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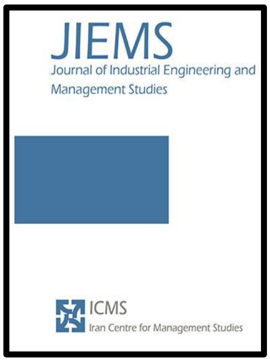


# Effects of inspection errors on economically design of CCC-r control chart

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## Abstract

*CCC-r* chart extended approach of *CCC* charts, is a technique applied when nonconforming items are rarely observed. However, it is usually assumed that the inspection process is perfect in the implementation control charts imperfect inspections may have a significant impact on the performance of the control chart and setting the control limits. This paper first investigates the effect of inspection errors on the formulation of *CCC-r* chart, then an economic model is presented in the presence of inspection errors to design control chart so that the average cost per item minimized. The *r* parameter in the chart is optimized with respect to the economic objective function, Modified Consumer Risk, and Modified Producer Risk.

**Keywords:** CCC-r control chart, Average Number of Inspected items, Inspection Errors, Economic design of control charts, Analytic Hierarchy Process.

## 1. Introduction

For decades, a lot of effort was devoted to improve quality of finished items. As a result, nowadays we often deal with high quality production in which the proportion of nonconforming items is very small. Hence, it is important to pay special attention to the control methods of such processes. Shewhart control charts such as *p* or *np* charts are ineffective when the nonconforming proportion reaches a low value. Bersimis *et al.* (2014) expressed that if a small or moderate shift occurs in these processes, then the out-of-control non-conforming fraction items will still be very small, and therefore, it is highly probable, that no defective item will be observed in the inspected sample. Recently, Emura and Lin (2015) compared five frequently used rules for *n* and *p* required for the normal approximation of the binomial distribution, which is relevant for the monitoring of nonconforming units. They also proposed a new rule for approximation, that is, $np \geq 10$ and $p \geq 0.1$ or $np > 15$, which works well compared with existing ones. However, the problem is not yet solved for other distributions. However, it is proven that for small nonconforming *p*, it is better to apply time-between-events (TBE) control charts (Liu *et al*, 2004), which consider the number of successes between failures. Consequently, the term geometric (or exponential) chart is also

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used (Yang *et al*, 2002). The idea of *CCC* chart based on geometric distribution first developed by Calvin (1983). In addition, Xie *et al*. (1998) and Ohta *et al*. (2001) speak about cumulative count of conforming (*CCC*) charts.

Yazdi and Nezhad (2015), and Yazdi and Nezhad (2016) presented an acceptance-sampling plan based on cumulative count of conforming using minimum angle method. In addition, they compared count of cumulative conforming sampling plans with Dodge-Romig single sampling plan. Their method had better performance in most of the cases.

Since the geometric probability distribution function is highly asymmetric, Acosta-Mejia (2012) discussed that this control chart is not very sensitive to signal when small to moderate shifts occurs in the nonconforming proportion. Several authors suggested methods for solving this problem. Acosta-Mejia (2014) suggested two geometric charts (simple and run sum chart) with runs rules. He concluded that the proposed charts could be compared favorably with the two-sided geometric chart based on probability limits when the fraction is very small. They observed that runs rules are appropriate for monitoring a distribution that is approximately unimodal and symmetric. Khilare and Shirka (2014) argued about m-of-m control chart based on cumulative count of conforming units for high-yield processes. They compared performance of the m-of-m control chart with control chart based on cumulative count of conforming units. Rather than deciding to stop the process after observation a single failure, it is better to postpone this decision until *r* failures have occurred. Hence a negative binomial chart is used as an alternative method to increase the sensitivity (the geometric chart is a special case when $r$ = 1). These charts are called *CCC-r* chart, and they have better performance in finding the shifts of the nonconforming proportion. Xie *et al.* (1999) was first who proposed the idea of *CCC-r* control.

For the efficient use of control charts, essential parameters of the control chart must be optimized properly. It is important to obtain the optimal value of parameter *r*. It is well known that as *r* becomes larger, the *CCC-r* chart becomes more sensitive to small upward shifts. However, too many observations are required to obtain a plotting point on the chart, and the related cost is fairly high (Ali *et al*, 2016) thus this parameter has a significant influence on the charts' effectiveness. Ohta *et al*. (2001) proposed a method to choose parameter *r*, by using a simplified optimal design method within a given profit objective function. Chan *et al.* (2003) presented a two-stage *CCC* control scheme, based on double sampling plans in order to improve the performance of the one stage *CCC* chart. They presented an economic model to minimize expected total cost in order to optimize the probabilities of false alarm at the first and second stages of the two-stage *CCC* control chart. Albers (2010) obtained approximate results on the Lower Control Limit and Average Run Length (ARL) values to determine the optimal value of parameter *r* for one-sided *CCC-r* charts. Di Bucchianico *et al*. (2005) presented a method to select the value of parameter *r* when the *CCC-r* chart is applied for monitoring the packing process in coffee production. The weakness of the CCC-*r* chart is that it shows an ARL biased performance. This undesirable phenomenon means that the ARL does not meet its target value $1/\alpha$. Joekes et al. (2016) performed a computational study of statistical validation to compare the two most outstanding procedures that are used to set control limits on the basis of an ARL-unbiased for the cases $r$ = 2, 3, and 4. The performance was evaluated based on the ARL under control. The application of the CCC- *r* chart is shown with data from an automobile parts plant.

Burke *et al.* (1995) showed that when the nonconforming proportion is estimated from the sample then the estimated value might deviate from the true value due to the presence of inspection errors. Based on the relationship between the true and observed values of the nonconforming proportion, Lu *et al.* (2000) computed the adjusted control limits for the *CCC* chart in the presence of inspection errors. Other studies on inspection errors can be found in Case (1985), Cheng and Chung (1994) and Suich (1988). Ranjan *et al.* (2003) designed a procedure to set control limits for *CCC*-1 charts in the presence of inspection errors to achieve the maximum ARL. Yazdi and Nezhad (2016) introduced a new sampling system based on the concept of Cumulative Count of Conforming. The inspection error was considered in the proposed model as well. This methodology was based on Markov modeling and a negative binomial distribution.

Nowadays with respect to the role of cost optimization in global market competition, economic factors must be taken into account for determining essential parameters. For designing an economic control chart, costs, such as those related to sampling, testing, investigating out of control signals, eliminating special cases, and sending nonconforming items to the customer, are affected by design of control charts. Duncan (1956) was the first who studied the economic design of control charts, and Lorenzen and Vance (1986) made the economic model applicable to many types of control charts. Kudo *et al*. (2004), proposed an economic design of a dynamic *CCC-r* chart with time-varying parameters. They explained a process for a Weibull distributed-shock model and determined the initial values and dynamic decision rules with the aim of maximizing the expected profit per unit time. Zhang *et al.* (2011) offered the economic design of control chart in a process based on time-between-events (TBE) data for monitoring multistage manufacturing processes to maximize the profit (or to minimize the cost). Yilmaz and Burnak (2012) discussed the importance of the cost consideration. Moreover, they debated that any related activities should be analyzed in the context of the cost saving in order to improve profitability. Fallahnezhad and Golbafian (2015) developed a mathematical model based on the average number of inspected items for the economic design of two sided *CCC-r* chart. The optimal Upper and lower control limits and *r* values, for different nonconforming fraction and different parameters in each iteration were calculated and sensitivity analysis of the model was carried out based on Type I error ($\alpha$) and Type II error ($\beta$) in the process.

Considering the importance of inspection errors in high quality processes, the goal in this research is first to study the effect of inspection error on the *CCC-r* chart. Then it is tried to design a new economic scheme with considering the formulation of one-sided *CCC-r* control chart in presence of inspection error where *r* is studied as decision variable. Our proposed objective function will optimize the average cost per “item produced” which is more convenient for high yield processes than average cost or net profit per unit time which is used in most of previous studies. Analysis on expected process control costs per produced item is more appropriate due to minimizing expected average cost in a cycle along with high rate of production which is the most important interest of producers in high quality productions.

ANI or Average Number of Inspected item is used as performance measure to determine chart parameter, containing more information about the process because it counts the number of inspected items and is more reliable than ARL in our problem condition.

Furthermore, to increase the ability of model in process statistical studies and determine *r* parameter more precisely, two other criteria are considered as Modified Producer Risk and Modified Consumer Risk except objective function criterion based on ANI so that economic model is modified to an economic-statistical one, as one of the advantages of the proposed model in high quality processes. Also, to choose the optimum value of *r* parameter based on these 3 criteria, the AHP method as multi criteria method is applied to estimate the levels of importance of each of the Modified Producer and Consumer Risk and objective function values.

The problem will be formulated in Section 2 with analyzing the effect of inspection errors on control chart. In section 3, the economic model will be presented. The optimization (and implementation) procedure and some comparative studies will be discussed in section 4 to explain and analyze the behavior of optimal solution. The conclusion comes in section 5.

# 2. The inspection errors and modification of *CCC-r* chart

## 2.1. Review of *CCC-r* chart

The idea of *CCC-r* chart can be applied for process monitoring by analyzing the number of items inspected until a fixed number of nonconforming items are observed. In a *CCC-r* chart, the number of items inspected ($x$) until the detection of $r_{th}$ nonconforming item is considered to control the nonconforming proportion of the process. Let $r$ be a fixed positive integer, then $x$ follows negative binomial distribution with the probability mass function (*pmf*) and the cumulative distribution function (*cdf*) as follows (Xie *et al*, 2012):

$$f_{r,p}(x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r \;; \qquad for\ x = r, r+1, \dots \tag{1}$$

$$F_{r,p}(n) = p(x \le n) = \sum_{i=r}^{x}\binom{i-1}{r-1}(1-p)^{i-r}p^r = = 1 - \sum_{k=0}^{r-1}\binom{x}{k}(1-p)^{x-k}p^k \;; \quad for\ x = r, r+1, \dots \tag{2}$$

For an acceptable risk of false alarm,$\alpha$, the lower control limit, LCL, in the one-sided *CCC-r* chart can be obtained as the solution of the following equation:

$$F(LCL, r, p) = \sum_{i=r}^{LCL}\binom{i-1}{r-1}p^r\,(1-p)^{i-r} = \alpha \tag{3}$$

Average run length (ARL), denotes the average number of samples taken before a signal is observed. Besides the ARL, the average number of inspected items (ANI) is defined as the expected number of inspected items before the chart signals an alarm. ANI is more applicable than ARL because the ANI counts the number of inspected items. ANI in one sided *CCC-r* chart can be approximately calculated as:

$$ANI = \frac{r}{p}ARL = \frac{r}{p} * \frac{1}{1-p(LCL \le x)} \tag{4}$$

Where $\frac{r}{p}$ is the mean value of negative binomial distribution with parameters $r$ and $p$.

## 2.2. Inspection Errors

With regards to Xie *et al*. (2012), because of the errors involved in the inspection of items, nonconforming proportion could deviate from its true value. These errors are catagorized in two types of: (1) classification of a conforming item as nonconforming, (2) classification of a nonconforming item as conforming. In this condition, relationship between the true and observed nonconforming proportion in presence of inspection errors, is as follows:

$$\acute{p} = p(1-e_2) + (1-p)e_1 \tag{5}$$

Where $\acute{p}$ and $p$ represent the observed $p$ value and true value of nonconforming proportion respectively, while $e_1$and $e_2$ denote, respectively, the probability of classifying a conforming item as nonconforming and the probability of classifying a nonconforming item as conforming.

By considering inspection errors, the false alarm probability and subsequently the average run length, will be change. In-control ARL can be calculated as following:

$$ARL = \frac{1}{\alpha_{actual}} = \frac{1}{1-p(LCL \leq x)} \tag{6}$$

This equation can be calculated based on the $\acute{p}$ equation. Also, ANI needs to be changed and it can be written as:

$$ANI_{(}\acute{p}, e_1, e_2) = \frac{r}{\acute{p}} * \frac{1}{1-p(LCL \leq x)} \tag{7}$$

## 3. Economic model introduction

The proposed economic model is based on the model of Chan *et al*. (2003). The *r* parameter is selected as design parameter and it will be determined in order to minimize the average cost per item produced in a specific production cycle. Assumptions of the model have been elaborated in the following.

### 3.1. Assumptions

(1) After start of the process and before the production of first item, or between productions of two successive items, one or more assignable causes will appear with probability $\pi$, which will cause the nonconforming proportion $p_0$ of the process to jump to a larger value, $p_1$. This shift does not occur during the period of investigation. The probability of this jump to occur before producing $i_{th}$ item follows a geometric distribution:

$$p(i) = \pi(1-\pi)^{i-1} \tag{8}$$

(2) All items are inspected after production. All the nonconforming items are reworked. $\boldsymbol{C_{rw}}$ is reworking cost of one nonconforming item.

(3) When an out of control signal is appeared on the chart, then the cost needed to perform an investigation is $\boldsymbol{C_{inv}}$.

(4) The production will be continued through the period of an investigation and at this time, *N* items will be produced. During this period, there will not be any other investigations.

(5) If no assignable cause is detected, then the out of control alarm that causes this investigation is a false alarm and the production process will be continued.

(6) $\boldsymbol{C_{rec}}$ is the rectification cost when one or more assignable causes are detected after investigation.

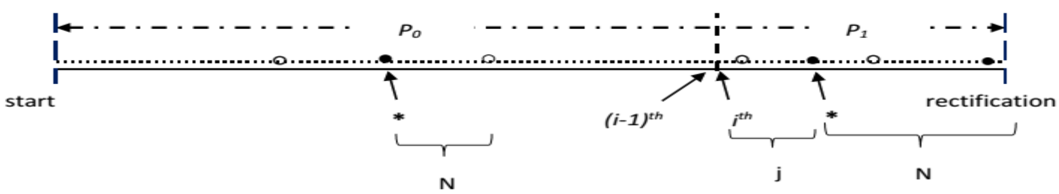


**Figure 1. Description of production cycle (Chan *et al*, 2003)**

According to Fig.1, the production cycle can be determined. A target nonconforming proportion $p$ jumps from $p_0$ to $p_1$ after the production of $(i-1)_{th}$ item. Including the $i_{\text{th}}$ item, altogether $j$ items are produced until an out of control alarm is observed. The small dots ''…'' denote production of items, the circle ''○'' denotes the production of a nonconforming item which does not give an out of control alarm, the heavy dots ''●'' denotes the production of a nonconforming item which gives an out of control alarm, and the star ''*'' denotes the start of an investigation (Chan *et al*, 2003).

Now the economic model is designed based on production cycle.

### 3.2. Notations

The parameters of the economic model are as following:

1- Design parameter (decision variable): $r$

2- Fixed parameters: $\pi$, $N$, $C_{rw}$, $C_{inv}$, $C_{rec}$, $C_1$, $C_2$, $C_3$, $e_1$, $e_2$

3- Process parameters: $p_0$, $p_1$, LCL, UCL, $\acute{p}_0$, $\acute{p}_1$

Notations:

$r$: required number of nonconforming items

$\pi$: probability that the nonconforming proportion jumps from $p_0$ to a larger value as $p_1$

$N$: the number of items produced during the period of investigation

$C_{rw}$: cost of reworking one nonconforming item

$C_{inv}$: investigation cost

$C_{rec}$: process rectification cost

$C_1$: the cost of one identified nonconforming item

$C_2$: the cost of classifying an item as conforming when it is nonconforming

$C_3$: the cost of classifying an item as nonconforming when it is conforming

$e_1$: the probability of classifying a conforming item as nonconforming

$e_2$: the probability of classifying a nonconforming item as conforming

$p_0$: in control nonconforming proportion

$p_1$: out of control nonconforming proportion

LCL: Lower control limit

$\acute{p}_0$: observed in control nonconforming proportion in the presence of inspection errors

$\acute{p}_1$: observed out of control nonconforming proportion in the presence of inspection errors

**Table 1. The value of fixed parameters**

| Fixed parameters | $C_{inv}$ | $C_{rw}$ | $C_{rec}$ | $C_1$ | $C_2$ | $C_3$ | $e_1$ | $e_2$ | $\pi$ | $N$ |
|---|---|---|---|---|---|---|---|---|---|---|
| **Value** | 0.5 | 50 | 400 | 5 | 10 | 7 | 0.01 | 0.05 | 0.2 | 10 |

### 3.3. Cost equations in economic model

Since $p(i) = \pi(1-\pi)^{i-1}$ thus, $\sum_{i=1}^{\infty} p(i) = 1 \quad and \quad \sum_{i=1}^{\infty}(i-1)\,p(i) = \frac{1-\pi}{\pi}$.

The different cost functions are obtained as following:

(1) The cost $C_I$ is incurred in order to **investigate** the process when an out of control alarm appears. The average number of out of control alarms that appear during the first $(i-1)$ produced items is equal to $\frac{i-1}{ANI(\acute{p}_0)}$. Nevertheless, the fraction $\left(\frac{ANI(\acute{p}_0)}{ANI(\acute{p}_0)+N}\right)$ of the alarms needs the investigation. Because when an out of control alarm is observed and the investigation is implemented, before the end of the investigation, there will not be any other investigation even though some other out of control alarms appeared.

Thus, during the first $(i-1)$ produced items, the average number of out of control alarms that need investigation is $\frac{i-1}{ANI(\acute{p}_0)+N}$ After the time that $p$ has jumped from $\acute{p}_0$ to an out of control alarm would appear which needs an investigation with the cost $C_{inv}$:

$$E[C_I] = C_{inv}\left(\sum_{i=1}^{\infty}\frac{(i-1)p(i)}{(ANI(\acute{p}_0)+N)}\right) + C_{inv} = C_{inv}\frac{(1-\pi)}{(\pi(ANI(\acute{p}_0)+N))} + C_{inv} \tag{9}$$

(2) The cost $C_R$ is incurred to **rework** all the nonconforming items found in the inspection: During the first $(i-1)$ inspected items, the nonconforming proportion of the items is $\acute{p}_0$. Hence, out of these $(i-1)$ inspected items, the average number of nonconforming items is equal to $(i-1)\acute{p}_0$, and these nonconforming items should be reworked. Such reworking process requires a cost:

$$C_{R1} = C_{rw}\sum_{i=1}^{\infty}(i-1)\,p(i)\acute{p}_0 = C_{rw}\frac{(1-\pi)}{\pi}\acute{p}_0 \tag{10}$$

When nonconforming proportion has jumped from $\acute{p}_0$ to $\acute{p}_1$ then total number of produced items is equal to $ANI(\acute{p}_1) + N$. Since proportion of nonconforming items is equal to $\acute{p}_1$, thus the number of nonconforming items is equal to $\acute{p}_1(ANI(\acute{p}_1) + N)$. Thus, the expected cost needed to rework nonconforming items is as follows

$$C_{R2} = C_{rw}(ANI(\acute{p}_1) + N)\acute{p}_1$$
(11)

Thus, the expected total cost of rework is as following

$$E[C_R] = C_{R1} + C_{R2} = C_{rw}\left(\frac{(1-\pi)\acute{p}_0}{\pi} + \acute{p}_1(ANI(\acute{p}_1) + N)\right)$$
(12)

(3) The cost $C_{rec}$ is incurred to **rectify** the process when one or more assignable causes are detected.

(4) The cost $C_{IEr}$ is referred to the cost related to the inspection errors in a cycle. The Inspection errors can be organized as follows:

1-$\left(\frac{1-\pi}{\pi}\right)p_0(1-e_2)C_1$: This term denotes the cost of detected nonconforming items when the process is in control.

2- $\left(\frac{1-\pi}{\pi}\right)p_0(e_2)C_2$: This term denotes the cost of classifying a nonconforming item as conforming when the process is in control.

3- $\left(\frac{1-\pi}{\pi}\right)(1-p_1)(e_1)C_3$: This term denotes the cost of conforming items classified as nonconforming when the process is in control.

4- $((ANI(\acute{p}_1)+N)p_1(1-e_2)C_1$: This term denotes the cost of detected nonconforming items when the process is out of control.

5- $((ANI(\acute{p}_1)+N)p_1(e_2)C_2$: This term denotes the cost of classifying a nonconforming item as conforming when the process is out of control.

6- $((ANI(\acute{p}_1)+N)(1-p_1)(e_1)C_3$: This term denotes the cost of conforming items classified as nonconforming when the process is out of control.

And finally,

$$C_{IEr} = \left(\frac{1-\pi}{\pi}\right)p_0(1-e_2)C_1 + \left(\frac{1-\pi}{\pi}\right)p_0(e_2)C_2 + \left(\frac{1-\pi}{\pi}\right)(1-p_1)(e_1)C_3 + (\ (ANI(\acute{p}_1) + N)p_1(1-e_2)C_1 + ((ANI(\acute{p}_1)+N)p_1(e_2)C_2 + \big((ANI(\acute{p}_1)+N)(1-p_1\big)(e_1)C_3 \qquad (13)$$

### 3.4. Objective function

The objective function is the average cost per produced item for one-sided *CCC-r* control chart. With regard to the cost parameters, the expected total cost in the cycle is $E[C_I] + E[C_R] + C_{rec} + C_{IEr}$. The expected value for the total number of items produced in a production cycle is:

$$E[(i-1)+j+N] = E(i-1) + ANI(\acute{p}_1) + N =$$

$$\sum_{i=1}^{\infty}(i-1)p(i) + ANI(\acute{p}_1) + N = \frac{1-\pi}{\pi} + ANI(\acute{p}_1) + N \qquad (14)$$

Finally, objective function is as following:

$$\underset{r}{Min}\ C_{avg} = \frac{C_{inv}\left(1+\frac{1-\pi}{\pi}\left(\frac{1}{ANI(\acute{p}_0)+N}\right)\right)+C_{rw}\left(\frac{1-\pi}{\pi}(\acute{p}_0)+\acute{p}_1(ANI(\acute{p}_1)+N)\right)+C_{rec}+C_{IEr}}{\frac{1-\pi}{\pi}+ANI(\acute{p}_1)+N} \qquad (15)$$

All the parameters are constant and by defining a search interval for decision variable of the model, the optimal solution is determined so that all related costs per one item, would be minimized. To illustrate the application of proposed model a numerical example is presented in next section.

## 4. The economic model analysis

### 4.1. Application of the model in an example

The Visual Basic is used to design an algorithm to find the optimum value of *r*, as decision variable within the range of 1 to 7. Since the model is designed for high quality processes, hence the values of $p_o = 0.001$ and $p_1 = 0.003$ are specified. The value of lower control limit is determined according to equation (3), and $\alpha$= 0.0027 is assumed as false alarm probability in this equation. The determined LCL in the presence of inspection errors is substituted in the objective function. To consider other performance aspects of control charts, two other criteria, Modified Producer Risk and Modified Consumer Risk, are calculated for different values of parameter *r* as follows:

$$Modified\ Producer\ Risk = \frac{1}{ANI\acute{p}_0} \qquad (16)$$

$$Modified\ Consumer\ Risk = 1 - \frac{1}{ANI\acute{p}_1} \quad (17)$$

The values of objective function, Modified Producer Risk, and Modified Consumer Risk for different values of parameter *r* are shown in Table 2.

**Table2. The values of objective function, Modified Producer, and Consumer Risks**

| *r* | Modified Producer Risk | Modified Consumer Risk | Objective Function |
|---|---|---|---|
| 1 | 0.010 | 0.988 | 4.538 |
| 2 | 0.004 | 0.995 | 2.064 |
| 3 | 0.001 | 0.998 | 0.779 |
| 4 | 0.00019 | 0.999 | 0.323 |
| 5 | 0.000017 | 0.99998 | 0.243 |
| 6 | 0.0000009 | 0.999999 | 0.236 |
| 7 | 0.00000003 | 0.99999996 | 0.235 |

As shown in Table 2, when *r*=7, minimum value of objective function is observed and in this case the Modified Consumer risk will be in its maximum value. Because of the high importance of the Modified Consumer Risk, the value of *r*=7 may not suitable despite of having the minimum value of objective function. The Modified Consumer Risk will have its lowest value for *r*=1 but its objective function in comparison with the other values is too large.

In order to choose the optimum value of *r* parameter, the AHP method is applied to estimate the levels of importance of each of the Modified Producer and Consumer Risk and objective function values, by designing and analyzing the matrixes of paired comparisons, as illustrated in Table 3. In the last column, there is the level of importance or weight of each criterion.

Analytic Hierarchy Process is one of the most comprehensive designed system for multi criteria decision making, since this technique can formulate problem in a hierarchical way and also consider quantitative and qualitative criteria in problem. So that it analyzes the attributes and criteria without the requirement of being based on a common scale for all of them.

This procedure involves different alternatives in decision making and has ability to sensitivity analysis on criteria and sub criteria. The most important advantages of this method are the use of paired comparison and showing decision compatibility and incompatibility.

Also, by using this method the criterion's and alternative's weight and ranking have been calculated easily while the other multi criteria decision making methods do not have this ability and consequently AHP should be used for this purpose.

Considering the nature of the problem in this research that need to compare between different alternatives and rank criteria, AHP method is chosen for this issue according to capabilities than other multi criteria decision making methods.

As it is shown in Table 3, in this case, the objective function represents 56.43% of the importance, followed by 13.12% of the Modified Producer Risk and 30.43% of the Modified Consumer Risk.

**Table 3. Matrix of paired comparisons of objective function, Modified Producer, and Consumer Risk**

| | **Objective Function** | **Modified Producer Risk** | **Modified Consumer Risk** | **Weight** |
|---|---|---|---|---|
| **Objective Function** | 1 | 4 | 2 | **0.5643** |
| **Modified Producer Risk** | 0.25 | 1 | 0.4 | **0.1312** |
| **Modified Consumer Risk** | 0.5 | 2.5 | 1 | **0.3043** |

In addition, paired comparisons are made for each of the 7 alternatives in relation with each 3 criteria in Table 2. The used scales to make the paired comparisons for Modified Producer Risk and Modified Consumer Risks, and Objective Function values are listed as table 4 to 6.

**Table 4. AHP used scales in the paired comparatives of Modified Producer Risks**

| **Importance** | **Definition** | **Explanation** |
|---|---|---|
| 9 | i is extremely preferred to j | $\frac{Pr_i}{Pr_j} < 10^{-5}$ |
| 7 | i is very strongly preferred to j | $10^{-5} \leq \frac{Pr_i}{Pr_j} < 10^{-4}$ |
| 5 | i is strongly preferred to j | $10^{-4} \leq \frac{Pr_i}{Pr_j} < 10^{-3}$ |
| 3 | i is moderately preferred to j | $10^{-3} \leq \frac{Pr_i}{Pr_j} < 10^{-2}$ |
| 2 | i is preferred to j | $10^{-2} \leq \frac{Pr_i}{Pr_j} < 10^{-1}$ |
| 1 | i is equally preferred to j | $1 \geq \frac{Pr_i}{Pr_j} > 10^{-1}$ |

**Table 5. AHP used scales in the paired comparatives of Modified Consumer Risks**

| **Importance** | **Definition** | **Explanation** |
|---|---|---|
| 9 | j is extremely preferred to i | $1 + 10^{-2} < \frac{Cr_i}{Cr_j} < 1 + 10^{-1}$ |
| 7 | j is very strongly preferred to i | $1 + 10^{-3} < \frac{Cr_i}{Cr_j} < 1 + 10^{-2}$ |
| 5 | j is strongly preferred to i | $1 + 10^{-4} < \frac{Cr_i}{Cr_j} < 1 + 10^{-3}$ |
| 3 | j is moderately preferred to i | $1 + 10^{-5} < \frac{Cr_i}{Cr_j} < 1 + 10^{-4}$ |
| 1 | j is equally preferred to i | $1 + 10^{-6} < \frac{Cr_i}{Cr_j} < 1 + 10^{-5}$ |

**Table 6. AHP used scales in the paired comparatives of Objective Function**

| Importance | Definition | Explanation |
|---|---|---|
| 9 | i is extremely preferred to j | $\frac{OF_i}{OF_j} < 0.1$ |
| 7 | i is very strongly preferred to j | $0.1 < \frac{OF_i}{OF_j} < 0.3$ |
| 5 | i is strongly preferred to j | $0.3 < \frac{OF_i}{OF_j} < 0.5$ |
| 3 | i is moderately preferred to j | $0.5 < \frac{OF_i}{OF_j} < 0.7$ |
| 2 | i is preferred to j | $0.7 < \frac{OF_i}{OF_j} < 0.95$ |
| 1 | i is equally preferred to j | $1 > \frac{OF_i}{OF_j} > 0.95$ |

To illustrate the procedure, paired comparison matrix of Modified Producer Risks and the levels of importance for each alternative according to this criterion are shown in Table 7.

**Table 7. Matrix of paired comparisons of Modified Producer Risks**

| Modified Producer Risk | $pr_1$ | $pr_2$ | $pr_3$ | $pr_4$ | $pr_5$ | $pr_6$ | $pr_7$ | Weight |
|---|---|---|---|---|---|---|---|---|
| $pr_1$ | 1 | 1 | 1 | 0.333 | 0.333 | 0.143 | 0.111 | **0.042** |
| $pr_2$ | 1 | 1 | 1 | 0.5 | 0.5 | 0.2 | 0.111 | **0.049** |
| $pr_3$ | 1 | 1 | 1 | 1 | 0.5 | 0.2 | 0.143 | **0.056** |
| $pr_4$ | 3 | 2 | 1 | 1 | 0.5 | 0.333 | 0.2 | **0.082** |
| $pr_5$ | 3 | 2 | 2 | 2 | 1 | 0.5 | 0.333 | **0.125** |
| $pr_6$ | 7 | 5 | 5 | 3 | 2 | 1 | 1 | **0.275** |
| $pr_7$ | 9 | 9 | 7 | 5 | 3 | 1 | 1 | **0.370** |

In addition, by analyzing this procedure for Modified Consumer Risk and Objective Function values of Table 2, the results in Table 8 are obtained. For example, the last column in Table 7 is substituted in the first column of Table 8. Finally, Table 9 is obtained by multiplying the level of importance of each criterion to its corresponding column's cells in order to choose the optimum value of *r* parameter value by calculating the weight of each alternatives. As shown in table 9, *r*=7 is selected as the maximum weight and *r*=7 is the optimum value.

**Table 8. Weights of each alternative according to paired comparisons**

| *r* | Modified Producer Risk | Modified Consumer Risk | Objective Function |
|---|---|---|---|
| 1 | 0.042 | 0.452 | 0.019 |
| 2 | 0.049 | 0.225 | 0.038 |
| 3 | 0.056 | 0.160 | 0.074 |
| 4 | 0.082 | 0.065 | 0.150 |
| 5 | 0.125 | 0.047 | 0.194 |
| 6 | 0.275 | 0.025 | 0.262 |
| 7 | 0.370 | 0.025 | 0.262 |

**Table 9. Final weight of each alternative**

| *r* | Weight |
|---|---|
| 1 | 0.154 |
| 2 | 0.097 |
| 3 | 0.098 |
| 4 | 0.115 |
| 5 | 0.140 |
| 6 | 0.192 |
| 7 | **0.204** |

## 4.2. Sensitivity Analysis

### 4.2.1. Sensitivity analysis of criteria's weights

In order to analyze the weight of each criterion the variation of their weights are examined and the results are shown in Table 10. By increasing the weights of Modified Consumer Risk, *r*=1 is selected as the optimum value because the Modified Consumer Risk in this case is minimum. In addition, when the weights of Modified Producer Risk or Objective Function increase, then *r*=7 is chosen as optimum value but when the weight of consumer risk increase then *r*=1 is selected as optimal.

We can conclude that optimal solution is determined based on the tradeoff between risks and costs.

**Table 10. Sensitivity analysis of criteria's weight**

| Exp. No | weight of Modified Producer Risk | weight of Modified Consumer Risk | weight of Objective Function | *r* parameter | LCL |
|---|---|---|---|---|---|
| 1 | **0.1312** | **0.3043** | **0.5633** | **7** | **1812** |
| 2 | 0.1 | 0.45 | 0.45 | 1 | 4 |
| 3 | 0.05 | 0.8 | 0.15 | 1 | 4 |
| 4 | 0.3 | 0.4 | 0.3 | 7 | 1812 |
| 5 | 0.3 | 0.6 | 0.1 | 1 | 4 |

### 4.2.2. Sensitivity Analysis of parameters

In this section, to evaluate the model performance, the effects of variations of the effective parameters such as cost and inspection errors are studied. In the other words, the value of a specified parameter has been changed while keeping all of the other parameters constant. The optimal design parameter, objective function, Modified Consumer Risk and Modified Producer

Risk have been obtained according to these different scenarios. The result in Table 11 can be used to analyze the sensitivity of the model.

As stated in Table 11, the variation of modified Producer Risk is directly related to the variation of $p_0$ but as $p_1$ value is decreased, and then the Modified Producer Risk increased. Furthermore, when $p_0$, $e_1$ and $e_2$ values are decreased (increased) the Modified Consumer Risk is increased (decreased). It is noted that increasing the value of $p_1$, $p_0$, $e_1$ and $e_2$ leads to decrease $r$ and LCL values. The variations of Objective function are directly related to all cost parameters changes as expected.

**Table11. Sensitivity Analysis of parameters**

| Experiment | | $p_0$ | $p_1$ | $e_1$ | $e_2$ | $c_1$ | $c_2$ | $c_3$ | Selected *r* (within the range of 1 to 7) | LCL | Objective function | Consumer risk | Producer Risk | Weight of selected *r* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **0.001** | **0.003** | **0.01** | **0.05** | **5** | **10** | **7** | **7** | **1812** | **0.2356** | **0.99999996** | **3.16375E-08** | **0.204** |
| Analyzed Parameter | $p_0$ | **0.0005** | 0.003 | 0.01 | 0.05 | 5 | 10 | 7 | 5 | 1858 | 0.236 | 0.999999998 | 1.29E-09 | 0.148 |
| | | **0.002** | 0.003 | 0.01 | 0.05 | 5 | 10 | 7 | 7 | 1426 | 0.2361 | 0.999999 | 1.30E-06 | 0.271 |
| | $p_1$ | 0.001 | **0.0015** | 0.01 | 0.05 | 5 | 10 | 7 | 7 | 1812 | 0.153 | 0.9999998 | 2.17198E-07 | 0.216 |
| | | 0.001 | **0.006** | 0.01 | 0.05 | 5 | 10 | 7 | 6 | 1348 | 0.401 | 0.99999993 | 4.7944E-08 | 0.174 |
| | $e_1$ | 0.001 | 0.003 | **0.005** | 0.05 | 5 | 10 | 7 | 7 | 1812 | 0.206 | 0.99999 | 1.01459E-05 | 0.224 |
| | | 0.001 | 0.003 | **0.02** | 0.05 | 5 | 10 | 7 | 4 | 562 | 0.308 | 0.999994 | 5.43657E-06 | 0.190 |
| | $e_2$ | 0.001 | 0.003 | 0.01 | **0.025** | 5 | 10 | 7 | 7 | 1812 | 0.235 | 0.99999997 | 2.8563E-08 | 0.195 |
| | | 0.001 | 0.003 | 0.01 | **0.1** | 5 | 10 | 7 | 6 | 1348 | 0.237 | 0.999999 | 1.03341E-06 | 0.172 |
| | $c_1$ | 0.001 | 0.003 | 0.01 | 0.05 | **2.5** | 10 | 7 | 7 | 1812 | 0.230 | 0.99999996 | 3.16375E-08 | 0.1991 |
| | | 0.001 | 0.003 | 0.01 | 0.05 | **10** | 10 | 7 | 7 | 1812 | 0.250 | 0.99999996 | 3.16375E-08 | 0.190878217 |
| | $c_2$ | 0.001 | 0.003 | 0.01 | 0.05 | 5 | **5** | 7 | 7 | 1812 | 0.235 | 0.99999996 | 3.16375E-08 | 0.190878217 |
| | | 0.001 | 0.003 | 0.01 | 0.05 | 5 | **20** | 7 | 7 | 1812 | 0.237 | 0.99999996 | 3.16375E-08 | 0.190878217 |
| | $c_3$ | 0.001 | 0.003 | 0.01 | 0.05 | 5 | 10 | **3.5** | 7 | 1812 | 0.201 | 0.99999996 | 3.16375E-08 | 0.190878217 |
| | | 0.001 | 0.003 | 0.01 | 0.05 | 5 | 10 | **14** | 7 | 1812 | 0.306 | 0.99999996 | 3.16375E-08 | 0.190878217 |

### 4.3. Model evaluation in different scenarios

To verify the effectiveness of proposed model, some different non conforming farctions are studied within the range of 1 to 5 for *r* parameter as decision variable. The value of lower control limit in the presence of inspection errors, objective function, Modified Producer Risk and Modified Consumer Risk as 3 criteria to choose optimal *r* parameter are calculated for different values. According to explained procedure in section 4-1 by using AHP method the final weight of each parameter *r* is calculated.

As shown in Table 12 and as expected in high quality processes, when nonconforming fraction is too little then higher value for r is chosen. By increasing the value of p0 and p1 the value for parameter r reduces.

**Table 12. Model analysis in different nonconforming fractions**

| | | | | | |
|---|---|---|---|---|---|
| **Scenario 1** | **$p_0$=0.0007** **$p_1$=0.002** | Final weight of *r* parameter | **Scenario 2** | **$p_0$=0.003** **$p_1$=0.007** | Final weight of *r* parameter |
| | r=1 | 0.17 | | r=1 | 0.171 |
| | r=2 | 0.105 | | r=2 | 0.118 |
| | r=3 | 0.103 | | r=3 | 0.124 |
| | r=4 | 0.147 | | r=4 | 0.149 |
| | **r=5** | **0.216** | | **r=5** | **0.182** |
| **Scenario 3** | **$p_0$=0.008** **$p_1$=0.02** | Final weight of *r* parameter | **Scenario 4** | **$p_0$=0.04** **$p_1$=0.08** | Final weight of *r* parameter |
| | **r=1** | **0.189** | | **r=1** | **0.186** |
| | r=2 | 0.115 | | r=2 | 0.141 |
| | r=3 | 0.132 | | r=3 | 0.132 |
| | r=4 | 0.151 | | r=4 | 0.156 |
| | r=5 | 0.181 | | r=5 | 0.174 |

## 5. Conclusion

The *CCC-r* control chart is one of the best methods to monitor high quality processes. Most of the researches on *CCC-r* chart has been based on the assumption of perfect inspection but this assumption is rarely met in practical conditions. In this paper, due to important effects of inspection errors on the performance of high quality processes, the presence of inspection errors is studied in the case of one-sided *CCC-r* charts. Modified Producer and Consumer Risk are analyzed in the presence of inspection errors too. Then, an economic model is presented with the aim of minimizing average cost per item produced in order to choose the optimum value of *r* parameter. In addition to the objective function, two Modified Risks are selected as the decision making criteria. So, AHP method is used to determine levels of importance of each criterion in the specified range of *r* parameter. To analyze the model performance, a numerical example is examined and sensitivity analysis on model's parameters and criteria's weights is experimented.

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