithms, we employed the full factorial design and the Taguchi parameter design methods. Applying the Taguchi method, the research cut down the original gigantic experiment combinations. To probe our idea which is based on the nature of the algorithm, we compare the results of all types of the proposed EM. Computational results showed the superiority performance of hybrid EM dealing with the problem in all problem sizes. From the obtained results, the revised EM performs approximately better than the original EM. But the hybrid EM shows superior performance with a big gap from two other types of EM.

There are potentially unlimited opportunities for research in FFCTP. For future researches, it is possible to investigate and develop new algorithms based on other metaheuristics. Another clue for future research is to present new version of the metaheuristic algorithms by developing capable factors and new operators. Besides some other realistic assumptions, such as dynamic environment and truck availability constraints can be utilized.

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