

A knowledge-based NSGA-II approach for scheduling in virtual manufacturing cells

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Abstract

This paper considers the job scheduling problem in virtual manufacturing cells (VMCs) with the goal of minimizing two objectives namely, makespan and total travelling distance. To solve this problem two algorithms are proposed: traditional non-dominated sorting genetic algorithm (NSGA-II) and knowledge-based non-dominated sorting genetic algorithm (KBNSGA-II). The difference between these algorithms is that, KBNSGA-II has an additional learning module. Finally, we draw an analogy between the results obtained from algorithms applied to various test problems. The superiority of our KBNSGA-II, based on set coverage and mean ideal distance metrics, is inferred from results.

Keywords: Multi-objective optimization; Non-dominated sorting genetic algorithm; Knowledge based algorithm; Virtual manufacturing cells; Job scheduling.

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1. Introduction

For most enterprises, it is of vital importance that the manufacturing processes to be optimally controlled. That is, to minimize manufacturing cost, satisfy the production schedule and to improve productivity. In some industries the optimal control of the system is a complex and hard task to carry out. Typically these companies specialize in a few processes and render their production facilities for the manufacturing of a variety of parts. A well-known method for improving planning and control in such industries is virtual manufacturing cells (VMCs). The virtual cell concept was first developed at the national bureau of standards as part of the control software for the automated manufacturing research facility (AMRF) project. A virtual cell is a logical grouping of workstations

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which are dynamically generated and controlled and are not necessarily transposed into physical proximity (McLean et al., 1982). The grouping is based on a predefined logic and only exists in the production control system and in the minds of the workers. Virtual manufacturing cells are created periodically, for instance every week or every month depending on changes in volumes and mix of demand (Slomp et al., 2005). A virtual cell controller takes over the control and identifies a set of required machines with spare capacity to process the jobs. In VMC a machine can be shared among several cells. This increases the performance of cells.

A new approach for layout in VMCs is scattered layout. This approach was first suggested by Montreuil et al. (1991), in which to increase accessibility, similar machines are scattered across the shop floor. Since the similar machines are not located in close proximity travelling distance between machines must be considered when scheduling jobs. Nomden et al. (2006) have surveyed an excellent taxonomy of past research devoted entirely to VMCs. We provide state of the art related to VMCs as follows.

Kesen et al. (2009) investigated the three different types of layouts namely cellular, process and virtual cells. They compared the performance of these layouts using simulation technique. They developed an ant colony optimization to reveal the behavioral characteristics of each layout. Kesen et al. (2010) presented a multi objective mixed integer programming formulation for job scheduling in VMCs, where the objective is to minimize the sum of the weighted makespan and total travelling distance. They also developed a genetic algorithm based on heuristic for scheduling of jobs in VMCs.

In this paper we employed two multi-objective meta-heuristic algorithms for solving the multi-objective mixed integer programming formulation which Kesen et al. (2010) have presented. First, we applied a non-dominated sorting genetic algorithm II (NSGA-II) and then a new algorithm called knowledge based non-dominated sorting genetic algorithm (KBNSGA-II) to solve Kesen's formulation. Both algorithms were tested on various problems and the results were compared using various comparison metrics.

This paper is organized as follows: In Section 2 we present the problem description with the assumptions and mathematical formulation. Section 3 introduces non-dominated sorting genetic algorithm for VMCs scheduling. A knowledge-based sorting genetic algorithm is presented in Section 4. Section 5 presents experimental results and comparison of the proposed algorithms in term of some metrics. Finally, Section 6 states our conclusions and future researches on this topic.

2. Problem description and formulation

This paper is concerned with the scheduling of jobs in VMCs where there are n jobs and m machine types. Each machine type can include at least two machines. $s(i)$ is an individual machine that is a member of machine type i ($s(i) \in i$). All the machines that belong to machine type i are identical. Similar machines are located at different areas in the shop floor. $O_{j,h}$ denotes the h th operation of job j . Each job has its specific route and the machines that can process $O_{j,h}$ are predefined. If $O_{j,h}$ is performed in machine type i , all $s(i)$ machines compete to perform the operation.

The problem consists of two parts: machine assignment and scheduling. Each job can only visit each machines once. Jobs are produced in batches. Processing times, batch sizes as well as precedence relationships are defined in advance. Batch splitting is not permissible, *i.e.* when $O_{j,h}$ is allocated to machine $s(i)$ all operations in the batch must be performed on that machine. Moreover, when $O_{j,h}$ starts all operations in the batch must be completed without interruption and preemption. All jobs are available at time zero. Machines are stationary and distances between each pair of machine are predefined. Breakdown and maintenance activities are ignored. . The notations for the problem definition and formulation are given as follow (Kesen et al. 2010).

Parameters

- j job index ($j = 1, 2, \dots, n$)
- i, k machine group index ($i, k = 1, 2, \dots, m$)
- l index for order on each machine ($l = 1, 2, \dots, l_{s(i)}$)
- h operation index ($h = 1, 2, \dots, h_j$)
- $O_{j,h}$ h th operation of job j
- $s(i)$ individual machine s belonging to machine group i
- N_j batch size of job j
- P_{jh} unit processing time of operation h of job j
- $D_{s(j),s(k)}$ unit transportation cost for each job from machine $s(i)$ to $s(k)$
- M very big number
- w_q weight of the q th objective function

Decision variables

- C_{\max} makespan or completion time of all jobs
- $Y_{s(i),j,h}$ 1 if machine $s(i)$ is selected for operation $O_{j,h}$, 0 otherwise
- $X_{s(i),j,h,l}$ 1 if $O_{j,h}$ is performed on machine $s(i)$ in the order l , 0 otherwise
- $t_{j,h}$ starting time of operation $O_{j,h}$
- $Tm_{s(i),l}$ starting time of the machine $s(i)$ for the order l

$$\min \left(C_{\max}, \sum_{j=1}^n \sum_{h=1}^{h_j} \sum_{i=1}^m \sum_{k=1}^m Y_{s(i),j,h} Y_{s(k),j,h+1} D_{s(i),s(k)} N_j \right) \tag{1}$$

Subject to:

$$C_{\max} \geq t_{j,h} + P_{j,h} N_j; j = 1, 2, \dots, n; h = 1, 2, \dots, h_j \tag{2}$$

$$t_{j,h} + P_{j,h} N_j \leq t_{j,h+1}; j = 1, 2, \dots, n; h = 1, 2, \dots, h_j - 1 \tag{3}$$

$$Tm_{s(i),l} + X_{s(i),j,h,l} P_{j,h} N_j \leq Tm_{s(i),l+1}; i = 1, 2, \dots, m; j = 1, 2, \dots, n; h = 1, 2, \dots, h_j; l = 1, 2, \dots, l_{s(i)} - 1 \tag{4}$$

$$t_{j,h} + M.(1 - X_{s(i),j,h,l}) \geq Tm_{s(i),l}; i = 1, 2, \dots, m; j = 1, 2, \dots, n; h = 1, 2, \dots, h_j; l = 1, 2, \dots, l_{s(i)} \tag{5}$$

$$Tm_{s(i),l} + M.(1 - X_{s(i),j,h,l}) \geq t_{j,h}; i = 1, 2, \dots, m; j = 1, 2, \dots, n; h = 1, 2, \dots, h_j; l = 1, 2, \dots, l_{s(i)} \tag{6}$$

$$\sum_{j=1}^n \sum_{h=1}^{h_j} X_{s(i),j,h,l} \leq 1; i = 1, 2, \dots, m; l = 1, 2, \dots, l_{s(i)} \quad (7)$$

$$\sum_i Y_{s(i),j,h} = 1; j = 1, 2, \dots, n; h = 1, 2, \dots, h_j \quad (8)$$

$$\sum_l X_{s(i),j,h,l} = Y_{s(i),j,h}; i = 1, 2, \dots, m; j = 1, 2, \dots, n; h = 1, 2, \dots, h_j \quad (9)$$

$$C_{\max} \geq 0; t_{j,h} \geq 0; Tm_{s(i),l} \geq 0; X_{s(i),j,h,l}, Y_{s(i),j,h} \in \{0, 1\} \quad (10)$$

The objective function in equation (1) minimizes the total travelling distance and the completion time of all jobs. Constraints in equation (2) assure that makespan must be greater than or equal to the completion times of all operations. The constraints in equation (3) secure that no operation can be started unless the preceding operations have been completed. The constraints in equation (4) indicate that successive operations of any machine must wait for the preceding operation to be completed. Constraint sets (5) and (6) enforce that if $X_{s(i),j,h,l}$ is equal to 1 the h th operation of job j and the l th order on machine $s(i)$ must start at the same time. Constraint (7) shows that only one operation can be assigned on each order of operation on any machine. Equation (8) indicates that every operation is assigned only to one machine. Equation (9) ensures that if an operation is assigned to any particular machine, this operation will be positioned only once. Constraint set (10) enforces that decision variable must be greater than or equal to zero.

3. A NSGA-II for the job scheduling in VMCs problem

Finding the global optimum for a general multi objective optimization problem (MOOP) is NP-Complete (Back, 1996). In a MOOP, finding a solution that optimizes all objectives is ideal but because of conflicting objectives, in most cases, a set of solutions is found. Consider a general multi-objective minimization problem with p decision variables and q objectives ($q > 1$):

$$\text{Minimize } y = f(x) = (f_1(x), f_2(x), \dots, f_q(x))$$

Subject to:

$$x = (x_1, x_2, \dots, x_p) \in X$$

$$y = (y_1, y_2, \dots, y_q) \in Y$$

Where x is the decision vector, y is the objective vector, X is the parameter space and Y is the objective space. We say solution a dominate solution b , if and only if: $f_i(a) \leq f_i(b); \forall i \in \{1, \dots, q\}$ and $f_i(a) < f_i(b); \exists i \in \{1, \dots, q\}$. Non-dominated solutions are a set of solutions that dominate the others but do not dominate themselves. A feasible solution which is not dominated by any other solution in all feasible space is called Pareto optimal. The Pareto optimal set, which is also called the efficient set, is the collection of all Pareto optimal solutions. Pareto optimal solutions in the objective space are called the Pareto optimal frontier.

NSGA-II developed by Deb et al. (2002) is an improvement of NSGA which was originally proposed by Srinivas and Deb (1994). In the each iteration of algorithm a new population is obtained by using four algorithmic components: a fast non-dominated method, an efficient crowded

comparison mechanism, genetic operators and an elitist approach. The NSGA-II ranks each solution based on dominance. The solutions of the current population are partitioned in non-dominated fronts. The first non-dominated front contains all non-dominated solutions in the population. The second non-dominated front contains all non-dominated solutions obtained after removing the first front. This process continues till all solutions are fitted in a front. The NSGA-II also calculates the crowding distance for each individual solution. Crowding distance gives the algorithm the ability to distinguish individual solutions that are in the same frontier. The individual solutions that are in a lower frontier are considered better than those in higher frontier. If the individual solutions are in the same frontier, then those that have upper crowding distance are considered better.

if $f_m^{(i+1)}$ and $f_m^{(i-1)}$ denote the values for objective function m for the nearest solution surrounding solution i with $f_m^{(i+1)} \leq f_m^i \leq f_m^{(i-1)}$, f_m^{\max} and f_m^{\min} the maximum and the minimum values for objective function m and M is the total number of objective, then crowding distance (d_i) for a solution i is calculated by $d_i = \sum_{m=1}^M \frac{f_m^{(i+1)} - f_m^{(i-1)}}{f_m^{\max} - f_m^{\min}}$ or set to $d_i = \infty$ if solution i is a boundary solution in any objective function.

NSGA-II starts with the creation of a random population of solutions P_0 of size N , where N is the population size. Solutions are ranked using the concept of non-dominated sorting. A new population of solutions Q_0 is created using the set of recombination operators that are appropriate for the problem considered. The old and the new generation of solutions are merged into a new population of solutions R_t of size $2N$. Non-dominated sorting is subsequently used to sort the solutions contained in R_t . According to the ranking priorities, fronts from the combined population (R_t) are moved to intermediate population P_{t+1} . If the last move would not fill the P_{t+1} completely and the next move is bigger than the size of space left in P_{t+1} then based on crowding distance the best solution from last front are moved to P_{t+1} . The new population of solutions Q_{t+1} is created from P_{t+1} through the typical recombination operators. The same process is repeated for a pre-specified number of generations. The specially designed components of the proposed NSGA-II are explained in following sections:

3.1. The representation mechanism and the schedule generation scheme

There are two parts to each solution (chromosome). One represents operation sequence vector and the other machine assignment vector. Elements in the first vector indicate the order of operations related to jobs and elements in the second vector represents the selected machine for the corresponding operation. To show the inner working of this representation mechanism Consider the case where there are three types of machines, namely A, B and C and each machine type has four, two and two machines respectively. Furthermore, there are four jobs namely 1, 2, 3 and 4 that consist of three, two, two and two operations respectively (see [Table 1](#) and [2](#)). For this case, one possible schedule could be $(O_{4,1} \rightarrow O_{2,1} \rightarrow O_{4,2} \rightarrow O_{1,1} \rightarrow O_{1,2} \rightarrow O_{3,1} \rightarrow O_{1,3} \rightarrow O_{2,2} \rightarrow O_{3,2})$, where O_{ij} is operation j of job i . In generating the offspring by genetic operators, it is possible to produce an infeasible solution. To avoid such incident the operation is replaced by its corresponding

job index (see Figure 1). To decode the solutions in the new representation, reading the data from left to right, for each occurrence of the job index, the operation index is increased by one.

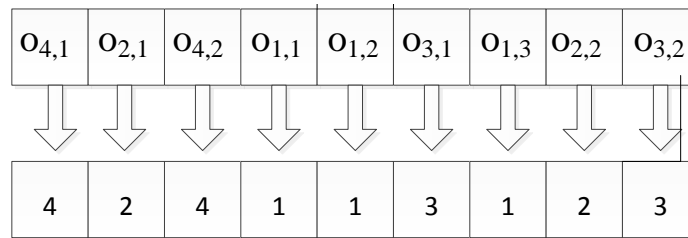


Figure 1: Operation order vector

Machine selection vector is presented using an array of values. For the problem shown in Table 1 and 2 one possible encoding of the machine selection vector is presented in Figure 2. This solution representation is derived from representation of Gen et al. (1994).

Operation	O _{1,1}	O _{1,2}	O _{1,3}	O _{2,1}	O _{2,2}	O _{3,1}	O _{3,2}	O _{4,1}	O _{4,2}
Machine selection	5	3	8	7	1	5	8	2	6

Figure 2: Machine selection vector

Table 1: Machine types and individual machines belonging to it

Machine type	Machine number
A	1, 2, 3, 4
B	5, 6
C	7, 8

Table 2: Operation sequences for the jobs

Job number	Operation sequence
1	B→A →C
2	C→A
3	B→C
4	A→B

In this paper, only active schedules are considered. An active schedule is defined as a feasible schedule, where no operation can be started earlier unless at least one other operation is delayed or some constraints are violated. In order to generate an active schedule, each operation is allocated to its predetermined machine in the order identified in the operation sequence vector. When operation O_{jh} is scheduled on machine $s(i)$ the blank intervals between operations that have already been scheduled on machine $s(i)$ are evaluated from left to right to detect the earliest feasible time available. If such a time interval is found, it will be allocated there. Otherwise it will be allocated at the end of scheduled operations.

3.2. Selection strategy

The selection strategy chooses a number of chromosomes as parents to implement the genetic operators on them. In this research, a crowded tournament selection operator with tour-size=2 is used. In the crowded tournament selection, first we select two chromosomes to attend the tournament randomly. If the two chromosomes are in the different frontiers, then the chromosome with the best frontier would be selected as winner. Otherwise the chromosome with the best crowding distance is selected.

3.3. Crossover operators

In this paper, two crossover operators are employed to create an offspring. One of them is to create the operation sequence part of child and the other to create the machine assignment part of child. These operators are defined as follows:

Crossover operator for operation sequence vector: In the proposed solution representation, there are similar numbers in the operation sequence vector but each number represents a different concept. This makes it difficult to define the crossover operator. To overcome this, the representation for operation sequence vector is transformed into the permutation representation.

In order to implement permutation representation all operations are assigned a fixed ID as shown in Figure 3. Then all operations in the operation sequence vector are shown with their respective ID number. For instance, permutation representation for operation sequence in Figure 1 can be represented as shown in Figure 4.

Operation	$O_{1,1}$	$O_{1,2}$	$O_{1,3}$	$O_{2,1}$	$O_{2,2}$	$O_{3,1}$	$O_{3,2}$	$O_{4,1}$	$O_{4,2}$
Fixed ID	1	2	3	4	5	6	7	8	9

Figure 3: Fixed ID for each operation

Operation referred	$O_{4,1}$	$O_{2,1}$	$O_{4,2}$	$O_{1,1}$	$O_{1,2}$	$O_{3,1}$	$O_{1,3}$	$O_{2,2}$	$O_{3,2}$
Operation sequence	8	4	9	1	2	6	3	5	7

Figure 4: Permutation operation sequence vector

The crossover operation is implemented as follows: First, we randomly select a substring of operation sequence vector from first parent. Next we produce a proto child by copying this substring into the corresponding positions in child. The corresponding operations of substring are deleted from the second parent. Finally, the remaining operations in the second parent are placed into the unfilled positions of the proto-child from left to right. After the crossover operator is implemented, the offspring in the operation sequence vector are converted into the prior representation and a feasible child is produced. This procedure has been proposed by Gao et al. (2004). An example of this crossover is illustrated in Figure 5.

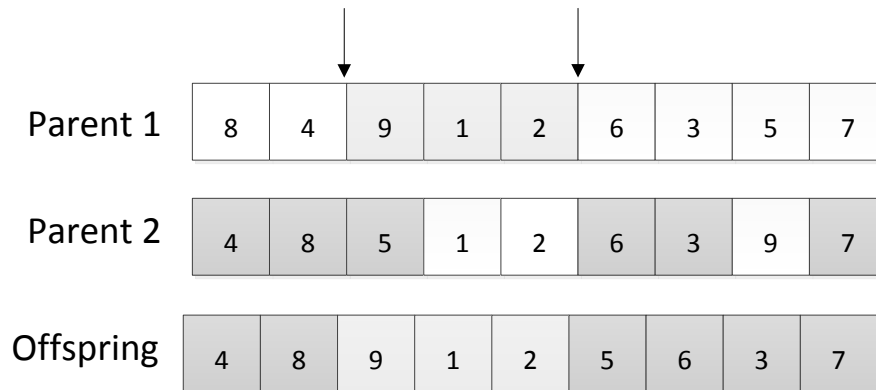


Figure 5: Order crossover on operation sequence vector

Crossover operator for machine assignment vector: the operation sequence vector of an offspring is produced by a Uniform crossover operator. Uniform crossover first generates a random crossover mask and then exchanges relative genes between parents according to the mask. A crossover mask is a binary string randomly generated and equal in size to the chromosomes. The mask is scanned from left to right. If the current bit is 1 then the child inherits the corresponding gene from first parent. Otherwise the genes are supplied from the second parent. The second offspring is produced in the similar way, except that when current position is 1 in the string the genes are selected from the second parent and when 0 the selection is from the first parent. It is illustrated in Figure 6.

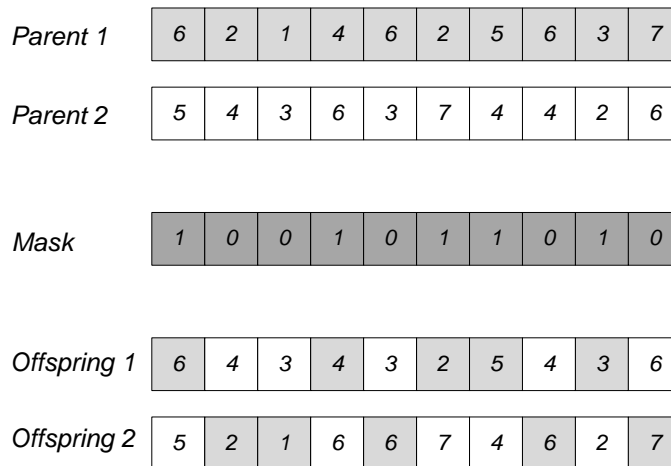


Figure 6: Illustration of the uniform crossover

3.4. Mutation operator

Mutation operator is implemented in both the operation sequence vector and the machine assignment vector, separately.

Mutation operator for machine assignment vector: An operation is selected randomly and from among the machines can process this operation, one is selected randomly and replaces previous machine.

Crossover operator for operation sequence vector: To mutate in operation sequence an operation is selected randomly and its position in the operation sequence vector is swapped with the position of its immediate prior operation.

4. The proposed knowledge based NSGA-II

4.1. Introduction

In recent years the study of interaction between evolution and learning in combinatorial optimization problems has been attracting much attention and several research have been done in this field (e.g. by Ho et al. (2007), Chung and Reynolds (1996), Branke (1999), Louis and McDonnell (2004), Michalski (2000), Xing et al. (2010)). The diversity of these methods and their positive results has persuaded us to opt for a holistic architecture called knowledge-based heuristic searching (KBHSA). KBHSA comprises the integration of two modules namely knowledge and heuristic search. Heuristic searching module searches through the wide solution space and finds good solutions. The knowledge module learns the available knowledge from the same good solutions and feeds it back to heuristic searching module. We demonstrate the performance of this architecture in a new algorithm called knowledge based non-dominated sorting genetic algorithm (KBNSGA-II). The general idea for this methods is to keep good and/or bad features of the previous solutions to improve the quality of next solutions. We use operational memory (OM) to save good features of previous solutions which initially was proposed by Ho et al. (2007) for flexible job-shop schedules in the algorithm named LEGA.

4.2. Operational memory

Operational memory is an array of bits that contains the set of suitable machines for generating improved solutions (see Figure 7).

O _{1,1}		O _{1,2}			O _{1,3}		O _{2,1}		O _{2,2}				O _{3,1}		O _{3,2}		O _{4,1}			O _{4,2}			
M ₅	M ₆	M ₁	M ₂	M ₃	M ₄	M ₇	M ₈	M ₇	M ₈	M ₁	M ₂	M ₃	M ₄	M ₅	M ₆	M ₇	M ₈	M ₁	M ₂	M ₃	M ₄	M ₅	M ₆
1	1	0	0	1	0	1	1	1	0	1	0	1	0	1	1	1	0	0	0	1	1	1	1

Figure 7: Operational memory

A set of machines that have generated good solutions are kept in the OM to limit machine selection. All bits in OM are initially set to zero. When a machine is considered to be suitable for an operation, the value of corresponding bit for that machine and that operation is set to 1. If the initial knowledge is available, the memory would be initialized. The operational memory would be updated when algorithm finds other suitable machine. The process of initializing OM and updating it are described as follows:

(i) *OM initialization*: In initializing OM we must consider two objectives, the makespan and the total traveling distance. We were able to extract the initial knowledge for the total traveling distance but the useful initial knowledge for minimizing the total completion time of the jobs was not attained. Initial knowledge for minimizing the total traveling distance of jobs is explained below.

Every job can choose different routes between machines for its completion. All possible routes for that job are evaluated and k number of best routes are discovered and OM is initialized by them (k is the number of operations related to that job). For instance, consider the problem shown by Table 1, 2 and 3. Table 3 shows the distance between machines. Job 1 in Table 2 consists of three operations namely $O_{1,1}$, $O_{1,2}$, $O_{1,3}$ which have to be processed by machine types B, A and C. According to Table 1, each machine types B, A and C consist of 2, 4 and 2 machines respectively. Therefore, Job 1 has 16 ($2 \times 4 \times 2 = 16$) routes to choose from. Using Table 3 the following three shortest routes are chosen.

$M_5 \rightarrow M_3 \rightarrow M_7$ TTD=46

$M_5 \rightarrow M_3 \rightarrow M_8$ TTD=60

$M_6 \rightarrow M_3 \rightarrow M_7$ TTD=62

Table 3. Distance matrix between each pair of machine

Machine	1	2	3	4	5	6	7	8
1	0	16	79	61	72	71	26	52
2		0	34	51	36	78	40	72
3			0	25	34	50	12	26
4				0	55	34	68	42
5					0	12	18	67
6						0	34	43
7							0	20
8								0

According to these three routes, machines M_5 and M_6 are suitable for operation $O_{1,1}$. Therefore bits related to machines M_5 and M_6 in OM for operation $O_{1,1}$ is set to 1. The machine that is considered for operation $O_{1,2}$ is M_3 . Similarly, machines M_7 and M_8 deemed suitable for operation $O_{1,3}$. The bit related to machine M_3 for operation $O_{1,2}$ and the bits related to machine M_7 and M_8 for operation $O_{1,3}$ are set to 1. The rest of the bits related to operations of Job 1 remain at zero. Likewise suitable routes for all jobs are identified and OM is initialized. Figure 7 depicts an initialized OM for the given example.

(ii) *Updating of OM*: With the repetition of algorithm, it is possible to discover other suitable machines and OM would be updated. In each generation all solutions in first front are used for updating OM and consequently the number of bits that have the value of 1 would increase.

4.3. Applying OM on the proposed KBNSGA-II

The difference between KBNSGA-II and NSGA-II is that KBNSGA-II has an additional mutation operator named intelligent mutation. In intelligent mutation when selecting a new machine for an operation, suitable machines are selected from OM. The steps of intelligent mutation operator in machine selection vector are presented below.

- 1) Select an operation O_{ij} randomly from operation sequence vector of a solution
- 2) Detect machine M_k that is currently selected to process O_{ij} .
- 3) Detect a set of machines $P(O_{ij})$ that can process O_{ij}
- 4) Identify a set of suitable machines $POM(O_{ij})$ in Operational Memory that can process O_{ij}
- 5) If $POM(O_{ij})$ contains only M_k then select a machine randomly from $P(O_{ij})$. Otherwise, select a random machine in $POM(O_{ij})$ (except M_k) to process O_{ij} .

5. Experimental design

5.1. Test problems

Thirty nine test problems were created to examine the performance of the algorithms. Data required for a problem are the distance between each pair of machines, processing time of jobs on machines and batch size. Distances between each pair of machines in the test problem are uniformly distributed from 10 to 80. Processing time of jobs on machines follows the uniform distribution with a lower bound of 2 and upper bound of 10. Batch size also corresponds to uniform distribution with a lower bound of 5 and upper bound of 40.

5.2. Evaluation metrics

To evaluate the effectiveness and performance of the proposed algorithms the following five comparison metrics are considered:

- (1) *Spacing metric*: This metric measures the uniformity of the spread of the points of the solution set. It is defined as follows:

$$S = \sqrt{\frac{1}{|Q|} \sum_{i=1}^{|Q|} (d_i - \bar{d})^2}$$

where $d_i = \min_{k \in Q, k \neq i} \sum_{m=1}^M |f_m^i - f_m^k|$, $\bar{d} = \sum_{i=1}^{|Q|} \frac{d_i}{|Q|}$, Q is the non-dominated solution set in first front, $|Q|$ is the number of non-dominated solution in Q , M is the number of objectives and f_j^i is the value of objective function j for solution i . The lower value of spacing metric the better solution quality we have.

- (2) *Set coverage metric*: This metric is applied for comparing the Pareto solutions A and B . Set coverage metric $C(A, B)$ shows the ratio of the number of solutions in set B that are dominated by solutions in set A . Set A is better than set B whenever $C(A, B)$ is greater than $C(B, A)$.

$$C(A, B) = \frac{|\{b \in B \mid \exists a \in A : a \prec b\}|}{|B|}$$

(3) *Mean ideal distance (MID)*: This metric shows the closeness between Pareto solution and ideal point (0,0). Lower values of MID are preferred.

$$MID = \frac{\sum_{i=1}^{|\mathcal{Q}|} D_i}{|\mathcal{Q}|}, \text{ where } D_i = \sqrt{(f_1^i)^2 + (f_2^i)^2 + \dots + (f_M^i)^2}$$

(4) *Maximum spread metric*: this metric measures the Euclidean distance between boundary objectives values in the non-dominated solutions. The higher value of set coverage metric, the better solution quality we have.

$$D = \sqrt{\sum_{m=1}^M (\max_{i=1}^{|\mathcal{Q}|} f_m^i - \min_{i=1}^{|\mathcal{Q}|} f_m^i)^2}$$

(5) *The number of Pareto solutions*: This metric shows the number of Pareto solutions that each algorithm obtain. We have the better solution quality, whenever this metric is higher.

5.3. Experimental results

The algorithms were coded in Borland C++ and executed on a computer with 2.1 GHz Intel Core 2 Duo processor and 2GB of RAM. We executed the algorithms 3 times for each problem. The computational time for the problems 1 to 18 and 19 to 39 are determined to be 60 and 180 seconds, respectively. The population size for both algorithms ranges from 300 to 5000 Depending on the complexity of the problems. Crossover probability is 0.7. Mutation probability in NSGA-II is 0.3. Intelligent and simple mutations in KBNSGA-II are set at 0.24 and 0.06 respectively.

In this paper *RPD* is applied as a common performance measure to compare the algorithms (Ruiz and Allahverdi, 2007). *RPD* for spacing and mean ideal distance metrics is calculated by the following formula:

$$RPD = \frac{Sol - LB}{LB} \times 100$$

Where, *sol* is the objective function value obtained from a given algorithm for each instances of its execution. *LB* is the best solution obtained from both algorithms. *RPD* For maximum spread and the number of Pareto solution metrics is calculate as follow:

$$RPD = \frac{UB - Sol}{UB} \times 100$$

Where *UB* is the best solution obtained from both algorithms. The *RPD* is not calculated for the set coverage metric, whereas the value of this metric is independent of the size of problem. Lower values of *RPD* and upper value of set coverage metric are preferred. Table 4 and Table 5 represent the set coverage values and the average of the relative percentage deviation (*RPD*) values for the four metrics. Notations used in the Table 4 and 5 are as follows:

: problem number

n : number of jobs

m : number of machine types

$Flex$: average number of equivalent machines per operation

Table 4. RPD values of the number of Pareto solutions and values of set coverage metric for algorithms.

#	$n \times m$	$\sum_i s(i)$	$Flex$	Number of Pareto solutions		Set coverage	
				NSGA-II	KBNSGA-II	NSGA-II	KBNSGA-II
1	4×2	4	2	0	0	0	0
2	4×2	5	2.5	0	0	0	0
3	6×2	4	2	0	0	0	0
4	6×3	8	2.71429	0	0	0	0
5	10×3	8	2.75	13.33333	6.66667	0.051282	0
6	10×3	12	4.375	28.57143	22.61905	0.561264	0.018519
7	12×3	10	3.26667	33.33333	17.77778	0.266667	0.264682
8	12×4	12	2.92308	4.761905	14.28571	0.611111	0.142857
9	12×4	16	4.06061	9.52381	28.57143	0.733333	0.269841
10	14×3	12	3.94118	22.91667	29.16667	0.253728	0.487179
11	14×3	14	4.66667	11.11111	22.22222	0.333333	0.666667
12	14×4	14	3.5	21.21212	21.21212	0.077922	0.333333
13	14×4	18	4.55263	31.57895	31.57895	0.296053	0.45202
14	16×4	14	3.52	17.70833	23.95833	0.510313	0.188194
15	16×4	20	5.02	12.5	14.58333	0.315152	0.485606
16	16×5	18	3.52727	23.80952	19.04762	0	0.275757
17	18×4	14	3.44828	11.11111	29.16667	0.319966	0.499687
18	18×5	14	2.55738	16.21622	15.31532	0.356322	0.114943
19	20×4	24	6.01587	13.33333	13.33333	0.100733	0.759907
20				37.03704	25.92593	0.037037	0.944444

21	20×6	26	4.11828	38.09524	32.38095	0.666667	0.333333
22	25×4	20	5.18667	24.24242	33.33333	0.333333	0.666667
23	30×4	22	5.58696	13.33333	40	0.333333	0.666667
24	30×4	30	7.47917	73.33333	25.33333	0.085185	0.722222
25	30×6	26	4.31707	17.7305	25.53191	0.049645	0.899493
26	35×4	20	5.0099	57.14286	45.2381	0	1
27	35×4	32	8.0381	20.68966	24.13793	0.594203	0.212946
28	35×6	26	4.4589	23.07692	22.4359	0.025458	0.912232
29	40×4	18	4.47009	47.12644	26.43678	0.451299	0.345977
30	40×4	32	8.41379	45.55556	20	0.422222	0.466667
31	45×4	18	4.62411	29.5082	27.86885	0.174881	0.731935
32	45×4	24	5.83459	50	38.09524	0.333333	0.666667
33	45×5	28	5.72152	33.33333	66.66667	0.25	0.470588
34	45×5	38	7.47651	17.64706	12.2549	0.22043	0.737296
35	50×4	24	5.9021	67.81609	32.95019	0.225976	0.578136
36	50×4	36	9.13194	33.33333	66.66667	0.416667	0.430556
37	50×5	30	5.98477	39.04762	71.42857	0.666667	0.333333
38	50×5	40	7.96571	45.45455	60.60606	0	1
39	50×6	34	5.63819	12.90323	16.12903	0.148008	0.762857
Mean	50×6	42	7.0625	25.57507	26.22886	0.26209	0.457467

Table 5. RPD values of Spacing, Maximum spread and MID metrics for the algorithms

#	$n \times m$	$\sum_i s(i)$	Flex	Spacing		Maximum spread		MID	
				NSGA-II	KBNSGA-II	NSGA-II	KBNSGA-II	NSGA-II	KBNSGA-II
1	4×2	4	2	0	0	0	0	0	0
2	4×2	5	2.5	0	0	0	0	0	0
3	6×2	4	2	0	0	0	0	0	0
4	6×3	8	2.71429	0	0	0	0	0	0
5	10×3	8	2.75	20.94268	13.21846	10.51305	1.277389	0.957165	0.35338
6	10×3	12	4.375	76.54691	38.27589	19.51546	21.04831	0.21248	0.415497
7	12×3	10	3.26667	82.50769	35.47882	16.71055	11.03461	1.239622	1.846815
8	12×4	12	2.92308	407.2025	1523.18	24.67841	35.43695	0.691722	0.351991
9	12×4	16	4.06061	81.18244	137.5273	9.671417	0	1.146281	1.535008
10	14×3	12	3.94118	188.6708	83.43242	8.601471	16.85833	2.887835	1.274911
11	14×3	14	4.66667	88.05328	126.8062	46.50126	22.93099	2.794737	1.491991
12	14×4	14	3.5	49.29817	76.92683	4.407064	6.725787	0.734786	0.385521
13	14×4	18	4.55263	80.60102	68.62553	18.69504	9.183134	2.60656	2.004345
14	16×4	14	3.52	122.588	40.29714	14.94865	25.12233	1.618057	2.727187
15	16×4	20	5.02	268.2142	107.0728	16.10448	17.07139	1.481653	1.357673
16	16×5	18	3.52727	45.68206	25.1819	4.957553	4.189876	0.380035	0.312821
17	18×4	14	3.44828	8.334391	58.39294	26.36492	37.49564	4.610128	2.976127
18	18×5	14	2.55738	53.08239	53.01306	12.67556	8.712305	0.237166	0.201854
19	20×4	24	6.01587	76.49178	86.4384	38.73056	29.15351	7.429662	3.400386
20	20×6	26	4.11828	455.4679	450.9746	43.34684	55.46685	2.307943	0.313855
21	25×4	20	5.18667	24.982	13.36687	36.82421	29.60836	1.741368	1.942954
22	30×4	22	5.58696	86.09455	478.688	36.09795	42.02699	5.073303	2.414434
23	30×4	30	7.47917	60.15118	54.54377	21.8859	23.68209	6.4029	3.065788
24	30×6	26	4.31707	117.8754	133.9127	82.60245	47.94758	3.450422	1.456793

25	35×4	20	5.0099	309.4261	926.9429	59.84582	44.18387	2.736112	1.636741
26	35×4	32	8.0381	82.79936	162.9596	67.80154	25.02028	2.86889	1.110642
27	35×6	26	4.4589	85.39437	1734.811	81.99853	51.70065	0.882059	1.680163
28	40×4	18	4.47009	235.8405	126.2618	11.11885	13.48464	1.547209	0.076514
29	40×4	32	8.41379	216.7817	122.1881	52.94436	41.41806	1.527537	2.935825
30	45×4	18	4.62411	503.1054	115.5011	28.85419	35.77596	1.811919	1.46918
31	45×4	24	5.83459	207.4439	502.9213	42.2813	33.84191	2.223352	1.798964
32	45×5	28	5.72152	236.6583	1662.493	66.15424	10.76106	2.352625	2.952191
33	45×5	38	7.47651	191.3561	89.24439	44.18482	75.32104	5.785695	4.198806
34	50×4	24	5.9021	19.89574	158.4972	71.08444	44.55521	3.612942	2.656749
35	50×4	36	9.13194	300.4056	1263.631	75.06103	32.30925	2.551022	1.304348
36	50×5	30	5.98477	398.2369	1009.05	35.94961	50.25078	4.237655	3.106266
37	50×5	40	7.96571	173.0098	237.1833	29.16576	44.40162	4.089312	3.145251
38	50×6	34	5.63819	800.9179	131.4423	36.63923	72.77698	5.889459	3.088123
39	50×6	42	7.0625	144.6526	967.1466	69.8149	42.69404	4.305791	4.153152
Mean				161.5357	328.6058	32.48029	27.2684	2.421164	1.670314

To compare the two algorithms in term of the five metrics, the two sample t –tests are performed and the results are shown in Tables 6-10. The results indicate that KBNSGA-II is preferred with respect to the set coverage and the mean ideal distance metrics. Moreover, in spacing metric the obtained results from NSGA-II are better than those of KBNSGA-II. This is because the knowledge module (OM) guides the algorithms toward promising space and therefore the search space is condensed. However, since NSGA-II searches a larger space, metrics related to diversity obtained from NSGA-II are preferred. Figure 8 depicts the convergence of both algorithms in minimizing the mean ideal distance metric for Problem 21.

Table 6. Two-sample t – test for NSGA-II vs KBNSGA-II in term of the number of Pareto solutions

	N	Mean	StDev	SE Mean
NSGA-II	39	25.6	18.1	2.9
KBNSGA-II	39	26.2	17.5	2.8

Difference = mu NSGA-II - mu KBNSGA-II
 Estimate for difference: -0.65
 95% upper bound for difference: 6.06
t-test of difference = 0 (vs <): *t*-value = -0.16 *p*-value = 0.436 df = 75

Table 7. Two-sample *t* – test for NSGA-II vs KBNSGA-II in term of set coverage

	N	Mean	StDev	SE Mean
NSGA-II	39	0.262	0.220	0.035
KBNSGA-II	39	0.457	0.307	0.049

Difference = mu NSGA-II - mu KBNSGA-II
 Estimate for difference: -0.1954
 95% upper bound for difference: -0.0944
t-test of difference = 0 (vs <): *t*-value = -3.23 *p*-value = 0.001 df = 68

Table 8. Two-sample *t* – test for NSGA-II vs KBNSGA-II in term of spacing

	N	Mean	StDev	SE Mean
NSGA-II	39	162	170	27
KBNSGA-II	39	329	494	79

Difference = mu NSGA-II - mu KBNSGA-II
 Estimate for difference: -167.1
 95% upper bound for difference: -26.8
t-test of difference = 0 (vs <): *t*-value = -2.00 *p*-value = 0.026 df = 46

Table 9. Two-sample *t* – test for NSGA-II vs KBNSGA-II in term of maximum spread

	N	Mean	StDev	SE Mean
NSGA-II	39	32.5	24.8	4.0
KBNSGA-II	39	27.3	20.3	3.2

Difference = mu NSGA-II - mu KBNSGA-II
 Estimate for difference: 5.21
 95% lower bound for difference: -3.34
t-test of difference = 0 (vs >): *t*-value = 1.01 *p*-value = 0.157 df = 73

Table 10. Two-sample *t* – test for NSGA-II vs KBNSGA-II in term of mean ideal distance

	N	Mean	StDev	SE Mean
NSGA-II	39	2.42	1.94	0.31
KBNSGA-II	39	1.67	1.24	0.20
Difference = mu NSGA-II - mu KBNSGA-II				
Estimate for difference: 0.751				
95% lower bound for difference: 0.134				
<i>t</i> -test of difference = 0 (vs >): <i>t</i> -value = 2.03 <i>p</i> -value = 0.023 df = 64				

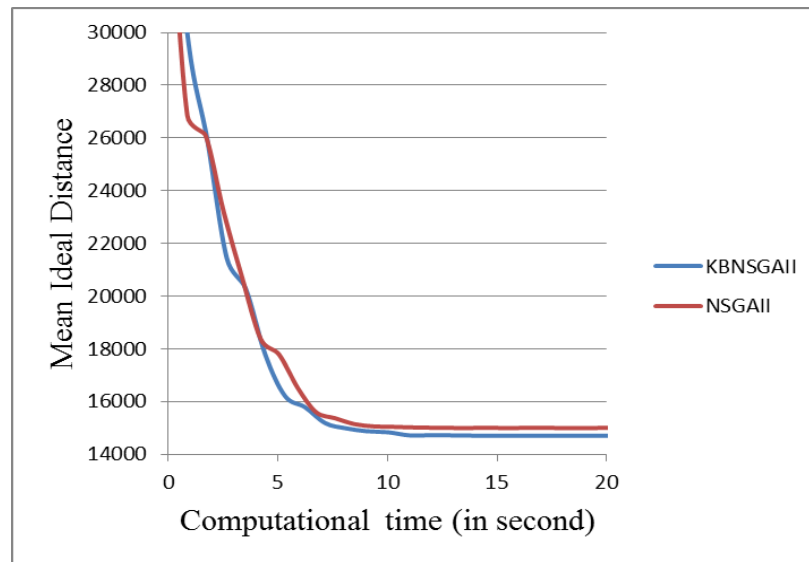


Figure 8: Convergence of proposed algorithms in minimizing the MID metric

6. Conclusion

In this paper, a multi-objective job scheduling in VMCs optimization problem was introduced. The objectives were minimizing the makespan and total travelling distance. A non-dominated sorting genetic algorithm II (NSGA-II) and a new algorithm named knowledge-based NSGA-II (KBNSGA-II) were proposed. The knowledge module learns available knowledge from the obtained good solutions and then applies the existing knowledge to guide the algorithm. In the proposed KBNSGA-II an operational memory is defined to save the knowledge. The operational memory is applied to mutation operator. Using this method, the searching process intelligently is guided toward promising space and therefore search space is reduced. Both algorithms were test on variety of problems and the results were compared. Different comparison metrics such as spacing, set coverage, mean ideal distance, maximum spread and the number of Pareto solutions were used to evaluate the algorithms. The results of experiment and the statistical test demonstrated KBNSGA-II is superior to NSGA-II with respect to set coverage and mean ideal distance metrics. In spacing metric NSGA-II was better. There was not any significant difference in the other metrics.

Further research can concentrate on developing the proposed knowledge-based procedure for other scheduling problems or applying the learning method in other meta-heuristics. Another future research may be the consideration of some other realistic assumptions such as machine availability constraints, batch splitting and sequence-dependent setup times. To add the other optimization objectives in the problem such as minimization of maximum machine workloads is another opportunity for research.

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