

A discrete-event optimization framework for mixed-speed train timetabling problem

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Abstract

Railway scheduling is a complex task of rail operators that involves the generation of a conflict-free train timetable. This paper presents a discrete-event simulation-based optimization approach for solving the train timetabling problem to minimize total weighted unplanned stop time in a hybrid single and double track railway networks. The designed simulation model is used as a platform for generating feasible conflict-free train timetables. It includes detailed infrastructure information, such as station characteristics, trains running time and praying intervals. The proposed approach has the capability of scheduling trains in large-scale networks subject to the capacity constraints and infrastructure characteristics. In optimization procedure, a path relinking meta-heuristic algorithm is utilized to generate near-optimal train timetables. A case study of Iran railway network is selected for examining the efficiency of the meta-heuristic algorithm. The computational result shows that the proposed approach has the capability of generating near-optimal timetable in real-sized train scheduling problems.

Keywords: Train scheduling; Simulation-based optimization; Object oriented model; evolutionary path re-linking

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1. Introduction

Railway optimization problems and especially train timetabling are complex and data-intensive applications of large-scale transportation networks. The quality of a published train timetable directly impacts the service cost and quality. Train scheduling involves sequencing of trains over the railway network and this problem belongs to the class of NP-complete problems (Caprara et al., 2002, Garey and Johnson, 1979). Railway operations including arrival and departure of trains are exposing to stochastic disturbances. Evaluation of timetable stability and punctuality in a large scale railway network is highly cost and time-consuming procedure. Railway timetable planning is extremely complex, mainly because it is a decision problem under many uncertainties. Several uncertain factors may eventually result in railway operations and create some minor or major deviations from the pre-planned timetable. These major disruptions may cause schedule deviation from the planned schedule and make the

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current solution infeasible. In Iranian railway system, the main sources of delays are capacity degrade due to accident or engine breakdown, and severe weather condition (Hassannayebi et al., 2017). Designing an optimal train timetable in the rail network is one of the most difficult activities in the railway companies. In Iranian railway company, the train schedules are produced manually by rail experts that ensure high cost and time. The generated timetable must include the detail departure/arrival times at stations as well as the stop time and location for pray services of the passengers. According to the Islamic regulation, trains have to stop in the religious horizon intervals at stations for prayer services. This makes the distinction between the Iranian rail and the railways in the other countries.

Mathematical modeling approaches, analytical models, and the simulation models are alternative methods used to evaluate the impact of timetabling procedure on reliability (Hajian-Heidary and Aghaie, 2015). In this regard, simulation models are flexible tools and can be used to create the disruptions and to study how the delays spread on the whole network (Vromans, 2005). In the absence of tractable mathematical models for this complex combinatorial problem, simulation-based optimization is a flexible and powerful approach to tackling with challenges. The capability of simulation models is to increase the reliability of the train timetable by decreasing the average traveling time of passenger trains (Azadeh et al., 2008). This study aims to provide a practical simulation-optimization approach to train timetabling problem on rail networks. More specifically, the scope of this research includes the optimization train services in a rail network subject to the capacity constraints. This paper deals with an investigation of stochastic simulation models to evaluate and optimize railway timetables. In this study, the advantages of applying a simulation-based optimization approach are demonstrated for the railway industry. The contributions of the article are twofold: (1) designing an effective simulation-based optimization framework for timetable assessment of rail operations, and (2) an efficient meta-heuristic algorithm is proposed to minimize the total excessive stop time of the trains.

The remaining of the article is structured as follows. The literature review is provided in Section 2. The problem statement along with the assumptions are given in Section 3. The developed simulation-optimization approach is described in section 4. Numerical results of a case study are presented in section 5. The simulation outputs will be validated by comparing to the solutions that determined by the human dispatcher or generated by standard simulation-based optimization software packages. Finally, the conclusion and recommendations for further research are summarized in Section 6.

2. Literature review

Because of the complexity and growing importance of rail transport, there have been a growing number of studies that deal with the railway timetable analysis. In what follows, the most relevant contributions are discussed. (Noordeen, 1996) developed a stochastic discrete-event simulation (DES) system called FASTA for stability analysis of periodic Swiss network timetables. In FASTA different constraints are modeled such as minimum headway restrictions between trains and also train connections at stations. Carey and Carville (2003) focus on the improvement of the generated timetables by reducing the consequences of delay propagation in large stations by simulation approach. (Rudolph and Demitz, 2003) investigates approaches to optimize the allocation and size of time supplements in railway systems. In (Vromans, 2005), a discrete-event simulation model (SIMONE) is used for timetable robustness analysis. SIMONE accounts for many complex details in railway systems such as interactions between trains, headway times on the tracks, platform occupations, and connections for travelers. (Goverde, 2005) developed analytical approaches based on max-plus algebra to evaluate and quantify critical network dependencies on capacity utilization and timetable stability. The algorithms proposed in (Goverde, 2005) have led to

the development of the software application PETER (Performance Evaluation of Timed Events in Railways). Mattsson (2007) studied the joint relationships between the capacity consumption and train delay using simulation and statistical approaches. Dingler et al. (2009) examined the influence of train heterogeneity on the accessible capacity of the single-track railway systems. A simulation-based train dispatching method was established to study the effect of heterogeneity on capacity and train delays of a North American railway. (Khan and Zhou, 2010) develop a stochastic optimization formulation to incorporate segment travel time uncertainty and dispatching policies into a medium-term train timetabling process. They proposed a sequential decomposition solution approach that aims to minimize both the total trip times and the expected schedule delay. Medeossi et al. (2011) presented an approach for using stochastic blocking times to support precise conflict resolution in timetable planning. The proposed approach describes train conflicts by assigning a probability with each conflict predicted as a function of process time variability.

Simulation techniques are the most practical approaches for analysis of the complex transport problems; see e.g. (Dessouky and Leachman, 1995), (Confessore et al., 2009), (Motraghi and Marinov, 2012), (Sajedinejad et al. (2011), Eskandari et al. (2013), Hassannayebi et al. (2014), (Ilati et al., 2014), (Pouryousef et al., 2015), and (Keiji et al., 2015). Fröidh et al. (2014) proposed a simulation model for capacity assessment in a mixed traffic railway. The model adopts skip-stop services for local trains with the purpose of increasing the running speed for rapid trains. The result of case studies proves that the optimized train formation can improve the capacity consumption and at the same time decrease the undesirable effects of disturbances. Warg and Bohlin (2016) addressed the assessment of capacity and economic aspects for railways by simulation approaches. The validation process was performed using the historical delay data of rapid trains on a double-track corridor with highly dense and mixed traffic in Sweden railway. The simulation model was utilized to quantify the delay parameters of train timetables. The result of the capacity assessment model supports the rail planner to test scenarios in economic view. Xu et al. (2016) proposed a discrete-event model for rescheduling of subway trains in the case of an incident on a track of a double-track segment. The discrete-event simulation model accounts for the train position where the state variables are considered as a sequence of discrete events. The simulation model was enhanced with an optimization module that includes a rescheduling strategy. In addition, to avoid deadlocks, a capacity check procedure was designed. The optimization model is capable of obtaining a near-to-optimal disposition timetable with the least total delay time. The model was tested in a real-world case study of Beijing Yi Zhuang subway line of China to prove the efficiency of the proposed simulation-based optimization algorithm. Hassannayebi et al. (2016) presented an integrated simulation and optimization for railway systems under capacity degraded modes. The model is capable of adjusting the train stop plan and short turn services to minimize the total delay caused by a blockage. The successful application of the proposed simulation model was confirmed using a real case study of Tehran metropolitan network.

Zhou et al. (2017) addressed integrated optimization of train timetables and trajectory calculations in high-speed railways using space-time-speed networks. The designed model accounts for the train location, speed, and acceleration/deceleration. The optimization problem was solved by a Lagrangian relaxation-based method and a dynamic programming algorithm. The tractability of the approach was tested on a real case of the Beijing-Shanghai rail corridor. The outcomes indicated the efficiency and computational proficiency of the proposed algorithms. Bhaskar et al. (2017) presented a transfer coordination model using event-based multi-agent simulation platform. The objectives were the minimization of the passenger waiting time as well as the missed connections. The simulation model is capable of modeling the dynamic interfaces between the train and passengers. The simulation model was

calibrated using the collected data of the Automatic Vehicle Location (AVL) and Automatic Fare Collection (AFC) systems. The result of experiment demonstrates that the tuned transfer strategies can significantly decrease the transfer waiting time and the probability of missing a transfer. Stankaitis et al. (2017) designed an advanced railway simulation model for train operating systems. The simulation platform was developed on the basis of innovative verification procedures to support mixed traffic condition and different signaling and driving systems.

Taxonomy of the related studies has been provided in

Table 1. As can be seen, the majority of the existing approaches ignored the combined optimization of train schedule, sequence, and stopping patterns. The main research questions are (1) how to schedule the train stopping pattern and the timetable, (2) how the traffic can be handled by the available capacity and (3) how much extra stop time will be incurred for the designed timetable.

Table 1. Taxonomy of the simulation-optimization approaches to train timetabling problem

References	Modeling approach	Solution method	Objective function(s)	Stopping pattern	Station capacity	Mixed speed	Network topology
Suhl et al., 2001	Simulation approach	Buffer allocation	delay	-	-	✓	N*
Khan and Zhou, 2010	Simulation approach	Sequential decomposition solution approach	Total trip times and the expected schedule delay	-	✓	✓	L
Xu et al., 2014	Discrete event model	genetic algorithm (GA)	total delay time	-	✓	-	L
Dingler et al. 2014	Simulation approach	-	Capacity	-	✓	✓	L
Larsen et al., 2014	Simulation approach	branch and bound algorithm	delay	-	✓	-	N
Xu et al., 2016	DES	Simulation-optimization	Total delay time	-	✓	✓	L
Jensen et al., 2017	Stochastic simulation model	-	Capacity	-	✓	✓	N
Current study	DES	Simulation-optimization	Total unplanned stop time	✓	✓	✓	N

* N: Network, L: Line

The validity of deterministic operation planning models has been critically questioned and seriously criticized. In the absence of tractable mathematical models for the complex train timetabling problem, simulation is a flexible and wide-range method in timetable design of a rail system. According to the literature review, relatively a few number of studies have considered the integrated train scheduling and stop planning in railway systems, while the simulation modeling has been recognized in some of the previous papers. The objective of this study is to narrow the existing gaps in this field by finding an optimized train meet-pass

and stopping plans where the minimization of the total unplanned stop time is of interest. For this purpose, this paper proposes a discrete-event simulation modeling framework for mixed-speed train scheduling problems.

This study has several contributions to the field of train scheduling problem in railway networks. First, it provides an integrated simulation-optimization model to generate train plans under capacity constraints. Second, the constraints associated with train stops within the praying intervals are practically handled. In addition, the designed simulation model can generate a valid timetable under different scenarios. Third, the proposed approach has the capability of generating near-optimal solutions for real-world instances of the train scheduling problems.

2. Problem definition

In this section, the formal definition of the train timetabling problem along with the representation of the input parameters, and model's constraints are described in details. The rail network includes a set of stations and block sections and a set of operating trains (Fig. 1). Every line between two adjacent stations consists of a single or double block section. This follows the principle of absolute fixed block operations between two stations. In this case, every track between two stations corresponds to a single block section. Each train has a predefined route in the network. The signaling system controls the train traffic by imposing a minimum headway time.

The problem is to decide the departure and arrival time of all train at relevant sections, as well as the stop locations for performing praying services. A primary objective is defined as the total travel time of passengers. This objective comprises of the unplanned stop time due to capacity constraints. In this study, the unplanned stop times of train at stations are divided into two types. In the first type, a train stops for crossing due to the single-track section. The second type is related to the situation when a train stops as a result of the limited capacity of the next station (track occupied by train in the same direction). In other words, the trains must wait at the former stations to be ready to dispatch when the next station has a free track (or a free platform in case of passenger load/unloading). In the optimization model, we aim to minimize the total unplanned stop time of all trains. Although, the second delay time is very annoying for the passengers, thus must be minimized. For this purpose, we set a higher weighting coefficient for the second type of delay.

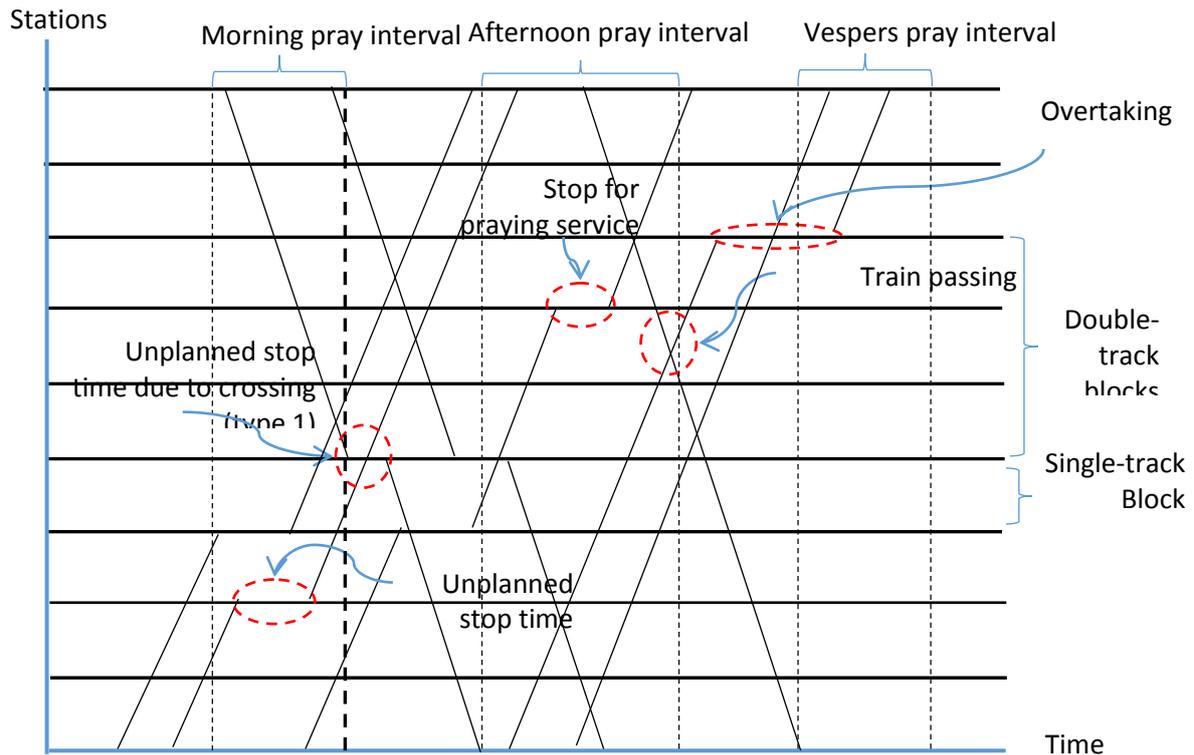


Figure 1. The time-station graph of train movements

3.1. Notations

Symbols used in the train timetabling model are summarized in Table 2. In order to accurately formulate the meet and crossing conditions of the trains in blocks and station lines, parameter v_{ij} is used to determine the direction of movement in any part of the rail network. Without loss of the generality of modeling, two directions (+1 for inbound, -1 for outbound) redefined for each segment. If the train does not pass block, the corresponding v_{ij} is equal to zero.

Table 2. The notation of the parameters and sets

Symbol	Definition
i	Index of train service
j	Block-station index
u	Index of track segment
k	Counter of train operations (running on the rail segments or dwelling at stations)
p	Index of praying services
J	Set of blocks/stations on the network
I	The set of scheduled trains that has not been dispatched at the moment of disruption
P	The set of religious services (morning, evening and Vesper)
J_i	The set of stations/blocks that i th train passes it ($J_i \subseteq J$)

E_i	Set of stations for passenger stop ($E_i \subseteq J_i$)
N	The total number of trains in rail network
H_j	The minimum headway between the arrival/departure of trains at the station j
α_{ik}	The kth operation of the train i
v_{ij}	The direction of movement of train i on section j {1,-1,0}
m_i	The total number of operations for ith train
p_j	The number of block section at the jth station
$t_{ij}^{min}, t_{ij}^{max}$	The minimum and maximum travel time or stop time of train i at jth block
d_i^{min}, d_i^{max}	Earliest and latest scheduled departure time of origin for ith train
q_{pj}	Stop time of trains at station j for prayer service within the pth religious horizon
γ_j	=1 If there is a mosque at station j, and 0 otherwise
L_p, U_p	The lower and upper bounds for performing pth religious services
M	A large positive number

Table 3. The definition of decision variables

Symbol	Definition
y_{ij}	Arrival time of train i to block/station j
e_{ij}	Departure time of train i from block/station j
t_{ij}	Running time or stop time of train i in block/station j
$S_{i_1, i_2, j}$	If the train i1 enters the block/station j before the train i2 is equal to 1 and 0 otherwise.
$B_{i_1, i_2, j}$	If the train i1 enters the block/station j after the train i2 is equal to 1, and 0 otherwise.
z_{iju}	If train i assigned to track number u in block/station j equals to 1, and 0 otherwise.
g_{pij}	If train i stops at the station j within the pth religious horizon is equal to 1, and 0 otherwise.
AP_{pi}	If train i have to stop within the pth religious horizon is equal to 1, and 0 otherwise.
τ_{pi}	If train i departs before the start time of pth religious horizon is equal to 1, and 0 otherwise.
φ_{pi}	If train i arrives at the destination after the end time of pth religious horizon is equal to 1, and 0 otherwise.

The objective function of the integrated train timetabling and stop planning model is the total travel time of trains which is calculated by Equation (1). The e_{i, α_i, m_i} represents the time of arrival for the train i , and $y_{i, \alpha_{i_1}}$ specifies the train departure time from the origin station. Equation (2) defines the running time constraint in the rail network. Equation (3) defines the departure time interval from the origin. According to equation (4), the start time of an operation is equivalent to the end of the previous operation. Based on the model assumptions, equation (5) states that the train running and dwelling times are within the minimum and maximum values. Conflict resolution constraints are described in Equations (6) and (7). The track allocation constraints are expressed in equation (8). Each train to pass through the station/block only can be assigned to one of the track segments. Finally, the logical relationship between the sequencing and assignment variables are defined in constraints (9) and (10). After recognizing whether a praying service is active for a train or not, the

optimization model determines the best stopping pattern for praying services. Equation (11) ensures that, in case of planned stop for praying, a specific station is assigned to each train to perform religious service. Constraints (12) imply that train can only stop at eligible stations for performing praying services. Constraints (13) define the conditions corresponding to the stopping pattern for pray services. Constraints (14) and (15) define the logical relationship that corresponds to the overlap between the train timetable and the praying interval. Constraints (16) define the lower and upper bounds for the time of praying services.

$$[\text{TTP}]: \text{minimize } \sum_{i \in I} (e_{i,\alpha_i,m_i} - y_{i,\alpha_{i,1}}) \quad (1)$$

$$e_{i,\alpha_{ik}} = y_{i,\alpha_{ik}} + t_{i,\alpha_{ik}} \quad i \in I, \quad 1 \leq k \leq m_i \quad (2)$$

$$d_i^{\min} \leq e_{i,\alpha_{i1}} \leq d_i^{\max} \quad i \in I \quad (3)$$

$$e_{i,\alpha_{ik}} = y_{i,\alpha_{i,k+1}} \quad 1 \leq k \leq m_i - 1 \quad (4)$$

$$t_{i,\alpha_{ik}}^{\min} \leq t_{i,\alpha_{ik}} \leq t_{i,\alpha_{ik}}^{\max} \quad i \in I, \quad 1 \leq k \leq m_i \quad (5)$$

$$y_{i_2,j} \geq e_{i_1,j} + H_j - M.(1 - S_{i_1,i_2,j}) \quad i_1, i_2 \in I, \quad i_1 < i_2 \quad v_{i_1,j} * v_{i_2,j} \neq 0 \quad (6)$$

$$y_{i_2,j} \geq e_{i_2,j} + H_j - M.(1 - B_{i_1,i_2,j}) \quad i_1, i_2 \in I, \quad i_1 < i_2 \quad v_{i_1,j} * v_{i_2,j} \neq 0 \quad (7)$$

$$\sum_{u=1}^{p_{\alpha_{ik}}} z_{i,\alpha_{ik},u} = 1 \quad i \in I, \quad 1 \leq k \leq m_i \quad (8)$$

$$z_{i_1,j,u} + z_{i_2,j,u} - 1 \leq S_{i_1,i_2,j} + B_{i_1,i_2,j} \quad i_1, i_2 \in I, \quad i_1 < i_2, \quad v_{i_1,j} * v_{i_2,j} \neq 0, \quad 1 \leq u \leq p_i \quad (9)$$

$$S_{i_1,i_2,j} + B_{i_1,i_2,j} \leq 1 \quad i_1, i_2 \in I, \quad i_1 < i_2, \quad v_{i_1,j} * v_{i_2,j} \neq 0 \quad (10)$$

$$\sum_{1 \leq k \leq m_i} g_{p,i,\alpha_{ik}} = AP_{pi} \quad i \in I, \quad p \in P \quad (11)$$

$$g_{pij} \leq \gamma_j, \quad \forall p, i \in I, \quad j \in J_i. \quad (12)$$

$$2.AP_{pi} \leq \tau_{pi} + \varphi_{pi} \leq 1 + AP_{pi}, \quad p \in P, \quad i \in I. \quad (13)$$

$$e_{i,\alpha_{i1}} \leq L_p + M.(1 - \tau_{pi}), \quad p \in P, \quad i \in I. \quad (14)$$

$$U_p \leq e_{i,\alpha_{i,m_i}} + M.(1 - \varphi_{pi}), \quad p \in P, \quad i \in I. \quad (15)$$

$$L_p - M.(1 - g_{pij}) \leq e_{ij} \leq U_p + M.(1 - g_{pij}) - q_{pj}, \quad p \in P, \quad i \in I, \quad j \in J_i. \quad (16)$$

$$S_{i_1,i_2,j}, B_{i_1,i_2,j}, z_{ijw}, g_{pij}, \varphi_{pi}, \tau_{pi}, AP_{pi} \in \{0,1\}, \quad y_{ij}, t_{ij}, e_{ij} \in \mathbb{R}^+. \quad (17)$$

The [TTP] model is formulated as an extended flexible job shop scheduling problem. It has been shown that the classic job shop scheduling problem belongs to the category of NP-Hard (Pinedo, 2015). Thus, the investigated problem belongs to the class of NP-Hard. Due to the computational complexity, the integrated simulation and optimization methods is proposed to solve the real-world instances of the train timetabling problems.

4. Simulation modeling approach

The railway network simulated in this paper consists of stations and links. The number of tracks and platforms inside the station modeled implicitly. The railway network topology is similar to a tree. Hence the routing of trains on the network is not really challenging part of modeling. Only just some bypass routes and limited connecting stations exist.

In this article, a simulation model is designed to model the operation characteristics of the rail systems. The simulation model has been developed in Enterprise Dynamics (ED) platform, which is a commercial object-oriented and discrete-event simulation software (Hullinger, 1999). The developed simulation model in is characterized as a discrete event and synchronous simulation to model dynamic and stochastic processes and it is very similar to SIMONE (Middelkoop and Bouwman, 2001). The developed timetable optimization model not only is capable of generating a timetable but also is can estimate the train delays. The detection of possible resource conflicts between trains is handled according to the reservation of tracks and platforms and upcoming trains.

The architecture of the simulation-based decision support system is illustrated in Fig. 2. Briefly, the system is designed for infrastructure modeling layout, a simulation module, optimization module, a schedule evaluation module, and a graphical user interface (GUI). The proposed simulation-based decision support system has the capability of generating a conflict-free train schedule through a fast simulation engine. The model developers can access standard libraries of atoms to build simulation models. ED's object-oriented architecture enhances the functionality of the simulation system. In addition to standard libraries, the modeler can design a customized library of objects (atoms) for modeling purposes. Successful applications of the Enterprise Dynamics in railway context can be found by (Hassannayebi et al., 2014, Hassannayebi et al., 2016, Middelkoop and Bouwman, 2001). In the language of ED, an atom is a multi-dimensional object. Each atom has different attributes e.g. size, location, speed, and dynamic behavior over time. Atoms can correspond to a resource, an entity, a data set, or a specific report (e.g. table or graph). As illustrated in Fig. 3, the developed simulation model of rail operations is composed of atoms that can be categorized as moving entities (vehicles), servers (rail segments), and queues (depots). Simulation engine implements an event handler that defines the behavior of an object. For example, when a train enters a station, the On-Entering event handler of the train is executed. The On-Entered event handler is executed when another moving object (e.g. a train) enters the container object (e.g. a block segment). Likewise, the On-Exiting and On-Exited event handlers are utilized to trigger the exit (or departure) events.

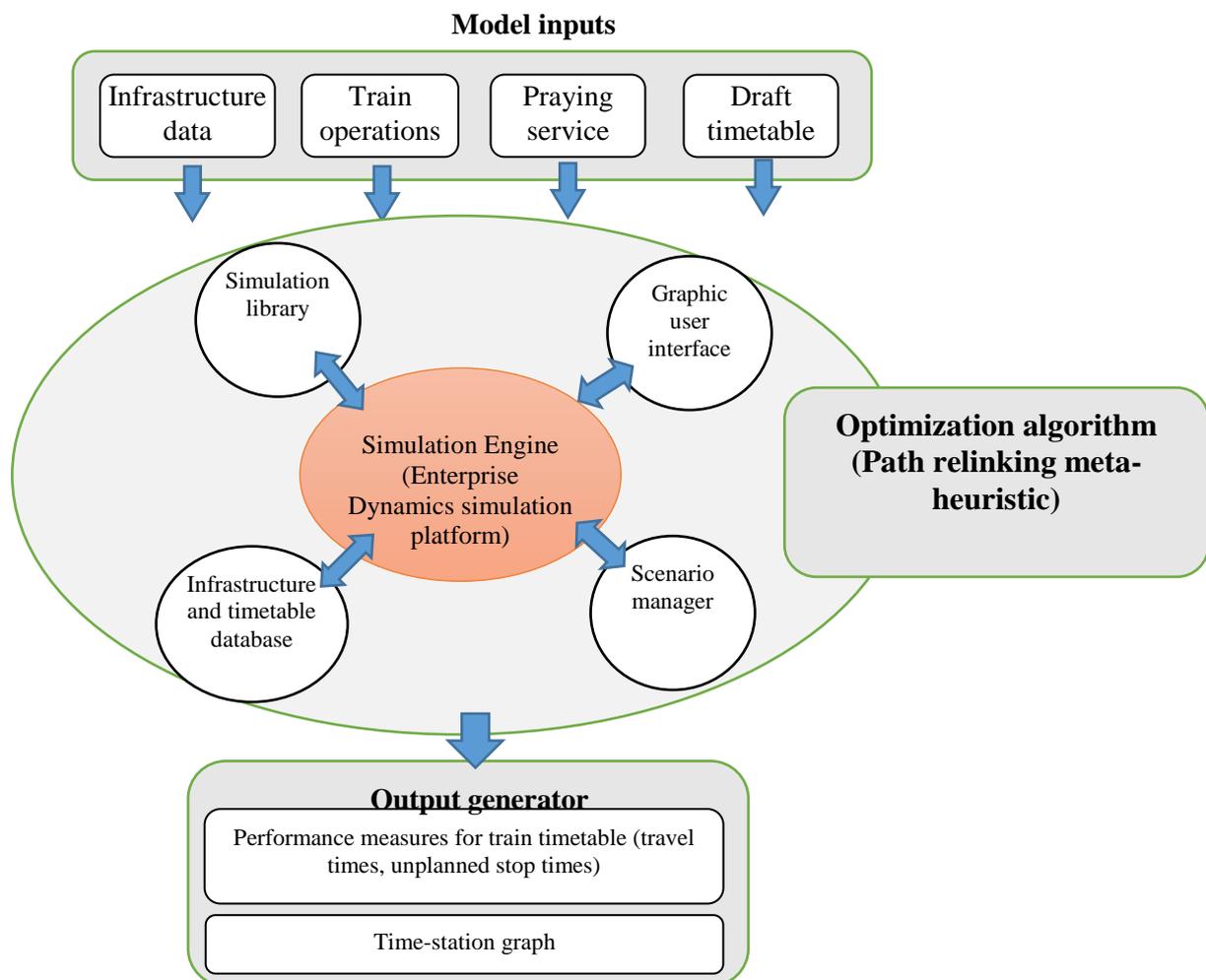


Figure 2. The architecture of the designed decision support system for train scheduling

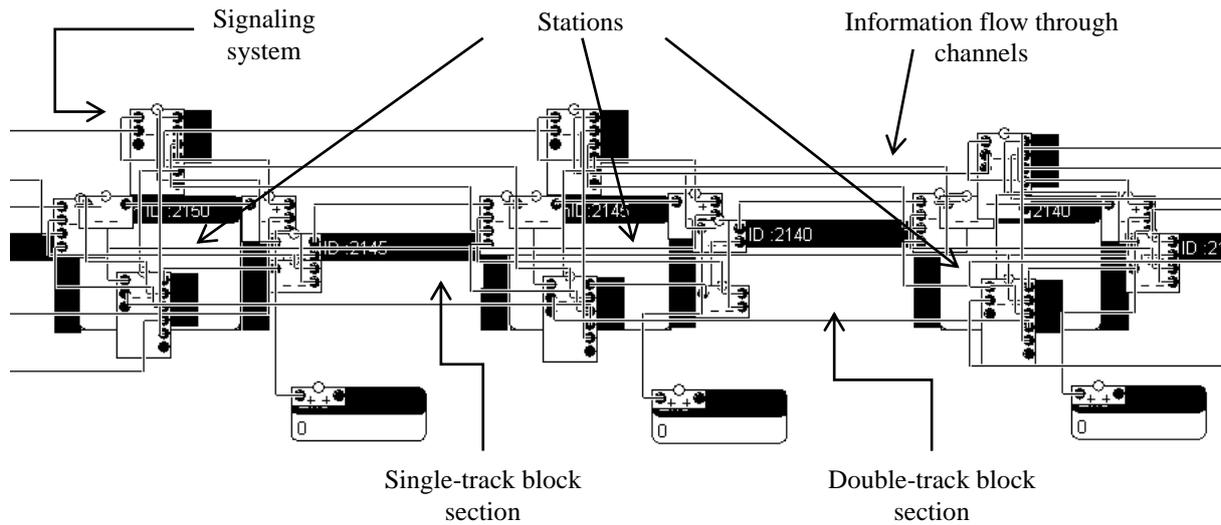


Figure 3. A partial view of the designed simulation model in two-dimensional layout of ED

5. Solution method: path relinking algorithm

This article presents a Path relinking meta-heuristic algorithm for designing a conflict free train timetable. Path relinking algorithm was first presented by Glover and Laguna (1993) as a search strategy of exploring solution space between references solutions (Laguna and Glover, 1993). Path relinking algorithm is based on the process of generating a series of reference solutions. The process of creating a path of solutions leads to the set of combined solutions which may have better quality compared with elite solutions. The reader is referred to (Lai et al., 2016) for a recent review of path relinking algorithm and its extensions. Path relinking uses logical rules for linking solutions by combining promising elements of elite solutions. Each path of solutions can be constructed by using a starting solution and a directorial solution (selected from the reference set of elite solutions). The idea of generating path is to gradually introduce the promising attributes of the elite solution into the intermediate solutions. The moving direction of the generated paths can be forward, backward or hybrid.

The decision variables in the optimization model are defined as dispatch times of trains at the origin. The dispatching time takes places within a specified interval (usually an hour) in 5-minute slots ($step = 5$), e.g. 8:00, 8:05, 8:10, etc. For solution representation, a series of departure time are selected. One potential solution to the problem is represented by a sequence of integers $S_i = [d_1, d_2, d_3, \dots, d_n]$. The initial departure time of j th train is denoted by d_j^1 and allowed time interval is defined within the range $[d_j^{\min}, d_j^{\max}]$. The fitness function (F) is obtained by equation (18), where C_{ij} and T_{ij} are the non-planned stop time of types I and II at station i , respectively.

$$Fitness = \lambda \cdot \sum_{j=1}^n \sum_{i=1}^{m_j} T_{ij} + (1 - \lambda) \cdot \sum_{j=1}^n \sum_{i=1}^{m_j} C_{ij} + \sigma \cdot \sum_{p=1}^k \sum_{j=1}^n pen_{pj}, \quad 0 \leq \lambda \leq 1 \tag{18}$$

Scaling factor (λ) determines the relative importance of the two delay types. According to Iranian railway preferences, the second delay type is of more importance as against the first one. In addition, the penalty coefficient (σ) calculates the amount of infeasibility regarding the non-regular stop time for praying services. The penalty coefficient, $pen_{pj} \in \{0,1\}$, determines the feasibility of stopping pattern for j th during the p th praying interval.

Suppose two solutions x and y . The set $\Delta(x, y)$ is constructed by including non-common elements of these solutions. In the path generated from x to y , their shared characteristics remain unchanged. The algorithm starts from an initial solution and the best move of set

$\Delta(x, y)$ that has not been used, is performed. This process continues until we reach to the solution y . The best intermediate solution is regarded as the output solution. The strategy of moving between initial solution and the guiding solution include forward, reverse, mix, and random. These differences rely on the selection of the initial and guiding solutions, the objective value as well as the moving direction between the two solutions.

The general framework of the integrated simulation-optimization method is illustrated in Fig. 4. In the proposed algorithm, a forward path relinking strategy is used. In this way, starting from the initial solution x , the target solution (y) is constructed progressively while keeping the common features of these two solutions. Featured from evolutionary optimization algorithms such as genetic algorithms and scatter search, path relinking can explore the problem space and generate a population of solutions. The population evolves by producing solutions that combine the solutions of the previous population. Thus, during the evolving process, a series of population P_0, P_1, P_2 are constructed. The best-obtained solutions are defined as the reference population. The evolution of solution population is performed based on the criteria of quality and a variety of search tricks. The proposed algorithm starts by generating random initial solutions. The solutions are evaluated within the simulation model to determine the average value of the fitness function. The solution population evolves until stop criteria have been met.

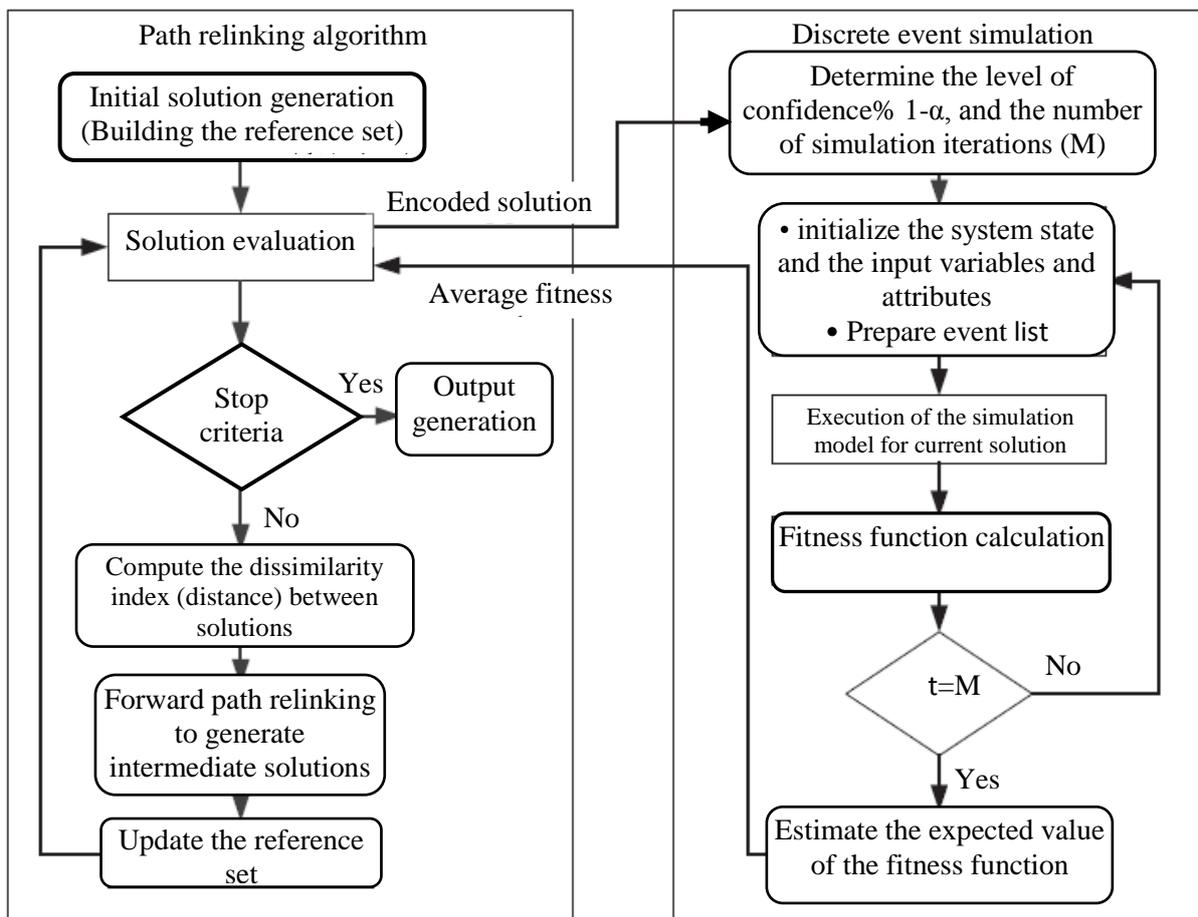


Figure 4. The general framework of the integrated simulation-optimization method

If one of the following conditions is satisfied, a candidate solution enters to the reference set: The solution is better than all the solutions in the reference set.

The objective function value of the candidate solution is within the range of the best and worst objective values of the solutions in the reference set as well as the distance between this solution and the solutions in the reference set is larger than a specified amount.

The candidate solution should be replaced by one of the solutions of the reference set, as w . The solution w is not better than the candidate solution. In addition, it should be the closest (most similar) one compared with other solutions in the reference set. To determine the similarity between the two solutions, one must measure the distance between two defined solutions. The distance between solutions x_1 and x_2 is defined by the symbol $d(x_1, x_2)$ that is obtained by equation (19). In this equation, the departure time of j th train for solution x is denoted by t_j^x .

$$d(x_1, x_2) = \sum_{j=1}^n |t_j^{x_1} - t_j^{x_2}| \tag{19}$$

6. Result and discussion

6.1. Numerical example

A numerical example is provided to verify the performance of the simulation model and the optimization algorithm. In this example, the 10 inbound and 10 outbound trains are scheduled on a partially single and double-track line with 21 stations. Two out of track segments are double track. The velocity class of the train is given which specify the running time on the sections. The stations with mosque belong to the set $N = \{5,7,8,11,13,16\}$. The input data of the numerical example is given in Table 4.

Table 4. The input data of the numerical example

Train number	Velocity class (km/hour)	Initial departure time
1	150	12:00
2	150	13:00
3	115	12:00
4	115	13:00
5	110	13:00
6	110	14:00
7	110	14:00
8	150	15:00
9	120	15:50
10	110	16:15
11	110	11:00
12	110	11:20
13	110	17:00
14	110	14:30
15	120	14:25
16	110	12:15
17	110	14:45
18	110	16:20
19	110	17:10
20	110	13:30

The data of the rail infrastructure, operational parameters, and religious horizon are given in Table 5. The departure tolerances are defined as ± 30 , ± 60 , and ± 120 minutes. Trains stop time intended for prayer is 20 minutes. All problem instances have been solved using the penalty coefficient $\sigma=50$ minutes. The simulation runs are executed via Enterprise Dynamics 8.2.5. All the experiments are performed on an Intel(R) Core 2 Duo personal computer with

3.3 GHz and 4 GB of RAM. The optimization algorithm was coded with a built-in scripting language (4DScript programming language) of ED software.

Table 5. The data of the rail infrastructure and operational parameters

Station number	Planned dwell time (minutes)	Total number of lines	Number of platforms	Noon prayer time	The end of the stoppage period	Vesper prayer time	The end of the stoppage period
1	0	2	2	13:00	14:30	19:50	21:20
2	0	2	1	-	-	-	-
3	2	2	2	-	-	-	-
4	0	2	2	-	-	-	-
5	2	2	2	13:00	14:30	19:50	21:20
6	0	2	1	-	-	-	-
7	0	2	1	13:00	14:30	19:50	21:20
8	0	2	1	13:00	14:30	19:50	21:20
9	0	2	1	-	-	-	-
10	0	3	1	-	-	-	-
11	5	3	2	13:05	14:30	19:55	21:20
12	0	3	1	-	-	-	-
13	2	3	1	13:05	14:30	19:55	21:20
14	0	3	1	-	-	-	-
15	0	3	1	-	-	-	-
16	2	3	1	13:05	14:30	19:55	21:20
17	0	3	1	-	-	-	-
18	0	3	1	-	-	-	-
19	0	3	1	-	-	-	-
20	0	3	1	-	-	-	-
21	0	3	1	13:10	14:30	20:00	21:20

In what follows, the performance of the meta-heuristic algorithm for various parameters is tested. For this purpose, 9 scenarios were designed in accordance with the data given in Table 7. In each scenario, the algorithm run for 30 minutes and the best-found solutions are reported. Furthermore, the initial population and the pop-size are selected in accordance with increased departure tolerance. In the problem instances, the number of possible choices for departure time of train j is $a_j = \left\lfloor \frac{d_j^{\max} - d_j^{\min}}{\text{step}} \right\rfloor$. As a result, the total number of potential solution to train dispatching problem is $\prod_{i=1}^n a_i$.

The total number of eligible stations to stop for praying services is denoted by L . In addition, the pray services are performed at H intervals over the planning horizon. Thus, the maximum number of stops during the praying interval equals to $LH.n$. Accordingly, the maximum number of feasible schedule based on the model parameters is denoted by $L^{H.n} \cdot \prod_{i=1}^n a_i$. It can be seen that by increasing the number of trains, the computational complexity is growing exponentially and thus the problem belongs to a class of NP-Hard

Table 6. The sensitivity of the optimization model against algorithm's parameters

Scenario	Departure tolerance	λ	K	Pop-Size	F(xbest)	Total unplanned stop time (hours)	
					(minute)	first type	second type
1	±30	9	300	10	62	10.03	2
2	±30	0.7	300	20	190	10.01	14
3	±30	0.5	300	30	291	9.11	35
4	±60	9	500	10	60	9.7	2
5	±60	0.7	500	20	144	7.84	4
6	±60	0.5	500	30	276	8.68	31
7	±120	9	1000	10	51	8.5	0
8	±120	0.7	1000	20	124	6.77	3
9	±120	0.5	1000	30	216	7.1	6

According to the obtained result, the performance of the algorithm has been improved by increasing the departure time tolerance. In addition, the choice of $\lambda = 0.7$ has a positive impact on the convergence of the algorithm. The result indicates that the scenarios 1,5, 8 are in high desirability due to the lowest unplanned stoppage time of trains. The time-space train diagram in scenarios #1, #5, and #8 are given in Figure 5 to Figure 7, respectively. As can be seen, in scenario 1, due to the compressed schedule, as anticipated, an overtaking has occurred between the train 2 and 16. But in scenarios 5 and 8, because of the increased range of traveling, there is no overtaking there.

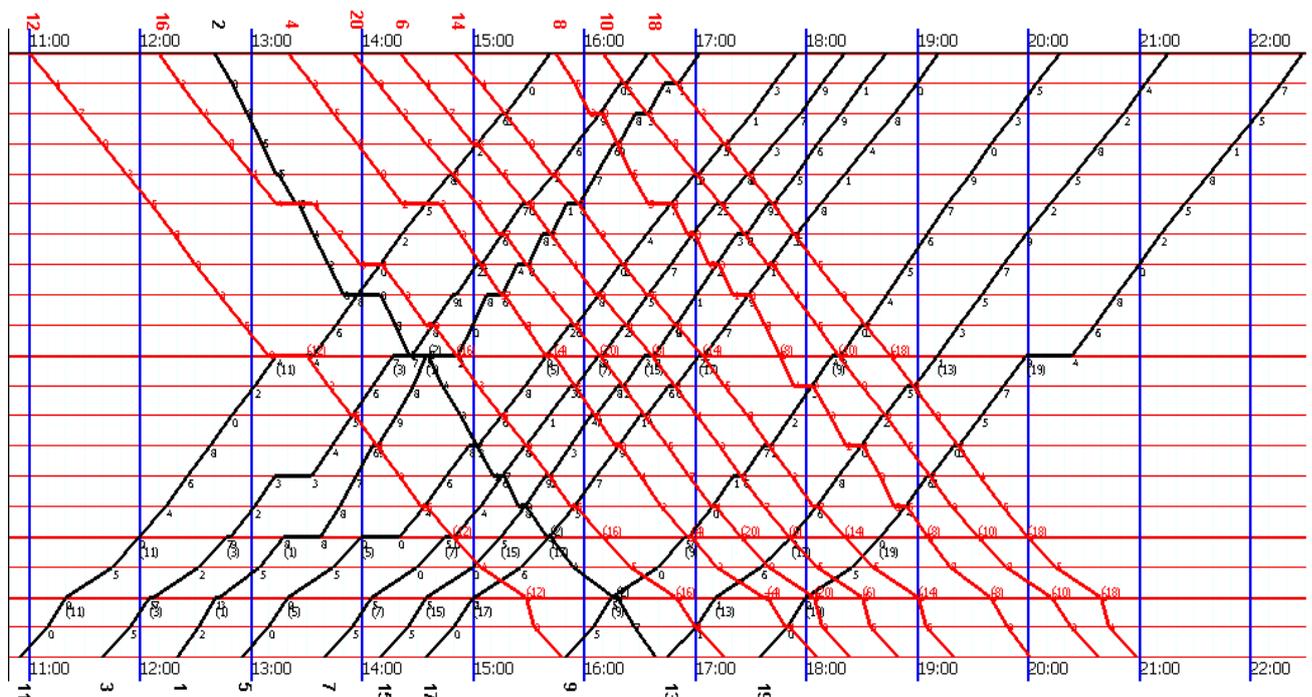


Figure 5. The time-space train diagram in scenario #1

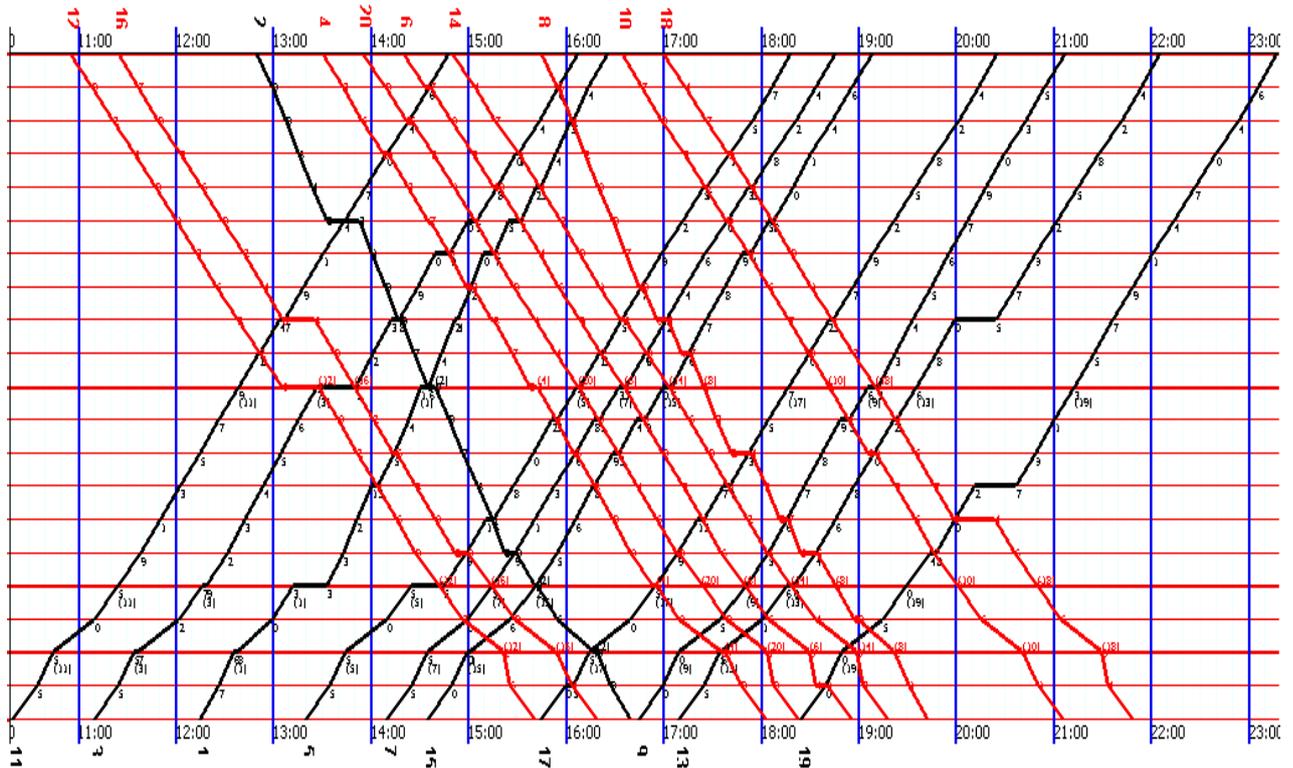


Figure 6. The time-space train diagram in scenario #5

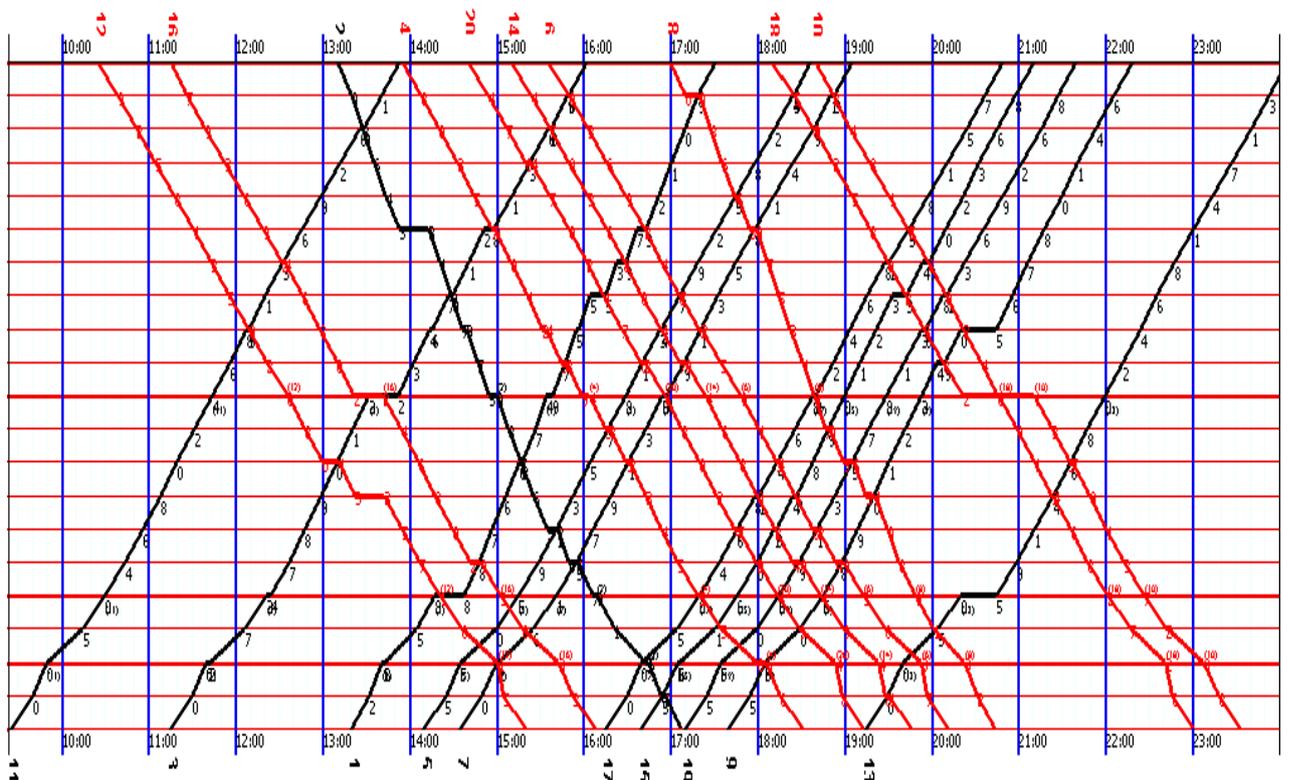


Figure 7. The time-space train diagram in scenario #8

6.2. Case study

In this section, the proposed simulation-optimization method is validated using a case study of Iranian railway network (Fig. 7). The Iranian railway's network is the national state-owned railway system sponsored by Ministry of Roads and Transportation. The railway network converges on capital (Tehran) and connects all major parts of the country.

In this study, Tehran-Razi corridor is selected as a case study to assess the performance of the simulation-based optimization algorithm. This corridor is one of the most important East-West corridors. It connects the capital to the Turkish at the Razi border. The corridor includes 56 stations, 55 blocks, and 46 trains. All stations with praying zones have been implemented in the simulation application. All trains depart with a tolerance interval of ± 60 minutes. All problems solved with penalty coefficient $\sigma = 50$ minutes. The optimization algorithm has been programmed with the 4DScript programming language of ED.

The result of the train scheduling model in Tehran-Razi route for various parameters is summarized in Table 7. The maximum execution time is set to 2 hours for all considered scenarios. In the best-found solution that has been manually generated by Railroad Company, the unplanned stop time of trains of type I and type II are 1030 and 30 minutes, respectively.

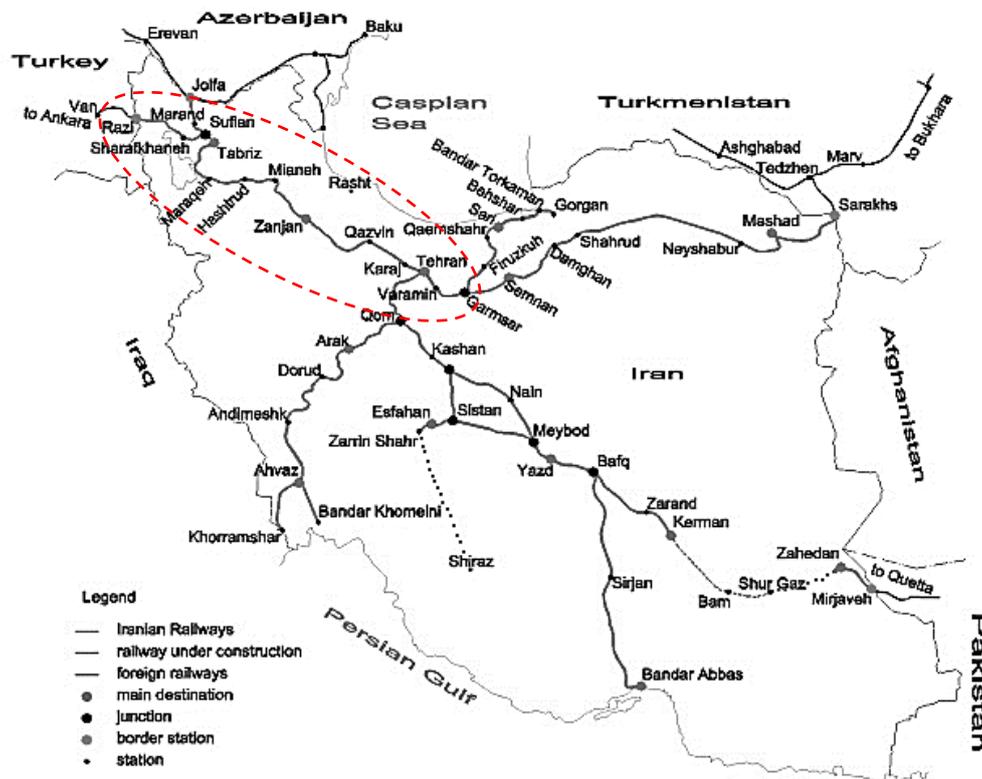


Figure 8. Iran railway network (Tehran-Razi corridor)

The improvement gained by implementing the solution of the proposed simulation-optimization approach are summarized in Table 7. In scenario 3, 4 and 6, the unplanned stop times have been reduced by 8.38% and 71.11% on average, respectively. The time-space diagram generated by the evolutionary Path-relinking algorithm is shown in Figure 9. It takes about 2 days to generate the train schedules by experts in Iranian railway. But the proposed system is capable of obtaining good quality solutions in a very reasonable time.

Table 7. The result of the train scheduling model in Tehran-Razi route

Scenario	λ	K	Pop-Size	F(xbest) (minutes)	Total unplanned stop time of first type (hours)	Total unplanned stop time of second type (hours)	Reduction% of the total unplanned stop time of the first type	Reduction% of the total unplanned stop time of the second type
1	0.9	2000	30	107	1052	2	-2.14%	93.33%
2	0.9	2000	20	119	1100	10	-6.80%	66.67%
3	0.9	2000	10	97	970	0	5.83%	100.00%
4	0.8	1000	10	194	950	5	7.77%	83.33%
5	0.8	3000	30	225	1089	9	-5.73%	70.00%
6	0.7	2000	20	288	911	21	11.55%	30.00%
7	0.7	100	10	378	1148	48	-11.46%	-60.00%
8	0.5	2000	20	471	900	42	12.62%	-40.00%
9	0.6	1000	30	475	1123	43	-9.03%	-43.33%

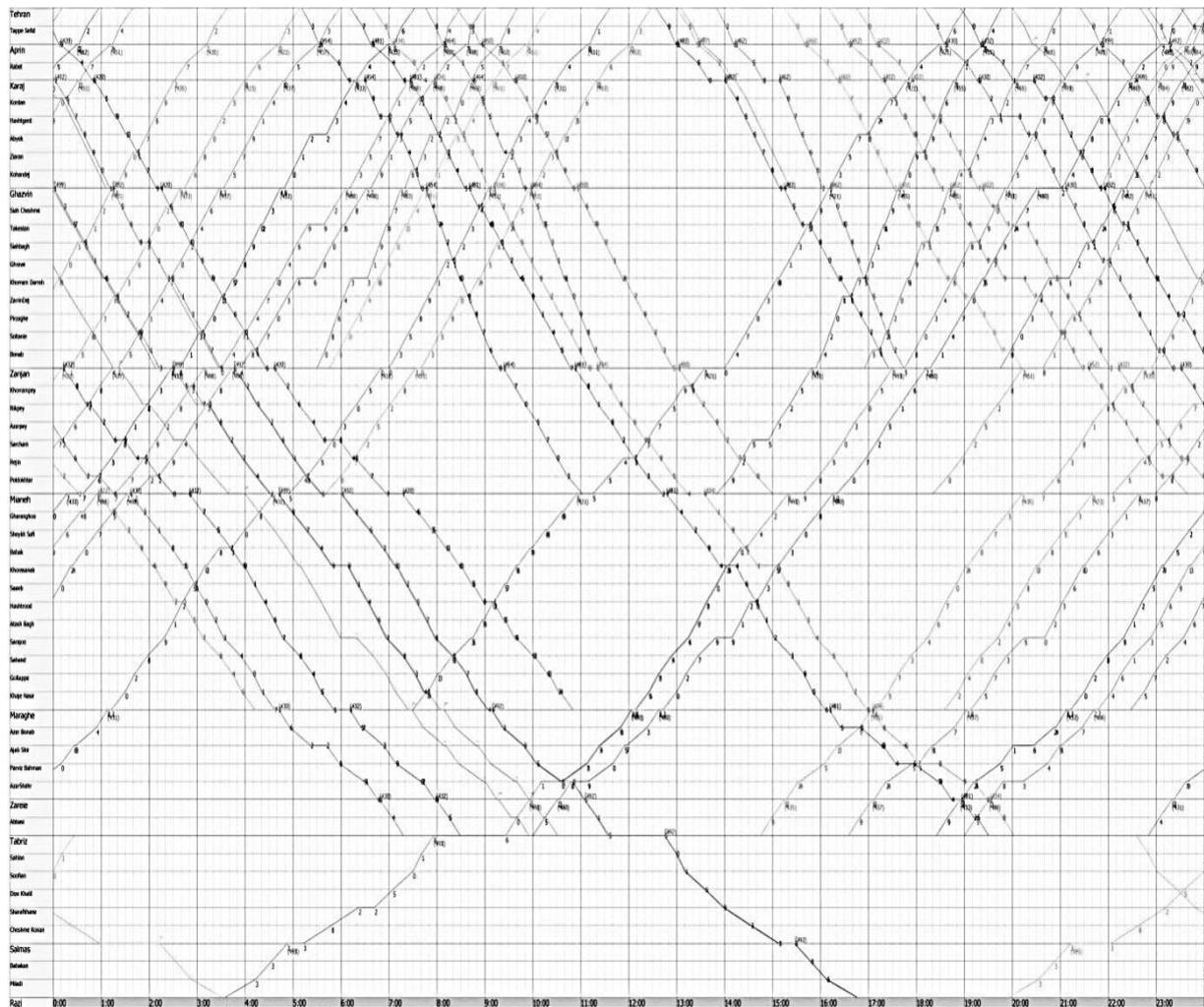


Figure 9. The time-space train diagram using Path-relinking algorithm in Tehran-Razi corridor

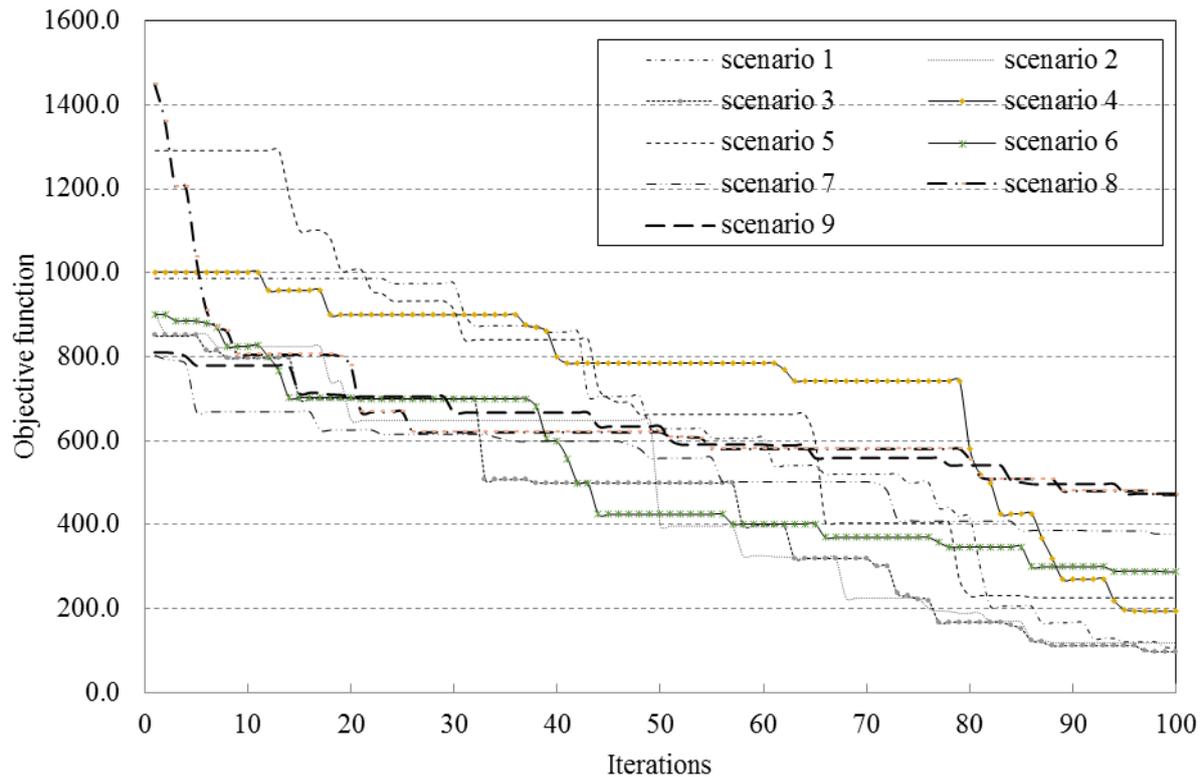


Figure 10. The convergence graph of Path-relinking algorithm

As a general rule, a faster convergence rate results in an improved and more realistic solution for the train scheduling model. A plot of solution quality progress against computational time is provided in Figure 10, which accounts for the convergence analysis of the proposed meta-heuristic algorithm. The search profiles are illustrated in Figure 10 in terms of different weight parameter. It should be noted that the computation time is essentially corresponding to the algorithm iterations, as each replication has an approximately constant time. The convergence graph of Path-relinking algorithm indicates diverse rates of convergence to the optimal solution in different scenarios. In all scenarios, the Path-relinking algorithm could converge to the near-optimal solution within the maximum execution time of two hours. As a conclusion, the computational results of the simulation-optimization method on a real case illustrate that the proposed approach is capable of finding an improved train timetable in reasonable computation times. The results highlight the importance of accounting for optimized stopping pattern in train schedules.

7. Conclusion

In this study, the integrated train scheduling and stop planning of trains on rail networks is addressed. The solution methodology is presented using a simulation-optimization approach based on a discrete-event model to generate near-optimal train timetables. The designed system is capable of modeling a rail network and adaptable to different infrastructure settings. In order to better representation of the problem, a mixed-integer linear programming formulation was proposed. However, the mathematical model cannot be solved to large-size instances using the commercial optimization packages. Due to the complexity of the problem, a simulation-optimization method and a meta-heuristic algorithm were employed to find the improved solutions. The simulation model has the ability to model high flexibility in terms of various operational constraints. The designed simulation model allows the users to change the schedule to fit their needs and produce a more reasonable train schedule. The results of implementing the optimization methodology in real-world instances demonstrate that the

system is able to produce the desired train schedule in large-scale railway systems in a reasonable time. Our findings will significantly contribute to the rail service planning as the simulation-optimization approach greatly supports timetabling process and allows moving from the theoretical experience to an applied one. This will benefit the rail companies by increasing efficiency and decreasing operational costs.

The combinatorial nature of the problem addressed in this study shows the difficulty of solving with exact methods. It is mainly because of the complexity and the large size of the train scheduling problem and the specific constraints in Iranian railway systems. Due to time constraint, the application of exact methods is limited. Correspondingly, we remark that no exact methods exist in the literature, dealing with stochastic factors and disruptions. The possibility to use exact methods for train scheduling problem in the network is left for future research.

Further research is required into integrated simulation-optimization approaches providing decision support for rail capacity planning procedures. In addition, the model can be extended by providing fast algorithms in the simulation model to reschedule trains in case of a disruption. Another area for future research is considering multi-objectives e.g. energy consumption, delay cost and the total traveling time in the optimization model.

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