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## A bi-objective model for the assembly flow shop scheduling problem with sequence dependent setup times and considering energy consumption

Seyed Mohammad Hassan Hosseini<sup>1,\*</sup>

### Abstract

The two-stage assembly flowshop scheduling problem has been studied in this research. Suppose that a number of products of different kinds are needed to be produced. Each product consists of several parts. There are  $m$  uniform machines in the first stage to manufacture the components (parts) of products and there is one assembly station in the second stage to assemble parts into products. Setup operation should be done when a machine starts processing a new part and setup times are treated as separate from processing times. Two objective functions are considered: (1) minimizing the completion time of all products (makespan) as a classic objective, and (2) minimizing the cost of energy consumption as a new objective. Processing speed of each machine is adjustable and the rate of energy consumption of each machine is dependent of its processing speed. At first, this problem is described with an example, and then needed parameters and decision variables are defined. After that, this problem is modeled as a mixed integer linear programming (MILP) and GAMS software is applied to solve small problems. To solve this bi-objective model, Epsilon Constraint algorithm is used on some test problems obtained from standard references. Data of test problems were obtained from previous references and new parameters have been adjusted for the considered problem. Conflicting of two considered objective functions has been validated through the result. In addition, the result of solving test problems and sensitivity analysis shows that how we can reduce energy consumption by adjusting completion times.

**Keywords:** Assembly flowshop; Uniform parallel machines; Sequence dependent setup times; Energy consumption; Bi-objective.

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\* Corresponding author; [sh.hosseini51@gmail.com](mailto:sh.hosseini51@gmail.com)

<sup>1</sup> Department of Industrial Engineering and management, Shahrood University of technology, Shahrood, Iran.

## **1. Introduction**

The scheduling issue is one of the most critical issues in production planning and control systems. This is because of strong competition and limitation of resources in our environment (Cummings & Egbelu, 1998; Maleki-Daroukolaei, Modiri, Tavakkoli-Moghaddam, & Seyyedi, 2012). Hence, after publishing the first paper of Johnson in 1954, there has been an increasing interest in solving scheduling problems during recent years. In particular, manufacturing of almost all items may be modeled as a two-stage production system in which that there is a fabrication stage for processing the components, and an assembly stage to joint components to each other and form products (Allahverdi & Al-Anzi, 2009; Seyed Mohamad Hasan Hosseini, 2019). The fabrication stage may be formatted as parallel machines, flowshop, and jobshop and so on. When the first stage is a form of parallel machines and after that, there is an assembly stage, this production system is known as assembly flowshop that has a high application in two-stage manufacturing systems. This production system that is a generalization of flowshop, has many practical applications in industries especially in assembly-driven manufacturing and hence, has received an increasing attention of researchers recently (Arroyo & Armentano, 2005; Loukil, Teghem, & Tuytens, 2005). Fire engine assembly plant was the first application of this problem that studied by Lee et al. (C.-Y. Lee, Cheng, & Lin, 1993). They studied this system as an assembly flow-shop scheduling problem with two stages. In their considered problem, each product is assembled from two types of parts and each type of part is processed by specific machine. There is a third machine in the second stage to assemble the two parts into a product. Makespan was considered as objective function and they proved that the problem with this condition is strongly NP-complete. Therefore, some several special cases of this problem were defined that can be solved in polynomial time.

After Lee et al., many researchers has been investigated assembly flowshop scheduling problem because of its applications in industries (Seyyed Mohammad Hassan Hosseini, 2016). Potts et al. presented another application of this problem in personal computer manufacturing (Potts, Sevast'janov, Strusevich, Van Wassenhove, & Zwaneveld, 1995). Their study aims to minimize the makespan as objective function and so, they developed some heuristic algorithm to solve it. Cheng and Wang also studied the two-machine flow-shop scheduling with a special structure and develop several properties of an optimal solution and obtain optimal schedules for some special cases (Cheng & Wang, 1999). They considered two kind of parts called "unique" and "common" components, which have been produced by the first machine in batches and so, a setup is needed to form each batch. However, the unique components are processed individually. The second machine assembles components into products.

Yokoyama and Santos introduced another study on flow-shop scheduling problem with assembly operations (Yokoyama & Santos, 2005). They supposed that, several products of different kinds are ordered to be produced and each product is formed by assembly its own components. They developed a solution procedure to obtain an  $\epsilon$ -optimal solution based on a branch-and-bound method.

The two-stage assembly scheduling problem was studied by Allahverdi and Al-Anzi (Allahverdi & Al-Anzi, 2009). They considered this system in condition that there are  $m$  machines at the first stage and an assembly machine at the second stage with set up times separated from the processing times. They proved this problem is NP-hard, and therefore presented a dominance relation and proposed some heuristics based on a hybrid approach of tabu search, a self-adaptive differential evolution (SDE), and a new self-adaptive differential evolution (NSDE). They show that the NSDE is the best heuristic for the problem even if setup times are ignored.

Al-Anzi and Allahverdi studied the same problem as Allahverdi et al. where setup times are ignored. They proposed heuristics based on tabu search (Tabu), particle swarm optimization (PSO), and self-adaptive differential evolution (SDE) along with the earliest due date (EDD)

and Johnson (JNS) heuristics to solve the problem. Computational experiment reveals that both PSO and SDE are much superior to tabu. Moreover, it is statistically shown that PSO performs better than SDE (Al-Anzi & Allahverdi, 2009).

Fattahi et al. have studied a two-stage production system that was consisted of a hybrid flowshop and an assembly stage. Different kinds of products were produced in their study and each of them has been assembled with a multipart set. They have used mathematical modeling for this problem to minimize the completion time of all products (makespan). In addition, a series of heuristic algorithms have been proposed by considering the Johnson algorithm in order to solve large-sized problems. Then, two lower bounds have been introduced and improved to evaluate the final solution obtained from heuristics algorithms (Fattahi, Hosseini, & Jolai, 2013). Also, a hybrid flow shop scheduling problem (HFSP) with assembly operations and considering setup times has been studied by Fattahi et al. They have produced a number of identical products while each of these products were assembled by using a multipart set. Initially, the parts have been created in a hybrid flow shop and after that they were put together to form the products. In fact, minimizing the completion time of all products has been considered as the main objective (makespan). Because of this problem has been proved strongly NP-hard, so a hierarchical branch and bound algorithm has been presented to solve it. Also, some lower and upper bounds are developed to increase the efficiency of the proposed algorithm (Fattahi, Hosseini, Jolai, & Tavakkoli-Moghaddam, 2014). Lee et al. studied a two-stage assembly-type flowshop scheduling problem in which that the first stage consists of two independent machines, and the second stage consists of a single machine to assemble products. Each product is formed by assembly of two types of components that are fabricated in the first stage. Some dominance properties and lower bounds have been developed, and a branch and bound algorithm has been presented. They tried to minimize the total tardiness as a new objective function (J.-Y. Lee & Bang, 2016). Lee proposed a branch and bound for a two-stage assembly problem to minimize the total completion time (I. S. Lee, 2018). He also developed six lower bounds to increase performance efficiency of B&B algorithm. Also, four efficient heuristic algorithms have been developed to generate near-optimal schedules in his study. The results show that the derived B&B and heuristic algorithms perform very well. It is also important that some study have been done on

Most of research in scheduling problems is concerned with the minimization of a single criterion. So, up to the 1980s, scheduling researches were mainly concentrated on optimizing single performance measures such as makespan ( $C_{max}$ ), total flow time (F), maximum tardiness ( $T_{max}$ ), total tardiness (T) and number of tardy jobs ( $n_T$ ) (Arroyo & Armentano, 2005). While, most real world problems cope with several conflicting objectives. Therefore, considering some conflicting objectives is vital to make an optimization problem more realistic (Salmasnia, Mousavi-Saleh, & Mokhtari, 2018). For example in a company, the production manager tends to deliver products as soon as possible and minimize makespan. Also the commercial manager is interested in satisfying customers and then minimizing the tardiness. On the other hand, the financial manager and also department of energy focus on energy consumption and its costs. Each of these objectives are valid from a general point of view. Since, these are considered conflicting objectives, some of the solutions might be desirable for an objective and others might be undesirable. Therefore, a scheduling problem as a difficult decision making problem has often a multi-objective nature.

Energy is one of the most important sources in production systems. A lot of world's electrical energy and fossil fuels are being consumed by production industries daily. So, industries play a key role in energy consumption and environmental impacts. On the other hand, cost of energy is an important element in manufacture and saving in energy consumption lead to more profitability.

There have been many reasons to place reduction of energy consumption as a new objective in production industries and organizations such as:

- To save their money and increase their profitability.
- To improve the national economy.
- To keep environment against pollution.
- To improve national security by providing demands of health centers, transportations, houses, schools, and so on.
- To help preserve various habitat

Therefore, the energy consumption of the operations has to be considered in every organization such as production systems. Scheduling of production is one of the most activity with a lot of influence on energy consumption (Merkert et al., 2015). Although integration of the energy-awareness into production scheduling is getting more and more attention (Biel & Glock, 2016; Dooren, Sys, Toffolo, Wauters, & Berghe, 2017; Mansouri, Aktas, & Besikci, 2016), there is still a gap between industrial needs and academic research (Plitsos, Repoussis, Mourtos, & Tarantilis, 2017).

Some methods and solutions have been used to solve the problems of energy waste during production processes during recent years. Also, many operational methods have been utilized to reduce energy consumption and environmental effects. Yan et al. (2005) conducted one of the most relevant studies about reduction of energy consumption in production systems. These researchers presented a mathematical model for minimizing energy consumption and the makespan for job-shop scheduling problem. They also developed a heuristic algorithm to identify a good solutions for the model based on the “Tabu search mechanism” (He, Liu, Cao, & Li, 2005). After that, Mouzon et al. considered a single machine-scheduling problem to minimize total energy consumption. They proposed operation dispatching rules as steps to minimizing energy consumption: the machine could be shut down if the energy consumption for turning it off or on was less than the idle energy consumption (Mouzon, Yildirim, & Twomey, 2007). Liu et al. studied the hybrid flowshop scheduling problem to minimize energy consumption. They proposed a mixed-integer nonlinear programming model for this problem. They also applied an improved genetic algorithm to solve this problem while energy consumption was mainly considered and the makespan was a key constraint (Xiang Liu, Fengxing Zou, & Xiangping Zhang, 2008). Some others mathematical models have also been proposed for dynamic scheduling in flexible manufacturing systems (FMS) with energy consumption minimization by Zhanga et al. (Zhang, Li, Gao, Zhang, & Wen, 2012). Yi et al. proposed an emission-aware multi-machine job shop-scheduling model for minimizing both carbon emissions and makespan. They used a multi-objective genetic algorithm (MOGA) to solve this bi-objective optimization problem (Yi, Li, Tang, & Wang, 2012). Luo et al. studied a multi-objective scheduling problem in a hybrid flow shop. Two objectives considered in their proposed model were to minimize makespan and energy consumption. They developed a Preference Vector Ant Colony System (PVACS) to search for a set of Pareto-optimal solutions. They compared performance of PVACS to two well-known multi-objective genetic algorithms: SPEA2 and NSGA-II. The experimental results shown that PVACS outperforms the other two algorithms (Luo et al. 2013).

Fadi et al. considered a single machine production system and proposed a mathematical model for scheduling job processing in order to minimize energy consumption costs for during production processes. Their solution approach minimizes total energy consumption costs by determination the launch times for job processing, idle time, when the machine must be shut down, “turning on” time, and “turning off” time. They introduced a genetic algorithm to obtain a ‘near’ optimal solutions (Shrouf, Ordieres-Meré, García-Sánchez, & Ortega-Mier, 2014).

Mansouri et al. addressed energy consumption as an explicit criterion in flowshop scheduling. They considered variable speed of machining operations leading to different energy consumption levels and then traded-off between minimizing makespan, and total energy consumption, as an indicator for environmental sustainability. A mixed integer linear multi-objective optimization model has been developed to find the Pareto frontier comprised of makespan and total energy consumption. They also developed a new constructive heuristic for fast trade-off analysis between makespan and energy consumption (Mansouri et al., 2016). Modos et al. studied a scheduling problem in a production system with large electricity consumption considering the total energy consumption limits. The aim of their study is to design a robust production schedules pro-actively guaranteeing that the energy consumption limits are not violated. They scheduled jobs on one machine with release times of the operations and considered total tardiness as the objective function. In order to solve this problem, they proposed a pseudo-polynomial algorithm for finding the optimal robust schedule. They also developed two exact (Branch-and-Bound and logic-based Benders decomposition) and one heuristic algorithm (tabu search) for this problem (Módos, Šůcha, & Hanzálek, 2017).

To the best of authors' knowledge, assembly flow shops considering cost of energy have not been explored yet. So, the energy consumption and its cost is considered as a new objective in this study for the assembly flowshop scheduling problem. We also consider sequence dependent setup times for all machines in stage 1 as a realistic assumption for this problem. Therefore, definition a new feature for the assembly flow shop scheduling problem considering more applicable conditions such as energy consumption is the main contribution of this study. The remaining sections of this paper are organized as follow: In section 2, the problem is described completely via a numeric example and the mathematical model is presented. Using epsilon constraint method to solve problem is illustrated in section 3. Adjusted test problems and experiments is described in section 4 while the evaluation and analysis of result is done in this section. Finally, a summary of the work and direction for the future research are given in section 5.

## 2. Problem description

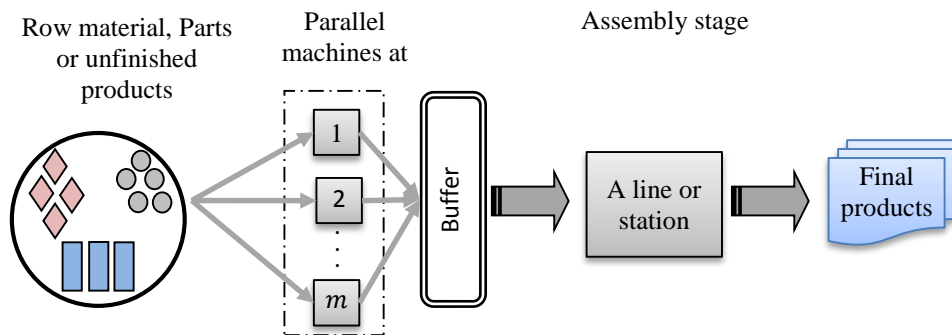
A two-stage assembly-type flowshop scheduling problem is studied in this paper. Figure (1) presents a schematic view of the considered problem. The first stage consists of  $m$  independent machines that fabricate parts, and the second stage consists of a single machine to assemble products. Each product is formed by assembly special parts and components of products are different to each other's. Setup operations and setup times are considered. Two objective functions are considered: (1) minimizing the completion time of all products (makespan) as a classic objective, and (2) minimizing the cost of energy consumption as a new objective. Machines in the first stage can process parts in three different speed and so, the rate of energy consumption of each machine is dependent of its processing speed.

The parallel machines considered uniform. As we know, the parallel machines in a scheduling problem can be classified in to the following types.

- Identical parallel machines scheduling problem.
- Uniform/proportional parallel machines scheduling problem.
- Unrelated parallel machines scheduling problem.

Let,  $t_{ij}$  be the processing times of the job  $j$  on the machine  $i$  with a specified and normal speed, for  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ . Then if the actual process time of part  $j$  on machine  $m$  is  $\frac{t_{ij}}{v_s}$  for all  $i$  and  $j$ , where  $v_s$  is the speed of the machine  $i$ , then a uniform (proportional) parallel

machines scheduling problem would be formed. This means that the parallel machines can have different speeds for processing parts.



**Figure 1. A schematic view of the considered problem**

The other basic assumptions of this problem are provided as below:

- ✓ All parts are available at time zero.
- ✓ The parallel machines in the first stage are unrelated.
- ✓ Each machine can process only one job at a time.
- ✓ Each job can be processed by only one machine at a time.
- ✓ Time processing of jobs and assembly of products is deterministic.
- ✓ Assembly operation for a product will not start until all parts of its product are completed.
- ✓ When assembly operation of a product is started, it doesn't stopped until completed (no preemption in assembly stage)
- ✓ There is no limited in buffer storages

In addition, the following notations are used to formulate the considered problem:

**Indices**

$h, h' = 1, 2, \dots, H$	Index for products
$i, j = 1, 2, \dots, n$	Index for parts
$J_h$	Set of parts for product $h$
$l = 1, 2$	Index for stage
$m = 1, 2, \dots, M$	Index for parallel machines in stage 1
$s = 1, 2, 3$	Index for processing speed

**Parameters**

$P_{im}$	Processing time of part $i$ on machine $m$ at normal speed ( $s = 2$ )
$A_h$	Assembly time of product $h$
$S_{ijm}$	Setup time for switching part $i$ to part $j$ on machine $m$
$S_{0jm}$	Setup time for the first part ( $j$ ) that is processed on machine $m$
$EV_{im}$	Binary variable taking value 1 if machine $m$ capable to process part $i$ , and 0, otherwise
$EN_{pms}$	Energy consumption per unit time of machine $m$ in processing mode with speed of $s$

$EN_{sm}$	Energy consumption per unit time of machine $m$ in set up mode
$v_s$	Processing speed that is fast, normal, and slow for $s = 1, 2, 3$ respectively
$u$	Cost of energy per time
$M$	A large number

**Decision variables**

$X_{ims}$	Binary variable taking value 1 if part $i$ is processed on machine $m$ at speed of $s$ , and 0 otherwise
$Y_{ijms}$	Binary variable taking value 1 if part $j$ is assigned to machine $m$ at speed of $s$ immediately after part $i$
$Y_{0jms}$	Binary variable taking value 1 if part $j$ is the first part on machine $m$ , and 0 otherwise
$Y_{i0ms}$	Binary variable taking value 1, if job $i$ is the last job on machine $m$ , and 0 otherwise
$AS_{h'h}$	Binary variable taking value 1, if assembly operation of product $h$ start immediately after that product $h'$ , and 0 otherwise
$AS_{0h}$	Binary variable taking value 1 if product $h$ is the first product that is assembled, and 0 otherwise
$AS_{h'0}$	Binary variable taking value 1 if product $h'$ is the last product that is assembled, and 0 otherwise
$F_j$	Finish time of part $j$ on stage 1
$TF_h$	Finish time of the set part of product $h$ at the first stage
$C_h$	Completion time of assemble the product $h$
$C_{max}$	Completion time of all products
$C_e$	Total cost of energy consumption

### 3. Mathematical modeling

Fattahi et al. presented the scheduling problem of a hybrid flowshop followed by an assembly stage and developed a mixed integer linear programming (MILP) model for this problem. Their mathematical is adopted for our considered problem in this study with modification the hybridflow shop to one stage consists of  $m$  uniform parallel machines. Also new parameters, new decision variables, and new operational constraints are added. According to this, and based on the present problem and notations, a new mathematical formulation for the problem is presented as follow:

$$\text{Min } Z_1 = (C_{max}) \tag{1}$$

$$\text{Min } Z_2 = (C_e) \tag{2}$$

Subject to:

$$\sum_{m=1}^M \sum_{s=1}^3 X_{ims} = 1 \quad i=1, 2, \dots, n \tag{3}$$

$$\sum_{i=0, i \neq j}^n \sum_{s=1}^3 Y_{ijms} = \sum_{s=1}^3 X_{jms} \quad j=1, 2, \dots, n, \quad m=1, 2, \dots, M \tag{4}$$

$$\sum_{j=0, j \neq i}^n \sum_{s=1}^3 Y_{ijms} = \sum_{s=1}^3 X_{ims} \quad i=1, 2, \dots, n, \quad m=1, 2, \dots, M \tag{5}$$

$$\sum_{i=0}^n \sum_{s=1}^3 Y_{ijms} \leq 1 \quad j=1, 2, \dots, n, \quad m=1, 2, \dots, M \tag{6}$$

$$\sum_{j=0}^n \sum_{s=1}^3 Y_{ijms} \leq 1 \quad i=1, 2, \dots, n, \quad m=1, 2, \dots, M \tag{7}$$

$$\sum_{j=1}^n \sum_{s=1}^3 Y_{0jms} = 1 \quad m=1, 2, \dots, M \quad (8)$$

$$\sum_{i=1}^n \sum_{s=1}^3 Y_{i0ms} = 1 \quad m=1, 2, \dots, M \quad (9)$$

$$\sum_{s=1}^3 X_{ims} \leq EV_{im} \quad i=1, 2, \dots, n, \quad m=1, 2, \dots, M \quad (10)$$

$$F_j \geq \sum_{m=1}^M \sum_{s=1}^3 \left( \frac{P_{jm}}{v_s} \times X_{ims} \right) + \sum_{m=1}^M \sum_{i=0}^n \sum_{s=1}^3 [(F_i + S_{ijm}) \times Y_{ijms}] \quad j=1, 2, \dots, n \quad (11)$$

$$TF_h \geq F_j \quad \forall j \in \{J_h\}, \quad h=1, 2, 3, 4, \dots, H \quad (12)$$

$$\sum_{h'=0, h' \neq h}^H AS_{h'h} = 1 \quad \forall h = 1, 2, 3, 4, \dots, H \quad (13)$$

$$\sum_{h=0, h \neq h'}^H AS_{h'h} = 1 \quad \forall h' = 1, 2, 3, 4, \dots, H \quad (14)$$

$$C_h \geq TF_h + A_h \quad h=1, 2, 3, 4, \dots, H \quad (15)$$

$$C_h \geq (AS_{h'h} - 1) \times M + C_{h'} + A_h \quad h, h' = 1, 2, 3, 4, \dots, H \quad (16)$$

$$C_{max} \geq C_h \quad \forall h=1, 2, 3, \dots, H \quad (17)$$

$$C_e = u \times \left[ \sum_{i=1}^n \sum_{m=1}^M \sum_{s=1}^3 \left( \frac{P_{im}}{v_s} \times EN_{pms} \times X_{ims} \right) + \sum_{i=0, i \neq j}^n \sum_j \sum_{m=1}^M \sum_{s=1}^3 (S_{ijm} \times EN_{sm} \times Y_{ijms}) \right] \quad (18)$$

$$X_{ims} \in \{0,1\} \quad i, j = 0, 1, 2, \dots, n ; \quad m = 1, 2, \dots, M \quad (19)$$

$$Y_{ijms} \in \{0,1\} \quad i, j = 0, 1, \dots, n ; \quad m = 1, 2, \dots, M \quad (20)$$

$$s = 1, 2, 3$$

$$AS_{h'h} \in \{0,1\} \quad h, h' = 0, 1, 2, \dots, H \quad (21)$$

According relation (1) of the mathematical model, completion time of all products is minimized (the first objective). Also relation (2) minimizes total cost of energy consumption. Constraint (3) indicates that each part is processed precisely once by one machine with a certain speed. Constraints (4) to (7) state that each machine can process just one part at each time. In particular, constraints (4) and (5) indicate that each part on each machine with a specified speed, inevitably is processed before or after a job which including dummy part 0. Also, Constraints (6) and (7) ensure that each part is assigned exactly to one machine with a specified speed. Constraints (8) and (9) show that the first and last part that are process by each machine are unique. Constraint (10) indicates the eligibility restrictions. Constraint (11) take care of the completion times for each part. Completion time of each part should be equal or greater than sum of its processing time and its previous part completion. Constraint (12) indicates that the assembly operation of each product can be starts after completion all needed parts. Constraints



(13) and (14) are used to determine assembly sequence of products. Constraints (15) ensure that the completion time of each product is at least equal to the sum of completion time of its parts and its assembly time. Constraint (16) ensure that the completion time of each product is at least equal to the sum of its assembly time and completion time of its before product. Constraint (17) is used to compute the first objective function (makespan). Equation of (18) is used to compute total cost of Energy consumption as the second objective function. Finally, constraints (19) to (21) are used to indicate the type of decision variables.

#### 4. Numerical example

In order to clarify and better understanding the problem, a numerical example is provided. Assume two products are needed to be produced. Product 1 is formed by assembling of three parts and product 2 is formed by assembling of two parts. There are two parallel machines in stage 1 to process and ready the parts, and after that, there are a single station to assemble the parts to form products. The input data for parts and their processing time of machining operation and assembly are given in table 1. In additional, the amount of the cost of energy per time is considered equal to 1 \$.

At first, we scheduled the products and assigned their parts to machines according their number (as a random scheduling). For simplicity, assume that machines process all parts in normal speed. The result is shown in figure (2) with value of  $Z_1 = 66$  and  $Z_2 = 491$ . Figure (3) shows another scheduling of this sample with value of  $Z_1 = 61$  and  $Z_2 = 430$ . So, the second solution has dominated the first. In addition, we have obtained another scheduling from solving model with value of  $Z_1 = 61$  and  $Z_2 = 385$  as the third solution that has been presented in figure (4). Although this solution is equal to the second in first objective function ( $Z_1 = 61$  for both of these two solutions) but it has been dominated because it is worse in the second objective function ( $Z_2$ ). In these three figures, the yellow color denote setup operations, the gray cells denote machining operations (process operations of the parts), and the blue shows assembly operations. Parts and products are shown with their numbers in these figures.

**Table 1. Data of a numerical example**

Product (h)	1			2	
	1	2	3	4	5
Part	1	2	3	4	5
$P_{i1}$ at low speed (s=1)	16	19	10	15	21
$P_{i1}$ at normal speed (s=2)	12	15	9	12	18
$P_{i1}$ at high speed (s=3)	9	12	7	10	15
$P_{i2}$ at low speed (s=1)	8	22	n	12	8
$P_{i2}$ at normal speed (s=2)	7	18	n	10	6
$P_{i2}$ at high speed (s=3)	5	15	n	7	5
$EV_{i1}$	1	1	0	1	1
$EV_{i2}$	1	1	1	1	1
$A_h$	20			12	

**Table 2. Setup time on machine 1 in the numerical example**

From part \ To part	1	2	3	4	5
0	4	1	2	1	3
1	0	3	5	6	4
2	5	0	1	3	4
3	2	4	0	2	6
4	2	2	0	0	4
5	1	2	2	1	0

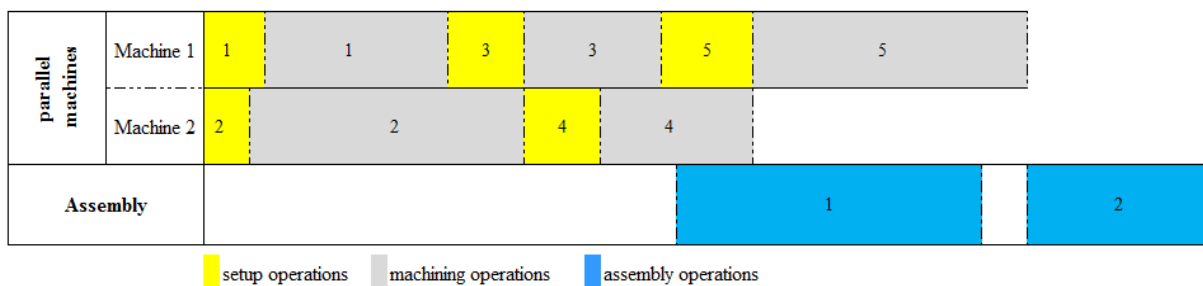
**Table 3. Setup time on machine 2 in the numerical example**

From part \ To part	1	2	3	4	5
0	2	3	n	3	3
1	0	1	n	2	1
2	1	0	n	5	1
3	3	3	n	1	3
4	2	2	n	0	1
5	3	1	n	2	0

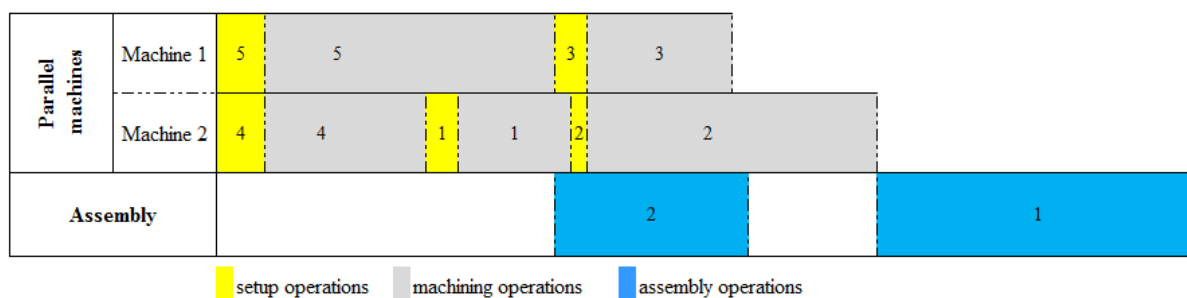
**Table 4. Energy consumption of machine  $m$  in the numerical example**

$m$	1			2		
$s$	1	2	3	1	2	3
$EN_{pms}$	2	5	7	3	7	10
$EN_{sm}$	4			5		

These results also are presented in figure (5). It is considerable that solution 1 is different in sequencing of product with solutions 2 and 3. And also, difference between solutions 2 and 3 is related to parts allocation to machines.



**Figure 2. A schedule for numerical example according products and parts number with  $Z_1 = 66$  and  $Z_2 = 491$**



**Figure 3. A new schedule for numerical example with  $Z_1 = 61$  and  $Z_2 = 430$**

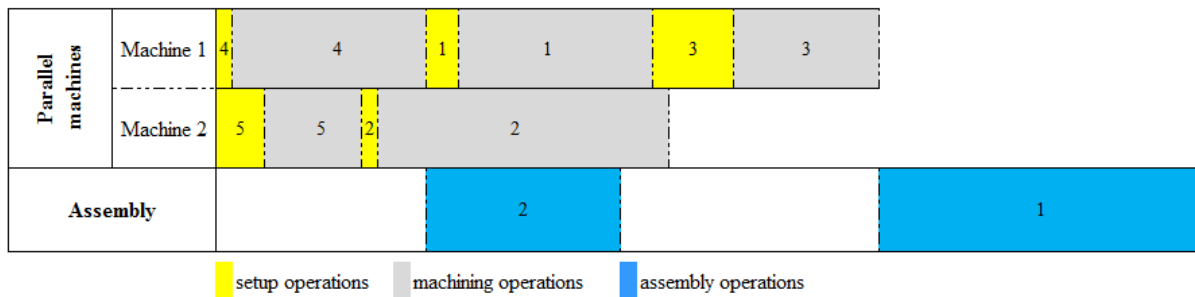


Figure 4. The best schedule for numerical example with  $Z_1 = 61$  and  $Z_2 = 385$

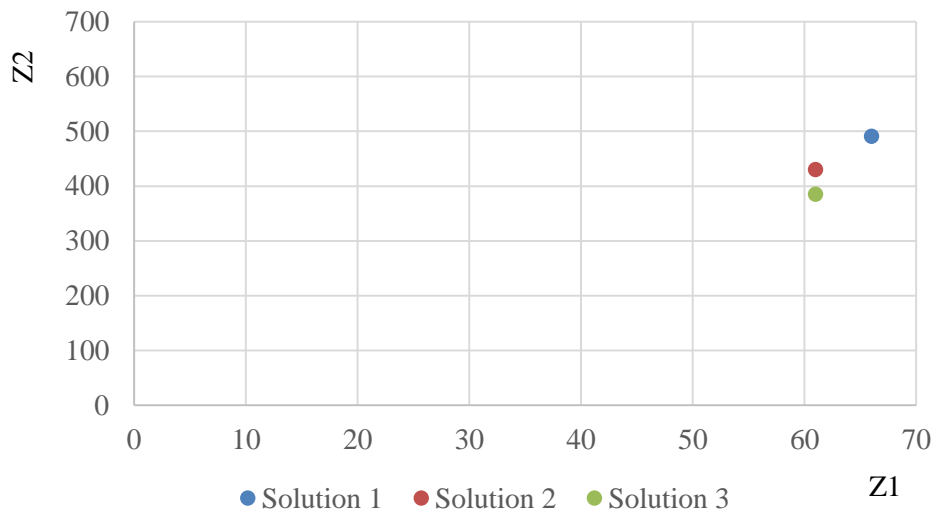


Figure 5. Comparison of three solutions the for numerical example

#### 4.1. Using epsilon constraint method to solve problem

Generally, the minimization of an objective function with respect to the constraints is considered to be the main constraint in an optimization problem. Some of which are an inherent part of the original problem, while others may be introduced by way of the multi-objective problem formulation. It should also be noted that the multi-objective optimization problem doesn't necessarily result in a single solution. Due to the existence of multiple objectives with their specific competitive nature, the possibility of obtaining an unlimited number of solutions, each of which prioritizes the objectives of the problem, is not entirely far from the mind. These kinds of solutions called Pareto points, which a collection of them comprises, the Pareto optimal set. The definition for the Pareto optimality is defined as the following:

**Pareto Optimality:** A point  $x^* \in X$  is Pareto optimal if and only if there does not an exist another point  $x \in X$ , such that  $F(x) \leq F(x^*)$ , and  $t F_i(x) \leq F_i(x^*)$  for at least one function.

The  $\epsilon$ -constraint method is a very intuitive and simple method to apply to bi-criterion problems to achieve Pareto optimal set. This method was first proposed by Haimes et al. in 1971 which emphasizes on selecting one of the objective functions and optimizing it while the other functions are transformed to additional constraints, leading to a solution that can be proven to always be weakly Pareto optimal (Chircop & Zammit-Mangion, 2013; Fattahi, Hosseini, Jolai, & safi Samghabadi, 2014). Indeed, generating an evenly distributed Pareto frontier has been originated from modifying the values of the objective functions, which form the additional constraints. In fact, it can be stated that an equidistant is a method for modifying the additional

constraints in a systematic way. Also, the general form of the  $\mathcal{E}$ -constraint method has been demonstrated as the following (Yahia, Felfel, Ayadi, & Masmoudi, 2015):

$$\min f_j(x)$$

$$f_i(x) \leq \varepsilon_i \text{ for } i = 1, 2, \dots, n; \quad i \neq j; \quad \varepsilon_i = RL_k$$

$$x \in X$$

It is important to note that this model has multiple run times. Each time an objective function has been considered as the main objective to be optimized while the other (s) are converted into additional constraints. There are two objective functions in considered problem of this study, included minimizing the completion time of all products as  $f_1$ , and minimizing total cost of energy consumption as  $f_2$ . In order to use the  $\mathcal{E}$ -constraint method for solving the bi-objective considered problem, 25 cut points have been considered for each objective. So, we will have 50 Pareto points totally. Since in some cut points just one objective has been worse and other point has not been better, so such these points are known as dominated points and are eliminated from continuing. Finally, the set points of non-dominated solutions are presented as an index of Overall Non-dominated Vector Generation (ONVG).

## **5. Result analysis on test problems**

This section presents the results of solving test problems using proposed mathematical model. The mathematical model was run in GAMS and experiments are executed on a Pc with a 2.0GHz Intel Core 2 Duo processor and 4GB of RAM memory. Data of test problems were obtained from Fattahi et al. (2013) as table (5) and new parameters have been adjusted for considered problem as table (6). Also in study of Fattahi et al. the machining section is a hybrid flow shop and so, their processing data of the first stage have been considered for parallel machines of this study. These data about processing (machining) times are considered for normal speed of machines and for fast and slow status its calculated based on  $v_s$ .

After solving test problems, the results include ONVG index and the best value of two objective functions have been presented in table (7). The best value of each objective function has been obtained by solving mathematical model without considering another objective and vice versa.

For additional analysis, the proposed model is applied to obtain the Pareto optimal for a sample of test problems. So, the problem number H-A19 has been considered and twenty Pareto optimal solutions have been presented in figure (6). These solutions have been obtained from 25 cut points of two objective functions and so 50 possible Pareto points totally. Because of some solutions have been dominated by the others, so the Overall Non-dominated Vector Generation (ONVG) is usually less than total possible Pareto points. For this test problem, 20 Pareto points were obtained in totally from 50 solution points.

**Table 5. Problem generation**

Problem name	H	$J_h$	$m$	$S_{ljm}$	$P_{im}$	$A_h$
H-A1	7	[2, 4]	2	[10, 50]	[30, 40]	[50, 100]
H-A4	7	[2, 4]	3	[10, 50]	[30, 40]	[50, 100]
H-A7	7	[2, 4]	4	[10, 50]	[30, 40]	[50, 100]
H-A10	10	[2, 4]	2	[10, 50]	[30, 40]	[50, 100]
H-A13	10	[2, 4]	3	[10, 50]	[30, 40]	[50, 100]
H-A16	10	[2, 4]	4	[10, 50]	[30, 40]	[50, 100]
H-A19	15	[2, 4]	2	[10, 50]	[30, 40]	[50, 100]
H-A22	15	[2, 4]	3	[10, 50]	[30, 40]	[50, 100]
H-A25	15	[2, 4]	4	[10, 50]	[30, 40]	[50, 100]
H-A28	25	[2, 4]	2	[10, 50]	[30, 40]	[50, 100]
H-A31	25	[2, 4]	3	[10, 50]	[30, 40]	[50, 100]
H-A34	25	[2, 4]	4	[10, 50]	[30, 40]	[50, 100]

**Table 6. Specification of machines for test problems**

	$v_s$	Chance of capability to process a part	$EN_{pms}$	$EN_{sm}$	$u$
S=1	1.2	0.9	[5, 7]	[1, 3]	1
S=2	1	0.9	[3, 5]	[1, 3]	1
S=3	0.8	0.9	[1, 3]	[1, 3]	1

**Table 7. The ONVG index and the best value of two objective functions**

Problem name	H	$J_h$	$m$	ONVG	The best value of $f_1$	The best value of $f_2$
H-A1	7	[2, 4]	2	5	790	5756
H-A4	7	[2, 4]	3	7	555	5043
H-A7	7	[2, 4]	4	8	438	4873
H-A10	10	[2, 4]	2	11	1092	7310
H-A13	10	[2, 4]	3	14	757	6938
H-A16	10	[2, 4]	4	18	589	6460
H-A19	15	[2, 4]	2	20	1595	9212
H-A22	15	[2, 4]	3	25	1092	8826
H-A25	15	[2, 4]	4	31	840	85497
H-A28	25	[2, 4]	2	37	2601	17229
H-A31	25	[2, 4]	3	41	1763	16732
H-A34	25	[2, 4]	4	46	1344	16541

Figures (7) and (8) show sensitivity of objectives to changes of the energy consumption per unit time of machines in processing mode and set up mode respectively. According to the part (a) of these two figures, energy consumption changes nearly directly according to changes of  $EN_{pms}$ , but it has a low variation rate by changing in  $EN_{sm}$ . Part (b) of these figures show that completion time of all products ( $C_{max}$ ) is not dependent to changes of the energy consumption per unit time of machines, when the energy consumption is not considered as an objective. According to the part (c),  $C_{max}$  has a sensitivity to changes of the energy consumption per unit time of machines when the total energy consumption is considered as a constraint. Although it's changing rate is more considerable when energy consumption per unit time of machines in processing mode is changed in comparison to the changing of the set up mode.

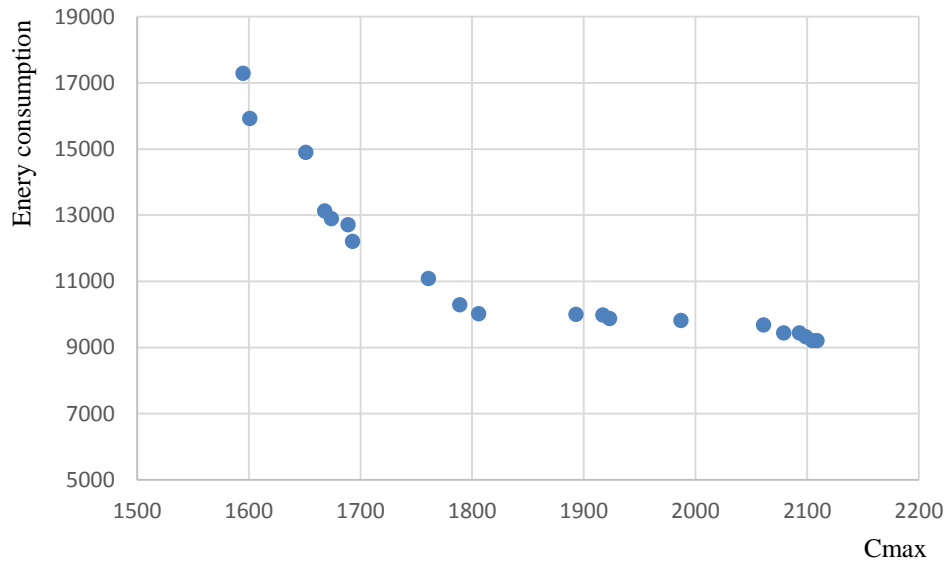


Figure 6. Pareto optimal solutions of the problem H-A19

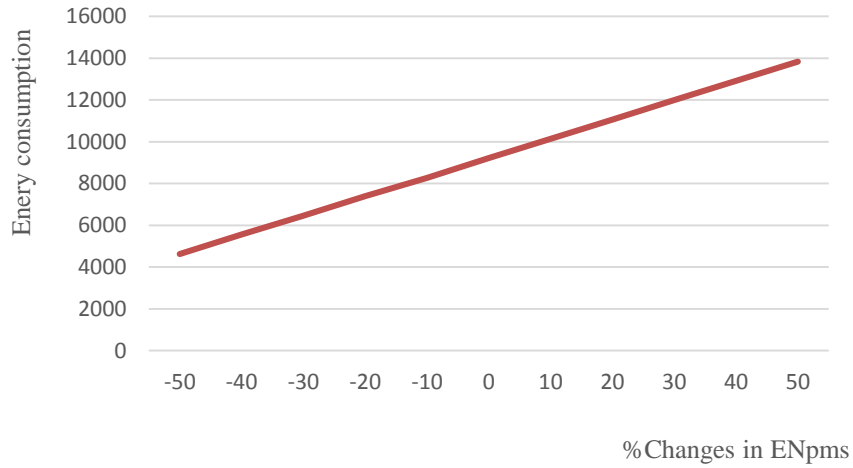


Figure 7 (a). Impact of the  $EN_{pms}$  rate variation on the total energy consumption

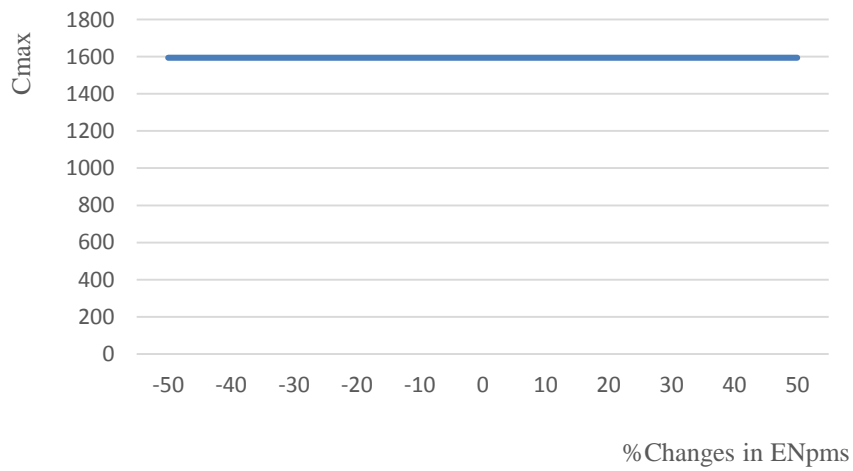


Figure 7 (b). Impact of the  $EN_{pms}$  rate variation on the makespan

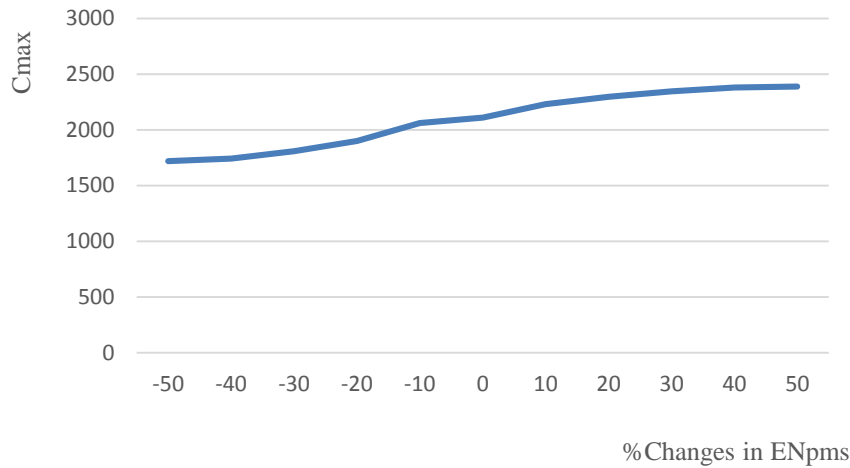


Figure 7 (c). Impact of the  $EN_{pms}$  rate variation on the makespan when total energy consumption is considered fixed

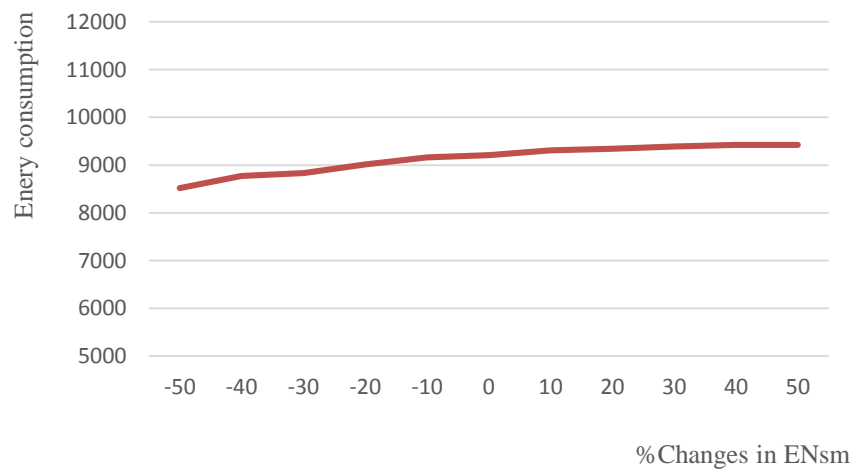


Figure 8 (a). Impact of the  $EN_{sm}$  rate variation on the total energy consumption

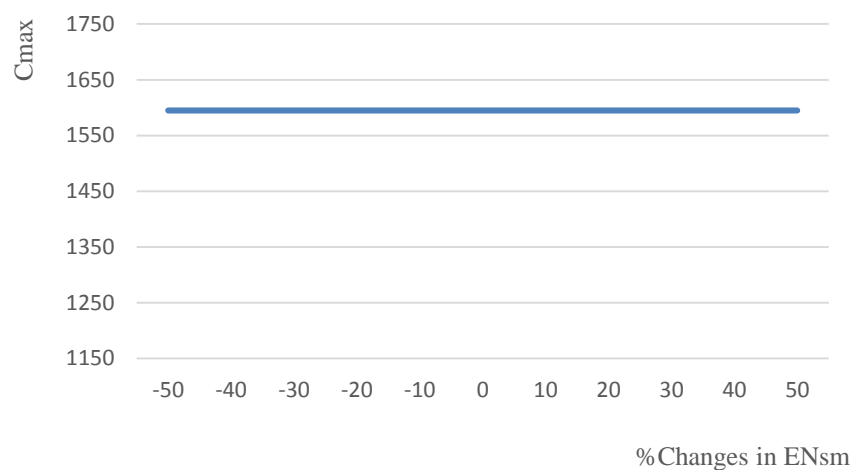


Figure 8 (b). Impact of the  $EN_{sm}$  rate variation on the makespan

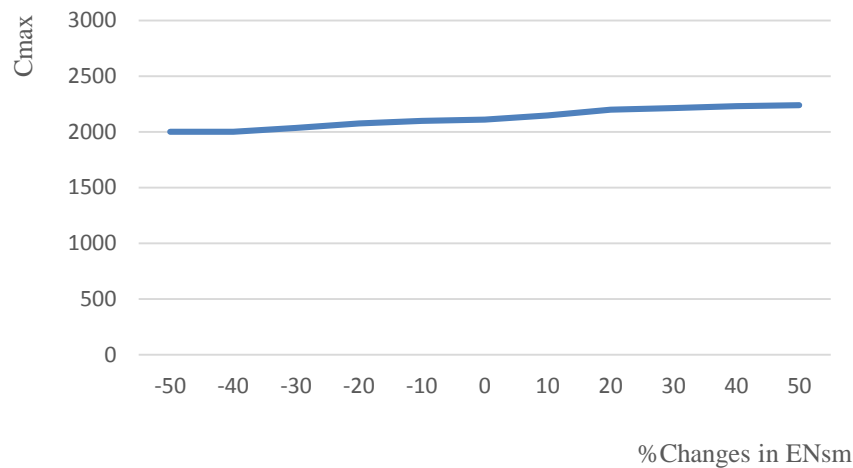


Figure 8 (c). Impact of the  $EN_{sm}$  rate variation on the makespan when total energy consumption is considered fixed

## 6. Conclusion

This paper focused on the problem of scheduling a given set of jobs in a two-stage assembly-type flowshop in which the first stage consists of  $m$  independent parallel machines and the second stage consists of a single machine as the assembly stage. In the first stage, the needed components are fabricated on different machines; then, they are assembled into the final product in the second (or assembly) stage. In this system, the components can be processed by one of the  $m$  independent parallel machines. Setup operation should also be done when a machine starts processing a new part and setup times are treated as separate from the processing times. Each machine can process parts with three different speeds. Energy consumption of machines can be reduced or increased by changing in processing speed. This problem was described via a numeric example. Also a mixed integer linear programming (MILP) as the mathematical model was developed for this problem with two objective functions: (1) minimizing the completion time of all products (makespan), and (2) minimizing the cost of energy consumption.

The methodology of  $\epsilon$  – *constraint* was used to solve this bi-objective problem. Some standard test problems that were introduced in previous studies were used to run the mathematical model. The solutions have been obtained from 25 cut points of each objective function and so 50 possible Pareto points were obtained totally. Because of some solutions were dominated by the others, so the final Overall Non-dominated Vector Generation (ONVG) was presented as possible Pareto points.

Some sensitivity analysis were conducted of main parameters. Due to these result, energy consumption per unit time of machines in processing mode with different speed, has the most impact on the makespan when total energy consumption is considered fixed. According the result, energy consumption can be managed and reduced by adjusting the speed of machines. So, production managers should take attention to energy consumption as one of the important environmental aspect and green production. They can reduce the energy consumption and energy costs by trade offing between two criteria of completion time and energy consumption. This goal can be earned by adjusting the speed if the speed of machines is adjustable and the rate of energy consumption of each machine is dependent of its processing speed. The result shown also that for an example problem, the energy cost can be reduced till 47% by adjusting the speed of processing machines.



Definition a new problem considering more applicable conditions was the main contribution of this study. According to this problem is NP-Hard, using a good heuristic or metaheuristic such as MOGA, NSGA-II and NSGA-III to solve this problem by large-scale dimensions can be a new subject for study. In additional, design and implementation a hybrid approach of metaheuristics (for example NSGAI+VNS algorithm) can improve quality solutions. Modelling and solving this problem under uncertainty can be a new interesting subject to study. For this, some parameters such as process times or assembly operation can be considered as stochastic to more close the problem to practical and real world. Considering different cost for energy consumption such as pick period may be another applicable feature for this problem.

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