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A Bootstrap Interval Robust Data Envelopment Analysis for Estimate Efficiency and Ranking Hospitals

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Abstract

Data envelopment analysis (DEA) is one of non-parametric methods for evaluating efficiency of each unit. Limited resources in healthcare economy is the main reason in measuring efficiency of hospitals. In this study, a bootstrap interval data envelopment analysis (BIRDEA) is proposed for measuring the efficiency of hospitals affiliated with the Hamedan University of Medical Sciences. The proposed method is capable to consider uncertainty and sampling errors. The inputs of this model include total number of personals, number of medical equipment, and number of operational beds. Also, outputs consist of number of inpatients, number of outpatients, number of special patients, bed-day, and bed occupancy rate. First, we estimate the efficiency by applying original DEA that does not consider any uncertainty and sampling error; then we utilize RDEA that considers uncertainty and after that we use BRDEA that consider both uncertainty and sampling error with an adaptation of bootstrapped robust data envelopment analysis and could be more reliable for efficiency estimating strategies.

Keywords: data envelopment analysis, robust optimization, hospital management, bootstrap.

1. Introduction

Hospitals are the original and the last section of the health care systems. Thus, evaluating the performance of hospitals is vital. In healthcare applications of DEA, hospitals often have various specializations. Data envelopment analysis (DEA) in several contexts including education systems, health care units, agricultural production, transportation and military has a great applicability. This method was first presented by Charnes, Cooper, and Rhodes (1978). DEA is a nonparametric method based on mathematical procedure according to linear programming, which is used to evaluate relative efficiency of multiple homogeneous decision making units (DMUs) with the same inputs and outputs.

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It can determine the set of weights that maximizes the efficiency of a DMU, it allow to incorporate multiple inputs and outputs into a single value, needles to convert them into a common unit of measure (Cooper, Ruiz, and Sirvent 2007) . DEA determines the relative efficiency of a group of DMUs that use the same types of input and produce the same types of output. This model depends on a number of factors, including the number of inputs and outputs in relation to the number of units, the type (variable or constant) of returns-to-scale which is assumed (VRS and CRS, respectively) and, more generally, the particular dataset that is under the investigation (Angulo-Meza and Lins 2002; Podinovski and Thanassoulis 2007).

Robust optimization, is able to tackle the decision makers prefer risk aversion or service level function, and has made a string of solutions that are gradually less sensitive to realizations of the data in a scenario set. The optimal solution presented by a robust optimization model is called robust if input data change it remains close to the optimal. This is considered as a solution robustness.

Bootstrapping inclusive the duplicated simulation of the data generation process and the application of the main estimator for each simulated sample so that the resulting guess imitation the sampling distribution of the main estimator. A bootstrapping procedure for DEA was developed by Simar and Wilson (1998). This approach has been successfully used to decreases the sample bias in a wide range of econometric studies which has applied a smoothed distribution of revenue values to generate bootstrap instance of revenue. In this paper, a bootstrapped robust DEA (BRDEA) model is developed, which solves the perturbation and instance error problem. The first method assumes that the output parameters are uncertain and can be changed in a distance. The robust DEA (RDEA) is then applied to find the robust solutions in the first level of the algorithm. Next, with this guess that the sampling error, a bootstrap RDEA is used to reduce the sample bias. The presented BRDEA model is used to find the exact efficiency ranking scores for the Hamedan's hospital. Moreover, information of 16 hospital under supervision of Hamedan University of medical sciences were collected by means of collect field information technique and then with DEA technique a lower and upper interval is defined for each hospital. Due to the sensitivity to the ratio of DEA, the basic model, CCR, is robust. Afterwards, we use it for the closer the performance obtained 16 specimens to the whole society we use bootstrap methods.

The reminder of paper is organized as follow: In Section 2, we review a related literature of DEA and relevant methods. In Section 3, we describe the DEA models including DEA, robust DEA (RDEA), interval DEA (IDEA) and Bootstrapped robust DEA (BRDEA). Section 4 contains the results of measure an efficiency for 16 hospitals. Finally, conclusion remarks of this paper is provided in Section 5.

2. Literature review

Rezaee and Karimdadi (2015) proposed a new approach for evaluating hospitals. In this approach, hospitals are classified into different groups and each group is equivalent to one province. It causes hospitals in each category (province) must be evaluated in homogenized environment. The data on 288 Iranian hospitals grouped under 31 provinces are used to show this model. Inputs of this model is total number of personals, number of medical equipment, number of operational beds and output include number of inpatients, number of outpatients, number of special patients, bed-day, and bed occupancy rate. The results show that the efficiency scores are changed a lot when hospitals are evaluated in own groups. Chowdhury and Zelenyuk (2016) analyzed production performance of hospital services in Ontario (Canada), by various review its key determinants. Specially, they used DEA at the first stage to estimate performance scores and then used truncated regression estimation with double bootstrap to test the significance of explanatory variables. Inputs of this model is

Administrative Staff Hours, Nursing Hours, Staffed Beds, Medical-surgical supplies costs, Non-medical supplies costs, Equipment Expenses and output include Ambulatory Visits, Case-mix weighted Inpatient Days. Dotoli et al. (2015) presented a novel cross-efficiency fuzzy DEA technique for assessment different DMUs under uncertainty. The proposed method is applied to the performance assessment of healthcare systems in a region of Southern Italy. Input of data include the number of doctors, the number of nurses, the number of other employees and administrative staff, the total number of available beds and outputs is the total number of discharges, the total yearly days of hospitalization of all patients, the total number of surgeries. To assessment the performance of several DMUs while dealing with uncertain input and output data, the presented decision making technique employs triangular fuzzy numbers. A fuzzy triangular efficiency is accompanied to each DMU through a cross assessment obtained by a compromise between suitably chosen objectives. Results are then defuzzified to provide a ranking of the DMUs.

To evaluate the performance of providers in a service industry such as health care, it is so important that the measurement tools take into account both the efficiency and the quality of service provided. Fiallos et al. (2015) developed a DEA model to help assessment the performance of emergency department (ED) physicians at a partner hospital. The model contains efficiency measures as inputs and quality measures as outputs. Patients were grouped according to their presenting complaint and ED of doctors were assessed on each group, separately. The original dataset used in this paper was comprised of 36,441 visits classified under the 25 highest content complaints. considering consultation with CHEO management, it was determined that the four major resources consumed by ED physicians in this period suitable for input that performing their duties are ED time, laboratory tests, diagnostic imaging and specialist consults. The task of an ED doctor is to stabilize and assess a patient as quickly as possible and then to correctly determine whether the patient requires admittance to the hospital. Hence, the output is Rate of non-return patient visits.

Misiunas et al. (2015) proposed establishment DEA for solving the problem of effectively preprocessing a dataset containing a large number of performance metrics and also for preprocessing the data to remove outliers and therefore, preserve monotonicity as well as in order to reduce the size of the dataset used to train the ANN. DEANN methodology is executed via the problem of predicting the functional status of patients in organ transplant operations. In this work, DMUs are individual transplant records. Pre and post-transplant variables considered as inputs and outputs. Lam (2014) introduced a new mixed integer linear programming (MILP) models for determining the most efficient decision making unit (DMU) in data envelopment analysis that has an objective similar to that of the super efficiency model. It has one input (total cost) and three outputs, namely, the number of teaching units (TU), regular patients (RP), and severe patients (SP) in 15 hypothetical hospitals. In this model entire of the inputs and outputs are considered to be strictly positive. Mitropoulos, Talias, and Mitropoulos (2015) proposed a combined application of a chance constrained DEA (CCDEA) model that is integrated with a stochastic mechanism from Bayesian techniques in order to combine statistical properties in a DEA analysis. The suggested method is conducted in two basic steps. In the first step they used Bayesian techniques on the data set and in the second step they used the sampling distribution of these measures as an approximation to the finite sample distribution. This paper discussed the statistical advantages of this method using cross-sectional data from a sample of 117 Greek public hospitals. Inputs in this paper are Doctors, Other personnel, Beds, Operating cost and output is Inpatient admissions, Outpatient visits. Kazley and ozkan (2008) Using two research approaches including a retrospective, cross-sectional design and a first differencing repeated measures design, find limited evidence that EMRs can improve hospital efficiency. In this research using DEA to investigate the relationship between hospital electronic medical record (EMR) use and efficiency in a national sample of acute care hospitals. Dataset includes the American Hospital Association (AHA), Health Information Management Systems Society (HIMSS), and Case Mix Index. The inputs are non-physician full-time equivalent employees (FTEs), beds set up and staffed, capital assets, and non-labor operating expenses. Outputs include case mix adjusted admissions and outpatient visits.

In fact, in healthcare applications of DEA, hospitals often have different specializations (in terms of treatments available), and may also have other non-clinical outputs reflected their engagement in research, education and community services (Ozkir and Demirel 2012). One of the main problems in measuring the relative efficiency of a telecommunication unit compared with other similar units is the uncertainty on input/output data. Sadjad and Omrani (2011) proposed a bootstrapped RDEA model to solve this problem that is capable of handling different issues such as the uncertainty in data or sampling errors.

Despite of Crisp input and output data which are fundamentally indispensable in traditional DEA, the input and output data in real-world problems are often vague. Shokouhi et al. (2010) proposed an approach based on a RDEA model that the input and output parameters are constrained to be within an uncertainty set with additional constraints based on the worst case solution with respect to the uncertainty set. Jalali Naini and Nouralizadeh (2012) proposed a DEA model in two-stage for analyzing the effects of entrance deregulation on the efficiency in the insurance market. The first stage is a robust optimization approach due to overcome the sensitivity of DEA results to any uncertainty in the output parameters and in the second one, they proposed a comprehensive statistical analysis using generalized estimating equations.

One of the privileges of DEA is to determine benchmarks for DMUs. However, determination of the benchmarks is the result of past performance of DMUs. In other words, the benchmarks do not provide any recommendation for improvement of future efficiency of DMUs. On the other hand, in dynamic DEA models often no DMU gets the efficiency score of unity. In this case, although we can rank the DMUs, we cannot introduce an efficient DMU and benchmarks. To overcome these shortcomings Yousefi, Shabanpour, Fisher, and Saen, (2016) proposed a dynamic ideal DMU using dynamic DEA and scenario-based model of robust.

Landete, Monge, & Ruiz, (2017) proposed robust efficiency scores for the scenario in which the profile of the inputs/outputs to be included in the DEA model is modelled with a probability distribution. This probabilistic approach allows us to obtain three different robust efficiency scores.

In order to achieve sustainable energy systems that involves complex decision-making processes Martín-Gamboa, Iribarren, García-Gusano, and Dufour, (2017)superscription the combination of life-cycle methods and DEA for the sustainability assessment of energy systems. The main focus of this article is to explore and transparent potentials of these combined approaches within sustainability-oriented MCDA in the area of energy. These potentials are mainly associated with the profitable of sustainability benchmarks for decision-makers and the wide accessibility of DEA models and LC methods. In this article proposed a novel methodological framework given the increasing role of MCDA tools in energy scenario analysis and the initiated trends found in LCDEA. The LCDEA concept emerges as a hopeful methodology to evaluate and ranking futuristic scenarios in energy planning, thereby facilitating decision-making processes in to a sustainable energy future.

Although health care reforms may improve efficiency at the macro level, less is known regarding their effects on the utilization of health care personnel. accordingly, Johannessen, Kittelsen, and Hagen, (2017)using panel analysis and non-parametric DEA to study physician productivity defined as patient treatments per full-time equivalent (FTE) physician. In the DEA, cost efficiency did not change in the study period, but allocative efficiency decreased significantly.

Given the importance of health care, Kalantary and Azar (2008) proposed a measurement of Tehran hospitals efficiency utilizing Crisp DEA and Robust DEA, and compared the results to show the effects of uncertain data on the performance of DEA outputs. The obtained results show that the robust DEA approach can be reasonably reliable method in order to efficiency estimation and ranking strategies. Omrani (2013) introduced a new model based on robust DEA with fuzzy perturbation to measure efficiency of DMUs. This model is intended fuzzy uncertainties for both input and output data supply an efficient fuzzy RDEA (FRDEA) that for solving problem, FRDEA is formulated as a nonlinear programming and incorporated as a parametric model. In many industries, the supplier's efficiency measurement often becomes the most significant concern for manufacturers. Hence, Hafezalkotob et al. (2014) used robust optimization approach of DEA (RDEA), and computed the relative efficiency of the suppliers. They also introduced the most efficient supplier as a benchmark due to large amounts of uncertainty regarding the suppliers' measurements. Since the Prevailing RDEA models are based on mirrored interval DEA models, Shokouhi et al.(2010) proposed a modified RDEA model that calculates the empirical distribution for the interval efficiency for the case of a random number of variables based on a flexible formulation for the number of variables perturbed. Shokouhi and Shahriari (2014) proposed a new model for measuring the efficiency of DMUs. In their presented model, the inputs and outputs take fixed values for each DMU. This model is integrated measuring all the DMUs performance, simultaneously.

The incorporation of probabilistic operators aims to diversify search directions or to escape from trapping in local optima. For this reason some researches have used the meta-heuristics features which result in non-deterministic output in solutions that vary from one run to another. Lu (2015) took into the account not only an average but also standard deviation of an algorithm's output for evaluating relative efficiencies of a set of algorithms develops RDEA models. This model wants to evaluate a set of distinct configurations, and uses a genetic algorithm and a set of parameter settings of a simulated annealing heuristic. O'Neill et al. (2008) proposed the first taxonomy of hospital efficiency studies that use DEA. They did a systematic review of 79 studies published. Comparisons show significant differences with respect to important study characteristics such as type of DEA model selected and choice of input and output categories. This approach can be used by policy makers and researchers to review past, and assemble new, DEA models. Table 1 provides a summarization about the related literature.

Table 1. Summarized related literature

Authors	Methodology	Sample	Inputs	Outputs
Rezaee and Karimdadi (2015)	multi-group DEA	288 Iranian hospitals	1.Total number of personals 2.Number of medical equipment 3.Number of operational beds	1.Number of inpatients 2.Number of outpatients 3.Number of special 4.patients 5.Bed-day 6.Bed occupancy rate
Chowdhury and Zelenyuk (2015)	DEA and bootstrap	hospital services in Ontario (Canada)	1.Administrative Staff Hours 2.Nursing Hours 3.Staffed Beds 4.Medical-surgical supplies costs 5.Non-medical supplies costs 6.Equipment Expenses	1.Ambulatory Visits 2.Case-mix weighted Inpatient Days
Dotoli et al.(2015)	cross-efficiency fuzzy DEA	healthcare systems in a region of Southern Italy	1.the number of doctors 2.the number of nurses 3.the number of other employees and administrative staff 3.the total number of available beds	1.the total number of discharges 2.the total yearly days of hospitalization of all patients 3.the total number of surgeries.
Fiallos et al. (2015)	SBM-VRS model	comprised of 36,441 visits classified under the 25 highest content complaints	1.ED time 2.laboratory tests 3.diagnostic imaging 4. Specialist consults.	1.Rate of non-return patient visits
Misiunas et al. (2015)	DEANN	a large number of performance metrics and an even larger number of records is crucial	1.Pre transplant variables	1.post transplant variables
Lam (2014)	mixed integer linear programming DEA	15 hypothetical hospitals	1.total cost	1.the number of teaching units (TU) 2.regular patients (RP) 3.severe patients (SP)
Mitropoulos et al.(2014)	DEA	117 Greek public hospitals	1.Doctors 2.Other personnel 3.Beds 4.Operating cost	1.Inpatient admissions 2.Outpatient visits
Kazley and ozkan (2008)	DEA and Window analysis	-American Hospital Association (AHA) -Health Information Management Systems Society (HIMSS) -Case Mix Index	1.non-physician full time equivalent employees (FTEs) 2.beds set up and staffed 3.capital assets 4. Non-labor operating expenses.	1.case mix adjusted admissions 2.outpatient visits

3. Data envelopment analysis (DEA)

Data envelopment analysis (DEA) was first proposed by Charnes, Cooper, and Rhodes (1978). DEA has been used in several contexts including education systems, health care units, agricultural production, transportation and military logistics. The application of the method in the transport sector is wide-spread. With respect to the work of Markovits-Somogyi (2011) some key features of DEA are summarized as follows:

- DEA is used to measure the efficiency of homogeneous units called decision making units (DMUs)
- DEA is a non-parametric method;
- DEA is a fractional mathematical programming method. However, it can be converted into a linear Programming model and solved by a standard LP solver;
- DEA generalizes the concept of a single-input, single-output technical performance measure of Farrell to the multiple-input and multiple-output to a virtual input;
- DEA is a method focusing on frontiers instead of central Orientation;
- DEA determines the relative performance at a time over all other DMUs by finding the most favorable weights from the viewpoint of that, Goal, DMU;
- Alternative for making each inefficient DMU can be done by projecting them into the efficient border.

The purpose of DEA methodology is to assess the relative performance of units that are comparable which are mentioned as Decision Making Units (DMU) (Seiford 1996). These DMUs are described according to some inputs and outputs. Score of relation of performance assigned to each DMU is defined as the ratio of weighted sum of outputs to weighted total of inputs. The common DEA framework was presented to evaluate the relative performance of a group of entities. The model can be defined by:

$$E_j = \max \sum_{k=1}^{p} \omega_K Y_{kj} / \sum_{i=1}^{q} t_i X_{ij} \tag{1}$$

$$\sum_{k=1}^{p} \omega_K Y_{ik} / \sum_{j=1}^{q} t_j X_{ij} \leq 1, \tag{2}$$

$$i = 1,2,\dots,s, \qquad \omega_k, t_j \geq 0 \tag{3}$$

where, X_{ij} is the value of the i-th input (i=1,2⋯⋯,q) of the j-th DMU, and Y_{kj} is the value of the kth output (k=1,2⋯⋯, p) of the j-th DMU, t_i is the weight given to the i-th input, ω_k is the weight given to the k-th output, and n is the total number of DMUs.

Research on DEA models with imprecise data may be classified into three main categories, namely, fuzzy DEA, interval DEA, and robust DEA. In this study, we provide a ranking for hospitals affiliated with the Hamedan University of Medical Sciences. The data were collected by field study and library. The information used by the questionnaire, in table format output, with titles, number of surgeries, the number of outpatients, and beds and inputs including doctors (the number of general practitioners, specialist doctors, Intern doctors in teaching hospitals, the number of resident doctors in Teaching hospitals) and personnel (nurses, nurse aid and other personnel). This Information obtained through directly referring to hospitals and health deputy statistics were collected from the central university campus. Using BRDEA to measure the effectiveness of the advantage that hospitals located in a region with the use of computer analysis in other words, the determined and compared with each other.

Table 2. Summary of inputs and outputs

	Maximum	Minimum	Mean
Input			
Doctor	[103,105]	[6,8]	[29.3125,31.9375]
Personnel	[672,680]	[45,12]	[210.75,213.687]
Beds	[452,460]	[20,28]	[121.8125,127.375]
Outputs			
Outpatient	[384169, 384207]	[41155,41200]	[100067.1875,100170.43]
Surgery	[19321,19406]	[0,10]	[3012.625,3211.31]

3.1. Interval DEA

In this section we introduce the basic concepts of a DEA model with vague data. in the interval DEA models, consider that all input and output data cannot be exactly obtained Due of uncertainty. They are only known within the upper and lower bounds indicated with distance $x_{ij}\epsilon[x_{ij}^{l}, x_{ij}^{u}]$, $y_{rj}\epsilon[y_{rj}^{l}, y_{rj}^{u}]$ respectively. The following pair of linear programming models was presented to measure the upper and lower bounds of the performance of DMUs:

$$Max\theta_0^u = \sum_{k=1}^{q} \omega_k y_{k0}^u, \tag{4}$$

Subject to: (5)

$$\sum_{i=1}^{p} t_i x_{i0}^l = 1 \tag{6}$$

$$\sum_{k=1}^{q} \omega_k y_{k0}^u - \sum_{i=1}^{p} t_i x_{i0}^l \leq 0 , \tag{7}$$

$$\sum_{k=1}^{q} \omega_k y_{kj}^l - \sum_{i=1}^{p} t_i x_{ij}^u \leq 0 , \qquad \forall\, j , k, k \neq j \tag{8}$$

$$t_i, \omega_k \geq \varepsilon \qquad \forall\, j , k \tag{9}$$

$$Max\theta_0^l = \sum_{k=1}^{q} \omega_k y_{k0}^l, \tag{10}$$

Subject to: (11)

$$\sum_{i=1}^{p} t_i x_{i0}^{u} = 1 \tag{12}$$

$$\sum_{k=1}^{q} \omega_k y_{k0}^{l} - \sum_{i=1}^{p} t_i x_{i0}^{u} \leq 0\,, \tag{13}$$

$$\sum_{k=1}^{q} \omega_k y_{kj}^{u} - \sum_{i=1}^{p} t_i x_{ij}^{l} \leq 0\,, \qquad \forall\, j\,, k, k \neq j \tag{14}$$

$$t_i, \omega_k \geq \varepsilon \qquad \forall\, j\,, k \tag{15}$$

3.2. Robust DEA

The robust DEA approach is based on the robust counterpart optimization (RCO) approach (Bertsimas and Sim 2004; Ben-Tal and Nemirovski 1999) which describes uncertain data using an uncertainty set, and aims to maximize a DMU's worst-case relative efficiency with respect to that uncertainty set. Sajjadi and Omrani (2008) proposed robust DEA models with consideration of uncertainty on output parameters for the performance assessment of electricity distribution companies. Shokouhi et al. (2014) developed robust DEA models which consider uncertainty on both input and output parameters. Note that both of the works focused on the adaptation of Bertsimas and Sim (2004) approach to the CCR model. More recently, Sadjadi et al. (2011) applied the RCO approach of Ben-Tal and Nemirovski (1999) to the super-efficiency DEA model of Andersen and Petersen (1993), which was also based on the CCR model. The non-linear RDEA model formulated as follow:

$$Max\ C \tag{16}$$

Subject to: (17)

$$-\sum_{k=1}^{q} \omega_k y_{k0}^{u} + z_0 G_0 + \sum_{k=1}^{q} L_{k0} \leq -C \tag{18}$$

$$\sum_{k=1}^{q} \omega_k y_{r0}^{u} - \sum_{i=1}^{p} t_i x_{i0}^{l} + z_0 G_0 + \sum_{k=1}^{q} L_{k0} \leq 0 \tag{19}$$

$$\sum_{k=1}^{q} \omega_k y_{rj}^{l} - \sum_{i=1}^{p} t_i x_{ij}^{u} + z_j G_j + \sum_{k=1}^{q} L_{kj} \leq 0 \quad \forall j, k, k \neq j \tag{20}$$

$$z_j + L_{kj} \geq t_i\left(y_{kj}^{u} - y_{kj}^{l}\right) \qquad \forall\, r, j \tag{21}$$

$$z_j, L_{kj} \geq 0 \qquad \omega_k, t_i \geq \varepsilon \tag{22}$$

3.3. Bootstrapped robust DEA (BRDEA) model

One of the main factor considered in DEA is that the input/output data are usual collected through a sampling system and the efficient frontier obtained from this model cannot necessarily be indicative of the actual one. In actually, the sample size is small and the existence frontier differs from the real frontier. Namely, the true production frontier is unknown and the estimation of the efficiency measure can be calculated only using the observed, or actual, input–output combinations.

To overcome the sampling error problem, using the bootstrap DEA that proposed by Simar and Wilson [31], [32]. The bootstrap possible incorporating the effects of other external factors, which could possibly play important roles in the DMU efficiency [33], and to overcome any existing perturbations in the data, the RDEA is applied instead of the original DEA and the method for each re-sampling is as follows. Given the observed output and input ratio, the resample data are constructed in two steps. First, the frontier inputs are estimated and bootstrap pseudo inputs are generated by replicating the DGP in using the estimated frontier inputs and the pseudo efficiencies drawn using some estimates for the distribution. Second, the bootstrapped efficiency estimate is obtained by evaluating the measure from the original input relative to the bootstrap estimate of the frontier.

The following steps summarize the method for the bootstrapped robust DEA algorithm :

Initialization: Use the RDEA Model 5 and calculate efficiency scores $z_1, \dots, z_n$ for n companies.

1. Let $\beta_i^*, \dots, \beta_n^*$ be the non-smooth sample generated with replacement from $z_1, \dots, z_n$.
2. Smooth the sampled values formulated as follow:

$$z_i^* = \begin{cases} \beta_i^* + h\varepsilon_i^* & \text{if } \beta_i^* + h\varepsilon_i^* \leq 1 \\ 2 - \beta_i^* - h\varepsilon_i^* & \text{otherwise} \end{cases} \qquad \varepsilon_i^* \sim N(0,1) \tag{23}$$

   where h is the bandwidth of a standard normal kernel density and ε_i^* is a random deviation from the standard normal. The value of bandwidth of the density estimate h is method of minimizing an approximation to the mean weighted integrated square error.
3. Calculate the final z_i^* by adjusting the smoothed sample value formulated as follows:

$$z_i^* = \bar{\beta}_i^* + \frac{z_i^* - \bar{\beta}_i^*}{\sqrt{1 + h^2/\hat{\sigma}_z^2}} \tag{24}$$

$$\bar{\beta}_i^* = \sum_{i=1}^{n} \bar{\beta}_i^*/n \tag{25}$$

4. Adjust the original inputs using the ratio $x_i^* = x_i \times \frac{z_i}{z_i^*}$.
5. Fix problem of the RDEA Model 5 with the justify inputs to obtain z_i^* for each company.
6. Repeat steps (2)–(6) for B times to yield B new RDEA efficiency scores z_{ib}^* where *i*=1…*n* and *b*=1…*B*.
7. Calculate the bootstrap bias and bias-corrected estimator formulated as follows:

$$Bias\, z_i = B^{-1} \sum_{i=1}^{B} \hat{z}_i^* - \hat{z}_i \tag{26}$$

$$\hat{z}_i^* = \hat{z}_i - Bias\ \hat{z}_i = 2\hat{z}_i = B^{-1}\sum_{i=1}^{B}\hat{z}_i^* \tag{27}$$

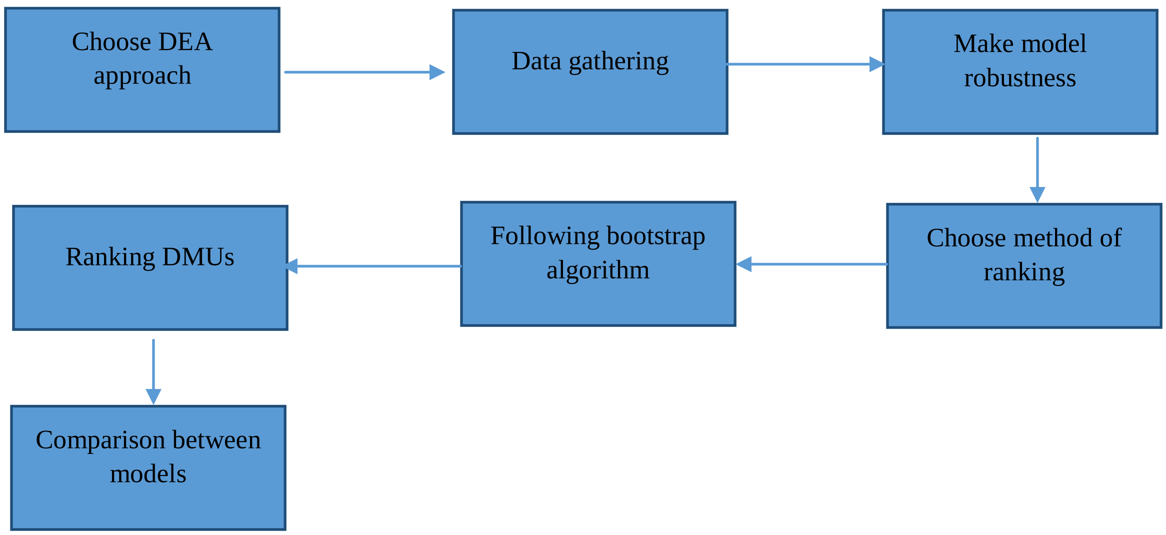

Figure1. steps of my approach

4. Numerical results

In this section, by using the upper and lower bound for 16 hospitals from Hamedan state, measure an efficiency for each hospital.

First we solved a DEA model without any uncertainty after that we solved the RDEA and BRDEA bootstrapped robust DEA model) are applied to investigate the hospitals' efficiency scores. The BRDEA model is based on the original bootstrap algorithm proposed by Simar and Wilson (2000). To show the effect of robustness we use 3 different gammas, G=0, 1, 2 and the result show in below table. After determining the efficiency of units for ranking them in terms of efficiency we use MRA approach. MRA method is in the form below:

We assume $A_i=[a_i^l, a^u{}_i]$ To be interval of efficiency for each DMU. Then, we define m and w as follows:

$$m(A_i)=(a_i^u+a^l{}_i)/2 \tag{28}$$

$$w(A_i)=(a_i^u-a^l{}_i)/2 \tag{29}$$

We use the equation below to find maximum regret approach and whichever has the lowest maximum regret approach will be chosen as first rank.

$$R(A_i)=\max_i[\max_j\{m(A_j)+w(A_j)\}-(m(A_j)-w(A_j)),0];\ i=1,2,3,\ldots,n \tag{30}$$

Then we delete first rank and repeat the above process for remaining DMUs until we rank all DMUs.

Table 3. The results obtained from DEA and RDEA approach

	G=0			*G*=1			*G*=2		
DMU	L	U	rank	L	U	rank	L	U	Rank
1	0.347923	0.378614	14	0.343813	0.378547	13	0.343808	0.378546	1
2	0.927014	1	2	0.91565	1	2	0.915639	1	2
3	0.506078	0.549699	11	0.499965	0.549604	11	0.499958	0.549603	3
4	0.289064	0.321667	16	0.286595	0.321602	16	0.286584	0.321593	4
5	0.31707	0.375787	15	0.317415	0.375686	15	0.317385	0.375649	5
6	0.53827	0.585831	10	0.531923	0.585729	10	0.531915	0.585726	6
7	0.702703	0.757646	8	0.693466	0.757522	8	0.693461	0.757526	7
8	0.422401	0.480792	12	0.420223	0.480683	12	0.420199	0.480657	8
9	0.583888	0.656446	9	0.579792	0.656307	9	0.579766	0.65628	9
10	0.715825	0.867931	4	0.719203	0.867676	4	0.71912	0.867569	10
11	1	1	1	0.999715	1	1	0.999629	1	11
12	0.735272	0.816473	5	0.728761	0.816311	5	0.728737	0.816289	12
13	0.697911	0.791409	7	0.693916	0.791233	7	0.693879	0.791193	13
14	1	1	3	0.98658	1	3	0.986575	1	14
15	0.727785	0.805427	6	0.720977	0.80527	6	0.720955	0.805251	15
16	0.321108	0.39548	13	0.32344	0.395357	14	0.323398	0.395302	16

As you see in Table 3 the base DEA model both DMU 11 and 14 for upper bound and lower bound have the best efficiency score and DMU 2 has efficient just for upper case and other DMUs have efficiency rating less than one. For accomplish robust DEA, consider the perturbation equal to 0.05 just for output and run model for G=1 and G=2. After we use RDEA model for the efficiency scores are for all DMUs decreasing and none of DMUs efficient for both lower and upper efficacies bound and ranking are the same except for DMU 1 and 16. After this we use BRDEA result of this show in Table 4.

Table 4. The result obtained from RDEA and BRDEA with *G*=2

	G=2			BRDEA with G=2			Bias		SD	
DMU	L	U	rank	L	U	rank	L	U	L	U
1	0.343808	0.378546	13	0.341091	0.375526	13	0.002717	0.003019	0.002105	0.002256
2	0.915639	1	2	0.908404	1	2	0.007235	0	0.005605	0
3	0.499958	0.549603	11	0.496007	0.545219	11	0.00395	0.004384	0.00306	0.003276
4	0.286584	0.321593	16	0.28432	0.319028	16	0.002264	0.002565	0.001754	0.001917
5	0.317385	0.375649	15	0.314878	0.372653	15	0.002508	0.002996	0.001943	0.002239
6	0.531915	0.585726	10	0.527712	0.581054	10	0.004203	0.004672	0.003256	0.003491
7	0.693461	0.757526	8	0.687982	0.751484	8	0.005479	0.006042	0.004245	0.004515
8	0.420199	0.480657	12	0.416879	0.476823	12	0.00332	0.003834	0.002572	0.002865
9	0.579766	0.65628	9	0.575185	0.651045	9	0.004581	0.005235	0.003549	0.003912
10	0.71912	0.867569	4	0.713438	0.860649	4	0.005682	0.00692	0.004402	0.005171
11	0.999629	1	1	0.99173	1	1	0.007898	0	0.006119	0
12	0.728737	0.816289	5	0.722979	0.809778	5	0.005758	0.006511	0.004461	0.004865
13	0.693879	0.791193	7	0.688396	0.784882	7	0.005483	0.006311	0.004248	0.004716
14	0.986575	1	3	0.978779	1	3	0.007795	0	0.006039	0
15	0.720955	0.805251	6	0.715258	0.798828	6	0.005696	0.006423	0.004413	0.004799
16	0.323398	0.395302	14	0.320843	0.392149	14	0.002555	0.003153	0.00198	0.002356
Mean	0.610063	0.673824		0.605243	0.669945					
Max	0.999629	1		0.99173	1					
Min	0.286584	0.321593		0.28432	0.319028					

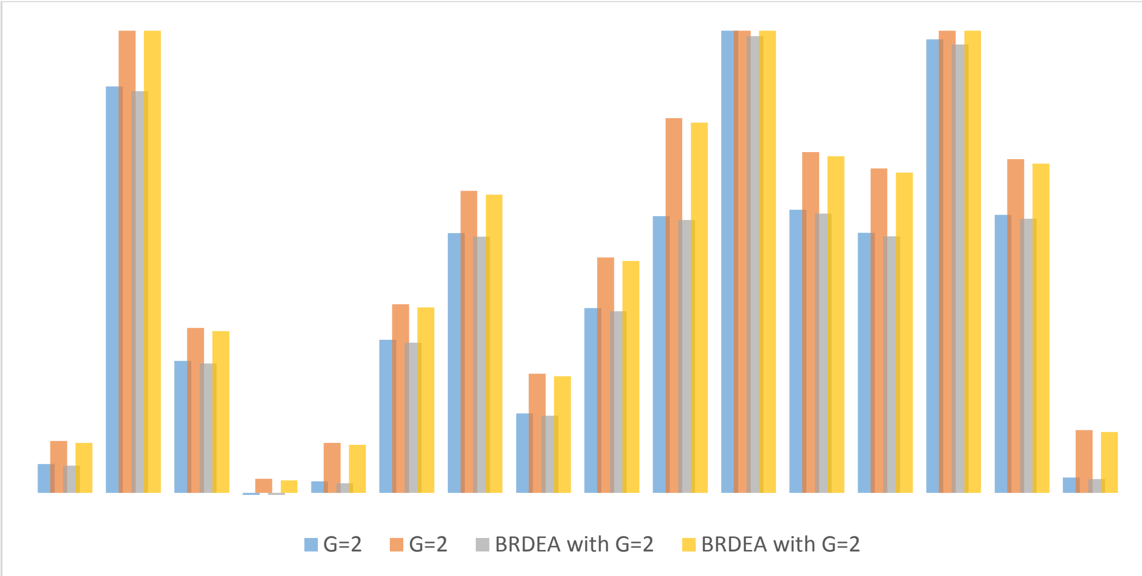

Figure2. Comparison between lower and upper bound for RDEA and BRDEA with *G*=2

As you see in Table 4, after bootstrapping with 1000 replacement all the efficiency scores decreasing and the mean of the bootstrapping efficiency is less than the original DEA efficiencies but the ranking are the same.

5. Conclusion

DEA is one of the most important and common methods for estimating assessment of DMUs The analysis compares the relative efficiency of organizational "units" such as bank branches, hospitals, vehicles, shops and other cases where units perform similar execution of work. The first assumptions with the main DEA are that there is no error or noise in input/output data and the information of all DMUs is available to be considered. However, as previously discussed, the goal of this paper is to propose a bootstrapped robust DEA model to achieve the correct efficiency and ranking for the telecommunication companies. The proposed Bootstrap DEA model can be used to overcome the disruption in the data and the sampling error for many real world case studies and is used achievement for the hospitals. To consider the effect of the confusion and the sampling error, we were used the robust optimization and bootstrap methods. In this study both robust concept and interval data were considered simultaneously for the first time. The performance of the suggested method is shown using the data from 16 hospitals in Iran. The results show that the RDEA efficiency scores are biased upwards and the bias-corrected bootstrap efficiency scores from a BRDEA model are lower. This indicates that the RDEA efficiency scores have to be designed to be biased for small samples. Also, the RDEA model results operation an average of [0.61,0.673], while the BRDEA generates an average bias-corrected score of [060,066]. The results show that the consideration of a confusion in the data and sampling error, and applying a bootstrapped robust data envelopment analysis model can be more reliable for efficiency assessment strategies. For future studies we suggest following directions to extend the current study:

- Ranking hospitals with common weight of DEA model
- Ranking hospitals with supper efficient DEA model
- Changing in constraint to show which index is more important

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