# JIEMS

**Journal of Industrial Engineering and Management Studies**

**Vol. 5, No. 1, 2018, pp. 85-105**

DOI: 10.22116/JIEMS.2018.66506

www.jiems.icms.ac.ir

# A statistical analysis framework for bus reliability evaluation based on AVL data: A case study of Qazvin, Iran

Amir Albadvi[1,*], Hossein Hashemi[1], Mohammad Reza Amin-Naseri[1], Babak Teimourpour[1]

**Abstract**

Reliability is a fundamental factor in the operation of bus transportation systems for the reason that it signifies a straight indicator of the quality of service and operator's costs. Todays, the application of GPS technology in bus systems provides big data availability, though it brings the difficulties of data preprocessing in a methodical approach. In this study, the principal component analysis is utilized to systematically assess the reliability indicators based on automatic vehicle location (AVL) data. In addition, the significant reliability indicators affecting the bus reliability are identified using a statistical analysis framework. The proposed bus reliability assessment framework can be applied to each bus route or a complete network. The proposed methodology has been validated using computational experiments on real-world AVL datasets extracted from bus system in Qazvin, Iran. The analysis indicates that 1) on-time performance, 2) headway regularity, 3) standard deviation of the travel time of the buses, and 4) 50$^{th}$ percentile travel time are key indicators the reliability of bus services. The potential of the proposed methodology is discussed to provide insights for bus operators. Using the proposed approach in the article, the desirable reliability status of bus lines is identifiable from the point of view of key stakeholders, and the ways to improve reliability can be more clearly defined.

**Keywords**: AVL data; bus reliability; regression analysis; mobility management.

*Received: March 2018-17*

*Revised: April 2018-20*

*Accepted: July 2018-05*

## 1. Introduction

Today, effective transportation management systems play an important role in the economic development of cities [1]. Due to the expense of the development of transportation systems, a better planning of these systems from the perspective of key stakeholders is a practical approach [2]. One of the areas of effective management of transportation systems is to identify the basic components of the quality of service index [3]. Improving the level of passenger satisfaction is only possible through analysis of these factors [4]. The city bus transportation system is considered as one of the most important transportation infrastructure systems for passenger mobility in the world [5]. As a systems approach, the use of mobility management can manage

* Corresponding author; albadvi@modares.ac.ir

[1] Faculty of Industrial and Systems Engineering, Tarbiat Modares University, Tehran, Iran.

transportation resources so as to improve the effectiveness, efficiency, and quality of the travel services being delivered [6].

Reliability concept has attracted increasing interest during last decades by quantitative researchers [7, 8]. It is one of the key indicators of the delivered quality in service industries [9]. According to Abkowitz and Tozzi [10], reliability is defined by the invariability of service characteristics that could have impacts on (a) the choices of passengers, (b) the behavior of the vehicles, and (c) the operator policies. Improvements in reliability may increase the service demand and, consequently, the profitability of the company [11].

Usually, the reliability of service is related to one of the following causes: (a) schedule deviations at the terminals, (b) irregular number of passengers, (c) running time variability, (d) meteorological factors, and (e) driver behavior [12]. When public transportation provides low reliability, there will be a decrease in passengers perception of the quality of the service [13]. One solution to increase the quality of bus services and to meet passengers' needs, is to change their attitude about bus service and make a shift from private transportation to pubic mobility [14]. Similarly, the owners of transportation organizations constantly work to increase the reliability of the service and attract more customers to the implementation of improvement strategies [15]. For example, an increase in the reliability of bus service could shorten the passengers' travel time. Furthermore, good bus service can persuade more people to choose the bus as their way to travel [16]. The different perceptions held by travelers and operators are of vital importance when the owners decide to establish improved levels of reliability criteria, especially since both parties are affected [17]. Typically, these differences exist because the travelers and operators may have a relatively different perception of service reliability and performance [18]. Therefore, it is essential to acknowledge passengers' perspectives of transit reliability under diverse circumstances, with the assumption of certain factors. In public transport systems, passenger perceptions of transport reliability are strongly connected with service frequency [19]. In other words, routes with higher frequency may be considered reliable by passengers even if the service is poor or erratic. Timetable adherence is the most commonly used reliability indicator for infrequent transit services [20]. On the other hand, for routes considered as high frequency, headway unevenness has been identified as the most important reliability indicator.

There are various ways to assess and predict schedule reliability [21]. Two main factors related to passengers' experience criteria are the unanticipated increases on the off-board waiting time at bus stops and the extra time incurred in congested conditions as a result of overloading [22]. However, variability in passengers' travel time and arrival time has negative effects on their satisfaction [23].

According to [24] low wait time at the origin station, on-time arrival at travel destinations, and variability in wait and travel times have the highest effect on travelers 'perceptions of reliability. Typically, operators and agencies mainly define transit reliability in terms of adherence to the planned schedules and on-time performance (OTP) regulations. These OTP indicators do not take into consideration the extent of delay or the range of departure/arrival deviations from the initial timetable [25]. Therefore, the timetable cannot account for the variations that occur in passengers' waiting times [26]. Accordingly, from the travelers' perspective, the OTP indicators represent, simply, a timetable of vehicles positioned in certain places at certain times, regardless of potential variability.

An effective bus reliability model must reflect the diverse and sometimes conflicting views among different stakeholders [27]. For example, it may be of interest to analyze the conflicting reliability indicators with use of the actual records bus data. The assessment and comparison of reliability indicators for different bus lines with various characteristics can be an important step in the identification of those vital factors, which influence reliability improvement.

Furthermore, the findings from this analysis can better inform managers about the factors of unreliable bus service. The bus service quality analysis is much more difficult in the case of limited or missing data. Hopefully, in recent years, the development of the Intelligent Transportation System (ITS), the data collected from Automatic Vehicle Location (AVL) systems can be analyzed in order to improve transportation services quality specifically, in Iran. However, typically the collected data
are not analyzed in regard to passenger satisfaction. Recently, AVL has been utilized in Iran to improve public transport services. A key issue is how this volume of data can be used to provide reliable information for passengers, as well as evaluate the performance of the urban bus system.
The purpose of the current research study is to identify the significant reliability factors of bus service. The research scope is to provide insights into bus system performance using large-scale AVL data. The research study was designed to analyze different bus reliability indicators in order to evaluate the reliability of several bus routes, in terms of both the passengers' and the operators' perspective. Principal component analysis (PCA) is utilized to find significant reliability indicators based on AVL data. Based on the use of the findings from these statistical models, it is anticipated that the manager of public transportation can utilize this information to improve bus reliability in several routes, and companies can improve, more efficiently, their network performance.
It is expected that the findings from the present study will be twofold: First, the findings from the factor analysis framework for bus reliability assessment will provide information, which will account for both passenger and operator satisfaction at the stop and route lines. Use of this method will provide an analysis of the AVL raw data, which is needed to measure the most suitable reliability indicators of bus routes at the bus stop or segment levels.
Moreover, the improved reliability condition can be identified using the characteristics of the best performing route. Second, it is estimated that use of the PCA will produce findings, which will be more accurate than current reliability indicators. The significant reliability factors affecting the bus reliability are identified using a statistical modeling framework. The implementation of this technique in a real-world case study should provide insight into the detailed classification of bus reliability factors on different routes. Consequently, the outcomes from this model could support the decisions made by managers of transport companies to improve the reliability of bus systems and satisfy the requirements of both travelers and operators.

## 2. Research background

A major part of the research on the assessment of public transportation systems can be categorized into one of four types, namely, long-term design performance evaluation, route planning assessment, operation planning assessment and real-time service evaluation [28]. Analyzing urban bus service reliability is of great importance for operators and thus much of the research has been devoted to the planning of bus lines. In spite of the importance of evaluating the reliability of the bus, from both the operator and user perspectives, a few studies have identified the factors that affect the optimum reliability conditions. Taxonomy of the related literature is provided in
Table **1**.
Transit operational service evaluation focuses on system performance, including operational efficiency and quality of service [27]. Fielding, Babitsky [29] aimed to determine the key factors affecting transit quality of service. For this purpose, a factorial study was proposed to analyze huge recorded of operational data. Karlaftis and McCarthy [30] suggested a similar methodology to assess transit service operations from both the standpoints of effectiveness and total quality of service.

They concluded a significant positive association between level of service and transit operation efficiency. De Borger, Kerstens [31] proposed a serviceability evaluation framework based on classical Availability Dependability Capacity (ADC) model. The designed model accounts for service quality, service functionality as well as transit reliability. Karlaftis [32] proposed a service evaluation approach for the analysis of the efficiency and effectiveness of urban transit network. For this purpose, an integrated data envelopment analysis (DEA) approach and rough set theory were utilized to find the key effective factors. Sheth, Triantis [33] provide a performance evaluation model for bus routes accounting for both operator and user’s perspectives. The proposed model considers objective and subjective indicators from the perspective of travelers, to evaluate the service quality of bus operations. Eboli and Mazzulla [34] presented an integrated methodology for evaluating public transport service quality on the basis of subjective and objective criteria of passenger’s attitude. The integration of these service quality perspectives arranges for a more effective reliability indicator of the transit performance. Deng and Nelson [35] proposed a performance evaluation model for analyzing the impacts of bus rapid transit (BRT) on traffic, travel behavior, and transportation system development in Beijing, China. The result of the model highlighted the role of ITS in the improvement of the mobility operational efficiency, reliability and transit performance of BRT systems.

Ma, Ferreira [17] proposed a probabilistic model called Gaussian mixture method to measure bus service reliability using operational AVL data. Traditionally, reliability buffer time (RBT) indicators account for passengers’ standpoints. The validity of the mixture model was confirmed using a real case study in Brisbane, Australia. Moreira-Matias, Mendes-Moreira [36] conducted an extensive survey study to highlight the existing gaps in the AVL-based bus reliability literature. A number of research fields on improving both planning and control on public transportation systems were identified. The result of literature review showed that the existing methods can be further extended to improve the accuracy of the long-term travel time prediction, and better selection of the efficient control strategies in case of disturbances. Gittens and Shalaby [37] presented a field study of the traveler’s perception of transit reliability in bus network in London, Ontario, Canada. The methodology involves the analysis of twenty reliability indicators. A linear regression model was also used to find the significant factors affecting the reliability. According to the results, the route length, route location, stop position, time of day and passenger load is of the highest effect on perceived reliability.

Sun, Chen [38] provided a DEA method for public transport was designed to help managers to develop urban transport systems. The method includes a comparative analysis to evaluate different bus routes in terms of service quality in an attempt to select candidates for further optimization. Shenzhen bus system in China was used as a case study. The computational results confirm the capability of the proposed assessment method. Hu and Shalaby [39] designed a statistical framework for selection and evaluation of both users- and operation-driven reliability indexes. First, a simple multi-criteria decision making was proposed to rank reliability measures against four criteria: 1) data quality, 2) data gathering cost, 3) recognition and 4) calculation easiness. Accordingly, a linear regression model was used to determine the significant factors correlated with transit reliability at route and segment levels. The proposed method was validated on a real case of Toronto bus system using AVL data. The outcomes demonstrated that different reliability measure is significantly related to traffic changes, route distance, the density of bus stops and signalized intersections, and flow of passengers.

Khalid, Haris [40] proposed a model for mobility pattern analysis using spatial-temporal data of vehicles. The efficiency of bus services in Lahore City was examined based on trajectory data collected via GPS and MYSQL databases. The outcomes indicated that a significant delay is caused at beginning or end of the route. Gkiotsalitis and Cats [41] presented an optimization model to determine the bus headways to obtain a trade-off between passenger demand coverage and operating costs. The designed model incorporates the variability of headway and travel time, as well as vehicle capacity and fleet size constraints. Empirical results show that the sensitivity of the optimized headways to the variations in passenger demand and bus running costs.

**Table 1. Taxonomy of the related articles**

| **Reference** | **Level of analysis** | **Reliability analysis methodology** | **Case study** | **Scope of analysis** | **Passengers and operator's perspectives** | **Correlation analysis** | **Number of bus routes** | **Number of criteria** | **AVL data** | **Index categorization and integration** |
|---|---|---|---|---|---|---|---|---|---|---|
| [33] | Macro | Network DEA and goal programming | Virginia, USA | Network | √ | - | 60 | 14 | - | √ |
| [42] | Micro-macro | Multivariate regression models | Minnesota, USA | Line | - | - | 1 | 4 | √ | - |
| [34] | Macro | Sample surveys | Cosenzaand Rende, Italy | Line | √ | - | 1 | 2 | √ | - |
| [43] | Macro | Cluster analysis | Brisbane, Australia | Line | √ | - | 1 | 6 | √ | √ |
| [44] | Macro | Fuzzy AHP and TOPSIS | Shiraz, Iran | Network | - | - | 71 | 16 | √ | - |
| [37] | Micro | Regression Analysis | Ontario, Canada | Network | √ | - | 41 | 21 | - | - |
| [38] | Macro | Super-efficient DEA | Shenzhen, China | Network | √ | - | 18 | 10 | - | - |
| [45] | Macro | Statistical method | Suzhou, China | Line | √ | √ | 1 | 4 | √ | - |
| [46] | Macro | DEA | Seoul metropolitan | Network | √ | - | 36 | 14 | √ | √ |
| [39] | Micro | Simple weighting method | Ontario, Canada | Network | √ | - | 13 | 19 | √ | - |
| [47] | Macro | Data mining | Not mentioned | Network | - | - | 3 | 2 | √ | - |
| [48] | Micro | Quantile Regression Analysis | Brisbane, Australia | Line | - | - | 1 | 1 | √ | - |
| Present study | Micro-macro | PCA and heat map | Qazvin, Iran | Network | √ | √ | 8 | 12 | √ | √ |

According to the above-mentioned articles, previous literature reviews have focused on either passengers' or transit agencies' perspectives on service reliability [49]. However, despite the most study focused on the reliability evaluation, only a few references are about statistical analysis of reliability indicators and the main factors contributing to the reliability improvement [50]. Previous research evaluating bus service reliability using AVL data focused on quantifying the potentials of AVL systems in improving reliability [51]; however, the main causes of unreliability have not been investigated. In summary, existing studies evaluating bus service quality mainly focus on empirical evaluation, while ignoring some important factors, such as the mutual influence of passenger perspectives and operators cost, and only a few propose the utilization of AVL data in the evaluation framework. To fill these research gaps, this study proposes a statistical evaluation framework to categorize existing bus routes and identify contributing factors for reliability improvement. This study purposes a reliability assessment framework to evaluate and compare different bus routes from the perspective of various reliability indicators. The contribution of this study is to provide insights of bus reliability in the perspectives of passengers and operators and to measure the performance of bus services in a real context where AVL data is available.

This paper is organized as follows. Section 3 provides the methodology undertaken in this paper for analyzing the existing status of a bus system in terms of an effective set of reliability indicators. Section 4 describes the Case study and datasets used to evaluate the performance of the reliability assessment model. Section 5 discusses the results of the proposed model for bus reliability analysis, and section 6 concludes.

## 3. Methodology

### 3.1. Bus reliability evaluation framework

This section provides a description of the data and method used to evaluate the bus reliability indicators. Focusing on transit reliability from both the perspectives of passengers and operator, a new multi-criteria framework is developed to assess the reliability performance indicators of different bus routes. In the proposed decision analysis model, public transportation manager and travelers can get city bus reliability information in several routs and in addition, bus transportation companies can rank bus routes and improve the overall bus network performance using this model. The result of this analysis also has potentials for improved perceived reliability by travelers.

As mentioned previously, the reliability of the urban bus transportation system can be defined using different approaches: from the operator and passengers' points of view. Frequently, random disturbances affect the traveling time of buses and accordingly the reliability of services in different aspects. Reliability of bus services can be measured by on-time performance or punctuality and regularity. Punctuality refers to the percentage of buses arriving on time with respect to a predefined schedule. The AVL data include GPS location, date, time, and operational data influenced by randomness or disruptions. Thus, the effects of random disturbances, e.g., severe weather conditions, accidents, and breakdowns have been considered by calculating the difference between the planned times and the actual arrival times. On the other hand, regularity accounts for the deviation regarding the scheduled headways. For example, CV of Running Time accounts for the randomness in traveling time of buses. Correspondingly, bus operations are characteristically unstable and will habitually deviate from the planned schedule. The proposed reliability assessment model considers a different type of reliability indicators so as to provide a comprehensive evaluation of bus system performance.

Most reliability indicators that have been used in various papers have been categorized based on the beneficiary's level, measurement method, and relevant application. These indicators are based on the latest research on the reliability of the bus, and with their integration, a new structure for assessing the quality of service of the city bus system and the ranking of bus routes is presented. The key point in this selection is identifying a minimal set of indicators that provide the most information about the existing status of reliability in the system. Therefore, it is necessary to consider these indicators both from the perspective of passengers and bus companies together. In this regard, by an interview with experts and doing field visits, appropriate indicators are selected for assessing the reliability of the bus system. At this stage, using a variety of sources in this area, the classification of key indicators of the reliability of the urban bus system from the perspective of stakeholders is discussed and after extracting different indicators, classification and integration of these indicators through correlation analysis are performed.

In Table 4 most of the reliability measures which have been applied recently in different papers are classified based on the level of the beneficiary, context, level of analysis, category and application. In addition, the notations used in the calculation of the bus reliability indicators are provided in Table 2. For example, for a transit service with short headways and riders arriving more randomly in relation to the schedule, reliability is better reflected by a transit agency’s ability to maintain headways and minimize a typical passenger’s waiting time. Thus, the deviation index based on stops (DIS) is designed to capture the operational characteristics at the station level.

**Table 2. The notations used in the calculation of the reliability indicators**

| Symbol | Description |
|---|---|
| $\theta_1$ , $\theta_2$ | Limits of acceptable headway deviation |
| $H_s$ | Observed headway at stop *s* |
| $H_0$ | Headway at which buses are dispatched from origin station |
| $\sigma_{TT,obs}$ | The standard deviation of observed travel time |
| $TT^{avg}$ | Average travel time duration |
| $TT^{95\%}$ | 95$^{th}$ percentile travel time |
| $TT^{50\%}$ | 50$^{th}$ percentile travel time |
| $h_{ij}$ | Observed headway of bus *i* at stop *j* |
| $\bar{h}_j$ | Mean headway at stop *j* |
| $n_j$ | Number of buses at stop *j* |

Generally, identifying important indicators in the evaluation of the bus system increases the efficiency of reliability assessment methods. For example, slack or buffer time criteria can measure the perceived reliability of travelers in the context of departure scheduling using operational data.

ATD is defined as the 50$^{th}$ percentile travel time under the recurrent service state instead of the whole service states for a specific time period over different days. Running Time Deviation (RTD) shows the actual travel time compliance with the planned value. Evenness index based on stops (EIS) is employed to capture the pattern of consistency or evenness of the headway between vehicles. The buffer time index (BTI) is calculated as the difference between the 95$^{th}$ percentile travel time and average travel time divided by average travel time.

**Table 3. Reliability indicators used in this article**

| Reference | Reliability index | Stakeholder | Formula | Level of analysis | | Category | | |
|---|---|---|---|---|---|---|---|---|
| | | | | Stop | Route | On-Time | Headway | Travel Time |
| [24] | Deviation Index (DIS) | O | $DIS = P\{\theta_1 \leq H_s - H_0 \leq \theta_2\}$ | √ | | | √ | |
| [52] | On-time Performance (OTP) | O,U | $OTP = \frac{number\ of\ on - time\ trips}{number\ of\ trips} * 100\%$ | | √ | √ | | |
| [17] | Headway Regularity (HR) | O | $SD_H = \sqrt{\frac{\sum_{i=1}^{n_j}\left(h_{ij} - \bar{h}_j\right)^2}{n_j - 1}}$ | √ | | | √ | |
| [17] | Reliability Buffer Time (RBT) | U | $RBT = TT^{95\%} - TT^{50\%}$ | | √ | | | √ |
| [17] | Reliability time index (RTI) | O | $RTI = \frac{RBT}{TT^{50\%}} * 100\%$ | | √ | | | √ |
| [43] | Standard deviation of travel time (SDT) | U | $\sigma_{TT}$ | | √ | | | √ |
| [17] | Average Trip Duration (ATD) | U | $ATD = (TT_{recurrent}^{50\%})$ | | √ | | | √ |
| [17] | Planning Time Index (PTI) | O,U | $PTI = \frac{TT^{95\%}}{TT^{avg}} * 100\%$ | | √ | | | √ |
| [52] | Running Time Deviation (RTD) | O,U | $RTD = \frac{TT^{actual}}{TT^{planned}} * 100\%$ | | √ | | | √ |
| [52] | CV of Running Time (CRT) | O | $C_{v,TT} = \frac{\sigma_{TT,obs}}{TT^{avg}}$ | | √ | | | √ |
| [53] | CV of Running Time Deviation (CVRTD) | O | $\frac{\sigma_{RTD}}{RTD} * 100$ | | √ | | | √ |
| [24] | Evenness Index (EIS) | O | $\frac{\sqrt{\frac{\sum_2^m (H_s - H_0)^2}{m-2}}}{H_0}$ | √ | | | √ | |
| [17] | Buffer Time Index (BTI) | O | $BTI = \frac{TT^{95\%} - TT^{avg}}{TT^{avg}} * 100\%$ | | √ | | | √ |

U: Users
O: Operator

## 4. Case study

Qazvin is the main city and capital of the Province of Qazvin in the Islamic Republic of Iran. The Qazvin transportation network is divided into two parts: inland and outskirts, a complex network of 733 kilometers of interconnecting routes, 23 bus lines, 6 thousand city taxi, and trams, cycling routes, three railway stations, airports, and subway system. The bus organization

of Qazvin was established in 1991. The organization covers up to 22 km around Qazvin. Buses, minivans, and schools in Qazvin are under the control of this organization. Currently, the organization operates 23 bus lines in the city of Qazvin. In Qazvin city, 220 buses are equipped with AVL systems in which their instant positions are online available throughout the day.
The important reason for selecting this city as a case study is the growing development of bus management systems that use to control buses fleet i.e. AVL and AFC systems. Moreover, the city of Qazvin is a leading provider of intelligent transportation systems among different cities of Iran.

### 4.1. AVL system

In the designed fleet management system, the urban buses are classified into 23 different routes where each route has different stations. The most important information of this system is the buses geographical positions including their latitude, altitude, and speed based on the station's GIS data which are sent to the center automatically every 30 seconds using the AVL system and then stored in the system database. Accordingly, for a period of two years, the total records of data are equal to 2000*365*2=1460000 for a bus. Assuming that there are about 220 bus units in the network, the database includes twenty million data records, indicating the huge amount of available data available for this research. Using the current data of the system gathered (instantly and historically) a wide range of analyses and reports can be generated. Figure 1 shows the overall scheme of the AVL system and its main page illustrating the movement of each bus. In this paper, the archived travel data of eight important and congested bus routes i.e. $L_5$, $L_6$, $L_7$, $L_9$, $L_{16}$, $L_{17}$, $L_{18}$, and $L_{23}$ are provided for reliability analysis as shown in Figure 3.

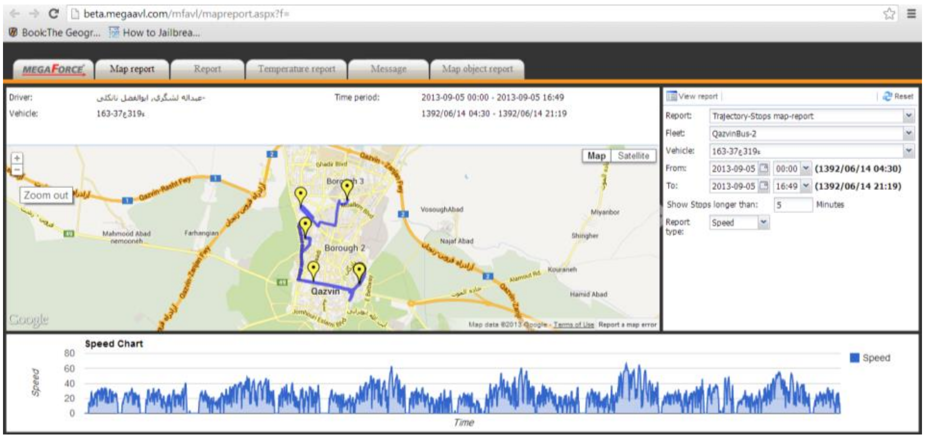


**Figure 1. The general scheme of the AVL system in Qazvin city**

### 4.2. Data definition and preprocessing

The up-to-date and actual data are critical in assessing the performance of systems and providing an appropriate indicator in transport analyzes. This data can be used in design planning and operations, and in the evaluation of the transportation system.
The data and information extracted from the bus operation in lines is not only necessary for the daily running of bus lines, but also is critical for the improvement of this system. In other words, bus managers need to monitor and control accurately the operations of picking passengers into a set of information collected by AVL systems so that they can quickly detect any errors in the system by observing and analyzing this information and, in order to eliminate them, take the necessary measures. On the other hand, some of the data collected can help senior executives make strategic decisions to improve citizen service.

The general structure of AVL data usage in urban bus systems indicates that different data from different sources are collected and then processed, categorized and summed up for decision making, planning, and evaluation. Ultimately, these decisions, plans, and assessments lead to a change in the current state (in order to achieve the desired status), and as a result, input data will change, resulting in a continuous improvement perspective in the system.

The most important part of the data preparation is the transformation of the data structure in order to implement the bus ranking model appropriately. In this paper, the data is extracted from the AVL database and is prepared based on the movement records of each bus on all the stations of each route. Since raw data of bus system is not operational, the data is cleaned so that the missing data is estimated or replaced by average values and the data format is prepared to perform the reliability analysis. Nearly 10% of the data has been deleted during the cleaning process, which is mainly due to the type of data that was missing or unregistered in the system. In the cleaned data file, each row of the data table contains the information of a unique bus code, in which the time of arrival and departure to each station is recorded along the route. In this research, the data related to the buses are considered for one month in which 8 routes of the bus transportations network are analyzed as the case study. A sample of the prepared data structure related to arrival/departure of 10 buses within route $L_{18}$ to/from the second station in a specific time is considered and their travel times from the first to the second stations and the real start time of the intervals between the arrivals of buses at the second station are calculated as shown in Table 3. After finalizing the data based on the data format shown in Table **4**, the reliability indicators given in Table 3 are calculated according to the archived data of the bus system during the one month period from 1 August until 31 August 2017.

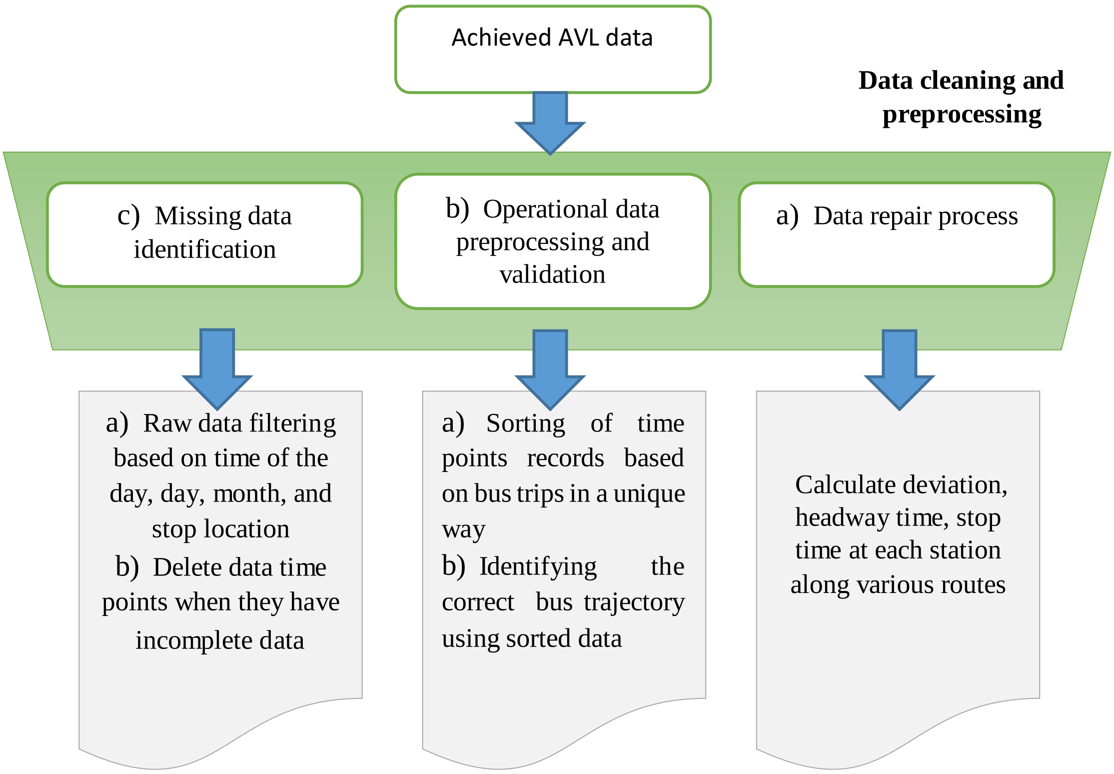


**Figure 2. The general framework proposed for AVL data analysis**

**Table 4. A sample of the prepared data (station #2)**

| Vehicle ID | Station ID | Enter Time | Exit Time | Year | Day | Month | Dwell Time |
|---|---|---|---|---|---|---|---|
| 2476 | 1625 | 5:16:05 | 5:16:42 | 2017 | 23 | 8 | 0:00:37 |
| 2476 | 848 | 5:18:10 | 5:18:23 | 2017 | 23 | 8 | 0:00:13 |
| 2281 | 1625 | 5:20:35 | 5:21:20 | 2017 | 23 | 8 | 0:00:45 |
| 2281 | 848 | 5:23:21 | 5:23:38 | 2017 | 23 | 8 | 0:00:17 |
| 2476 | 852 | 5:37:48 | 5:38:25 | 2017 | 23 | 8 | 0:00:37 |
| 2016 | 1625 | 5:25:45 | 5:26:47 | 2017 | 23 | 8 | 0:01:02 |
| 2346 | 1625 | 5:27:23 | 5:28:10 | 2017 | 23 | 8 | 0:00:47 |
| 2349 | 1622 | 5:56:55 | 5:57:48 | 2017 | 23 | 8 | 0:00:53 |
| 2016 | 848 | 5:28:31 | 5:28:59 | 2017 | 23 | 8 | 0:00:28 |
| 2346 | 848 | 5:30:06 | 5:30:23 | 2017 | 23 | 8 | 0:00:17 |

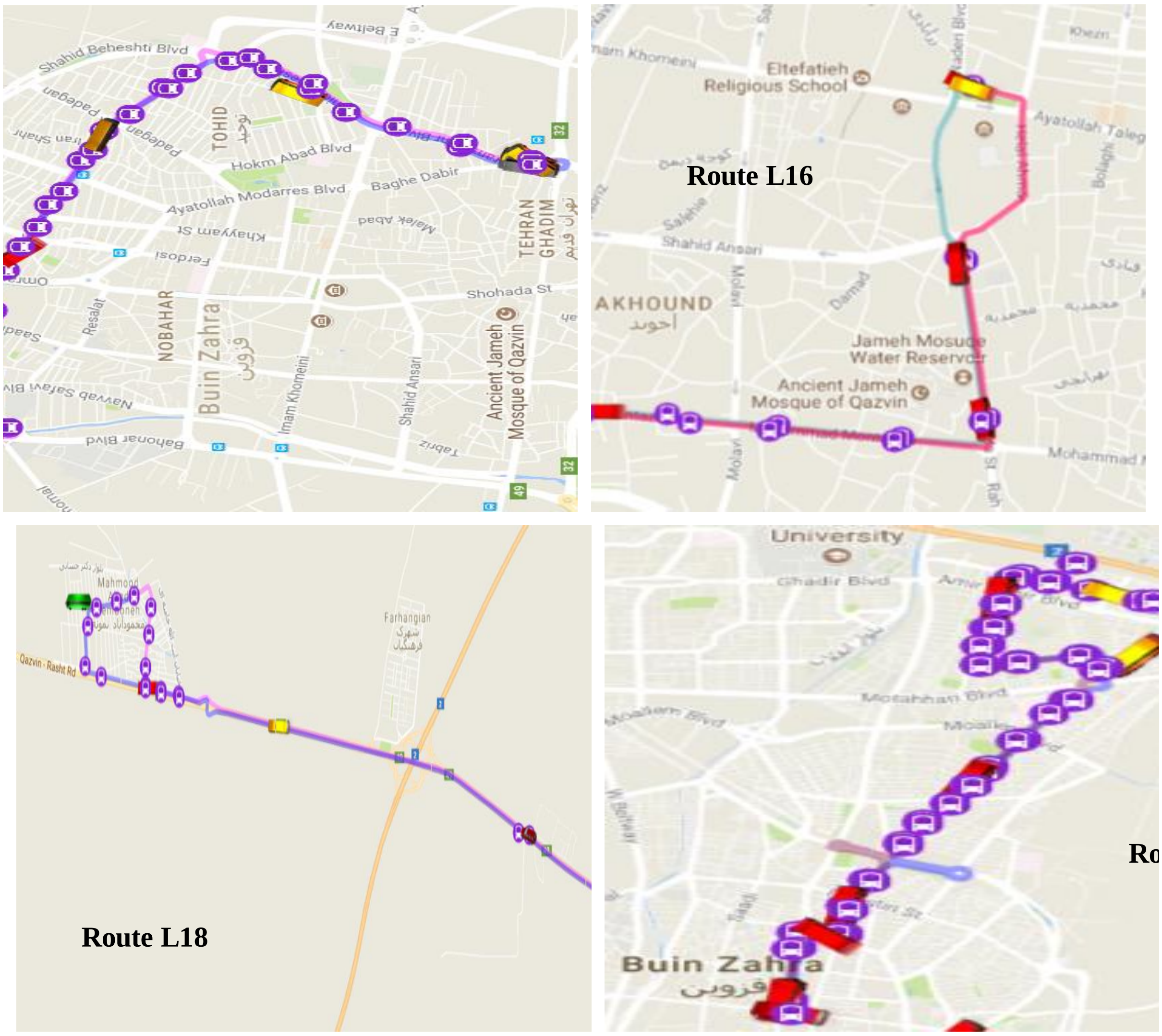


**Figure 3. The map of the important bus routes in Qazvin transportation system**

## 5. Results and discussion

As mentioned previously, the implementation of the methodology involves the evaluation of the reliability measures for each bus route. The reliability measures for Qazvin bus transportation network are calculated as given in Table 5. To calculate these indices, specific stop data including bus departure, arrival along the route and initial plan are utilized. The reliability assessment model presented in this article starts with statistical analysis for calculated reliability measures to obtain a minimum set of uncorrelated criteria representing the reliability of service in both perspectives of passengers and operator.

**Table 5. The reliability measures calculated for Qazvin bus transportation system**

| Bus route\reliability indicators | RBT | OTP% | SDT | HR | ATD | PTI% | BTI% | RTI% | RTD% | CRT | CVRTD | DIS% | EIS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $L_5$ | 3.9 | 61.2 | 3.1 | 13.6 | 24.8 | 117 | 15.50 | 15.70 | 83 | 0.12 | 4.6 | 22.2 | 3.2 |
| $L_6$ | 3.3 | 72.3 | 2.7 | 14.2 | 44.2 | 111 | 7.20 | 7.50 | 92 | 0.06 | 6.6 | 25.6 | 4.1 |
| $L_7$ | 4.9 | 53.1 | 4.3 | 13.8 | 25.3 | 124 | 18.60 | 19.40 | 81 | 0.16 | 3.7 | 30.2 | 3.6 |
| $L_9$ | 6.4 | 64.5 | 3.6 | 15.5 | 45.7 | 119 | 13.30 | 14.00 | 88 | 0.07 | 4.6 | 24.6 | 4.7 |
| $L_{16}$ | 2.8 | 62.6 | 2.7 | 15.3 | 24.7 | 115 | 10.90 | 11.30 | 87 | 0.11 | 5.4 | 32.54 | 3.7 |
| $L_{17}$ | 2.1 | 83.2 | 4.1 | 14.7 | 15.1 | 115 | 13.80 | 13.90 | 86 | 0.27 | 5 | 35.7 | 3.4 |
| $L_{18}$ | 1.6 | 74.5 | 2.2 | 13.3 | 32.8 | 107 | 4.80 | 4.90 | 92 | 0.07 | 5.8 | 25.67 | 4.3 |
| $L_{23}$ | 3.1 | 67.8 | 3.4 | 14.5 | 51.3 | 109 | 5.90 | 6.00 | 84 | 0.06 | 4.7 | 32.6 | 4.9 |

## 5.1. Heat map analysis

In this section, the heat map is used a graphic encryption method of data with the purpose of providing an overview of the entire reliability data set. Data as-is for reliability indicators are shown in Figure 4. It is basically related to multivariate analysis based on distance matrices for characterizing associations between related variables. In the designed model, heat map shows correlated reliability indicators against different bus routes. The heat map allows decision-makers the in-depth analysis of the fundamental associations between reliability indicators. The distance matrices based on Pearson and Euclidian are shown in Figure 5 and Figure 6 respectively. Pairwise distance matrices illustrate all pairwise distances between the points in a data set. Likewise, the pairwise correlation matrix shows the correlations between all pairs of variables in a data set. The results obtained by Euclidian distance matrix indicate the highest dissimilarity between ADT and other reliability indicators.

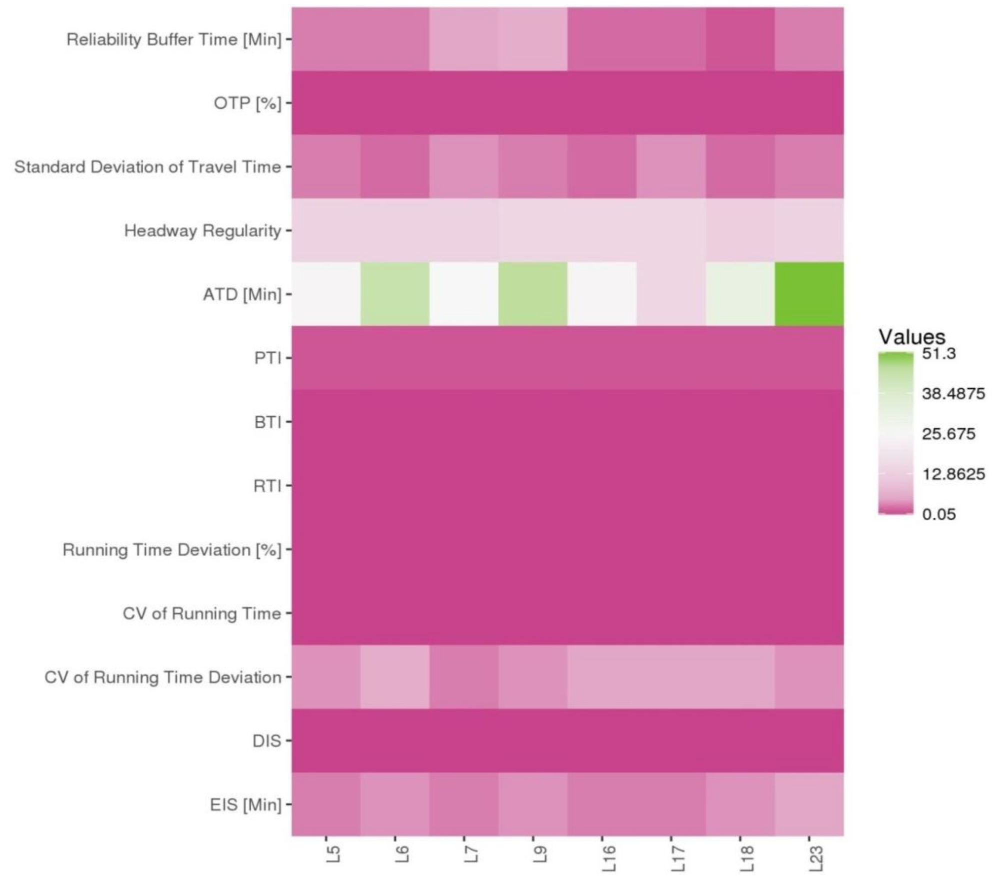


**Figure 4. Data as-is for reliability indicators**

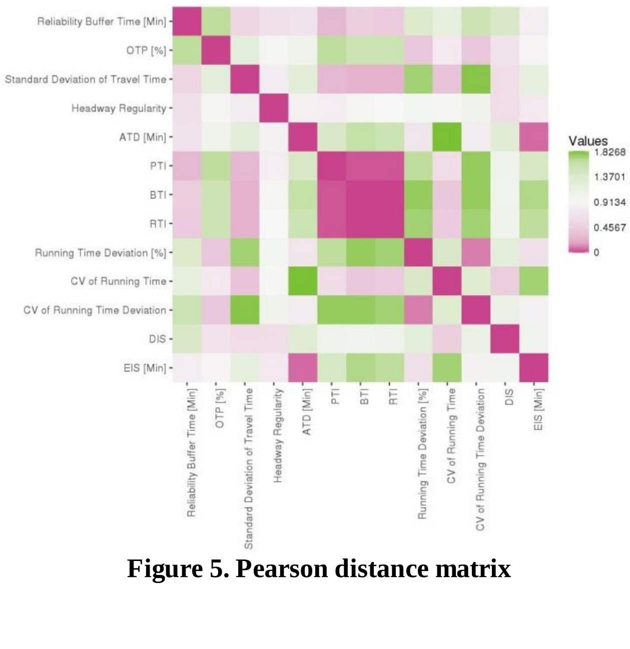


**Figure 5. Pearson distance matrix**

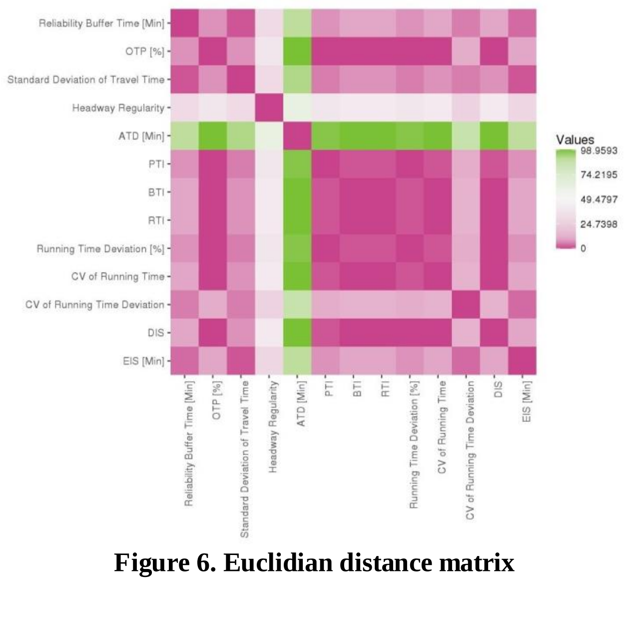


**Figure 6. Euclidian distance matrix**

## 5.2. Principle component analysis (PCA)

PCA is an exploratory data analysis method to study the structure of the data, with emphasis on determining the patterns of covariance among variables. Thus, PCA is the study of the structure of the variance-covariance matrix. This procedure is achieved by calculating a matrix

of coefficients whose columns are called eigenvectors of the variance-covariance or of the correlation matrix of the data set.
More specifically, PCA is a method to identify variable or sets of variables that are highly correlated with each other. The results can be used for identifying variables or factors, underlying the original variables, which are responsible for the variation in the data [54]. PCA does not have any model to be tested, although it is assumed that the variables are linearly related. The analysis can be thought of as looking at the same set of data from a different perspective. The perspective is changed by moving the origin of the coordinate system to the centroid of the data and then rotating the axes [55].
Given a set of *p* variables ($X_1$, ..., $X_p$), PCA calculates a set of *p* linear combinations of the variables ($PC_1$, ..., $PC_p$) such that: The total variation in the new set of variables or principal components is the same as in the original variables [56]. Also, the first PC contains the most variance possible, e.g. as much variance as can be captured in a single axis. The second PC is orthogonal to the first one (their correlation is 0) and contains as much of the remaining variance as possible. The third PC is orthogonal to all previous PC's and also contains the most variance possible. In this study, the calculated eigenvalues against relevant principal component are depicted in Figure 7.

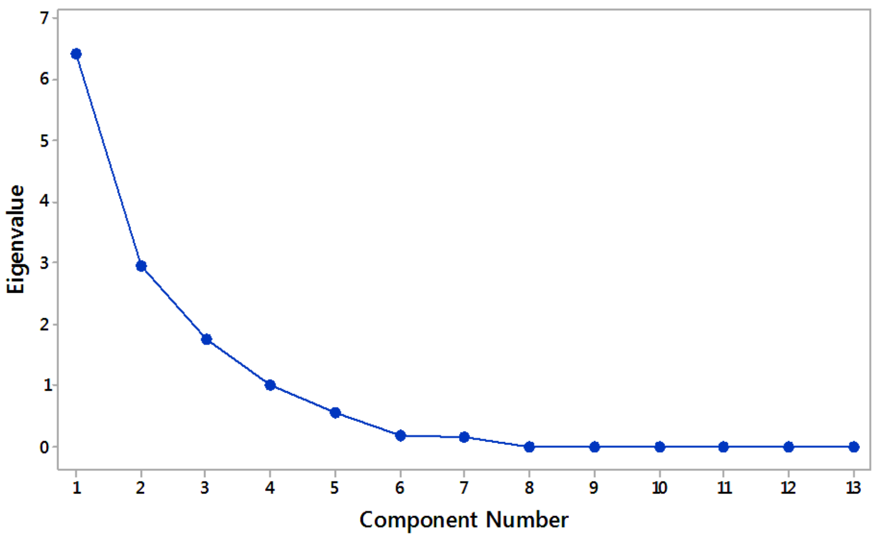


**Figure 7. Eigenvalues against relevant principal component**

The classification of the reliability indices from the PCA analysis is shown in Figure 8. Component interpretation is performed by computing the Pearson correlation coefficients between the components with the main variables (Loading values). Variables that have higher correlation coefficients with the extracted component play a more important role in defining the desired component.

**Table 6. Component matrix generated using PCA for Reliability Indicators**

| Reliability Indicators | Component | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| Reliability Buffer Time [Min] | .519 | .795 | .078 | .259 | .157 |
| OTP [%] | -.528 | -.635 | .288 | .194 | .424 |
| Standard Deviation of Travel Time | .818 | -.022 | .442 | -.115 | .318 |
| Headway Regularity | .103 | .192 | .731 | .554 | -.292 |
| ATD [Min] | -.542 | .740 | .285 | -.136 | .148 |
| PTI | .923 | .266 | -.076 | .228 | -.020 |
| BTI | .974 | .013 | -.136 | .173 | .043 |
| RTI | .974 | .047 | -.129 | .178 | .036 |
| Running Time Deviation [%] | -.834 | -.037 | -.099 | .465 | .127 |
| CV of Running Time | .644 | -.713 | .183 | .073 | .196 |
| CV of Running Time Deviation | -.824 | -.251 | -.139 | .416 | -.047 |
| ADIS | .175 | -.554 | .714 | -.209 | -.272 |
| AEIS [Min] | -.566 | .590 | .499 | -.176 | .140 |

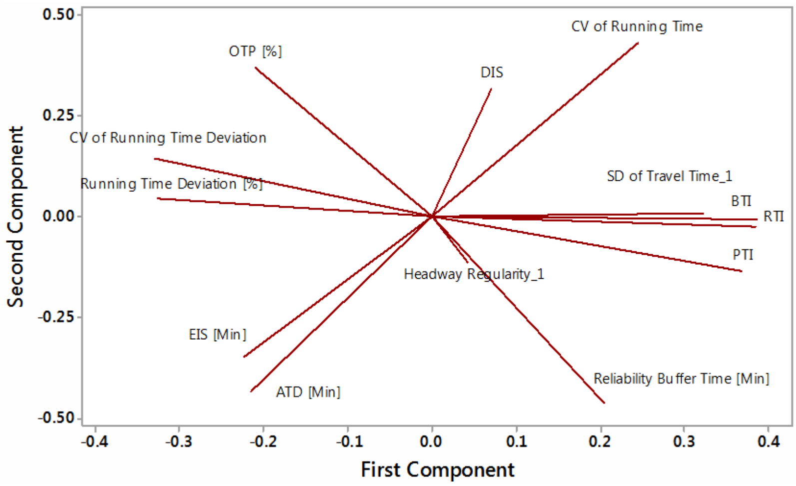


**Figure 8. Loading plot of reliability Indicators from PCA Analysis**

**Table 7. Total variance explained using PCA**

| Component | Initial Eigenvalues | | | Extraction Sums of Squared Loadings | | |
|---|---|---|---|---|---|---|
| | Total | % of Variance | Cumulative % | Total | % of Variance | Cumulative % |
| **1** | 6.413 | 49.329 | 49.329 | 6.413 | 49.329 | 49.329 |
| **2** | 2.920 | 22.461 | 71.790 | 2.920 | 22.461 | 71.790 |
| **3** | 1.763 | 13.562 | 85.352 | 1.763 | 13.562 | 85.352 |
| **4** | 1.027 | 7.900 | 93.252 | 1.027 | 7.900 | 93.252 |
| **5** | .566 | 4.357 | 97.609 | .566 | 4.357 | 97.609 |

## 5.3. Correlation analysis

The Pearson correlation coefficient is a parametric method and the statistical tool for determining the type and degree of the relationship between quantitative. The Pearson correlation coefficient between the two random variables is defined by their covariance divided by their standard deviation. The Pearson Correlation Coefficient varies between 1- and 1. If r=1 represents the complete direct relationship between the two variables, the direct or positive relationship means that if one of the variables increases or decreases, the other also increases (or decreases). r =-1 also shows a complete inverse relationship between the two variables. When the correlation coefficient is zero, this indicates that there is no linear relationship between the two variables.

Table **8** shows the Pearson correlation coefficients between different reliability indicators. The highlighted parts in this table indicate the pairwise weak correlation ($-0.3<r<0.3$) between the variables. Correspondingly, the p-value values for Pearson correlation tests between the various reliability indices are summarized in Table 9. To determine whether the correlation between reliability indices is significant, p-values are compared with the level of decision-making importance. Usually, the significance level ($\alpha$) is 0.05. The value of $\alpha = 0.05$ shows that the risk of the conclusion for the existence of a correlation is 5% in the absence of any significant correlation between the two indices.

| Variable | Total Count | Mean | StDev |
|---|---|---|---|
| Reliability Buffer Time [Min] | 8 | 3.513 | 1.549 |
| OTP [%] | 8 | 0.674 | 0.092 |
| SD of Travel Time | 8 | 3.262 | 0.727 |
| Headway Regularity | 8 | 14.362 | 0.789 |
| ATD [Min] | 8 | 32.987 | 12.747 |
| PTI | 8 | 1.146 | 0.056 |
| BTI | 8 | 0.113 | 0.049 |
| RTI | 8 | 0.116 | 0.051 |
| Running Time Deviation [%] | 8 | 0.866 | 0.040 |
| CV of Running Time | 8 | 0.115 | 0.072 |
| CV of Running Time Deviation | 8 | 5.050 | 0.882 |
| DIS | 8 | 0.286 | 0.048 |
| Total | 8 | 62.491 | 13.394 |

Cronbach’s alpha = 0.07633

| Omitted Variable | Adj. Total Mean | Adj. Total StDev | Item-Adj. Total Corr | Squared Multiple Corr | Cronbach’s Alpha |
|---|---|---|---|---|---|
| Reliability Buffer Time [Min] | 58.98 | 12.80 | 0.33115 | * | -0.00388 |
| OTP [%] | 61.82 | 13.40 | -0.11269 | * | 0.07856 |
| SD of Travel Time | 59.23 | 13.53 | -0.20976 | * | 0.10025 |
| Headway Regularity | 48.13 | 13.20 | 0.22178 | * | 0.05011 |
| ATD [Min] | 29.50 | 2.05 | 0.24384 | * | -0.04376 |
| PTI | 61.35 | 13.41 | -0.27116 | * | 0.07926 |
| BTI | 62.38 | 13.42 | -0.47483 | * | 0.08053 |
| RTI | 62.38 | 13.42 | -0.45049 | * | 0.08047 |
| Running Time Deviation [%] | 61.63 | 13.38 | 0.26739 | * | 0.07533 |
| CV of Running Time | 62.38 | 13.45 | -0.78390 | * | 0.08554 |
| CV of Running Time Deviation | 57.44 | 13.28 | 0.09819 | * | 0.06397 |
| DIS | 62.20 | 13.41 | -0.29730 | * | 0.07913 |

**Table 8. The results of Pearson correlation test between different reliability indices**

| | RB | OTP | SDTT | HR | ADT | PTI | BTI | RTI | RTD | CV_RT | CV_RTD | DIS | EIS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **RB** | | | | | | | | | | | | | |
| **OTP** | -0.641 | | | | | | | | | | | | |
| **SDTT** | 0.465 | -0.194 | | | | | | | | | | | |
| **HR** | 0.359 | 0.040 | 0.229 | | | | | | | | | | |
| **ADT** | 0.322 | -0.070 | -0.251 | 0.172 | | | | | | | | | |
| **PTI** | 0.722 | -0.644 | 0.718 | 0.205 | -0.379 | | | | | | | | |
| **BTI** | 0.559 | -0.512 | 0.733 | 0.083 | -0.570 | 0.955 | | | | | | | |
| **RTI** | 0.585 | -0.534 | 0.732 | 0.097 | -0.547 | 0.967 | 0.999 | | | | | | |
| **RTD** | -0.345 | 0.582 | -0.718 | 0.031 | 0.307 | -0.651 | -0.712 | -0.703 | | | | | |
| **CV_RT** | -0.206 | 0.283 | 0.641 | 0.042 | -0.827 | 0.410 | 0.591 | 0.568 | -0.410 | | | | |
| **CV_RTD** | -0.538 | 0.591 | -0.766 | -0.040 | 0.218 | -0.728 | -0.733 | -0.734 | 0.894 | -0.343 | | | |
| **DIS** | -0.388 | 0.284 | 0.430 | 0.367 | -0.306 | -0.015 | 0.017 | 0.008 | -0.275 | 0.574 | -0.160 | | |
| **EIS** | 0.188 | 0.090 | -0.202 | 0.276 | 0.898 | -0.445 | -0.636 | -0.614 | 0.358 | -0.689 | 0.151 | -0.065 | |

**Table 9. P-values for Pearson correlation test between different reliability indices**

| | RB | OTP | SDTT | HR | ADT | PTI | BTI | RTI | RTD | CV_RT | CV_RTD | DIS | EIS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **RB** | | | | | | | | | | | | | |
| **OTP** | 0.086 | | | | | | | | | | | | |
| **SDTT** | 0.246 | 0.646 | | | | | | | | | | | |
| **HR** | 0.382 | 0.926 | 0.586 | | | | | | | | | | |
| **ADT** | 0.437 | 0.868 | 0.549 | 0.684 | | | | | | | | | |
| **PTI** | 0.043 | 0.085 | 0.045 | 0.626 | 0.354 | | | | | | | | |
| **BTI** | 0.150 | 0.194 | 0.039 | 0.844 | 0.141 | 0.000 | | | | | | | |
| **RTI** | 0.128 | 0.173 | 0.039 | 0.820 | 0.160 | 0.000 | 0.000 | | | | | | |
| **RTD** | 0.402 | 0.130 | 0.045 | 0.942 | 0.459 | 0.081 | 0.048 | 0.052 | | | | | |
| **CV_RT** | 0.625 | 0.497 | 0.087 | 0.922 | 0.011 | 0.314 | 0.123 | 0.142 | 0.313 | | | | |
| **CV_RTD** | 0.169 | 0.123 | 0.027 | 0.925 | 0.604 | 0.041 | 0.039 | 0.038 | 0.003 | 0.406 | | | |
| **DIS** | 0.342 | 0.495 | 0.288 | 0.371 | 0.461 | 0.971 | 0.969 | 0.985 | 0.510 | 0.137 | 0.705 | | |
| **EIS** | 0.656 | 0.832 | 0.631 | 0.509 | 0.002 | 0.269 | 0.090 | 0.106 | 0.384 | 0.059 | 0.720 | 0.878 | |

The pairwise correlation indices can be represented in the form of a graph *G(V,E)* so that if there is a weak correlation between two indices (two nodes of *V*), one arc is drawn from the set of *E* in the graph. Graph analysis can be used to find the largest subset of reliability indicators that are not statistically correlated. In this case, a complete graph is a simple graph in which each vertex is connected to the other vertices by an edge.

For the purpose of finding the largest set of independent reliability indicators, an algorithm is proposed for finding the largest subgraph in a complete graph. This algorithm was programmed in MATLAB software to identify the largest subset of weakly correlated indicators. According to the results of the statistical correlation test, the largest subsets of reliability indices that are not statistically correlated with each other include the following indices:

A={OTP, HR, SDT, ATD}

B={OTP, HR, SDT, EIS}

From the two sets A and B, the sum of the correlation coefficients between the variables set A is less than of those of the set B, so the set A is chosen. Thus, in the final set of uncorrelated reliability indicators there exist 1) on-time performance (from the perspective of the passenger and the operator), 2) headway regularity (from the perspective of the operator), 3) standard deviation of travel time of the buses (from the passenger's point of view), and 4) $50^{th}$ percentile travel time under the recurrent service state (from the passenger's point of view).

## 6. Conclusion remarks

The bus service reliability indexes are normally associated with both beneficiaries' points of view (i.e. passenger and operator) within an urban bus transportation system. Measuring and evaluating the bus service reliability of alternative bus routes requires a comprehensive multi-attribute decision framework. In this paper, a statistical analysis method is utilized based on the AVL data to categorize the reliability indicators and to find the significant factors affecting the reliability of the alternative bus routes. This study has evaluated the reliability of bus transit lines from two main viewpoints, namely operator, and user preferences, using factor analysis method.

The outcomes of the model presented in this paper are used to meet the citizens' requirements and improve the performance of system bus. The results of this research can be used for performance analysis of the buses network routes as well as for decision making about improving the reliability of both main beneficiaries of the urban bus transportation systems. Thus, the proposed methodology is a potential for analyzing other bus routes facing similar reliability problems.

Future studies should consider passenger data including APC to assess the effects of other causal elements on bus service reliability, thus proposing more effective policies to improve the quality of bus service. Further research can also be directed to a investigate and explore the effects of various parameters affecting the reliability of the bus system at the micro level with large data such as time of day, month, the day of the week, peak hours, passenger flows, etc.

## Reference


1. Srinidhi, S., and A.K. Manrai, (2013). "A conceptual model for demand forecasting and service positioning in the airline industry", Journal of Modelling in Management, Vol. 8, No. 1, pp. 123-139.
2. López-Ramos, F., (2014). "Integrating network design and frequency setting in public transportation networks: a survey", SORT-Statistics and Operations Research Transactions, Vol. 38, No. 2., pp. 181-214.
3. Hu, K.-C., and P.T.-W. Lee, (2017). "Novel 3D model for prioritising the attributes of port service quality: cases involving major container ports in Asia", International Journal of Shipping and Transport Logistics, Vol. 9, No. 6, pp. 673-695.
4. Gajewska, T., and E. Grigoroudis, (2017). "Estimating the performance of the logistics services attributes influencing customer satisfaction in the field of refrigerated transport", International Journal of Shipping and Transport Logistics, Vol. 9, No. 5, pp. 540-561.
5. Monzon, A., S. Hernandez, and R. Cascajo, (2013). "Quality of bus services performance: benefits of real time passenger information systems", Transport and Telecommunication, Vol. 14, No. 2, p. 155-166.
6. Mátrai, T., J. Tóth, and M.T. Horváth, (2016). "Route Planning Based on Urban Mobility Management", Hungarian Journal of Industry and Chemistry, Vol. 44, No. 2, pp. 71-79.
7. Bashir, M., M.T. Afzal, and M. Azeem, (2008). "Reliability and validity of qualitative and operational research paradigm", Pakistan journal of statistics and operation research, Vol. 4, No. 1.
8. Zanjani, Y., Z. Rafie Majd, and A. Mirzazadeh, (2015). "Calculation of the fuzzy reliability in Neishabour train disaster; a case study", Journal of Industrial Engineering and Management Studies, Vol. 2, No. 1, pp. 1-15.
9. Sorensen, P., and R. Church, (2010). "Integrating expected coverage and local reliability for emergency medical services location problems", Socio-Economic Planning Sciences, Vol. 44, No. 1, pp. 8-18.
10. Abkowitz, M., and J. Tozzi, (1987). "Research contributions to managing transit service reliability", Journal of advanced transportation, Vol. 21, No. 1, pp. 47-65.
11. Hassannayebi, E., (2017). "Train timetabling at rapid rail transit lines: a robust multi-objective stochastic programming approach", Operational Research, Vol. 17, No. 2, pp. 435-477.

12. Fattouche, G., (2007). "Improving high-frequency bus service reliability through better scheduling, Massachusetts Institute of Technology.
13. Strathman, J., (1999). "Automated bus dispatching, operations control, and service reliability: Baseline analysis", Transportation Research Record: Journal of the Transportation Research Board, pp. 28-36.
14. Sandim, M., (2016). "Using GPS-based AVL data to calculate and predict traffic network performance metrics: A systematic review", in Intelligent Transportation Systems (ITSC), 2016 IEEE 19th International Conference on. 2016. IEEE.
15. Cham, L.C., (2006). "Understanding bus service reliability: a practical framework using AVL/APC data", Massachusetts Institute of Technology.
16. Maeso-González, E., and P. Pérez-Cerón, (2014). "State of art of bus rapid transit transportation", European Transport Research Review, Vol. 6, No. 2, pp. 149-156.
17. Ma, Z., L. Ferreira, and M. Mesbah, (2014). "Measuring service reliability using automatic vehicle location data", Mathematical Problems in Engineering.
18. Odeck, J., (2006). "Congestion, ownership, region of operation, and scale: Their impact on bus operator performance in Norway", Socio-Economic Planning Sciences, Vol. 40, No. 1, pp. 52-69.
19. Rahman, M.M., S. Wirasinghe, and L. Kattan, (2016). "The effect of time interval of bus location data on real-time bus arrival estimations", Transportmetrica A: Transport Science, Vol. 12, No. 8, pp. 700-720.
20. Byon, Y.-J., (2017). "Bunching and Headway Adherence Approach to Public Transport with GPS", International Journal of Civil Engineering.
21. Tahmasseby, S., (2009). "Reliability in urban public transport network assessment and design".
22. Wardman, M., and G. Whelan, (2011). "Twenty years of rail crowding valuation studies: evidence and lessons from British experience", Transport Reviews, Vol. 31, No. 3, pp. 379-398.
23. Hassannayebi, E., A. Sajedinejad, and S. Mardani, (2016), "Disruption management in urban rail transit system: a simulation based optimization approach", Handbook of Research on Emerging Innovations in Rail Transportation Engineering, pp. 420-450.
24. Chen, X., (2009). "Analyzing urban bus service reliability at the stop, route, and network levels", Transportation research part A: policy and practice, Vol. 43, No. 8, pp. 722-734.
25. Camus, R., G. Longo, and C. Macorini, (2005). "Estimation of transit reliability level-of-service based on automatic vehicle location data", Transportation Research Record: Journal of the Transportation Research Board, pp. 277-286.
26. Hassannayebi, E., (2017). "Timetable optimization models and methods for minimizing passenger waiting time at public transit terminals", Transportation Planning and Technology, Vol. 40, No. 3, pp. 278-304.
27. Xue, R., D.J. Sun, and S. Chen, (2015). "Short-term bus passenger demand prediction based on time series model and interactive multiple model approach", Discrete Dynamics in Nature and Society.
28. Hensher, D.A., and T.F. Golob, (2008). "Bus rapid transit systems: a comparative assessment", Transportation, Vol. 35, No. 4, pp. 501-518.
29. Fielding, G.J., T.T. Babitsky, and M.E. Brenner, (1985). "Performance evaluation for bus transit", Transportation Research Part A: General, Vol. 19, No. 1, pp. 73-82.
30. Karlaftis, M.G., and P.S. McCarthy, (1997). "Subsidy and public transit performance: A factor analytic approach", Transportation, Vol. 24, No. 3, pp. 253-270.
31. De Borger, B., K. Kerstens, and A. Costa, (2002). "Public transit performance: what does one learn from frontier studies?", Transport reviews, Vol. 22, No. 1, pp. 1-38.
32. Karlaftis, M.G., (2004). "A DEA approach for evaluating the efficiency and effectiveness of urban transit systems", European Journal of Operational Research, Vol. 152, No. 2, pp. 354-364.
33. Sheth, C., K. Triantis, and D. Teodorovic, (2007). "Performance evaluation of bus routes: A provider and passenger perspective", Transportation Research Part E: Logistics and Transportation Review, Vol. 43, No. 4, pp. 453-478.
34. Eboli, L., and G. Mazzulla, (2011). "A methodology for evaluating transit service quality based on subjective and objective measures from the passenger's point of view", Transport Policy, Vol. 18, No. 1, pp. 172-181.
35. Deng, T., and J.D. Nelson, (2013). "Bus rapid transit implementation in Beijing: An evaluation of performance and impacts", Research in Transportation Economics, Vol. 39, No. 1, pp. 108-113.

36. Moreira-Matias, L., (2015). "Improving mass transit operations by using AVL-based systems: A survey", IEEE Transactions on Intelligent Transportation Systems, Vol. 16, No. 4, pp. 1636-1653.
37. Gittens, A., and A. Shalaby, (2015). "Evaluation of bus reliability measures and development of a new composite indicator", Transportation Research Record: Journal of the Transportation Research Board, pp. 91-99.
38. Sun, D., (2016). "A bus route evaluation model based on GIS and super-efficient data envelopment analysis", Transportation Planning and Technology, Vol. 39, No. 4, pp. 407-423.
39. Hu, W.X., and A. Shalaby, (2017). "Use of Automated Vehicle Location Data for Route-and Segment-Level Analyses of Bus Route Reliability and Speed", Transportation Research Record: Journal of the Transportation Research Board, pp. 9-19.
40. Khalid, M.B., (2017). "Spatio-temporal Network-Constrained Trajectory Data Model and Service Reliability Assessment", IETE Journal of Research, pp. 1-6.
41. Gkiotsalitis, K., and O. Cats, (2018). "Reliable frequency determination: Incorporating information on service uncertainty when setting dispatching headways", Transportation Research Part C: Emerging Technologies, Vol. 88, pp. 187-207.
42. El-Geneidy, A.M., J. Horning, and K.J. Krizek, (2011). "Analyzing transit service reliability using detailed data from automatic vehicular locator systems", Journal of Advanced Transportation, Vol.45, No. 1, pp. 66-79.
43. Ma, Z., L. Ferreira, and M. Mesbah, (2013). "A framework for the development of bus service reliability measures", in Proceedings of the 36th Australasian Transport Research Forum (ATRF), Brisbane, Australia.
44. Soltani, A., E.Z. Marandi, and Y.E. Ivaki, (2013). "Bus route evaluation using a two-stage hybrid model of Fuzzy AHP and TOPSIS", Journal of Transport Literature, Vol. 7, No. 3, pp. 34-58.
45. Yan, Y., Z. Liu, and Y. Bie, (2016). "Performance evaluation of bus routes using automatic vehicle location data", Journal of Transportation Engineering, Vol. 142, No. 8.
46. Hahn, J.-S., (2017). "Sustainability evaluation of rapid routes for buses with a network DEA model", International Journal of Sustainable Transportation, Vol. 11, No. 9, pp. 659-669.
47. Barabino, B., M. Di Francesco, and S. Mozzoni, (2017), "Time Reliability Measures in bus Transport Services from the Accurate use of Automatic Vehicle Location raw Data", Quality and Reliability Engineering International, Vol. 33, No. 5, pp. 969-978.
48. Ma, Z., (2017). "Quantile Regression Analysis of Transit Travel Time Reliability with Automatic Vehicle Location and Farecard Data", Transportation Research Record: Journal of the Transportation Research Board, Vol. 26, No. 52, pp. 19-29.
49. Diab, E.I., M.G. Badami, and A.M. El-Geneidy, (2015). "Bus transit service reliability and improvement strategies: Integrating the perspectives of passengers and transit agencies in North America", Transport Reviews, Vol. 35, No. 3, pp. 292-328.
50. Sun, Y., (2017). "Reliability prediction model of further bus service based on random forest", Journal of Algorithms & Computational Technology.
51. Yu, B., Z. Yang, and S. Li, (2012). "Real-time partway deadheading strategy based on transit service reliability assessment", Transportation Research Part A: Policy and Practice, Vol. 46, No. 8, pp. 1265-1279.
52. Bertini, R., and A. El-Geneidy, (2003). "Generating transit performance measures with archived data", Transportation Research Record: Journal of the Transportation Research Board, Vol. 18, No. 41, pp. 109-119.
53. Diab, E.I., and A.M. El-Geneidy, (2013). "Variation in bus transit service: understanding the impacts of various improvement strategies on transit service reliability", Public Transport, Vol. 4, No. 3, pp. 209-231.
54. Holland, S.M., (2008). "Principal components analysis (PCA). Department of Geology", University of Georgia, Athens, GA.
55. Jolliffe, I.T., (1986). "Principal Component Analysis and Factor Analysis, in Principal component analysis", Springer. p. 115-128.
56. Abdi, H., and L.J. Williams, (2010). "Principal component analysis", Wiley interdisciplinary reviews: computational statistics, Vol. 2, No. 4, pp. 433-459.