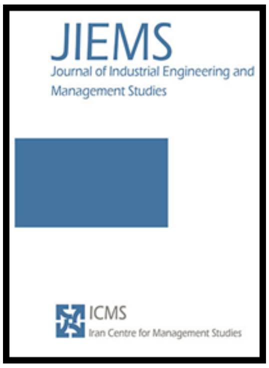
JIEMS

Journal of Industrial Engineering and Management Studies

Vol. 6, No. 1, 2019, pp. 79-110

DOI: 10.22116/JIEMS.2019.87661

www.jiems.icms.ac.ir



A cost sharing-based coordination mechanism for multiple deteriorating items in a one manufacture-one retailer supply chain

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Abstract

In this paper, an inventory model for deterioration items in a two-echelon supply chain including one retailer and one manufacturer is proposed by considering the stock and price dependent demand and capacity constraint for holding inventories. First, the model is presented as a leader-follower game in which the manufacturer announces wholesale prices. Second, the retailer decides for the order quantity and price of the items based on the wholesale prices. Then, by introducing the integrated model of the supply chain, a cost-sharing contract is applied to coordinate the manufacturer and the retailer. On the other hand, by using the convergence properties of the model and proving non-concavity of the problem, a meta-heuristic algorithm, namely iterative local search (ILS) is proposed to solve the models. The results show the determinant role of the capacity constraint on the optimal decisions and the ability of the proposed contract to coordinate the supply chain. Moreover, it is shown that the proposed algorithm outperforms the well-known interior point algorithm as the results of the initiations embedded in it for the special problem.

Keywords: Deterioration items; Stock and price dependent demand; Two echelon supply chain; Supply chain coordination; Cost sharing contract.

Received: September 2018-06

Revised: November 2018-12

Accepted: December 2018-17

1. Introduction

Many products are subject to degradation phenomena during time due to the reasons such as failure, decomposition, obsolesce or even reduction in the economic value. On the other hand, the rapid change in the technology level leads to the introduction of new products which make the older products Obsolete (Wee, 1993). Therefore in the inventory systems, the growth in the deterioration items are increasing and the inventory models of such products are in development (Bakker et al., 2012).

In this paper, the problem of inventory management for deteriorating items is studied by considering some assumptions about these items. We assume that the demand for the items is price and stock-level dependent. This type of demand function is common for items which will

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encourage more customers when they are supplied with higher amounts in stores' shelves. However, in this case, taking into account the deterioration rate will increase the deterioration related cost despite growing the demand. Hence, it is necessary to decide about the deteriorating items by considering the effects of stock-level on the demand and the deterioration cost. On the other hand, when demand is stock-dependent, there might be some cases for which exposing the maximum possible level of inventory are optimal. However, other restrictions such as the storage capacity prevent tending the inventory level to infinity. Therefore, studying the problem of inventory management for deteriorating items by considering the price and stock-level dependent demand, as well as the capacity constraint, make the analysis more realistic as done in this paper.

In the literature, the deterioration inventory models are categorized based on the type of demand functions, deterioration rates and other common assumptions in the inventory models such as allowing shortage, delay in payment, discount, time value of money and etc. (Backer, 2012). In this paper, we extend the literature in the deterioration inventory models by regarding new assumptions which are explained in the model representation section of the paper.

The demand for deterioration items has been studied with different assumptions such as constant demand, time-dependent demand, price dependent demand, stock dependent demand or combinations of them. In this regard, Mandal et al. (2006) have developed an inventory model by considering the capacity constraint of the warehouse for deterioration items with fixed demand. Teng et al. (2011) have proposed an inventory model for deterioration items by regarding stock-dependent demands, constant deterioration rates and delays in payment. The news vendor model for deterioration items with stock and price dependent demand has been developed by Sana (2011) and Sana (2013). Gosh, et al. (2015) have extended the model of Mandal et al. (2006) to the case where the demand is stock-dependent and have proposed an exact method for solving it after proving the model convexity. Singh (2016) has investigated the optimal order quantity of stock dependent deteriorating items by allowing the delay in payment. Lin et al. (2016) have considered pricing and inventory decisions of deteriorating items by regarding the price as a function of the demand and amount of deterioration. Sarkar et al. (2017) have been modeled an inventory system in the viewpoint of a retailer which aims to find the optimal order quantity and the optimal level of investment such that the deterioration rate is decreased. In the present paper, the problem of pricing and inventory management of a one-retailer and one-manufacturer supply chain is studied by considering the demand as a function of price and inventory level. The retailer aims to maximize its profit by choosing the proper value for price and order quantity of items. These decision variables together the deterioration rate of products specify the demand rate of products at any time. This type of demand function seems to be more general in real-world problems, and such assumptions make the model more applicable.

In the context of the deterioration items, some papers studied the multiple items (Akçay et al., 2010; Maiti and Maiti, 2009 and Gosh et al., 2015). In this case, it is clear that the storage capacity has a determinant role in the optimal solution. Some papers such as Gosh et al. (2015) and Tiwari et al. (2018) considers the capacity constraint for deterioration items in our models. Adding this constraint increases the complexity of the model. Therefore, finding the closed-form solution of the models would be more difficult. In this paper, the capacity constraint is also regarded in the models of the deterioration items, and as shown in the following, this constraint is not a convex constraint, and the optimality of the solutions couldn't be proved. Therefore, we utilize a Meta-Heuristic algorithm to extract the solution of the models after providing some properties for special cases.

It is notable that the idea behind the integration of a supply chain is that considering a set of players as a whole could lead to the decisions that improve the performance of the overall system (Frazelle, 2002). However, without a coordination mechanism which provides enough motivations for all members, there is no guarantee that the integrated decisions are implemented in practice. Moreover, due to the specific characters of the deterioration items, there is more conflict between members of a supply chain concerning deteriorated items (Lin et al., 2010). In the context of the supply chain management, coordination mechanism such as the supply chain contracts, the information technology, the information sharing mechanism, and the joint decision making could be referred (Kanda and Deshmukh, 2008). In the inventory problems, the contract based coordination mechanism have found many applications and among such contracts, the wholesale price contract, the buy-back contract, the revenue sharing contract, the cost-sharing contract, the quantity flexibility contract, and the discount contract could be mentioned (Kanda and Deshmukh, 2008). In the present paper, the cost-sharing contract is considered as the coordination mechanism of a one-retailer-one-manufacturer supply chain. In this mechanism, the manufacturer is committed to paying a part of its revenue to the retailer in order to motivate it for buying more items, and by doing so, the benefits of the supply chain as a whole is improved.

Several studies have investigated the integration and coordination of the deterioration items supply chain. Wu and Sarker (2013) have used an integrated model for a two-echelon supply chain comprising one manufacturer and multiple buyers. They have assumed the same production cycle and the replenishment time for the manufacturer and the buyers. Chen and Chang (2010) have studied the problem of determining the price, the replenishment cycle time and the number of transportation for multiple deterioration items with price and time-dependent demand in a one-manufacturer- multiple-retailers supply chain. Chen and Wei (2012) have examined the effect of three coordination contracts namely the revenue sharing contract, the price based contract and the combinations of them. Chen (2016) have developed a coordination model for a one-manufacturer – one-retailer supply chain by regarding the decisions about the price, the quantity order and the recycling times of deterioration products. Kaasgari et al. (2016) have modeled a supply chain including one vendor and multiple buyers in which the vendor manages the inventory decisions and a discount policy is utilized to sell the items before deterioration of them. Another reviewed study is the work of Pandey et al. (2017) which has proposed a discount contract proportional to the number of deterioration items purchased. In the considered problem of the current study, the cost-sharing mechanism is a good choice for coordinating the supply chain. Here, we have multiple items and a closed-form solution couldn't be obtained to interpret the parameters-based contract such as the discount contract. This contract distributes the resulting surplus gain of the integration between the manufacturer and the retailer in a fair manner. Furthermore, we show under what conditions the integration and coordination of the considered supply chain are noteworthy. For example, we will answer the questions of whether the storage capacity of the retailer emphasizes the importance of the coordination between the retailer and the manufacturer or what strategies the manufacturer could follow to motive the retailer when the coordination has not any significant outcomes for the manufacturer and the retailer.

In Table 1, the differences between some of the related papers in the literature and the proposed paper are outlined.

Table 1. Taxonomy of the related articles

Paper	Number of items		Storage capacity	Coordination mechanism		Demand			Item		Supply chain echelons				Replenishment cycle time	
	Multiple items	Single item		Cost sharing	Other	Fixed	Stock dependent	Price dependent	Deterioration items	Non deterioration items	Supplier	Manufacturer	Distributer	Retailer	Discrete value	Continues
Teng and Chang, (2005)		✓					✓	✓	✓					✓		✓
Chen et al., (2006)		✓			✓	✓			✓			✓		✓		✓
Lin and Lin, (2007)		✓				✓			✓			✓		✓		✓
Roy and Chaudhuri, (2009)		✓					✓		✓					✓		✓
Lin et al., (2010)		✓			✓	✓			✓		✓			✓	✓	
Sana, (2010)		✓					✓		✓					✓		✓
Wang et al., (2011)		✓				✓			✓			✓	✓	✓	✓	
(Wu and Sarker, 2013)		✓						✓	✓			✓		✓	✓	
(Yan et al, (2011)		✓				✓			✓			✓		✓		✓
Giri and Bardhan, (2012)		✓		✓				✓	✓			✓		✓	✓	
Ghosh et al. (2015)	✓		✓				✓		✓					✓		✓
Chen, (2016)	✓				✓			✓	✓			✓		✓		✓
Lin et al., (2016)		✓					✓	✓	✓					✓	✓	
Pandey, (2017)		✓						✓	✓					✓		✓
Sarkar et al., (2017)		✓					✓	✓	✓					✓		✓
Chen, (2017)	✓							✓	✓			✓		✓	✓	
Chen and Bidanda, (2018)		✓			✓		✓		✓			✓		✓		✓
Tiwari et al., (2018)		✓	✓				✓	✓	✓		✓			✓		✓
Pakhira et al., (2018)	✓				✓	✓				✓	✓			✓		✓
The present paper	✓		✓	✓			✓	✓	✓			✓		✓		✓

As shown in Table 1, the present study differs from the previous studies either in its assumptions or its complexity. In this regard, most studies considered the single-item models and those who regarded multiple items usually neglect the capacity storage as a determinant factor for order quantity variables and optimal decisions of models. Moreover, the coordination mechanism of supply chain members seem to be a recent issue for the inventory management problems of the deterioration items. Also, assuming the price and stock level dependent demand though makes the problem more realistic but increases the complexity of the model and therefore this assumption was neglected in most of the studies. Finally, some studies which are relevant to the present paper considered the replenishment cycle time as a discrete variable to simplify the model and reduce the complexity of it in discussing the convexity properties. We don’t use such simplification and instead provide in-depth analysis about the replenishment cycle time of the retailer. Generally, the present study is distinguished from the mentioned studies or extend them by considering the additional assumption shown in Table 1.

The rest of the paper is organized as follow. In Section 2, the models of the paper are illustrated. The numerical results are described in Section 3. Finally, in Section 4, the conclusions of the paper, the managerial insights of the paper and some suggestions for the future studies are presented.

2. Model

We study a one-retailer-one-manufacturer supply chain in which deterioration items are produced. For this supply chain, the proposed models are developed in two cases. In the first case, a one-item inventory model is generated and in the second case, the first model is extended to the multiple-item inventory model. Also, each case consists of three stages. The first stage includes the decisions of the retailer which aims to find the replenishment cycle time and the retail price such that its profit function is maximized. Base on the first stage results, the manufactures maximize its profit function by setting its wholesale price in the second stage. Furthermore, the integrated model of the supply chain is developed in the third stage and a cost-sharing contract is introduced as the coordination mechanism of the supply chain.

The following assumptions are considered throughout the paper:

- The demand for items or products is price and stock dependent.
- The stock out aren't allowed.
- The manufacturer uses the lot-for-lot policy for providing the retailer orders.
- The manufacturer doesn't hold any inventory.
- The deterioration rate of the items is fixed.
- The retailer has limited shelf space for exposing the items.

Also, the notations of the paper are as the following:

Parameter	*Description*
c_m	the marginal cost of the manufacturer per each unit of the item
c_r	the ordering cost of the retailer per each order
H	the holding cost of the retailer per unit quantity of the item per time unit
α	the market scale of the item in the demand function
β	the retail price elasticity of the item in the demand function
δ	the stock level elasticity of the item in the demand function
θ	the deterioration rate of the item
υ	the bargaining power parameter of the retailer
V	the storage capacity of the retailer
U	the required storage capacity per unit quantity of the item
T_l	the minimum value for the replenishment cycle time of the item

Variable	*Description*
P	the retailer selling price per unit quantity of the item
W	the manufacture wholesale price per unit quantity of the item
Q	the order quantity of the retailer for the item
T	the replenishment time of the retailer for the item
HC	the retailer holding cost
π_r^d	the retailer profit in the non-integrated model
π_m^d	the manufacturer profit in the non-integrated model
π_s	the profit of the supply chain as a whole
π_r^c	the profit of the retailer in the integrated model
π_m^c	the profit of the manufacturer in the integrated model
π_r	the distributed profit of the retailer based on the cost-sharing contract
π_m	the distributed profit of the manufacturer based on the cost-sharing contract
ϕ	The side payment paid to the retailer by the manufacturer

It is notable that in the multiple-item inventory model, the subscript *i* is added the notation above to denote the related notation for item *i*.
When the demand is dependent on the stock level and the price, the demand function could be described by Eqn. (1) where $0 < \delta < 1$, $\alpha, \beta > 0$ (Urban, 2005):

$$D(p, I(t)) = \alpha - \beta p + \delta I(t) \tag{1}$$

According to Eqn. (1), the demand has a positive relationship with the inventory level $I(t)$ and a negative relationship with price *p*. considering the deterioration rate θ, the instantaneous inventory $I(t)$ at any time *t* is governed by the differential equation Eqn. (2):

$$\frac{d\,I(t)}{d\,t} = -D - \theta . I(t) \tag{2}$$

The differential equation Eqn. (2) denotes that the response to the demand and the deterioration of the item decrease the inventory level at any time. By replacing *D* in Eqn. (1) into Eqn. (2), the changes of inventory over time is described as a function of the price and the inventory level in Eqn. (3). Also, Figure 1 shows the inventory level over time.

$$\frac{d\,I(t)}{d\,t} = -(\alpha - \beta p) - (\theta + \delta) . I(t) \tag{3}$$

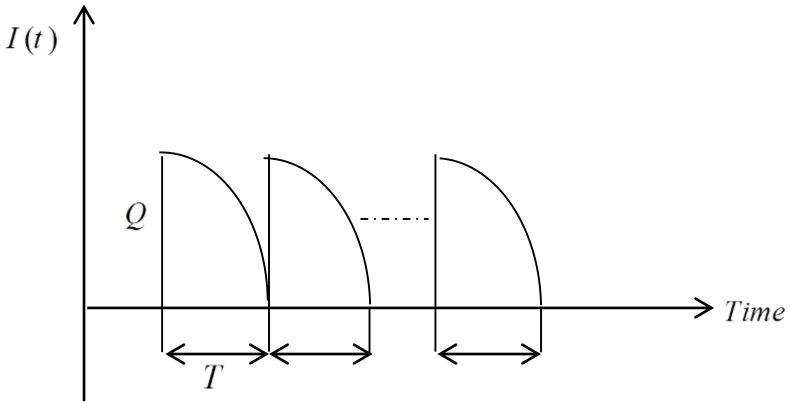

Figure 1. The inventory level over time

Regarding Eqn. (3), Eqn. (4) is used to characterize the replenishment cycle time *T*:

$$T = \int_Q^0 d\,t = \int_Q^0 \frac{d\,I(t)}{-(\alpha - \beta p) - (\theta + \delta) . I(t)} = \int_0^Q \frac{d\,I(t)}{(\alpha - \beta p) + (\theta + \delta) . I(t)} = \frac{1}{(\theta + \delta)} . \ln\left(\frac{(\alpha - \beta p) + (\theta + \delta) Q}{(\alpha - \beta p)}\right) \tag{4}$$

Now, the variable *Q* could be specified as Eqn. (5) by rewriting Eqn. (4):

$$Q = \frac{\alpha - \beta p}{\theta + \delta}(e^{(\theta+\delta)T} - 1) \tag{5}$$

Therefore, taking into account Fig 1 and Eqns. (1)-(5), the retailer holding cost is given by Eqn. (6):

$$HC = \int_0^T I(t).dt = -\int_Q^0 \frac{I(t).dI}{(\alpha - \beta p) + (\theta + \delta).I(t)} = \frac{\alpha - \beta p}{(\theta + \delta)^2}(e^{(\theta+\delta).T} - (\theta + \delta)T - 1) \tag{6}$$

2.1. The single-item inventory models

2.1.1. The single-item non-integrated inventory models

In this case, by regarding the Eqn. (5) and Eqn. (6), the non-integrated profit function of the retailer π_r^d is be described by Eqn. (7). This profit consists of the sales revenue minus the ordering cost and the holding cost of the retailer.

$$\begin{aligned} \pi_r^d &= \max \ \frac{1}{T}\left((p - w).Q - c_r - h.\frac{\alpha - \beta p}{(\theta + \delta)^2}.(e^{(\theta+\delta).T} - (\theta + \delta)T - 1) \right) = \\ &\max \ \frac{1}{T}\left((p - w).\frac{\alpha - \beta p}{\theta + \delta}(e^{(\theta+\delta)T} - 1) - c_r - h.\frac{\alpha - \beta p}{(\theta + \delta)^2}.(e^{(\theta+\delta).T} - 1) \right) + h.\frac{\alpha - \beta p}{(\theta + \delta)} \end{aligned} \tag{7}$$

Also, the storage capacity is given by equation (8):

$$u.Q = u.\frac{\alpha - \beta p}{\theta + \delta}(e^{(\theta+\delta)T} - 1) \le V \tag{8}$$

In the other so, the profit function of the manufacturer π_m^d which includes the sale income minus the production cost is according to Eqn. (9):

$$\pi_m^d = \max \frac{1}{T}(w - c_m).Q = \frac{1}{T}(w - c_m)(\frac{\alpha - \beta p}{\theta + \delta}(e^{(\theta+\delta)T} - 1)) \tag{9}$$

It is notable that the cost function components of the retailer and the manufacturer are based on the proposed inventory models of Saha and Goyal (2015). Based on the above discussion, the non-integrated leader-follower model is declared as the following:

The non-integrated leader-follower model of the two-echelon supply chain for a single deteriorating item:

$$\begin{aligned}
&\max_{w} \quad \pi_m^d = \frac{1}{T}(w - c_m).Q = \frac{1}{T}(w - c_m)\left(\frac{\alpha - \beta p}{\theta + \delta}(e^{(\theta+\delta)T} - 1)\right) \\
&s.t. \\
&\quad c_m \le w \le \frac{\alpha}{\beta} \\
&\quad \max_{T,p} \quad \pi_r^d = \frac{1}{T}\left((p - w).\frac{\alpha - \beta p}{\theta + \delta}(e^{(\theta+\delta)T} - 1) - c_r - h.\frac{\alpha - \beta p}{(\theta + \delta)^2}.(e^{(\theta+\delta).T} - 1) \right) \\
&\qquad + h.\frac{\alpha - \beta p}{(\theta + \delta)} \\
&\qquad\quad s.t. \\
&\qquad\qquad w \le p \le \frac{\alpha}{\beta} \\
&\qquad\qquad T \ge T_l \\
&\qquad\qquad u.\frac{\alpha - \beta p}{\theta + \delta}(e^{(\theta+\delta)T} - 1) \le V
\end{aligned} \tag{10}$$

In this model, the constraints $c_m \le w \le \frac{\alpha}{\beta}$, $w \le p \le \frac{\alpha}{\beta}$ and $T \ge T_l$ are logical thresholds or valid inequality for the variables. Here, the parameter T_l is a technical constraint that specifies the lowest value of the replenishment cycle time.

The common procedure for solving the shown model in (10) is as follow: First, the retailer finds the optimal value of the price *p* and *T* as a function of *w*. Then, the manufacturer substitutes the parametric optimal value of *T* and *p* in its profit function to find the optimal value of *w*. Finally, after determining the optimal value of *w*, the optimal value of *p* and *T* will be determined accordingly. Therefore, to explore the solution, the model of the retailer should be optimized firstly. To do so, the profit function of the retailer in Eqn. (7) is rewritten as Eqn. (11):

$$\pi_r^d = \frac{1}{T.(\theta+\delta)^2}\Big(((p - w)(\theta+\delta) - h))(\alpha - b.p)(e^{(\theta+\delta)T} - 1) - c_r\Big) + h.\frac{\alpha - \beta p}{(\theta+\delta)} \tag{11}$$

It is clear from Eqn. (11) that for values of *p* that inequality $(p - w)(\theta+\delta) - h \ge 0$ $\Rightarrow p \ge w + \frac{h}{\theta+\delta}$ is hold, the objective function in Eqn. (11) is an ascending function of the variable *T*. So, the optimal value of *T* that maximize the objective function will be obtained when $T \to \infty$ (The proof is shown in Appendix (A)) and this is equivalent to infinitive order quantity based on Eqn. (5). In this regard, one could rearrange the demand function such that some restrictions on the effect of the stock level are imposed. However, doing so makes the integral and differential equations of the model very complicated such that the closed form representation of the models is very difficult. Hence, the assumed demand function in this paper and even other common stock dependent demand functions in the literature inherit such unrealistic behavior. This is a note which couldn't be neglected in the pricing and inventory studies with stock and price dependent demand function. Therefore, without loss of generality, we turn our attention to the case where the storage capacity acts as a restriction and don't allow *T* to tend infinity. Also, this is the reason for our claim which states the capacity storage has a determinant role in the optimal decisions of the model. Based on the above discussion, *Proposition (1)* is stated as below:

Proposition (1): If $w+\frac{h}{\theta+\delta}\le\frac{\alpha}{\beta}$, then the inequality $p>w+\frac{h}{\theta+\delta}$ is valid for partitioning the solution space (Because $p\le\frac{\alpha}{\beta}$). Now, if we consider the constraint $p\ge w+\frac{h}{\theta+\delta}$ in the model, the storage capacity constraint will be active at the optimal solution and Eqn. (12) is valid.

$$u.\frac{\alpha-\beta p}{\theta+\delta}(e^{(\theta+\delta)T}-1)=V\Rightarrow T_1^*=\frac{1}{(\theta+\delta)}Ln\left(\frac{(\theta+\delta).V}{u.(\alpha-\beta p)}+1\right) \tag{12}$$

Therefore, by substituting T_1^* into the profit function of the retailer (Eqn. (12)), $\pi_r^d(T_1^*(\mathrm{p}),\mathrm{p})$ will be a continuous and bounded function of *p*. So in this case, the optimal value of *p* is among the extreme values of *p*. However *p*=*w* and $p=\frac{\alpha}{\beta}$ as the extreme value of *p* are never optimal because from Eqn. (7), it is clear that such values lead to negative values for the retailer profit function. So, *Proposition* (2) of the paper is stated as the following:

Proposition (2): If the constraint $(p-w)(\theta+\delta)-h\ge 0\Rightarrow p\ge w+\frac{h}{\theta+\delta}$ is considered for the non-integrated profit function of the retailer, the optimal value of *p* is determined among the extreme values of *p* including $p_1=w+\frac{h}{\theta+\delta}$ and $\frac{\partial\pi_r^d(T_1^*(p),p)}{\partial p}(p_2)=0\,|w+\frac{h}{\theta+\delta}\le p_2\le\frac{\alpha}{\beta}$. Each of this extreme values that maximizes $\pi_r^d(T_1^*(p),p)$, is the optimal *p* i.e., p^*. Also, by replacing p^* into Eqn. (12), the optimal value of the replenishment cycle time is determined.

In the other so, if the constraint $(p-w)(\theta+\delta)-h\le 0$ is imposed in the problem, it is proved that π_r^d in Eqn. (11) is a concave function of *p* and *T*. The proof of this is presented in Appendix (B). However in this case, one could reject the convexity of the storage capacity using the numerical examples. Hence to find the optimal solution, first, the necessary condition of the optimality i.e. $\frac{\partial\pi_r^d}{\partial p}=0$ and $\frac{\partial\pi_r^d}{\partial T}=0$ is examined and if the resulting solution satisfies the constraints, then the optimal solution has been found. Otherwise, again the extreme values of the variables are examined. In this case, the extreme values of the variables are as follow:

$$\begin{cases} T_1 = T_l\,;\, p_1 \,|\, \dfrac{\partial \pi_r^d}{\partial p}(p_1,T_1) = 0;\, w \le p_1 \le w + \dfrac{h}{\theta+\delta} \\ T_2 = T_l\,;\, p_2 = \dfrac{\alpha}{\beta} - \dfrac{(\theta+\delta).V}{u.\beta.(e^{(\theta+\delta).T_1}-1)} \,|\, w \le p_1 \le w + \dfrac{h}{\theta+\delta} \\ T_3 = \dfrac{1}{(\theta+\delta)} Ln\left(\dfrac{(\theta+\delta).V}{u.(\alpha-\beta p)}+1\right),\, p_3 \,|\, \dfrac{\partial \pi_r^d}{\partial p}(p_3,T_3) = 0; w \le p_3 \le w + \dfrac{h}{\theta+\delta}, T_3 \ge T_l \\ T_4 \,|\, \dfrac{\partial \pi_r^d}{\partial T}(p_4,T_4) = 0\,;\; T_4 \ge T_l\,,\, p_4 = \dfrac{\alpha}{\beta} - \dfrac{(\theta+\delta).V}{u.\beta.(e^{(\theta+\delta)T}-1)} \end{cases} \tag{13}$$

Based on the above discussion, *Proposition* (3) is represented as follow:

Proposition (3): If the constraint $p \le w + \dfrac{h}{\theta+\delta}$ is considered for the non-integrated profit function of the retailer and solving the system equation $\left\{ \dfrac{\partial \pi_r^d}{\partial p} = 0, \dfrac{\partial \pi_r^d}{\partial T} = 0 \right.$ results in feasible values for p and T then the resulting solution is optimal, otherwise, the optimal solution is among the solutions shown in (13).

The above discussions show that the retailer couldn't find closed form solutions for p and T. Therefore the concavity of the manufacturer profit π_m^d isn't proved straightforwardly. So, to find the solution of the model shown in (10), an iterative search process as the following is suggested:

Algorithm (1): *The procedure for solving the non-integrated leader-follower model of the two-echelon supply chain for a single deteriorating item.*

Initiate $\mathit{convergence\ controller} = \varepsilon$, $\mathit{step\ size} = \gamma$, w=$c_m$ and $\pi_m^{d.best} = 0$.

Consider the constraint $p \ge w + \dfrac{h}{\theta+\delta}$ and find the values of T and p according to *Proposition* (2). In this case, denote the optimal value of the retailer profit function, T and p as $\pi_r^{d,I}$, T^I and p^I respectively.

Consider the constraint $p \le w + \dfrac{h}{\theta+\delta}$ and use *Proposition* (3) to find the values of p and T. The solution of this step is denoted by $\pi_r^{d,II}$, T^{II} and p^{II}.

If $\pi_r^{d,I} > \pi_r^{d,II}$ then set $T' = T^I$, $p' = p^I$ and $\pi_r^{d,best} = \pi_r^{d,I}$. Otherwise, set $T' = T^{II}$, $p' = p^{II}$, $\pi_r^{d,best} = \pi_r^{d,II}$ and go to step 5.

Calculate the manufacturer profit function in (9) by using T' and p' and denote it as $\pi_m^{d\,'}$.

If $\pi_m^{d\,'} < \pi_m^{d,best}$ and $\dfrac{\pi_m^{d\,'} - \pi_m^{d,best}}{\pi_m^{d,best}} < \varepsilon$, stop the algorithm. Otherwise, set $w = w + \gamma$, $\pi_m^{d,best} = \pi_m^{d\,'}$ and return to step 2.

2.1.2. The single-item integrated inventory model

In this case, the integrated model of the supply chain is introduced as (14):

The integrated model of the two-echelon supply chain for a single deteriorating item:

$$\pi_S = \max \ \frac{1}{T}\left((p-c_m).\frac{\alpha-\beta p}{\theta+\delta}(e^{(\theta+\delta)T}-1)-c_r-h.\frac{\alpha-\beta p}{(\theta+\delta)^2}.(e^{(\theta+\delta).T}-(\theta+\delta)T-1)\right)$$

$$\begin{aligned} & s.t. \\ & \quad w \le p \le \frac{\alpha}{\beta} \\ & \quad T \ge T_l \\ & \quad u.\frac{\alpha-\beta p}{\theta+\delta}(e^{(\theta+\delta)T}-1) \le V \end{aligned} \qquad (14)$$

The profit function π_s in (14) consists of the sales revenue minus the production cost, the holding cost, and the ordering costs. This arrangement shows the total profit of the supply chain as a whole. This model resembles the retailer non-integrated model only with the difference that here, the wholesale price *w* is replaced by the production cost c_m. Therefore, *Propositions* (2) and (3) could be used to find the solution of the integrated model.

2.2. The multi-item inventory models

The multi-item inventory models are obtained by summing up the terms of the single-item inventory models over items. Here, the subscript *i* is added to the notations of the single-item inventory models to indicate the related parameters and variables for the item *i*. Also, we assume that the number of items is *n*.

2.2.1. The multi-item non-integrated inventory models

The non-integrated bi-level inventory model of the multiple deteriorating items is given as follow:

The non-integrated leader-follower model of the two-echelon supply chain for multiple deteriorating items:

$$\max \ \pi_m^d = \sum_{i=1}^{n}\frac{1}{T_i}(w_i-c_{m,i}).Q_i = \sum_{i=1}^{n}\frac{1}{T_i}(w_i-c_{m,i})(\frac{\alpha_i-\beta_i p_i}{\theta_i+\delta_1}(e^{(\theta_i+\delta_i)T_i}-1))$$

$$\begin{aligned} & s.t. \\ & \quad w_i \ge c_{m,i} \\ & \quad \pi_r^d = \max\sum_{i=1}^{n}\frac{1}{T_i}\left((p_i-w_i).\frac{\alpha_i-\beta_i p_i}{\theta_i+\delta_i}(e^{(\theta_i+\delta_i)T_i}-1)-c_{r,i} \right. \\ & \qquad\qquad \left. -h_i.\frac{\alpha_i-\beta_i p_i}{(\theta_i+\delta_i)^2}.(e^{(\theta_i+\delta_i).T_i}-1)\right)+h_i.\frac{\alpha_i-\beta_i p_i}{(\theta_i+\delta_i)} \\ & \quad s.t. \\ & \qquad \sum_{i=1}^{n}u_i.Q_i \le V \Rightarrow \sum_{i=1}^{n}u_i.\frac{\alpha_i-\beta_i p_i}{\theta_i+\delta_i}(e^{(\theta_i+\delta_i)T_i}-1) \le V \end{aligned} \qquad (15)$$

$$w_i \le p_i \le \frac{\alpha_i}{\beta_i} \quad \forall i$$
$$T_i \ge T_{l,i} \qquad \forall i$$

As the single-item model, it is assumed that first, the manufacturer declares the wholesale prices of items to the retailer and then, the retailer decides to determine the retail price and the replenishment cycle time of the items. Also, the storage capacity and the thresholds for the variables are considered as the model constraint.

It should be pointed out that the concavity proof of the retailer profit function π_r^d in the non-integrated model is possible for some special cases. However, the convexity of the storage capacity isn't proved like the one discussed for the single-item inventory model. This entails the need for a heuristic or Meta heuristic algorithm exploring the optimal solution. However, if we fix the values of $T_1, T_2, ..., T_n$, then the capacity constraint in (15) is converted to a linear and convex constraint. Also, when $T_1, T_2, ..., T_n$ are regarded fixed, it is proved that the retailer profit function in (15) is a concave function of the variables $p_1, ..., p_n$. The proof of this is shown in Appendix (C). So, *Proposition* (4) is expressed as below:

Proposition (4): In the non-integrated model of the retailer for the multiple deteriorating items, if the values of $T_1, T_2, ..., T_n$ are fixed then the profit function of the retailer is concave with respect to the variables $p_1, ..., p_n$ and the storage capacity is a convex constraint.

Hence in an iterative algorithm that explained later, by initiating the values of $T_1, T_2, ..., T_n$ and changing them in the subsequent iterations, the *K.K.T* condition is held for obtaining the optimal values of $p_1, ..., p_n$. This condition will be as follow:

The K.K.T condition for the multi-item non-integrated inventory model of the retailer when the values of T_i's are fixed.

$$\begin{cases} g_{1,i} = p_i - \min(w_i - \frac{h_i}{\theta_i + \delta_i}, \frac{\alpha_i}{\beta_i}) & \forall i \\ g_{2,i} = w_i - p_i & \forall i \\ g_3 = \sum_i u_i \cdot \frac{\alpha_i - \beta_i . p_i}{\theta_i + \delta_i} (e^{(\theta_i + \delta_i)T_i} - 1) - V & \\ \nabla \pi_r^d (p, T) = \sum_i (\lambda_{1,i} . \nabla g_{1,i} + \lambda_{2,i} . \nabla g_{2,i}) + \lambda_3 . \nabla g_3 & \\ \lambda_{1,i} . g_{1,i} = 0 & \forall i \\ \lambda_{2,i} . g_{2,i} = 0 & \forall i \\ \lambda_3 . g_3 = 0 & \\ \lambda_{1,i}, \lambda_{2,i} \ge 0 & \forall i \\ \lambda_4 \ge 0 & \\ g_{1,i}, g_{2,i} \le 0 & \forall i \\ g_4 \le 0 & \end{cases} \tag{16}$$

The Iterative local search algorithm (*ILS*) is utilized to find the solution of the non-integrated model of the multiple deteriorating items in this paper. The *ILS* algorithm has operators which enable it for deep exploration of the solution space without getting stuck in the local optimal solutions (Lourenço et al., 2001). This algorithm is based on two operators, namely Local search (*LS*) and Perturbation operators (*PO*). The algorithm starts with an initial solution including m variables, namely $P_{initial} = (x_1, x_2, \ldots, x_m)$ and improves it in the subsequent iterations through *LS* and *PO* operators until reaching some predetermined final conditions. For the application of this algorithm, solving the problems of travel salesman problem (Katayama and Narihisa, 1999) and the quadratic assignment problem (Martin and Otto, 1995) are referred. The following Pseudo-code describes the *ILS* algorithm (Lourenço et al., 2003):

The meta-heuristic ILS algorithm:

1. Generate the initial solution P_{initial}.
2. Find the new solution P' using *LS* operator: P^*= local search (P).
3. Repeat the following stages for the predetermined iterations.
   a. Perturb the solution P^*: P'=perturb (P^*).
   b. Do a local search on P': P'^*=local search (P')
   c. if the profit of P'^* is more than that of P^*, then set $P^*=P'^*$

Furthermore, the Local search operator in the *ILS* algorithm is as follow:

Local search procedure in ILS Algorithm: local search (P):

1. Generate initial parameters *step*, $x_{\text{i,min}}$, $x_{\text{i,max}}$ and *time*.
2. Randomly select a set of variables $x_1, x_2, \ldots, x_m$ from P and place them in the set C_{ls}.
3. For each $x_i \in C_{ls}$ do the following stages:
   a. Set $x_i^{down} = x_i$; $\quad x_i^{up} = x_i$.
   b. Repeat while profit is increased, $x_i^{down} > x_{i,\min}$ and the current time is less than *time*
   c. - $x_i^{down} = x_i^{down} - step$
   d. Repeat while profit is increased, $x_i^{up} > x_{i,\max}$ and the current time is less than *time*
   e. - $x_i^{up} = x_i^{up} + step$
   f. Set x_{i} equal to the element of $\{x_i, x_i^{down}, x_i^{up}\}$ which yields to the best profit.

The local search operator is a simple search process that frequently increases or decreases the values in C_{ls} as long as the profit function is increased. This procedure has two controller parameters *step* and *time*. The *step* parameter shows the minimum change in x_{i} during the search process and the parameter *time* controls the maximum time of doing the search process. The interval of variable x_i in which the local search modifies the solution is specified as $[\mathrm{x}_{i,min}, x_{i,max}]$. Moreover, the random selection of C_{ls} ensures a better exploration of the solution space.
Also, the Perturbation operator in the *ILS* algorithm is as follow:

Perturbation procedure in the ILS algorithm: perturbation (P):

1. Initiate the parameter k and d_{i}
2. Determine C_{p} as a set containing k randomly selected element of C_{ls}
3. For each x_{i} in C_{p} do the following stage:
   a. Set Δx_i equals to a value randomly generated in the interval $[-d_i, d_i]$.

b. Modify x_i by $x_i = \min\{\max\{x_i + \Delta x_i, x_{i,\min}\}, x_{i,\max}\}$.

The *PO* operator has two parameters. The first one is *k* which determines the number of elements which are perturbed during the operator. The second parameter d_i shows the maximum allowable domain for the variable x_i in which could be perturbed.

The *ILS* algorithm proposed for the multi-item non-integrated inventory models consists of two loops. In the first loop, the value of $w_1, w_2, ..., w_n$ is changed. In this loop, when the local search operator is implemented, we need to evaluate the manufacturer profit function. To do so, the resulting values of $w_1, w_2, ..., w_n$ are fed as the input values for the second loop. Then in the second loop, the *ILS* algorithm is implemented by choosing the initial value for $T_1, T_2, ..., T_n$ and changing them in the *ILS* iterations. Furthermore, the *K.K.T* condition in (16) is used to find the optimal value of $p_1, ..., p_n$ when $T_1, T_2, ..., T_n$ are regarded fixed. For the aforementioned problem, we set $[p_{i,\min}, p_{i,\max}] = [w_i, \frac{\alpha_i}{\beta_i}]$, $T_{i,\min} = T_l$ and $T_{i,\max}$ equal to the maximum value of T_i reached based on the *Proposition*s (2) and (3). Also, the *ILS* parameters *step* and *time* and the perturbation parameters *k* and *d* could be determined using the trial and error or other methods such as the experimental design.

2.2.2. The multi-item integrated inventory model

By summing up the terms of the single-item integrated model over the items, the integrated model of multiple deteriorating items is identified as the following:

The integrated model of the two-echelon supply chain for multiple deteriorating items:

$$
\begin{aligned}
\pi_S = \max \sum_{i=1}^{n} \frac{1}{T_i} \Bigg(& (p_i - c_{m,i}).\frac{\alpha_i - \beta_i p_i}{\theta_i + \delta_i}(e^{(\theta_i+\delta_i)T_i} - 1) - c_{r,i} \\
& - h_i . \frac{\alpha_i - \beta_i p_i}{(\theta_i + \delta_i)^2}.(e^{(\theta_i+\delta_i).T_i} - 1) \Bigg) + h_i . \frac{\alpha_i - \beta_i p_i}{(\theta_i + \delta_i)} \\
s.t. \quad & \\
& \sum_{i=1}^{n} u_i . Q_i \le V \Rightarrow \sum_{i=1}^{n} u_i . \frac{\alpha_i - \beta_i p_i}{\theta_i + \delta_i}(e^{(\theta_i+\delta_i)T_i} - 1) \le V \\
& c_{m,i} \le p_i \le \frac{\alpha_i}{\beta_i} \quad \forall i \\
& T_i \ge T_{l,i} \qquad \forall i
\end{aligned} \tag{17}
$$

In this model, by replacing w_i with $c_{m,i}$, the *K.K.T* condition in (16) are valid for obtaining the optimal values of $p_1, ..., p_n$ when $T_1, T_2, ..., T_n$ are considered as fixed values. So to solve this model, the second loop of the *ILS* algorithm for the non-integrated model is, with the difference that here, we have $c_{m,i}$ parameter instead of w_i.

2.4. The integrated model

In the previous sections, it was explained how we could find the solutions of the non-integrated and the integrated models of the supply chain. The integrated model as the numerical results show has a better profit function than the non-integrated model. Also, one could rewrite the objective function in the integrated model (11) as Eqn. (18):

$$\pi_S = \max\ \frac{1}{T}\left((p-c_m).\frac{\alpha-\beta p}{\theta+\delta}.(e^{(\theta+\delta)T}-1)-c_r-h.\frac{\alpha-\beta p}{(\theta+\delta)^2}.(e^{(\theta+\delta).T}-(\theta+\delta)T-1)\right)=$$

$$\max\ \frac{1}{T}\left((p-w).\frac{\alpha-\beta p}{\theta+\delta}.(e^{(\theta+\delta)T}-1)-c_r-h.\frac{\alpha-\beta p}{(\theta+\delta)^2}.(e^{(\theta+\delta).T}-(\theta+\delta)T-1)\right) \tag{18}$$

$$+\frac{1}{T}(w-c_m)(\frac{\alpha-\beta p}{\theta+\delta}(e^{(\theta+\delta)T}-1))=\max\ \pi_r^d+\pi_m^d$$

From Eqn. (18), it is obvious that the optimal solution of the non-integrated model is also a solution to the integrated model. Hence, the optimal solution of the integrated model has an objective function equal to or greater than the solution of the non-integrated model.

Furthermore, it should be specified how the profit of the integrated model is distributed between the retailer and the manufacturer such that they motivate toward the adoption of the integrated model. As mentioned earlier, the cost-sharing contract is used for coordination of the supply chain in this paper. Such a coordination mechanism is based on the arrangement between partners which entails the partners receiving more profit as a result of the integration to pay the losses of the other partners and even more. Doing so, the losses partners are also committed to adopting the integrated decision. For example, if the results of the integrated model are such that the retailer should hold more inventories than that of the non-integrated approach, then the more holding cost of the retailer must be compensated by the manufacturer (Saha and Goyal, 2015). So, the coordination contract in this paper is based on the side payment paid by the gainer partner to the loser partner as a result of integration.

The proposed coordination mechanism is as follow. We consider the optimal wholesale price $w_1,...,w_n$ from the non-integrated model and $p_1,...,p_n,T_1,...,T_n$ from the integrated model and substitute these values to the profit functions of the retailer and the manufacturer. Doing so, the resulting profit of the retailer and the manufacturer in the integrated model is determined as π_r^c and π_m^c respectively. Then the side payment ϕ is introduced which should be paid by the manufacturer to the retailer. As mentioned, the side payment aims to ensure that the partners could obtain more profit than that of the non-integrated approach. So, if the profit of the retailer and the manufacturer after the side payment are indicated by π_r and π_m respectively, then the following equations ensure the desirability of the coordination mechanism by the partners:

$$\pi_r=\pi_r^c+\phi;\ \pi_m=\pi_m^c-\phi;\ \pi_r+\pi_m=\pi_S;\pi_r\geq\pi_r^d;\ \pi_m\geq\pi_m^d \tag{19}$$

We refer to the difference between π_S and $\pi_r^d+\pi_m^d$ as the synergy derived from the integration approach. The maximum value of ϕ is such that all synergy of the integration is distributed to the retailer i.e. $\pi_m=\pi_m^d$; $\pi_r=\pi_S-\pi_m^d$; $\phi=\pi_m^c-\pi_m^d$. On the other hand, if all the synergy is considered for the manufacturer, the side payment value is minimum and we have $\pi_r=\pi_r^d$; $\pi_m=\pi_S-\pi_r^d$; $\phi=\pi_r^d-\pi_r^c$. Therefore, the threshold $\pi_r^d-\pi_r^c\leq\phi\leq\pi_m^c-\pi_c^d$ could be taken into account for ϕ and this threshold satisfies the relations in Eqn. (19). The precise value of ϕ depends on the negotiation power of the partners. Actually, the more negotiation power of the retailer leads to more value for ϕ in practice.

In this situation, the Nash bargaining solution could be used as a fair solution for distributing the integration synergy between the partners. If the Nash bargaining solution is adopted, the distributed profit of the retailer π_r and that of the manufacturer π_m are based on the shown model in (12) (Saha and Goyal, 2015):

$$\max\ (\pi_r - \pi_r^d)^{\nu} (\pi_m - \pi_m^d)^{1-\nu}$$
$$s.t \tag{20}$$
$$\pi_r + \pi_m = \pi_S$$

In the Nash bargaining model, the parameter ν shows the relative bargaining power of the retailer such that the more value of ν indicates the more negotiation power of the retailer. Solving the above model results in the coordination solution as Eqn. (21):

$$\pi_r = \pi_r^d + \nu(\pi_S - (\pi_r^d + \pi_m^d));\ \pi_m = \pi_m^d + (1-\nu)(\pi_S - (\pi_r^d + \pi_m^d)) \tag{21}$$

It should be pointed out that the Nash bargaining solution which seems to be fair could ensure the long-term relationship of the partners. Moreover, if the Nash bargaining solution is accepted by the partner, the side payment ϕ will be as Eqn. (22):

$$\phi = \pi_r^d - \pi_r^c + \nu(\pi_S - (\pi_r^d + \pi_m^d)) \tag{22}$$

3. Numerical results

In this section, the numerical examples are provided to illustrate the results of the proposed models and algorithms. First, an example in the case of single deteriorating item is presented, and then the problem is extended to the case of multiple items. It should be pointed out that all the proposed algorithms have been coded and implemented using Matlab software v.R2015 on a computer with Core i5 CPU and 4.0 GB RAM.

3.1. single-item example

For the single deteriorating item example, the input parameters as Table 2 is considered. All parameters value are according to the paper of Gosh et al. (2015) except the parameter δ which shows the stock level elasticity of the demand function. The value of this parameter is similar to the one used in Saha and Goyal (2015).

Table 2. The numerical example of the single-item models

Parameter	Values	Parameter	Values
α	100	h	0.8
β	0.5	c_r	100
δ	0.3	c_m	80
θ	0.4	T_l	0.01
V	350	u	0.2

We use the step size $\gamma = 1$ and the convergence controller $\varepsilon = 0.0001$ in *Algorithm* (1) to solve the non-integrated model of this example. Also, the integrated model could be solved using the *Propositions* (2) and (3). The results of solving this problem have been presented in Table 3. The results in Table 3 show that the manufacturer profit in the integrated approach is more than the non-integrated approach while this is vice versa for the retailer. This entails a side payment from the manufacturer to the retailer to persuade it to adopt the optimal decisions of the integrated model. The threshold of the side payment accordingly will be $559.3 \le \phi \le 2284.5$. Moreover, if the bargaining power of the partner is assumed to be equal i.e., $\nu = 0.5$, then the side payment from the manufacturer to the retailer will be 851.55 based on the Nash bargaining solution in Eqn. (22).

Table 3. The results of the single-item inventory models

The variable	Non-integrated model	Integrated model
The retailer profit	1700	1140.7
The manufacturer profit	2642	3785.8
The supply chain profit	4342	4926.5
The wholesale price	144	-
The retail price	186.7	164.65
The order quantity	175	175
The replenishment cycle	4.23	2.95

3.2. Multi-item example

For the multiple deteriorating items, the example shown in Table 4 is regarded. In this example, an inventory system of three items with storage capacity V=1000 is studied.

Table 4. The numerical example of the multi-item models

Parameter	Product (1)	Product (2)	Product (3)
α_i	100	120	110
β_i	0.5	0.5	0.5
δ_i	0.3	0.5	0.4
θ_i	0.4	0.3	0.35
h_i	0.8	0.6	0.7
$c_{r,i}$	100	100	100
$c_{m,i}$	80	90	70
u_i	2	4	3
$T_{l,i}$	0.01	0.01	0.01

As mentioned earlier, the *ILS* algorithm is used to solve the multi-item models. In the *PO* operation of the algorithm, we set *k*=2 and $d_i = 0.01 \times [p_{i,\max} - p_{i,\min}]$ where d_i is the perturbation parameter of the regarded variable i.e. p_i. Also, the *step* parameter of the *LO* operator in the first and second loop of the algorithm is considered 1 and 0.01, respectively and *m*=3 is regarded for both loops. Finally, we adjust the *LO* operator to last at most 300 seconds (*time*=300). In this example, the main loop of the *ILS* algorithm was iterated 20 times to solve the non-integrated models of the supply chain. Doing so, the *ILS* algorithm resulted in $\pi_r^d = 2037, \pi_m^d = 9324$ and $\pi_S^d = 11361$. Also, the best value of the variables was according to Table 5 in this case.

Table 5. The results of the multi-item non- integrated inventory model using the *ILS* algorithm

variable	Product (1)	Product (2)	Product (3)
The wholesale price	163.7	210.5	185.19
The retail price	187.2	232.5	211
The replenishment cycle time	4.13	2.9	4.5
The order quantity	155.19	44.73	170.2

Moreover, the profit function of the manufacturer during the *ILS* iterations has been shown in Figure 2. This figure illustrates the ability of the *ILs* algorithm for improving the solution and exploring the solution space deeply. Next, the integrated model of the supply chain was solved using the second loop of the *ILS* algorithm and by replacing w_i with c_{mi}. The resulting profit of the supply chain in the integrated model was 15636. Also, the best value of the variables in the integrated model has been shown in Table 6. As expected, the profit of the supply chain in the integrated model is more than the non-integrated model. Finally, the cost-sharing mechanism is used to coordinate partners and aligns them toward the integrated approach. The Nash bargaining solution of the cost-sharing mechanism is as following: $\pi_r = 4174.5$ and $\pi_m = 11461.5$.

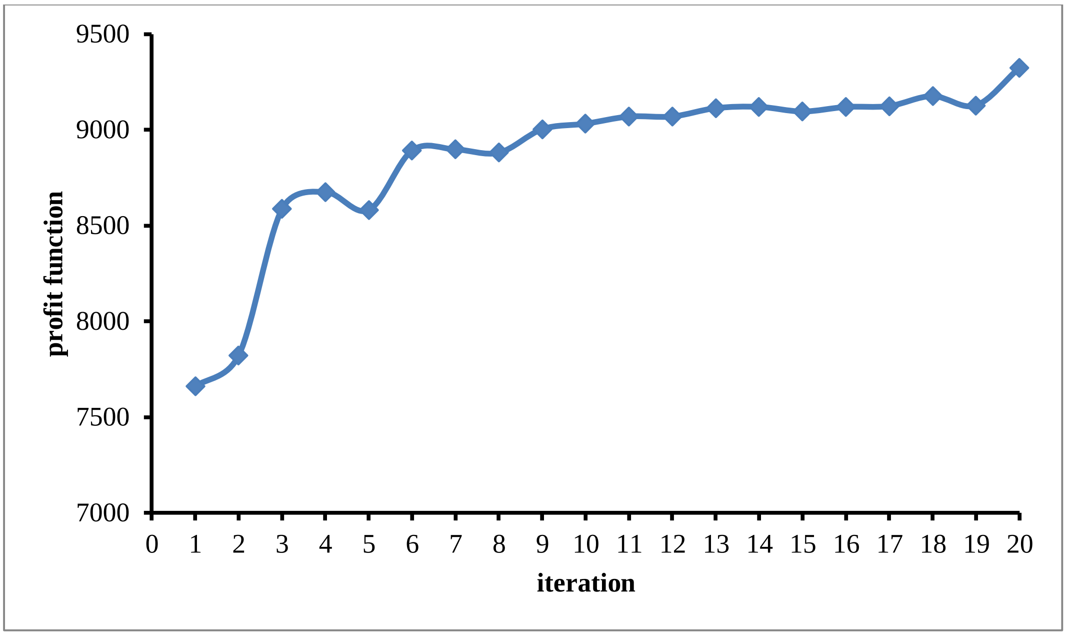

Figure 2. The profit function of the manufacturer during *ILS* iterations

Table 6. The results of the multi products example in case of integrated model using *ILS* algorithm

variable	Product (1)	Product (2)	Product (3)
The retail price	167.2	177.19	170.6
The replenishment cycle time	3.4	0.77	2.15
The order quantity	233.5	33.5	132.9

3.3. Sensitivity analysis

In this section, the sensitivity analysis on the parameters of the proposed models is done to investigate different changes in the results of models. To do so, we take into account the multi-item example in Table 4. First, we consider the capacity constraint, the production cost and the deterioration rate as the parameters which stand for the technical capabilities of the inventory system. Second, we change the demand function parameters in the integrated model to examine the effect of the market structure on the results of the model. It is notable that, when we do the sensitivity analysis on a parameter, the regarded parameter is multiplied in a coefficient *m* while other parameters are kept without change.

3.3.1. Changes in the storage capacity of the retailer (*V*)

In this section, we change the storage capacity and examine the effects of this change on the model results. The results of changes in the storage capacity have been shown in Table 7. It is notable that we add the columns "CS_d" and "CS_S" to the table to indicate whether the capacity constraint is active in the non-integrated model and the integrated model, respectively, or not. Moreover, Figure 3 is used to express a graphical explanation of the results.

Table 7. The models results versus the storage capacity

m	p1	p2	p3	T1	T2	T3	π_r^d	π_m^d	$\pi_r^d + \pi_m^d$	π_S	CSd	CSs
0.25	147.4	168.93	146.82	1.08	0.39	0.69	1964	8912	10876	10876	active	active
0.5	155	169.75	154.3	2.01	0.49	1.3	2020	9012	11032	13470	active	active
1	170.6	177.19	167.2	2.15	0.77	3.4	2037	9324	11361	15636	not-active	active
1.25	164.5	172.02	168.05	3.43	0.7	2.5	3443	11235	14678	20240	not-active	active
1.5	166.19	172.91	170.62	3.73	0.78	2.86	3422	12899	16321	22327	not-active	active
2	168.63	174.85	174.34	4.22	0.94	3.31	3800	13565	17365	26342	not-active	active

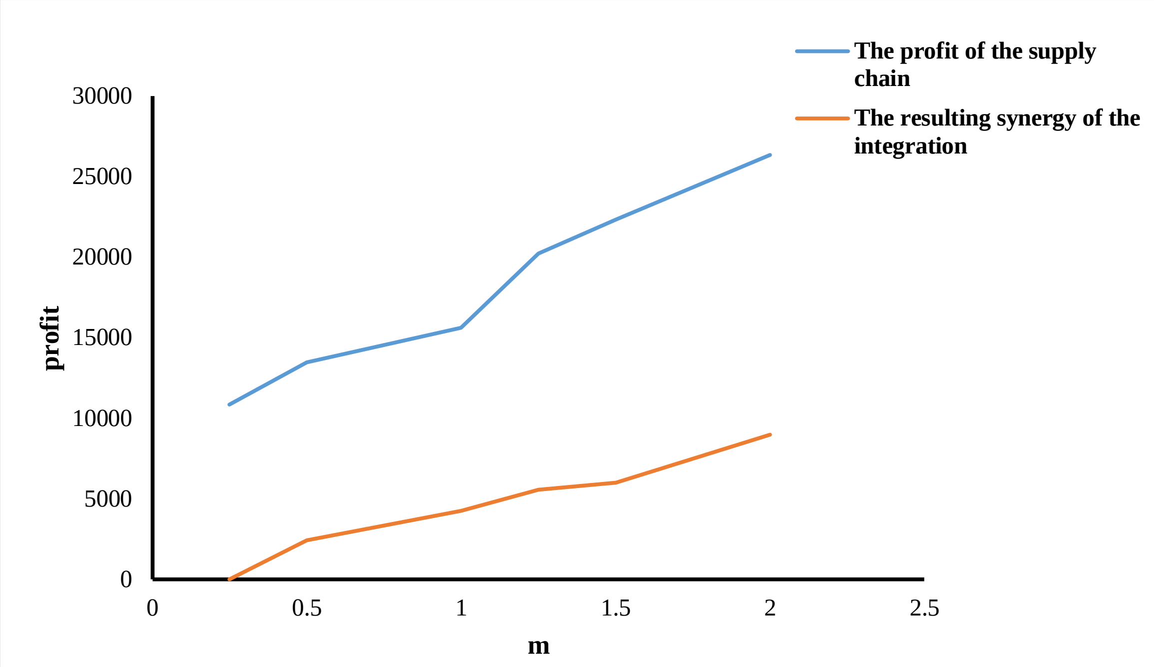

Figure 3. The profit and synergy of the integrated supply chain versus the storage capacity

The results of Table 7 and Figure 3 reflect the fact that when the capacity storage is increased, the coordination between the retailer and the manufacturer become more important. For example, when the capacity is set at the current value (V=1000) or more, the capacity constraint is not active in the non-integrated model. This means that the retailer is reluctant to fill its warehouse for the reasons such as the high holding cost or losses of the items due to the deterioration. However, in this case, the manufacturer could motivate the retailer to order more items and compensates its losses by a cost-sharing contract. On the other hand, when the capacity of the retailer is low for the manufacture's items, the coordination of the retailer and the manufacturer loses its importance. This fact suggests that the manufacturer discards the cooperation with such retailer or encourages it to increase its capacity for the manufacturer's items (For example by marketing strategies). Furthermore, this suggests that the manufacturer invests in the capacity of the retailer and whereby, develops its market share. Another noteworthy point is that even if the capacity of the retailer is active for the manufacturer's items, a coordination mechanism could improve the performance of the supply chain (For example when m=0.5 in Table 7). This could be due to the fact that by a coordination contract, the retailer changes the order quantities of the items which have more profit for the supply chain as a whole, though its warehouse remains in the full capacity again.

3.3.2. Changes in the production cost of the manufacturer (c_m)

By changing the production cost of the manufacturer, we expect that the total cost of the supply chain is reduced. Hence, the results of such changes have been illustrated in Table 8. Also, the graphical representation of the results is given in Figure 4.

Table 8. The models results versus the production cost

m	p_1	p_2	p_3	T_1	T_2	T_3	π_r^d	π_m^d	$\pi_r^d+\pi_m^d$	π_S	CS_d	CS_S
0.25	148.83	141.99	137.43	3.28	0.69	2.56	7034	20681	27715	28183	active	active
0.5	153.67	151.87	146.91	3.27	0.74	3.47	7171	15462	22633	24547	not-active	active
1	170.6	177.19	167.2	2.15	0.77	3.4	2037	9324	11361	15636	not-active	active
1.25	166.11	180.63	172.48	2.82	0.78	2.64	2130	9052	11182	15280	not-active	active
1.5	169.5	189.88	179.19	2.42	0.71	3.03	2280	8065	10345	12777	not-active	active
2	178.58	208.35	189.59	2.56	0.77	3.64	1674	6912	8586	8664	not-active	not-active

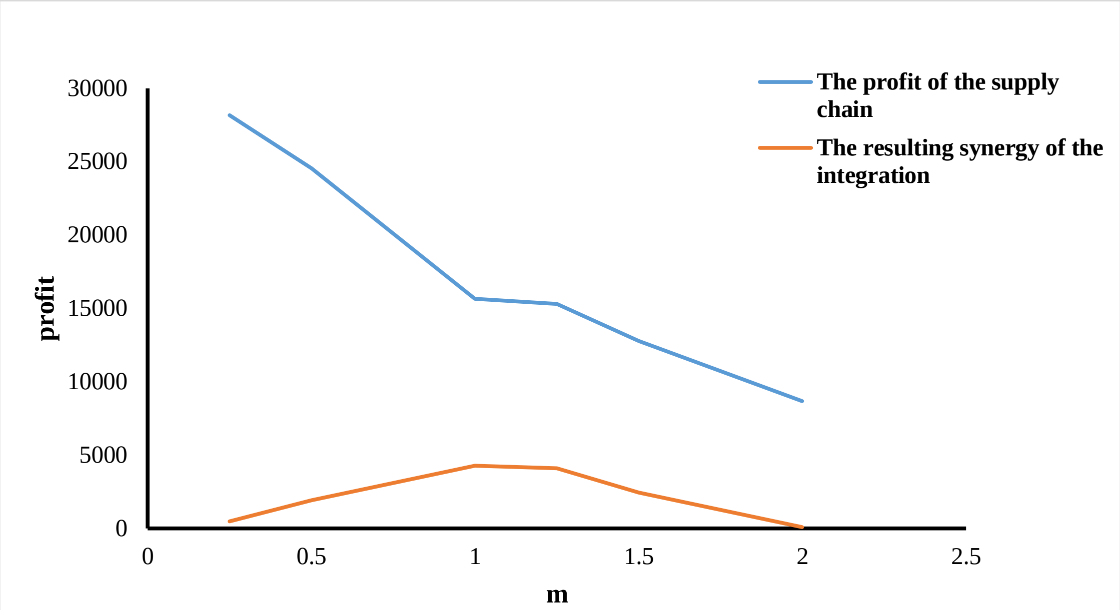

Figure 4. The profit and synergy of the integrated supply chain versus the production cost

The results of Table 8 and Figure 4 could be explained as the following. As the production cost is increased, the price of the items and their replenishment cycle time are increased. The overall effect of such changes is the reduction in the order quantity of the items. Therefore, by an increase in the production cost, the storage capacity is less binding and the coordination mechanism becomes less important. On the other hand, if the production cost is decreased, the order quantity of the items is increased in the non-integrated model. This increment is continued until the capacity constraint becomes active in the non-integrated model. If this occurs, then the retailer will purchase the maximum amounts of the items which fill the warehouse. Therefore, as much as the production cost decreases, the importance of the coordination contract will be reduced.

3.3.3. Changes in the deteriration rate of the items (θ)

The deterioration rate of the items leads to the loss of the items and imposes additional cost including the purchasing cost of the items. Clearly, decreasing the deterioration rate increases the profit of the supply chain. Furthermore, we investigate other effects of the deterioration rate on the model's results by doing the sensitivity analysis. These results are declared in Table 9 and Figure 5. As the results in Table 9 and Figure 5 show, the reduction in the deterioration rate of the items has a significant effect on the improvement of the supply chain profit. Also, the effects of cooperation between the manufacturer and the retailer are remarkable in this case. In this regard, the supply chain members could regard the investment in the reduction of the deterioration rate as a worthy strategy which could provide significant saving in the cost and improve the efficiency of the supply chain.

Table 9. The models results versus the deterioration rate

m	p_1	p_2	p_3	T_1	T_2	T_3	π_r^d	π_m^d	$\pi_r^d + \pi_m^d$	π_S	CS_d	CS_S
0.25	150.3	171.86	161.71	2.54	1	3	1931	13951	15882	22328	not-active	active
0.5	165.91	171.6	166.52	2.87	0.54	1.95	2250	12011	14261	20229	not-active	active
1	170.6	177.19	167.2	2.15	0.77	3.4	2037	9324	11361	15636	not-active	active
1.25	155.9	171.29	162.84	2.99	0.82	2.68	2108	9058	11166	15100	not-active	active
1.5	150.3	171.86	161.71	2.54	1	3	1762	8827	10589	14811	not-active	active
2	146.16	172.56	160.66	2.06	1.18	3.2	1917	8048	9965	14171	not-active	active

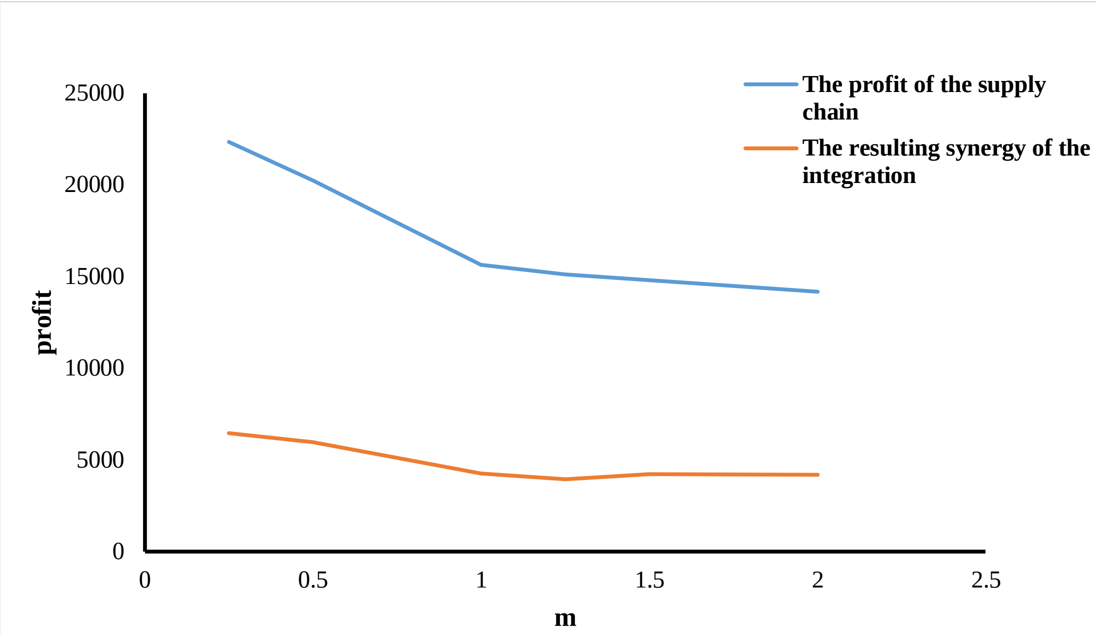

Figure 5. The profit and synergy of the integrated supply chain versus the deterioration rate

Also, we showed in the previous section that the high production cost decreases the demand, the order quantities of the items and the profit of the supply chain. Therefore, when the production cost is high, the reduction in the deterioration rate could compensate for the losses of the supply chain or vice versa. So, we have investigated the joint effects of the production cost and the deterioration rate on the performance of the supply chain in Figure 6 and Table 10. According to Figure 6, when the production cost is high, reductions in the deterioration rate could improve the profit of the supply chain. However, reduction in the production cost or the deterioration rate require some investments in the supply chain. The incurred cost of investments in addition to the saving results should be balanced properly to specify the required amount of investment. We didn’t include such an investment strategy in this paper, but this issue could be suggested for the future research.

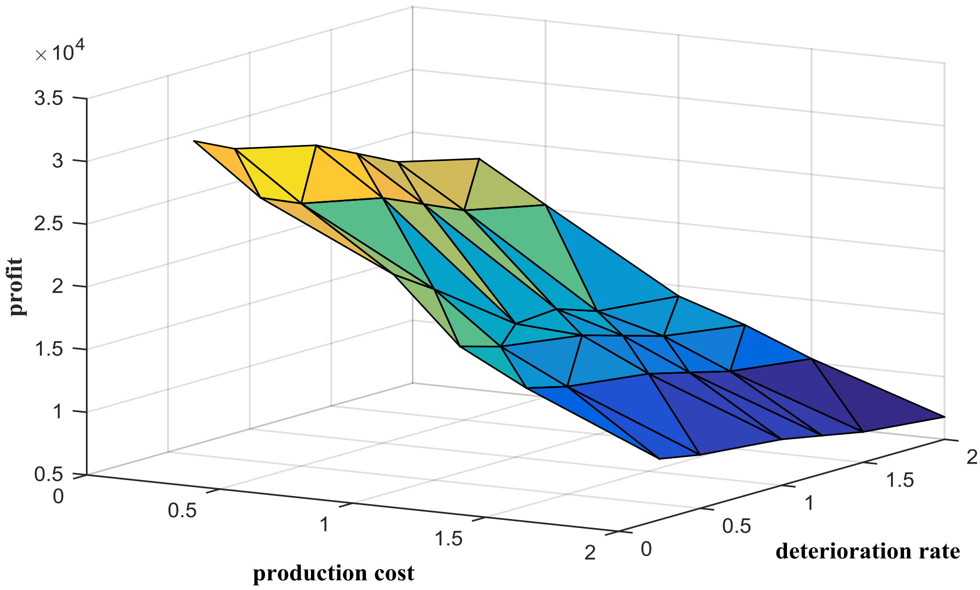

Figure 6. The joint effects of the production cost and the deterioration rate on the supply chain profit

Table 10. The models results versus the deterioration rate and the production cost

The multiplier of the production cost	**The multiplier of the deterioration rate**	π_S	**The multiplier of the production cost**	**The multiplier of the deterioration rate**	π_S
0.25	0.25	31278	1.25	0.25	17142
0.25	0.5	29740	1.25	0.5	16216
0.25	1	28183	1.25	1	15280
0.25	1.25	26608	1.25	1.25	14335
0.25	1.5	25022	1.25	1.5	13384
0.25	2	23446	1.25	2	12434
0.5	0.25	27314	1.5	0.25	14382
0.5	0.5	25939	1.5	0.5	13584
0.5	1	24547	1.5	1	12777
0.5	1.25	23140	1.5	1.25	11961
0.5	1.5	21725	1.5	1.5	11137
0.5	2	20321	1.5	2	10307
1	0.25	22328	2	0.25	9862
1	0.5	20229	2	0.5	9267
1	1	15636	2	1	8664
1	1.25	15900	2	1.25	8050
1	1.5	14811	2	1.5	7425
1	2	14171	2	2	6786

3.3.4. Changes in the market scale parameter (α_i)

The results of the changes in the market scale parameter are shown in Table 11. The results can be explained as follows: when the market scale is increased, the demand and the marginal profit per unit of the items are increased. So, if the price of the items is increased, the higher market size could compensate the effect of the price on the reduction of the demand. Therefore, the supply chain could benefit from increasing the prices.

Table 11. The integrated model results versus the market scale

m	p_1	p_2	p_3	T_1	T_2	T_3	Q_1	Q_2	Q_3	π_S
0.5	95.4	111.8	101.6	3.5	2.3	5.2	34.8	27.2	271	7919.8
1	170.6	177.19	167.2	2.15	0.77	3.4	170.2	44.73	155.19	15636
1.5	234.2	237	225.2	2.9	0.39	1.1	310.8	28	89	39228
2	298	296.5	276	2.5	0.25	0.61	346.4	25.3	63.4	72219

3.3.5. Changes in the retail price elasticity parameter (β_i)

The results of the changes in the price elasticity parameter are shown in Table 12. As the results illustrate, the profit and the optimal prices are decreased by increasing the retail price elasticity. Actually, when the price elasticity is increased, the marginal profit per each unit of the items is decreased, and consequently, the profit and the optimal prices of the items are decreased.

Table 12. The integrated model results versus the retail price elasticity of the demand function

m	p_1	p_2	p_3	T_1	T_2	T_3	Q_1	Q_2	Q_3	π_S
0.5	312.8	304	298.7	3.4	0.45	1.4	305.3	23.8	87.5	46762
1	170.6	177.19	167.2	2.15	0.77	3.4	170.2	44.73	155.19	15636
1.5	118.2	133.4	125.6	3	1.14	3.14	116.2	37.1	201	6271.7
2	94.5	110.3	100.8	2.5	1.5	4.1	37.3	28.1	253.3	2391

3.3.6. Changes in the stock level elasticity parameter (δ_i)

The results of the changes in the stock level elasticity parameter are shown in Table 13. The results could be explained in the following ways. By increasing the stock level elasticity, if we don't change the prices, then the demand for the items will be increased. Now, increasing the demands without changing the prices lead to more befits for the supply chain. Clearly, if we raise the prices, then we could reach the previous level of the demand and benefit from the increase in the prices. So, the overall effect of increasing the stock elasticity is that the profit of the supply chain will be increased.

Table 13. The integrated model results versus the stock level elasticity of the demand function

m	p_1	p_2	p_3	T_1	T_2	T_3	Q_1	Q_2	Q_3	π_S
0.5	137	173.4	164	4.3	0.67	2	552.3	26.9	102.3	13719
1	170.6	177.19	167.2	2.15	0.77	3.4	170.2	44.73	155.19	15636
1.5	167.5	181.5	174.5	2.8	0.86	2	187.4	40.8	136.2	17534
2	168	185.5	177.1	2.5	0.92	1.9	178.9	48.3	147.2	19402

Finally, we sum up the results of the sensitivity analysis on the market structure parameters in Figure 7. These results indicate that the supply chain profit is highly sensitive for the market scale and retail price parameters while it is moderately sensitive for the stock level elasticity parameter.

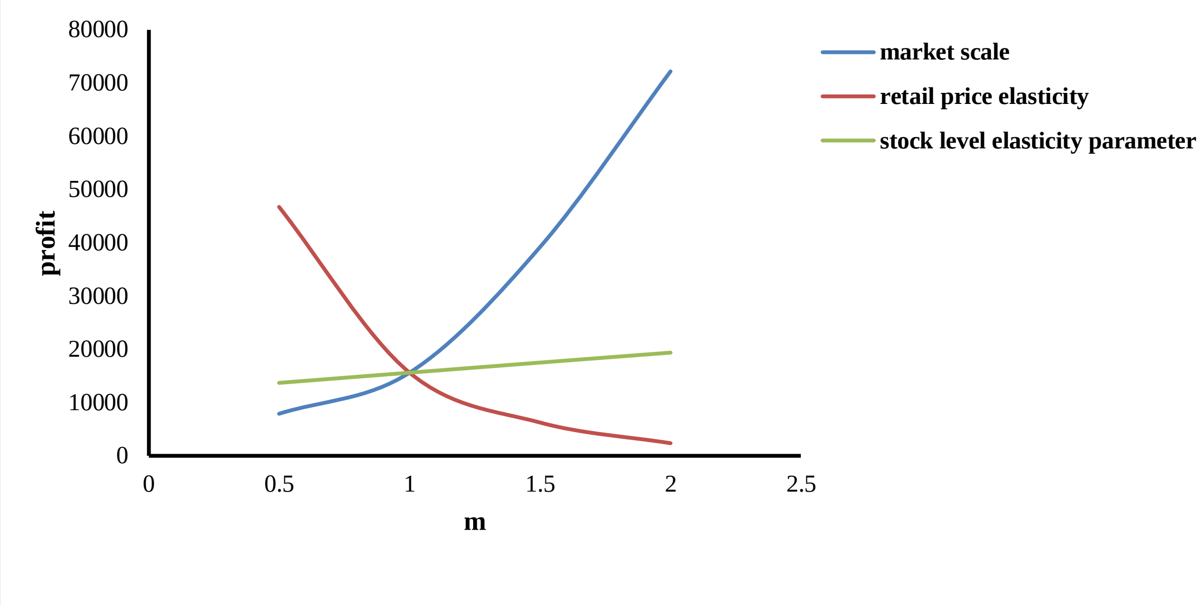

Figure 7. The profit of the supply chain versus the market structure parameters

3.4. Models and algorithms validation

To validate the proposed Models, some notes should be considered. First, the one-retailer-one-manufacturer models proposed in this paper are based on the two-echelon model proposed in Saha and Goyal (2015). Actually, the profit functions of the manufacturer and the retailer are the same in both papers. The profit function of the retailer includes the sales income, the purchasing cost, the holding cost, and the ordering cost. On the other hand, the profit function of the manufacturer in two studies comprises of the sales income minus the production cost and the manufacturer doesn't incur any holding cost. However, the Saha and Goyal (2015) considered one-item inventory models whilst the models of the present paper have been developed for multiple items which are more general in reality. Besides that, unlike the Saha and Goyal (2015), we include the deterioration rate and the capacity constraint in our analysis and such modifications are due to the aims of the current paper and reflect the novelty of it. It is notable that the equations of the paper which deal with the deterioration of items and the capacity constraint are in line with the paper of Gosh et al. (2015) who studied an inventory model for multiple deteriorating items in the viewpoint of just one retailer, not a supply chain.
In the non-integrated model, we show that if the values of $T_1, T_2, \ldots, T_n$ are fixed then the objective function of the retailer is concave and the capacity constraint is convex. So, the *K.K.T* condition is valid in this case. Moreover, the iterative algorithm of the non-integrated model searchs the values of $T_1, T_2, \ldots, T_n$ among many solutions and select the one which results in the best value of the objective function. So, the iterative algorithm of the non-integrated model is valid locally, though we couldn't ensure the optimality of it globally.
Furthermore, we could compare the results of the *ILS* algorithm with a well-known algorithm which is common in the optimization toolbox, i.e. the *interior point algorithm* . This algorithm could be found in the optimization toolbox of the Matlab software. In the following, we first generate some random problems based on Table 14. Second, we report the gaps of two algorithms for the randomly generated test problems in Table 15. The control parameters of the *ILS* algorithm are similar to Section (3.2). Note that the gap is calculated using Eqn. (23).

$$gap = \frac{\pi_{s,ILS} - \pi_{s,INP}}{\pi_{s,ILS}} \times 100 \tag{23}$$

Table 14. The value of the parameters in the randomly generated test problems

Parmeters	Value
α_i	uniform[50,150]
β_i	uniform[0.1,0.7]
θ_i	uniform[0.2,0.8]
h_i	uniform[0.1,0.9]
$c_{r,i}$	uniform[50,150]
$c_{m,i}$	uniform[20,100]
u_i	uniform[1,5]
$T_{l,i}$	0.01
V	300**n*

Table 15. The comparison of the *ILS* algorithm with the *INP* algorithm

Test problem	Number of items	The *ILS* solution		The *INP* solution		Gap (%)
		Time (Sec)	π_S	Time (Sec)	π_S	
Test problem 1	3	91.56	16342	101.2	15421	5.64
Test problem 2	4	97.68	18616	105.6	17899	3.85
Test problem 3	5	103.89	19635	112.5	19878	-1.24
Test problem 4	10	156.25	24312	184.23	22781	6.3
Test problem 5	15	189.1	27654	204.24	27545	0.39
Test problem 6	20	227.66	30015	297.45	29182	2.78
Test problem 7	25	285.48	34095	302.42	32891	3.53
Test problem 8	30	317.18	38202	419.15	37704	1.3

According to Table 15, the *ILS* algorithm outperforms the *INP* algorithm in the solving time and the value of the objective function. This is due to the fact that the *ILS* algorithm has more ability to get away from the local optimal solution than *INP* algorithm. Moreover, we discussed that when the values of $T_1, T_2, ..., T_n$ are fixed, the concavity of the objective function and the convexity of the constraint is proved straightforwardly. Hence, it is enough to search the optimal solutions through the feasible values of $T_1, T_2, ..., T_n$ and use the *K.K.T* conditions to find the best values of the other variables, namely $p_1, ..., p_n$. This initiation improves the performance of the *ILS* algorithm in term of the objective function and saves the time required to search the feasible region of the solutions.

4. Conclusion

In this paper, we studied an inventory system for deteriorating items in a two-echelon supply chain including one retailer and one manufacturer. This problem was investigated in the cases of single-item and multi-item inventory systems where the demand for the items was price and stock level dependent. First, the problem was modeled as a non-integrated leader-follower game in which the manufacturers declared the wholesale price of the items and then, the retailer decided the retail price and replenishment cycle time of the items by considering the manufacturer's wholesale prices.

Then, the supply chain was regarded as a whole, and the joint decisions of the supply chain were investigated for determining the optimal decisions which maximized the sum of the retailer and the manufacturer profit. Furthermore, a cost sharing-based coordination mechanism was proposed to distribute the resulting synergy or the surplus profit of the integration between the retailer and the manufacturer. This mechanism provided them the sufficient incentive for integration.

In the context of the supply chain management studies, the coordination mechanisms and contracts have been regarded as a base tool to ensure the integration of the supply chain. In these mechanisms, the independent nature of the partner is preserved, and sufficient incentives for the partners is provided to align them with the integrated decisions. The cost-sharing mechanism presented in this paper is among the mechanisms that could coordinate the supply chain. The simple form of such contract is suitable for complex problems in which, the lack of a closed-form solution prevents the use of other mechanisms such as discounted price or revenue- sharing contract. Moreover, the Nash bargaining solution which was used to determine the side payment in this mechanism could provide a fair distribution of the surplus profit between the partners. In the business context, such equitable distribution is somehow necessary for the long-term relationship of the partners and could increase the trust in the supply chain.

Moreover, we explored the convexity properties of the proposed model for the stock-level and price dependent demand function. We showed that assuming a linear relationship between the demand and the stock level could provide unrealistic results. In this regard, we reviewed some papers that examined a discrete replenishment cycle time of items for the sake of the simplicity or ignoring the infinite values of the quantity orders. Instead, we turn our attention to the case where the storage capacity limits the quantity order of the items. Doing so, we proved that the resulting models don't lead to concave objective functions and convex constraints for all values of the parameters. Hence, we proposed a heuristic algorithm and a Meta heuristic algorithm for solving the single-item and the multi-items models, respectively. These algorithms were based on some properties which facilitate exploring the solution space and improve the efficiency of the solutions. Such initiatives cause that the proposed algorithm outperforms a general and well-known algorithm namely, the interior point algorithm. The proposed algorithms were implemented in the numerical examples, and as we expected, the results indicated that the integrated supply chain has better performance than the non-integrated supply chain.

Several managerial insights could be derived from the results of the paper. First, it was shown that the common stock and price dependent demand functions in the aforementioned problem lead to unrealistic optimal decisions such that without considering the capacity storage, the order quantity that maximizes the retailer profit, tends to infinity. Second, we examined the importance of the coordination mechanism for the various cases. In this regard, we showed that when the storage capacity of the retailer is low, the importance of the coordination is reduced. This was explained due to the fact that in the case of the low capacity, the retailer will order the items in the amounts which make the warehouse full and the integration of the manufacturer with the retailer doesn’t change the retailer orders. Therefore in such cases, the manufacturer should seek strategies which increase the capacity of the retailer warehouse for the manufacturer's items such as marketing strategies. Doing so, the manufacturer will be able to extend or retrieve its market. Also, the results of the sensitivity analysis disclosed the positive effect of the market scale on the profit of the supply chain. Therefore, these strategies not only should target the retailer storage capacity but also should develop the market scale of the items. Third, we discussed that when the production cost is high, the order quantity of the items is decreased and the price of them is increased. The overall effect is such that the profit of the supply chain is decreased. The same analysis was true for the increase in the deterioration rate.

These parameters reflected the technical characters of an inventory system; therefore, we explored the joint effects of the production cost and the deterioration rate. In this context, it was shown that when one of the parameters is high, a reduction in the other parameter could compensate some losses of the other parameter. For example, when the production cost is high, the wholesale price will be high logically. This by self, leads to a low marginal profit of the retailer per items which consequently decreases the order quantities of the retailer. Also, the high wholesale price results in significant deterioration cost and the deterioration cost doubles the effects of the high production cost on reducing the supply chain profit. So, the members of the supply chain could invest in reducing two cost type or one of them to discount the negative effect of them. However, as mentioned in the paper, the balance between the investment cost and the saving cost requires a detailed evaluation which is proposed for the future research.
For future studies, it is useful to extend the model of this paper to a supply chain with more than two echelons. Furthermore considering uncertainty in the model parameters for example in the deterioration rate and the demand function of items could be studied. Also, the models of the paper could be developed by regarding other assumptions in the inventory systems such as considering the time value of money, payback, shortage, etc.

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This article can be cited: Arab Momeni, M., Yaghubi, S., Mohammad Aliha, M.R. (2019). "A cost sharing-based coordination mechanism for multiple deteriorating items in a one manufacture-one retailer supply chain", *Journal of Industrial Engineering and Management Studies*, Vol. 6, No. 1, pp. 79-110.

Appendix A.

The proof that π_r^d in Eqn. (11) is an ascending function of *T* when *p* is fixed and $((p-w)(\theta+\delta)-h))\geq 0$.

Consider the profit function of the retailer π_r^d in Eqn. (11). Using the Taylor expansion, π_r^d could be rewritten as followed:

$$\pi_r^d = \frac{1}{T.(\theta+\delta)^2}\left(((p-w)(\theta+\delta)-h))(\alpha-b.p)((\theta+\delta)T+\frac{(\theta+\delta)^2T^2}{2}+\ldots)-c_r\right)$$
$$+h.\frac{\alpha-\beta p}{(\theta+\delta)} = \frac{((p-w)(\theta+\delta)-h))(\alpha-b.p)}{(\theta+\delta)}(T+\frac{(\theta+\delta)T^2}{2}+\frac{(\theta+\delta)^2T^3}{3!}+\ldots) \qquad \text{(A-1)}$$
$$-\frac{c_r}{T}+h.\frac{\alpha-\beta p}{(\theta+\delta)}$$

Also, Eqn. (A-2) shows the derivative of π_r^d in (A-1) with respect to *T*. Clearly, $\frac{\partial \pi_r^d}{T}$ is positive which indicates the higher value of *T* resuls in higer values for π_r^d.

$$\frac{\partial \pi_r^d}{T} = \frac{((p-w)(\theta+\delta)-h))(\alpha-b.p)}{(\theta+\delta)}(1+(\theta+\delta)T+\frac{(\theta+\delta)^2T^2}{2!}+\ldots)+\frac{c_r}{T^2}\geq 0 \qquad \text{(A-2)}$$

Appendix (B)*:* The proof that π_r^d in Eqn. (11) is concave when $p<w+\frac{h}{\theta+\delta}$.

To do so, it should be shown that the hessian of π_r^d is a semi-definite negative matrix. This matrix is as follow:

$$H=\begin{bmatrix} \frac{\partial^2\pi_r^d}{\partial p^2} & \frac{\partial^2\pi_r^d}{\partial T.\partial p} \\ \frac{\partial^2\pi_r^d}{\partial T.\partial p} & \frac{\partial^2\pi_r^d}{\partial T^2} \end{bmatrix} \qquad \text{(A-3)}$$

Therefore to show the concavity of (24), it is enough to show $\frac{\partial^2\pi_r^d}{\partial p^2}\leq 0$, $\frac{\partial^2\pi_r^d}{\partial T^2}\leq 0$ and $\begin{vmatrix} \frac{\partial^2\pi_r^d}{\partial p^2} & \frac{\partial^2\pi_r^d}{\partial T.\partial p} \\ \frac{\partial^2\pi_r^d}{\partial T.\partial p} & \frac{\partial^2\pi_r^d}{\partial T^2} \end{vmatrix}\geq 0$ · $\frac{\partial^2\pi_r^d}{\partial p^2}$ is determined as follow:

$$\frac{\partial^2 TC}{\partial p^2} = -\frac{2\beta\left(e^{(\theta+\delta)T}-1\right)}{T.(\theta+\delta)} \qquad \text{(A-4)}$$

From (A-4), it is clear that $\frac{\partial^2\pi_r^d}{\partial p^2}$ always is negative. Also $\frac{\partial^2\pi_r^d}{\partial T^2}$ is as Eqn. (A-5):

$$\frac{\partial^2 TC}{\partial T^2} = \frac{(\alpha-\beta.p).(h-(p-w).(\theta+\delta))}{T^3.(\theta+\delta)^2}(2T(\theta+\delta)e^{T(\theta+\delta)}$$
$$-2e^{T(\theta+\delta)}+2-T^2(\theta+\delta)^2e^{T(\theta+\delta)})-\frac{2c_r}{T^3} \qquad \text{(A-5)}$$

Now, to prove the negativity of $\frac{\partial^2 TC}{\partial T^2}$, it is sufficient to determine the term $(2T(\theta+\delta)e^{T(\theta+\delta)}-2e^{T(\theta+\delta)}+2-T^2(\theta+\delta)^2e^{T(\theta+\delta)})$ is negative. To show this, considering $x=T(\theta+\delta)$ and using the Taylor expansion $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots+\frac{x^n}{n!}$, we could write $(2T(\theta+\delta)e^{T(\theta+\delta)}-2e^{T(\theta+\delta)}+2-T^2(\theta+\delta)^2e^{T(\theta+\delta)})$ as follow:

$$(2T(\theta+\delta)e^{T(\theta+\delta)} - 2e^{T(\theta+\delta)} + 2 - T^2(\theta+\delta)^2 e^{T(\theta+\delta)}) = (2xe^x - 2e^x + 2 - x^2 e^x) =$$
$$(2x(1+x+\frac{x^2}{2}+\frac{x^3}{3!}+...) - 2(1+x+\frac{x^2}{2}+\frac{x^3}{3!}+...) + 2 - x^2(1+x+\frac{x^2}{2}+\frac{x^3}{3!}+...) =$$
$$(2x+2x^2+\frac{2x^3}{2}+...) - (2+2x+\frac{2x^2}{2}+\frac{2x^3}{3!}+...) + 2 - (x^2+x^3+\frac{x^4}{2}+\frac{x^5}{3!}+...) = \quad \text{(A-6)}$$
$$2x^2(1-\frac{1}{2!}-\frac{1}{2}) + 2x^3(\frac{1}{2!}-\frac{1}{3!}-\frac{1}{2}) + ... + 2u_n.x^n$$

Where $u_n = (\frac{1}{(n-1)!} - \frac{1}{n!} - \frac{1}{2(n-2)!})$. Rewriting u_n as Eqn. (A-7), it is observed that $u_n < 0$ for $n \geq 1$. This indicates $(2T(\theta+\delta)e^{T(\theta+\delta)} - 2e^{T(\theta+\delta)} + 2 - T^2(\theta+\delta)^2 e^{T(\theta+\delta)})$ is negative.

$$(2T(\theta+\delta)e^{T(\theta+\delta)} - 2e^{T(\theta+\delta)} + 2 - T^2(\theta+\delta)^2 e^{T(\theta+\delta)}) \quad \text{(A-7)}$$

Furthermore, the determinant of *H* i.e. det (*H*) is as follow:

$$\det(H) = -\frac{\left(1-e^{(\theta+\delta).T}\right)^2 \left(\beta.h - (\theta+\delta)(\alpha-\beta.w)\right)^2}{\mathrm{T}^4.(\theta+\delta)^4} + \frac{2\beta^2(e^{(\theta+\delta).T}\left(e^{(\theta+\delta).T}-1\right)(h-(\theta+\delta)(p-w))^2}{T^3(\theta+\delta)^3}$$
$$= \frac{2\beta^2 T(\theta+\delta)(e^{(\theta+\delta).T}\left(e^{(\theta+\delta).T}-1\right)(h-(\theta+\delta)(p-w))^2 - \beta^2\left(1-e^{(\theta+\delta).T}\right)^2\left(h-(\theta+\delta)\left(\frac{a}{\beta}-w\right)\right)^2}{\mathrm{T}^4.(\theta+\delta)^4} \quad \text{(A-8)}$$

First, it should be noted that the minimum value of $h-(p-w)(\theta+\delta)$ occurs when *p* takes its maximum value i.e. $p = \frac{\alpha}{\beta}$. Therefore, the inequality (A-9) is always true:

$$h-(p-w)(\theta+\delta) \geq h - (\frac{\alpha}{\beta}-w)(\theta+\delta) \Rightarrow$$
$$\beta(h-(p-w)(\theta+\delta)) \geq \beta.h - (\alpha-\beta.w)(\theta+\delta) \quad \text{(A-9)}$$

Now regarding (A-8) and (A-9), the following inequality is also held:

$$\det(H) \geq \frac{\beta^2\left(2T(\theta+\delta)e^{(\theta+\delta).T}\left(e^{(\theta+\delta).T}-1\right) - \left(1-e^{(\theta+\delta).T}\right)^2\right)(h-(\theta+\delta)(p-w))^2}{\mathrm{T}^4.(\theta+\delta)^4} \quad \text{(A-10)}$$

Hence, to show the positivity of $det(H)$, it is enough to prove that $2T(\theta+\delta)e^{(\theta+\delta).T}\left(e^{(\theta+\delta).T}-1\right) - \left(1-e^{(\theta+\delta).T}\right)^2 \geq 0$. To do so, again regarding $x = T(\theta+\delta)$ and using the Taylor expansion, the following equation is valid:

$$2xe^x\left(e^x-1\right) - \left(1-e^x\right)^2 = (e^x-1)(xe^x+1-e^x) =$$
$$2(e^x-1)\left(x.(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...) + 1 - (1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...)\right) =$$
$$2(e^x-1)\left(2x+2x^2+\frac{2x^3}{2!}+\frac{2x^4}{3!}...-x-\frac{x^2}{2!}-\frac{x^3}{3!}-...\right) = \quad \text{(A-11)}$$
$$2(e^x-1)\left((2-1)x+(2-\frac{1}{2!})x^2+(\frac{2}{2!}-\frac{1}{3!})x^3+...+u_n x^n\right)$$

Where $u_n = (\frac{2}{(n-1)!} - \frac{1}{n!})$. Now, because u_n is always positive, $\det(H) \geq 0$ and the proof is completed.

Appendix (C): The proof that π_r^d in (15) is a concave function of $p_1,...,p_2$ when $T_1,...,T_n$ are regarded fixed.

To prove this claim, it should be shown that the hessian matrix of the profit function of the retailer in (15) is a semi-definite negative matrix. This matrix is as follow:

$$H(\pi_r^d) = \begin{bmatrix} \frac{\partial^2 \pi_r^d}{\partial p_1^2} & 0 & \cdots & 0 \\ 0 & \frac{\partial^2 \pi_r^d}{\partial p_2^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial^2 \pi_r^d}{\partial p_n^2} \end{bmatrix} \quad \text{(A-12)}$$

The first order principal minor of $H(\pi_r^d)$ is $\frac{\partial^2 \pi_r^d}{\partial p_1^2} = -\frac{2\beta_i . \left(e^{(\theta_i+\delta_i)T_i} - 1\right)}{T_i . (\theta_i + \delta_i)}$ which is clearly negative. The second order principle minor of $H(\pi_r^d)$ is $\frac{\partial^2 TC}{\partial p_1{}^2} \times \frac{\partial^2 TC}{\partial p_2{}^2}$ that is a multiplication of two negative values and therefore is a positive value. Similarly, the *m* th principle minor of $H(\pi_r^d)$ is $\frac{\partial^2 TC}{\partial p_1{}^2} \times \frac{\partial^2 TC}{\partial p_2{}^2} \times \ldots \times \frac{\partial^2 TC}{\partial p_m{}^2}$. It is notable that the $(2m+1)$ th principle minor of $H(\pi_r^d)$ is multiplication of (2m+1) negative values which results in a negative value for the multiplication. Also, $(2m)$ th principle minor of $H(\pi_r^d)$ is determined by multiplication of (2m) negative values and therefore is a positive value. This shows that $H(\pi_r^d)$ is a semi-definite negative matrix.