A quantitative model for optimization and disruption mitigation in a supply chain (Auto parts case study)

Seyed Ali Alavikia¹, Mohammad Taghi TaghaviFard²*, Maghsoud Amiri², Parham Azimi³

Abstract
However, there is a lot of capital and plenty of manpower in the auto spare part industry, the enterprises and supply chains of this industry do not perform well in our country. This research models a three-level supply chain with multiple manufacturers, distribution centers and retailers, to minimize the total cost by taking into account various disruptions. The database of two active car spare parts companies for five strategic products in one year has been used. Then, the mathematical model is analyzed by considering disruptions based on three different sales policies: back orders, lost sales and outsourcing. Besides, to evaluate the performance of the model some numerical examples are used and analyzed to determine that algorithm works. Model solved efficiently by MATLAB software. The results show that the proposed algorithm of this research can neutralize the effect of the disruptions and cause a significant reduction in total cost of the system. The model is useful for helping decision makers to adopt an active approach to maintaining business benefits when disruptions take place in the supply chain.

Keywords: Optimization model, Fuzzy, Auto parts supply chain, Disruption.

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1. Introduction

Iran's automobile industry turnover was $ 25 billion in 1395, including $ 16 billion in domestic production and $ 3 billion in imports of automobiles and related parts, and $ 6 billion in spare parts for after-sales services. (ISNA, 2017) The $ 6 billion annual volume of spare parts represents the position of this sector of the industry. Given today's competitive environment, supply chain management is essential in order to reduce costs, improve customer service and achieve a balance between costs and services, and thereby to give a competitive advantage to a company.

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(Fakhrzad & Alidoosti, 2018) Delivering the outputs depends on organization's ability to manage the flow of materials, information, and money inside and outside the organization. This flow is known as the supply chain. In today's competitive market, the supply chain coordination (SCC) has great significance. In the lack of coordination, the SC members optimize their own decisions independently. (Johari, Hosseini-Motlagh, & Nematollahi, 2016) Increasing globalization, the rapid development of technology and the creation of competitive advantages hinder the organization's ability to predict and manage the behavior of supply chain partners. Supply chains may be complex and long and involve a large number of different business partners and their problems and issues are inevitable. Failure to resolve these problems will result in customers’ dissatisfaction and loss of sale and impose lots of costs to organizations. Companies in the world-class owe many of their successes to supply chain management, hence the supply chain has become a popular management discourse in recent decades. Since supply chain loops are independent entities of economics, creating a mechanism that can coordinate the activities and objectives of these entities in order to optimize the overall system performance is a key issue in the management of the supply chain. Generally, traditional researches in supply chain focused on making decisions in the ideal conditions, where demand was defined, and the producer had complete information about the market; but in the real world, the complete market information is rarely available and obtaining demand distributions may be hard and even impossible. In the dynamic and complex business environment, uncertainty is considered as an important concern. The demand uncertainty is also an important issue in the science and industry. These uncertainties of demand posed major challenges to supply chain management, leading to a reduction in real income. In order to take uncertainty and its effects into account, it is necessary to pay attention to the sources of uncertainty, which are disruptions or interruptions in all parts of the supply chain.

Many studies have been carried out on the main field of this research, the supply chain; many of the existing studies carried out in this field are qualitative articles, and there aren’t any quantitative modeling issues. (Giannakis, 2011) Although such studies play a major role in explaining new topics or in learning the key concepts, their application is restricted to the above and is less applicable to solving problems that impress competition of supply chains. Qualitative studies with quantitative statistical approaches are another category of research in the supply chain, although these researches have pretended quantitative due to the use of statistical tools, but the principles of research are based on qualitative concepts such as chain performance measurement (Maestrini, Luzzini, Maccarrone, & Caniato, 2017), supply chain culture (Zhi, Baofeng, Yuan, & Xiande, 2015), supply chain interoperability (Veronica, 2013), supply chain stability (Tamplin, 2017), lean supply chain (Kanchan, 2018), agile supply chain (Minkyun & Sangmi, 2017), and so on. Quantitative studies in the supply chain are the third category of research in the supply chain, which the present study is in this category, and attempts to simulate real supply chains and solve problems and concerns of different parts of chains. Due to the extensive research, this section can be divided into three main sub-domains: a) studies focused on demand forecasting and inventory control; b) pricing-focused studies; and c) studies focused on disruptions and uncertainties. Each of these sub-domains also deals with the study of two-level, three-level, and etc. To clarify the perspective of each subcategory, relevant literature is classified.

1) Studies focused on demand forecasting and inventory control: In recent research, Rezayi's research (Rezayi, 2012) has been based on the principles of artificial intelligence science and data mining to provide models for forecasting sales of the next month. The proposed models of this study are of a hybrid type and include steps 1- The reduction of dimension 2- clustering and 3- prediction. Although the results indicate the higher accuracy of these models compared to traditional models of prediction, there are limitations such as limited time intervals. In inventory control, one of the most important initial tasks in the field of multi-level inventory
systems, with emphasis on the concept of "level inventory", which deals with the analysis of the cost function of multilevel systems, is Clarke and Scarf's paper, "Optimal Policies for Multi-Level Inventory Issues". [12] In another study [13], it has been forecasted using the Bayesian network of gasoline in Iran. The model presented in this study has been expanded with the use of annual data for the years 1347-1386 and GDP, vehicle numbers, population and actual price of gasoline. The results of this research show that the Bayesian network has less error than the regression method.

2) Pricing-focused studies, uses concepts such as game theory to find the best pricing strategy. (Aust & Buscher, 2012) (Giri & Sharma, 2014)

3) In studies focused on distortions and uncertainties, the goal is to design supply chain network and develop models that are both acceptable and efficient in both normal and disruptive situations. (Liang & Renbin, 2017) A systematic literature review and a comprehensive analysis of the decision-making models for supply chain risk (SCR) mitigation was also provided by Rajagopal et al. (2017) (Rajagopal, Venkatesan, & Goh, 2017) A review of the literature indicates that most of the available articles ignore the potential for breakdowns and disruptions in the supply chain, and generally assume the demand and supply parameters in a definitive manner. Other research in the literature has often overlooked the severity of the disruption and often focuses solely on complete disruption (Shishebori & Yousefi Babadi, 2015) (Peng, Snyder, Lim, & Liu, 2011) and few studies have focused on minor disruption (Azad, Saharidis, Davoudpour, Maleky, & Yektamaram, 2013); A minor disorder is also modeled definitively in (Azad, Saharidis, Davoudpour, Maleky, & Yektamaram, 2013).

There are some gaps in the literature. First, it is clear that most researches focused on supply chain optimization and collaboration problems under ideal conditions, although some studies developed reactive and recovery mitigation models after the occurrence of a sudden disruption. However, no study has been found that predicts the possible changes in future demand that is used as input to the mitigation planning model in supply chain. On the other hand, most past studies focused mainly on a single disruption in production and a very few focused on a series of disruptions on a real-time basis, but they are for a single manufacturer and a retailer, which limits their applicability in real-life situations. In addition, most studies have simplified the chain as a single product. According to the above mentioned, the present research tries to fill the scientific and research gaps by using actual data to present an applicable model.

To fill the gaps in the current literature, this paper attempts to carry out the following novelties in modelling and solution approach:

Our proposed heuristics for reactive mitigation are capable of dealing with following: (i) single occurrence of sudden disruption, (ii) a series of sudden independent disruptions, and (iii) a series of a mix of independent and dependent disruptions.

The three different policies for managing a sudden disruption are considered: Back orders, lost sales and Outsourcing.


Using the fuzzy sets theory to model and solve problems. When the problem parameters are uncertain, using fuzzy theory creates a real flexibility.
2. Problem Description

In this research, a three-level supply chain model is developed under ideal conditions for a limited planning time. This ideal program used to determine the cycle length that is supposed to be the length of time required for programming and analysis. The three-level supply chain consists of production factories, distributors and retailers with several parts per level. For this system, supply chain develops three different approaches. 1- An ideal program for a limited planning time that updates the program if changes are made to data. 2- A preventive avoidance reduction planning approach for managing demand changes that can predict future demand by using an appropriate tool. 3- A program to reduce the impact of a response disruption based on real-time results for managing sudden production disruption. The existence of assumptions in designing all issues in the operation research is very important. The following assumptions have been considered in designing the proposed model for this research:

1. Several independent products (G) are produced in the system and the system is a three-level supply chain system (including multiple producers (I), multiple distributors (J), and several retailers (K))
2. In the reduction of preventive risks, only changes in demand are taken into account.
3. Disruption of production in different manufacturing plants is considered independently.
4. There is no safety stock in the system.
5. The number of turns during the recovery period is determined by the management of the production organization.
6. The demand process is discrete and is based on the Poisson process in every retailer. (The customer demand function is a type of Poisson distribution function.)
7. No horizontal link between retailers is allowed.
8. All retailers are considered the same.

2.1. Notations and parameters
The following notations are used in this study to formulate the mathematical model.
The following parameters are used in this study to formulate the mathematical model.

- **$C_P^g_i$**: Maximum production capacity of g in plant i under ideal conditions
- **$C_D^g_i$**: Maximum handling capacity of DC j
- **$D_{gk}$**: Demand for g in retailer k
- **$p_{gi}$**: Production cost per unit of product g at plant i (currency per unit)
- **$H_{1gi}$**: Holding Cost per unit per interval at plant i (currency per unit per interval)
- **$H_{2gj}$**: Handling cost per unit of product g at DC j (currency per unit)
- **$H_{3gk}$**: Holding cost per unit of product g per period at retailer k (currency per unit per period)
- **$T_{1gij}$**: Transportation cost of product g per unit from plant i to DC j (currency per unit)
- **$T_{2gjk}$**: Transportation cost of product g per unit from DC j to retailer k (currency per unit)

The following variables are decision variables.

- **$P_{gi}$**: production quantity of product g at plant i
- **$X_{gij}$**: transportation quantity of product g from plant i to DC j
- **$Y_{gjk}$**: transportation quantity of product g from DC j to retailer k
- **$P'_{gim}$**: production quantity of product g after disruption at plant i in period m
- **$X'_{gijm}$**: transportation quantity of product g from plant i to DC j after disruption in period m
- **$Y'_{gjkm}$**: transportation quantity of product g from DC j to retailer k after disruption in period m

### 3. Model formulation

In this section, the mathematical model (Paul, Sarker, & Essam, 2017) is developed for both an ideal and a disrupted supply chain system. The ideal plan is updated if there are any changes in the data and is also reviewed according to any prediction of future changes for a finite planning period. In the case of managing a disruption, the model is formulated again to incorporate the effect of a disruption and the production and distribution plan is revised for a finite planning period. After the recovery window, the production and distribution plan reverts to its ideal plan.

#### 3.1. Formulation in ideal plan

In this section, different costs are calculated to formulate the mathematical model for the ideal case. The production cost is determined as the per unit production cost multiplied by the production quantity, the average holding cost as the unit holding cost multiplied by the total
inventory, the transportation cost as the unit transportation cost multiplied by the transportation quantity, the total operating cost as the sum of the operating cost of each DC, and the handling cost of distribution is the unit handling cost multiplied by the total handling quantity. Finally, the different costs are summed to obtain the objective function to be minimized subject to capacity, distribution and demand constraints, where decision variables are $P_{gi}$, $X_{gij}$ and $Y_{gjk}$. The final model is considered in a constrained programing problem form.

\[
\text{Minimize: } TC = TCF + TCW + TCR
\]

\[
TC = \sum_{i=1}^{l} \sum_{g=1}^{G} p_{gi} P_{gi}
+ \frac{1}{2} \sum_{i=1}^{l} \sum_{g=1}^{G} H_{1gi} P_{gi}
+ \sum_{j=1}^{J} \sum_{i=1}^{l} \sum_{g=1}^{G} T_{1gij} X_{gij}
+ \sum_{j=1}^{J} OC_{j}
+ \sum_{j=1}^{J} \sum_{i=1}^{l} \sum_{g=1}^{G} H_{2gj} X_{gij}
+ \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{l} \sum_{g=1}^{G} T_{2gjk} Y_{gjk} + \frac{1}{2} \sum_{k=1}^{K} \sum_{g=1}^{G} H_{3gk} \tilde{D}_{gk}
\]

**Costs at plant:**

Production cost = \[
\sum_{i=1}^{l} \sum_{g=1}^{G} p_{gi} P_{gi}
\] (3)

Average holding cost = \[
\frac{1}{2} \sum_{i=1}^{l} \sum_{g=1}^{G} H_{1gi} P_{gi}
\] (4)

Transportation cost = \[
\sum_{j=1}^{J} \sum_{i=1}^{l} \sum_{g=1}^{G} T_{1gij} X_{gij}
\] (5)

**Costs at DCs:**

Operating cost = \[
\sum_{j=1}^{J} OC_{j}
\] (6)

Handling cost = \[
\sum_{j=1}^{J} \sum_{i=1}^{l} \sum_{g=1}^{G} H_{2gj} X_{gij}
\] (7)
Costs at retailer:

Transportation cost \( T_{2gjk} Y_{gjk} \)  
Average holding cost \( H_{3gk} \bar{D}_{gk} \)  

Here, \( P_i, X_{ij} \) and \( Y_{jk} \) are decision variables, subject to the following constraints.

Constraint (10) ensures that the number of \( g \) products in each plant is smaller or equal to the maximum production capacity of \( g \) in that factory. Constraint (11) ensures that the number of \( g \) products per plant is equal to the total number of \( g \) shipped from the plant to the distributors in addition to the number of safety stock. Constraint (12) ensures that the total number of \( g \) products shipped from the factories to the distributors is equal to the total number of \( g \) shipped from the distributors to the retailers. Constraint (13) ensures that the total number of \( g \) products shipped from the factories to distributors is smaller or equal to the maximum capacity of the number of \( g \) products shipped from the factories to the distributors. Constraint (14) ensures that the total number of \( g \) products shipped from distributors to the retailer \( k \) is equal to the demand of \( g \) from retailer \( k \). Constraint (15) ensures that the total quantity of \( g \) products produced is equal to the demand of retailers. Constraint (16) The number of \( g \) product per plant, the number of products shipped from factories to distributors, the number of products shipped from distributors to retailers, and the number of safety stock are all integers equal to or greater than zero.

3.2. Formulation for disruption mitigation plan

In this section, the mathematical model is developed to revise the production and distribution plan for a limited planning timeline after a disruption of production, with the goal of minimizing the cost of the entire supply chain. Recovery strategies include back orders, lost sales, and outsourcing. Back order costs are deducted from the amount of delayed sales costs per unit of product in the number of products postponed by their sales, lost sales cost, multiplied
by the cost of sales lost per unit of product in the number of products whose sales have been lost. And the cost of outsourcing is the quantity outsourced products at the cost of purchasing each unit.

If there is a disruption in the nth factory for the duration of $T_{dn}$ and the start time of $t_n$, the production loss after a disruption is calculated by the following:

If: $\frac{P_n}{CP_n} > t_n + T_{dn}$

$$D' = CP_n \times T_{dn} - \min \left\{ \sum_{g=1}^{G} \sum_{i=1}^{I} SC_{gi} \cdot \sum_{g=1}^{G} \sum_{i=1}^{I} CP_{gi} \times (1 - t_n - T_{dn}) \right\}$$ (17)

If: $\frac{P_n}{CP_n} < t_n + T_{dn}$

$$D' = CP_n \times \left( \frac{P_n}{CP_n} - t_n \right) - \min \left\{ \sum_{g=1}^{G} \sum_{i=1}^{I} SC_{gi} \cdot \sum_{g=1}^{G} \sum_{i=1}^{I} CP_{gi} \times (1 - t_n - T_{dn}) \right\}$$ (18)

If this quantity needs to be filled during the recovery window, we consider back orders, lost sales and outsourcing options so that the total supply chain cost during this time can be minimized.

Production cost

$$\text{Production cost} = \sum_{m=1}^{M} \sum_{l=1}^{L} \sum_{g=1}^{G} p_{gi} P_{gim}$$ (19)

Average holding cost

$$\text{Average holding cost} = \sum_{i=1}^{I} \sum_{g=1}^{G} \sum_{m=1}^{M} \frac{1}{2} H_{1i} P_{gim}$$ (20)

Transportation cost

$$\text{Transportation cost} = \sum_{m=1}^{M} \sum_{j=1}^{J} \sum_{l=1}^{L} \sum_{g=1}^{G} T_{1ij} X'_{gijm}$$ (21)

DCs Operating cost

$$\text{DCs Operating cost} = M \times \sum_{j=1}^{J} \sum_{l=1}^{L} \sum_{g=1}^{G} OC_{j} Y_{gjk}$$ (22)

DCs Handling cost

$$\text{DCs Handling cost} = \sum_{m=1}^{M} \sum_{j=1}^{J} \sum_{l=1}^{L} \sum_{g=1}^{G} H_{2j} X'_{gijm}$$ (23)

DCs Transportation cost

$$\text{DCs Transportation cost} = \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{l=1}^{L} \sum_{g=1}^{G} T_{2jk} Y'_{gjkm}$$ (24)

Retailers Average holding cost

$$\text{Retailers Average holding cost} = \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{g=1}^{G} \frac{1}{2} H_{3k} D'_{gkm}$$ (25)

Back orders cost

$$\text{Back orders cost} = B \times \left[ \sum_{m=1}^{M} m \left( \sum_{l=1}^{L} \sum_{g=1}^{G} P'_{gim} - \sum_{i=1}^{I} \sum_{g=1}^{G} P_{gi} \right) \right]$$ (27)

Outsourcing cost

$$\text{Outsourcing cost} = S \times \left[ M \sum_{l=1}^{L} \sum_{g=1}^{G} P_{gi} + D' - \sum_{m=1}^{M} \sum_{l=1}^{L} \sum_{g=1}^{G} P'_{gim} \right]$$ (28)
Lost sales cost = \( L \times \left[ M \sum_{i=1}^{I} \sum_{g=1}^{G} P_{gi} + D' - \sum_{m=1}^{M} \sum_{i=1}^{I} \sum_{g=1}^{G} P'_{gim} \right] \)

If \( S \leq L \), the cost of lost sale is zero, otherwise the cost of outsourcing is zero.

The total cost of the supply chain, which is our objective function, is derived from equations above and equals the total plant cost + total DC cost + total retailer cost + back orders cost + outsourcing cost + lost sales cost, where \( P'_{gim}, X'_{gijm}, Y'_{gjkm} \) and \( D'_{gkm} \) are decision variables.

\[
P'_{gim} = CP'_{gi}; \forall i, m, g \quad (29)
\]

\[
P'_{gim} \leq CP'_{gi}; \forall i, m, g \quad (30)
\]

\[
\sum_{i=1}^{I} X'_{gijm} = \sum_{k=1}^{K} Y'_{jkm}; \forall j, m, g \quad (31)
\]

\[
\sum_{i=1}^{I} X'_{gijm} \leq CD_{gjm}; \forall g, j, m \quad (32)
\]

\[
\sum_{j=1}^{J} Y'_{gjkm} = D'_{gkm}; \forall k, m, g \quad (33)
\]

\[
M \sum_{i=1}^{I} \sum_{g=1}^{G} P_{gi} + D'_{gkm} \geq \sum_{m=1}^{M} \sum_{i=1}^{I} \sum_{g=1}^{G} P'_{gim} \quad (34)
\]

\[
P'_{gim}, X'_{gijm}, Y'_{gjkm}, SS'_{gim} \geq 0. \quad \in \mathbb{Z}; \forall i, j, k, m, g \quad (35)
\]

Constraint (29) ensures that the number of product after the disruption in each plant at m period is less than or equal to the maximum production capacity in the revised plan. Constraint (30) ensures that the number of \( g \) product after the disruption in each plant at m period equals the total number of products shipped from the factories to the distributors in addition to the number of safety stock. Constraint (31) ensures that the total number of \( g \) products shipped from factories to distributors at the m period after the disruption is equal to the total number of \( g \) products shipped from distributors to retailers at the m period after the disruption. Constraint (32) ensures that the total number of \( g \) products shipped from factories to distributors at m period after a disruption is less than or equal to the maximum capacity of \( g \) products from the factories to the distributors at m period after disruption. Constraint (33) ensures that the total number of products shipped from distributors to the retailer \( k \) at m period after the disruption is equal to the retailer \( k \) demand. Constraint (34) ensures that the total number of \( g \) products shipped from distributors to retailers in the m period after the disruption is equal to the demand of retailers. Constraint (35) ensures that the number of \( g \) product of each plant during the m period after the disruption, the number of products shipped from the factories to the distributors at the m period after the disruption, the number of products shipped from the distributors to the retailer in the m period after the disruption are all integers and equal to or greater than zero.
for a given \( P_{gi} \), \( CP_{gi} \), \( CD_j \), and \( D_{gk} \), and the \( n^{th} \) disrupted plant, if \( B \ll \), \( S \), the recovery plan will use only the back orders option if \( D' \leq M \sum_{i=1}^{l} S_{Ci} \) and the recovery plan will use both the lost sales/outside and back orders options if \( D' > M \sum_{i=1}^{l} S_{Ci} \).

\[
D' = M \sum_{g=1}^{G} \sum_{i=1}^{l} P_{gi} + D' = \sum_{g=1}^{G} \sum_{m=1}^{M} \sum_{i=1}^{l} P'_{gim}
\]

\[
D' = M \sum_{g=1}^{G} \sum_{i=1}^{l} CP_{gi} - M \sum_{i=1}^{l} P_{gi}
\]

\[
D' \leq M \left( \sum_{g=1}^{G} \sum_{i=1}^{l} CP_{gi} - \sum_{g=1}^{G} \sum_{i=1}^{l} P_{gi} \right)
\]

\[
D' \leq M \sum_{g=1}^{G} \sum_{i=1}^{l} SC_{gi}
\]

Therefore, it can be said, that if \( D' \leq M \sum_{i=1}^{l} S_{Ci} \), the system will utilize only the back orders option in the recovery plan. From the opposite consequence of Eq. (36), it also can be said that the recovery plan will use both the back orders and lost sales/outside options if \( D' > M \sum_{i=1}^{l} S_{Ci} \).

4. Solution approaches

In this section, solution approaches for both ideal and disrupted systems are developed. A standard solution technique for solving the ideal supply chain system is proposed and applied to obtain updated and predictive mitigation plans for changes in the data and future predictions respectively. An efficient heuristic for managing a single disruption in the system is developed and then extended to be implemented for managing multiple disruptions on a real-time basis. The presented algorithm solves the problem by MATLAB software. The criteria for comparing the results obtained from the algorithm are also described and numerical results are presented in table and chart formats. All examples run in the 3.2GHz personal computer with 4GB of RAM through the Windows 10 operating system. In this research, the ranking method provided by Jimenez (Jimenez, Arenas, Bilbao, & Rodriguez, 2007) has been used for Defuzzification.

4.1. Heuristic for managing single disruption

A heuristic is designed to obtain the revised plan after an occurrence of a single disruption at any plant. Firstly, both the ideal and disrupted systems are solved using the MATLAB software and then the heuristic efficiently solves the disruption management model through the following steps.

Step 1: Input all the information about production and distribution under ideal conditions.

Step 2: Obtain an ideal production and distribution plan by solving the mathematical model for ideal situations and also determine the spare capacity in each plant.

Step 3: Input a production disruption scenario involving a disrupted plant, disruption start time \((t_s)\) and disruption duration \((T_{dn})\).

Step 4: Determine the production plan.
4.1. If $B \leq L, S$:
   
   4.1.1. If $D' \leq M \sum_{i=1}^{l} SC_i$, use the spare capacity to revise the plan until the unfilled demand is met;
   
   4.1.2. If $D' > M \sum_{i=1}^{l} SC_i$, use both the spare capacity and lost sales/outsource options;

   4.1.2.1. If $L \geq S$, use the outsourcing option and
   
   4.1.2.2. If $L < S$, use the lost sales option.

4.2. If $B > L, S$:
   
   4.2.1. If $L \geq S$, use the outsourcing option to revise the plan or
   
   4.2.2. If $L < S$, use the lost sales option to revise the plan.

Step 5: Determine the distribution plan.
   
5.1. If $D' \leq M \sum_{i=1}^{l} SC_i$, determine the distribution plan by varying only the transportation quantity while using the same path as the ideal plan.

5.2. If $D' > M \sum_{i=1}^{l} SC_i$, determine the distribution plan by varying only the transportation quantity while using the same path as that obtained from the proposed algorithm for $D' > M \sum_{i=1}^{l} SC_i$.

Step 6: Record the results and determine the different costs.

Step 7: Stop.

4.2. Proposed heuristic for multiple disruptions

In this section, the heuristic that was first developed to manage a single disruption is extended to manage multiple production disruptions on a real-time basis. When a disruption occurs, a revised plan can be generated by solving the mathematical model using the proposed heuristic for a single disruption. Then, if another disruption occurs, the plan should be revised again to consider the effects of both disruptions. This can be done by simply updating some of the parameters in the same mathematical model to represent the changed scenario; for example, the newly disrupted plant, start time of the disruption, disruption duration, quantity produced before starting the revised plan and demand to be filled in the revised plan. The objective function and constraints are also updated for the changed situation. Therefore, the heuristic for a single disruption can still be used but must be slightly modified for the changed situation to be capable of dealing with a series of disruptions on a real-time basis. In the proposed approach, the heuristic must be run every time a disruption occurs to re-optimize the revised plan whenever there are disruptions in the system. For a series of disruptions, the production quantity loss after the $sth$ disruption can be determined using following equations:

\[
I_f: \frac{P_n}{CP_n} > t_n + T_{dn}
\]

\[
D' = D'_{s-1} - \left( \sum_{m=1}^{l} \sum_{i=1}^{j} P'_m - l \sum_{i=1}^{j} P_i \right) + CP_n \times T_{dn} - m \min \left\{ \sum_{i=1}^{j} SC_i, \sum_{i=1}^{j} CP_i \times (1 - t_n - T_{dn}) \right\}
\]
If: \( \frac{p_n}{CP_n} < t_n + T_{dn} \)

\[
D' = D'_{s-1} - \left( \sum_{m=1}^{l} \sum_{i=1}^{l} p'_{im} - l \sum_{i=1}^{l} P_i \right) + CP_n \times \left( \frac{p_n}{CP_n} - t_n \right) - \min \left\{ \sum_{i=1}^{l} SC_i, \sum_{i=1}^{l} CP_i \times (1 - t_n - T_{dn}) \right\}
\]

Here, \( l \) is the new disrupted period since the previous disruption. The main steps in the proposed heuristic for a series of disruptions on a real-time basis can be presented as follows.

Step 1: Input all the information about production and distribution under ideal conditions.

Step 2: Determine the optimal plan under ideal conditions.

Step 3: Input the disruption scenario (disrupted plant, disrupted period since the previous disruption, disruption start time \( t_n \) and disruption duration \( T_{dn} \)).

Step 4: Update the loss of production quantity using Eqs. (37) and (38).

Step 5: Revise the production plan for the corresponding disruption using the proposed heuristic developed in Section 4.2.

Step 6: Record and update the optimal production and distribution plan from Step 5 after the disruption occurs. Step 7: If there is any other disruption, go to Step 3.

Step 8: Stop.

4.3. Experimentation and analysis of results

In order to investigate the effectiveness of the research initiative in achieving results in real-world conditions, available information used from automobile parts supply chain. In this example, 5 pieces of product from two companies in the industry are used as follows.

<table>
<thead>
<tr>
<th>Table 2. Products data for two companies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product description</td>
</tr>
<tr>
<td>Peugeot 405 lower Ball joint</td>
</tr>
<tr>
<td>Peugeot 405 left Ball joint</td>
</tr>
<tr>
<td>Peugeot 405 right Ball joint</td>
</tr>
<tr>
<td>Peugeot 206 left control arm</td>
</tr>
<tr>
<td>Peugeot 206 right control arm</td>
</tr>
</tbody>
</table>

All the presented information is about the components of the Peugeot 206 and 405 and other data needed to solve the problem are generated hypothetically. Table 3 shows the values of the problem parameters.

<table>
<thead>
<tr>
<th>Table 3. Problem parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>parameter</td>
</tr>
<tr>
<td>( CP_{gi} )</td>
</tr>
<tr>
<td>( CD_{gij} )</td>
</tr>
<tr>
<td>( D_{gk} )</td>
</tr>
<tr>
<td>( H_{1gi} )</td>
</tr>
<tr>
<td>( H_{2gj} )</td>
</tr>
<tr>
<td>( H_{3gk} )</td>
</tr>
<tr>
<td>( T_{1gij} )</td>
</tr>
<tr>
<td>( T_{2gjk} )</td>
</tr>
</tbody>
</table>

Table 4 presents cases with different random combinations of a disrupted plant, disruption start times and disruption durations. Although disruptions can happen continuously within a production cycle, it presents ten disruptions as a sample representation. The production and distribution plans were revised immediately after each disruption occurred in the system.
Table 4. A case problem for a series of disruptions.

<table>
<thead>
<tr>
<th>Disruption number</th>
<th>Disrupted plant</th>
<th>Disrupted period since previous disruption</th>
<th>Disruption start time</th>
<th>Disruption duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>-</td>
<td>0.02</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0.21</td>
<td>0.09</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0.31</td>
<td>0.10</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>0.42</td>
<td>0.16</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>3</td>
<td>0.58</td>
<td>0.10</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>2</td>
<td>0.68</td>
<td>0.05</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>2</td>
<td>0.74</td>
<td>0.05</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td>0.80</td>
<td>0.08</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>4</td>
<td>0.90</td>
<td>0.02</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>3</td>
<td>0.93</td>
<td>0.05</td>
</tr>
</tbody>
</table>

After solving the problem with the proposed algorithm, the results are presented in Table 5, which includes the total cost, the cost of back orders, the cost of lost sales and the cost of outsourcing. It was observed that the system utilized the spare capacity, lost sales and outsourcing options for the first six disruptions for revising the plan, and it was capable of revising the plan by using only the back orders and outsourcing option for the 7th disruption. After the 7th disruption, the system was capable to recover by utilizing the spare capacity of the other plant so no back orders, lost sales or outsourcing are presented in the recovery plan then.

Table 5. Summary of results for a series of disruptions.

<table>
<thead>
<tr>
<th>Disruption number</th>
<th>Total cost</th>
<th>Total back orders cost</th>
<th>Total lost sales cost</th>
<th>Total outsourcing cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>173807643</td>
<td>1414306</td>
<td>4348706</td>
<td>124405</td>
</tr>
<tr>
<td>2</td>
<td>172530731</td>
<td>1036844</td>
<td>3319837</td>
<td>255417</td>
</tr>
<tr>
<td>3</td>
<td>172368030</td>
<td>1004562</td>
<td>3318837</td>
<td>40154</td>
</tr>
<tr>
<td>4</td>
<td>170594654</td>
<td>831437</td>
<td>3318837</td>
<td>40154</td>
</tr>
<tr>
<td>5</td>
<td>168921429</td>
<td>718232</td>
<td>3318837</td>
<td>40154</td>
</tr>
<tr>
<td>6</td>
<td>168755262</td>
<td>581107</td>
<td>3318837</td>
<td>40154</td>
</tr>
<tr>
<td>7</td>
<td>168257875</td>
<td>213244</td>
<td>3318837</td>
<td>40154</td>
</tr>
<tr>
<td>8</td>
<td>168044631</td>
<td>0</td>
<td>3318837</td>
<td>40154</td>
</tr>
<tr>
<td>9</td>
<td>168044631</td>
<td>0</td>
<td>3318837</td>
<td>40154</td>
</tr>
<tr>
<td>10</td>
<td>168044631</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

5. Analysis of the model solution results

Ten disruptions are considered at different times in the two plants. Each of these disruptions causes problems in each of the manufacturer's factories. After solving the problem with the aid of the proposed quotation, it is observed that until the disruption of number 7, the cost of selling lost is unpaired. In fact, the presence of the disruption causes the problem in the line produced, and some products appear in the form of a deferred order, outsourcing or lost sales. But in the seventh disruption, the remainder is not a sale, and there are only deferred orders and outsourcing costs. Otherwise, the production system is able to respond to all requests, but some consider them as outsourced orders, and some are outsourced. In disruptions 8 to 10, all remaining orders, lost sales, and outsourced costs are zero and there is only the cost of product production. In fact, the algorithm is able to neutralize the effect of disruptions and reduce the cost of the system.

Figures 1, 2, 3 and 4 present changes in the cost (total, back orders, lost sales, and outsourcing) with different disruption durations. In Fig.1, the graph shows the total cost of the system for different disruptions duration. According to the diagram, the cost of the system is initially the
same in the two schemes. But gradually, with increasing duration of disruption, system costs in factory 1 are greater than factory 2. But in the last disruptions, costs are again equal.

![Graph 1: Total cost vs disruption duration](image1)

**Figure 1. Total cost vs disruption duration**

In Fig. 2, the graph shows the total back orders cost for disruptions duration. But the important point in this chart is the close-up of the cost of back orders in the two factories. This only slightly increased in 0.5 and 0.56 disruptions, but at the other level they still have little distance. This indicates the efficiency level of proposed algorithm in creating a balance in controlling back orders costs in the two factories.

![Graph 2: Total back orders cost vs disruption duration](image2)

**Figure 2. Total back orders cost vs disruption duration**

In Fig. 3, the lost sales costs are shown for disruptions durations. In this chart, same as Fig. 1, the lost sales costs in the two factories are very close to each other and presents the existence of a proper control of costs levels at different disruptions.

![Graph 3: Lost sales costs vs disruption duration](image3)
Figure 3. Total lost sales cost vs disruption duration

In Fig. 4, outsourcing costs are shown at various levels of the disruption. In this diagram, the outsourcing costs in the two factories is very close to each other, this indicates the efficiency of proposed algorithm in creating a balance in controlling outsourcing costs in the two factories. Of course, at disruption that start at a longer time interval, outsourcing costs differ at the two factories.

Figure 4. Total outsourcing cost vs disruption duration

In order to illustrate the effect of applying the proposed method to reduce the effect of disruptions, we solved the problem in two ways: the existence and absence of a proposed method and the level of costs have been compared. In fact, it can be shown that in the event of disruptions, their prediction and attempts to deal with them can make it easier to reduce system costs. The following diagrams are formed.

As it can be seen in Fig. 5, if the proposed method is used to control the disruptions, all the total back orders cost of the system decreases. This increase is quite predictable, since any disruption always causes some of the system performance on meeting the demand to be eliminated and forced to take actions such as back orders.
A quantitative model for optimization and disruption mitigation in a supply chain...

In Fig. 5, it can be seen if the proposed method is not used to control the disruptions, the total lost sales cost of the system increases. This increase is quite predictable, since any disruption always causes some of the system performance on meeting the demand to be eliminated and forced to take actions such as lost sales.

**Figure 5. Total back orders cost: research results vs normal results**

In Fig. 6, it can be seen if the proposed method is used to control the disruptions, the outsourcing cost of the system decreases. This increase is quite predictable, since any disruption always causes some of the system performance on meeting the demand to be eliminated and forced to take actions such as outsourcing.

**Figure 6. Total lost sales cost: research results vs normal results**

In Fig. 7, it can be seen if the proposed method is used to control the disruptions, the outsourcing cost of the system decreases. This increase is quite predictable, since any disruption always causes some of the system performance on meeting the demand to be eliminated and forced to take actions such as outsourcing.
Figure 7. Total outsourcing cost: research results vs normal results

6. Conclusions

Developing a quantitative approach for managing any changes in data and for creating a predictive and reactive mitigation solution in a multiproduct supply chain was main objective of this paper. In the case of any data changes that were known in advance, the supply chain plan was revised according to them. Also, this paper developed a quantitative reactive mitigation plan for managing sudden production disruptions that were not known and cannot be predicted. The supply chain plan was revised for a limited interval into the future, just after happening a disruption. The objective is to minimize the effect of the disruption and to return the system to its ideal plan as soon as possible. For this purpose, this research developed an impressive heuristic to obtain the reactive plan. Also an extended heuristic was developed for managing a series of disruptions, because they are common in real-life supply chain practice. Several numerical examples were presented to analyze the results and usefulness of the models. As the developed approaches have great practical implications, so it can be said that the proposed quantitative and heuristic approaches offer a potentially useful quantitative means of helping decision makers arrive at prompt and accurate decisions regarding both predictive and reactive mitigation plans. Organizations can immediately revise their plans (a) for any data changes, (b) for any future prediction of demand changes, and/or (c) after the occurrences of any single and/or series of sudden production disruptions in the organizations. Moreover, this paper presented an innovative method for solving the minimization of the total cost of the whole supply chain. Three-level supply chain consists of three manufacturing plants, four distributors and five retailers whose products are strategic car spare parts. The results showed that, using proposed model in considering the chain as a whole unit and minimizing the cost of the chain, the chain profit would be increased and the chain survival would be guaranteed. It is suggested that the model presented in this study can be used to optimize the cost of supply chain in different industries. In particular, because of dealing with spare parts for automobiles in this study, model usage has become more tangible for decision makers in the supply chain of auto spare parts, as well as experts in the planning departments of auto parts production factories. However, the application of the model is not limited to this area of the industry and is applicable to all three-level supply chains by applying the necessary inputs. Companies operating in this competitive industry should understand the necessity of product production based on the actual demand of customers. The advancements in technology, the changing market conditions, and the competition of competitors change the buyer’s minds fast, and rational action to improve the position of the product is needed more than ever. Because, customers tend to buy the products that offer them more value at a lower price. Any competitor who can provide more value to customers at a lower price can bring more market share. Hence, it is imperative that each member minimizes costs by updating models and patterns because
today's increasingly complex forms of production and sales cannot be followed by traditional patterns.
The following approaches can be proposed for the future studies:
The proposed approaches can be extended by considering sudden disruptions in supply, distribution and delivery.
Expansion of the model for example the number of supply chain levels can be increased.
Addressing the demand uncertainty and the supply of returned products in a multi-product integrated logistics network.
Consideration of green supply chain (addressing environmental issues) is also proposed.

References


