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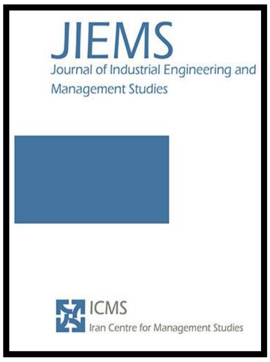

A Simulation Approach to Evaluate Performance Indices of Fuzzy Exponential Queuing System (An M/M/C Model in a Banking Case Study)

Mohammad Ehsanifar[1,*], Nima Hamta[2], Mahshid Hemesy[1]

Abstract

This paper includes a simulation model built in order to predict the performance indices such as waiting time by analyzing queue's components in the real world under uncertain and subjective situation. The objective of this paper is to predict the waiting time of each customer in an M/M/C queuing model. In this regard, to enable decision makers to obtain useful results with enough knowledge on the behavior of system, the queuing system is considered in fuzzy environment in which the arrival and service times are represented by fuzzy variables. The proposed approach for vague systems can represent the system more accurately, and more information is provided for designing queueing systems in real life. Furthermore, simulation method is applied successfully for modeling complex systems and understanding queuing behavior. Finally, a numerical example as a case study in a banking system is solved to show the validity of developed model in the real situation.

Keywords: Queuing theory; Fuzzy set; M/M/C model; Simulation; Uncertainty.

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1. Introduction

Queuing theory has been developed to provide models in order to predict the behavior of systems that attempt to provide service for randomly arising demands (Gross 2008). In many realistic situations, the statistical information is represented linguistically with qualitative data such as cold, cool, worm or hot rather than by probability distribution. Thus, fuzzy queues that can accommodate the ambiguities of real-world human language and logic are much more practical than the classical ones. For example, in queuing theory it is usually supposed that the time between two successive arrivals and servicing time follows special probability distribution, while in the real world this kind of information is expressed by words (Jolai et al. 2016). Most of the time queues formed by waiting customers are too long and nowadays this issue has become a major challenge for banks to manage time spent by customers in queues to increase the customers' satisfaction and decrease costs to stay competitive in today's economy.

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The fuzzy queuing model has been used parametric planning method using α-cut approach to analyze the fuzzy queue system (Bouazzi, Bhar, and Atri 2017). The problem of fuzzy queues has been analyzed using the extension principle (Barak and Fallahnezhad 2012). However, these approaches are fairly complicated and are not generally appropriate to computational purposes. Recent developments on fuzzy numbers by random variables can be used to analyze the queuing system. For example, the concept of fuzzy probabilities has been introduced and the properties of fuzzy probability Markov chains were discussed (Zhu, Ching, and Guu 2016). Fuzzy queuing models have been described by such researchers who have analyzed fuzzy queues using Zadeh's extension principle (Gou, Xu, and Liao 2017) and the membership functions of the system characteristics for the fuzzy queues using parametric linear programming has constructed (Chen 2004; Chen 2007).

In spite of the wide range of studies on queuing systems and applying fuzzy theory to them in some cases, there are a few researches have focused their attentions on that much time a customer has to spend in a queue so far, while by knowing that people can take great advantages of it to spend in other staffs; hence here it is tried to find an easy-to-use way to estimate this losing time.

In this paper, a model is presented to provide fuzzy performance measures for queues with fuzzified arrive rate and service rate and transform a fuzzy queue to a family traditional crisp queue by applying the α-cut approach based on Zadeh's extension principle.

2. Literature Review

2.1. Queuing theory

Queuing theory is a study of long waiting lines done to estimate queue lengths and waiting time. It uses probabilistic methods to make predictions used in the field of operational research, computer science, telecommunications, traffic engineering etc.

Yingjie Fu provided a waiting time prediction method for handling failed admission request in the mobile telecommunication system by neural networks (NN-WTP) (Yingjie Fu et al 2005). In this paper, an integrated adaptive framework, Qespera, for prediction of queue waiting times on parallel systems is presented. A novel algorithm based on spatial clustering for predictions using history of job submissions and executions is proposed (Murali et al. 2016). Quintas, Ana et al work aims at the development of a decision support system that allows one to predict how long a patient should remain at an emergency unit. It is built on top of a Logic Programming approach to knowledge representation and reasoning, complemented with a Case Base approach to computing (Ana et al 2016). Yoshida et al (2014) performed waiting time prediction using the data collected from prescription pick-up. A regression equation was used to predict waiting time. An explanatory variable was adopted as "a number of retention prescriptions at the time of acceptance", "the variety of drug to dispense" and "the presence or absence of a particular task of time-consuming work in computing multiple medications into one package". Analysis of the models helps to increases the performance of the system. In this paper various models of the Single server queuing system with necessary implementation using MATLAB Software are analyzed (Sameer 2014). The paper is concerned with the construction of the best linear predictor (in the mean-square sense) of the waiting time process in a simple delay system. Unlike in the usual formulation, it is only assumed that the process is stationary with known covariance function. Explicit expressions are obtained for a simplified joint distribution of waiting time of two calls (R. Syski).

Queuing theory predicts behavior of queuing systems due to variable demands by using mathematical calculations such as probability of having N customers in system, the average queue length, the average waiting time, maximum system capacity, etc. There are different applications of queuing theory, most of which have been categorized in areas such as probability, operations research, management science, and industrial engineering. Some instances are scheduling (patients in hospitals, jobs on machines, programs on a computer), traffic flow (aircraft, vehicles, people, communications), and facility design (banks, post offices, amusement parks, fast-food restaurants) (Gou, Xu, and Liao 2017).

Evaluation criteria of a queuing system are:

1. Queue length: the average number of customers in system.

2. Waiting time: the average time that a customer spends in the queuing system.

3. Occupation rate: the percentage of time that shows how busy a service is.

In this paper, the highly suitable modeling tool for MMC queueing model, the stochastic Birth-death Markov process is used. The Model M/M/C is a multi-channel queueing system with Poisson arrival and exponential distribution. Queue discipline is FCFS. This model is applied to HDFC Bank Chennai (Sundari et al 2011). In this paper, a QuickPass model is proposed to improve bank queuing system based on queuing theory. The current bank queuing system is modeled with M/M/c and analytical approach is adopted to guide optimization strategy. Through adjusted greedy algorithm, the multiple-optimization model is simulated with MATLAB. Simulation result shows that the introduction of QuickPass system considerably reduces the average waiting time of customers both in regular queue and QuickPass queue (Yu-Bo Wang et al 2010). (Abou-El-Ata et al 1992) discussed multi-server M / M / C / N queues with balking and reneging and they have derived steady state probabilities. (F.S.Q. Alves et al 2011) analyzed upper bound on performance measures of Heterogeneous M / M / C queues.

2.2. Fuzzy queue systems

Queuing theory is a mathematical conventional method to represent the different kind of queuing systems those appear in practice. For that reason, queuing models are very useful for determining how to control a queuing system in the most effectual way (Hillier and Lieberman 2005).

In recent decades, discrete event simulation (DES) has been widely used for analyzing construction projects. Recently, fuzzy discrete event simulation (FDES), which is an integration of fuzzy set theory with DES, has been proposed for simulating construction projects. FDES provides a framework to consider subjective uncertainty (uncertainty due to vagueness, subjectivity, and linguistic expression of knowledge) in construction simulation models. Using fuzzy logic to consider the subjective uncertainty of service time and the inter-arrival time of systems' queues may improve such simulation models by more realistically representing their results. This paper provides a novel methodology to consider subjective uncertainty in analyzing the fuzzy queues in construction FDES models (Sadeghi et al 2015).

Many models have been developed to solve the problem of queuing system while the inter arrival time and service time are supposed to follow certain probability distribution with fixed parameters, but finding statistical data for calculation of random variables is too difficult and impossible, like the time we have new system to expand when some information about the system is unavailable like linguistic expression. The concept of linguistic probability is not a new one and has been used consistently, for example, according to (Bonissone 1980) psychological studies in workplace; people seem reluctant to estimate exact probabilistic numbers so it’s reasonable that there is a possibility to estimate linguistically.

Generally, in practical applications that are based on subjective judgments, perceptions and inaccurate people's knowledge, it is reasonable for statistical data to be assumed uncertain and ambiguous (Ke, Huang, and Lin 2007).
In this situation, fuzzy queuing models can be much more realistic and more rational than common crisp models and by representing fuzzy sets or fuzzy variables instead of real numbers, data have wider and more useful application.
The reason why the concept of fuzzy logic is regarded as the considerable one is that human's interpretations and reasons are uncertain and approximate.
The paper considers a queuing system that has k servers and its interarrival times and service times are random fuzzy variables. A theorem concerning the average chance of the event "r servers ($r \leq k$) are busy at time t", provided that all the servers work independently is obtained. The average chance using fuzzy simulation method is simulated and some results on the number of servers that are busy are obtained (Fathi 2016). Fuzzy Queueing Theory (FQT) is a powerful tool to model queueing systems taking into account their natural imprecision. The traditional FQT model assumes a fixed number of servers with constant fuzzy arrival and service rates. It is quite common, however, to encounter situations where the number of servers, the arrival rate, and the service rate change with time. In those cases, the variability of these parameters can significantly impact the behavior of the queueing system and complicate its analysis (Enrique H. 2014).

2.3. Simulation

Simulation can be defined as a Process of designing a model of a real system and conducting experiments with this model for the purpose of understanding the behaviour of the system. Simulation can be applied for small and large systems. The cost of building the system is more and simulation provides a replica of the exact model with the behaviour of the system. Direct experimentation would cost more when compared with the simulated model of the system which is the main motive behind simulation. Simulation helps in delivering some very important decisions to be made by the system. It is a very efficient methodology to solve complicated problems. Simulation provides good strategy to analyze the client-server systems and help in better implementation of feasible solutions (Deshpande et al 1996). The behavior of a system as it evolves over time is studied by developing a simulation model. This model usually takes the form of a set of assumptions concerning the operation of the system. These assumptions are expressed in mathematical, logical, and symbolic relationships between the entities, or objects of interest, of the system. Once developed and validated, a model can be used to investigate a wide variety of "what if" questions about the real-world system. Potential changes to the system can first be simulated, in order to predict their impact on system performance. Simulation can also be used to study systems in the design stage, before such systems are built. Thus, simulation modeling can be used both as an analysis tool for predicting the effect of changes to existing systems and as a design tool to predict the performance of new systems under varying sets of circumstances (Carson et al 2005). Mageed A. Ghaleb et al ranked these alternatives to choose the best alternative to improve the efficiency of our system to get better service quality during rush hours. The Arena Process Analyzer to rank and select the best scenario beside the D&D procedures for ranking and selecting the best alternative is used (Mageed A. Ghaleb et al 2015). In this project, two types of queuing systems are examinated: the single-channel and the multiple-channels queues which are commonly seen in banks and fast food restaurants respectively. Computer programs are used to simulate the queues and predict the queue length, waiting time and wait probability (Joel et al., 2000). One of the popular topics in simulation science is queuing system simulation. For example, consider a bank queuing system, it is a complex system that its analysis by mathematical relations because of uncertainty in its queuing model is not easy. Simulation is a tool to simplify analysis of queuing systems.

Steps in Simulations

Designing a simulation consists at least of the following steps:

Setting objectives/Overall design

– Problem formulation

– Setting objectives and overall project plan

Model building

– Model conceptualizations

– Data collection (for input)

– Model translation

– Verification of the (computer) model

– Validation (accurate description of the real world)

Running the model

– Experimental design

– Production runs and analysis

Documentation and reporting

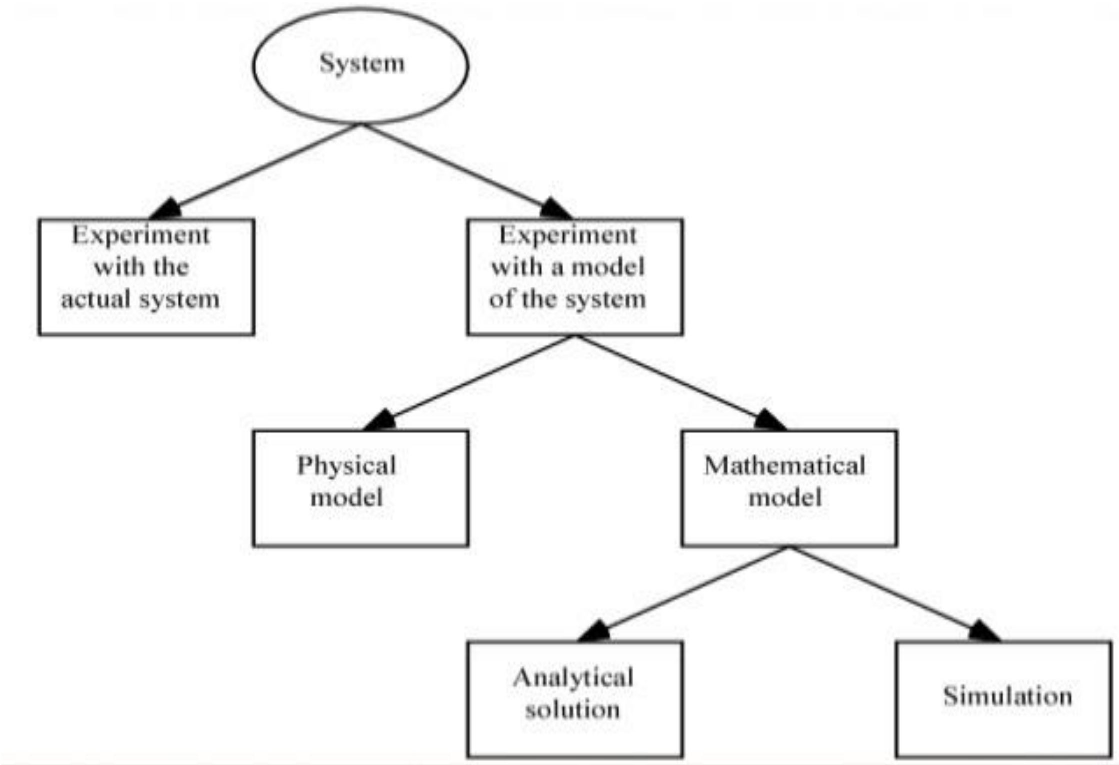

Figure 1. Basic Simulation Model (Güneş, 2012)

3. Preliminaries

3.1. Fuzzy theory

The theory of the fuzzy set was firstly introduced by (Zadeh 1965) with the membership function and then it has been well developed in an extensive range of real problems.

In classical logic being member of a set is considered as 0 and 1, this indicates that if a member entirely belongs to the given set, the value of 1 is assigned to that member, otherwise if a member is entirely out of set 0 is assigned (Yazdanbakhsh and Dick 2017). In fact, membership value (or degree of membership) a function that its range belongs to set {0, 1}.

A fuzzy set on an origin set X is written as a set of pairs as follow (Zadeh 1965):

$$A = \{(\mu_A(x), x) | x \in X, \mu_A(x) \in [0,1] \in R\} \quad (1)$$

In this paper, our fuzzy numbers will be triangular fuzzy numbers. A triangular fuzzy number $\widetilde{N}$ is defined by three numbers a < b < c where the base of the triangle is the interval [a, c] and its vertex is at x = b. Triangular fuzzy numbers will be written as $\widetilde{N} = [a, b, c]$ (Buckley 2005).

All of fuzzy sets have membership function as follows (Shavandi 2006):

$$\mu_N(X) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ \frac{x-c}{b-c} & b \le x \le c. \\ 0 & others \end{cases} \quad (2)$$

3.2. M/M/C Queue

The M/M/C queue is a multi-server queuing model. It describes a system where arrivals form a single queue and are governed by a Poisson process, there are C servers and job service times are exponentially distributed (Pardo and Fuente 2010).

Performance measures of M/M/C queue are (Cooper 1981):

L_s = Number of customers in the system= L_q + r

W_s = Sojourn time in the system= W_q + 1/μ

L_q = Average queue length= $\frac{\rho/C}{\left(1-\rho/C\right)^2} \pi_C$

W_q = Average waiting time = L_q / λ

Π = Average number of busy server, p = π

So M/M/C queue equations are:

$$L = \frac{\pi_0}{m!}\left(\frac{\lambda}{\mu}\right)^m \frac{p}{(1-p)^2} + \frac{\lambda}{\mu} = L_q + \frac{\lambda}{\mu}. \quad (3)$$

$$L_q = \frac{\pi_0}{m!}\left(\frac{\lambda}{\mu}\right)^m \frac{p}{(1-p)^2}. \quad (4)$$

$$W_q = \frac{\left(\frac{\lambda}{\mu}\right)^m \mu}{(C-1)!(C\mu-\lambda)^2} \pi_0 = \frac{L_q}{\lambda}. \quad (5)$$

$$W = \frac{1}{\mu}\left[\frac{\left(\frac{\lambda}{\mu}\right)^{m}\mu}{(C-1)!(C\mu-\lambda)^{2}}\right]\pi_0 = W_q\frac{1}{\mu}. \qquad (6)$$

4. Solution Procedure

If $\tilde{x} = [a, b, c]$ and $\tilde{y} = [p, q, r]$ are two fuzzy numbers their triangular membership functions are defined as follows (Shavandi, 2006):

$$\mu_x(n) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ \frac{c-x}{c-b} & b \le x \le c\cdot \\ 0 & \text{others} \end{cases} \qquad (7)$$

$$\mu_y(y) = \begin{cases} \frac{y-p}{q-p} & p \le x \le q \\ \frac{r-y}{r-q} & q \le x \le r. \\ 0 & \text{others} \end{cases} \qquad (8)$$

α-cuts are slices through a fuzzy set producing regular (non-fuzzy) sets. If A is a fuzzy subset of some set Ω, then an α-cut of A, written A[α], is defined as follows (Shavandi 2006):

$$A[\alpha] = \{x \in \Omega | A(x) \ge \alpha\}. \qquad (9)$$

For all α, $0 < \alpha \le 1$.

The first thing to do is determine the lower and upper bound of α-cut of $\tilde{x}$ and $\tilde{y}$. These values are calculated as follows (Pardo and Fuente, 2007):

$$\mu(X) = \frac{x-a}{b-a} = \propto. \qquad (10)$$

$$\mu(X) = \frac{c-x}{c-b} = \propto. \qquad (11)$$

Therefore:

$$X_\alpha = [a + (b-a)\propto, c - (c-b)\propto]. \qquad (12)$$

And for $\tilde{y}$ similarly we have:

$$\mu_{(Y)} = \frac{y-p}{q-p} = \propto. \qquad (13)$$

$$\mu_{(y)} = \frac{r-y}{r-q} = \propto. \qquad (14)$$

$$Y_\alpha = [(q-p)\propto + p\ , r - (r-q)\propto]. \qquad (15)$$

Note that Y_α and X_α are crisp sets rather than fuzzy sets.

To gain domains of $\tilde{Y}, \tilde{X}$ we set $\propto = 0$ and $\propto = 1$.

5. Application of Solution Procedure in Exponential Queuing Models

According to previous definitions consider a queuing system with a fuzzy arrival rate $\tilde{\lambda}$ and fuzzy service rate $\tilde{\mu}$. Since the distribution of times between arrivals and service times are exponential. The numbers of servers are considered to be independent and identically distributed. Customers are served under FIFO (First-In-First-Out) discipline and the system capacity is assumed infinite.

The random variable X is said to have an exponential distribution with parameter λ, (λ>0) (Hillier and Lieberman 2005):

$$F_{(x)} = \begin{cases} 0 & x < 0 \\ \int_0^x \lambda e^{-\lambda t}\,dt = 1 - e^{-\lambda x}, & x \geq 0 \end{cases} . \quad (16)$$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{others} \end{cases} . \quad (17)$$

$$E(X) = \frac{1}{\lambda} . \quad (18)$$

$$V(X) = \frac{1}{\lambda^2} . \quad (19)$$

Also lack of memory can be stated as (Hillier and Lieberman 2005):

$$P(X > s + t \mid X > S) = P(X > t) \quad s, t \geq 0. \quad (20)$$

This distribution is frequently used to model service time and arrival time. Therefore, for arrival time, it is defined as follows:

$$f(x) = \tilde{\lambda} e^{-\widetilde{\lambda x}} . \quad (21)$$

And similarly for service time:

$$f(x) = \tilde{\mu} e^{-\tilde{\mu} x} . \quad (22)$$

Relating to expressed algorithm, we consider two triangular fuzzy numbers for $\tilde{\mu}, \tilde{\lambda}$ as:

$$\lambda_{\tilde{\alpha}} = [a + (b - a) \propto, c - (c - b) \propto]. \quad (23)$$

$$\mu_{\tilde{\alpha}=}[p + (q - p) \propto , r - (r - q) \propto]. \quad (24)$$

In this step, we calculate α-cuts as follows:

$$\lambda = \frac{(a+(b-a)\propto)+(c-(c-b)\propto)}{2} = \frac{(a+c)+\propto(2b-(a+c))}{2} \quad 0 \leq \propto \leq 1. \quad (25)$$

$$\mu = \frac{(p+r)+\propto(2q-(p+r))}{2} \quad 0 \leq \propto \leq 1. \quad (26)$$

6. Numerical Example

To illustrate how expressed model can be used to analyzing fuzzy queue we consider a real situation in one of our local bank in which there are five bank counters with completely similar type of service, so it can be concluded that all of arrival times and service times are entirely random.

In this example, the waiting time that every customer averagely spends in bank's queue is to be predicted. To make a situation in which customers can estimate how long they would wait in queue, they can use that much time do their other activities. There are few studies about this subject has been done so far.

After determining sample size to collect data with observation and recording times in some days (7 hours per day) in row and analyzing those with *Arena*, following results were gained:

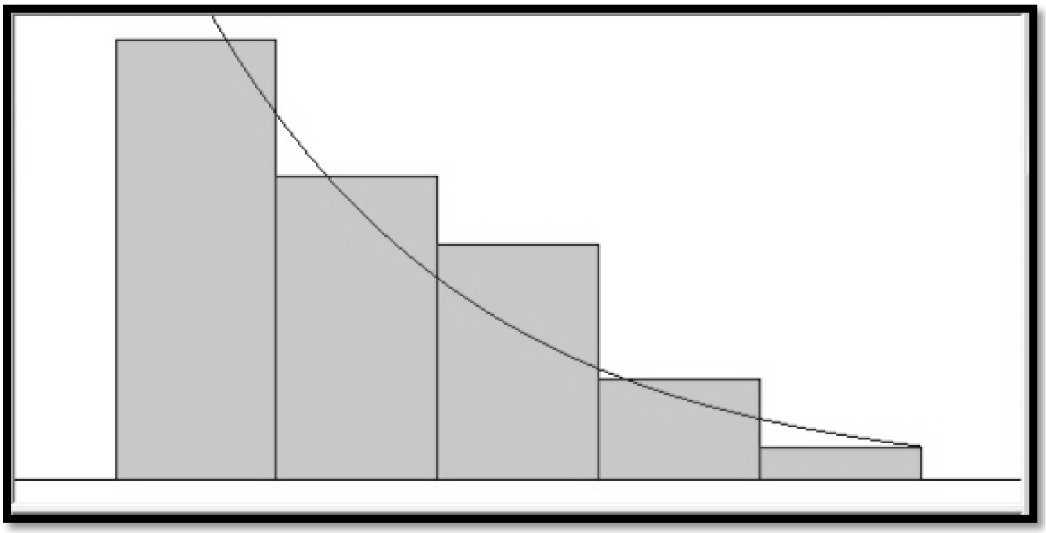

Figure 2. Service times' histogram

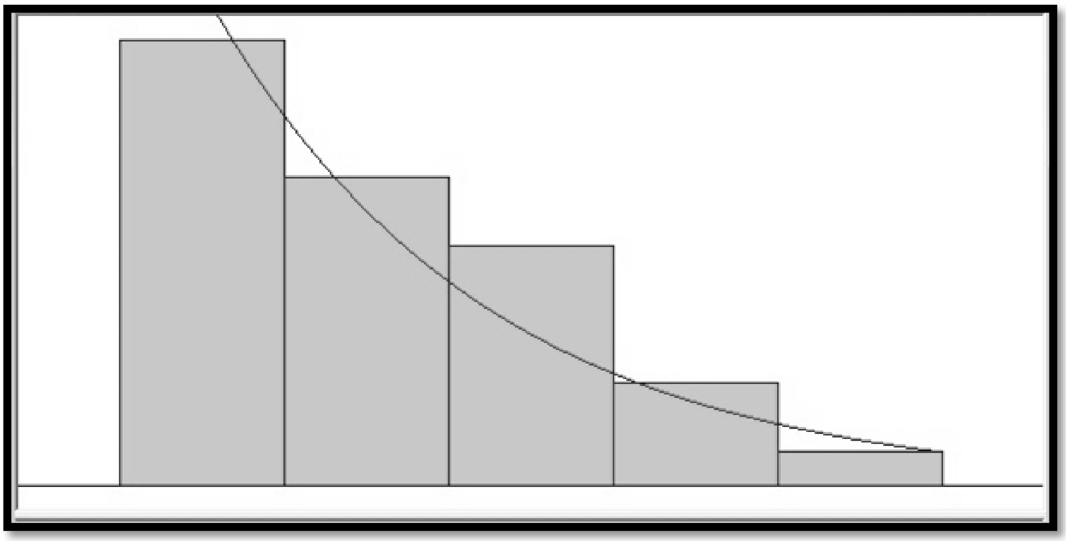

Figure 3. Arrival times' histogram

According to *Arena*'s calculation that estimates closest probability distribution for behavior of data by means of Chi-square test, the distribution of the times between arrivals and the service times as indicated in figure 2 and 3 are exponential. Given that all arrival times all service times are independently and identically distributed according to an exponential distribution and the number of servers is *C*, the bank's queuing system follows *M/M/C* model shown in Fig.4:

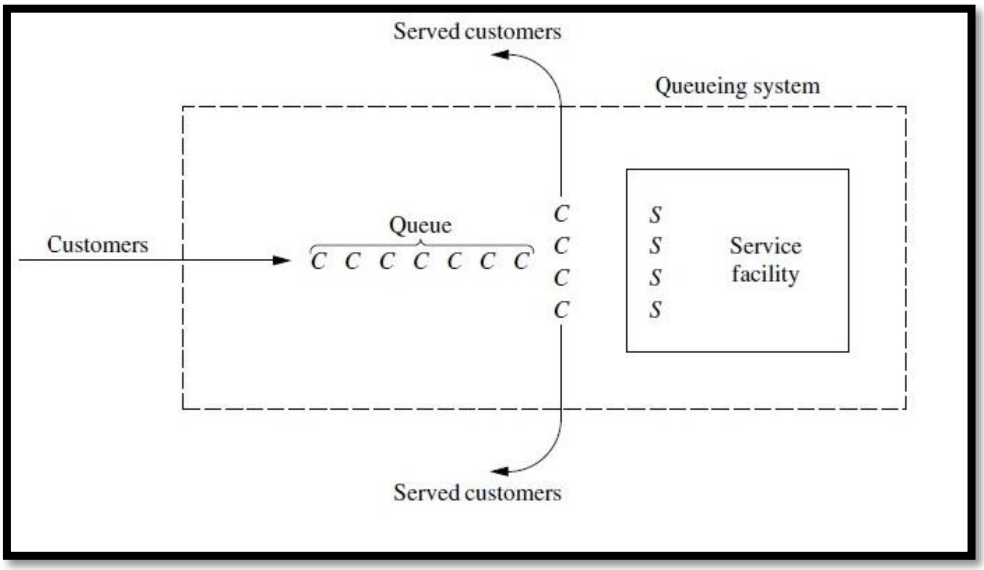

Figure 4. An elementary queuing system (each customer is indicated by a *C* and each server by an *S*) (Liberman 1967)

Considering mentioned method and Arena's analysis, the fuzzy numbers of λ and μ in our example are:
Fuzzy arrival rate:
$\tilde{\lambda} = (1.49, 2.3256, 2.94)$

Fuzzy service rate:
$\tilde{\mu} = (5, 7.8, 10)$

Now for α=0.1 we have:

$$\lambda = \begin{cases} \dfrac{x - 1.49}{2.3256 - 1.49} = 0.1 \quad => \quad x = 1.57356 \\ \qquad\qquad \lambda_{0.1} = \dfrac{1.57356 + 2.92356}{2} = 2.24856 \\ \dfrac{2.94 - x}{2.94 - 2.3256} = 0.1 \quad => \quad x = 2.92356 \end{cases}$$

$$\mu = \begin{cases} \dfrac{y - 5}{7.8 - 5} = 0.1 \quad => \quad y = 5.28 \\ \qquad\qquad \lambda_{0.1} = \dfrac{5.28 + 9.78}{2} = 7.53 \\ \dfrac{10 - y}{10 - 7.8} = 0.1 \quad => \quad y = 9.78 \end{cases}$$

For other amounts of α, λ and μ are computed similarly.

Following bellow in Table (1) we summarized L_q (Average number of customers waiting for service) and W_q (Average time customers wait in queue) computed from these λ_s and μ_s either mathematically or by simulation:

Table1. The results of traditional and fuzzy computing

α	λ	μ	L_q	W_q	Semi-L_q	Semi-W_q
0.1	2.24856	7.53	3.98127	1.77059	6.699	1.5055
0.2	2.28212	7.56	3.54904	1.55515	6.278	1.43
0.3	2.31568	7.59	3.16836	1.36822	5.915	1.3685
0.4	2.34924	7.62	2.83251	1.20571	5.619	1.3185
0.5	2.3828	7.65	2.53575	1.06419	5.233	1.2457
0.6	2.41636	7.68	2.27313	0.940724	4.959	1.1974
0.7	2.44992	7.71	2.04036	0.832828	4.679	1.1452
0.8	2.48348	7.74	1.83376	0.738381	4.403	1.0925
0.9	2.51704	7.77	1.65011	0.655576	4.171	1.0492
1	2.5506	7.8	1.48665	0.582861	3.94	1.0043

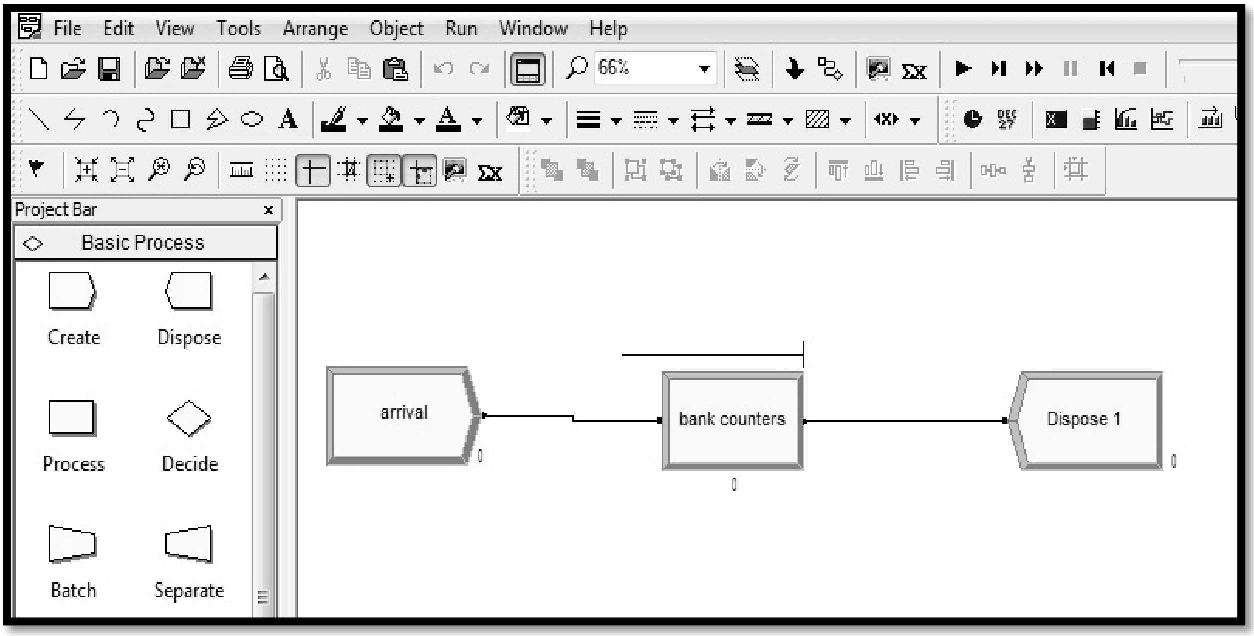

Figure 5. Simulation model of $\tilde{M}/\tilde{M}/5$

The case-study developed model by Arena is demonstrated in figure 5.

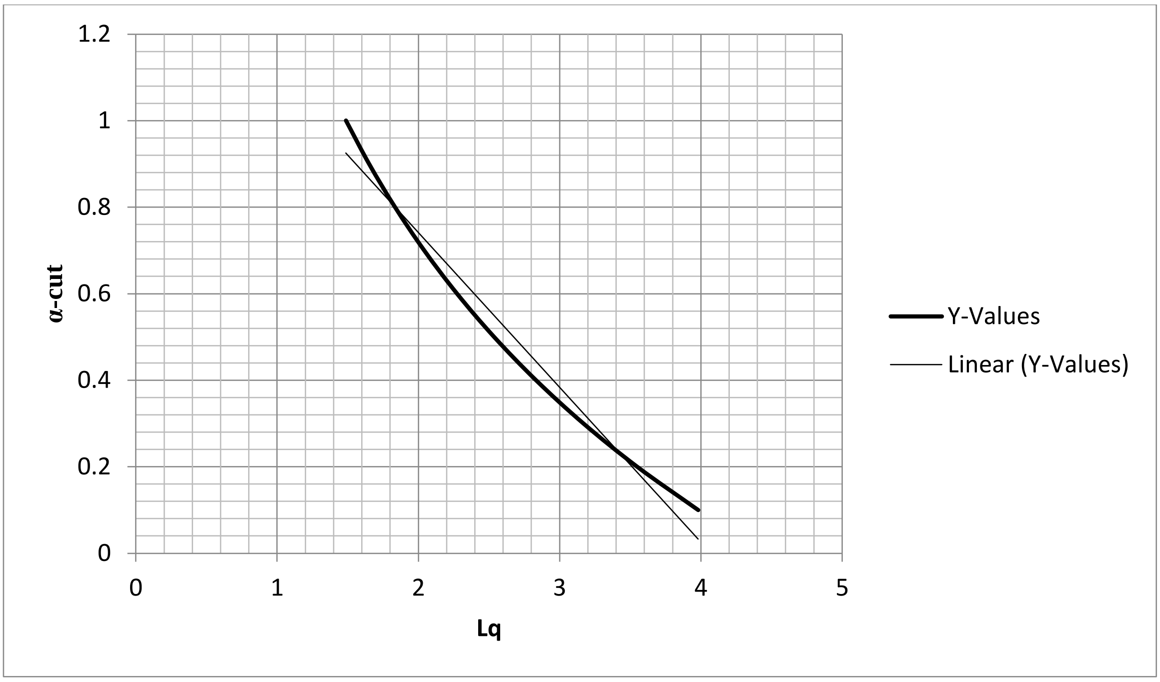

Figure 6. Average number of customers waiting for service in $M/M/5$

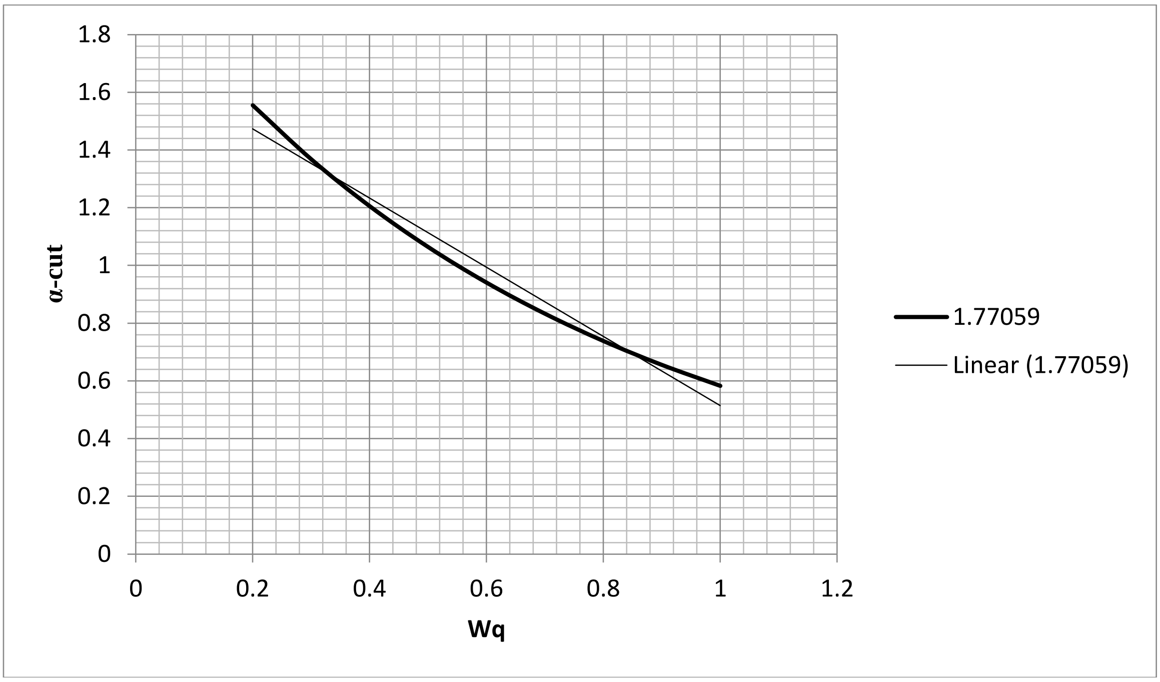

Figure 7. Average time customers wait in queue in $M/M/5$

The final results of Lq and Wq achieved from mathematical computation are indicated in figure 6 and 7 respectively.

Figure 8. Average number of customers waiting for service $\widetilde{M}/\widetilde{M}/5$

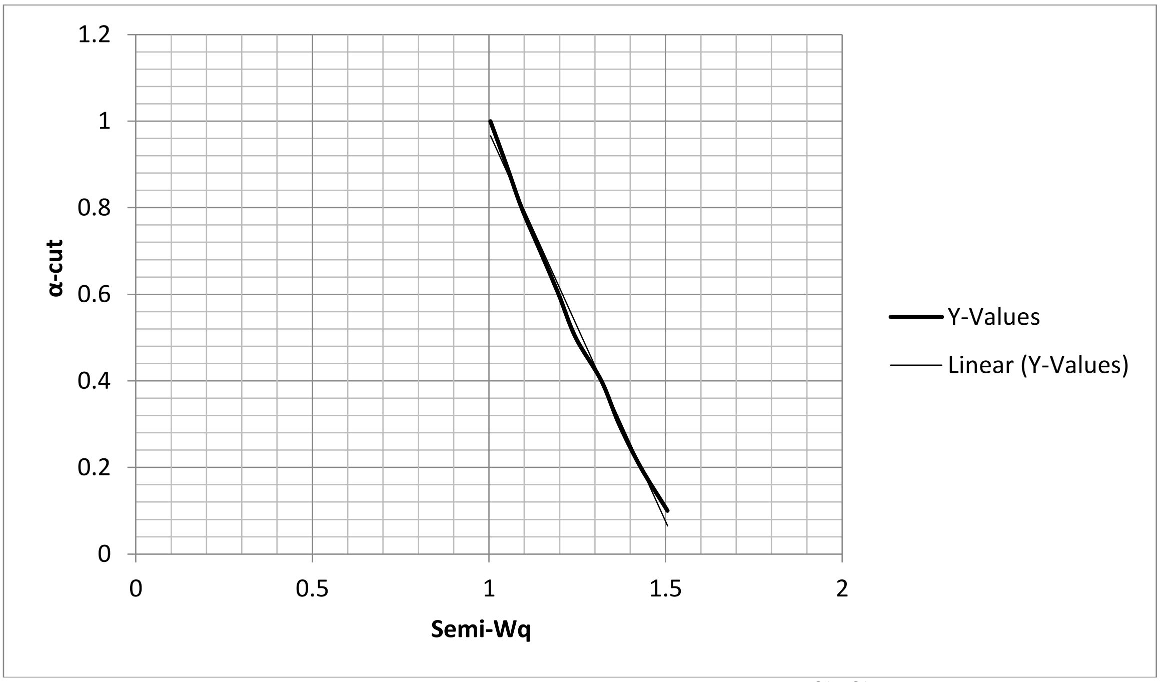

Figure 9. Average time customers wait in queue $\widetilde{M}/\widetilde{M}/5$

The final results of Lq and Wq obtained from simulation model are indicated in figure 6 and 7 respectively.

7. Discussion

When arrival and service rate are uncertain, performance measures will be uncertain too. The range of performance measures can be divided into different levels by applying α-cut approach.

Considering Lq in the numerical example, achieved range [1.48665, 3.98127] indicates that the number of customers vary from 1.48665 to 3.98127. Comparing this result with achieved Lq range from simulation, that is [6.699, 3.94], shows both ranges are very close to each other and this slight difference is due to limitation to replicate the simulated model to infinity, while the queuing formula are considered for infinite period of time as a default. For mentioned example, 1000 replications are set as default. The more simulated model is replicated and approached to infinity, the closer answer to theatrical computation is achieved. By continuing replication, finally the model reaches a point that from this point onwards the answer stays steady. This answer is exactly equal to theoretical answer. Comparing Lq and Wq charts obtained from two methods clarify this matter perfectly.

8. Conclusion

This paper attempted to develop a more practical distribution of queuing models by means of fuzzy set theory, when the arrival times and service times are fuzzy variable. It can be used to predict the delay that makes customers unsatisfied with delivery while in bank. Applying a software to get results instantaneously with running live data of customer arrival pattern, service delivery duration and identification of dissatisfied customers makes the analytics more lively. In this study because of uncertainly and complexity of these systems, we used simulation modeling with *Arena*.

To reduce a fuzzy queue to a family of crisp queue, α-cut approach has been applied and in order to confirm the validity of our model we have used triangular fuzzy numbers.

The validity of the model that have been mentioned is proved by solving an example of real queue system in one of our local banks.

After gathering and studying data, testing on arrival times and service times by *Arena* simulation software indicates the interaction between customers and servers is a typical queuing model as *M/M/C* model.

Service time (μ) and arrival time (λ) were computed by organizing an observational study at the particular bank. Comparing performance measures resulted from simulation process and mathematical calculating shows that *Lq* and *Wq* obtained from both methods are very close to each other. This slight difference is due to our limitation to run our model for infinitive times. The values are well-matched in infinity.

Study outcome indicates our expected result to predict a period of time that every customer spends in lines, are accomplished. By having this period particularly by utilization of *Arena* simulation software which is quick and easy to understand through a simple flowchart modeling approach, we can prevent queues from growing too long, So that customers' satisfaction and quality of services are increased; It is obvious that people would prefer to be informed about the average waiting time to spend time in more useful affairs instead of waiting in long queues and also for service providers, making delay announcements is a relatively inexpensive way of reducing customer uncertainty about delays and can manage their systems better, as a consequence customers can have long-term positive effects on the nation's economy while earlier queuing system for instance in railways is so rigid and decision to open additional counter is so slow, the result is passengers miss their trains, loose time, effort, money and satisfaction and hence less rise in train travel is observed however applying our model can make ticket servicing efficient and cost effective.

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Appendices A.

Table2. A sample of data are gathered from the bank in numerical example

Time	Numbers of arrival	Numbers of customer in queue
7:56’	2	2
8:06’	4	2
8:08’	2	3
8:38’	8	0
8:43’	4	0
8:53’	6	1
8:56’	1	1
9:01’	3	1
9:05’	7	1
9:17’	12	3
9:20’	4	4
9:34’	16	4
9:38’	5	0
9:44’	8	3
9:47’	2	1
10: 04’	19	3
10: 07’	6	7
10: 15’	8	5
10: 21’	3	0
10:25’	5	1
10: 28’	2	2
10: 39’	8	3
10: 42’	4	4
10: 43’	3	6
10: 51’	12	9
10: 57’	6	15
11:07’	16	13
11:09’	2	14
11:20’	11	16
11:26’	6	12

Appendices B.

Estimating closest probability distribution for behavior of data:
One application of *Arena* is to estimate closest probability distribution for behavior of data. The first step for this purpose is to create the list of data that must be save as a text, next we open data file from Input analyzer, at this time histogram of input data is drown automatically and some result of goodness-of –fit are appeared like: Chi squar, Kolmogorov-Smirove tests and P-value that is the most important part to guess a purpose distribution. If P-value is less than for example 0.05, results that the considered distribution is not acceptable.