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A fuzzy approach to reliability analysis

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Abstract

Most of the existing approaches for fuzzy reliability analysis are based on fuzzy probability. The aim of this paper is to describe fuzzy reliability using fuzzy differential equation. The reliability of a system in real world applications is affected by some uncertain parameters. Fuzzy reliability is a way to present the reliability function uncertainly using fuzzy parameters. In the proposed fuzzy differential equation for reliability, two types of fuzzy derivative: Hukuhara derivative and generalized differentiability are used. It is proved that the Hukuhara differentiability is not adequate for fuzzy reliability analysis. Finally, using the fuzzy integration, the concept of fuzzy mean time to failure (FMTTF) will be introduced. Some numerical simulations are presented to show the applicability and validity of generalized differentiability, in comparison with the Hukuhara differentiability results for fuzzy reliability analysis.

Keywords: Fuzzy reliability; Fuzzy differential equation; Fuzzy derivative.

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1. Introduction

Reliability is the probability that the network performs its assigned functions properly in a specified period (He and Zhang (2016)). Several authors have proposed different methods for analyzing the fuzzy reliability of industrial systems. Chen (1994) has presented a new method to analyze fuzzy system reliability using fuzzy number arithmetic operations, where the reliability of each system component is represented by a triangular fuzzy number. Mon and Cheng (1994) used fuzzy distributions instead of the classical probability distribution for the components, and calculated the functions of fuzzy numbers to use for the fuzzy system reliability via non-linear programming techniques where the commercial packages are available to use. Kumar and Lata (2012) explored the concept of fuzzy differential equations (FDEs) and extended vague differential equations (VDEs) also they proposed a new method to find the analytical solution of VDEs with new representation of trapezoidal vague set (TrVS), named as

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JMD TrVS. A lot of other references exist which employed the theory of fuzzy systems for classical problems to model the uncertainty of real world situations (see for example Effati and Pakdaman (2010); Effati et al. (2011); Sadoghi Yazdi et al. (2008); Pakdaman and Effati (2016)).

Reliability analysis is one of the most challenging topics for the system analysts or plant personnel such that the system should run for a longer period of time without failures. Unfortunately, failure is an unavoidable phenomenon in an industrial system, and hence the role of the system analyst becomes critical for maintaining the performance of the industrial systems so as to increase the performance and productivity of the system. As the complexity of a systems grows, the uncertainty of information about the reliability increases. To overcome this issue, Garg (2015) proposed the nth-order fuzzy Kolmogorovs differential equations, by using a fuzzy Markov model of a repairable industrial system. He evaluated the fuzzy reliability of the system both in transient and steady state, using Runge-Kutta method.

In the process of applying the Weibull analysis (testing a component), the data obtained Sometimes cannot be recorded or collected precisely due to unexpected case scenarios. Since the failure times in many cases are not observable and the censoring mechanism may or may not be known, new approaches have been developed. Gonzalez-Gonzalez et al. (2014) proposed the use of fuzzy probability theory to account for the uncertainty and the prior knowledge of the process in the parameters' estimation, for censored data. The proposed method was applied to risk based inspection. Results demonstrate that the method represents a reliable option for using the expert knowledge about the component and the physics of the failure mode.

Lee et al. (2012) examined the reliability of a parallel system. They used the level (λ, ρ) interval-valued fuzzy numbers to find the fuzzy reliability of parallel systems and obtain the estimated reliability of the systems in the fuzzy sense by employing the signed distance method (Ahmad et al. (2013)). Aghili and Hajian- Hoseinabadi (2017) focused on practical applications of fuzzy reliability; impacts of repair and data uncertainty. Findings of their research demonstrated that fuzzy outputs were computationally tractable and properly estimated.

He and Zhang (2016) developed a new method based on cellular automata and fuzzy logic following different types of fuzzy failure rates. The proposed method has two separate processing procedures: the computing of fuzzy numbers and the decomposing model based on cellular automata. The combination of these procedures can be used to topology decomposition and the fuzzy reliability evaluation of any real network.

Purba (2014) proposed a fuzzy-based reliability approach to evaluate basic events of system fault trees whose failure precise probability distributions of their lifetime to failures are not available. It applies the concept of failure possibilities to qualitatively evaluate basic events and the concept of fuzzy sets to quantitatively represent the corresponding failure possibilities. Due to the difficulties in obtaining sufficient load specimen, it is imprecise to construct the stress as random variables. Therefore, dynamic fuzzy reliability models were developed in Gao et al. (2014), which resolve the problems in dealing with the interaction between the fuzzy stress process and the stochastic strength process.

Jamkhaneh (2011) considered the problem of the evaluation of system reliability, in which the lifetimes of components are described using a fuzzy exponential distribution. The fuzzy reliability functions of k-out-of-m system, series systems and parallel systems and their FMTTF were discussed, respectively. He modeled the lifetime of electronic component by an exponential distribution with fuzzy parameter. In another paper Jamkhaneh (2014) described Fuzzy Weibull distribution and lifetimes of components. Also Formulas of a fuzzy reliability function, fuzzy hazard function and their α-cut set were presented.

Furthermore, the fuzzy functions of series systems and parallel systems were discussed, respectively. The Bayesian reliability estimation under fuzzy environments was proposed in Wu's paper (Wu (2004)). In order to apply the Bayesian approach, the fuzzy parameters were assumed to be fuzzy random variables with fuzzy prior distributions. He transformed the original problem into a non-linear programming problem. This non-linear programming problem was then divided into four sub-problems to simplify the computation. Finally, He obtained the failure rate and reliability by solving the proposed sub-problems. Regarding the fuzzy mean time to failure (FMTTF), Liu et al. (2007) defined the MTTF of non-repairable systems with fuzzy random lifetimes.

As it was studied in the above-mentioned literature review, the ability of fuzzy differential equations was not employed to define the uncertain reliability. On the other hand, in the literature of fuzzy mathematics, there are several definitions for fuzzy derivative and consequently, several approaches for fuzzy differential equations. Thus, it could be very important from both theoretical and practical points of view, to study the fuzzy reliability form a fuzzy differential equation perspective. In this paper, we introduce and derive the fuzzy reliability function based on the concept of fuzzy differential equations and fuzzy derivative. It will be shown that between two existing definitions of fuzzy derivative, just one will result in a valid fuzzy reliability function while the other one results an invalid fuzzy reliability function. The paper is organized as follows: In next section, some preliminaries are presented. Sections 3 and 4 introduce the fuzzy reliability analysis based on two types of fuzzy derivative. Section 5 introduces the concept of FMTTF based on fuzzy integration. To illustrate the validity and efficiency of the proposed approach, some numerical examples are presented in Section 6. Finally, Section 7 contains conclusions and some remarks.

2. Preliminaries

In this section we present some necessary preliminaries from fuzzy set theory, which will be used in this paper.

Definition 2.1 (Gomes et al. (2015)) Let U be the universe of discourse. A fuzzy subset $\tilde{A}$ is determined by the following membership function:

$$\mu_{\tilde{A}} : U \to [0,1] \tag{2.1}$$

Note that in (2.1), if we replace the interval $[0,1]$ with the binary set $\{0,1\}$, then $\tilde{A}$ characterizes a crisp subset of U.

Definition 2.2 (Khastan and Rodrguez-Lpez (2015)) A fuzzy set $\tilde{A}$ in the universe U, with an upper-semicontinuous membership function $\mu_{\tilde{A}}$ is said to be a fuzzy number if, $\tilde{A}$ be fuzzy convex (i.e. $\mu_{\tilde{A}}(\lambda x + (1-\lambda)y) \le \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}$ for each $x, y \in U$ and $\lambda \in [0,1]$), fuzzy normal (i.e. $\exists x \in U$ such that $\mu_{\tilde{A}}(x) = 1$) and compact support (i.e. $cl\{x \in U : \mu_{\tilde{A}}(x) > 0\}$ is compact). We denote by F(R) the set of all fuzzy numbers defined over the real set R.

Definition 2.3 (Khastan and Rodrguez-Lpez (2015)) For $0 < \alpha \le 1$ the α-cut of a fuzzy number $\tilde{A}$ is defined as $[\tilde{A}]^{\alpha} = \{x \in U : \mu_{\tilde{A}}(x) \ge \alpha\}$ and for $\alpha = 0$ we define: $[\tilde{A}]^{0} = cl\{x \in U : \mu_{\tilde{A}}(x) > 0\}$, where 'cl' denotes the closure of the set. Usually, the α-cut of a fuzzy number $\tilde{A}$ is denoted by $\left[\tilde{A}\right]^{\alpha}=\left[A^{L}(\alpha),A^{U}(\alpha)\right]$ where $A^{L}(\alpha)$ and $A^{U}(\alpha)$ demonstrate the lower and upper bounds of $\tilde{A}$ respectively.

Note that $\left[A^{L}(\alpha),A^{U}(\alpha)\right]$ determines a valid α-cut of a fuzzy number, just when for each $\alpha\in[0,1]$ we have $A^{L}(\alpha)\le A^{U}(\alpha)$

Definition 2.4 Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers with α-cuts $\left[A^{L}(\alpha),A^{U}(\alpha)\right]$ and $\left[B^{L}(\alpha),B^{U}(\alpha)\right]$ respectively. Then we define:

- $\left[\tilde{A}+\tilde{B}\right]^{\alpha}=\left[A^{L}(\alpha)+B^{L}(\alpha),A^{U}(\alpha)+B^{U}(\alpha)\right],$
- $\left[\tilde{A}-\tilde{B}\right]^{\alpha}=\left[A^{L}(\alpha)-B^{U}(\alpha),A^{U}(\alpha)-B^{L}(\alpha)\right],$
- $\left[\tilde{A}.\tilde{B}\right]^{\alpha}=\left[\min_{r,s\in\{L,U\}}A^{r}(\alpha).B^{s}(\alpha),\ \max_{r,s\in\{L,U\}}A^{r}(\alpha).B^{s}(\alpha)\right],$
- For $\lambda\in R,\left[\lambda\tilde{A}\right]^{\alpha}=\lambda\left[\tilde{A}\right]^{\alpha}.$

Definition 2.5 (Gomes et al. (2015); Bede (2013)) The Hukuhara difference (H-difference) of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as $\left[\tilde{A}-_{H}\tilde{B}\right]^{\alpha}=\left[A^{L}(\alpha)-B^{L}(\alpha),A^{U}(\alpha)-B^{U}(\alpha)\right].$

However, the H-difference for two arbitrary fuzzy numbers doesn't exist, other differences are defined such as generalized Hukuhara difference
$\left[\tilde{A}-_{gH}\tilde{B}\right]^{\alpha}=\left[\min\left\{A^{L}(\alpha)-B^{L}(\alpha),A^{U}(\alpha)-B^{U}(\alpha)\right\},\max\left\{A^{L}(\alpha)-B^{L}(\alpha),A^{U}(\alpha)-B^{U}(\alpha)\right\}\right]$
and also generalized difference.

Definition 2.6 (Gomes et al. (2015)) the distance between two fuzzy numbers $\left[\tilde{A}\right]^{\alpha}=\left[A^{L}(\alpha),A^{U}(\alpha)\right]$ and $\left[\tilde{B}\right]^{\alpha}=\left[B^{L}(\alpha),B^{U}(\alpha)\right]$ can be defined as follows:

$$d(\tilde{A},\tilde{B})=\sup_{0\le\alpha\le1}\left\{\max\left[\left|A^{L}(\alpha)-B^{L}(\alpha)\right|,\left|A^{U}(\alpha)-B^{U}(\alpha)\right|\right]\right\} \tag{2.2}$$

Definition 2.7 (see Ahmad et al. (2013)) A mapping $\tilde{f}:(a,b)\to F(R)$ which assigns a fuzzy Number $\tilde{f}(t)$ to each $t\in(a,b)$ with α-cut $\left[\tilde{f}(t)\right]^{\alpha}=\left[f^{L}(t,\alpha),f^{U}(t,\alpha)\right]$ is called a fuzzy function (i.e. for each $t\in(a,b)$, $\left[\tilde{f}(t)\right]^{\alpha}$ denotes valid α-cut). Then $\tilde{f}$ is called differentiable at $\hat{t}\in(a,b)$ if there exists an element $\tilde{f}'(\hat{t})\in F(R)$ such that:

for all $h>0$ sufficiently small, the H-differences $\tilde{f}(\hat{t}+h)-\tilde{f}(\hat{t}),\tilde{f}(\hat{t})-\tilde{f}(\hat{t}-h)$ exist and:

$$\lim_{h\to0^{+}}d\left(\frac{\tilde{f}(\hat{t}+h)-\tilde{f}(\hat{t})}{h},\tilde{f}'(\hat{t})\right)=\lim_{h\to0^{+}}d\left(\frac{\tilde{f}(\hat{t})-\tilde{f}(\hat{t}-h)}{h},\tilde{f}'(\hat{t})\right)=0 \tag{2.3}$$

(II) for all $h>0$ sufficiently small, the H-differences $\tilde{f}\left(\hat{t}\right)-\tilde{f}\left(\hat{t}+h\right)$ and $\tilde{f}\left(\hat{t}-h\right)-\tilde{f}\left(\hat{t}\right)$ exist and:

$$\lim_{h\to 0^+} d\left(\frac{\tilde{f}\left(\hat{t}\right)-\tilde{f}\left(\hat{t}+h\right)}{h},\tilde{f}'\left(\hat{t}\right)\right)=\lim_{h\to 0^+} d\left(\frac{\tilde{f}\left(\hat{t}-h\right)-\tilde{f}\left(\hat{t}\right)}{h},\tilde{f}'\left(\hat{t}\right)\right)=0 \tag{2.4}$$

In this situation, $\tilde{f}'\left(\hat{t}\right)$ is called the type-I and type-II fuzzy derivative of $\tilde{f}$ at $\hat{t}$ respectively.

Theorem 2.1 (see Ahmad et al. (2013)) let $\tilde{f}:(a,b)\to F(R)$ and suppose that $\left[\tilde{f}(t)\right]^{\alpha}=\left[f^{L}(t,\alpha),f^{U}(t,\alpha)\right]$ for $\alpha\in[0,1]$.

(i) If $\tilde{f}$ is (I)-differentiable at all $t\in[a,b]$ then $f^{L}(t,\alpha)$ and $f^{U}(t,\alpha)$ are differentiable functions and we have $\left[\tilde{f}'(t)\right]^{\alpha}=\left[\frac{d}{dt}f^{L}(t,\alpha),\frac{d}{dt}f^{U}(t,\alpha)\right]$.

(ii) If $\tilde{f}$ is (II)-differentiable at all $t\in[a,b]$ then $f^{L}(t,\alpha)$ and $f^{U}(t,\alpha)$ are differentiable functions and we have $\left[\tilde{f}'(t)\right]^{\alpha}=\left[\frac{d}{dt}f^{U}(t,\alpha),\frac{d}{dt}f^{L}(t,\alpha)\right]$.

Definition 2.8 Suppose that $\lambda(t)$ is the hazard function and $R(t)$ is the reliability at time t.

Then, the reliability can be calculated by solving the following ordinary differential equation:

$$\frac{dR(t)}{dt}=-\lambda(t)R(t) \quad , \quad R(0)=1 \tag{2.5}$$

Where $\lambda(t)$ is a non-negative function.

Theorem 2.2 (See Stefanini et al. (2006)): Suppose that $\tilde{f}(t)$ is a fuzzy function with α-cut

$\left[\tilde{f}(t)\right]^{\alpha}=\left[f^{L}(t,\alpha),f^{U}(t,\alpha)\right]$ then:

$$\left[\int_a^b \tilde{f}(t)dt\right]^{\alpha}=\left[\int_a^b f^{L}(t,\alpha)dt,\int_a^b f^{U}(t,\alpha)dt\right] \tag{2.6}$$

Theorem 2.2 will be used for deriving the definition of FMTTF.

3. Fuzzy reliability analysis I

In real world, reliability of a product, which determines the quality of a product during the time without failure, is not a crisp value. Indeed, the reliability of a product is affected by several unknown and known parameters. Since the reliability is calculated using hazard function and this function is not also a crisp value, thus considering the reliability as a fuzzy value is a natural way to model the imposed uncertainty. There are several approaches to define fuzzy reliability. In this section, we use (2.5) to introduce the fuzzy reliability function.

To introduce the fuzzy reliability function, based on (2.5), we must solve the following fuzzy differential equation:

$$\frac{d\tilde{R}(t)}{dt}=-\tilde{R}(t)\lambda(t) \tag{3.7}$$

Wherein $\tilde{R}(t)$ is the fuzzy reliability function with α-cut $\left[\tilde{R}(t)\right]^{\alpha}=\left[R^{L}(t,\alpha),R^{U}(t,\alpha)\right]$.

We also consider the initial condition $\tilde{R}(0)=\tilde{1}$ where $\tilde{1}$ is fuzzy one with alpha-cut $\left[\tilde{1}\right]^{\alpha}=\left[c^{L}(\alpha),c^{U}(\alpha)\right]$.

In comparison with other existing approaches for fuzzy reliability (e.g. Gonzalez-Gonzalez et al. (2014)) which are based on fuzzy probability density function, in our proposed approach we introduce the fuzzy reliability function based on fuzzy differential equation.

To solve (3.7), if we use the first definition of fuzzy derivative (and noting that $\lambda(t)$ is positive) then:

$$\begin{cases}\dfrac{dR^{L}(t,\alpha)}{dt}=-R^{U}(t,\alpha)\lambda(t),R^{L}(0,\alpha)=c^{L}(\alpha)\\ \dfrac{dR^{U}(t,\alpha)}{dt}=-R^{L}(t,\alpha)\lambda(t),R^{U}(0,\alpha)=c^{U}(\alpha)\end{cases} \tag{3.8}$$

Equations (3.8) present a system of ordinary differential equations which can be solved analytically.

If $\lambda(t)=\lambda$ be a positive constant, then (3.8) is a time-invariant linear system. Also when λ a function of time, then (3.8) is describes a time-variant linear system. In the case of $\lambda(t)=\lambda$, system (3.8) can be presented as follows:

$$\begin{bmatrix}\dot{R}^{L}(t,\alpha)\\ \dot{R}^{U}(t,\alpha)\end{bmatrix}=\begin{bmatrix}0 & -\lambda\\ -\lambda & 0\end{bmatrix}\begin{bmatrix}R^{L}(t,\alpha)\\ R^{U}(t,\alpha)\end{bmatrix} \quad , \quad \begin{bmatrix}R^{L}(0,\alpha)\\ R^{U}(0,\alpha)\end{bmatrix}=\begin{bmatrix}c^{L}(\alpha)\\ c^{U}(\alpha)\end{bmatrix} \tag{3.9}$$

In this case the eigenvalues of the coefficient matrix are $-\lambda$ and λ . Thus the final solution is:

$$\begin{cases}R^{L}(t,\alpha)=\dfrac{1}{2}k_{1}e^{\lambda t}+\dfrac{1}{2}k_{2}e^{-\lambda t}\\ R^{U}(t,\alpha)=\dfrac{1}{2}k_{2}e^{-\lambda t}-\dfrac{1}{2}k_{1}e^{\lambda t}\end{cases} \tag{3.10}$$

Where $k_{1}=c^{L}(\alpha)-c^{U}(\alpha)$ and $k_{2}=c^{L}(\alpha)+c^{U}(\alpha)$. As it can be observed, in (3.10), when $t\to+\infty$, the values of upper and lower reliability functions tends to $+\infty$. Thus the final fuzzy reliability function is not valid. To illustrate the solution, consider the following example.

Example 3.1 Suppose that $\lambda=10^{-1}$ and $\left[c^{L}(\alpha),c^{U}(\alpha)\right]=\left[0.9+0.1\alpha,1\right]$. Then solution of (3.10) is:

$$\begin{cases}R^{L}(t,\alpha)=\dfrac{1}{2}k_{1}e^{\lambda t}+\dfrac{1}{2}k_{2}e^{-\lambda t}\\ R^{U}(t,\alpha)=\dfrac{1}{2}k_{2}e^{-\lambda t}-\dfrac{1}{2}k_{1}e^{\lambda t}\end{cases} \tag{3.11}$$

As it can be observed in Figure 1, the reliability decreases by time and the fuzzy reliability solution is a valid fuzzy function. Also the proposed fuzzy reliability function is a valid fuzzy function (since $R^{L}(t,\alpha)\le R^{U}(t,\alpha)$ for each $t\in[0,10]$). But if we extend the time horizon we observe that the values of reliability function tends to $+\infty$ (see Figure 2). Also, based on Figure 2 it can be observed that we have negative values and also values more than one for the lower and upper reliability functions which are incorrect. This also indicates that the first definition of fuzzy derivative is inconsistent for fuzzy reliability.

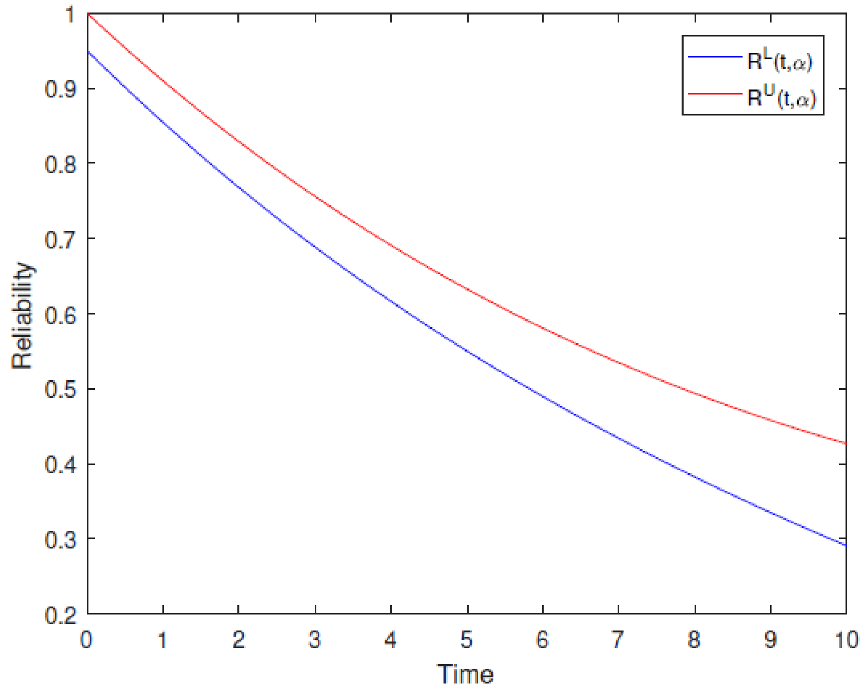

Figure 1. Fuzzy reliability in Example 3.1 in the time horizon [0; 10] for $\alpha = 0.5$

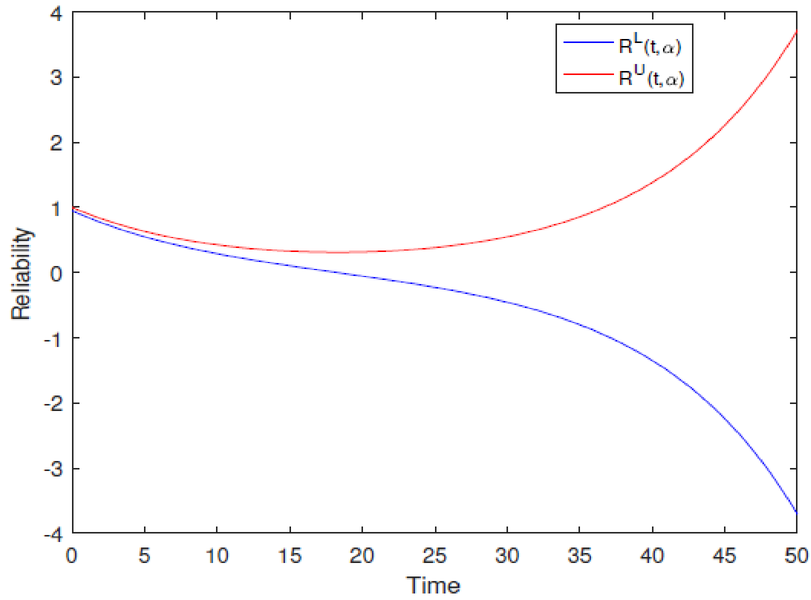

Figure 2. Fuzzy reliability in Example 3.1 in the time horizon [0; 50] for $\alpha = 0.5$

4. Fuzzy reliability analysis II

As it was discussed in previous section, the first definition of fuzzy derivative is not suitable for fuzzy reliability analysis. Now consider equation (3.7) based on the second definition of fuzzy derivative. In this sense, we have the following system of differential equations:

$$\begin{cases} \dfrac{dR^L(t,\alpha)}{dt} = -R^L(t,\alpha)\lambda(t), R^L(0,\alpha) = c^L(\alpha) \\ \dfrac{dR^U(t,\alpha)}{dt} = -R^U(t,\alpha)\lambda(t), R^U(0,\alpha) = c^U(\alpha) \end{cases} \tag{4.12}$$

Each equation in (4.12) can be solved separately. In the case of $\lambda(t) = \lambda$ the final analytical solution is:

$$\begin{cases} R^L(t,\alpha) = c^L(\alpha)e^{-\lambda t} \\ R^U(t,\alpha) = c^U(\alpha)e^{-\lambda t} \end{cases} \tag{4.13}$$

As it can be observed, in (4.13), when $t \to +\infty$ the values of upper and lower reliability functions tends to zero. Since $c^L(\alpha) \le c^U(\alpha)$ for each $\alpha \in [0,1]$, thus $R^L(t,\alpha) \le R^U(t,\alpha)$. To illustrate the solution, consider the following example.

Example 4.1 In Example 3.1, if we apply (4.13), then:

$$\begin{cases} R^L(t,\alpha) = (0.9 + 0.1\alpha)e^{-\lambda t} \\ R^U(t,\alpha) = c^U(\alpha)e^{-\lambda t} \end{cases} \tag{4.14}$$

The solution for time horizon [0; 10] and [0; 50] can be observed in Figures 3 and 4 respectively.

5. Fuzzy mean time to failure

In this section we try to introduce and define the fuzzy mean time to failure (FMTTF) based on definition of fuzzy integral. In crisp case, the MTTF can be calculated as follow:

$$MTTF = \int_0^{+\infty} R(t)dt \tag{5.15}$$

In real world applications, since the reliability is not crisp, consequently the value of MTTF could not be determined clearly. Indeed, fuzzy reliability function will result fuzzy values for MTTF.

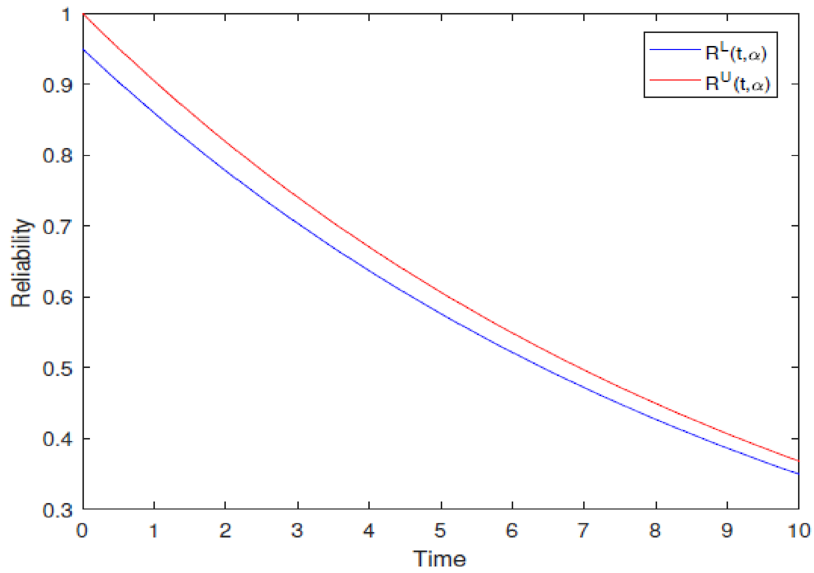

Figure 3. Fuzzy reliability in Example 4.1 in the time horizon [0; 10] for $\alpha = 0.5$

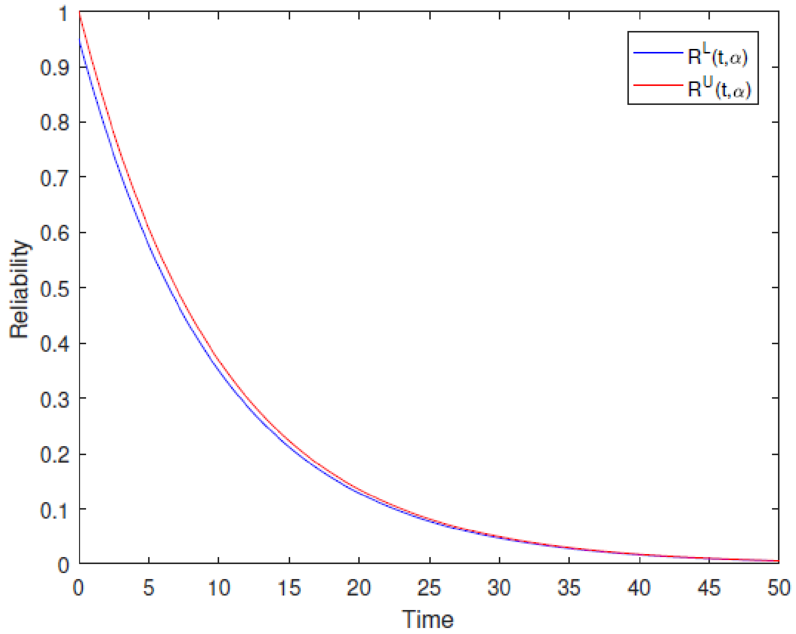

Figure 4: Fuzzy reliability in Example 4.1 in the time horizon [0; 50] for $\alpha = 0.5$

In (5.15) if we replace crisp reliability function R(t) with its fuzzy version $\tilde{R}(t)$ which now is calculated from (3.7), then we have:

$$FMTTF = \int_0^{+\infty} \tilde{R}(t)dt \tag{5.16}$$

which gives a fuzzy number. Suppose that for each $t \in [0,+\infty)$ we have $\left[\tilde{R}(t)\right]^{\alpha} = \left[R^L(t,\alpha), R^U(t,\alpha)\right]$. Since the upper and lower values of fuzzy reliability function are positive, thus, the α-cut of FMTTF can be calculated as follows:

$$[MTTF]^{\alpha} = \left[\int_0^{+\infty} R^L(t,\alpha)dt, \int_0^{+\infty} R^U(t,\alpha)dt\right] \tag{5.17}$$

Example 5.1 Suppose that we are going to calculate the FMTTF for Example 4.1. From Example 4.1 we had $R^L(t,\alpha) = (0.9+0.1\alpha)e^{-\lambda t}$ and $R^U(t,\alpha) = e^{-\lambda t}$. Thus we have:

$$[MTTF]^{\alpha} = \left[\int_0^{+\infty} (0.9+0.1\alpha)e^{-\lambda t}dt, \int_0^{+\infty} e^{-\lambda t}dt\right] = \left[\frac{(0.9+0.1\alpha)}{\lambda}, \frac{1}{\lambda}\right] \tag{5.18}$$

For $\lambda = 10^{-6}$ we have $[MTTF]^{\alpha} = 10^6\left[(0.9+0.1\alpha),1\right]$ which is plotted in Figure 5. As it can be observed in Figure (5), the values of FMTTF can be calculated for each $\alpha \in [0,1]$

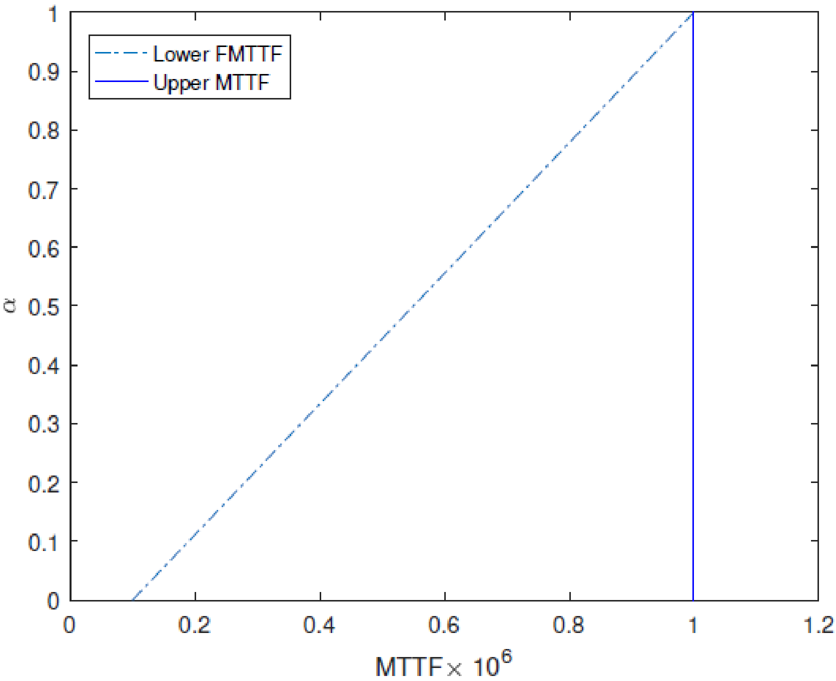

Figure 5. The value of FMTTF in 5.1

6. Illustrative examples

For the purpose of illustration, two numerical examples are provided here.
Example 6.1 In previous numerical examples, we considered a constant failure rate $\lambda(t) = \lambda$. Now, consider a special product, which has a failure factor $\lambda(t) = 0.0003t$. In this case the failure factor is a linear function of time which means that as time goes on, the failure increases with rate 0:0003. Also suppose that $\left[\tilde{R}(0)\right]^{\alpha} = \left[0.99 + 0.01\alpha, 1\right]$.We are going to calculate the reliability. The proposed fuzzy differential equation is as follows:

$$\begin{cases} \dfrac{dR^L(t,\alpha)}{dt} = -R^L(t,\alpha)\lambda(t), R^L(0,\alpha) = 0.99 + 0.01\alpha \\ \dfrac{dR^U(t,\alpha)}{dt} = -R^U(t,\alpha)\lambda(t), R^L(0,\alpha) = 1 \end{cases} \tag{6.19}$$

The solution can be observed in Figure 6. As it can be observed in Figure 6, both lower and upper bounds of fuzzy reliability function tend to zero as time tends to infinity. The different values of upper and lower reliability functions at t = 0 is due to fuzzy initial value of fuzzy reliability.
To illustrate the solution, the final fuzzy reliability solution of (6.19), is plotted in Figure 7 for time interval [0, 25]. As it can be observed in Figure 7, the final fuzzy reliability function has a triangular shape similar to the triangular initial condition. For example, when *t=25,* the fuzzy reliability $\tilde{R}(25)$ is a triangular fuzzy number which can be obtained from Figure 7. This fuzzy number is plotted in Figure 8 for different values of α.

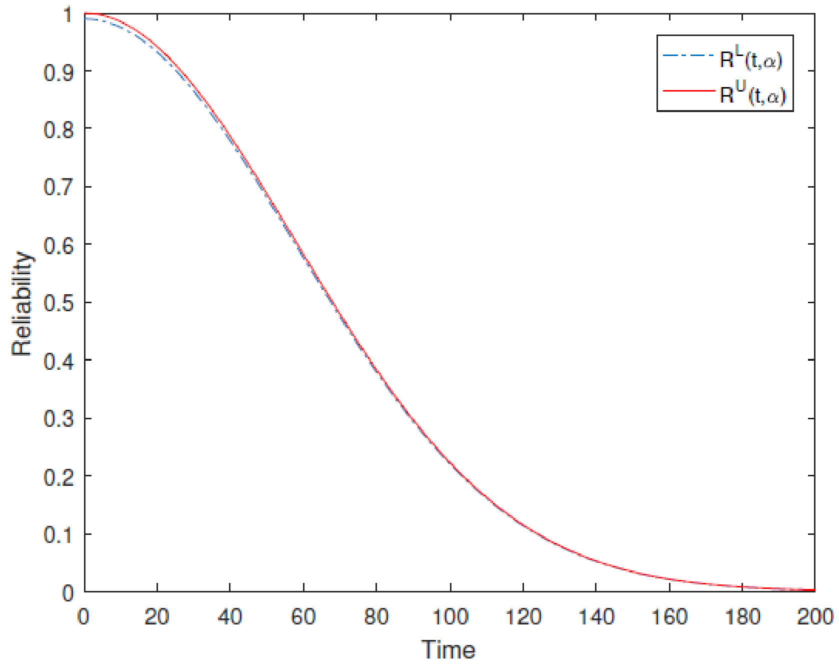

Figure 6. Fuzzy reliability in Example 6.1 in the time horizon [0; 200] for $\alpha = 0.5$

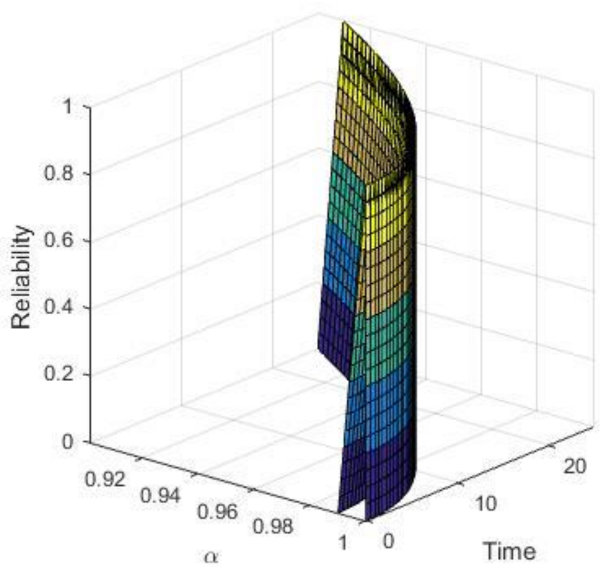

Figure 7. Final solution of Example 6.1

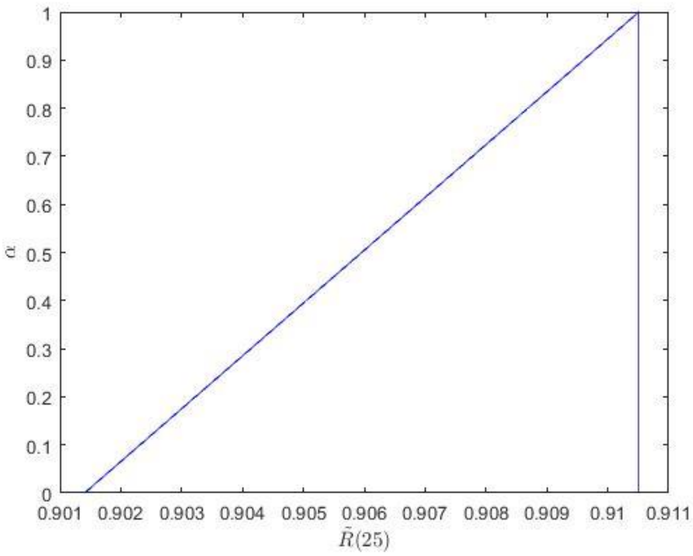

Figure 8. The fuzzy number $\tilde{R}(25)$ for Example 6.1

Example 6.2 In this numerical example, we use the proposed problem in Jamkhaneh (2011). Instead of a FDE approach, the author considered a fuzzy hazard value "about 0.7 to 0.85". Since, in this paper we use a crisp hazard function, suppose that $\lambda = 0.775$ which is the mean value of the fuzzy hazard function. Based on the second definition of fuzzy derivative, the fuzzy reliability solution for $\alpha = 0.5$ is plotted in Figure 9. Also, Figure 10 presents the fuzzy reliability $\tilde{R}(5)$. Note that $\tilde{R}(5)$, is a fuzzy number which shows the value of fuzzy reliability at time *t=5*.

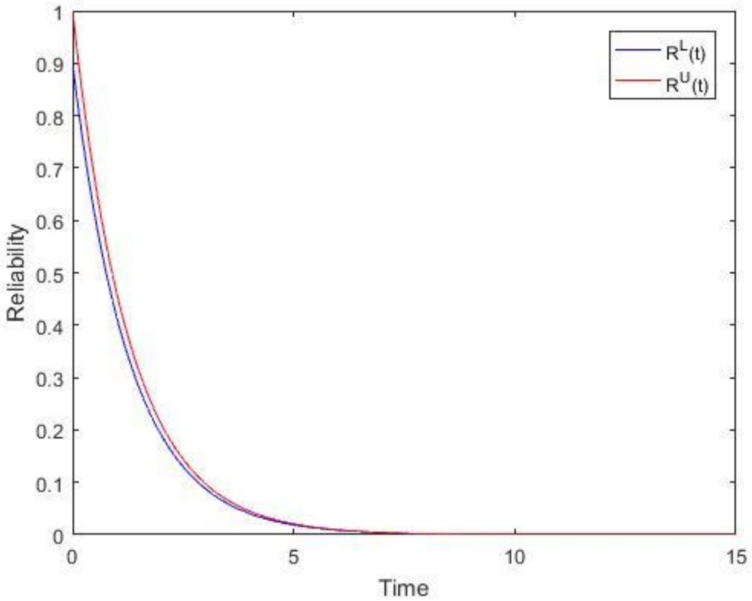

Figure 9. Fuzzy reliability in Example 6.2 in the time horizon [0; 200] for $\alpha = 0.5$

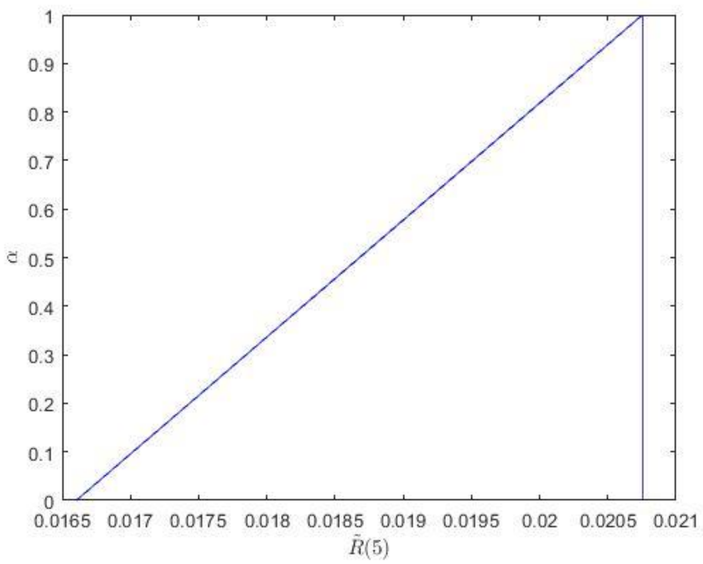

Figure 10. The fuzzy number $\widetilde{R}(5)$ for Example 6.2

Note that in comparison with Jamkhaneh (2011), we used the ability of fuzzy differential equation to obtain a fuzzy reliability function and fuzzy MTTF. Also, in this paper we used two different definitions of fuzzy derivative and proved that the second one is more accurate and applicable for fuzzy reliability analysis, thus, our approach is more general.

7. Conclusions and future works

In this paper, fuzzy reliability function was presented by employing fuzzy differential equations. Two types of fuzzy derivative were used to interpret and present the fuzzy reliability function. It was proved that just one of these types is valid for fuzzy reliability. Also several numerical simulations presented to illustrate and validate the proposed approach. Also fuzzy mean time to failure (FMTTF) was introduced by using fuzzy integral. In comparison with other existing approaches which are based on fuzzy probability distribution, in this paper, we introduced the concept of fuzzy reliability by using the concept of fuzzy derivative and fuzzy differential equation. For future work, the proposed approach can be applied for analyzing the reliability of parallel systems as well as for serial systems.

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