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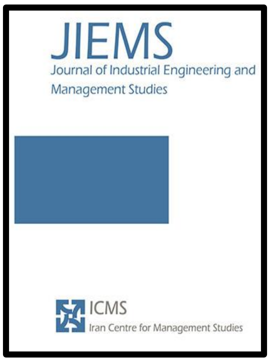

Designing the optimum plan for regenerating the pedestrian network of historic districts using bi-level programming (Case study: Historical-Cultural district of Tehran, Iran)

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Abstract

Motorized transportation systems in the urban areas witnessed huge developments in the infrastructures thanks to the advances in various aspects of technology. This urbanization revolution has its own pros and cons. The resulting dominance of vehicles has limited the presence of people in public places and their participation in social activities, threatening the human based lifestyle of the cities. Historic districts are of most affected areas which withstand the unwanted consequences of such an experience. These areas play a substantial role in urban activities by providing great social activity and walking zones for pedestrians. Hence, in recent years, urban management has paid attention to this endanger regions in order to sustain and enhance their properties by introducing some pedestrianization plan as urban regeneration policies. To design an effective plan, it is necessary to figure out how people behave in response to their environment. Pedestrian modeling is the key to the problem and is studied in the past few decades, mostly in microscopic scale. In addition, a logical decision-making process is required to choose the option with the best outcome in this complex system, considering financial limits of strategic urban planning. In this paper, a macroscopic multi-class user equilibrium pedestrian assignment algorithm is proposed to anticipate the route choice behavior of the pedestrians in a network, and a decision making platform for the pedestrian network design is presented using bi-level mathematical mixed-integer programming and genetic algorithm. The presented model determines the best possible projects to be implemented on the network, considering the constraints of the historic districts. The model brings forward an intelligent framework to help the urban planners in spending the minimum cost, while maximizing some predefined objectives. The proposed method is applied to solve the problem in a test network and in a real case scenario for the historic district of the city of Tehran. The results prove the validity and the efficiency of the algorithm.

Keywords: Complex System; Pedestrian Flow Modeling; mathematical modeling; Bi-level programming; multi-objective optimization; Genetic Algorithm.

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1. Introduction

In the last decades, the fast growing population and rapid developments of the cities caused some undesirable problems such as heavy traffic streams and social-environmental issues. These side effects are more critical in the historic districts, which are usually located at the central part of the urban areas and play a significant role in preserving the identity of the cities. One of the main characteristics of historic districts is the physical movement of pedestrians and their participation in social activities. The performance of these areas is highly vulnerable to the subject. To cope with the problem, more pedestrian-oriented trends are introduced in urban planning. In order to come up with these walking-friendly environment schemes, urban planners need to understand and analyze the behavior of the pedestrians in response to their environment. This calls for including the pedestrian modeling in transport studies which was until recently exclusively devoted to motorized transports as the main system of the urban transportation (Kuzmyak et al., 2014). As a result, many researchers has paid attention to the pedestrian modeling in the last years. There is a comprehensive literature on pedestrian modeling which can be classified into three main categories: microscopic, mesoscopic and macroscopic models. Microscopic approach models pedestrian with high details of their movements, distinguishes individuals and accounts for their physical-spatial interactions. On the other hand, the macroscopic method considers the pedestrians as a whole and models their behavior according to the parameters of flow, density and velocity as functions of time and space (Bellomo et al., 2012). Mesoscopic models come between the other two and take into account the velocity distribution.

First attempts were focused on microscopic simulation of pedestrian dynamics. Cellular automata (CA), agent based models, queuing models, social force models and magnetic force models are among the most famous methods in pedestrian microscopic modeling. CA modeling is a grid based discrete microscopic model, first introduced and formulated by Von Neuman (1963). To apply the CA in the pedestrian modeling, the 2-D floor space is divided into analogous units, called as cells, and pedestrians are allowed to move from each cell to another according to the condition of its four adjacent units. Many researches such as Blue and Adler (2001), Roland and Kirely (2006), Gou et al. (2012), Pereira et al. (2013) and Abdelghany et al. (2016) have studied and developed models based on the CA. Agent based models explain the walking of a crowd as a result of micro level behavior among a set of interacting agents. Each agent may not necessarily have a perfect knowledge of the whole system and interacts with the other agents and his environment based on behavioral rules and his cognitive abilities. The models provide for assessment of different hypothesizes of agent attributes and their behavioral rules. Another advantage of these models lies in the possibility of defining intelligent agents who have the ability to control and make their individual decisions according to their own perceptions. This allows for more realistic simulation of the edestrian movements. Some pedestrian modeling software such as Legion (Still, 2000), NetLogo (Wilensky, 2000) and Repast Simphony (North et al., 2013) are developed using agent based approach. Collins et al. (2015) and Enciso et al. (2016) are among those who studied agent based approach for the pedestrian modeling. In the queuing models, the environment is considered as a queuing network with its source nodes (rooms), transportation nodes and destination nodes (doors). The process of the network is built upon the movements of the pedestrians in the environment as the queuing network costumers. The approach is suitable and most applied for evacuation plans in emergency situations. Watts (1987) studied queuing approach for evacuation analysis. Drager et al. (1992) developed EVACSIM software for pedestrian evacuation simulation using queuing model. Løvås (1994) presented a stochastic queuing model and developed an evacuation model in 1995. He extended his previous proposed model for different way-finding models in 1998. Social force model, introduced by Helbing and Molnar (1995), considers the movement of a pedestrian as the superposition of three main forces: attraction by destination (self-driving force), repulsive force of other pedestrians and the repulsive force of the obstacles. The model incorporates the socio-psychological and physical forces to anticipate the behavior of a crowd. Helbing et al. (2000) extended the model and proposed a method proper for escape panics. Parisi and Dorso (2005), Lin et al (2006) and Song et al. (2006) presented models for the evacuation scenarios. Langeston et al. (2006) and Qu et al. (2014) extended the model to consider the rotation of pedestrians. Yuen et al. (2012) included overtaking behavior into the model. Gou et al. (2016) developed a force model to analyze uni- and bi-directional pedestrian flow in limited view condition. Despite the ability of these models in predicting the pedestrian behavior, the underlying assumptions oversimplify the dynamics of the pedestrian movements (Al-Habashna and Wainer, 2016). Magnetic force model is another microscopic approach for analyzing the movements of the pedestrians, introduced by Okazaki and Matsushita (1993). The proposed model is based on Coulomb's law where the pedestrians and the obstacles are assumed as positive charged poles, and the destinations as the negative ones. Each pedestrian tries to reach his destination by the attraction force between different magnetic poles, while keeping his distance from other pedestrians and obstacles as a result of the repulsive magnetic force. All the mentioned microscopic models are capable of predicting the behavior of a single pedestrian, providing valuable information on the individual behavior. However, some of these models suffer from the heavy computational effort, due to their complexity. The mesoscopic models often consider individuals, but evaluate interactions among groups of the pedestrians. Hanisch et al. (2003) and Tolujew and Alaca (2004) discussed mesoscopic approach for the pedestrian modeling.

Macroscopic models do not present pedestrians as individuals, but instead, describe them using average quantities such as density and mean velocity. These models are consisted of a system of partial differential equations, defining the relationships between average speed, flow, and density as a function of location and time. Macroscopic models are mostly inspired from the equations governing fluid dynamics, resulting into expressing the crowd behavior in terms of aggregate. Helbing (1992), Hoogendoorn and Bovy (2002), Daamen (2004), Daamen et al. (2005), Colombo and Rosini (2005), Colombo et al. (2010) and Bruno et al. (2011) studied and developed macroscopic models. Reviews on the macroscopic models are given by Karmanova (2013), Twarogowska et al. (2014) and Gupta and Pundir (2015).

All the above models are only applicable to a corridor, an intersection or at most for some crowded places such as subways, railway stations, airports, stadiums and museums. Optimistically, a few of them may be able to solve the problem for an area as big as a medium size city block. Nevertheless, they lack the ability to anticipate the distribution of the pedestrian traveling flow in a certain large-scale passing network. It is noted that sidewalks of a region are somehow connected together and consist a systematic network of passages, not small enough to be analyzed by the above models. Historic districts have high density textures, covering a wide area with considerable pedestrian demand. To design an effective pedestrianization plan, one should be able to anticipate the distribution of the demand on the network. This requires a macroscopic origin-destination pedestrian assignment model. Such a model is not yet proposed in the literature. Utilizing the structures of all the existing models, the goal cannot be achieved. In fact, a new platform and definition of the problem should be established. In this paper, by thinking out of the box, we propose a model inspired from the motorized transportation modeling to construct an appropriate tool for the problem.

Another important topic is the utilization of all the available pedestrian models in the urban planning. All the mentioned models are more or less capable of analyzing the behavior of the pedestrians in a defined condition. Hence, it is possible to anticipate the reactions of the pedestrians to a desired change of the network. Disregarding the incapability of the available models in a real world scenario for a relatively large-scale network, the urban planners come across a challenging question when trying to improve some pre-defined objectives in the urban areas, especially in historic district where there are some additional limitations. The question is how to choose among numerous sets of alternatives (projects) with limited financial sources, to get the best possible result. Because of the high complexity of such a system and the limits of the human mind, it is mandatory to evaluate every possible option to find the optimum one. The total enumeration of the vast solution space impose some computational burden and may be practically impossible. Accordingly, it is required to devise a supplementary tool beside the analyzing model (Vermuyten et al. 2016). This problem is mathematically equivalent to the problem known as the Network Design Problem (NDP) in the literature of the motorized transportation. The problem is proven to be a NP-hard problem and is widely studied. A popular method to deal with the problem is to define a bi-level platform using metaheuristic methods as the supplementary tool.

The main purpose of this paper is to tackle three drawbacks in the literature: 1- No model exists to anticipate the distribution of the pedestrians with different behavior patterns in a large scale network. 2- There is no systematic decision making platform to choose the optimize option of urban projects in a sidewalk network, considering the limitations and objectives. In fact, the urban planners lay the schemes based on their experience and knowledge. The effectiveness of the process is bounded by the constraints of the human mind. 3-The decisions are not made systematically, since their effects are not exactly and quantitatively measured. Considering these problems, we first aim to propose a macroscopic origin-destination pedestrian model to consider the heterogeneity of the travelers and applicable to the large-scale networks. Second, we present a model as a decision making structure for urban planners. In this paper, a bi-level programming model is presented in which the lower level is a multi-class user equilibrium pedestrian assignment algorithm and the upper level problem is a multi-objective optimization model based on the Non-dominated Sorting Genetic Algorithm (NSGA-II). The paper is structured as follows. In the next section, the pedestrian network design problem and the proposed model are presented. Section 3 describes the framework and algorithmic design of the proposed model. In order to evaluate the performance of the method, numerical examples are provided in section 4. Finally, some concluding remarks are given in the last section.

2. Mathematical formulation

In this section, a mathematical model is presented as a decision making platform for urban pedestrianization planning. Nowadays, urban planners have difficulties for allocating limited resources of strategic urban planning to improve the walkability in particular areas like historic districts. They usually face totally different options. They may choose between widening the passage, modifying the pavement, improving the lightning, expansion of the greenness or any combination of these actions. Adding up for all the sidewalks and nodes in a network, an immense solution space is created. As mentioned before, this problem is equivalent to the conventional transportation network design problem and is categorized as a non-deterministic polynomial-time hard problem. There are generally three main categories of solution methods for the problem: Branch and Bound, sensitivity analysis and bi-level programming. The Branch and Bound method struggles with the large scale problems and becomes inefficient and time-consuming. The sensitivity analysis is not suitable for the problems with too many local optimum solutions in a non-convex space. In the other hand, the bi-level programming is superior to the two others and is the most appropriate choice, considering the properties of the problem. Hence, a bi-level programming model for Pedestrian Network Design Problem (PNDP) is presented. The model can be stated as follows:

$$[PNDP]:\begin{cases} ULP: \; Min \; (F(\mu), \mu) \\ s.t. \quad \mu \in \eta \\ \\ LLP: \; h(\mu) = \{\arg\min G(\mu, h(\mu))\} \\ s.t. \quad h \in H \end{cases} \tag{1}$$

where $(F(\mu), \mu)$ is the set of objective functions of the upper level problem (ULP), μ is the set of decision variables of the problem and $h(\mu)$ is the solution of the lower level problem (LLP) based on μ. η and H are the feasible space for μ and h, respectively. The ULP is a multi-objective optimization model which seeks a feasible μ and the LLP is a user equilibrium macroscopic origin-destination pedestrian assignment problem to estimate the pedestrian flows for any given μ. This model is similar to the Stackelberg game (1934). The leader (urban planners as system manger) sets the decision variables on the network and followers (pedestrians) response by choosing their routes according to the dictated situation.

2.1. Multi-class pedestrian assignment as the LLP

In this paper, a macroscopic multi-class pedestrian flow distribution model is presented. The original idea of the proposed model is derived from the last step of the well-known four-stage motorized modeling, where an origin-destination (OD) demand matrix is assigned to a directed graph as the network based on the user equilibrium of Wardrop (Wardrop, 1952). According to the Wardropian law, the travel cost of all used paths for a specific OD pair should be equal to the minimum cost and equal or less than the cost of the unused paths. In the other words, no user (driver) can reduce his travel cost by changing his route unilaterally. This principle corresponds to the Nash equilibrium state in the game theory as the stabilized state of the competitive game (Grange and Munoz, 2009). This process is known as the traffic assignment or network loading. Extending the assumptions to the pedestrian flow, the underlying concept of the proposed model is formed. It is noted that some former studies have discussed the similarity of the pedestrian modeling to the conventional four stage modeling (Clifton et al. 2016a-2016b), but mainly focused on the demand estimation step. Adopting the assignment procedure is not yet addressed in the literature.

However, using the method for pedestrian modeling, some precautions should be in mind. There are two main differences between motorized and pedestrian modeling. First, in contrary to the motorized modeling where the route choice behavior of drivers is mainly derived from the travel time, environmental qualities have noticeable influence on the perception of travel cost for pedestrians (Saelens and Handy, 2008; Ewing and Ceravero, 2010). The second difference is the undeniable diversity of the users in the pedestrian modeling compared to the other. In motorized transportation modeling, drivers are assumed to follow a certain pattern (time-volume function); thus can be fairly considered as identical users of a single class. But the behavior of a pedestrian is heavily based upon the purpose of the trip (Kerridge, et al., 2001). A pedestrian with a work or personal business purpose is mostly concerned with the traveling time, while those who travel with the recreational purpose may prefer a longer route with higher levels of environmental qualities and visual appeals. Hence, it is of importance to consider the variety of travels in the pedestrian modeling. In fact, considering all the pedestrians to have similar behavior in macroscopic modeling does not fit to reality. Therefore, we present a macroscopic user equilibrium multi-class pedestrian assignment model (MPAM) where it is possible to consider different types of pedestrians. Assume *G=(N,A)* to be a graph representing the pedestrian passage network with

A as the set of sidewalks and *N* as the set of the nodes. Consider $I = \{i\}$ as set of OD pairs in the network and $d = \{d_i\}$ as the pedestrian demand matrix in which each element is assigned to OD pair *i*. For each OD pair *i*, there exist a set of path K_i and each path is denoted by $k \in K_i$. The MPAM can be stated by a system of non-linear equations as below:

$$\begin{cases} \sum_{k \in K_i} f_k^m = d_i^m & \forall i \in I, m \in M & (2) \\ f_k^m.\left(T_k^m(v, Q(\mu), \mu) - u_i^m\right) = 0 & \forall i \in I, k \in K_i, m \in M & (3) \\ T_k^m(v, Q(\mu), \mu) \geq u_i^m & \forall i \in I, k \in K_i, m \in M & (4) \\ f_k^m \geq 0 & \forall k \in K_i, m \in M & (5) \\ v_a^m = \sum_{i \in I} \sum_{k \in K_i} f_k^m.\delta_{ak}^m & \forall a \in A, m \in M & (6) \\ v_a = \sum_{m \in M} v_a^m & \forall i \in I, k \in K_i, m \in M & (7) \\ T_k^m = \sum_{a \in A} c_a^m(v_a, Q_a(\mu), \mu).\delta_{ak}^m + \sum_{n \in N} c_n^m(Q_n(\mu)).\delta_{nk}^m & \forall i \in I, k \in K_i, m \in M & (8) \end{cases}$$

Where $M = \{m\}$ is the set of pedestrian classes, f_k^m is the flow of class *m* on the path *k* and d_i^m is the demand of the class *m* between the OD pair *i*. T_k^m and u_i^m are the travel cost of the class *m* in the path $k \in K_i$ and minimum travel cost of the class *m* for the OD pair *i*, respectively. $Q_a = \{q_a(\mu)\}$ and $Q_n = \{q_n(\mu)\}$ are the set of environmental qualities in the sidewalk *a* and the node *n*, respectively. $Q = \{Q_a, Q_n\}$ is the vector of the environmental qualities in the network. The details of quantifying the qualities and its relation to the decision variables are described in Appendix A. v_a^m and $c_a^m(v_a, Q_a(\mu), \mu)$ represent the pedestrian volume and the travel cost of class *m* on the link (directed-sidewalk) *a*, in that order. $c_n^m(Q_n(\mu))$ is the travel cost of the class *m* in the node *n*. δ_{ak}^m is an indicator with the value of 1 if the arc *a* is located in the path *k* of the class *m*, and 0 otherwise. Similarly, δ_{nk}^m is an equivalent term for the node *n*. Equation (2) guarantees the demand conservation constraint. Equations (3) and (4) satisfy the Wardrop principle and equation (5) shows the non-negativity of pedestrian flows.

In this paper, the pedestrians are divided into two classes with different traveling purposes: work and leisure. The pedestrian of each class walk through the network with their own perception of travel cost. This distinction appears in the form of the function c_a^m, as shown below:

$$c_a^m = \alpha^m.\bar{t}_a(v_a, \mu) - \beta^m.\overline{Q}_a(\mu) \qquad (9)$$

Where $\bar{t}_a$ and $\overline{Q}_a(\mu)$ are the normalized walking time and normalized integrated environmental quality of the sidewalk *a*, respectively. α^m and β^m are two weighted coefficients which determine the importance of each term of the equation (9) in the travel cost on each link for the class *m*. In a comparison point of view, the value of β^m for leisure purpose trips is higher than the travels with the purpose of work, and opposite statement exists for the coefficient α^m.The travel time of the arc *a*, t_a, is derived from the fundamental diagrams of the pedestrian flow theory. The details and equations are given in the appendix B. moreover, the travel cost of class *m* in the node *n*, c_n^m, is calculated as follows:

$$c_n^m = \alpha^m.d_n^{mk} - \beta^m.\overline{Q}_n(\mu) \tag{10}$$

Where d_n^{mk} is a constant normalized delay time for the mode *m* in the path *k* at the node *n*, and $\overline{Q}_n(\mu)$ is the normalized integrated environmental quality for the node *n*, respectively.

It is noted that the vector of decision variables, μ, is set to assign multiple levels for improving a variable, not only a binary state of selecting or rejecting the project. Furthermore, the variables can be divided into two main category:

1- Widening the width of sidewalks, $\mu^W = \{\mu_a^w\}$, where $W = \{w\}$ is the set of feasible widening options on the links.

2- Enhancing the qualities of both the sidewalks and nodes, $\mu^R = \{\mu_a^r, \mu_n^r\}$ where $R = \{r\}$ is the set of pre-defined environmental indices considered for different levels of improvements.

2.2. A novel algorithm for solving MPAM

Although there are many algorithms in the literature for solving a single class assignment, dealing with multi-class assignment brings up more theoretical and computational complexity. In the interest of solving the proposed MPAM, we applied a single-class path-based complementary traffic assignment algorithm in an iterative process inspired from the Ortuzar and Willumsen (1990). The proposed scheme is depicted in the figure 1.

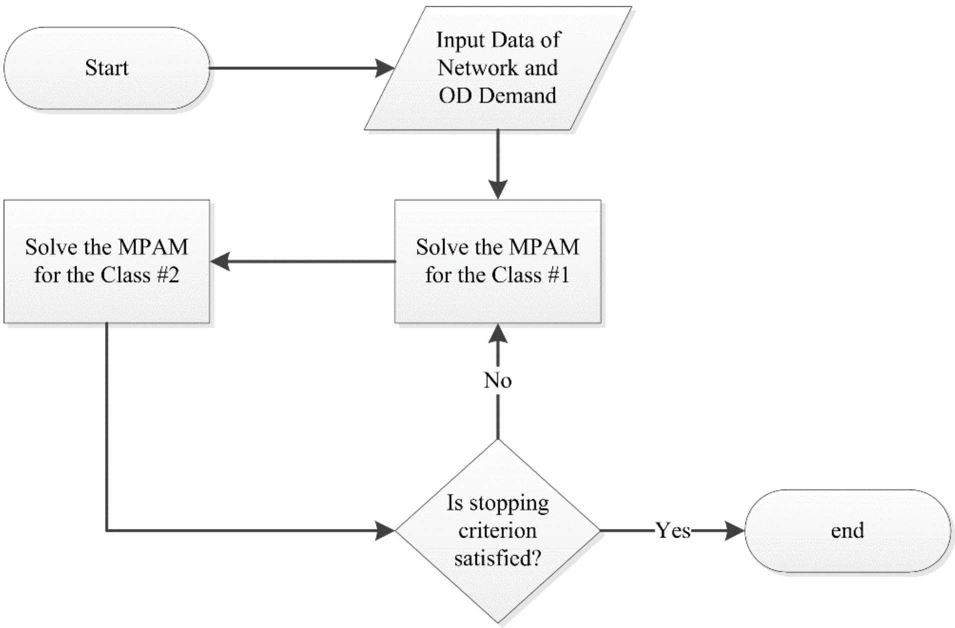

Figure 1. The proposed method for solving the MPAM

As shown in figure 1, the procedure starts with solving the MPAM for class #1 (leisure or work) based on a complementary traffic assignment algorithm and obtains the equilibrium pedestrian flows, ignoring the presence of class #2. Then, the class #2 is assigned to the network considering the class #1 pedestrian flows as fixed volumes of arcs. Afterward, the flows of class #1 are updated according to the volumes of class #2. This iterative process is performed until a stopping criterion is met. Applying such a method allows to account for the marginal effects of each class on the other.

The stopping criterion in flowchart of figure 1 can be set to CPU running time, repetition of the process or any convergence measurement of the problem. In this paper, the Relative Gap (RG) is selected as a criterion of the accuracy for solving the problem. Here, the process would goes on until both classes reach a desired level of RG. This measure is calculated for each class as below:

$$RG^m = \frac{\sum_{i\in I}\sum_{k\in K_i} f_k^m.T_k^m - \sum_{i\in I} d_i^m.u_i^m}{\sum_{i\in I} d_i^m.u_i^m} \tag{11}$$

Using the RG as the stopping criterion ensures the convergence of the solution, regardless of the CPU time.

2.3. Applying NSGA-II as the ULP

There is a fundamental difference in the objectives of the motorized and pedestrian modeling. In motorized transportation planning, the main interest is to overcome the worst case of the network as a kind of crisis management. Consequently, the peak hour conditions are considered in the planning progress. On the other hand, urban planners are not just concerned with the peak hour when laying out the pedestrian plans. They aim to consider all the pedestrians in the network in different times of the day. Thus, they can apply a scheme in the favor of all the travelers, regardless of their trip time interval. That is believed to be the genuine goal of a reasonable and equitable urban planning process. Therefore, it is of necessity to recognize various time periods in the PNDP. The ULP of the PNDP is a multi-objective optimization method and can be stated as below:

$$\begin{cases} F1 = \min \sum_{t\in T}\sum_{m\in M}\sum_{a\in A} v_a^{mt}.c_a^{mt}(v_a^t.Q_a(\mu)) + \sum_{t\in T}\sum_{m\in M}\sum_{n\in N} v_n^{mt}.c_n^{mt}(Q_a(\mu)) & (12) \\ F2 = \min \sum_{r\in R}\sum_{a\in A} \mu_a^r.S_a^r + \sum_{r\in R}\sum_{n\in N} \mu_n^r.S_n^r & (13) \\ F3 = \min \sum_{r\in R}\sum_{a\in A} \mu_a^w.S_a^w & (14) \\ F4 = \max \sum_{t\in T}\sum_{i\in I}\left(\sum_{k\in K_i}\sum_{a\in A}\sum_{m\in M} f_k^m.q_a(\mu).\delta_{ak}^m + \sum_{k\in K_i}\sum_{n\in N}\sum_{m\in M} f_k^m.q_n(\mu).\delta_{nk}^m \right) \quad \forall q\in Q & (15) \end{cases}$$

$s.t.$

$$\sum_{a\in A} \mu_a^w.S_a^w \le B_w \tag{16}$$

$$\sum_{a\in A} \mu_a^r.S_a^r + \sum_{n\in N} \mu_n^r.S_n^r \le B_r \tag{17}$$

$$g(\mu) > 0 \tag{18}$$

Where $T = \{t\}$ is the set of time intervals. v_a^{mt} and c_a^{mt} represent the flow and travel cost of the class *m* in the time interval *t* for the arc *a*, respectively. μ_a^r and μ_n^r are the chosen levels of improvements for environmental indices, *r*, in the link *a* and the node *n*, in that order. S_a^r and S_n^r are the financial expenses of implementing the projects of μ_a^r and μ_n^r. B_w and B_r are the budget limits at hand for widening the sidewalks and enhancing the qualities, respectively. $g(\mu)$ shows the constraints of μ for a historic district with its own unique boundaries. The objective function F1 tries to minimize the total travel cost of the network. The objective functions F2 and F3 aim to reduce the construction cost of improving the qualities of the network and widening the sidewalks, respectively. Although the nature of both functions is monetary, the attitude of the system manager may differ in spending money for each type according to the strategic planning of the vision state. Finally, in the pursuit of enhancing the social surplus of the network, objective function F4 tries to maximize the beneficial of pedestrians from each environmental quality of security, safety, walkability, sociability, sense of richness and permeability. To obtain the flow and travel cost of each link in different time intervals, the LLP is decomposed and solved for each time period individually.

It is noteworthy that the PNDP is an ill-posed NP-hard problem and its objective functions are non-convex in respect to the decision variables. Accordingly, most exact methods usually gradient based algorithms, encounter computational difficulties and are not suitable for the problem. They even may stuck in a local optimum solution. In the contrary, metaheuristic algorithms are much faster and simpler against the exact methods for this kind of problems and have the ability to evade local optima. In light of that, a multi-objective metaheuristic algorithm, embedded as the upper level of the bi-level model, seems a reasonable choice to deal with such a problem. Among the very multi-objective algorithms in the literature, a genetic evolutionary algorithm known as the NSGA-II is considered for the ULP of the PNDP. NSGA-II was proposed by Deb et al. (2000) and is broadly used in different fields of engineering for solving multi-objective optimization problems. This method is an extension of the Genetic algorithm for generating pareto optimal frontier, inspired from the field of evolutionary computation. This algorithm incorporates two main operators known as crossover and mutation, and determines the pareto front of each generation by crowding distance rule. Although it does not guarantee to find the best solution, it can reach a satisfactory near optimal result for sure. Its great performance in different areas of engineering, the flexibility of its operators to suit with the PNDP and reliable ability of the algorithm for evading the local-optima in a NP-hard non-convex problem justifies the logic of using the algorithm. Details of the NSGA-II is extensively discussed in the literature and are omitted here for brevity. One can find further data in Deb et al (2000).

3. Proposed framework

In this section, the algorithmic design of the proposed model is described. Figure 2 illustrates the framework presented for the PNDP. As it can be seen in this figure, first generation of the NSGA-II algorithm is produced using Monte-Carlo method. Taking into account the vast non-convex solution space, applying this technique helps the algorithm in the sense of solutions quality, requiring less population or iterations of the NSGA-II. The stopping criterion of the proposed algorithm is set to the maximum iteration number of the NSGA-II. The bi-level model is a game with one leader and numerous followers. The leader solves an optimization problem and the followers are engaged in a non-cooperative game among themselves.

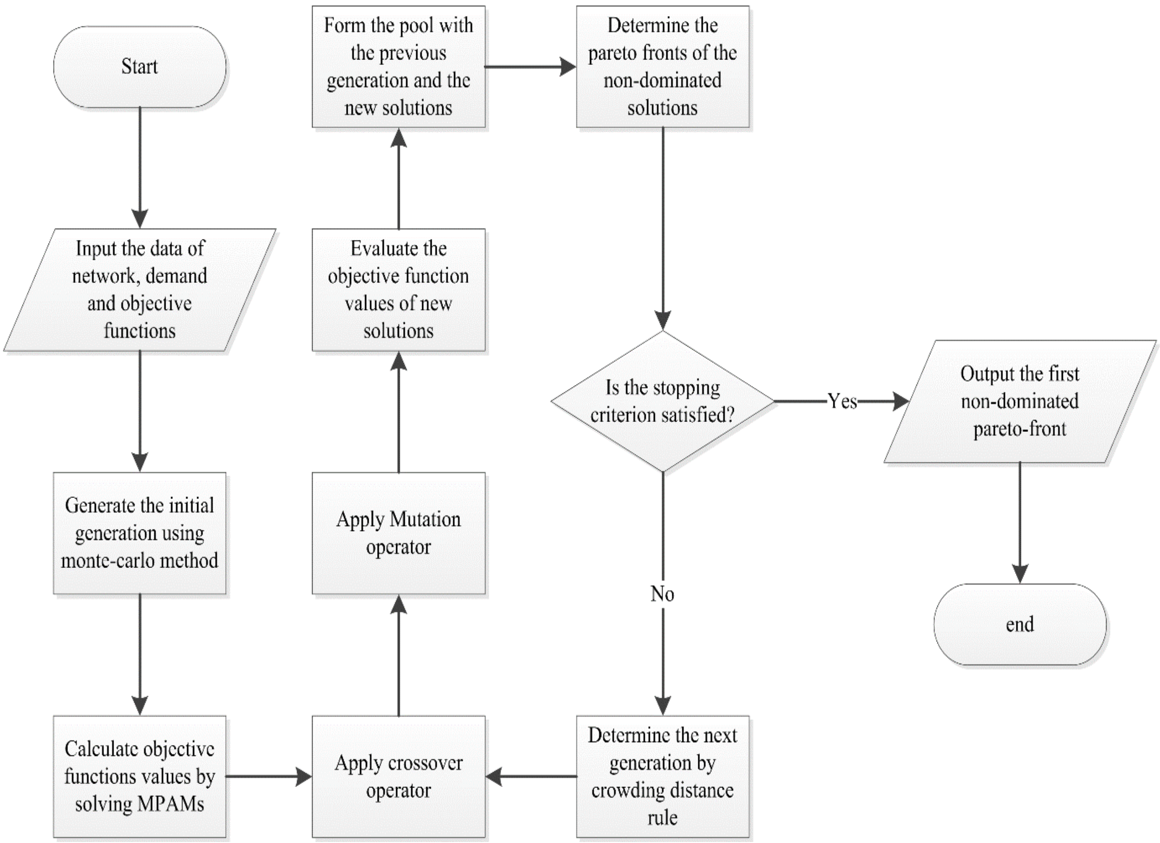

Figure 2. The Proposed genetic based algorithm for solving the PNDP

As depicted in figure 2, the first generation is produced by the Monte-Carlo method. Afterwards, the new offsprings are produced using the crossover and the mutation operators. It is worth mentioning that these operators are especially designed and tuned to suit the PNDP, resulting in an intelligent process for generating better offsprings. In the next step, the new and the old generations are combined in a so called "pool" of the NSGA-II. The solutions of the pool are then sorted in pareto-fronts. The solutions are selected until they meet the maximum population of the next generation. Using this technique and applying enhanced operators ensures that each generation has higher qualities, compared to the previous one. This procedure is repeated until the stopping criterion is met and the first pareto-front of the last generation is selected as the solutions of the PNDP.

4. Numerical experiments

In order to assess the performance and the efficiency of the proposed algorithm, the PNDP is solved for a small test network and a real large-case scenario of the historic district of the city of Tehran. In the test network, the algorithm is validated and proven to be able to solve the problem in general. In the network of Tehran historic district, the efficiency of the algorithm is evaluated by comparing the results to the lower and upper bound of the solution and some analysis on the results are also provided. The proposed algorithm is implemented in C# programming language. The MPAMs are solved with the accuracy equal to the RG of 10^{-4} for the test network and 10^{-3} for the large scale problem. In order to choose among the dominant solutions of the first pareto front in the last generation, the Technique of Order of Preference by Similarity to Ideal Solution (TOPSIS) is applied as a multi-criteria decision analysis. The weights of the objective functions are set via a preprocessing. The numerical experiments are executed on a desktop computer with an Intel 3.90 GHz CPU and 32GB memory. Due to the stochastic nature of the ULP, the algorithm is performed 20 times and the average results are reported. Table 1 shows the settings of the ULP in solving the problem for the two networks.

Table 1. settings of the ULP in solving the problem

Subject	Value/rate		Type
	Test network	Tehran network	
Crossover	0.6	0.6	multiple cut point
Mutation	0.2	0.2	substitution
Population	100	1000	-
Maximum Generation	100	2500	-
Pareto frontier	-	-	Crowding distance
$\frac{B_w}{TC_w}$	0.6	0.5	-
$\frac{B_r}{TC_r}$	0.6	0.5	-

Where TC_w and TC_r represent the maximum cost of developing the sidewalks and the qualities in the network, respectively.

4.1. Small test network

Here, the PNDP is solved for a test network with 6 nodes, 8 arcs and 36 OD pairs, each consisted of leisure and work trips. The network is depicted in the Figure 3.

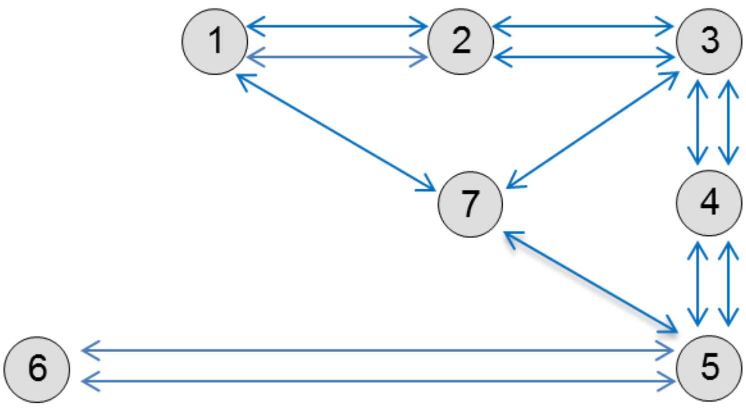

Figure 3. The considered test pedestrian network

The corresponding solution space of the test network problem contains 1048576 different states. Since solving the MPAM takes about 50×10^{-3} seconds, total enumeration on the solution space was performed and the optimal non-dominated solutions were obtained.

Table 2 shows the results for the problem. As shown in this table, the proposed algorithm found the exact solution of the total enumeration process. This result verifies the capability of the method in finding the best solution of the non-convex problem. Moreover, the algorithm reached the solution of the problem by solving 8100 MPAM, against the 1048576 MPAM of the total enumeration method.

Table 2. The results for the small size network

Function	Current value	Proposed algorithm		Total enumeration		Improvement of the best solution (%)
		Best solution	Pareto average	Best solution	Pareto average	
F1	3211976.5	1533757.858	2753355.15	1533757.858	2753355.15	52
F2	0	10.9358	8.3655	10.9358	8.3655	-
F3	0	1.3430	1.0914	1.3430	1.0914	-
F4 (Security)	55129871	91324905	74737354	91324905	74737354	66
F4 (Safety)	15876512	30759566	19773326	30759566	19773326	94
F4 (Walkability)	29098712	59126971	39470970	59126971	39470970	103
F4 (Sociability)	12720829	35152628	16616693	35152628	16616693	55
F4(Sense of Richness)	20662309	40553398	28206426	40553398	28206426	96
F4 (Permeability)	9349910	14167603	11628117	14167603	11628117	52

4.2. The case of Tehran historic district

In order to evaluate the capability of the proposed method in a large scale network, the algorithm is applied for a part of the historic district of Tehran network. The primary area is about 2000 hectares and located at the center of the city, operating as a CBD district. This district contains relatively ancient textures surrounded by many memorial and historical building and plazas, dating back to the 15th century. This irrefutable significance asks for certain management plans. In last years, several successful pedestrianization projects have been implemented for the sake of restoration and revival of the region. A part of the region in the southwest with area of 233 hectares is considered for solving the PNDP.

The defined pedestrian network has 55 nodes, 98 arcs, 906 OD pair of leisure trips and 922 OD pair of work trips, which counts for a relatively large-scale network. The corresponding graph of the network is shown in the Figure 4.

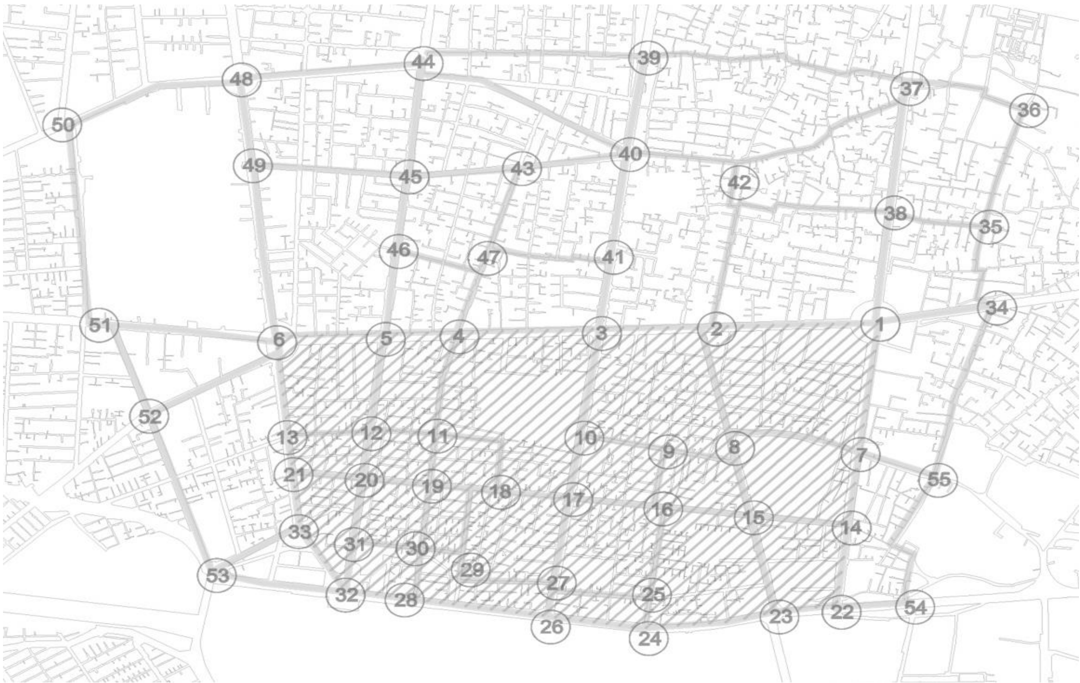

Figure 4. Graph of the historic district zone

4.2.1 The OD demand matrix

The first step for solving the PNDP in a real case scenario is to estimate the true OD matrix of the network. For the case study of this paper, the trips were divided into home-based and non-home-based groups. Each group was then subdivided into recreational, shop, eat, educational, work and personal business categories. The first three were supposed to form the leisure class, while the other three combine together as the work class. To obtain the data of the trips in the region, household survey and field observations (counting people coming out and going through the subway stations, bus stops, etc.) were performed. The procedure is similar to the method in Clifton and Muhs (2012). In addition, pedestrian flows were observed on a sub-set of the links. Based on these data and the preprocessing of zoning the district, the following procedure was applied to achieve the OD matrix for each $t \in T$:

Step (1) Fitting trip generation and attraction models of zones by linear or nonlinear regression for each group and each trip purpose with acceptable statistical measures (developing ten models).

Step (2) Trip distribution of each purpose, using doubly constrained gravity model (the maximum walking distance of 450 and 650 meters for work and leisure class are considered in tuning the parameters of the models).

Step (3) Construct work and leisure OD matrices by summing the corresponding sub-matrices.

Step (4) Assign the OD matrices of work and leisure to the network by solving the MPAM.

Step (5) If the coefficient of determination (R-squared) between the assigned and observed volumes are less than 0.95, alter the parameters of the gravity model and go to the step (2); otherwise go the step (6).

Step (6) output the OD matrices of work and leisure.

The models of step (1) are built upon some parameters such as population, car ownership, retail, shopping, etc. In the step (5), if the stopping criterion is not satisfied, a try and error process is executed. The parameters of the impedance functions are adjusted in a way that no irrational result appears. Although the mathematical model of the above procedure may not have a unique solution, the proposed expert judgmental method will result in a reliable one. The high value of the desired R-square is an acceptable proof to the claim.

4.2.2 How big is the problem?

For the historic district of Tehran, widening the sidewalks and 19 environmental indices are considered as variables of the PNDP, each of which can be set to 10 different values between selection of preserving the current situation or maximum level of improvement. Ignoring the limitations of the historic district, there are 2.54 $\times 10^{735}$ possible states for the solution. Involving $g(\mu)$ in the problem eliminates 93.6% of the alternatives and the solution space of problem shrinks to 1.63 $\times 10^{734}$ feasible states. Since solving the MPAM for the network takes about 1.5 seconds, evaluating all feasible solutions needs at least 7.75 $\times 10^{726}$ years, which is obviously impossible. This evaluation demonstrates the big scale of the NP-hard problem, its complexity and justifies the application of a metaheuristic algorithm.

4.2.3 Numerical results

Here, the numerical results of solving the PNDP in Tehran historic district are presented. Figure 5 illustrates the average values of the first pareto front for the objective functions in the successive generations. As it can be seen in this figure, all objective functions are enhanced and have converged after 2230 generations.

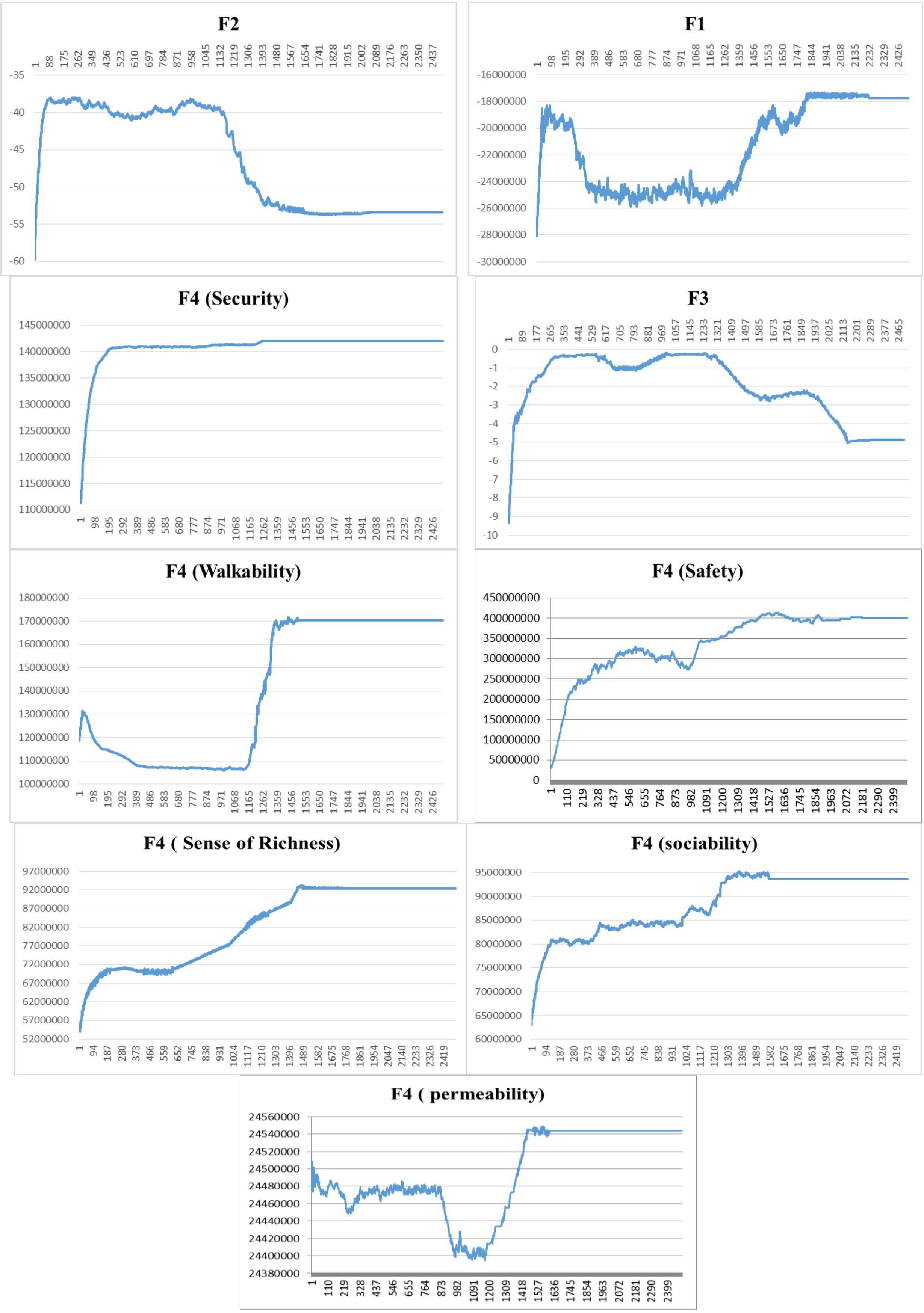

Figure 5. The value of objective functions in successive generations

Table 3 demonstrates the results for the found solutions, as well as the best state for each objective function.

Table 3. Results of the proposed for the large-scale problem

Objective function	Best solution value	Pareto average value	Best possible value	Welfare gain (%)
F1	14553385	17726679	9864537	84.24
F2	58.30	53.43	0	_
F3	5.09	4.88	0	_
F4 (Security)	168407304	142102449	177419770	82.01
F4 (Safety)	487941060	400906985	509174634	87.21
F4 (Walkability)	221624434	170279259	230156698	91.47
F4 (Sociability)	102622066	93675581	106840235	86.63
F4(Sense of Richness)	100789896	92487688	103543351	78.70
F4 (Permeability)	24930633	24543578	25004917	77.99

The welfare gain of the function (*i*) is calculated as below:

$$Welfare\,Gain\,(i) = \left| \frac{OFV_i - OFV_i^c}{OFV_i^c - OFV_i^b} \right| \tag{19}$$

Where OFV_i is the value of objective function *i* in the best solution found, OFV_i^c is the value of objective function *i* in the current situation and OFV_i^b is the best possible value for the function *i*, ignoring the other objective functions. The average welfare gain of the proposed algorithm is 84.0 percent. It implicitly verifies the performance and the efficiency of the algorithm, since spending half of the maximum possible construction cost has led to a welfare gain of 84.0 percent.

5. Conclusion and results

Historic districts are one of the most precious elements of the cities, which have been neglected in the process of urbanization revolution. This has dramatic consequences on the identity of these fast growing cities. In this paper, urban regeneration and revival of these areas are investigated by considering the movements of the pedestrians as the key factor in the dynamics of such places. First, we proposed a novel macroscopic user equilibrium multi-class pedestrian assignment model (MPAM), inspired from motorized modeling, to provide the prediction of behavior of the pedestrians. This is a unique model, since it is the first one applicable for large-scale networks. In addition, we presented a bi-level mathematical mixed-integer programming model as a decision making tool for urban planners. The model can output the optimum budgeting plan for selecting the projects in public places, while satisfying the financial limits and constraints of the historic districts. The upper level problem is a multi-objective optimization model solved by the NSGA-II, and the lower level problem is the proposed MPAM. The problem is solved for a small test network and a real case of historic district of Tehran. The results of the small network validated the ability of the algorithm in solving the problem and the large scale one proved its efficiency. The results are promising and encourage for further investigations of the proposed method in the pedestrian modeling and the pedestrian network problem.

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Appendix A: Environmental qualities

The environmental qualities of sidewalks and nodes of a pedestrian network have a great influence on the response of the pedestrians, especially on their route selecting. They also bring out social interactions as prerequisite of healthy social life. This performance becomes more vital in the historic districts, where the exotic surrounding fabrics designate the sense of belonging to the citizens. Thus, it is inappropriate to neglect these qualities in the pedestrian modeling of these remarkable areas. But how to measure something which is intrinsically qualitative? To answer the problem, we chose the effective qualities in route choice behavior of pedestrians, presented as $Q = \{q\}$, according to our observation data. Analyzing these qualities, we determined a set of sub-qualities $P = \{p\}$. Each quality is a function of the considered sub-qualities:

$$Q(q) = Z(p)$$

where Z is a polynomial function of degree 1. The coefficients of Z are obtained by an Analytical Hierarchy Process (AHP). In the numerical experiments of this paper, some sub-qualities are considered as constant parameters since no decision variable alter their values. As a result, there is a constant term in $Q(q)$ for each quality. Since the corresponding sub-qualities for the quality of permeability are all independent from the decision variables, the quality becomes a constant value itself. Table A.1 shows the effective sub-qualities which will be changed by decision variables for the other qualities.

Table A.1. Effective sub-qualities of each quality

Quality	Sub-qualities
Safety	- Design Speed of the adjacent street(s) - Median of the adjacent Street(s) - Lighting of the sidewalk/node - Traffic Signs in the adjacent Street(s) - Crosswalk for pedestrians
Sociability	- Furniture in the sidewalk/node - Cleanness of the sidewalk/node - Human made equipment - Vegetation of the sidewalk/node
Sense of Richness	- Pavement of the sidewalk/node - Greenness of the sidewalk - Visual excrescences in the sidewalk/node
Security	- Lighting of the sidewalk/node - Hidden corners in the sidewalk/node
Walkability	- Pavement of the sidewalk/node - Greenness of the sidewalk - Obstacles in the passage - Pedestrian facilities of the sidewalk/node Design Speed of the adjacent street(s) - Median of the adjacent Street(s) - Lighting of the sidewalk/node - Traffic Signs in the adjacent Street(s) - Crosswalk for pedestrian

Each sub-quality itself is a resultant of a set of environmental indices, $R\{r\}$. These indices are the subjects that projects can be defined for, such as number of trees, light poles, etc. This is the link between the ULP and the LLP, where decision variables are set to alter these indices and pedestrians act based on their perception from the indices. Accordingly, it is concluded that the decision variables on indices decide the value of qualities. Finally, all qualities are integrated to shape a single value as the general quality used in the LLP. This unified quality is obtained using a linear equation. The weight of each quality is determined by the field observation and surveys. It is noted that there has been presented some qualitative measurements and quantification techniques for evaluating an environmental quality in the literature. Since the value of the environmental qualities is the input of our proposed algorithm, we applied a precise quantification technique, $Q(q)$, to improve the accuracy of the results. It is possible to use different methods, since it has no effect on the mathematical formulation of the model.

Appendix B. Travel Time on sidewalks

Here the details of estimating the relation for t_a is presented. For this purpose, it is sufficient to determine the velocity of the pedestrians in the sidewalk *a*. Then, t_a can be calculated by dividing the length of the sidewalk to the corresponding velocity. To do so, fundamental diagrams of pedestrian flows are applied. This diagrams exhibit the statistical relations of the flow, density and speed in a macroscopic level. In this paper, it is assumed that a linear relation exists between speed and density, similar to the greenshields model in the motorized traffic flow theory. The model can be described by the following equations:

$$\begin{cases} v = k \times u & (B.1) \\ u = u_f - \dfrac{u_f}{k_j} \times k & (B.2) \end{cases}$$

where *v* is the pedestrian flow, *k* is the density of pedestrian in a specific length of the sidewalk and *u* is the traveling speed. u_f and k_j are the free flow speed and the jamming density, respectively. The diagrams of above equations are tabulated in the figure B.1.

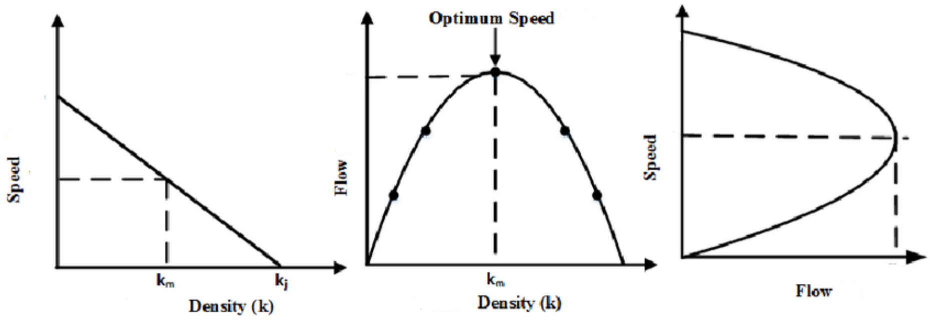

Figure B.1: the fundamental diagrams of pedestrian flow

It is noted that the in the greenshields model, maximum possible flow occurs at the speed of $\frac{u_f}{2}$ and the density equal to $\frac{k_j}{2}$. This maximum value is known as the passage capacity with the value of $\frac{u_f \times k_j}{4}$. As it can be seen in the figure B.1, as the flow increases, the speed reduces from the free flow speed until it reaches the capacity. This range is known as the uncongested or undersaturated flow. From this point, if more pedestrian attempt to enter the passage, queues will be formed because of the inadequacy of capacity in respect to the demand; and congested or oversaturated flow appears. Hence, t_a is estimated for each range separately.

<u>The uncongested state</u>

For the uncongested pedestrian flow, the relation between speed and flow is established. Substituting equation (B.1) in equation (B.2), we get:

$$u = u_f - \frac{u_f}{k_j} \times \frac{v}{u} \qquad (B.3)$$

Now, multiplying both side in u :

$$u^2 = u_f.u - \frac{u_f}{k_j} \times v \qquad (B.4)$$

Solving the quadratic equation (B.4) in respect to u , the speed can be obtain as follows:

$$u = \frac{u_f + \sqrt{u_f^2 - 4 \times \left(\frac{u_f \times v}{k_j} \right)}}{2} \qquad (B.5)$$

Substituting v_a as v in equation (B.5), u_a can be derived for each sidewalk *a*. Finally, the travel time, t, can be calculated as below:

$$t_a = \frac{2 \times L_a}{\left(u_f + \sqrt{u_f^2 - 4 \times \left(\frac{u_f \times v}{k_j} \right)} \right)} \qquad (B.6)$$

where L_a is the length of sidewalk *a*.

<u>*The congested state*</u>

In the congested zone, it is assumed that the travel time increases linearly proportional to the increase in the upstream pedestrian flow. Hence the relation can be stated as follows:

$$u = \frac{u_f}{2} + a \times \left(v - \frac{u_f \times k_j}{4} \right) \qquad (B.7)$$

Accordingly, t_a can be stated as follows:

$$t_a = \frac{L_a}{u = \frac{u_f}{2} + a \times \left(v - \frac{u_f \times k_j}{4} \right)} \qquad (B.8)$$

Calibrating the parameters of the greenshields model for the historic district of the city of Tehran, the following function is obtained for calculating the travel time on the sidewalk *a*.

$$t_a = \begin{cases} 32.35 \times L_a \times (v_a - 1.8425) + \dfrac{L_a}{0.678185} & if \quad v_a > 1.8425 \qquad (B.9) \\ \dfrac{L_a}{\left(0.67 + \left(0.5 + \sqrt{\left(1.7956 - (0.9744 \times \bar{v}_a)\right)}\right)\right)} & if \quad v_a \leq 1.8425 \qquad (B.10) \end{cases}$$