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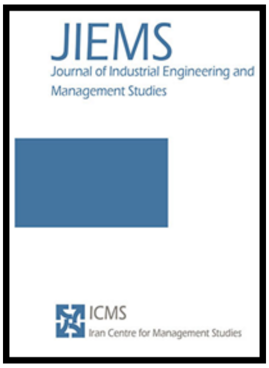

Optimal flexible capacity in Newsboy problem under stochastic demand and lead-time

Kamran Karimi Movahed[1], Ali Ghodrat Nama[1,*]

Abstract

In this paper, we consider a newsvendor who is going to invest on dedicated or flexible capacity, our goal is to find the optimal investment policy to maximize total profit while the newsvendor faces uncertainty in lead-time and demand simultaneously. As highlighted in literature, demand is stochastic, while lead-time is constant. However, in reality lead-time uncertainty decreases newsvendor's performance and increases purchasing cost. Analytical results suggest an approach for decision makers to decide which situation is optimal to invest in flexible capacity. Furthermore, we derive a closed-form solution for optimal production and capacity under dedicated and flexible policy when demand and lead-time follow uniform and normal distribution. An approximation method introduced in this paper to find the optimal production quantity and investment policy results show that this approximation is useful when the coefficient of variation is low under uniform distribution, and it is useful when the coefficient of variation is high under normal distribution. Finally, we show a threshold, considering the fact that it is optimal for the newsvendor to invest on flexible capacity when flexible capacity cost is less than the threshold. To sum up, we measure the effect of lead-time variability on optimal solution.

Keywords: Flexible capacity; Stochastic demand; Stochastic lead-time; Newsboy problem; Lagrangian optimization.

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1. Introduction

Supply chain consists of all the stages in fulfilling a customer request. These stages include supply, manufacturing, transporting, warehousing and distributing. Each product should pass all these stages to be available for the customer on time. On the other hand, the competitive and uncertain environment of the modern economy has forced the newsvendor to respond to a wide range of future demands. This has led the newsvendor to invest in flexible capacity, which incur more cost in comparison with dedicated capacity; consequently, the decision about flexible and dedicated capacity in uncertain environment is a big challenge for decision makers.

* Corresponding author; ghodratnama@khu.ac.it

[1] Department of Industrial Engineering, Kharazmi University, Tehran, Iran.

On the other hand, dealing with lead-time issues, especially for long lead-time industries, makes the problem even more complicated, as lead-time may be higher than the length of selling season. Nowadays, many U.S. wholesalers are sourced from Chinese manufacturers with the lead-time of approximately three months, but there is significant uncertainty around this (Wang and Tomlin, 2009). So, if US wholesalers intend to sell the product in fall, they should place order at least three months before fall, so they will face a type of newsvendor problem with uncertain lead-time. The following reports by The Economist and Boston Consulting Group (BCG) show that lead-time uncertainty is such a big managerial concern:

"Last autumn some 80m items of clothing were impounded at European ports and borders because they exceeded the annual import limits that the European Union and China had agreed on only months earlier. Retailers had ordered their autumn stock well before that agreement was signed, and many were left scrambling to find alternative suppliers" (Economist, 2006).

Studying the related literature on the effect of lead-time uncertainty on supply chain shows that Lead-time (LT) variance affects the overall performance of supply chain through inventory and ordering system, and it has significant effect on coordination among SC members (Ryu and Lee, 2003). Lead-time variance increases costs in supply chain by increasing Bullwhip rate, inventory, and stock out. In many studies, lead-time reduction has been viewed as an investment strategy (Bookbinder and Cakanyildirim, 1999). Ben-Ammar, Dolgui and Wu (2017) determined planned lead-time in multi-level assembly systems with stochastic lead-time of different partners of supply chains. Diabat, Dehghani and Jabbarzadeh (2018) presented a joint location-inventory model for network design of a supply chain with multiple Distribution Centers (DCs) and retailers to determine the quantity and location of DCs under stochastic demand and replenishment time. Díaz-Madroñero, Mula, Jiménez and Peidro (2017) proposed a fuzzy multi-objective integer linear programming (FMOILP) approach to model a material requirement planning (MRP) problem with fuzzy lead-times. Christensen et al. (2007) conducted a survey on a list for manufacturers, consisting of 1,264 individuals of the Institute of Supply Management, and they concluded that lead-time variance has more effect on financial performance of supply chain in comparison with lead-time mean. They also concluded that average supply chain lead-time has no direct impact on financial performance. He et al. (2011) resulted that lead-time variability affect supply chain costs and ordering policy more than lead-time mean.

Swenseth and Buffa (1991) studied the effect of lead-time variability on JIT cost and they concluded that lead-time variability, associated with uncertain transit time in JIT, is critical in the determination of order cycle time, order point, safety stock and holding cost. Heydari et al. (2009) studied the effect of lead-time variation on supply chain performance, and they resulted that by increasing lead-time variance, order variances increases. Furthermore, these results show that increasing in lead-time variance will lead to high inventory fluctuations. Fisher and Raman (1996) reported that lead-time between ordering and delivery of fashion goods can be as long as 12 months, while in toy industries it lasts for 18 months. Chopra et al. (2004) studied the effect of lead-time uncertainty on safety stock and they resulted that for cycle service levels above 50%, reducing lead-time variability reduces reorder point and safety stock, and for cycle service levels above 50%, reducing lead-time variability is more effective than reducing lead-times because it decreases safety stock by a larger amount.

In this paper, we consider a newsvendor who is going to make an investment decision between flexible and dedicated capacity to maximize his profit in the presence of stochastic demand and lead-time; the main contribution of this paper is:

(1) From the viewpoint of modeling, we model a Newsboy problem under capacity constraints and stochastic demand and lead-time, which has an extensive application in supply chain capacity and production planning. To our best knowledge, it is first research on the capacitated Newsboy problem with uncertainty in both stochastic demand and lead-time.

(2) From the viewpoint of solution method, we derive a closed-form solution for optimal production and capacity under dedicated and flexible policy when demand and lead-time follow uniform and normal distribution. (3) From the viewpoint of managerial insights, we prove that the flexible capacity cost threshold to invest in flexible capacity is always between dedicated capacity cost of two products $c_1 < \theta < c_2$.

The rest of this paper is organized as follows: In Section 2, we give a review of previous literature; in Section 3, we introduce problem formulation and modeling; in Section 4, we investigate optimality conditions and derive optimal solution; in Section 5, numerical examples are performed, and in Section 6, conclusion and future studies are investigated.

2. Literature Review

Capacity flexibility is used as an effective way to hedge against demand variability in short-term (Bish et al., 2005). Higher demand uncertainty motivates newsvendor to invest more on flexibility (Goyal and Netessine, 2007). Simchi-Levi, Wang and Wei (2017) proposed a two-stage robust optimization problem to choose flexibility decision strategy under uncertain demand. Chatzikontidou, Longinidis, Tsiakis and Georgiadis (2017) proposed a flexible supply chain network design (SCND) model that uses generalized production/warehousing nodes under demand uncertainty, and they used a scenario-based approach to solve it. Fan, Schwartz and Voß (2017) investigated the application of diverse transportation modes for flexible global supply chain (SC) in stochastic environments. Goyal and Netessine (2007) studied the effect of demand uncertainty and competitive pressures on newsvendor’s decision of technology investment. They insisted that demand uncertainty is the most important driver of technology choice in flexible manufacturing decision. Rodriguez et al. (2014) developed a non-linear programming with stochastic demand to find optimal inventory level in supply chain. Kulkarni and Francas (2017) investigated on capacity investment strategies and the optimal value of flexibility in food and chemical industry in the presence of uncertainty of input materials. Yongheng et al. (2013) developed Lagrangian decomposition algorithm to decide the optimal capacity in electric motor industry under stochastic demand and constant lead-time. Kaya et al. (2013) developed a robust optimization method for optimal capacity planning under stochastic demand and return in a closed loop supply chain. Ho and Fang (2013) studied the capacity allocation of multiple products under uncertain demand, they resulted that inventory holding cost, shortage cost, loss of excess production, and market demands should be considered in an effort to discover the optimal capacity allocation, concerning multiple products. The review of literature reveals that many capacity optimization models ignore stochastic characteristics of production time and just focus on stochastic demand in order to hedge against variability (Dangl, 1999; Bish et al., 2005; Birge and Louveaux, 2011; Rodriguez et al., 2014; Sting and Huchzermeier, 2014). Moreover, literature in capacity investment strategies take c_1, $c_2 < c_f < c_1 + c_2$ into notice. They believe that the problem becomes trivial, and the newsvendor never invests in the flexible resource if this assumption does not hold (Bish et al., 2005; Fine and Freund, 1990; Van Mieghem, 1998; Biller et al., 2006; Goyal and Netessine, 2007; Cattani et al., 2008; Bassamboo et al., 2010; Chod and Zhou, 2013). Yang and Ng (2014) studied flexible capacity strategy model under stochastic demand uncertainty and investment constraints, and they concluded that there are two thresholds for unit capacity cost. They found that when the unit capacity cost is low, optimal capacity is determined by its constraint; whereas when the capacity cost is high, optimal capacity is equal to safety production level. Equally important, when the cost is between the above two thresholds, optimal capacity is determined by capacity cost. In this paper, we show that when capacity cost is very high, although flexible capacity profit is more than dedicated capacity, but total profit is negative. Demand uncertainty is a common source of variability, which puts decision makers in a dilemma. In one hand, they like to maintain the high level of capacity to satisfy the demand of all customers; on the other hand, they want to reduce supply chain costs to increase profit. Demand variability, accompanied by lead-time uncertainty, makes it more difficult and complex for decision makers to manage inventory (Chandra and Grabis, 2008; Pan et al., 2009; Hsieh, 2011). Lukas, Spengler, Kupfer and Kieckhäfer (2017) studied the effect of uncertain technological life cycle on the decision to invest in new product introduction, taking into account the combined effects of flexible investment timing and optimal capacity choice. Dolgui et al. (2013) have done extensive literature review on supply chain planning under uncertain lead-time, and they suggested that future studies in supply chain should consider both demand and lead-time as uncertain parameters.

Most literature considering stochastic lead-time with constant demand (see more: Zipkin (1986); Bookbinder and Cakanyildirim (1999); Sajadieh et al. (2009); Hoque (2013)). Sajadieh et al. (2009) studied the inventory model of supply chain with constant demand rate and stochastic exponential lead-time, while they suggested inventory model with stochastic lead-time in their future research topics. Hoque (2013) developed a manufacturer-buyer inventory model with stochastic normal lead-time and constant lead-time, and they focused that the normal distribution of lead-time provides a better fit. Kim et al. (2004) developed a simple approximate optimal solution for (*s*, Q) inventory model with Erlang lead-time and deterministic demand. They insisted that their solution is as easy as the EOQ's, with an accuracy rate of 99.41% when prior information on lead-time distribution is available and 97.54 - 99.09% when only computer-generated sample information is available. Silver and Zufferey (2005) considered an inventory model with uncertain and seasonal lead-time and constant demand rate. They proposed heuristic methods to minimize expected cost. Chaharsooghi and Heydari (2010) investigated the effect of lead-time mean and variance on supply chain performance metrics, and they resulted that the effect of lead-time (LT) variance on SC performance measures is greater than the effect of the LT mean.

Although exist literature with stochastic lead-time with constant demand, there are a limited number of researches which consider both stochastic demand and lead-time simultaneously. Ben-Ammar and Dolgui (2018) examined an optimization problem for component replenishment in two-level assembly systems under stochastic lead-times. Sun and Guo (2017) proposed an inventory optimization model with fuzzy random demand in order to maximize revenue. Van Kampen et al. (2010) studied the effect of safety stock and safety lead-time on the delivery performance of a multi-product newsvendor. They resulted that safety lead-time leads to higher delivery performance when supply is variable. Zhao and Simchi-Levi (2006) studied the multi-product and multicomponent assemble-to-order (ATO) systems where replenishment lead-time of the components are stochastic and demand of the product follows poison process. They developed a numerical method to analyze performance based on Monte Carlo simulation. Movahed and Zhang (2013) developed robust optimization model under stochastic demand and lead-time to decide optimal inventory parameters using MILP programming. Das and Hanaoka (2014) developed a humanitarian disaster relief inventory model. They assumed that demand and lead-time are uniformly distributed.

According to above-mentioned literature survey, we can conclude that it is not applicable to ignore the important role of lead-time uncertainty in supply chain decisions, based on the mentioned we intend to expand our research into supply chain management under stochastic demand and lead-time.

3. Problem statement and formulation

The following notations are used thorough the paper:

Sets:

P : Set of products $P = 1, 2$

D : Set of decisions $D = 1, 2, f$

Parameters:

d_i : Stochastic demand for the product *i*, assumed to be uniform, $d_i \sim U(a_i, b_i)$

l_i : Newsvendor's stochastic lead-time for the product *i*, assumed to be uniform, $l_i \sim U(y_i, z_i)$

μ_{Di} : Mean of stochastic demand for the product *i*, under normal distribution

μ_{li} : Mean of stochastic lead-time for the product *i*, under normal distribution

σ_{Di}^2 : Variance of stochastic demand for the product *i*, under normal distribution

σ_{li}^2 : Variance of stochastic lead-time for the product *i*, under normal distribution

θ : Threshold for investment

p_i : Selling price per unit of product

c_i : Dedicated capacity investment cost per unit of product *i*

c_f : Flexible capacity investment cost per unit of product *i*

r_i : Holding cost of excessive production per unit of product *i*

v_i : Penalty cost per unit shortage of product *i*

Decision variables:

k_i : Total capacity of product *i* under dedicated strategy

k_f : Total capacity of product *i* under flexible strategy

$\vec{K}$: The vector of capacity k_i, k_f

q_i^d : Production quantity of product *i* under dedicated capacity

q_i^f : Production quantity of product *i* under flexible capacity

$q_i^{f|d}$: Production quantity of product *i* under flexible or dedicated capacity

A lot literature in capacity investment strategies such as Bish et al. (2005); Fine and Freund (1990); Van Mieghem (1998); Bassamboo et al. (2010); Chod and Zhou (2013) pointed that $c_i < c_f < \sum_{i \in P} c_i, \forall i \in P$. They believed that the problem becomes trivial, and the newsvendor never invests in the flexible resource if this assumption does not hold. Unlike previous literature in this paper, we show that there exists a threshold θ, $c_i < \theta < c_{3-i}$, which is optimal for the newsvendor to invest in flexible capacity when $c_f < \theta$. So, we have no assumption for c_f in this paper.

We consider a newsvendor, who produces two products which are indexed by $i = 1,2$. He decides to invest in two dedicated resources that enables him to satisfy the demand of only one product, or one flexible and more expensive resource, which enables him to satisfy the demand of both products. The newsvendor seeks the best investment portfolio to maximize his expected profit. Two kinds of demands and lead-time distributions are considered in this paper, which are uniform and normal distribution.

We model the problem as a two-stage stochastic program. In the first stage, the newsvendor identifies his resource policy (dedicated or flexible) investment to maximize the expected profit. He makes capacity investment decision in this stage, $\vec{K} = (k_1, k_2, k_f)$ where $\vec{K}$ is decision variable, the vector of dedicated capacity k_1, k_2 and flexible capacity k_f. In the second stage, the newsvendor finds optimal production quantity q_i to maximize his revenue before the realization of demand and lead-time. After the realization of the demand and lead-time if demand during the lead-time exceeds production quantity, a shortage cost v_i is incurred. Otherwise, excessive production results holding cost of r_i. Figure below depicts the proposed problem and the timeline associated with it:

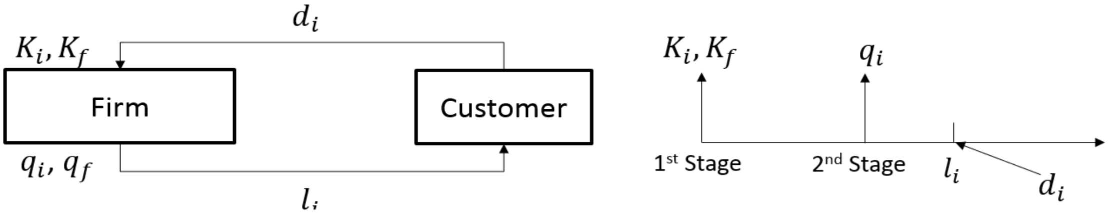

Figure 1. The proposed model of the newsvendor decision-making process

The capacity decision problem can be formulated as the following two stage stochastic program:

Stage 1:

$$\max_{\vec{K}} \; V\left(\vec{K}\right) = E\pi^*\left(\vec{K},\vec{D}\right) - \sum_{i\in D} c_i k_i \tag{1}$$

Subject to:

$$k_i \geq 0, \forall i \in D \tag{2}$$

Stage 2:

$$\pi^*\left(\vec{K},\vec{D}\right) = \max_{q_i^{f|d}} \; \pi\left(\vec{K},\vec{D}\right) = \sum_{i\in P}\int_0^{q_i^{f|d}} \left[p_i x - r_i(q_i^{f|d} - x)\right] f_i\left(x\right)dx + \sum_{i\in P}\int_{q_i^{f|d}}^{\infty} \left[p_i q_i^{f|d} - v_i(x_i - q_i^{f|d})\right] f_i\left(x\right)dx \tag{3}$$

Subject to:

$$q_i^d \leq k_i \tag{4}$$

$$\sum_{i\in P} q_i^f \leq k_f \tag{5}$$

$$q_i \geq 0, \forall i \in P \tag{6}$$

The objective of the second stage problem (3) is to maximize the summation of total revenue minus holding and shortage cost. Constraint (4) is production quantity under dedicated capacity constraint; constraint (5) is production quantity under flexible capacity constraint. Also, constraint (6) is the non-negativity constraint for production quantity under dedicated and flexible capacity.

Using Leibniz rule the second stage objective function (3) can be written as follows:

$$\pi\left(\vec{K},\vec{D}\right) = \sum_{i=1}^{2}\left(\left(p_i + v_i\right)q_i^{f|d} - v_i\mu_{DL_i} - \left(p_i + r_i + v_i\right)\int_0^{q_i^{f|d}} F_{DL_i}.(x)dx\right) \tag{7}$$

From (7), $\pi\left(\vec{K},\vec{D}\right)$ is concave in $q_i^{f|d}$, the Hessian $H(q_1^{f|d}, q_2^{f|d})$ matrix can be written as:

$$H\left(q_1^{f|d}, q_2^{f|d}\right) = \begin{bmatrix} \dfrac{\partial^2\pi\left(\vec{K},\vec{D}\right)}{\partial q_1^{f|d^2}} & \dfrac{\partial^2\pi\left(\vec{K},\vec{D}\right)}{\partial q_1^{f|d}\partial q_2^{f|d}} \\ \dfrac{\partial^2\pi\left(\vec{K},\vec{D}\right)}{\partial q_1^{f|d}\partial q_2^{1|d}} & \dfrac{\partial^2\pi\left(\vec{K},\vec{D}\right)}{\partial q_2^{f|d^2}} \end{bmatrix} = \begin{bmatrix} -\left(p_1 + r_1 + v_1\right) f_{DL_1}(q_1^{f|d}) & 0 \\ 0 & -\left(p_2 + r_2 + v_2\right) f_{DL_2}\left(q_2^{f|d}\right) \end{bmatrix} \tag{8}$$

Since $\frac{\partial^2 \pi\left(\vec{K},\vec{D}\right)}{\partial q_1^{f|d^2}} \leq 0$, $\frac{\partial^2 \pi\left(\vec{K},\vec{D}\right)}{\partial q_2^{f|d^2}} \leq 0$ and $detH\left(q_1^{f|d}, q_2^{f|d}\right) \geq 0$, then $\pi\left(\vec{K},\vec{D}\right)$ is concave in $q_1^{f|d}$ and $q_2^{f|d}$.

In this section, we assume that the demand and lead-time for each product are stochastic and follow uniform distribution with $l_i \sim U(y_i, z_i), d_i \sim U(a_i, b_i)$ (The normal distribution coefficients are also approximated at the end of this section). We can obtain PDF of demand during the lead-time as follows (Glen et al., 2004):

$$f_{DL_l}(x) = \begin{cases} \frac{\ln x - \ln y_i a_i}{(b_i - a_i)(z_i - y_i)} & y_i a_i \leq x < \min\left(a_i z_i, b_i y_i\right) \quad (8a) \\ \frac{\ln z_i - \ln y_i}{(b_i - a_i)(z_i - y_i)} & \min(a_i z_i, b_{iy_i}) \leq x < \max(a_i z_i, b_{iy_i}) \quad (8b) \\ \frac{\ln z_i b_i - \ln x}{(b_i - a_i)(z_i - y_i)} & \max(a_i, z_i, b_i y_i) \leq x < b_i z_i \quad (8c) \end{cases}$$

And, CDF of the demand during the lead-time can be obtained:

$$f_{DL_l}(x) = \begin{cases} \frac{y_i a_i + x(\ln x - \ln y_i a_i - 1)}{(b_i - a_i)(z_i - y_i)} & y_i a_i \leq x < \min\left(a_i z_i, b_i y_i\right) \quad (9a) \\ \frac{x(\ln z_i - \ln y_i) - a_i(z_i - y_i)}{(b_i - a_i)(z_i - y_i)} & \min(a_i z_i, b_{iy_i}) \leq x < \max(a_i z_i, b_{iy_i}) \quad (9b) \\ \frac{x(\ln z_i b_i - \ln x + 1) - a_i(z_i - y_i) - b_i y_i}{(b_i - a_i)(z_i - y_i)} & \max(a_i, z_i, b_i y_i) \leq x < b_i z_i \quad (9c) \end{cases}$$

Figure 2 shows pdf and CDF function of product of the demand and lead-time, based on different values of lead-time variation (*CV*).

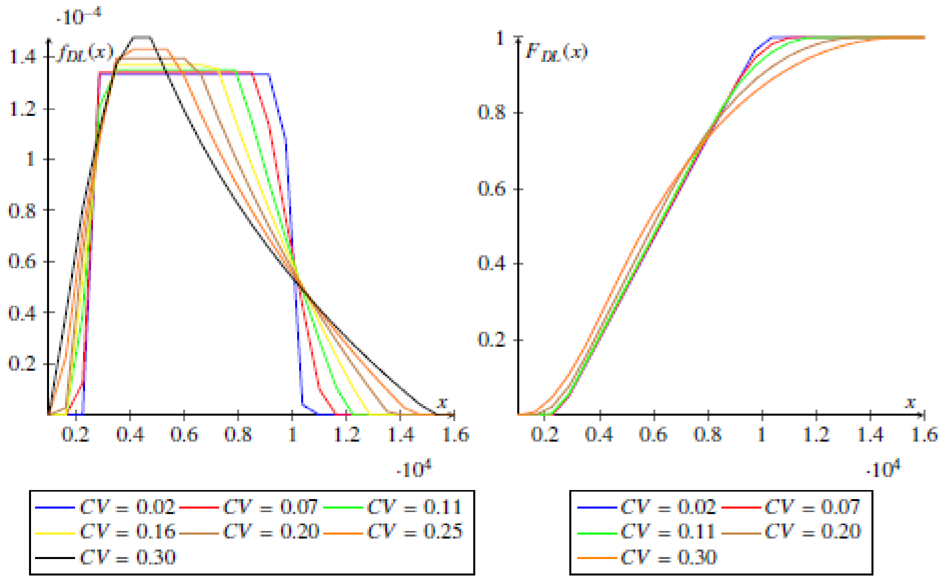

Figure 2. Example: PDF and CDF function of product of two uniform distribution

Areeratchakul and Abdel-Malek (2006) introduced a method to approximate $\int_0^t F(x)dx$ for triangle shape cumulative distribution functions. They proved that uniform, normal and exponential distributions can be approximated using this method. We are going to use this method to estimate *CDF* function of the demand during lead-time, introduced in (9a), (9b) and (9c). In the above *CDF*, the lower bound is $y_i a_i$ so we need to approximate:

$$S_i(t) = \int_{y_i a_i}^{t} F_{DL_i}(x)dx \approx \frac{1}{2}\Delta_i (t - y_i a_i)^2 \tag{10}$$

Where Δ_i is the slope of the approximated line and can be obtained as follow (Areeratchakul and Abdel-Malek, 2006):

$$\Delta_i = \frac{0.9 - 0.001}{F_{DL_i}^{-1}(0.9) - F_{DL_i}^{-1}(0.001)} \tag{11}$$

In addition, the approximation error can be found as follows:
Figure 3, shows the real (solid line) and the corresponding approximated distribution functions (dashed line) based on an example data in Section 5.

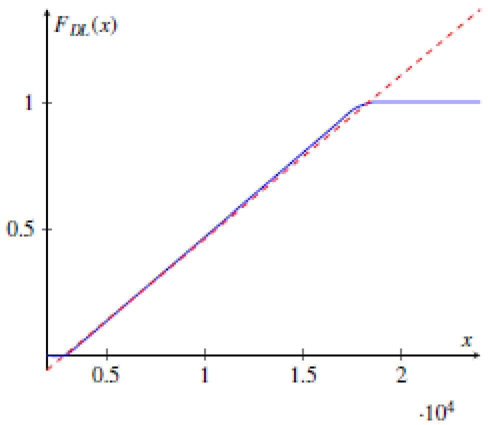

Figure 3. Estimated vs. cumulative distribution function

Using the triangular approximation, approximation error of the area under the curve is estimated as follow:

$$e(t) = \frac{U(t) - \int_{ya}^{t} F_{DL_s}(x)dx}{\int_{ya}^{t} F_{DL_s}(x)dx} \tag{12}$$

Figure 4 shows the approximation error using Equation (12) when $t = bz$. We can find that the approximated distribution function can match the real function accurately.
When cumulative distribution of the demand during lead-time is triangular based, as estimated in (10), we can approximate the integration of cumulative function using triangular approach introduced by (Areeratchakul and Abdel- Malek, 2006)). In figure 3, we have shown that the estimation error is too low, then by using equation (10) with $t = q_i^{f|d}$ and $\mu_{DL_l} = \frac{(a_l + b_l)(y_l + z_l)}{4}$ the objective function of (7) can be written in the following quadratic form:

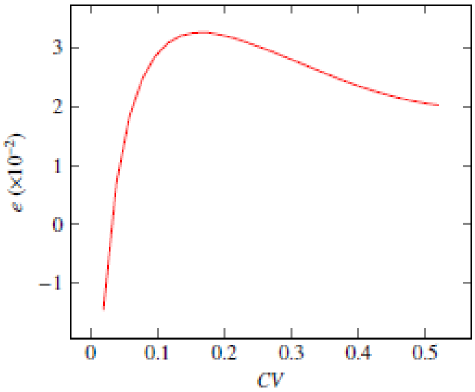

Figure 4. The approximation error of the area under the curve $F_{DL}(x), CV = \frac{\sigma}{\mu}$

Table 1. Summary of the coefficients of the objective function under uniform and normal distributions

Distribution Function	**A_i**	**B_i**	**C_i**
Uniform Distribution	$\frac{-(p_l + r_l + v_l)\Delta_l}{2}$	$(p_i + v_i) + \Delta_i a_i y_i (p_i + r_i + v_i)$	$-\frac{\Delta_l a_l^2 y_l^2 (p_l + r_l + v_l)}{2} - \frac{v_l (a_l + b_l)(y_l + z_l)}{4}$
Normal Distribution	$\frac{-(p_l + r_l + v_l)\Delta_l}{2}$	$(p_i + v_i)$	$-v_i \mu_{Di} \mu_{li}$

$$\pi(\vec{K},\vec{D}) = \sum_{i \in P}\left(A_i q_i^{f|d^2} + B_i q_i^{f|d} + C_i\right), \forall i \in P \tag{13}$$

Where A, B, and C, are defined in Table 1, and A is the slope of approximated line based on (11) using the method introduced in section 3. From the table, it can be noted that A and C, are always negative, while B is always positive. We use these notations for the future analysis.

4. Optimal production quantity and investment under dedicated and flexible capacities

If the newsvendor invests in flexible capacity, the objective function in (13) can then be written in the following Lagrangian form:

$$L\left(q_i^f, k_f, \lambda_f\right) = \sum_{i \in P}(A_i q_i^{f^2} + B_i q_i^f + C_i) - \lambda_f (\sum_{i \in P} q_i^f - k_f) \tag{14}$$

Under dedicated investment, we can write:

$$L\left(q_i^d, k_i, \lambda_i\right) = \sum_{i \in P}(A_i q_i^{d^2} + B_i q_i^d + C_i) - \sum_{i \in P} \lambda_i \left(q_i^d - k_i\right) \tag{15}$$

Proposition 4.1. Optimal production quantity under flexible capacity q_i^f and dedicated capacity q_i^d for the problem introduced in section 3 can be obtained as follow:

For flexible capacity:

$$q_i^{f^*}=\begin{cases}-\dfrac{B_i}{2A_i} & k_f\geq-\sum\limits_{i\in P}\dfrac{B_i}{2A_i} \\ \dfrac{2A_{3-i}k_f-B_i+B_{3-i}}{2\sum\limits_{i\in P}A_i} & \max\left(\dfrac{B_i-B_{3-i}}{2A_{3-i}},\dfrac{B_{3-i}-B_i}{2A_i}\right)\leq k_f<-\sum\limits_{i\in P}\dfrac{B_i}{2A_i},\forall i\in P \\ 0 & k_f\leq\dfrac{B_i-B_{3-i}}{2A_i}\end{cases}\qquad \begin{matrix}(16a)\\(16b)\\(16c)\end{matrix}$$

And, for dedicated capacity:

$$q_i^{d^*}=\begin{cases}-\dfrac{B_i}{2A_i} & k_i\geq-\dfrac{B_i}{2A_i} \\ k_i & k_i\leq-\dfrac{B_i}{2A_i}\end{cases},\forall i\in P\qquad \begin{matrix}(17a)\\(17b)\end{matrix}$$

Proof: see Appendix A.

We can get some insights from this proposition:
First, the optimal production quantity under flexible capacity is equal to zero when $k_f\leq\frac{B_i-B_{3-i}}{2A_i}$, and based on (13) the optimal second stage profit is negative; it means the manufacturer should never invest on flexible technology when $k_f\leq\frac{B_i-B_{3-i}}{2A_i}$.
Also, the optimal production quantity under flexible and dedicated capacity is equal to: $-\frac{B_i}{2A_i}$ when $k_f\geq-\sum_{i=1}^{2}\frac{B_i}{2A_i}$ and $k_i\geq-\frac{B_i}{2A_i}$.
Figure 5 shows the relationship between capacity and optimal production quantity in dedicated and flexible investment.
By substituting optimal production quantity from the above-mentioned equations into the second stage objective function (3), we can obtain the following results.
Under flexible investment,

$$\pi^*(k_f)=\begin{cases}\sum\limits_{i\in P}(-\dfrac{B_i^2}{4A_i}+c_i) & k_f\geq-\sum\limits_{i\in P}\dfrac{B_i}{2A_i} \\ \dfrac{\prod\limits_{i\in P}A_ik_f^2+\sum\limits_{i\in P}A_iB_{3-i}k_f-\frac{1}{4}(B_i-B_{3-i})^2}{\sum\limits_{i\in P}A_i}+\sum\limits_{i\in P}c_i & \max\left(\dfrac{B_i-B_{3-i}}{2A_{3-i}},\dfrac{B_{3-1}-B_i}{2A_i}\right)\leq k_f<-\sum\limits_{i\in P}\dfrac{B_i}{2A_i} \\ \sum\limits_{i\in P}c_i & k_f<\max\left(\dfrac{B_i-B_{3-i}}{2A_{3-i}},\dfrac{B_{3-1}-B_i}{2A_i}\right)\end{cases}\qquad \begin{matrix}(18a)\\(18b)\\(18c)\end{matrix}$$

And, under dedicated investment,

$$\pi^*(k_1,k_2)=\begin{cases}\sum_{i\in P}\left(-\frac{B_i^2}{4A_i}+c_i\right) & k_i\ge -\frac{B_i}{2A_i} \quad (19a)\\ \sum_{i\in P}\left(A_ik_i^2+B_ik_i+C_i\right) & k_i< -\frac{B_i}{2A_i} \quad (19b)\end{cases}$$

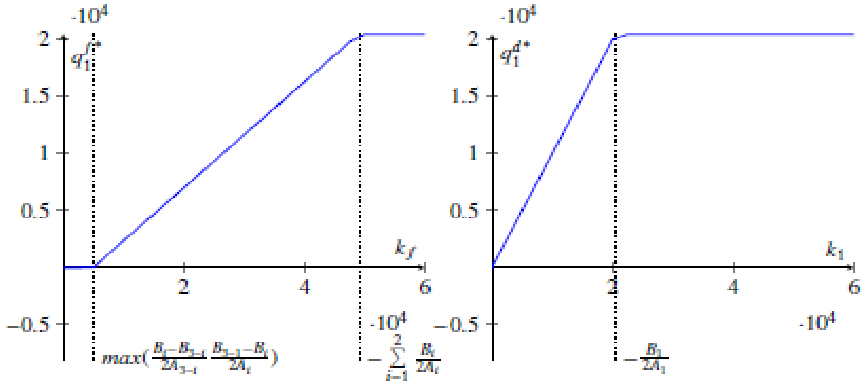

Figure 5. Example: The relationship between capacity and optimal production quantity under dedicated and flexible investment

Consequently, by substituting values of equation (18a), (18b) and (18c) into (1), we can obtain the first stage objective function under flexible capacity as follows:

$$V_f(k_f)=\begin{cases}\sum_{i\in P}\left(-\frac{B_i^2}{4A_i}+c_i\right)-c_fk_f & k_f\ge -\sum_{i\in P}\frac{B_i}{2A_i} \quad (20a)\\ \frac{\prod_{i\in P}A_ik_f^2+\sum_{i\in P}A_iB_{3-i}k_f-\frac{1}{4}(B_i-B_{3-i})^2}{\sum_{i\in P}A_i}+\sum_{i\in P}c_i-c_fk_f & \max\left(\frac{B_i-B_{3-i}}{2A_{3-i}},\frac{B_{3-1}-B_i}{2A_i}\right)\le k_f< -\sum_{i\in P}\left(\frac{B_i}{2A_i}\right) \quad (20b)\\ \sum_{i\in P}c_i-c_fk_f & k_f<\max\left(\frac{B_i-B_{3-i}}{2A_{3-i}},\frac{B_{3-1}-B_i}{2A_i}\right) \quad (20c)\end{cases}$$

Then, the first stage objective function under dedicated policy can be written as:

$$V_d(k_1,k_2)=\begin{cases}\sum_{i\in P}\left(-\frac{B_i^2}{4A_i}+C_i-c_ik_i\right) & k_i\ge -\frac{B_i}{2A_i} \quad (21a)\\ \sum_{i\in P}\left(A_ik_i^2+B_ik_i+C_i-c_ik_i\right) & k_i< -\frac{B_i}{2A_i} \quad (21b)\end{cases}$$

Proposition 4.2. If $k_f\ge -\sum_{i\in P}\frac{B_i}{2A_i}$ and $k_i\ge -\frac{B_i}{2A_i}$; it is optimal for the newsvendor to invest in flexible technology only when

$$c_fk_f<\sum_{i\in P}c_ik_i \quad (22)$$

Proof: By comparing (20a) and (21a), we can obtain the above proposition. Based on Proposition 4.2, we can conclude that under these conditions, $k_f\ge -\sum_{i\in P}\frac{B_i}{2A_i}$ and $k_i\ge -\frac{B_i}{2A_i}$, if the total investment cost of flexible capacity is lower than the total investment cost of two dedicated capacity, it is optimal for the newsvendor to invest in flexible capacity.

We have shown in Appendix B. that $V_f(k_f)$ is concave in (20b) and its optimal value is in (20b). Figure 6 shows $V_f(k_f)$ versus k_f. Based on the figure we can see that $V_f(k_f)$ is concave.

In Appendix D. also we have indicated that $V_d(k_1,k_2)$ is concave in (21b) and its optimal value is in (21b).

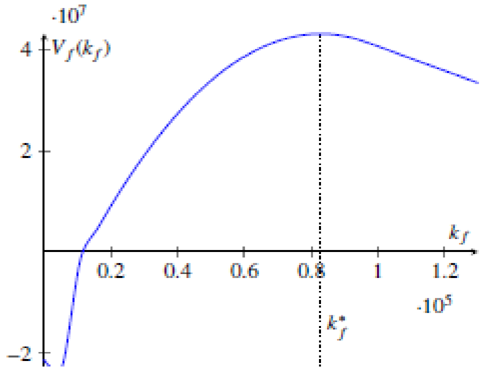

Figure 6. The first stage objective function under flexible policy vs. flexible capacity

Proposition 4.3. Under flexible policy investment, it is always optimal for the newsvendor to invest in flexible capacity as below:

$$k_f^* = \frac{\sum_{i \in P} \left(c_f - B_{3-i}\right) A_i}{2 \prod_{i \in P} A_i} \tag{23}$$

Proof: see Appendix B.

Proposition 4.3 shows the analytical solution for optimal flexible capacity to maximize the optimal expected profit. Based on this proposition, we can conclude that the optimal flexible capacity is increasing in selling price p_i, decreasing in c_f. This result is shown in Figure 7.

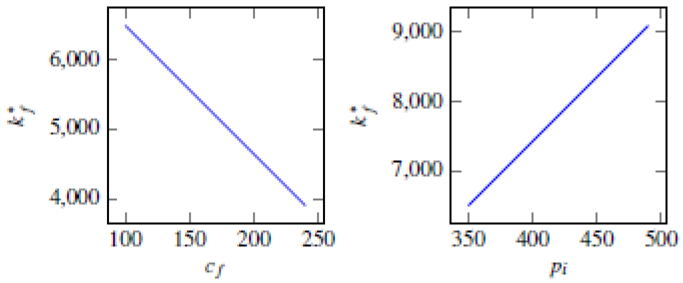

Figure 7. The effect of capacity cost c_f and price p_i on the optimal flexible capacity k_f^*

Based on (23) and this fact that A_i is always negative, we can conclude that k_f^* is decreasing on c_f. More, k_f^* should always be positive, so we can write the following proposition.

Proposition 4.4. The newsvendor never invests in flexible capacity when

$$c_f \geq \frac{\sum_{i \in P} A_i B_{3-i}}{\sum_{i \in P} A_i} \tag{24}$$

Proof: see Appendix C.

Using the objective function obtained in (21a) and (21b), now we are able to find the optimal capacity under dedicated policy. In (Appendix D), we have proved the concavity of $V_d(k_1,k_2)$, based on k_1 and k_2.

Proposition 4.5. Under dedicated policy investment, optimal capacity can be obtained as follow:

$$k_i^* = \frac{c_i - B_i}{2A_i}, \forall i \in P \tag{25}$$

Proof: see Appendix D.

Proposition 4.5 shows the analytical solution for optimal dedicated capacity to maximize the optimal expected profit. Based on this proposition, we can conclude that the optimal dedicated capacity is increasing in selling price p_i, decreasing in capacity cost c_i. From (25), k_i^* should always be positive, we can write the next proposition.

Proposition 4.6. The newsvendor never invests in dedicated technology when $c_i > B_i$.

By substituting k_f^* from (23) into $V_f(k_f)$ and k_i^* from (25) into the first stage objective function, we can obtain the final objective function under flexible and dedicated policy as follows:

$$V\left(k_f^*, k_1^*, k_2^*\right) = \begin{cases} -\dfrac{\sum_{i\in P} A_i (c_f - B_{3-i})^2}{4\prod_{i\in P} A_i} + \sum_{i\in P} c_i & \max\left(\dfrac{B_i - B_{3-i}}{2A_{3-i}}, \dfrac{B_{3-1} - B_i}{2A_i}\right) \le k_f^* < -\sum_{i\in P}\dfrac{B_i}{2A_i} \quad (26a) \\ -\dfrac{\sum_{i\in P} A_i (B_{3-i} - c_{3-i})^2}{4\prod_{i\in P} A_i} + \sum_{i\in P} c_i & k_i^* < -\dfrac{B_i}{2A_i} \quad (26b) \end{cases}$$

Proof is given in Appendix F. By comparing the above functions (26a), (26b), we would be able to obtain the threshold for flexible capacity cost c_f, to figure out the conditions under which flexible policy is preferred.

Proposition 4.7. There exists a threshold θ, which is always optimal for the newsvendor to invest in flexible capacity when $c_f \le \theta$, and:

$$\theta = \frac{\sum_{i\in P} A_i B_{3-i} + \sqrt{\sum_{i\in P} (B_i - c_i)^2 A_{3-i}^2 + 2\prod_{i\in P} A_i \left(\sum_{i\in P} c_i \left(c_i / 2 - B_i\right) + \prod_{i\in P} B_i\right)}}{\sum_{i\in P} A_i} \tag{27}$$

Proof: see Appendix F.

Proposition 4.7. shows the analytical threshold for the newsvendor to invest on flexible or dedicated capacity. Based on this proposition, the threshold is decreasing in capacity c_i, increasing in p_i, and increasing in v_i and r_i.

Table 2. Example Data

Product i	d_i	l_i	p_i	c_i	r_i	c_i	c_f
1	uniform(50,200)	uniform(200,300)	900 $	200 $	100 $	150 $	220 $
2	uniform(100,300)	uniform(300,400)	1000 $	250 $	200 $	100 $	

Table 3. Result Data

Product *i*	q_i^{d*}	q_i^{f*}	k_i^*	k_f^*	$V_d(k_1^*, k_2^*)$	$V_f(k_f^*)$	θ
1	70,370.4	68,261.1	70,370.4	82,732.6	45,635,307.94$	43,007,132.48$	208.5$
2	14,419.5	14,471.5	14,419.5				

Proposition 4.8. The value of threshold θ in (27) is always between the unit cost of two dedicated capacity:

$$c_i < \theta < c_{3-i}, \forall i = P \qquad (28)$$

Proof: see Appendix G.

An example of this proposition is illustrated in Figure 7.

5. Numerical Examples

Validity of the approximation is shown in Table 5.

To illustrate the validity of the proposed model and the usefulness of the proposed estimation method, several numerical experiments are performed, and the related results are reported. We have done numerical experiments based on uniform and normal distribution, and approximation validity has been performed for two distributions.

5.1. Uniform Distribution

In this section, two products are considered and their parameters are shown in Table 2. We assumed that both demand and lead-time are stochastic and follow uniform distribution. Then, we obtained the optimal capacity and profit under dedicated and flexible policy. We have shown the result data in Table 3. Based on the result, we can conclude that flexibility policy is not always profitable for the newsvendor. In fact, it depends on the flexible capacity cost. In this example, if flexible capacity cost is less than the threshold $\theta = 208.5,$ it is optimal for the newsvendor to invest in flexible capacity; otherwise, dedicated capacity is preferred. Moreover, we can conclude that the value of threshold θ is always between the dedicated capacity cost of two products $c_i < \theta < c_{3-i}$. Unlike previous literature which assumed $c_i < c_f < \sum_{i \in P} c_i$, based on this result, we show that this assumption is not always necessary.

Figure 8 shows the effect of flexible capacity cost c_f on the optimal profit $V_f(k_f^*)$ under flexible policy. Based on the chart, we can realize that when c_f is less than threshold $\theta = 208.5,$ flexible policy is preferred; otherwise, it is optimal for the newsvendor to invest on dedicated capacity.

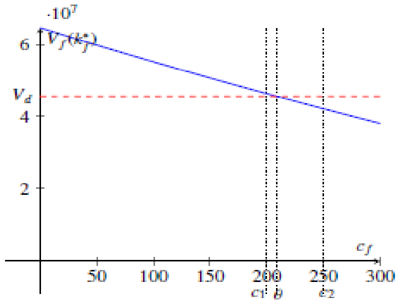

Figure 8. The effect of flexible capacity cost on the optimal profit under flexible policy

To illustrate the performance of approximation method presented in section 3, numerical simulation is performed. To do this, first we need to obtain the optimal value of the objective function numerically and then compare numerical results with the analytical approximated result. Numerically obtaining the optimal value of the objective function introduced in (1) is a very time-consuming task, so we utilized the Particle swarm optimization (PSO) method to overcome difficulties.

5.2. Particle swarm optimization

Particle swarm optimization (PSO) Kennedy (2010) is a computational method, which optimizes a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality. It solves a problem by having a population of candidate solutions.

A basic variant of the PSO algorithm works by having a population (called a swarm) of candidate solutions (called particles). These particles are moved around in the search-space according to a few simple formulae. The movements of the particles are guided by their own best known position in the search-space as well as the entire swarm's best known position. When improved positions are being discovered these will then come to guide the movements of the swarm. The process is repeated and by doing so it is hoped, but not guaranteed, that a satisfactory solution will eventually be discovered.

Formally, let $-L(\vec{A})$ be the cost function as defined in 14, which must be minimized, where $\vec{A}=(q_i^f, k_f, \lambda_f)$ is the vector of decision variables. The function takes a candidate solution as an argument in the form of a vector of real numbers and produces a real number as output which indicates the objective function value of the given candidate solution. The gradient of L is not known. The goal is to find a solution $\vec{A}$ for which $L(\vec{A}) \le L(\vec{B})$ for all $\vec{B}$ in the search-space, which would mean $\vec{A}$ is the global minimum.

Let S be the number of particles in the swarm, each having a position $x_i \in R^n$ in the search-space and a velocity $v_i \in R^n$. Let p_i be the best known position of particle *i* and let g be the best known position of the entire swarm. A basic PSO algorithm is then:

for each particle $i=1,\ldots,S$ do

  Initialize the particle's position with a uniformly distributed random vector: $x_i \sim U(b_l, b_u)$

  Initialize the particle's best known position to its initial position: $p_i \leftarrow x_i$

  if $L(p_i) < L(g)$ then

    update the swarm's best known position: $g \leftarrow p_i$

  Initialize the particle's velocity: $v_i \sim U(-|b_u - b_l|, |b_u - b_l|)$

while a termination criterion is not met do:

  for each particle $i=1,\ldots,S$ do

Pick random numbers: $r_p, r_g \sim U(0,1)$

Update the particle's velocity: $v_i \leftarrow \omega.v_i + \phi_p.r_p(p_{i,d} - x_i) + \phi_g.r_g(g_d - x_i)$

Update the particle's position: $x_i \leftarrow x_i + v_i$

if $L(x_i) < L(p_i)$ then

Update the particle's best known position: $p_i \leftarrow x_i$

if $L(p_i) < L(g)$ then

Update the swarm's best known position: $g \leftarrow p_i$

The values b_l and b_u are respectively the lower and upper boundaries of the search-space. The termination criterion can be the number of iterations performed, or a solution where the adequate objective function value is found. The parameters ω, ϕ_p, and ϕ_g are selected to control the behavior and efficacy of the PSO method.
For numerical studies the particle size was set to $S = 500$, and the number of iterations was set to 10,000 iterations. $\omega = 0.2, \phi_p = 0.1, \phi_g = 0.1$.

Table 4 shows input data for numerical studies based on different values of *CV*.

Table 4. Input data for numerical studies

CV	a_1	b_1	y_1	z_1	a_2	b_2	y_2	z_2
0.02	145	155	40	60	290	310	40	70
0.06	135	165	40	60	270	330	40	70
0.1	125	175	40	60	250	350	40	70
0.13	115	185	40	60	230	370	40	70
0.17	105	195	40	60	210	390	40	70
0.21	95	205	40	60	190	410	40	70
0.25	85	215	40	60	170	430	40	70
0.29	75	225	40	60	150	450	40	70
0.33	65	235	40	60	130	470	40	70
0.37	55	245	40	60	110	490	40	70
0.4	45	255	40	60	90	510	40	70
0.44	35	265	40	60	70	530	40	70
0.48	25	275	40	60	50	550	40	70
0.54	10	290	40	60	20	580	40	70

Table 5 shows optimal values based on numerical study and approximation. Based on this table, we realize that the performance of the approximation method is high when *CV* coefficient of variation is low.

Table 5. The comparison between approximated and real result (Approximated values are signed by *a* and real values signed by *)

CV	q_1^{fa}	q_2^{fa}	k_f^a	V^a	q_1^{f*}	q_2^{f*}	k_f^*	V^*	Objective error %(E)
0.02	7933.04	18315.19	26248.22	4926961.72	8184	18966	27150	4962116.65	0.71
0.06	7942.75	18332.18	26274.93	4958974.3	7934.31	18288.21	26222.53	4959039.56	0
0.1	8030.74	18573.83	26604.57	4873053.85	7913.03	18167.84	26080.87	4877922.81	0.1
0.13	8147.27	18897.58	27044.85	4757635.82	7974.68	18252.83	26227.51	4767482.23	0.21
0.17	8278.62	19264.12	27542.73	4628051.82	8082.46	18452.3	26534.76	4641225.65	0.28
0.21	8418.8	19656.28	28075.08	4489939.07	8219.11	18721.98	26941.09	4505059.23	0.34
0.25	8564.62	20064.89	28629.51	4346049.48	8376.11	19039.44	27415.54	4362127.08	0.37
0.29	8714.13	20484.37	29198.51	4197953.76	8546.11	19393.12	27939.22	4214307.22	0.39
0.33	8866.02	20911	29777.03	4046642.64	8553.7	19771.96	28325.67	4049804.69	0.08
0.37	9019.32	21342.02	30361.34	3887822.65	8713.38	20171.43	28884.81	3892786.76	0.13
0.4	9173.21	21775.13	30948.34	3722839.72	8858.04	20587.08	29445.12	3736865.26	0.38
0.44	9326.88	22208.11	31535	3555524.66	8991.52	21015.85	30007.37	3579236.59	0.66
0.48	9479.34	22638.24	32117.58	3386354.34	9115.29	21455.43	30570.72	3420183.59	0.99
0.8	9631.79	23068.37	32700.16	3217184.01	9233.33	21895.01	31128.34	3261130.59	1.35
1	9784.24	23498.5	33282.74	3048013.68	9350.43	22334.58	31685.02	3102077.59	1.74[1]

We have also calculated the effect of lead-time variation on the optimal production quantity, optimal flexible capacity, and optimal profit. Subsequently, the result is shown in Figure 9. Figure 10 also shows the profit error caused by approximation. According to this figure, the error is low when the *CV* is low. Based on these figures, we can realize that lead-time variation increases the optimal production quantity and capacity, while it increases the optimal profit.

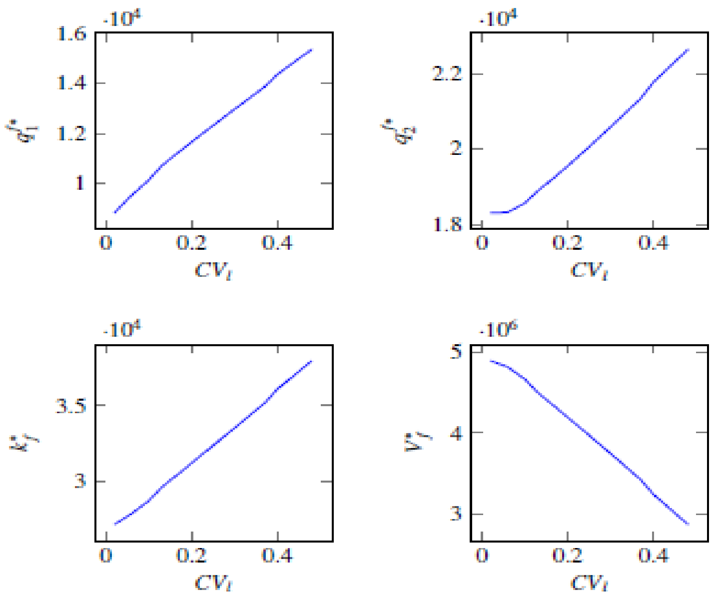

Figure 9. The effect of lead-time variation on the optimal production quantity, flexible capacity and profit using uniform distribution

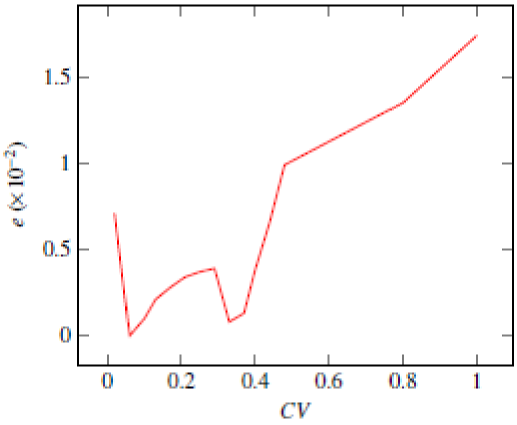

Figure 10. The approximation error of the objective function based on *CV* under uniform distribution

Table 6, shows the threshold value obtained by using approximation method and numerical studies. $c_1 = 300$, $c_2 = 500$, and $c_f = 100$. Based on this table, the threshold θ is always between the two dedicated capacity cost c_1 and c_2. On the other hand, this table shows the validity of the proposition according to Appendix G. It shows that maximum approximation error is 6%, and error increases with *CV*. Which means the higher *CV* is associated with the higher error.

Table 6. The comparison between approximated and real threshold (Approximated values are signed by *a* and real values signed by *)

CV	θ^a	θ^*	Error %(*E*)
0.02	367.07	357.26	2.74
0.06	374.63	371.19	0.93
0.1	380.22	381	-0.2
0.13	385.95	389.03	-0.79
0.17	390.48	395.86	-1.36
0.21	395.27	401.86	-1.64
0.25	417.08	407.19	2.43
0.29	404.19	412	-1.9
0.33	408.34	408.37	-0.01
0.37	412.29	410.7	0.39
0.4	416.05	420.8	-1.13
0.44	419.63	430.91	-2.62
0.48	423.03	441.02	-4.08
0.8	426.43	451.12	-5.47
1	429.82	461.23	-6.81

5.3. Normal Distribution

In this section, we used normal probability distribution instead of uniform distribution. Then, we obtained the product of two normal distributions numerically. To be able to achieve the optimal solution, we used PSO algorithm introduced in section 5.1 with MATLAB written objective function; the particle size was set to 10 together with the number of iteration to 100 times. The input data and results are shown in Table 7. The approximated flexible capacity (k_f^a) is also calculated.

Table 7. The optimal solution under flexible capacity using normal distribution

CV_i	μ_{D1}	μ_{l1}	σ_{D1}	σ_{l1}	μ_{D2}	μ_{l2}	σ_{D2}	σ_{l2}	q_1^{f*}	q_2^{f*}	k_f^*	k_f^a	V_f^*
0.1	100	30	30	3	200	20	40	2	3595.29	4194.26	7789.55	5909.9	1416316.18
0.2	100	30	30	6	200	20	40	4	3633.07	4342.51	7975.57	6499.33	1344616.53
0.3	100	30	30	9	200	20	40	6	3676.09	4367.19	8043.28	7088.77	1244995.16
0.4	100	30	30	12	200	20	40	8	3492.07	4705.6	8197.68	7678.2	1147803.22
0.5	100	30	30	15	200	20	40	10	3562.63	5009.16	8571.79	8267.63	1055923.13
0.6	100	30	30	18	200	20	40	12	3865.51	5109.59	8975.1	8857.06	966052.5
0.7	100	30	30	21	200	20	40	14	4004.21	5316.24	9320.45	9446.5	905129.82
0.8	100	30	30	24	200	20	40	16	4051.23	5769.6	9820.83	10035.93	848813.07
0.9	100	30	30	27	200	20	40	18	4262.36	5809.31	10071.67	10625.36	801996.09
1	100	30	30	30	200	20	40	20	4755.92	6476.25	11232.17	11214.79	760381.49
1.1	100	30	30	33	200	20	40	22	4906.06	6223.07	11129.13	11804.23	728097.5
1.2	100	30	30	36	200	20	40	24	4939.33	6925.3	11864.63	12393.66	11864.63

We have also figured out the results in Figure 11. Based on the figures, we can view similar trends as uniform distribution. Accordingly, the lead-time variation increases the optimal production quantity and capacity, while decreases the optimal profit.

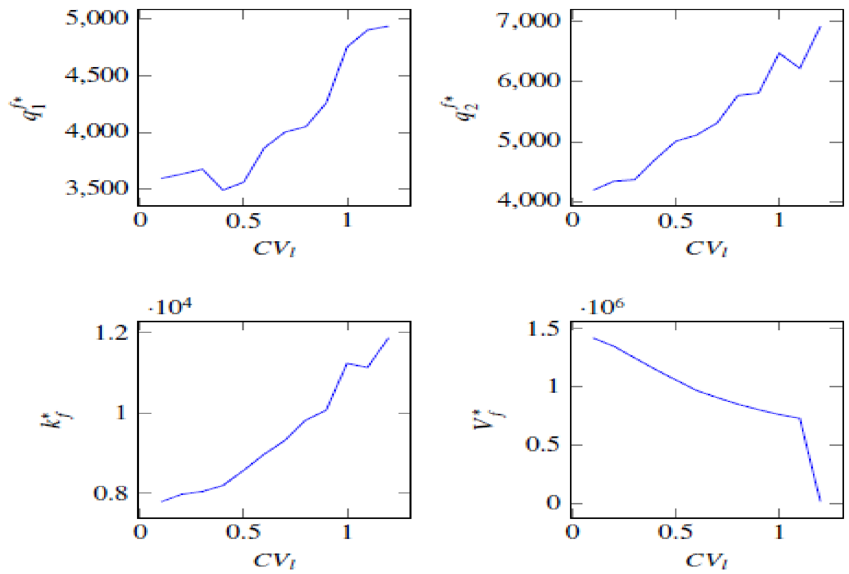

Figure 11. The effect of lead-time variation on the optimal production quantity, flexible capacity and profit under normal conditions.

Based on the calculated approximation error in Figure 12, we can conclude that under normal distribution, the approximation is useful when the lead-time (*CV*) is high, while as shown in the previous section, under uniform distribution, the approximation is useful when the (*CV*) is low.

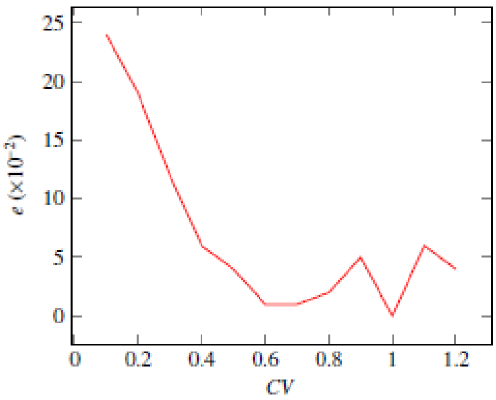

Figure 12. The approximation error of the objective function based on *CV*, under Normal distribution

6. Conclusion

In this paper, we found the optimal capacity and production quantity under dedicated and flexible capacity policy when both demand and lead-time are stochastic and follow uniform and normal distribution. We used the approximation method introduced by Areeratchakul and Abdel-Malek (2006) to simplify the objective function, and proved that under uniform distribution, this estimation is useful when variance of demand and lead-time is low, while under normal distribution the approximation is useful when variance of the demand and lead-time is high. Next, we analytically derived the threshold for flexible capacity cost which offers a criterion for managers to be aware that under which conditions, it is optimal to invest in flexible capacity. Later, we proved that this threshold is always between dedicated capacity costs of two products, which is not considered in previous literature. For future studies, we wish to obtain the optimal dedicated and flexible capacity, considering the multi-product model. Obtaining the accurate approximation under normal distribution is also an interesting topic for future studies.

References

Abdel-Malek, L. L., and Otegbeye, M., (2013). "Separable programming/duality approach to solving the multi-product Newsboy/gardener problem with linear constraints", *Applied Mathematical Modelling*, Vol. 37, No. 6, pp. 4497-4508.

Areeratchakul, N., and Abdel-Malek, L., (2006). "An approach for solving the multi-product Newsboy problem", *International Journal of Operational Research*, Vol. 3, No. 3, pp. 219-227.

Bagchi, U., Hayya, J. C., and Chu, C.-H., (1986). "The effect of lead-time variability: The case of independent demand", *Journal of Operations Management*, Vol. 6, No. 2, pp. 159-177.

Bassamboo, A., Randhawa, R. S., and Van Mieghem, J. A., (2010). "Optimal flexibility configurations in newsvendor networks: Going beyond chaining and pairing", *Management Science*, Vol. 56, No. 8, pp. 1285-1303.

Ben-Ammar, O., Dolgui, A. and Wu, D.D., (2017). "Planned lead-times optimization for multi-level assembly systems under uncertainties", *Omega*.

Ben-Ammar, O. and Dolgui, A., (2018). "Optimal order release dates for two-level assembly systems with stochastic lead-times at each level", *International Journal of Production Research*, pp.1-17.

Biller, S., Muriel, A., and Zhang, Y., (2006). "Impact of price postponement on capacity and flexibility investment decisions", *Production and Operations Management*, Vol. 15, No. 2, pp. 198-214.

Birge, J. R., and Louveaux, F., (2011). *Introduction to stochastic programming*. Springer.

Bish, E. K., Muriel, A., and Biller, S., (2005). "Managing flexible capacity in a make-to-order environment", *Management Science*, Vol. 51, No. 2, pp. 167-180.

Bookbinder, J. H., and Cakanyildirim, M., (1999). "Random lead-times and expedited orders in (q, r) inventory systems", *European Journal of Operational Research*, Vol. 115, No. 2, pp. 300-313.

Cattani, K. D., Dahan, E., and Schmidt, G. M., (2008). "Tailored capacity: speculative and reactive fabrication of fashion goods", *International Journal of Production Economics*, Vol. 114, No. 2, pp. 416-430.

Chaharsooghi, S. K., and Heydari, J., (2010). "Lt variance or lt mean reduction in supply chain management: Which one has a higher impact on sc performance? ", *International Journal of Production Economics*, Vol. 124, No. 2, pp. 475-481.

Chatzikontidou, A., Longinidis, P., Tsiakis, P. and Georgiadis, M.C., (2017). "Flexible supply chain network design under uncertainty", *Chemical Engineering Research and Design*, Vol. 128, pp.290-305.

Chandra, C., and Grabis, J., (2008). "Inventory management with variable lead-time dependent procurement cost", *Omega*, Vol. 36, No. 5, pp. 877-887.

Chod, J., and Zhou, J., (2013). "Resource flexibility and capital structure", *Management Science*.

Chopra, S., Reinhardt, G., and Dada, M., (2004). The effect of lead-time uncertainty on safety stocks. Decision Sciences 35 (1), 1-24.

Christensen, W. J., Germain, R. N., and Birou, L., (2007). "Variance vs average: supply chain lead-time as a predictor of financial performance", *Supply Chain Management: An International Journal*, Vol. 12, No. 5, pp. 349-357.

Clay Whybark, D., (2007). "Issues in managing disaster relief inventories", *International Journal of Production Economics*, Vol. 108, No. 1, pp. 228-235.

Dangl, T., (1999). "Investment and capacity choice under uncertain demand", *European Journal of Operational Research*, Vol. 117, No. 3, pp. 415-428.

Das, R. and Hanaoka, S., (2014). "Relief inventory modelling with stochastic lead-time and demand", *European Journal of Operational Research*, Vol. 235, No. 3, pp.616-623.

Diabat, A., Dehghani, E. and Jabbarzadeh, A., (2017). "Incorporating location and inventory decisions into a supply chain design problem with uncertain demands and lead-times", *Journal of Manufacturing Systems*, Vol. 43, pp.139-149.

Díaz-Madroñero, M., Mula, J., Jiménez, M. and Peidro, D., (2017). "A rolling horizon approach for material requirement planning under fuzzy lead-times", *International Journal of Production Research*, Vol. 55, No. 8, pp.2197-2211.

Dolgui, A., Ammar, O. B., Hnaien, F., and Louly, M.-A., (2013). "A state of the art on supply planning and inventory control under lead-time uncertainty", *Studies in Informatics and Control*, Vol. 22, No. 3, pp. 255-268.

Economist, (2006). *When the chain breaks.* (http://www.economist.com/node/7032258)

Fan, Y., Schwartz, F. and Voß, S., (2017). "Flexible supply chain planning based on variable transportation modes", *International Journal of Production Economics*, Vol. 183, pp.654-666.

Fine, C. H., and Freund, R. M., (1990). "Optimal investment in product-flexible manufacturing capacity", *Management Science*, Vol. 36, No. 4, pp. 449-466.

Fisher, M., and Raman, A., (1996). "Reducing the cost of demand uncertainty through accurate response to early sales", *Operations research*, Vol. 44, No (1), 87-99.

Glen, A. G., Leemis, L. M., and Drew, J. H., (2004). "Computing the distribution of the product of two continuous random variables", *Computational statistics & data analysis*, Vol. 44, No. 3, pp. 451-464.

Goyal, M., and Netessine, S., (2007). "Strategic technology choice and capacity investment under demand uncertainty", *Management Science*, Vol. 53, No. 2, pp. 192-207.

He, X. J., Xu, X., and Hayya, J. C., (2011). "The effect of lead-time on the supply chain: the mean versus the variance", *International Journal of Information Technology & Decision Making*, Vol. 10 , No. 1, pp. 175-185.

Heydari, J., Kazemzadeh, R. B., and Chaharsooghi, S. K., (2009). "A study of lead-time variation impact on supply chain performance", *The International Journal of Advanced Manufacturing Technology*, Vol. 40, No. (11-12), pp. 1206-1215.

Ho, J.-W., and Fang, C.-C., (2013). "Production capacity planning for multiple products under uncertain demand conditions", *International Journal of Production Economics*, Vol. 141, No. 2, pp. 593-604.

Hoque, M., (2013). "A manufacturer-buyer integrated inventory model with stochastic lead-times for delivering equal-and/or unequal-sized batches of a lot", *Computers & Operations Research*, Vol. 40, No. 11, pp. 2740-2751.

Hsieh, Y.-J., (2011). "Demand switching criteria for multiple products: An inventory cost analysis", *Omega*, Vol. 39, No. 2, pp. 130-137.

Kaya, O., Bagci, F., Turkay, M., (2013). "Planning of capacity, production and inventory decisions in a generic reverse supply chain under uncertain demand and returns", *International Journal of Production Research* (ahead-of-print), pp. 1-13.

Kennedy, J., (2010). "Particle swarm optimization. In: Encyclopedia of Machine Learning", *Springer*, pp. 760-766.

Kim, J. G., Sun, D., He, X. J., and Hayya, J. C., (2004). "The (s, q) inventory model with Erlang lead-time and deterministic demand", *Naval Research Logistics (NRL)*, Vol. 51, No. 6, pp. 906-923.

Kulkarni, S.S. and Francas, D., (2017). "Capacity investment and the value of operational flexibility in manufacturing systems with product blending", *International Journal of Production Research*, pp.1-27.

Lukas, E., Spengler, T.S., Kupfer, S. and Kieckhäfer, K., (2017). "When and how much to invest? Investment and capacity choice under product life cycle uncertainty", *European Journal of Operational Research*, Vol. 260, No. 3, pp.1105-1114.

Movahed, K. K., and Zhang, Z.-H., (2013). "Robust design of (s, s) inventory policy parameters in supply chains with demand and lead-time uncertainties", *International Journal of Systems Science* (ahead-of-print), pp. 1-11.

Pan, K., Lai, K., Liang, L., and Leung, S. C., (2009). "Two-period pricing and ordering policy for the dominant retailer in a two-echelon supply chain with demand uncertainty", *Omega*, Vol. 37, No. 4, pp. 919-929.

Rodriguez, M. A., Vecchietti, A. R., Harjunkoski, I., and Grossmann, I. E., (2014). "Optimal supply chain design and management over a multi-period horizon under demand uncertainty. Part i: MINLP and MILP models", *Computers & Chemical Engineering*, Vol. 62, pp. 194-210.

Ryu, S. W., and Lee, K. K., (2003). "A stochastic inventory model of dual sourced supply chain with lead-time reduction", *International Journal of Production Economics*, Vol. 81, pp. 513-524.

Sajadieh, M. S., Jokar, M. R. A., and Modarres, M., (2009). "Developing a coordinated vendor-buyer model in two-stage supply chains with stochastic lead-times", *Computers & Operations Research*, Vol. 36, No. 8, pp. 2484-2489.

Silver, E. A., and Zufferey, N., (2005). "Inventory control of raw materials under stochastic and seasonal lead-times", *International journal of production research*, Vol. 43, No. 24, pp. 5161-5179.

Simchi-Levi, D., Wang, H. and Wei, Y., (2017). "Increasing supply chain robustness through process flexibility and inventory".

Sirkin, H. L., and Stalk, G., (2005). "Avoiding supply chain shipwrecks", (https://www.bcgperspectives.com)

Sting, F. J., and Huchzermeier, A., (2014). "Operational hedging and diversification under correlated supply and demand uncertainty", *Production and Operations Management*.

Sun, Y. and Guo, Z., (2017). "Optimal Inventory Policy for Newsboy Problem Based on Fuzzy Random demand", In Proceedings of the Fourth International Forum on Decision Sciences, Springer, Singapore, pp. 737-745.

Swenseth, S. R., and Buffa, F. P., (1991). "Implications of inbound lead-time variability for just-in-time manufacturing", *International Journal of Operations & Production* Management, Vol. 11, No. 7, pp. 37-48.

Thomas, A., (2008). *Humanitarian logistics: enabling disaster response*. Fritz Institute.

Van Kampen, T. J., Van Donk, D. P., and van der Zee, D.-J., (2010). "Safety stock or safety lead-time: coping with unreliability in demand and supply", *International Journal of Production Research*, Vol. 48, No. 24, pp. 7463-7481.

Van Mieghem, J. A., (1998). "Investment strategies for flexible resources", *Management Science*, Vol. 44, No. 8, pp. 1071-1078.

Wang, Y., Tomlin, B., (2009). "To wait or not to wait: Optimal ordering under lead-time uncertainty and forecast updating", *Naval Research Logistics (NRL)*, Vol. 56, No. 8, pp. 766-779.

Yang, L., and Ng, C., (2014). "Flexible capacity strategy with multiple market periods under demand uncertainty and investment constraint", *European Journal of Operational Research*.

Yongheng, J., Rodriguez, M. A., Harjunkoski, I., and Grossmann, I. E., (2013). "Optimal supply chain design and management over a multi-period horizon under demand uncertainty Part ii: A Lagrangian decomposition algorithm", *Computers & Chemical Engineering*.

Zhao, Y., and Simchi-Levi, D., (2006). "Performance analysis and evaluation of assemble-to-order systems with stochastic sequential lead-times", *Operations Research*, Vol. 54, No. 4, pp. 706-724.

Zipkin, P., (1986). "Stochastic lead-times in continuous-time inventory models", *Naval Research Logistics Quarterly*, Vol. 33, No. 4, pp. 763-774.

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Appendix A.

Proof of proposition 4.1

By differentiating (14) with respect to q_i^f the optimum production quantity under flexible capacity q_i can be found as follow:

$$\frac{\partial L\left(q_i^f, k_f, \lambda_f\right)}{\partial q_i^f} = 2A_i q_i^f + B_i - \lambda_f = 0 \Rightarrow q_i^{f^*} = \max\left(0, \frac{-B_i + \lambda_f}{2A_i}\right), \forall i \in P \qquad \text{(A.1)}$$

Substituting q_i^{f*} into (5), we can obtain λ_f as follow:

$$\lambda_f = \frac{k_f + \sum_{i \in P} \frac{B_i}{2A_i}}{\sum_{i \in P} \frac{1}{2A_i}} \qquad \text{(A.2)}$$

Subsequently, by differentiating (15) with respect to q_i^d optimal production quantity under dedicated capacity q_i^d can be found as follow:

$$q_i^{d*} = \max\left(0, \frac{-B_i + \lambda_i}{2A_i}\right), \lambda_i = 2A_i k_i + B_i, \forall i \in P \qquad \text{(A.3)}$$

On obtaining $\lambda_i, \lambda_f \geq 0$ optimal production quantity can be obtained using equation (A.1), (A.2) and (A.3). The following situations may arise in computation of λ , q_i^{f*} and q_i^{d*}:

When $\lambda_i, \lambda_f < 0,$ constraints (4), (5) are not binding then q_i^{f*}, q_i^{f*} can be obtained by setting λ_i, λ_f equal to zero in equations (A.1), (A.3) . If q_i^{f*} is less than zero then Abdel-Malek and Otegbeye (2013) suggested that, we can take the lower bound for the production quantity and remove that product from calculation of λ_f in (A.2) . Based on the above discussions we can obtain the optimal production quantity under flexible investment as given in preposition 4.1.

Appendix B.

Proof of proposition 4.3

Proof by substituting values of equation (18a), (18b) and (18c) into the first stage objective function (1), we can obtain:

$$V_f\left(k_f\right) = \begin{cases} \sum_{i \in P}\left(-\frac{B_i^2}{4A_i} + C_i\right) + c_f k_f & k_f \geq -\sum_{i \in P} \frac{B_i}{2A_i} \qquad \text{(B.1a)} \\ \frac{\prod_{i \in P} A_i k_f^2 \sum_{i \in P}(A_i B_{3-i}) k_f - \frac{1}{4}(B_i - B_{3-i})^2}{\sum_{i \in P} A_i} + \sum_{i \in P} C_i - c_f k_f & \max\left(\frac{B_i - B_{3-i}}{2A_{3-i}}, \frac{B_{3-1} - B_i}{2A_i}\right) \leq k_f < -\sum_{i \in P} \frac{B_i}{2A_i} \qquad \text{(B.1b)} \\ \sum_{i \in P} C_i - c_f k_f & k_f < \max\left(\frac{B_i - B_{3-i}}{2A_{3-i}}, \frac{B_{3-1} - B_i}{2A_i}\right) \qquad \text{(B.1c)} \end{cases}$$

From the above equations, (B.1a) is not concave but its decreasing on k_f so under (B.1a) conditions, maximum of $V_f^*(k_f)$ in (B.1a) can be obtained when $k_f = \sum_{i=1}^{2} \frac{B_i}{2A_i}$. (B.1c) is always negative, and its optimal when: $k_f^* = 0$

(B.1b) is strictly concave in the given region the proof is provided in (Appendix H), and its optimal value can be obtained by using differentiation.

Based on the previous discussions and by substituting optimal value of (B.1a) and (B.1b) into the objective function, we can get:

$$V_f\left(k_f^*\right)=\begin{cases}\sum_{i\in P}\left(-\frac{B_i^2}{4A_i}+c_i\right)+c_f\sum_{i\in P}\frac{B_i}{2A_i} & k_f\geq-\sum_{i\in P}\frac{B_i}{2A_i} \quad \text{(B.2a)}\\ -\frac{\sum_{i\in P}A_i(c_f-B_{3-i})^2}{4\prod_{i\in P}A_i}+\sum_{i\in P}C_i & \max\left(\frac{B_i-B_{3-i}}{2A_{3-i}},\frac{B_{3-1}-B_i}{2A_i}\right)\leq k_f<-\sum_{i\in P}\frac{B_i}{2A_i} \quad \text{(B.2b)}\\ \sum_{i\in P}C_i & k_f<\max\left(\frac{B_i-B_{3-i}}{2A_{3-i}},\frac{B_{3-1}-B_i}{2A_i}\right) \quad \text{(B.2c)}\end{cases}$$

In (Appendix I) we have shown that (B.2b) is the maximum of $V_f(k_f^*)$, and the optimal k_f^* associated with (B.2b) is given in (23), so we can conclude that under flexible policy it is always optimal for the newsvendor to invest on the amount of capacity as given in (23) and (4.3) is proved.

Appendix C.

Proof of proposition 4.4

Proof Based on (23) and this fact that k_f^* should be positive $k_f^*\geq 0$ we can obtain (24).

Appendix D.

Proof of proposition 4.5

Proof by substituting values of equation (19a) and (19b) into the first stage objective function (1), we can obtain:

$$V_d\left(k_1,k_2\right)=\begin{cases}\sum_{i\in P}\left(-\frac{B_i^2}{4A_i}+C_i-c_ik_i\right) & k_i\geq-\frac{B_i}{2A_i} \quad \text{(D.1a)}\\ \sum_{i\in P}\left(A_ik_i^2+B_ik_i+C_i-c_ik_i\right) & k_i<-\frac{B_i}{2A_i} \quad \text{(D.1b)}\end{cases}$$

From the above equations (D.1a) is not concave but its decreasing in k_i, so under (D.1a) conditions, maximum of $V_d(k_1,k_2)$ in (D.1a) can be obtained when $k_i=-\frac{B_i}{2A_i}$. (D.1b) is strictly concave in k_i and its maximum can be found by using its derivative equal to zero, which results to $k_i^*=\frac{c_i-B_i}{2A_i}$. In (Appendix J) we have shown that $V_d(k_1,k_2)$ is concave and its maximum is equal to maximum of (D.1b). So under dedicated capacity investment it is always optimal for the manufacturer to invest as (25) so (4.5) is proved.

Appendix F.

Proof of proposition 4.7

Proof in (Appendix I) we have shown that (B.1b) is strictly bigger than (B.1a) in optimal solution. So when the newsvendor invests in flexible capacity, (B.1b) is the corresponding optimal profit. At the other hand when the newsvendor invests in dedicated capacity in (Appendix J) we have shown that under dedicated technology maximum profit can be found through (D.1b).

Based on the above discussion we can write optimal $V(k_f^*,k_1^*,k_2^*)$ as follow:

$$V\left(k_f^*,k_1^*,k_2^*\right)=\begin{cases}-\frac{\sum_{i\in P}A_i(c_f-B_{3-i})^2}{4\prod_{i\in P}A_i}+\sum_{i\in P}C_i & \max\left(\frac{B_i-B_{3-i}}{2A_{3-i}},\frac{B_{3-1}-B_i}{2A_i}\right)\leq k_f^*<-\sum_{i\in P}\frac{B_i}{2A_i} \quad \text{(F.1a)}\\ -\frac{\sum_{i\in P}A_i(B_{3-i}-c_{3-i})^2}{4\prod_{i\in P}A_i}+\sum_{i\in P}C_i & k_i^*<-\frac{B_i}{2A_i} \quad \text{(F.1b)}\end{cases}$$

Based on (F.1a),(F.1b) we can find the values of c_f in which profit of investment in flexible capacity is more than investment in dedicated capacity, by using equation: $V(k_f^*)-V(k_1^*,k_2^*)=0.$ Based on (F.1a) we can conclude that (F.1a) is a second order function of c_f. So the mentioned equation has two solutions.

If c_f is bigger that the bigger root and smaller than smaller root, flexible capacity is always preferred. But from (24), c_f should be less than $\frac{\sum_{i\in P} A_i B_{3-i}}{\sum_{i\in P} A_i}$, which is always between the two mentioned roots. So the threshold value should be always less than the smaller root of the equation $V(k_f^*)-V(k_1^*,k_2^*)=0$, which is given in (4.7).

Appendix G.

Proof of proposition 4.8
Proof Based on (27) we can define:

$$f\left(c_1,c_2\right)=\frac{\sum_{i\in P} A_i B_{3-i}+\sqrt{\sum_{i\in P}(B_i-c_i)^2 A_{3-i}^2+2\prod_{i\in P} A_i\left(\sum_{i\in P} c_i\left(c_i/2-B_i\right)+\prod_{i\in P} B_i\right)}}{\sum_{i\in P} A_i}$$

Solving equation $f(c_1,c_2)-c_2=0$ results to $c_1=c_2$. So we can conclude that: $f(c_1,c_1)=c_1$ and $f(c_2,c_2)=c_2$. .

Now we assume that $c_2 > c_1$, so we can write: $c_2=c_1+\alpha,\alpha\geq 0$. We should prove that $c_1<f(c_1,c_2)<c_2$, it can be seen that $f(c_1,c_1)$ is increasing in c_1,c_2:

From proposition 4.6 we know that $c_i<B_i$ and A_i is always negative, so:

$$\frac{\partial^f\left(c_1,c_2\right)}{\partial c_i}=\frac{1}{2\sum_{i\in P} A_i}\frac{2\left(c_i-B_i\right)\prod_{i\in P} A_i-2A_{3-i}^2\left(B_i-c_i\right)}{\sqrt{\sum_{i\in P}(B_i-c_i)^2 A_{3-i}^2+2\prod_{i\in P} A_i\left(\sum_{i\in P} c_i\left(c_i/2-B_i\right)+\prod_{i\in P} B_i\right)}}>0,\forall i\in P$$

Based on the above $f(c_1,c_2)$ is increasing in c_1,c_2, now we can obtain:

$$f\left(c_1,c_1+\alpha\right)>f\left(c_1,c_1\right)=c_1$$
$$f\left(c_2-\alpha,c_2\right)<f\left(c_2,c_2\right)=c_2$$

Now we assume that $c_2>c_1$, based on the above result we can conclude: $c_1<f(c_1,c_2)<c_2$, we can prove the reverse also when $c_1>c_2$ using the same procedure. So proposition 4.8 is proved.

Appendix H.

(B.1b) is strictly concave and its optimal value is always inside the given region of (B.1b), let:

$$f\left(k_f\right)=\frac{A_i A_{3-i} k_f^2+\sum_{i\in P}\left(A_i B_{3-i}\right)k_f\frac{1}{4}(B_i-B_{3-i})^2}{\sum_{i\in P} A_i}+\sum_{i\in P} c_i-c_f k_f,\forall i\in P$$

Then, $\frac{\partial^2 f\left(k_f\right)}{\partial k_f^2}=\frac{\prod_{i\in P} A_i}{\sum_{i\in P} A_i}<0$

To find the optimal value of $f(k_f)$ we use:

$$\frac{\partial^f\left(k_f\right)}{\partial k_J}=0\Rightarrow k_f^*=\frac{\sum_{i\in P}\left(c_f-B_i\right)A_{3-i}}{\prod_{i\in P} A_i}$$

Then, we have to show that its optimal value is $\max\left(\frac{B_i - B_{3-i}}{2A_{3-i}}, \frac{B_{3-1} - B_i}{2A_i}\right) \le k_{f^*} < -\sum_{i=1}^{2}\frac{B_i}{2A_i}$

For the second part we have:

$$k_f^* + \sum_{i\in P}\frac{B_i}{2A_i} = \frac{c_f\sum_{i\in P}A_i}{2\prod_{i\in P}A_i} < 0$$

We suppose that $B_1 > B_2$ then:

$$\max\left(\frac{B_i - B_{3-i}}{2A_{3-i}}, \frac{B_{3-1} - B_i}{2A_i}\right) = -\frac{B_2 - B_1}{2A_1} \quad \text{and:}$$

$$k_f^* - \frac{B_2 - B_1}{2A_1} = \frac{\left(c_f - B_1\right)\sum_{i\in P}A_i}{2\prod_{i\in P}A_i} > 0$$

So, $\max\left(\frac{B_l - B_{3-i}}{2A_{3-i}}, \frac{B_{3-1} - B_i}{2A_i}\right) \le k_f^* < -\sum_{i\in P}\frac{B_i}{2A_i}$

Appendix I.

First, we have to show that (B.2b) is bigger than (B.2a), we can write:

$$-\frac{\sum_{i\in P}\left(c_f - B_i\right)^2 A_{3-i}}{4\prod_{i\in P}A_i} + \sum_{i\in P}C_i - \sum_{i\in P}\left(-\frac{B_i^2}{4A_i} + C_i\right) - c_f\sum_{i=1}^{2}\frac{B_i}{2A_i} = -\frac{c_f^2\sum_{i\in P}A_i}{4\prod_{i\in P}A_i} > 0$$

Second, we have to show that (B.2b) is bigger than (B.2c), we can write:

$$-\frac{\sum_{i\in P}\left(c_f - B_i\right)^2 A_{3-i}}{4\prod_{i\in P}A_i} + C_1 + C_2 - C_1 - C_2 = -\frac{\sum_{i\in P}\left(c_f - B_i\right)^2 A_{3-i}}{4\prod_{i\in P}A_i} > 0$$

Based on the previous results we can conclude that maximum value of $V_f(k_f^*)$ can be obtained through (B.2b).

Appendix J.

(D.1a) is maximized when $k_i = -\frac{B_i}{2A_i}$, by substituting we can find:

$$\sum_{i=1}^{2}\left(-\frac{B_i^2}{4A_i} + C_i - c_i k_i\right) = -\frac{\sum A_i B_{3-i}(B_{3-i} - 2c_{3-i})}{4\prod_{i\in P}A_i} + \sum_{i\in P}C_i$$

(D.1b) is maximized when $k_i = \frac{c_i - B_i}{2A_i}$, by substituting we can find:

$$\sum_{i\in P}(A_i k_i^2 + B_i k_i + c_i - c_i k_i) = -\frac{\sum_{i\in P}A_i(B_{3-i} - c_{3-i})^2}{4\prod_{i\in P}A_i} + \sum_{i\in P}C_i$$

Difference between (D.1b) and (D.1a) can be found as:

$$-\frac{\sum_{i\in P}A_i(B_{3-i} - c_{3-i})^2}{4\prod_{i\in P}A_i} + \sum_{i\in P}C_i - \left(-\frac{\sum_{i\in P}A_i B_{3-i}(B_{3-i} - 2c_{3-i})}{4\prod_{i\in P}A_i} + \sum_{i\in P}C_i\right) = -\frac{\sum_{i\in P}A_i c_{3-i}^2}{4\prod_{i\in P}A_i} > 0$$

Based on the above result we can conclude than maximum of (D.1b) is strictly bigger than the maximum of (D.1a) , so $V_d(k_1, k_2)$ is concave ((D.1b) is concave) and its maximum is equal to the maximum of (D.1b).