JIEMS

Journal of Industrial Engineering and Management Studies

Vol. 6, No. 1, 2019, pp. 46-67

DOI: 10.22116/JIEMS.2019.87660

www.jiems.icms.ac.ir

Robust economic-statistical design of the EWMA-R control charts for phase II linear profile monitoring

Ali Salmasnia[1,*], Ali Tavakoli[1], Maryam Noroozi[1], Behnam Abdzadeh[1]

Abstract

Control charts are powerful tools to monitor quality characteristics of services or production processes. However, in some processes, the performance of process or product cannot be controlled by monitoring a characteristic; instead, they require to be controlled by a function that usually refers as a profile. This study suggests employing exponentially weighted moving average (*EWMA*) and range (*R*) control charts for profile monitoring, simultaneously. For this purpose, the parameters of these control charts should be determined in a way that the expected total cost is minimized. In order to evaluate the statistical performance of the proposed model, the in-control and out-of-control average run lengths are applied. Moreover, the existence of uncertain parameters in many processes is a barrier to attain the best design of control charts in practice. In this paper, the economic-statistical design of control charts for linear profile monitoring under uncertain conditions is investigated. A genetic algorithm is used for solving the proposed robust model, and the Taguchi experimental design is employed for tuning its parameters. Furthermore, the effectiveness of the developed model is illustrated through a numerical example.

Keywords: Exponentially weighted moving average chart; Range chart; Economic-Statistical design; Profile monitoring; Robust optimization; Genetic algorithm; Average run length.

Received: June 2018-05

Revised: January 2019-02

Accepted: January 2019-30

1. Introduction

Control charts as the most important tools in statistical process monitoring (SPM) are usually used to detect the occurred assignable cause in manufacturing processes. Sample size (n), sampling interval (h) and control limit coefficient (l) are three main decision variables associate with control charts which quality engineers must take some technical and behavioral decisions about them. The main approaches that have been presented in the literature to determine these decision variables are heuristic, economic design (ED) and economic-statistical design (ESD).

* Corresponding author; a.salmasnia@qom.ac.ir

[1] Department of Industrial Engineering, Faculty of Technology and Engineering, University of Qom, Qom, Iran.

Heuristic approach recommends sample size of 4 or 5, three sigma control limit coefficient and 1-hour sampling interval. This method has attracted great attention due to its simplicity in industrial practice. ED approach minimizes the expected total cost of using control chart per time unit in a process. On the other hand, ESD approach searches a scheme that minimizes the expected total cost subject to statistical quality constraints.

In the context of ED, Duncan (1956) proposed the first economic design scheme for control charts. He recommended replacing general guidelines for the design parameters of control charts with process-specific design to minimize the average cost of applying $\bar{X}$ control chart. Montgomery and Klatt (1972) discussed an economic design of T^2 Hoteling control chart. Lorenzen and Vance (1986) proposed another general model for economic design of control charts.

In order to improve statistical performance of ED models, Saniga (1989) proposed economic-statistical design of $\bar{X}$ and R charts. Chen (1995) used the model proposed by Montgomery and Klatt (1972) to develop economic-statistical design of T^2 Hoteling control chart. Montgomery et al. (1995) discussed economic-statistical design of exponentially weighted moving average (*EWMA*) control chart with considering the Lorenzen-Vance cost function. In conjunction with simultaneous monitoring of multiple quality characteristics, Linderman and Love (2000) employed the Lorenzen-Vance cost function to develop economic-statistical design of multivariate exponentially weighted moving average (*MEWMA*) control chart. Tolley and English (2001) investigated the impact of constraining the in-control average run length on the optimal cost performance of both economic design of *EWMA* and combined *EWMA*- $\bar{X}$ control schemes. In order to monitor both process mean and variability, Serel and Moskowitz (2008) presented economic-statistical design of *EWMA* control chart. Niaki et al. (2013) employed a genetic algorithm for economic-statistical design of variable sampling interval $\bar{X}$ chart for non-normal correlated samples. Faraz and Saniga (2013) used multi-objective genetic algorithm for economic-statistical design of $\bar{X}$ and S^2 charts. Amiri et al. (2014) developed economic-statistical design of *MEWMA* chart in an uncertain environment based on Markov chain. In many cases, the quality of a product or a process can be effectively monitored by a function or profile. Woodall (2007) reviewed the researches on the use of control charts to monitor process and product quality profiles. Amiri et al. (2010) presented a case study of profile application in the automotive industry. Saghaei et al. (2009) used a cumulative sum (*CUSUM*) control chart for simple linear profile monitoring. Lee and wang (2010) used *EWMA* control charts with variable sampling intervals to monitor linear profile. Noorosaana et al. (2014) developed economic and economic-statistical design of phase II linear profile monitoring. Khedmati and Niaki (2015) developed an approach for phase II linear profile monitoring in the presence of auto correlated profiles. Ershadi et al. (2015) presented economic-statistical design of a simple linear profile with variable sampling interval. Khedmati and Niaki (2016) presented a new control approach for phase II monitoring of a simple linear profile in multi-stage processes. In their proposed approach, a single max-*EWMA*-3 statistic is applied for simultaneous monitoring of the parameters of a simple linear profile. Although the effectiveness of a control chart design depends on the estimation accuracy of input parameters, data with high accuracy are rarely available in practice. In other words, some parameters usually deviate from their estimated values due to unanticipated disruptions. In such situations, the robust economic-statistical design of control charts is necessary in practical applications. Actually, the aim of the robust optimization approach is to find a solution that is robust to uncertainty of input data. Pignatiello and Tsai (1988) used the robust idea for the first time when accurate estimations for the cost parameters are not available in design of control charts. Linderman and Choo (2002) developed the concept of robust economic design of control charts where multiple economic and process scenarios are considered. Vommi and Seetala (2007) designed $\bar{X}$ control chart when the input parameters are unknown but bounded. Safaei et al. (2015) presented robust economic-statistical design of $\bar{X}$ control chart. They developed a heuristic algorithm to obtain the robust scheme of $\bar{X}$ control chart. Amiri et al. (2014) considered an interval robust optimization for the economic-statistical design of *MEWMA* control chart. Li et al. (2016) proposed a one-sided nonparametric monitoring procedure using the single sample sign statistic for detecting a shift in the location parameter of a continuous distribution. In Wu et al. (2017) an optimal design procedure for robust-likelihood cumulative sum control schemes is presented and several corresponding enhancements are considered.

Considering descriptions mentioned in the last paragraphs, a model is presented in this paper consisting of the following properties: (1) According to the best of our knowledge, so far the problem of uncertain data has not been considered for profile monitoring procedures. In this paper, to fill the mentioned research gaps, the economic-statistical design of *EWMA-R* control charts to monitor profile characteristic is developed. (2) In this model, parameters of profile monitoring procedure are determined in a way that the expected total cost is minimized subject to the statistical constraints. (3) Then, in order to deal with uncertain input parameters, a robust counterpart model for the ESD profile monitoring approach that hereafter called RESD model is presented. This paper presents a new framework to deal with the uncertainty in chart parameters within the context of economic-statistical design of a simple linear profile.

The rest of the paper is organized as follows: A brief discussion of liner profile is presented in Section 2. In Section 3, the total expected cost function is presented and the economic-statistical model for profile monitoring procedure is given. A robust counterpart for economic-statistical design is developed in Section 4. In Section 5, the genetic algorithm that is used for solving the proposed robust economic-statistical model is presented. In Section 6, to better clarify the proposed model, a numerical example is considered. Also, a Taguchi orthogonal array design is used for tuning the GA parameters. Our concluding remarks are assigned in the final section.

2. Description of linear profiles

Before designing the *EWMA-R* charts to monitor profile characteristics, the used notations to formulate the problem are introduced in Table1. As demonstrated in Table 1, notations are divided into three parts: indices, decision variables and parameters. After that, the concept of profile monitoring is defined briefly and eventually the mathematical formulation is presented.

Table1. Notations

Notation	Description
Indices	
i	Index of set points
j	Index of samples
s	Index of scenarios
Decision variables	
h	The time between two successive samples
l	Control limit coefficient
n	Number of set points
r	The weighting parameter in combined EWMA-R chart
Parameters	
A_0	Intercept parameter of profile function
A_1	Slope parameter of profile function
a_{0j}	The least square estimation of A_0 at j^{th} profile

a_{1j}	The least square estimation of A_1 at j^{th} profile
a	Fix cost of sampling
b	Variable cost of sampling
C_0	Quality loss cost in per time unit while the process is in the in-control state
C_1	Quality loss cost in per time unit while the process is in the out-of-control state
d_2 *and* d_3	Constants related to control limits of range chart
E	Expected time to sample and construct the profile
F	Fix cost of each false alarm
m	The number of parameters that may deviate from their nominal values
R_j	R statistic for the j^{th} profile
S_{in}	Average number of samples taken while the process is in the in-control state
T_0	Expected time to search when a false alarm signal takes place
T_1	Expected time for detecting an assignable cause
T_2	Expected time to repair the process
W	Cost of locating and repairing an assignable cause
X	Independent Variable at profile function
Y	Response variable at profile function
$\hat{Y}$	Predicted value for Y
Z_j	$EWMA$ statistic for the j^{th} profile
Γ	The budget of uncertainty
γ_1	A binary variable that equals zero if the process is stopped during the search for an assignable cause and equals one if the process continues to operate
γ_2	A binary variable that equals zero if the process is stopped during the repair and equals one if the process continues to operate
τ	Expected time between last sample in the in-control period and occurrence of an assignable

2.1. Concept of profile

To understand the concept of profile monitoring, an example from Kang and Albin (2000) is presented as follows:

Suppose that an artificial sweetener (aspartame) is produced at a plant, and what is considered as a quality characteristic is the amount of sweetener that can be dissolved per liter of water at different temperatures. After analyzing several samples of aspartame, it was observed that there is a non-linear relationship between the water temperature and the amount of dissolved sweetener. So, in such processes, we should monitor a relationship (linear or non-linear), which is usually referred to as profile, instead of one or multiple quality characteristics.

2.2. Linear profile formulation

Assume that the quality of a process or product is characterized by a linear function between random variable Y and independent variable X. In such situation, the performance of process or product can well modeled by the following linear relationship:

$$Y = A_0 + A_1 X + \varepsilon \qquad , \qquad X_l < X < X_h \tag{1}$$

Where A_0 and A_1 are defined as the intercept and slope parameters while X_l and X_h specify the range of X. Moreover, it is assumed that the random variable ε is independent and normally distributed with mean 0 and variance σ^2. With the purpose of estimating A_0 and A_1

parameters, n set points $X_1, X_2, ..., X_{n-1}, X_n$ in the range $\left(X_l, X_h\right)$ are randomly selected. For the j^{th} profile, process outputs are $Y_{1j}, Y_{2j}, ..., Y_{(n-1)j}, Y_{nj}$. Since it is supposed that the relationship between the response variable and the independent variable is linear, the least square estimations for A_0 and A_1 are obtained as follows:

$$a_{1j} = \frac{S_{xy(j)}}{S_{xx}} \tag{2}$$

$$a_{0j} = \bar{y} - a_{1j}\bar{x} \tag{3}$$

Where $\bar{y} = n^{-1}\sum_{i=1}^{n} y_{ij}$ and $\bar{x} = n^{-1}\sum_{i=1}^{n} x_{ij}$ are the sample mean of the response $\left(Y\right)$ and independent variable $\left(X\right)$, respectively, also $S_{xy(j)}$ and S_{xx} are defined as follows:

$$S_{xy(j)} = \sum_{i=1}^{n} y_{ij}(x_i - \bar{x}) \tag{4}$$

$$S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2 \tag{5}$$

It is well known that statistics a_{0j} and a_{1j} are normally distributed with means A_0 and A_1 the following variances:

$$\sigma_0^2 = \sigma^2\left(n^{-1} + \bar{x}^2 S_{xx}^{-1}\right) \tag{6}$$

$$\sigma_1^2 = \sigma^2 S_{xx}^{-1} \tag{7}$$

Furthermore, the covariance between a_{1j} and a_{0j} is defined as:

$$\sigma_{01}^2 = -\sigma^2 S_{xx}^{-1} \tag{8}$$

According to the independent variable X, the predicted value for dependent variable Y specified by $\hat{y}$ is:

$$\hat{y} = a_{0j} + a_{1j}x \tag{9}$$

Also the residual as the difference between the observed and predicted values of dependent variable that is given as:

$$e_{ij} = y_{ij} - a_{0j} - a_{1j}x_i \tag{10}$$

The independent random variables e_{ij} are normally distributed with mean 0 and variance σ^2 where an unbiased estimator for σ^2 is:

$$MSE_j = (n-2)^{-1}\sum_{i=1}^{n} e_{ij}^2 \tag{11}$$

Since the estimators a_{1j} and a_{0j} are not independent from each other, we cannot control these characteristics with two separated control charts. With considering this issue, Kang and Albin (2000) presented the multivariate and the residual strategies to monitor a simple linear profile. In the multivariate approach, T^2 Hoteling control chart is employed for monitoring the intercept and slope parameters while in the residual approach, *EWMA* and *R* charts are used for monitoring average residuals between the reference profile and sample profile. In this paper, we apply the residual approach to develop a robust economic-statistical model that presented in the next subsection.

2.3. The residual approach for monitoring the simple linear profile

In this approach, *EWMA* chart is used for monitoring the residuals $\left(e_{ij}\right)$. For sample j, we have n residuals $e_{ij}, i=1,2,...,n$ corresponding to the n set points. The average of residuals for sample j is obtained as follows:

$$\bar{e}_j = \frac{\sum_{i=1}^{n} e_{ij}}{n} \qquad (12)$$

The j^{th} sample statistic for the *EWMA* chart, Z_j is as Equation (13) in which $0<r<1$ is the weighting constant and $Z_0 = 0$.

$$Z_j = r\,\bar{e}_j + (1-r)\,\bar{e}_{j-1} \qquad (13)$$

The lower and upper control limits for the *EWMA* control chart are defined as:

$$LCL = -l\sigma\sqrt{\frac{r}{(2-r)n}} \qquad (14)$$

$$UCL = l\sigma\sqrt{\frac{r}{(2-r)n}} \qquad (15)$$

Where l and n are the control limit coefficient and the number of set points, respectively. The *R* chart is considered along with *EWMA* chart to monitor the residuals for two reasons: (1) To detect changes in the process standard deviation, and (2) To take care of the unusual situation where the absolute values of the residuals are large, but, canceling out caused by the signs of the residuals results in small values for the average of the residuals (Kangs and Albin, 2000). For the *R* chart, the sample statistic, the lower and upper control limits are given as:

$$R_j = \max_i(e_{ij}) - \min_i(e_{ij}) \qquad (16)$$

$$LCL = \sigma(d_2 - l d_3) \qquad (17)$$

$$UCL = \sigma(d_2 + l d_3) \qquad (18)$$

Where d_2 and d_3 in Equations (17) and (18) are constants that relate the range and standard deviation, which are dependent on the number of set points.

3. Economic-statistical model

With respect to the mentioned explanations earlier, the objective function and constraints related to the model are described as follows:

$$\min c\ (n,h,l,r) \qquad (19)$$

$$\textit{Subject to}:$$

$$ARL_0 \geq ARL_l \qquad (19.1)$$

$$ARL_1 \leq ARL_u \qquad (19.2)$$

$$n \in N^+ \qquad (19.3)$$

$$0 < r \leq 1 \quad (19.4)$$

$$h \text{ and } l > 0 \quad (19.5)$$

Where $c(n,h,l,r)$ is the expected total cost per time unit that should be minimized. Also, ARL_l and ARL_u are the lower and upper bounds for ARL_0 and ARL_1, receptively. In order to decrease the number of false alarms, the in-control average run length ARL_0 must be bigger than a pre-determined value of ARL_l, as illustrated in Equation (19.1). On the other hand, the control chart should detect the occurrence of the assignable cause as soon as possible. This is possible when out-of-control average run length ARL_1 is less than a pre-determined value of ARL_u, as shown in Equation (19.2). In this paper, we use the Markov chain approach to calculate ARL_0 and ARL_1 as are explained in Appendix A. As previously mentioned, n is the number of set points and Equation (19.3) clarify that n must be a positive integer. Equation (19.4) ensures that the weighting constant parameter r of *EWMA* chart is selected in the interval [0,1]. Moreover, the sampling interval and control limit coefficient must be real positive numbers as shown in Equation (19.5).

Also, the economic design (ED) model for profile monitoring is obtained by eliminating the constraints (19.1) and (19.2) from the model (19).

3.1. The expected total cost

In this study, the expected total cost per time unit is considered as ratio of the expected cycle cost to the expected cycle time while a quality cycle is defined as the time between the start of successive in-control periods as illustrated in Figure 1. In the next subsections, the expected cycle time and the expected cycle cost are introduced.

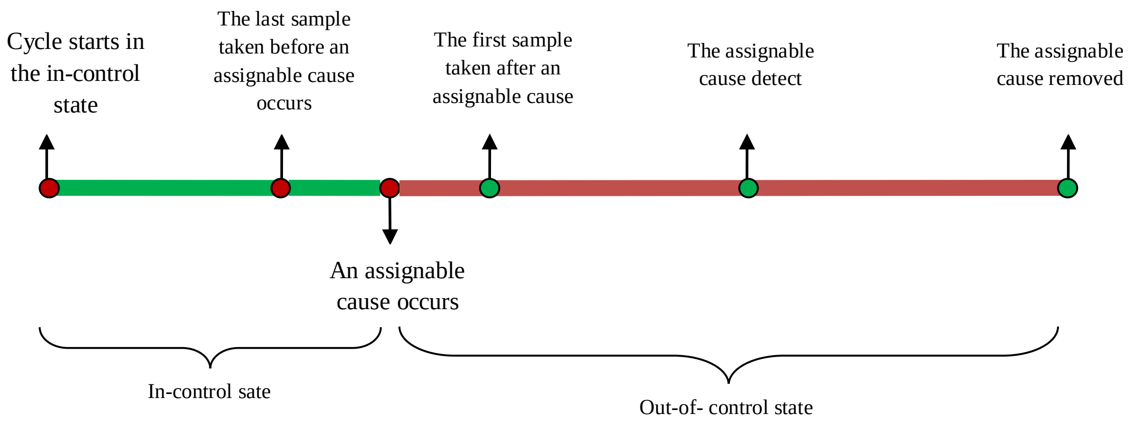

Figure 1. A quality cycle

3.1.1. The expected cycle time

In this model, it is assumed that the process starts its operation in the in-control state, and after a period of time shifts to an out-of-control state due to the occurrence of an assignable cause. Hence, the expected cycle time is the sum of in-control and out-of-control periods. In this study, it is assumed that time to occur an assignable cause follows a negative exponential random variable with mean $\frac{1}{\theta}$. In this situation, if production process continues during the search for assignable cause, the expected in-control time is simply $\frac{1}{\theta}$. On the other hand, if production ceases during the search state, the expected in-control time is sum of $\frac{1}{\theta}$ and the expected time spent for searching false alarms. Let T_0 be the expected search time when a false alarm signal issues. Then the expected time for searching the false alarms is T_0 times the expected number of false alarms, which equals to $\frac{S_{in}T_0}{ARL_0}$, where S_{in} is the expected number of taken samples while the process is in the in-control state.

$$S_{in} = \frac{e^{-\theta h}}{1-e^{-\theta h}} \tag{20}$$

Hence, the expected in-control time can be obtained as Equation (21).

$$E(T_{in}) = \frac{1}{\theta} + \frac{(1-\gamma_1)S_{in}T_0}{ARL_0} \tag{21}$$

The expected out-of-control time (T_{out}) consists of five time periods as follows:
The expected time between occurrence of an assignable cause and the first taken sample in the out-of-control period $(h-\tau)$ in which τ denotes the expected time between the last sample in the in-control period till the occurrence of an assignable cause.

$$\tau = \frac{\int_{jh}^{(j+1)h} e^{-\theta h}(t-jh)\,dt}{\int_{jh}^{(j+1)h} e^{-\theta h}\,dt} = \frac{1-(1+\theta h)e^{-\theta h}}{\theta\left(1-e^{-\theta h}\right)} \tag{22}$$

2. The expected time until issuing a true alarm $[h(ARL_1-1)]$.
3. The expected time for constructing a profile (nE).
4. The expected time to discover the occurred assignable cause (T_1).
5. The expected time to repair the process (T_2).

Therefore, the expected out-control period in a cycle is attained as follows:

$$E(T_{out}) = -\tau + nE + h(ARL_1) + T_1 + T_2 \tag{23}$$

According to the mentioned explanations, the expected cycle time is obtained as Equation (23)

$$E[\text{cyle time}] = \frac{1}{\theta} + \frac{(1-\gamma_1)S_{in}T_0}{ARL_0} - \tau + nE + h(ARL_1) + T_1 + T_2 \tag{24}$$

3.1.2. The expected cycle cost

The expected cycle cost consists of four parts: (1) expected quality loss cost in both in-control and out-control states, (2) expected false alarm cost, (3) expected cost of detection of the occurred assignable cause and repair process and (4) expected sampling cost.

1. Expected quality loss cost: The quality loss cost is imposed to the manufacturer in both in-control and out-of-control periods. However, it is an obvious fact that the quality loss cost extremely increments when the process goes to out-of-control state because the probability of producing non-conforming items increases. To calculate the quality loss cost when the process is in-control (out-of-control), the expected in-control (out-of-control) time must be multiplied

by the quality loss cost in per time unit. Consequently, the expected quality loss cost is formulated in Equation (25).

$$E[\text{quality loss cost}] = \frac{C_0}{\theta} + C_1\left(-\tau + nE + h\left(ARL_1\right) + \gamma_1 T_1 + \gamma_2 T_2\right) \tag{25}$$

2. The expected false alarm: This cost is obtained by multiplying the cost associated with each false alarm F by the expected number of false alarms which depends on the average numbers of sampling in the in-control period and the probability of type I error. Equation (26) indicates the expected false alarm cost.

$$E[\text{false alarm cost}] = \frac{S_{in} F}{ARL_0} \tag{26}$$

3. Cost of repairing the process: In each cycle, process goes to out-of-control state and W is the expected cost for locating and repairing the assignable cause.

4. The expected sampling cost: The expected cost of each profile construction consists of the fixed and variable sampling costs and is defined as a linear function of the number of set points as $a + bn$

To compute the profile construction cost in a given cycle, the expected cost of each profile construction must be multiplied by the number of sampling points, which can be obtained through dividing the expected cycle time by sampling interval. Hence, the expected cost for constructing the profile is obtained according to Equation (27).

$$E\ [\text{profile construction cost}] = \left[\frac{a+bn}{h}\right]\left[\frac{1}{\theta} - \tau + nE + h\left(ARL_1\right) + \gamma_1 T_1 + \gamma_2 T_2\right] \tag{27}$$

Combining four mentioned costs gives the expected cycle cost as follows:

$$\begin{aligned} &\frac{C_0}{\theta} + C_1\left(-\tau + nE + h\left(ARL_1\right) + \gamma_1 T_1 + \gamma_2 T_2\right) + \frac{S_{in} F}{ARL_0} + W \\ &+ \left[\frac{a+bn}{h}\right]\left[\frac{1}{\theta} - \tau + nE + h\left(ARL_1\right) + \gamma_1 T_1 + \gamma_2 T_2\right] \end{aligned} \tag{28}$$

Finally, the expected total cost per time unit is obtained by dividing the expected cycle cost by the expected cycle time as follows:

$$\begin{aligned} C(n,h,L,r) = &\frac{\left\{\dfrac{C_0}{\theta} + C_1\left(-\tau + nE + h\left(ARL_1\right) + \gamma_1 T_1 + \gamma_2 T_2\right) + \dfrac{S_{in} F}{ARL_0} + W\right\}}{\left\{\dfrac{1}{\theta} + \dfrac{(1-\gamma_1) S_{in} T_0}{ARL_0} - \tau + nE + h\left(ARL_1\right) + T_1 + T_2\right\}} + \\ &\frac{\left\{\left[\dfrac{a+bn}{h}\right]\left[\dfrac{1}{\theta} - \tau + nE + h\left(ARL_1\right) + \gamma_1 T_1 + \gamma_2 T_2\right]\right\}}{\left\{\dfrac{1}{\theta} + \dfrac{(1-\gamma_1) \mathrm{S}_{in} T_0}{ARL_0} - \tau + nE + h\left(ARL_1\right) + T_1 + T_2\right\}} \end{aligned} \tag{29}$$

4. Robust economic-statistical model

The effectiveness of the economic-statistical design of control charts depends on estimation accuracy of input parameters, but data with high accuracy are rarely available in practice.

Ignoring the uncertainty in the input data may leads to severely infeasible solution. In such situations, it seems to be a reasonable way to accept a suboptimal solution for the nominal values of the data to certify that the solution remains feasible and near optimal when the data varies. Therefore, formulation of robust counterpart for mathematical programming (19) is necessary to tackle the uncertain input data. The used robust optimization approach in this paper is based on heuristic approach developed by Safaei et al. (2015).

4.1. The robust counterpart for ESD of linear profile monitoring

In the suggested ESD model for profile monitoring, it is assumed that all parameters are deterministic. But as mentioned earlier, some parameters may deviate from their nominal values due to unanticipated disruptions. In this study, the set parameters $\{E,F,W,C_0,C_1,a,b\}$ are considered as uncertain parameters, while the other cost and process parameters are set on their nominal estimated values.

To deal with the uncertainty in the input data of mathematical programming (19), it is supposed that the values of uncertain parameters are unknown but bounded, thereby one can specify a suitable uncertainty set U for possible values of uncertain parameters. Moreover, to avoid having an infinite number of constraints, the set of uncertain values separated into a set of scenarios (S). Thus, the robust counterpart of the uncertain economic-statistical model is formulated as follows:

$$\min_{x\in X} \max_{s\in S} E(C^s) \tag{30}$$

$$Subject\,to:$$

$$ARL^s_0 \geq ARL_L \quad \forall s \in S \tag{30.1}$$

$$ARL^s_1 \leq ARL_U \quad \forall s \in S \tag{30.2}$$

$$n \in N^+ \tag{30.5}$$

$$0 < r \leq 1 \tag{30.4}$$

$$h\,and\,L > 0 \tag{30.5}$$

Where $E(C^s)$ is the expected total cost for scenarios s per each solution (x). Equations (30.1) till (30.5) are the same as the constraints described in the economic-statistical model. The descriptions of these constraints are explained in model (19).

A concern with the proposed model is that it might be too conservative. In this regard, Bertsimas and Sim (2004) were introduced the concept of *budget of uncertainty* to deliver less conservative solutions. The budget of uncertainty gives to designer capability of trading off between robustness and performance entered into several different uncertainties set formulations. Interested readers are referred to Bertsimas et al. (2011) for details. The parameter Γ, bounded below 0 and above by the number of parameters that can deviate from their nominal values, is called the budget of uncertainty. If $\Gamma = 0$, all of the parameters are fixed on their nominal values, which is equivalent to solving the deterministic ESD model. In this situation, there is no protection against uncertainty and the obtained solution may be completely meaningless from a practical viewpoint. In contrast, if Γ takes the maximum possible value ($\Gamma = m$), all uncertain parameters can deviate from their estimated values, simultaneously. In this situation, the problem is completely protected against uncertainty and a very conservatism solution is obtained. If $\Gamma \in (0,m)$, the designer can makes a trade-off between the performance and the degree of conservative of the solution. Most of the robust optimization approaches in the literature has been developed for linear and quadratic models.

Hence, in the next section a heuristic algorithm for robust economic-statistical design of profile monitoring as a non-linear mathematical programing is explained.

4.2. Heuristic algorithm for solving the RESD model

In this algorithm, a set of scenarios are generated to avoid having an infinite number of constraints. Scenarios are classified into two groups: (1) random scenarios, and (2) extreme-valued scenarios. To generate the random scenarios, one needs to consider all possible combinations of uncertain parameters. This will result in $\binom{m}{\Gamma}$ combinations when the budget of uncertainty and the maximum number of uncertain parameters are Γ and m, respectively. For each possible combination, the values of Γ considered uncertain parameters are randomly generated within their

bounds, while $(m-\Gamma)$ remained uncertain parameters are set on their nominal values. This procedure are repeated Sc times, which leads to generation of $Sc\times\binom{m}{\Gamma}$ random scenarios.

In the extreme-valued scenarios, the parameters that can be deviate from their nominal values are set on extreme values of their bounds. This results in generating $2^{\Gamma}\binom{m}{\Gamma}$ extreme-valued scenarios. Therefore, total number of generated scenarios is:

$$S = Sc\binom{m}{\Gamma} + 2^{\Gamma}\binom{m}{\Gamma} \quad (31)$$

This procedure searches a solution that leads to minimization of maximum cost within all scenarios (i.e, $\min_{x\in X} \max_{s\in S} E(C^s)$) and is feasible for all scenarios.

5. A GA for optimizing ESD and RESD models

Both the ESD and RESD models explained in the previous section are mixed integer non-linear mathematical models and have several complexities that prevent the model to be solved with exact methods. Three of these complexities are: (1) The mentioned models consists of both continuous and discrete decision variables; (2) the solution space is discontinuous and non-convex; and (3) in the both objective function and constraints, some decision variables are in limits of an integral. There is a variety of solution procedures in the literature for solving such problems. Genetic algorithm is one of the most popular evolutionary algorithms that is applied to solve various problems. Molavi and Rezaee Nik (2016) and Fakhrzad and Alidoosti (2018) applied this algorithm to optimize a project scheduling and a location-inventory-routing problem, respectively. Also several studies in the control chart design literature such as Niaki et al. (2010), Noorossana et al. (2014), Faraz et al. (2016), and Ershadi et al. (2015) were employed genetic algorithm to solve their models.

5.1. Solution representation

Encoding (representation) of a given solution called chromosome is a key factor in developing genetic algorithm. For this purpose, four-dimensional strings are employed in which each dimension refers to a certain decision variable. In the presented model, the sample size (*n*) is an integer, while the sampling interval (*h*), control chart limit (*l*) and weight (*r*) are real numbers. Figure 2 depicts a given chromosome that is generated during the process solving.

Chromosome	12	1.5	2.6	0.645
	↑	↑	↑	↑
	The first gene: sample size (n)	The second gene: sampling interval (h)	The third gene: control chart limit (l)	The forth gene: weight (r)

Figure 2. Chromosome representation

5.2. Initial population

The first step of GA is generation of initial population. In this regard, PS (population size) chromosomes are constructed by generating random values according to the defined intervals in Equation (32).

$$0 \le n \le 1 \,, 0.5 < h \le 10 \,, 0 < l \le 4 \,, 0 < r \le 1 \tag{32}$$

It is notable that to generate an initial value for discrete decision of sample size, a random value from a uniform distribution in the interval [0, 1] is generated as shown in equation (32). The discrete corresponding value of the sample size (n) is obtained according to Equation (33).

$$n = \min((2 + \text{floor}((15 - 2 + 1) \times R_1)), 15) \tag{33}$$

In this situation, the generated sample size can vary between 2 and 15.

5.3. Crossover

In this study, the uniform crossover is used for combination of two parents. To do this, a uniformly distributed random vector in the interval [0,1] with the same size of a chromosome is generated and then, output chromosomes are generated with using linear combination of input chromosomes and random vector according to Equations (34) and (35). The process of uniform crossover is illustrated in Figure 3.

Output chromosome 1= Random vector × Input chromosome 1+ (1- Random vector) × Input chromosome 2 (34)

Output chromosome 2= Random vector × Input chromosome 2+ (1- Random vector) × Input chromosome 1 (35)

Input chromosome 1	4	0.68	1.59	0.78
Random vector	0.84	0.91	0.13	0.91
Input chromosome 2	11	1.24	1.64	0.48

Uniform crossover →

Output chromosome 1	6	0.73	1.63	0.75
Output chromosome 2	10	1.18	1.59	0.51

Figure3. Two output chromosomes of the uniform crossover operation.

5.4. Mutation

In order to increase the diversification of genetic algorithm and to avoid trapping into local optimal, a Gaussian mutation operator is used with probability P_m. For this purpose, firstly a discrete random variable between 1 and 4 is generated to select the gene that must be mutated. Next, a random value from a normal distribution with mean the current value of the selected gene and standard deviation $SigV$ is generated.

$$DV = MaxV - MinV \tag{36}$$

$$SigV = 0.1 \times DV \tag{37}$$

Where $MaxV$ and $MinV$ are the vectors consisting of the maximum and minimum possible values of decision variables in the feasible space, respectively. Finally, output chromosome is obtained as illustrated in Figure 4.

nput chromosome	8	0.68	1.59	0.78
$MaxV$	1	10	4	1
$MinV$	0	0.5	0	0
DV	1	9.5	4	1
$SigV$	0.1	0.95	0.4	0.1

Gaussian Mutation →

Output chromosome	8	0.68	1.42	0.78

Figure 4. The output chromosome of the Gaussian mutation operation.

6. Numerical example

In order to understand the effectiveness of the proposed model, a numerical example is presented in this section. Suppose that an *EWMA-R* approach is used to monitor the quality characteristic y that has a linear relationship with the characteristic x as $y = 2x + 1$ in phase II . It is assumed that the slope and the intercept parameters have been specified during an in-control process in phase I. In addition, we need to construct RESD model of the linear profile to detect a shift in the residual average of size 0.5, 0.75 and 1. Moreover, the parameters used in the expected total cost function are as Table 2.

Table 2. The parameter values

Parameter	λ	E	T_0	T_1	T_2	γ_1	
Value	0.1	0.05	0.0	2	2	1	
Parameter	γ_2	C_0	C_1	F	W	a	b
Value	2	10	100	50	25	0.5	0.01

An upper bound for out-of-control average run length (ARL_u) and a lower bound for in-control average run length (ARL_l) are considered to improve the statistical performance of the proposed model. Selection of a greater value for ARL_l results in the lower Type-I error probability of the designed profile monitoring. On the other hand, selection of a greater value for ARL_u leads to the lower Type-II error probability. In this numerical example, ARL_l and ARL_u same as Linderman and Love (2000), Niaki and Ershadi (2012) and Noorosana et al. (2013) are set on 200 and 10, respectively. Therefore, the mathematical programming (19) can be rewritten as follows:

$$\min c\ (n,h,L,r) \tag{38}$$

Subject to :

$$ARL_0 \geq 200 \tag{38.1}$$

$$ARL_1 \leq 10 \tag{38.2}$$

$$n \in N^+ \tag{38.3}$$

$$0 < r \leq 1 \tag{38.4}$$

$$h \text{ and } L > 0 \tag{38.5}$$

6.1. Tuning of GA parameters

Since the GA parameters have significant effects on the quality of the obtained solutions, the important parameters consisting of population size (PS), The probability of crossover operation (P_C), The probability of mutation operation (P_m) and the number of iterations (NG) are tuned. To do this, three levels are considered for each parameter as shown in Table 2 and an orthogonal array experimental design including nine run experiments is employed. For each experiment, the algorithm is run three times, which are denoted by C_1, C_2 and C_3 in Table 3. Next, the signal-to-noise (SN) ratio values are obtained as given in the last column of Table 3.

$$SN = -10\log\left(\frac{1}{3}\sum_{u=1}^{3} C_u^2\right) \tag{39}$$

The values of SN ratio for each experiment are listed in Table (4). Next, the sum of SN ratios of each parameter level is calculated and is given in Table (5). According to Table (5), the best combination of the GA parameters based the Taguchi design is $Ps = 10$, $P_c = 0.7$, $P_m = 0.8$ and $NG = 20$.

Table 3. Levels of each parameter considered in orthogonal experiment

GA parameter	Range	Level 1	Level 2	Level 3
Population size (PS)	8-12	8	10	12
Crossover probability (P_C)	0.5-0.9	0.5	0.7	0.9
Mutation Probability (P_m)	0.5-0.8	0.5	0.65	0.8
Number of generation (NG)	10-30	10	20	30

Table 4. Experiment layout of L_9 orthogonal for GA parameters

Runs	PS	PC	Pm	NG	C_1	C_2	C_3	SN
1	8	0.5	0.5	10	60.531	58.093	60.531	-35.524
2	8	0.7	0.65	20	52.97	52.743	52.97	-34.468
3	8	0.9	0.8	30	53.624	52.3	52.3	-34.388
4	10	0.5	0.65	30	52.745	52.36	52.562	-34.402
5	10	0.7	0.8	10	52.962	54.345	53.884	-34.605
6	10	0.9	0.5	20	53.144	52.669	54.68	-34.568
7	12	0.5	0.8	20	52.864	52.864	52.864	-34.463
8	12	0.7	0.5	30	52.313	56.554	56.554	-34.835
9	12	0.9	0.65	10	69.478	61.72	61.72	-36.179

Table 5. Sum of SN ratios at each level of GA parameters

	PS	PC	Pm	NG
Level 1	-104.38	-104.389	-104.927	-106.308
Level 2	-103.575_{max}	-103.908_{max}	-105.049	-103.499_{max}
Level 3	-105.477	-105.135	-103.456_{max}	-103.625

Max specifies the largest sum of SN ratios for each parameter.

6.2. Optimization of ESD model

The GA with its tuned parameter values is employed to optimize economic and economic-statistical designs. The results of the ESD model for profile monitoring are presented in Table 6. The ED model is considered for comparison between the economic and the economic-statistical designs of profile monitoring. As previously mentioned, the ED model is obtained from the ESD model by eliminating the statistical constraints in model (19).
The results in Table 6 clarify that adding the statistical constraints to the optimization model improves significantly statistical measures while the difference between costs is negligible.

Table 6. The comparison between the ED and ESD models for profile monitoring.

Model cost	n	h	r	l	ARL_0	ARL_1	$Cost$
Economic	15	0.5	0.452	2.438	74.237	2.625	47.64
Economic-statistical	9	0.6	0.277	1.825	252.33	2.857	48.835
Improvement of performance indicator					289.899%	8.838%	-2.51%

6.3. Robust optimization of the ESD model of profile monitoring

In the previous subsection, it is assumed that parameters are deterministic and no uncertainty considered. However in this section to make the model more adapted to real manufacturing situation, it is considered that the parameters $\{E, F, W, C_0, C_1, a, b\}$ can deviate from their nominal values while the other parameters are set on their estimated nominal values. For this purpose, two scenarios are investigated. In the first scenario, it is supposed that each uncertain parameter takes a random value from a uniform distribution with mean equal to its nominal value and interval size of 10% of the nominal value. In the second scenario, 20% deviation of the uncertain parameters from their corresponding nominal values is considered.
The results of robust optimization for different values of budget uncertainty under shift size of 0.5 in the mean residual are listed in Table 6. Results show that as the budget of uncertainty is increased from 0 to 2, the expected total cost increases significantly while increasing of the budget of uncertainty from 2 to 7 does not have tangible impact on the expected total cost and only a small increase in the expected total cost is visible. Table 7 presents similar results when the robust solutions are obtained under 20% shift scenario with this difference that increasing of the expected total cost is more evidence 10% shift scenario as the budget of uncertainty is increased. Comparing the robust solutions under 10% and 20% shift scenarios yields some interesting results. As expected, it can be seen that the total expected costs in 20% shift scenario are larger than their corresponding values in 10% shift scenario. Figure 5 shows the impact of the budget of uncertainty parameter on the expected total cost under both shift scenarios.

Table 7. The robust solutions under 10% shift scenario with mean residual shift from μ to $\mu+0.5\sigma$.

Γ	n	h	r	l	$Cost$
0	9	0.6	0.277	1.85	48.835
1	15	0.523	0.398	3.068	52.78
2	15	0.511	0.4	2.849	53.031
3	15	0.53	0.4	2.866	53.226
4	15	0.5	0.392	3.052	53.329
5	14	0.5	0.37	2.867	53.324
6	15	0.52	0.285	2.913	53.4
7	13	0.534	0.431	2.823	53.521

Table 8. The robust solutions under 20% shift scenario with mean residual shift from μ to $\mu+0.5\sigma$.

Γ	n	h	r	L	$Cost$
0	9	0.6	0.277	1.85	48.835
1	13	0.5	0.546	3.1	55.133
2	13	0.5	0.344	3	56.969
3	13	0.512	0.344	3.003	57.12
4	14	1.96	0.359	3.223	57.275
5	11	0.839	0.86	3.396	57.338
6	11	0.5	0.344	3	57.67
7	10	1.96	0.6	3.22	58.578

Also, in order to investigate the impact of mean residual shift on the expected total cost, In Tables 8 and 9, we give the results of the expected total cost under 0.75σ and 1σ out-of-control step shifts in the 10% shift scenario, respectively. The results represent that increasing the shift size leads to smaller expected total cost. This issue is graphically confirmed by Figure 6.

Table 9. The robust solutions under 10% shift scenario with mean residual shift from μ to $\mu+0.75\sigma$

Γ	n	h	r	l	$Cost$
0	13	0.5	0.69	2.853	45.547
1	12	0.89	0.569	3	48.13
2	11	0.5	0.342	2.97	49.57
3	11	0.5	0.448	3.44	49.62
4	7	0.7	0.316	2.92	50.08
5	11	0.51	0.4	3.22	51.19
6	11	0.51	0.4	3.74	51.3
7	11	0.52	0.745	3.286	51.78

Table 10. The robust solutions under 10% shift scenario with mean residual shift from μ to $\mu+1\sigma$.

Γ	n	h	r	l	$Cost$
0	12	0.5	0.573	3.04	44.435
1	12	0.745	0.542	3.082	45.428
2	8	0.503	0.607	3.01	47.994
3	10	0.5	0.42	3.074	48.21
4	6	0.5	0.419	2.92	48.54
5	11	0.5	0.57	2.86	48.97
6	11	0.5	0.427	2.86	49.239
7	11	0.5	0.36	2.86	49.44

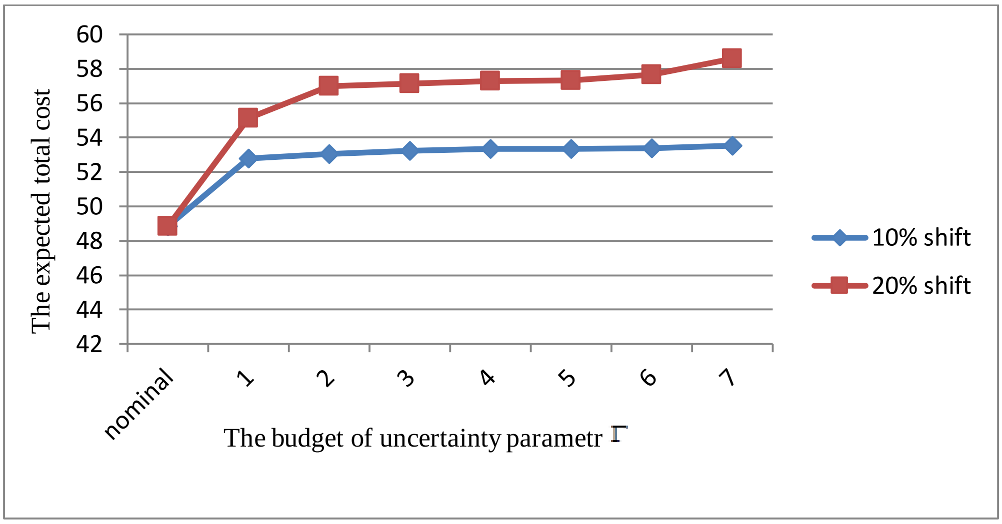

Figure 5. Impact of Γ on the expected total cost under different scenarios (10% and 20%)

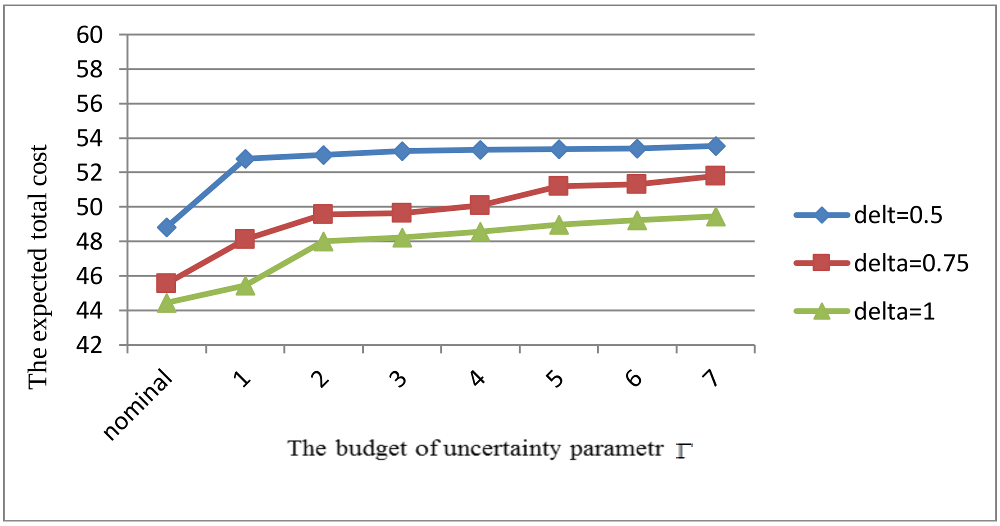

Figure6. The expected total cost of RESD model under different shift values (0.5, 0.75 and 1) in the mean residual

7. Conclusions

A small uncertainty in the input data can make the usual optimal solution completely infeasible from a practical viewpoint. In order to deal with the uncertainty in input data at profile monitoring, a robust economic-statistical design for profile monitoring in phase II was investigated. In order to improve the statistical performance of the proposed model, the in-control and out-of-control average run lengths were used as two statistical measures. The suggested robust optimization approach provides alternative designs for the users to select the most preference design of *EWMA* and *R* charts based on superiority in cost and protection against uncertainty for profile monitoring. To solve both ESD and RESD models, a GA was employed and its parameters were tuned with using Taguchi L_9 experimental design. The comparison between the ED and ESD models clarified that that adding the statistical constraints to the ED model improves significantly statistical measures while the difference between costs is negligible. Also, the results of solving the RESD model showed that a small increase in cost is visible as the budget of uncertainty becomes larger. In other words, the robust optimization for the ESD profile monitoring procedure can be used without incurring a significant increase in the expected total cost. Eventually, the impact of mean residual shift on the expected total cost in RESD model was investigated. The results showed that the scenarios with the bigger shift leads to less cost than the scenarios with smaller shift.

As future research we suggest to extend the presented model in two directions: first, extension of the proposed model for monitoring non-linear profiles, and second, developing a RESD with variable parameters.

References

Amiri, A., Willis, A., and Kazemzadeh, RB, (2010). "A Case Study on Monitoring Polynomial Profiles in the Automotive Industry", *Quality and Reliability Engineering International*, Vol. 26, pp. 509–520.

Amiri, A., Moghadam, A.S., and Aghababaee, Z., (2015). "Robust economic-statistical design of multivariate exponentially weighted moving average control chart under uncertainty with interval date", *Scientia Iranica*, Vol. 22, pp. 1189-1202.

Bertsimas, D., and Sim, M., (2004). "The Price of Robustness", *Operations Research*, Vol. 52, pp. 35–53.

Bertsimas, D., Brown, D., and Caramanis, C., (2011). "Theory and Applications of Robust Optimization", *SIAM Review*, Vol. 53, pp. 464–501.

Ben-Tal., A., Nemirovski, A., (2000). "Robust Solutions of Linear Programming Problems Contaminated with uncertain data", *Mathematical Programming*, Vol. 88, pp. 411-424.

Chen, Y., (1995). Economic and Economic-Statistical Designs of Hotelling's T2 Control Chart, MS thesis, Department of Statistics, National Tsing-Hua University, Hsinchu, Taiwan.

Duncan, A. J., (1956). "The Economic Design of x-bar charts Used to Maintain Current Control of a Process", *Journal of the American Statistica*l Association, Vol. 51, pp. 228–242.

Ershadi, M. J., Noorossana, R., and Niaki, S. T. A., (2015). "Economic-Statistical Design of Simple Linear Profiles with Variable Sampling Interval", *Journal of Applied Statistics*, Vol. 43, pp. 1400-1418.

Fakhrzad, M. B., and Alidoosti, Z., (2018). "A realistic perishability inventory management for location-inventory-routing problem based on Genetic Algorithm", *Journal of Industrial Engineering and Management Studies*, Vol. 5, No. 1, pp. 106-121.

Faraz, A., Chalaki, K., Saniga, E., and Heuchenne, C., (2016). "The robust economic-statistical design of the Hoteling's T^2 chart", *Communications in statistics-Theory and methods*, Vol. 45, pp. 6989-7001.

Faraz, A., and Saniga, E., (2013). "Multi-objective Genetic Algorithm Approach to the Economic Statistical Design of Control Charts with an Application to X-bar and S2 Charts", *Quality and Reliability Engineering International*, Vol. 29, pp. 407–415.

Kang, L., and Albin, S. L., (2000). "On-Line Monitoring when the Process Yields a Linear Profile", *Journal of Quality Technology*, Vol. 32, pp. 418–426.

Khedmati, M., and Niaki, S. T. A., (2015). "Phase-II monitoring of general linear profiles in the presence of between profile autocorrelation", *Quality and Reliability Engineering International*, Vol. 32, pp. 443-452.

Khedmati, M., and Niaki, S. T. A., (2016). "A new control scheme for Phase-II monitoring of simple linear profiles in multistage processes", *Quality and Reliability Engineering International*, Vol. 32, pp. 443-452.

Li, C., Mukherjee, A., Su, Q., and Xie, M., (2016). "Robust algorithms for economic designing of a nonparametric control chart for abrupt shift in location", *Journal of Statistical Computation and Simulation*, Vol. 86, pp. 306-323.

Linderman, K., and Choo, A. S., (2002). "Robust Economic Control Chart Design", *IIE Transactions*, Vol. 34, pp. 1069–1078.

Linderman, K., and Love, T. E., (2000). "Economic and Economic Statistical Designs for MEWMA Control Charts", *Journal of Quality Technology*, Vol. 32, pp. 410–417.

Lorenzen, T. J., and Vance, L. C., (1986). "The Economic Design of Control Charts: A Unified Approach", *Technometrics*, Vol. 28, pp. 3–10.

Mahmoud, M., Morgan, J. P., and Woodall W., (2010). "The Monitoring of Simple Linear Regression Profiles with Two Observations per Sample", *Journal of Applied Statistics*, Vol. 37, pp. 1249–1263.

McWilliams, T. P., Saniga, E. M., and Davis, D. J., (2001). "Economic Statistical Design of x-bar and R or x-bar and S x-bar", *Journal of Quality Technology*, Vol. 33, pp. 234–241.

Mestek, O., Pavlik, J., and Suchánek, M., (1994). "Multivariate Control Charts: Control Charts for Calibration Curves", *Journal of Analytical Chemistry*, Vol. 350, pp. 344–351.

Molavi, F., and Rezaee Nik, E., (2018). "A stochastic model for project selection and scheduling problem", *Journal of Industrial Engineering and Management Studies*, Vol. 3, No. 1, pp. 77-88.

Molnau, W. E., Montgomery, D. C., and Runger, G. C., (2001). "Statistically Constrained Designs of the Multivariate Exponentially Weighted Moving Average Control Chart", *Quality and Reliability Engineering international*, Vol. 7, pp. 39–49.

Montgomery, D. C., (2012). Introduction to Statistical Quality Control, 7th edition, *John Wiley & Sons*, Inc., New York.

Montgomery, D. C., Klatt, P. J., (1972). "Economic Design of T^2 Control Charts to Maintain Current Control of a Process", *Management Science*, Vol. 19, pp. 76–89.

Montgomery, D. C., Torng, J. C. C., Cochran, J. K., and Lowerance, F. P., (1995). "Statistically Constrained Economic Design of the EWMA Control Chart", *Journal of Quality Technology*, Vol. 27, pp. 250–256.

M. S., Saccucci, and J. M., Lucas, (1990). "Average run lengths for exponentially weighted moving average control schemes using the Markov chain approach", *Journal of Quality Technology*, Vol. 22, pp.154-162.

Niaki, S. T. A., and Ershadi, M. J., (2012). "A Hybrid Ant Colony, Markov Chain, and Experimental Design Approach for Statistically Constrained Economic Design of MEWMA Control Charts", *Expert Systems with Applications*, Vol. 39, pp. 3265–3275.

Niaki, S. T. A., Ershadi, M. J., and Malaki, M., (2010). "Economic and economic-statistical designs of MEWMA control charts–a hybrid Taguchi loss, Markov chain, and genetic algorithm approach", *International Journal of Advanced Manufacturing Technology*, Vol. 48, pp. 283-296.

Niaki, S. T. A., Gazaneh, F. M., and Toosheghanian, M., (2013). "A Parameter-tuned Genetic Algorithm for Economic- statistical Design of Variable Sampling Interval X-bar Control Charts for Non-normal Correlated Samples", *Communications in Statistics - Simulation and Computation*, Vol. 43, pp. 1212–1240.

Noorossana, R., Niaki, S. T. A., and Ershadi, M. J., (2014). "Economic and Economic-Statistical Design of Phase II Profile Monitoring", *Quality and Reliability Engineering International*, Vol. 30, pp. 645-655.

Noorossana, R., Saghaei, A., and Amiri, A., (2011). *Statistical Analysis of Profile Monitoring*, John Wiley & Sons, Inc., New York.

Park, C., Lee, J., and Kim, Y., (2004). "Economic Design of a Variable Sampling Rate EWMA Chart", *IIE Transactions*, Vol. 36, pp. 387–399.

Pignatiello, J. J., and Tsai, A., (1988). "Optimal Economic Design of X-control Charts when Cost Model Parameters are not precisely known", *IIE Transactions*, Vol. 20, pp. 103-110.

Lucas, J. M., and Saccucci, M. S., (1990). "Exponentially weighted moving average control schemes: Properties and enhancements", *Technometrics*, Vol. 32, pp. 1-12.

Safaei, A. S., Kazemzadeh, R., and Gan, H. S., (2015). "Robust Economic-Statistical Design f X-bar Control Chart", *International Journal of Production research*, Vol. 14, pp. 4446-4458.

Saghaei, A., Mehrjoo, M., and Amiri, A., (2009). "A CUSUM-based Method for Monitoring Simple Linear Profiles", *The International Journal of Advanced Manufacturing Technology*, Vol. 45, pp. 1252–1260.

Saniga, E. M., (1989). "Economic Statistical Control Chart with an Application to X-bar and R Charts", *Technometrics*, Vol. 31, pp. 313–320.

Serel, D. A., and Moskowitz, H., (2008). "Joint Economic Design of EWMA Control Charts for Mean and Variance", *European Journal of Operational Research*, Vol. 184, pp. 157–168.

Tolley, G. O., and English, J. R., (2001). "Economic Design of Constrained EWMA and Combined EWMA- x-bar Control Schemes", *IIE Transactions*, Vol. 33, pp. 429–436.

Vaghefi, A., (2003). "Phase Two Monitoring of Nonlinear Profiles", *Communications in statistics-Theory and methods*, Vol. 38, pp. 1834-1851.

Vommi, V. B., and Seetala, M. S. N., (2007). "A New Approach to Robust Economic Design of Control Charts", *Applied Soft Computing*, Vol. 7, pp. 211–228.

Williams, J. D., Woodall, W. H., and Birch, J. B., (2003). "Phase I Monitoring of Nonlinear Profiles", Quality and Productivity Research Conference, Yorktown Heights, New York.

Woodall, W. H., (2007). "Current Research on Profile Monitoring", *ProduBao*, Vol. 17, pp. 420–425.

Wu, C., Yu, M., and Zhuang, F., (2017). "Properties and enhancements of robust likelihood CUSUM control chart", *Computers & Industrial Engineering*, Vol. 114, pp. 80-100.

Zhonghua, L., and Zhaojun, W., (2010). "An Exponentially Weighted Moving Average Scheme with Variable Sampling Intervals for Monitoring Linear Profiles", *Computers and Industrial Engineering*, Vol. 59, pp. 630–637.

This article can be cited: Salmasnia A., Tavakoli, A., Noroozi, M., and Abdzadeh, B., (2019). "Robust economic-statistical design of the EWMA-R control charts for phase II linear profile monitoring", *Journal of Industrial Engineering and Management Studies*, Vol. 6, No. 1, pp. 46-67.

Appendix A.

The in-control and out-of-control *ARL* of the EWMA control chart;
In this study, ARL_0 and ARL_1 of *EWMA* chart are calculated based on the Markov chain introduced in the paper of Saccuci and Lucas (1990). In this approach, the interval between the UCL and LCL of *EWMA* chart is divided into M equally spaced subintervals, so that M is an odd integer, which are defined as follows:

$$R_1 = [d_0, d_1], R_2 = [d_1, d_2], ..., R_K = [d_{K-1}, d_K], ..., R_M = [d_{M-1}, d_M] \qquad A.1$$

Where $d_K = LCL_{ewma} + K\Delta d$ and $\Delta d = (LCL_{ewma} - UCL_{ewma}) / M$. Each of these M subintervals is considered as a transitional state in the Morkov chain method and the transition probability from state K to state L is calculated by setting the *EWMA* statistic Z_t to the midpoint of the subinterval R_K when $d_{k-1} < Z_t \le d_k$. Therefore, $p_{K,L}$ is obtained as follows:

$$\begin{aligned} p_{K,L} &= P(Z_t \in R_L \mid Z_{t-1} \in R_K) \\ &= P(d_{L-1} < Z_t \le d_L \mid Z_{t-1} = (d_{K-1} + d_K)/2) \end{aligned} \qquad A.2$$

Note that $d_{(M+1)/2} = \mu_0$. Morais and Pacheco (2000) showed that the transition probabilities can be calculated iteratively as follows:

$$p_{k,l} = f_{k,l} - f_{k,l-1} \qquad k,l = 1,...,m \qquad A.3$$

$$f_{k,l} = \phi\{[2L(l-(1-r)(k-0.5)-0.5rM]/(M[r(2-r)]^{0.5} - \delta n^{0.5})\} \qquad A.4$$

where $\phi(.)$ is the cumulative distribution function for the standard normal probability distribution. Moreover, the control statistic is in the absorbing state if Z_t falls outside the control limits. According to the motioned definitions, process is in-control whenever the *EWMA* statistic Z_t falls in a transient state, while is out-of-control whenever Z_t moves to the absorbing state. The run length of the *EWMA* chart is obtained based on its initial probability vector p_{in} and the transition probability matrix P. The initial probability vector includes probabilities of Z starting in each state of the Markov chain. In practice, the initial probability vector either consists of a single element equal to 1, or it will be a vector of steady state probabilities (Saccuci and Lucas, 1990). In this paper, it is assumed that the process starts its operations at state $\frac{M+1}{2}$. The ARL of the *EWMA* mean chart when the process mean shifts to $\mu_0 + \delta\sigma$ is given as follows:

$$ARL_{ewma} = p_{in}{}^T (\mathrm{I} - P)^{-1} 1 \qquad A.5$$

Where $P = [p_{k,l}]$ is the $M \times M$ matrix of transition probabilities, p_{in} is the initial probability vector and 1 is a column vector of ones. For computing the in-control ARL we set $\delta = 0$ in Equation $A.3$ and the out-of-control ARL can be obtained using the steady-state probability vector (p_s) as the initial vector in Equation A.5. The p_s is determined by solving $p_{in} = P^T p_{in}$ subject to $1^T p_{in} = 1$.