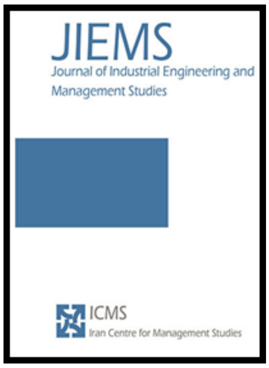
JIEMS

Journal of Industrial Engineering and Management Studies

Vol. 5, No. 2, 2018, pp. 38-60

DOI: 10.22116/JIEMS.2018.80684

www.jiems.icms.ac.ir



Adaptive aggregate production planning with fuzzy goal programming approach

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Abstract

Aggregate production planning (APP) determines the optimal production plan for the medium term planning horizon. The purpose of the APP is effective utilization of existing capacities through facing the fluctuations in demand. Recently, fuzzy approaches have been applied for APP focusing on vague nature of cost parameters. Considering the importance of coping with customer demand in different periods at different and variable rates, in this research, demand is considered fuzzy and the APP decisions modeled through a bi-objective LP model optimizing production and workforce level costs. The APP decisions are taken in two rounds, First The fuzzy model is transformed to a crisp goal programming counterpart and in the second round as the principal contribution of this paper, the APP decisions for rest of the horizon are updated based on actual demand occurred during starting periods. By generating several sample problems and using the Lingo, the validity of the proposed model is shown.

Keywords: Aggregate Production Planning; Adaptive planning; Fuzzy goal programming; Lingo Software.

Received: June 2018-26

Revised: August 2018-12

Accepted: September 2018-18

1. Introduction

Aggregate planning gives a comprehensive plan to respond to forecasted demand employing capacity of the organization effectively. For a 6 to 24-month planning horizon, mid-term production plans are provided in a framework dictated by long-term plans. Medium-term planning inputs are: demand forecasts, financial constraints, capacity constraints, strategic goals, and company policies. The output of mid-term production programs includes: labor force level, production rates, inventory levels that support the production plan, and the subcontracting and overtime levels and back orders. This plan sets the way in which the organization should devise its operational activities including operations, finance and marketing for a typical year.

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In this respect the advantage will be achieving a competitive match between capacity and demand. Such a match is not guaranteed in practice unless the plan updates itself according to real responses received from customers during the execution in the planning horizon. As our knowledge this is not addressed by the authors in related fields of research. Therefore, in this research the adaptive aggregate production planning is studied. Also, instead of cost parameters of the model which are usual to be assumed fuzzy in the existing literature, the demand is chosen to behave in a fuzzy mode because of its incomplete and uncertain nature.

In the rest of the paper, first the literature review is presented, and in the section 3 the research methodology and model and its constraints in fuzzy and equivalent crisp modes will be introduced. In the fourth section, the model's validity will be examined through discussion on some numerical examples. Finally conclusion and proposed future research will be presented.

2. Literature review

As Jamalnia (2017) reported in his thesis, the highest frequency of the published research on APP under uncertainty belongs to 2010-2016 with total frequency of 51.22%. Among the applied methodologies, the fuzzy mathematical programming is the most studied one with close to 50% relative frequency. In some real-world applications, parameters and input data such as demand, coefficient of resources and costs, are fuzzy because of incomplete or vague nature of data (Wang & Liang 2004). Fuzzy is a kind of ambiguity and there is no well-defined definition for its description, especially in places where people's judgments, evaluations, reasoning and learning play a critical role (Bellman and Zade, 1970). In 1976 Zimmerman (1978) first put forward the theory of fuzzy sets in conventional linear programming problems. Narasimhan (1980) described the application of the "fuzzy subset" methods for goal programming in fuzzy environments. This research proposed a fuzzy goal programming with multiple equal weight goals and developed a solution based on linear programming. Wang and Fang (2001) presented a fuzzy linear programming approach for solving the multi-objective aggregate planning in which product prices, subcontract costs, labor levels, production capacity, and market demand are fuzzy. Putting forward a close linkage between production and distribution, Aliev et. al. (2007) studied aggregate production–distribution planning in fuzzy environment. The goal is to maximize profits throughout the planning horizon, while market demand and production and storage capacities are identified as fuzzy random parameters. In their model, the fill rate or service level has been regarded as a fuzzy constraint. The authors claimed genetic algorithm (GA) as a better choice for tackling optimization problems containing fuzzy parameters and proposed a GA with binary coding and multipoint binary operators. Hu et al. (2007) investigated the application of fuzzy goal programming in multi-objective aggregate planning decisions with different priorities, and presented a modified genetic algorithm to solve the transformed model. Jamalnia et al. (2009) developed a hybrid fuzzy multi-objective nonlinear programming model with prioritization of various objectives in a fuzzy environment. They optimized customer satisfaction as a qualitative objective and costs of production, back order, inventory, and labor level changes as quantitative ones. In this research, the effect of learning is considered and the capacity of machinery and storage space is limited. The prioritization of the objectives is deterministic. The model has been modified with fuzzy goal programming and solved by GA. Bykasoglou and Genk (2010) presented a direct solution approach based on fuzzy ranking method and tabu search for solving fuzzy multi-objective APPs. This article shows how a multi-objective model of fuzzy APP can be solved directly without the need for the transformation process. Taghizadeh et al. (2011) introduced the application of the fuzzy multi-objective linear programming model in a multi-product and multi-period production planning problem, which simultaneously minimizes the net present value of production costs and maximizes the utilization of existing capacities. The proposed model includes production constraints such as available labor level, and maximum sub contract level. Ramezanian et al.

(2012) developed a mixed integer linear programming model for two-stage production planning systems to minimize costs and instability in workforce and inventory levels. A GA and tabu search algorithm are presented to solve this problem. Ghasemy Yaghin et al. (2012) proposed a hybrid fuzzy multi-objective programming model for determining optimal pricing policies and aggregate planning decisions in a two-stage supply chain in which the objectives of the model are to maximize total production profit, total retailer profit, and improve the aspects of retailer services simultaneously. After using the appropriate strategy for defuzzification, a crisp multi-objective model is created and then solved by goal programming. Iris (2014) presented a mathematical programming framework for solving APP with ambiguous data. In this paper, the demand is fuzzy and a test problem is solved with different amounts of α cut. Madadi et al. (2014) proposed a multi-objective fuzzy APP model for achieving quality and customer service level simultaneously. They implemented the model in an automotive parts manufacturing company and analyzed the effect of production lines performance. Gholamian et al. (2016) presented a mathematical model for production planning in a supply chain under demand uncertainty and solved it by a fuzzy multi-objective optimization method in GAMS environment. They considered fuzziness in a constraint that includes an uncertain demand. Zaidan et al. (2017) developed a hybrid algorithm called fuzzy–SASD to achieve efficient and effective solutions for APP with fuzzy operating costs, data capacities, and demand. They used Zimmerman's approach for handling all fuzzy parameters. Zhu et. al. (2018) incorporated the risk factor based on decision maker's preference. In this study an interval programming approach has been used to solve a multi-product APP with two objectives.

In the field of production planning, the advanced planning systems focus on two key features; Integration and adaptability. These features are considerably interrelated aspects in order to achieving the effectiveness of planning efforts. While integration aims at breaking the hierarchies among long, medium and short term plans, adaptability searches for any quick and responsive change in an under execution plan. Table 1 summarizes the most important recent studies to explore the status quo of research on these features and context and optimization aspects as well.

Table 1. the summery of the recent studies in the field of APP

Author(year)	Context		Advanced planning		Other Decisions	Objectives		Solution Approach
	Production	SCM	Integration	Adaptability		Multiple	Single	
Entezaminia et. al. (2016)		✓	×	×	Quantity of vehicles	✓		Exact-CPLEX
Modarres et. al. (2016)	✓		×	×	Energy consumption	✓		Goal attainment technique/Robust Optimization
Vogel et. al. (2017)	✓		✓	×	Number of production lines		✓	Exact
Entezaminia et. al. (2017)		✓	×	×	Quantity of vehicles		✓	Robust Optimization
Makui et. al.(2016)	✓		×	×	Under-fulfillment of market demand		✓	Robust Optimization
Jabbarzadeh et. al. (2018)		✓	×	×	Postponement strategy	✓		Robust Optimization
Altendorfer et. al. (2016)	✓		×	×	---		✓	Exact-CPLEX/Simulation
Aghezzaf ET. AL. (2016)	✓		×	×	Preventive maintenance		✓	An iterative MILP-based Solution Heuristic
Nobari ET. AL. (2018)		✓	×	×	Supplier selection	✓		MOICA; NSGA-II
Khemiria et. al. (2017)		✓	×	×	Supplier selection		✓	analytical modelling /multi-criteria approach
This Research	✓		×	✓	---	✓		A comprehensive framework regarding adaptability/ fuzzy goal programming

As it is clear, there is little research on the adaptability of aggregate plan based on real data in conditions of ambiguity in demand. However, under these conditions and in practice, during the implementation of the plan, as Vogel et. al. (2017) emphasized, feedback from the master production schedule (MPS) to the APP is inevitable and the APP must be flexible according to the emerging conditions and at the same time it should guarantee the economic utilization of the resources. Therefore, the model presented in this paper can cover the existing gap in the related literature.

3. Research methodology

As shown in Fig. 1, the framework for achieving adaptive APP is formed in six main steps. These steps are explained in the following sections.

3.1. APP fuzzy LP model

In this section, formulation of a linear programming model for the APP in fuzzy environments is discussed. This model consists of two minimization objective functions, taking into account the costs of producing products, inventory, backorder, setup, normal and overtime labor, hiring and firing. In this model, market demand is considered as a fuzzy parameter. In the following, the indices, parameters, and variables used in the model are defined.

3.1.1. Indices

i : The index of products (i = 1, 2... N)

t : The index of periods in the planning horizon (t = 1, 2... T)

3.1.2. Model parameters

$\widetilde{D}_{it}$: Fuzzy demand for product i in period t

C_{it} : Production cost of each unit of product i in period t

h_{it} : Cost to hold a unit of product i from period t to period $t+1$

π_{it}: Cost of a unit of product i backordered in the period t

S_{it} : Setup cost to produce product i in period t

r_t : Cost of one man-hour of labor on regular time in period t

ov_t : Cost of one man-hour of labor on overtime in period t

hi_t : Cost of hiring a man-hour of labor in period t

fi_t : Cost of firing a man-hour of labor in period t

cp_i : Man-hour required to produce a unit of product i

cs_i : Man-hour required to setup the production system for producing a unit of product i

ρ: Proportion of regular working hours that is allowed to be employed on overtime

3.1.3. Decision variables

X_{it} : Level of production of product i in period t

I_{it} : Level of net inventory of product i in period t

I_{it}^{+} : Level of on-hand ending inventory of product i in period t

I_{it}^{-} : Level of back order of product i in period t

R_t : Level of man-hour employed on regular time in period t

O_t : Level of man-hour employed on overtime in period t

H_t : Number of man-hour hired in period t

F_t : Number of man-hour fired in period t

y_{it} : Binary variable to decide to produce product i in period t ($y_{it} = 1$) else 0

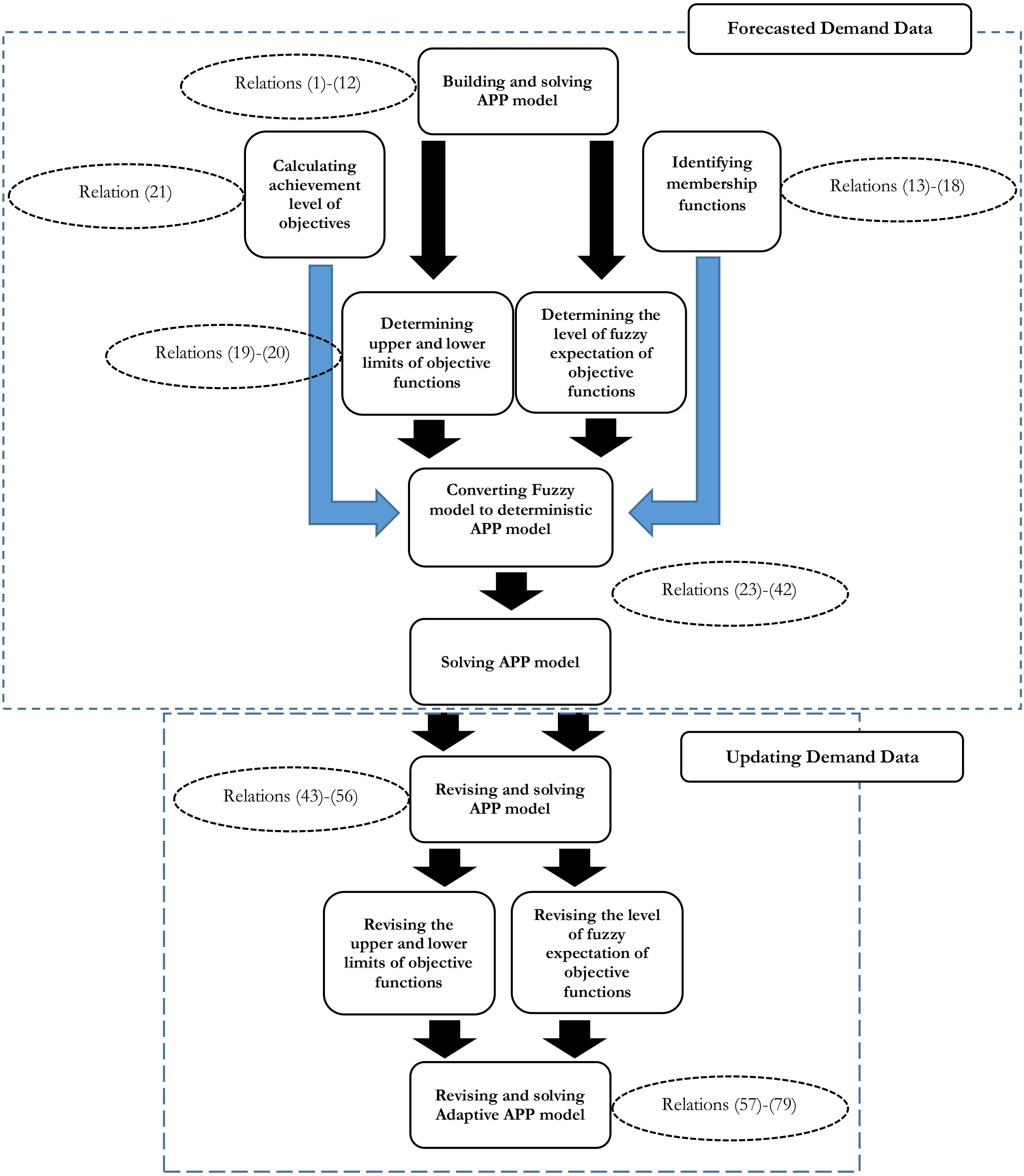

Figure 1. The research stages and its relevant methods

Using the above mentioned notations, the fuzzy LP model of research is presented in equations (1)-(12):

$$min\ Z_1 \cong \sum_{i=1}^{N}\sum_{t=1}^{T}(C_{it}X_{it} +\ h_{it}I_{it}^{+} +\ \pi_{it}I_{it}^{-} +\ S_{it}y_{it}) \qquad (1)$$

$$min\ Z_2 \cong \sum_{t=1}^{T}(r_tR_t +\ ov_tO_t +\ h_tH_t +\ f_tF_t) \qquad (2)$$

Subject to:

$$X_{it} + I_{it-1} - I_{it}\ =\ \widetilde{D}_{it} \qquad \forall\, i\ ;\ \forall\, t \qquad (3)$$

$$I_{it} = I_{it}^{+} - I_{it}^{-} \qquad \forall\, i\ ;\ \forall\, t \qquad (4)$$

$$R_t -\ R_{t-1} -\ H_t +\ F_t = 0 \qquad \forall\, t \qquad (5)$$

$$\sum(cp_iX_{it} +\ cs_iy_{it}) \le R_t +\ O_t \qquad \forall\, t \qquad (6)$$

$$O_t -\ \rho R_t\ \le 0 \qquad \forall\, i\ ;\ \forall\, t \qquad (7)$$

$$X_{it}\ \le My_{it} \qquad \forall\, i\ ;\ \forall\, t \qquad (8)$$

$$X_{it},, I_{it}^{+}, I_{it}^{-}, R_t, O_t, H_t, F_t \ge 0 \qquad \forall\, i\ ;\ \forall\, t \qquad (9)$$

$$R_t, O_t, H_t, F_t \ge 0 \qquad \forall\, t \qquad (10)$$

$$I_{it}\ : FIS \qquad \forall\, i\ ;\ \forall\, t \qquad (11)$$

$$Y_{it} \in \{0,1\} \qquad \forall\, i\ ;\ \forall\, t \qquad (12)$$

In this model, two categories of objectives are considered to optimize the APP decisions which are total production and total labor costs. Equation (1) is the objective function to minimize production and inventory costs. The four terms in this objective function are: production cost, inventory holding costs, cost of backorder, and set-up cost calculated for each product in each period and their total sum must be minimized. Equation (2) is the objective function to minimize labor costs. The three terms in this objective function are: cost of man-hour in regular times, cost of man-hour in overtimes, as well as costs associated with hiring and firing labor during the planning horizon. The constraint (3) is the material balance equation for each product in each period, in which the sum of input and output flows of each product in each period, are equal to the demand for that period. Constraint (4) calculates the net inventory variable in terms of on-hand inventory and the shortage variables in each period for each product. Constraint (5) establishes the balance of labor in each period. If we have a hiring period, we will have $R_t -\ R_{t-1} =\ H_t$; and in case of firing, it is obvious that $R_{t-1} -\ R_t = F_t$. Constraint (6) provides an upper limit for the number of hours required to produce all products in terms of man-hours of regular and overtime in each period. Constraint (7) formulates the allowed upper limit for the use of overtime as a percentage of regular working hours. Constraint (8) ensures that if a production system is not launched for a particular product in each period, the level of production of that will be zero. Constraint (9) - (11) reflect the condition of being non-negative for all decision variables and being free-in sign for net inventory variable. Constraint (12) adjusts the set up variables to be binary.

3.2. Identifying membership functions

The above fuzzy multi-objective linear programming APP model can be transformed into a linear programming APP model using the membership functions provided for fuzzy constraints and fuzzy objective functions. Two types of membership functions for fuzzy constraint and fuzzy objectives will be introduced.

3.2.1. Creating membership function for right hand side

For the equation constraints in the form of $a_{ij}x_j = \tilde{d}_i \quad \forall\, i$; the membership function of vector X is calculated based on the value of the left hand side which is labeled as y for convenience as follows:

$$\mu_d(x) = \begin{cases} 0 & y \leq d_i - p_i^- \\ \frac{y-(d_i-p_i^-)}{p_i^-} & d_i - p_i^- < y \leq d_i \\ \frac{(d_i+p_i^+)-y}{p_i^+} & d_i < y \leq d_i + p_i^+ \\ 0 & y \geq d_i + p_i^+ \end{cases} \tag{13}$$

For any vector X the membership of $D_i(X)$ which gives the membership degree of realizing or satisfying the constraint for a particular, is calculated through equation (14)

$$D_i(x) = \mu_i\left(\sum a_{ij}x_j\right) \tag{14}$$

For each, $D_i(X)$ is a fuzzy set and the intersection of these sets for all i ($\cap D_i$) is the solution space.

3.2.2. Creating membership function for objective functions

The use of fuzzy sets in goal programming has the advantage that decision makers are allowed to set a vague expectation level. An objective with an ambiguous expectation level can be considered as a fuzzy goal. Three types of fuzzy goal programming can be considered, each containing a kind of fuzzy goal in the form of relations (15).

$$\begin{aligned} &G_k\,(x) \lesssim g_1 && k = 1, \dots, m \\ &G_k\,(x) \gtrsim g_2 && k = m+1, \dots, n \\ &G_k\,(x) \cong g_3 && k = n+1, \dots, l \end{aligned} \tag{15}$$

s.t:

$$AX \leq b$$

$$X \geq 0$$

The optimal point, X, meets all fuzzy goals and is still justified. g is called the level of fuzzy expectation and $AX \leq b$ is the set of deterministic constraints. m objectives are to be at last equal to the desired goal of g_{1k} , $n-m$ objectives are to be at least equal to the desired goal of g_{2k} and finally $l-n$ objectives are to be exactly equal to the desired goal of g_{3k} . Three types of linear membership function of fuzzy goals are defined as:

$$\mu_{Z_k}(x) = \begin{cases} 1 & G_k(x) \leq g_{1k} \\ \frac{U_k - G_k(x)}{U_k - g_{1k}} & g_{1k} \leq G_k(x) \leq U_k \\ 0 & G_k(x) \geq U_k \end{cases} \qquad k = 1, \dots, m \tag{16}$$

$$\mu_{Z_k}(x) = \begin{cases} 1 & G_k(x) \geq g_{2k} \\ \frac{G_k(x) - L_k}{g_{2k} - L_k} & L_k \leq G_k(x) \leq g_{2k} \\ 0 & G_k(x) \leq L_k \end{cases} \qquad k = m+1, \dots, n \tag{17}$$

$$\mu_{Z_k}(x) = \begin{cases} 0 & G_k(x) \leq L_k \\ \frac{G_k(x) - L_k}{g_{3k} - L_k} & L_k \leq G_k(x) \leq g_{3k} \\ \frac{U_k - G_k(x)}{U_k - g_{3k}} & g_{3k} \leq G_k(x) \leq U_k \\ 0 & G_k(x) \geq U_k \end{cases} \qquad k = n+1, \dots, l \tag{18}$$

3.3. Determining upper and lower bounds and the level of fuzzy expectation of objective functions

In equations (16)-(18) the lower and upper bounds of each objective function L_k , U_k are calculated by solving the research model which is discussed in section 3-1 converting any fuzzy constraint to its crisp equivalent. To do this, the right hand side will be replaced by its bounds to get L_k , U_k as presented in equations (19) and (20) respectively and the generated crisp models will be solved to optimize each of objective functions separately.

$$\sum_{j=1}^{n} a_{ij} x_j = d_i - p_i^- \tag{19}$$

$$\sum_{j=1}^{n} a_{ij} x_j = d_i + p_i^- \tag{20}$$

The level of fuzzy expectation is obtained similarly but by replacing the right hand side of fuzzy constraint with d_i .

3.4. Converting Fuzzy model to deterministic one

Using the method proposed by Chen and Tsai (2001), which allows decision makers to determine a desired achievement level or importance (or weight) for each of the fuzzy goals, the linear programming model for fuzzy APP will be converted to a deterministic problem maximizing the intersection of the objective functions and the feasible space. This method can ensure that higher-importance goals have higher degrees of achievement. To do this, a set of optimal achievement levels as constraints to the set of constraints is added in the form of $\mu_k \geq \alpha_k$; where α_k is the optimal achievement level for the kth objective.

I is defined as a set of linguistic values for the importance of different objectives as follows:

$$I = \begin{Bmatrix} VLI: Very\ Low\ Importance, LI: Low\ Importance, SLI: Slightly\ Low\ Importance, M: Medium, \\ SHI: Slightly\ High\ Importance\ , HI: High\ Importance, VHI: Very\ High\ Importance \end{Bmatrix}$$

Then, for each linguistic value, a trapezoid fuzzy number is considered as shown in Table 2.

Table 2. Trapezoid fuzzy number for each linguistic value

VLI	LI	SLI	M	SHI	HI	VHI
(0,0,0.05, 0.10)	(0.05,0.10,0.20,0.25)	(0.20,0.25,0.35,0.40)	(0.35,0.40,0.50,0.5)	(0.50,0.55,0.65,0.70)	(0.65,0.70,0.80,0.85),	(0.80,0.85,0.95,0.95)

Now the Liu and Wang (1992) method is applied to rank fuzzy numbers in order to determine precisely the degree of achievement for different goals. Trapezoidal fuzzy numbers are defined for each linguistic value as $\tilde{A} = (\alpha, \beta, \gamma, \delta)$, so we have:

$$\mathrm{I}_{\mathrm{T}}^{1}(\tilde{A}) = \frac{1}{2}[\gamma + \delta] \tag{21}$$

$\alpha_k = I_T^1(\tilde{A}_k)$ is assumed as the optimal achievement level of kth fuzzy objective. The optimal solution to the problem is obtained by maximizing the intersection of the objective functions and the feasible space. The intersection of fuzzy sets of objective functions and solution space, the min operator is used as shown in equation (22).

$$\text{Max min } [\cap_{i=1}^{m} D_i(x),\ G_i(x)] \tag{22}$$

Let $\lambda = \cap_{i=1}^{m} D_i(x),\ G_i(x)$, so λ is maximized subject to $\lambda \leq G_i(x)$ and$\lambda \leq D(x)$. Finally, the fuzzy linear programming model becomes deterministic one as follows:

$$Max\ \lambda \tag{23}$$

$s.t$:

$$\lambda \leq \frac{U_{Z_1} - \sum_{i=1}^{N}\sum_{t=1}^{T}\left(C_{it}X_{it} + h_{it}I_{it}^{+} + \pi_{it}I_{it}^{-} + S_{it}Y_{it}\right)}{(U_{Z_1} - g_{Z_1})} \tag{24}$$

$$\lambda \leq \frac{U_{Z_2} - \sum_{t=1}^{T}(r_tR_t + ov_tO_t + h_tH_t + f_tF_t)}{(U_{Z_2} - g_{Z_2})} \tag{25}$$

$$\lambda p_{it} \leq \beta_{it}(X_{it} + I_{it-1} - I_{it}) - (d_{it} - p_{it})) + (1 - \beta_{it})((d_{it} + p_{it}) - (X_{it} + I_{it-1} - I_{it})) \quad \forall i = 1,2,\dots,N \quad \forall t = 1,2,\dots,T \tag{26}$$

$$X_{it} + I_{it-1} - I_{it} = \theta_{1\mathrm{it}}(d_{it} - p_{it}) + \theta_{2\mathrm{it}}d_{it} + \theta_{3\mathrm{it}}(d_{it} + p_{it}) \quad \forall i,t \tag{27}$$

$$\theta_{1\mathrm{it}} \leq \beta_{it} \quad \forall i,t \tag{28}$$

$$\theta_{2\mathrm{it}} \leq 1 \quad \forall i,t \tag{29}$$

$$\theta_{3\mathrm{it}} \leq 1 - \beta_{it} \quad \forall i,t \tag{30}$$

$$\theta_{1\mathrm{it}} + \theta_{2\mathrm{it}} + \theta_{3\mathrm{it}} = 1 \quad \forall i,t \tag{31}$$

$$I_{it} = I_{it}^{+} - I_{it}^{-} \quad \forall i,t \tag{32}$$

$$R_t - R_{t-1} - H_t + F_t = 0 \quad \forall t \tag{33}$$

$$\sum(cp_iX_{it} + cs_iy_{it}) \leq R_t + O_t \quad \forall t \tag{34}$$

$$O_t - \rho R_t \leq 0 \quad \forall t \tag{35}$$

$$X_{it} \leq M(y_{it}) \quad \forall i,t \tag{36}$$

$$\alpha_{Z_1} \leq \frac{U_{Z_1} - \sum_{i=1}^{N}\sum_{t=1}^{T}\left(C_{it}X_{it} + h_{it}I_{it}^{+} + \pi_{it}I_{it}^{-} + S_{it}Y_{it}\right)}{(U_{Z_1} - g_{Z_1})} \tag{37}$$

$$\alpha_{Z_2} \leq \frac{U_{Z_2} - \sum_{t=1}^{T}(r_tR_t + ov_tO_t + h_tH_t + f_tF_t)}{(U_{Z_2} - g_{Z_2})} \tag{38}$$

$$X_{it}, I_{it}^{+}, I_{it}^{-}, \theta_{1\mathrm{it}}, \theta_{2\mathrm{it}}, \theta_{3\mathrm{it}} \geq 0 \quad \forall i,t \tag{39}$$

$$R_t, O_t, H_t, F_t \geq 0 \quad \forall t \tag{40}$$

$$\beta_{it} \in \{0,1\} \tag{41}$$

$$I_{it}: Free\ In\ Sign \quad \forall i,t \tag{42}$$

The objectives of the fuzzy problem are simplified according to the membership function in relation (16) in the form of $\lambda \leq G_i(x)$ and added to the constraints of the deterministic model as constraints (23) and (24). The values of g_{Z_1} and g_{Z_2} will be obtained by solving the model presented in section 3-1 for optimizing equation (1) and (2) separately. To do so, the fuzzy numbers of demand will be assumed crisp by setting p_{it} values to zero. The fuzzy constraint of the basic model should be converted through the relation (13) to the form of $\lambda \leq D(x)$. To do this in constraint (25), whenever the third rule of the relation (13) is the matter of the case, it means $\beta = 0$ and consequently the term $\frac{(d_i + p_i) - (X_{it} + I_{it-1} - I_{it})}{p_i}$ will be greater than or equal to λ . By setting β to 1, the term $\frac{X_{it}+I_{it-1}-I_{it}-(d_i-p_i)}{p_i}$ will be greater than or equal to λ as the same way. The right hand side of relation (26) gives a linear combination of $d_{it}-p_{it}$ and d_{it} which is a general representative for values fallen between those extremes. Similarly this equation sets the term $X_{it}+I_{it-1}-I_{it}$ as a linear combination of d_{it} and $d_{it}+p_{it}$. The condition to do so, is that at most two adjacent θ_{k} can be positive. According to constraints (27) – (29) when $\beta=0$, θ_2 and θ_3 are active and equation (30) force their sum to be one. The same is returned for $\beta=1$ in which θ_1 and θ_2 are active their sum is forced to be one. Other added relations to the basic model are labeled (36) and (37) which respectively do the same of relations (23) and (24) considering the importance given by managers($\alpha_{Z_1},\alpha_{Z_2}$)for the two objective functions.

3.4. Adaptive model

Now it is assumed that after τ periods the real data of demand occurred is available. Johnson and Montgomery (1978) called τ as the production lead time which is the number of periods between a decision to change the production rate and the time the change becomes effective. In other words the plan given by the model presented in section 3-3 is a preliminary one and it will be adjusted after τ periods.

Hereafter, the solved variables for periods $t=1,2,\dots,\tau$ will be labeled as ${X^{*real}}_{it}$, ${I^{*real}}_{it}$, ${R^{*real}}_t, {O^{*real}}_t, {H^{*real}}_t, {F^{*real}}_t$ and real demands will be labeled as ${d^{*real}}_{it}$. To revise the bounds on objective functions ($U_{Z_1}, U_{Z_2}, L_{Z_1}, L_{Z_2}$) and the desired goals (g_{Z_1}, g_{Z_2}), it is required to solve the revised model as follows.

$$\min Z_1=\sum_{i=1}^{N}\sum_{t=1}^{\tau}\left(C_{it}{X^{*real}}_{it}+h_{it}{I^{+real}}_{it}+\pi_{it}{I^{-real}}_{it}+S_{it}{y^{*real}}_{it}\right)+\sum_{i=1}^{N}\sum_{t=\tau+1}^{T}(C_{it}X_{it}+h_{it}I_{it}^{+}+\pi_{it}I_{it}^{-}+S_{it}y_{it}) \quad (43)$$

$$\min Z_2=\sum_{t=1}^{\tau}(r_t{R^{*real}}_t+ov_t{O^{*real}}_t+h_t{H^{*real}}_t+f_t{F^{*real}}_t)+\sum_{t=\tau+1}^{T}(r_tR_t+ov_tO_t+h_tH_t+f_tF_t) \quad (44)$$

Subject to:

$X_{it}+I_{it-1}-I_{it}=d'_{it}$	$\forall\, i;\ \forall\, t=\tau+2,\dots,T$	(45)
$X_{it}+{I^{*real}}_{i\tau}-I_{it}=d'_{it}$	$\forall\, i;\ t=\tau+1$	(46)
$I_{it}=I_{it}^{+}-I_{it}^{-}$	$\forall\, i;\ \forall\, t=\tau+2,\dots,T$	(47)
$R_t-R_{t-1}-H_t+F_t=0$	$\forall\, t=\tau+2,\dots,T$	(48)
$R_t-{R^{*real}}_\tau-H_t+F_t=0$	$t=\tau+1$	(49)
$\sum(cp_iX_{it}+cs_iy_{it})\le R_t+O_t$	$\forall\, t=\tau+1,\dots,T$	(50)
$O_t-\rho R_t\le 0$	$\forall\, t=\tau+1,\dots,T$	(51)
$X_{it}\le My_{it}$	$\forall\, i;\ \forall\, t=\tau+1,\dots,T$	(52)
$X_{it},, I_{it}^{+}, I_{it}^{-}, R_t, O_t, H_t, F_t\ge 0$	$\forall\, i;\ \forall\, t=\tau+1,\dots,T$	(53)
$R_t, O_t, H_t, F_t\ge 0$	$\forall\, t=\tau+1,\dots,T$	(54)
I_{it} : FIS	$\forall\, i;\ \forall\, t=\tau+1,\dots,T$	(55)
$Y_{it}\in\{0,1\}$	$\forall\, i;\ \forall\, t=\tau+1,\dots,T$	(56)

By setting $d'_{it} = d_{it}$ the model revised g_{Z_1} and g_{Z_2} to $g_{Z_1}^{revised}$ and $g_{Z_2}^{revised}$ through optimizing Z_1 and Z_2 separately. By setting $d'_{it} = d_{it} + p_{it}$ the model revised U_{Z_1} and U_{Z_2} to $U_{Z_1}^{revised}$ and $U_{Z_2}^{revised}$ through optimizing Z_1 and Z_2 separately. Similarly, setting $d'_{it} = d_{it} - p_{it}$ leads to to $L_{Z_1}^{revised}$ and $L_{Z_2}^{revised}$.

Finally the adaptive and deterministic model is presented below.

$$Max\ \lambda \tag{57}$$

Subject to:

$$\lambda \le \frac{U_{Z_1}^{revised} - \sum_{i=1}^{N}\sum_{t=1}^{\tau}\left(C_{it}X^{*real}{}_{it} + h_{it}I^{+real}{}_{it} + \pi_{it}I^{-real}{}_{it} + S_{it}y^{*real}{}_{it}\right) - \sum_{i=1}^{N}\sum_{t=\tau+1}^{T}\left(C_{it}X_{it} + h_{it}I_{it}^{+} + \pi_{it}I_{it}^{-} + S_{it}y_{it}\right)}{\left(U_{Z_1}^{revised} - g_{Z_1}^{revised}\right)} \tag{58}$$

$$\lambda \le \frac{U_{Z_2}^{revised} - \sum_{t=1}^{\tau}\left(r_tR^{*real}{}_t + ov_tO^{*real}{}_t + h_tH^{*real}{}_t + f_tF^{*real}{}_t\right) - \sum_{t=\tau+1}^{T}\left(r_tR_t + ov_tO_t + h_tH_t + f_tF_t\right)}{\left(U_{Z_2}^{revised} - g_{Z_2}^{revised}\right)} \tag{59}$$

$\lambda p_{it} \le \beta_{it}(X_{it} + I_{it-1} - I_{it}) - (d_{it} - p_{it})) + (1 - \beta_{it})((d_{it} + p_{it}) - (X_{it} + I_{it-1} - I_{it}))$	$\forall i = 1,2,\dots,N \quad \forall t = \tau+2,\dots,T$	(60)
$\lambda p_{it} \le \beta_{it}(X_{it} + I^{*real}{}_{i\tau} - I_{it}) - (d_{it} - p_{it})) + (1 - \beta_{it})((d_{it} + p_{it}) - (X_{it} + I^{*real}{}_{i\tau} - I_{it}))$	$\forall\, i;\ \ t = \tau + 1$	(61)
$X_{it} + I_{it-1} - I_{it} = \theta_{1\mathrm{it}}(d_{it} - p_{it}) + \theta_{2\mathrm{it}}d_{it} + \theta_{3\mathrm{it}}(d_{it} + p_{it})$	$\forall\, i;\ \ \forall\, t = \tau+2,\dots,T$	(62)
$X_{it} + I^{*real}{}_{i\tau} - I_{it} = \theta_{1\mathrm{it}}(d_{it} - p_{it}) + \theta_2 d_{it} + \theta_3(d_{it} + p_{it})$	$\forall i\,; t = \tau + 1$	(63)
$\theta_{1\mathrm{it}} \le \beta_{it}$	$\forall\, i;\ \ \forall\, t = \tau+1,\dots,T$	(64)
$\theta_{2\mathrm{it}} \le 1$	$\forall\, i;\ \ \forall\, t = \tau+1,\dots,T$	(65)
$\theta_{3\mathrm{it}} \le 1 - \beta_{it}$	$\forall\, i;\ \ \forall\, t = \tau+1,\dots,T$	(66)
$\theta_{1\mathrm{it}} + \theta_{2\mathrm{it}} + \theta_{3\mathrm{it}} = 1$	$\forall\, i;\ \ \forall\, t = \tau+1,\dots,T$	(67)
$I_{it} = I_{it}^{+} - I_{it}^{-}$	$\forall i, \forall t = \tau+1,\dots,T$	(68)
$R_t - R_{t-1} - H_t + F_t = 0$	$\forall t = \tau+2,\dots,T$	(69)
$R_t - R^{*real}{}_{\tau} - H_t + F_t = 0$	$t = \tau + 1$	(70)
$\sum(cp_iX_{it} + cs_iy_{it}) \le R_t + O_t$	$\forall\, t = \tau+1,\dots,T$	(71)
$O_t - \rho R_t \le 0$	$\forall t = \tau+1,\dots,T$	(72)
$X_{it} \le M(y_{it})$	$\forall\, i;\ \ \forall\, t = \tau+1,\dots,T$	(73)

$$\alpha_{Z_1} \le \frac{U_{Z_1}^{revised} - \sum_{i=1}^{N}\sum_{t=1}^{\tau}\left(C_{it}X^{*real}{}_{it} + h_{it}I^{+real}{}_{it} + \pi_{it}I^{-real}{}_{it} + S_{it}y^{*real}{}_{it}\right) - \sum_{i=1}^{N}\sum_{t=\tau+1}^{T}\left(C_{it}X_{it} + h_{it}I_{it}^{+} + \pi_{it}I_{it}^{-} + S_{it}y_{it}\right)}{\left(U_{Z_1}^{revised} - g_{Z_1}^{revised}\right)} \tag{74}$$

$$\alpha_{Z_2} \leq \frac{U_{Z_2}^{revised} - \sum_{t=1}^{\tau}\left(r_t R^{*real}{}_t + ov_t O^{*real}{}_t + h_t H^{*real}{}_t + f_t F^{*real}{}_t\right) - \sum_{t=\tau+1}^{T}\left(r_t R_t + ov_t O_t + h_t H_t + f_t F_t\right)}{\left(U_{Z_2}^{revised} - g_{Z_2}^{revised}\right)} \tag{75}$$

$$X_{it}, I_{it}^{+}, I_{it}^{-}, \theta_{1\mathrm{it}}, \theta_{2\mathrm{it}}, \theta_{3\mathrm{it}} \geq 0 \qquad \forall\, i\,;\ \ \forall\, t = \tau+1, \dots, T \tag{76}$$

$$R_t, O_t, H_t, F_t \geq 0 \qquad \forall t = \tau+1, \dots, T \tag{77}$$

$$\beta_{it} \in \{0,1\} \tag{78}$$

$$I_{it}: Free\ In\ Sign \qquad \forall\, i\,;\ \ \forall\, t = \tau+1, \dots, T \tag{79}$$

4. Numerical example and analysis

To come up with possible situations, the case with two scenarios is discussed, taking in to account bellow assumptions and rules:

Assumption 1: It is assumed that the sales function tries his best to absorb orders from the market and to realize the sales plan.

Assumption 2: It is assumed that the sales and operations planners hand over the plan to each other in advance to achieve a realistic one considering the appropriateness for market and resources.

Assumption 3: At the aggregate sales and operations planning level, getting operations function a plan which misses coverable forecasted demands is contrary to the fundamental principle of full utilization of existing capacities of operations facilities. It is assumed that any decision to adjust system to a load under its normal capacity level means "plan to fail" and shrinks the effectiveness. Except the case in which permitting lost sales gives much better gains, for other cases this assumption will be valid.

Rule of changing SOP: Based on the above assumptions, if the revised values of expectations and bounds are not acceptable for the managers, the sales plan shall be revised and accordingly the operations plan shall be changed too. Otherwise, just the operations plan should be revised to achieve updated expectations and bounds.

Rule of acceptance: The absolute of difference between initial and revised values of expectations and bounds should be considered. Because the sign of difference is misleading and the absolute value can indicate to the fact that the existing plan becomes unrealistic. Hence, all reductions in revised values are not acceptable.

In all discussions, we are talking in behalf of operations function, so we argue the cases in which changing operations plan is required based on "rule of changing SOP".

Scenario 1:

If the real demands during periods $t = 1,2, \dots, \tau$ are less than forecasted ones, the sales plan will not be changed unless the revised values of expectations and bounds are not acceptable for the managers. Changing operations plan may lead to using overtime and sub-contracting to fulfil revised expectations and bounds.

In the case that the revised expectations and bounds are improved, the initial arrangement of resources (which is mathematically optimal) are might infeasible.

Scenario 2:

If the real demands during periods $t = 1,2, \dots, \tau$ are greater than forecasted ones, the sales plan will not be changed unless the revised values of expectations and bounds are not acceptable for the managers. Changing operations plan leads to using overtime and sub-contracting to fulfil revised expectations and bounds.

The lag in occurring demands leads to load of more work in operations system and consequently the initial arrangement of resources (which is mathematically optimal) is not feasible. So using overhead and subcontracting are necessitated and shall be used to cover the overall plan.

In this section a randomly generated problem is considered in which the decisions of sales and operations planning are to be made for 3 products in a planning horizon of 12 months. The initial man-hour is assumed 800, and the rate of allowed work in overtime is 25 percent. 9 man-hours of ordinary work for producing each of products and 3 man-hours of ordinary work for setup per each output of the production system are assumed. The other required data is presented in Table 3.

Table 3. The given data for a sample problem

Parameters	Periods											
	1	2	3	4	5	6	7	8	9	10	11	12
d_{it}	250	550	1150	2350	1000	700	800	900	1000	1500	1700	500
	400	660	400	350	200	800	1000	1200	700	600	500	300
	1000	800	720	630	480	300	210	700	800	900	1010	1500
ov_t	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5
hi_t	375	375	375	375	375	375	375	375	375	375	375	375
fi_t	599	599	599	599	599	599	599	599	599	599	599	599
C_{it}	169	195	198	153	179	154	186	196	177	180	189	169
	208	229	218	249	258	259	228	233	206	230	251	249
	45	92	88	45	90	45	49	74	77	84	50	63
h_{it}	149.3	70.31	174.0	89.56	175.0	125.3	65.81	177.0	94.87	126.3	143.0	96.45
	81.54	136.6	184.1	28.48	89.03	82.39	74.08	28.17	197.2	155.3	245.5	178.2
	28.50	3.085	82.82	33.51	0.723	40.48	44.24	7.550	76.58	39.38	12.61	39.65
π_{it}	51	58	58	61	70	61	67	60	61	52	64	70
	46	41	48	60	55	56	67	53	44	42	55	64
	93	100	85	85	99	96	100	87	95	96	86	95
S_{it}	1032	1105	1171	1089	1077	1178	1081	1128	1145	1076	1055	1139
	697	557	510	527	681	696	573	585	555	557	602	510
	443	301	333	422	366	466	481	419	378	381	305	350
p_{it}	60	59	59	55	51	66	61	63	52	52	70	50
	134	109	138	137	118	101	138	105	108	116	108	112
	136	124	135	125	145	131	159	159	159	136	133	129

For determining the level of fuzzy expectation of objective functions, the Relations (1)-(12) are solved separately for optimizing equation (1) and (2). The values of $\widetilde{D}_{it}$ in equation (3) are set as crisp numbers which are shown as d_{it} in Table 3. Similarly, the upper and lower limits of objective functions have been calculated by setting the values of $\widetilde{D}_{it}$ according to Relations (19) and (20). The values of p_{it} are shown in the last rows in Table 3. Achievement level of objectives are calculated by relation (21) after gathering the decision makers' linguistic values for the importance of objectives which are transformed through the fuzzy numbers in Table 2. The results are summarized in Table 4. All optimization efforts have been done in Lingo 17 on a 64-Bit system with 3.60 GHZ processor and 4.00 GB RAM.

Table 4. expectations/bounds/achievement level of objective functions

The level of fuzzy expectations		The bounds of objectives		Achievement level of objectives	
g_{Z_1}	g_{Z_2}	U_{Z_1}	U_{Z_2}	α_{Z_1}	α_{Z_2}
4424080	8607833	4976495	9785388	0.650	0.525

Applying the new calculated data in Table 4, the crisp counterpart of our studied fuzzy model can be solved through entering Relations (23)-(42) in the Lingo 17 environment. Its global solver was enabled and found the global optimum for these relations in about 3 hours of computations. According to this, the level of satisfying constraints and objectives is 60% ($\lambda = 0.604$). Table 5 summarizes the output for variables listed in section 3.1.3.

Table 5. Optimal values of decision variables in Non-Adaptive Crisp Model for the sample problem

	Periods											
	1	2	3	4	5	6	7	8	9	10	11	12
x_{it}	213.3675	539.5243	724.4766	1898.972	1591.779	985.9604	824.1479	924.9396	1020.585	1479.415	1672.289	480.2066
	228.8601	1080.314	0	0	449.0539	839.9826	1054.63	1158.434	687.983	554.0794	0	0
	946.1621	849.0875	773.4421	679.4834	537.4007	752.2902	699.4556	494.8596	869.6651	544.7389	906.1661	2098.249
R_t	10001.21	10001.21	18568.08	18568.08	18568.08	18568.08	18568.08	18568.08	18568.08	18568.08	18568.08	18568.08
O_t	2500.301	2500.301	4642.02	4642.02	4642.02	4642.02	4642.02	4642.02	4642.02	4642.02	4642.02	4642.02
H_t	9201.205	0	8566.873	0	0	0	0	0	0	0	0	0
F_t	0	0	0	0	0	0	0	0	0	0	0	0
I_{it}^{+}	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	400.4316	826.9443	558.8611	565.5833	156.4842	0	547.1819
I_{it}^{-}	12.88051	0	402.1672	831.4225	259.8332	0	0	0	0	0	0	0
	118.0937	734.9441	0	295.7662	0	0	0	0	0	0	457.2463	712.9092
	0	0	0	0	0	0	0	0	0	0	0	0

According to section 3-4 it is assumed that after 2 periods ($\tau = 2$), the plan can be updated. The real data from sales department for these two last periods (${d^{*real}}_{it}$) are as follows:

Table 6. Updated data of demand for two periods

	Product 1		Product 2		Product 3	
	Period 1	Period 2	Period 1	Period 2	Period 1	Period 2
Real Demand Occurred	110	80	140	170	1160	720
Forecasted Demand	(190,250,310)	(491,550,609)	(266,400,534)	(551,660,769)	(864,1000,10136)	(676,800,924)
Deviation from the most likely forecasts	-56.00%	-85.45%	-65.00%	-74.24%	16.00%	-10.00%

From Table 6, it is clear that the **"scenario 1"** is brought up. Replacing the real demand data and the solved values for decision variable for periods 1, 2 which are shaded in Table 5, first the expectations and bounds of objective functions should be revised through Relations (43)-(56). The results are obtained by Lingo 17 branch and bound solver as shown in Table 7.

Table 7. expectations/bounds/achievement level of objective functions

	The level of fuzzy expectations		**The bounds of objectives**	
	g_{Z_1}	g_{Z_2}	U_{Z_1}	U_{Z_2}
Initial values	4424080	8607833	4976495	9785388
Revised values	4180498	5008142	4605327	5008142
Deviation from initial plan	-5.51%	-41.82%	-7.46%	-48.82%

As it is shown by the last row in Table 7 the sign of deviations is negative. This means that deviation from the most likely forecasts led to the situation in which the better level of fuzzy expectations and bounds of objective functions should be pursued. So, after updating data, the initial values give a misleading perspective of the potential expectations and upper bounds. In this example, the operations function used its resources to produce the quantities which have not been sold at the end of the second period. Thus, inventories will be helpful to cover the rest of periods if the sales plan (for periods 3 to 12) remains unchanged. This has reduced the level of costs comparing to the initial ones. It is assumed that these deviations are accepted by managers.so in line with "**Rule of changing SOP"** the operations plan shall be updated. Also regarding **Assumption 3,** no lost sales throughout the planning horizon is permitted.

Replacing updated data which are bolded in Table 5 and from Table 6 and Table 7 with initial data of the sample problem and run the relations (57)-(79) in Lingo 17 environments leads to infeasibility. Examining the situation in Lingo 17 environment shows that the infeasibility has been caused by constraint (74). In fact production and inventory costs reached their revised upper bound and consequently the importance forced by managers for these costs couldn't be satisfied. It is predominantly due to transferred inventories from periods 1 and 2 which are caused by the lower level of real demand comparing to that of forecasted demand in initial SOP. So, as indicated in "scenario 1" the higher level of overtime and/or using sub-contracting could be useful. Using sub-contractors' capacities frees up the storages from pushed inventories which have been produced in ordinary or overtime hours for overcoming future demand peaks. Table 8 summarizes the result of various strategies might be considered by managers to be implemented.

Table 8. λ values obtained by different strategies

		sub-contracting	
		employed	not employed
lost sales	permitted	0.56-0.67(**S1**)	0.53(**S3**)
	not permitted	0.04-0.09(**S2**)	infeasible(**S4**)

For strategies which employs sub-contracting some modifications in the relations (57)-(79) have been done. The level of production in each period by sub-contracting and the related cost have been added to the model. The best value of λ is obtained by strategy **S1** (permitted lost sales/ employed sub-contracting**)** assuming 25 percent increase comparing to ordinary costs for sub-contractor production. For 150 percent increase, strategy **S1** obtained 0.56 which is close to the initial value of 0.604 . Strategy **S3** (permitted lost sales/ not employed sub-contracting**)** obtained 0.53 which is almost equal to employing rather expensive sub-contractor. Lingo 17 performed computations of strategy **S2** for about 10 hours and concluded non encouraging results far from the initial value of 0.604. So, strategy **S1** is recommended to the decision makers. Table 9 summarizes the output of strategy **S1** for variables listed in section **3.1.3.** The required production prepared by sub-contracting is 576.0555 of product 3 in period 4.

Table 9. summary of strategy S1

	Periods											
	1	2	3	4	5	6	7	8	9	10	11	12
x_{it}	213.3675	539.5243	795.253	2532.382	1101.158	671.5173	773.6751	872.812	977.5591	1184.464	0	0
	228.8601	1080.314	1075.389	0	439.9532	756.4128	940.4453	1154.687	0	0	0	0
	946.1621	849.0875	661.7399	0	417.4244	243.4662	267.8814	504.8837	731.3826	841.3084	2025.773	371.1598
R_t	10001.21	10001.21	18237.95	18237.95	18237.95	18237.95	18237.95	18237.95	18237.95	18237.95	18237.95	18237.95
O_t	2500.301	2500.301	4559.488	4559.488	0	0	0	4559.488	0	0	0	0
H_t	0	0	8236.743	0	0	0	0	0	0	0	0	0
F_t	0	0	0	0	0	0	0	0	0	0	0	0
I_{it}^{+}	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	126.4989	0	0	0	1073.169	0
I_{it}^{-}	12.88051	0	329.2852	123.1673	0	0	0	0	0	293.0949	1962.886	2441.308
	118.0937	734.9441	0	290.8768	0	0	0	0	653.3919	1203.331	1656.723	1908.389
	0	0	0	0	0	0	0	0	0	0	0	0

To have a better understanding of the case, for **S1** strategy the computations are done for three rates of overtime(0.20%, 25% and 30%) and ten cost rates of contracting(from 125% to 325%

by 25% steps and 1000%). It is interesting to know about the effect of a change in overtime rate (namely, a 5% increase) in any level of contracting cost. As shown in Fig. 2, the effect has a cyclic behavior. However, the least effect of any 5% increase of using normal capacity as overtime, is seen where the contract cost is 25% greater than that of using normal capacities. In addition, the greatest effect is observed in 300% and 325% levels of contract cost (slightly more than that of 1000%).

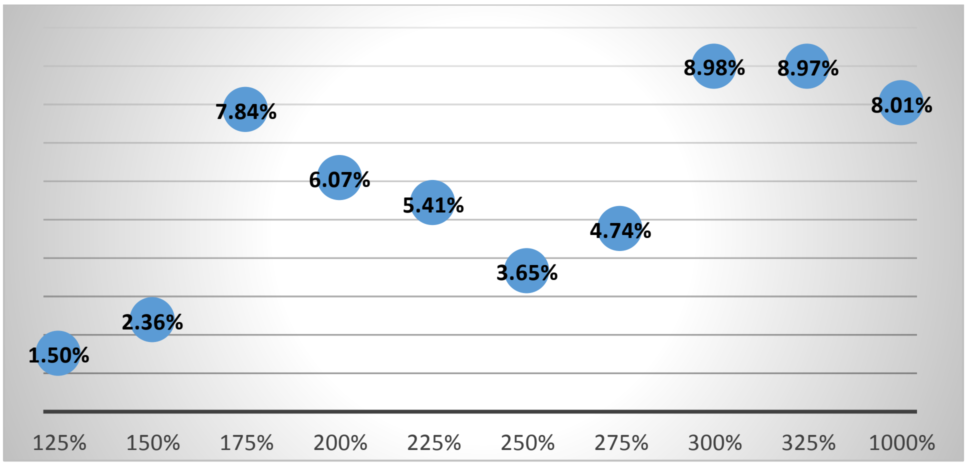

Figure 2. the effect of a change in overtime rate in any level of contracting cost

Similar analysis would be sought for the effect of any 25% increase in contract cost in any level of allowed percentage of overtime. It is concluded that the effect of each 25% increase in contract cost would be greater where the system uses the lesser percentage of normal capacity as overtime. Fig. 3 presents the observations summarized on average.

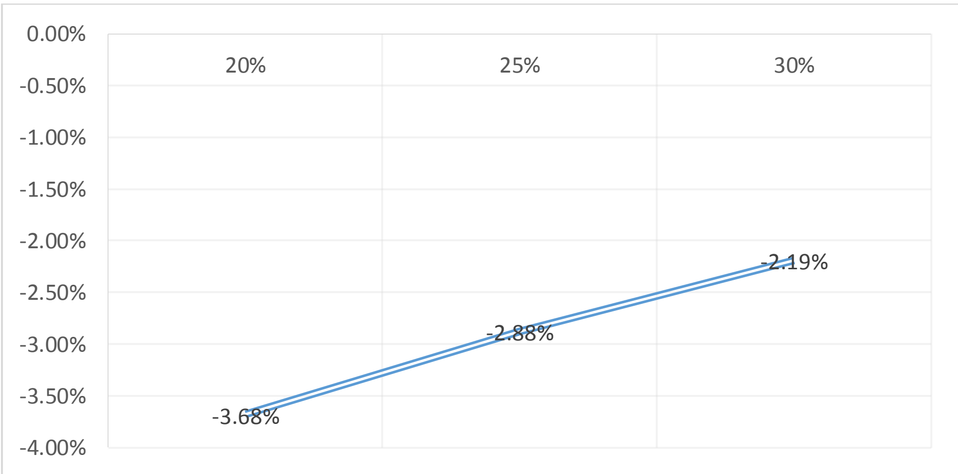

Figure 3. the effect of increase in contract cost in any level of allowed percentage of overtime

The further insight would be clustering of the various combination of overtime and contract levels which serves as a mechanism for a manager to make an executable optimum decision and to screen the alternatives. Each of these 30 alternatives is a combination of two percentages. The pair of (Ov. %, Cont. %) is representative for alternatives. As shown in Table 10, the observed range of Landa for 30 experiments is divided to 5 partitions each forms a cluster of homogenous combinations in term of Landa. For instance, the pairs (20%, 125%), (25%, 125%) and (30%, 125%) are homogenous and a manager would choose the lower level of overtime while obtaining the similar result is guaranteed for him.

Table 10. Clustering alternatives in strategy S1

Cluster	Pair		Range of Landa
	Ov. %	Cont. %	
Cluster #1	20%	125%	0.65-0.69
	25%	125% , 150%	
	30%	125% , 150% , 175%	
Cluster #2	20%	150%	0.60-0.64
	25%	175% , 200%	
	30%	200% , 225%	
Cluster #3	20%	175% , 200%	0.55-0.59
	25%	225% , 250% , 275%	
	30%	250% , 275% , 300%, 325% , 1000%	
Cluster #4	20%	225% , 250% , 275%	0.50-0.54
	25%	300% , 325% , 1000%	
	30%	-----	
Cluster #5	20%	300% , 1000%	0.45-0.49
	25%	-----	
	30%	-----	

About 67 percent of Experiments of S1 strategy are infeasible. Table 11 shows the details of results for solving the feasible pairs. As is shown, the best alternative is (30%, 125%) but because of the pressure of inventory and capacity costs during the planning horizon which is forced by the required 100% service level at the end of the planning, the obtained value is just about 0.1.

Table 11. Feasible alternatives in strategy S2

Exp. No.	Allowed percentage of overtime	Inflation percentage for contractor costs	Value of objective function	Local/ Global	Computation time(Seconds)
1	20%	125%	0.072	Global	6410.53
2	20%	150%	0.016	Global	8.75
1	25%	125%	0.087	Global	11196.98
2	25%	150%	0.04	Global	15.15
3	25%	175%	0.006	Global	12.07
1	30%	125%	0.1	Global	16333.02
2	30%	150%	0.067	Global	3889.42
3	30%	175%	0.04	Global	64.99
4	30%	200%	0.01	Global	29.84

5. Conclusion and future research

In this study, the dynamic nature of aggregate production plans and relative flexibility required for operations function were emphasized. Also the fuzziness of demand was incorporated in model building. The decisions of inventory and production as well as man hour levels during a planning horizon, were considered to be optimized. The fuzzy LP approach was adopted along with a multi stage analysis method to address the adaptability of the plan. Three assumptions were applied, two rules and two possible scenarios which would be happened during updating process. Discussing a sample problem according to the analysis method revealed that the updated operations plan might violate expectations and importance of objective functions and new strategies should be developed. 3 different strategies were analyzed to come up with the infeasibility of the adapted plan and showed how to define adaptive strategies to attain closer objective value to the initial one. The main insight based on the presented assumptions and rules is validating the necessity of updating the aggregate plans and the fact that it is better to prepare initial plans without sub-contracting and determine the required level of subcontracting during the adaptation process. For future researches, the lengths of golden time to revise the plan (τ) can be analyzed in various circumstances. Also the subject of adaptability is very crucial in multi stage production systems. In addition the role of adaptive aggregate plans in effectiveness of production-distribution systems in which there exists variations in sales performance of retailors is very important.

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This article can be cited: Ayough, A., (2018). "Adaptive aggregate production planning with fuzzy goal programming approach", *Journal of Industrial Engineering and Management Studies*, Vol. 5, No. 2, pp. 38-60.