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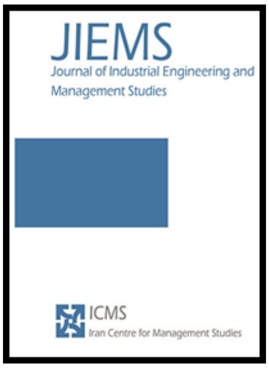

Queuing approach and optimal inventory decisions in a stochastic supply chain network design

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Abstract

This paper addresses an integrated multi-echelon location-allocation-inventory problem in a stochastic supply chain. In a bid to be more realistic, the demand and lead time are considered to be hemmed in by uncertainty. To tackle the proposed supply chain network design problem, a two-phase approach based on queuing and optimization models is devised. The queuing approach is first deployed, which is able to cope with inherent uncertainty of parameters. Afterwards, the proposed supply chain network design problem is formulated using a mixed-integer nonlinear model. Likewise, the convexity of the model is proved and the optimal inventory policy as closed-form is acquired. Inasmuch as the concerned problem belongs to NP-hard problems, two meta-heuristic algorithms are employed, which are capable of circumventing the complexity burden of the model. The numerical examples evince the efficient and effective performance of the solving algorithms. Lastly, sensitivity analyses are conducted through which interesting insights are gained.

Keywords: Supply chain; Stochastic modeling; Queuing theory; Inventory theory; Operations research.

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1. Introduction

A supply chain (SC) is a system comprising activities, people, organizations, resources, and information which is involved services or moving a product from manufacturers to customers (Nagurney, 2006). The competitive conditions of today's world eventuate in the companies focus on improving their SC performance in order to reduce costs and increase the customer satisfaction (Diabat and Theodorou, 2015). One of the most prominent issues in logistics and SC is designing the integrated multi-echelon distribution systems. That is, considering strategic, tactical and operational decisions in an incorporated manner can ameliorate the management across the SC and yield a full-optimized network (Miranda and Garrido, 2009; Sadjadi et al., 2016). Facility location and inventory management problems are the two most outstanding problems in SC optimization that have salient impacts on efficient design of SC networks (Gunasekaran et al., 2001; Stevens, 1989).

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Facility location problems, as strategic decisions, encompass determining the optimal number of open plants and their locations. On the other side, inventory management problems identify the responsiveness of systems in terms of stock availability as tactical decisions. Now, in view of pervious speaking, considering the integrated model of both decisions can lead to better solutions (Diabat, Battaïa, et al., 2015; Ramezani and Naderi, 2018).

In this research, we have presented a joint location-inventory problem in a three-level SC, including a single manufacturer, multiple potential distribution centers (DCs) and retailers. The aim is to identify a set of open DCs to meet the demands coming from retailers. The objective of the presented model is to minimize the total costs which is composed of the fixed cost of locating DCs, transportation cost from DCs to retailers and inventory costs. To make the problem more similar to real-life problems, the demand and lead-time are assumed to be hemmed in by uncertainty. Additionally, there is facility capacitation restriction. Accordingly, our model is an extension of capacitated facility location model, known as a NP-hard problem (Mirchandani and Francis, 1990). To circumvent the complexity burden of the model, two meta-heuristic algorithms, i.e. simulated algorithm (SA) and genetic algorithm (GA), have been devised to solve the problem, especially for large instances to reach near optimal solutions in a logical CPU time. In accordance with the best of our knowledge and the literature review, the main contributions of this research can be considered as follows. 1) This research is one the first studies in the field of location-inventory problems that cope with uncertainty with regard to queuing approach. 2) We have obtained a closed-form for optimal inventory policy of DCs by proving the convexity of objective function. Optimal control policies for stochastic inventories are the contribution of some researches such as Federgruen and Zheng (1993), Chao et al. (2012) and Chen and Feng (2006) in the field of inventory control.

Each open DC holds inventory and replenishes its inventory by ordering to the manufacturer under *(S-1, S)* policy. In current policy, the peak level of stock is *S* (i.e., base stock level) and an order will be placed, whenever the stock level (the summation of on hand and on order quantities) decreases to *S*-1. Speaking intuitively, an order will be placed, whenever a demand is received and satisfied (Hill, 1999). Some applications of (*S*-1, *S*) inventory policy are as follows; 1) military, airlines and computer industries, 2) the case of unit size and recoverable-item inventory systems, 3) the products with expensive stock level controlling, low demand and high holding cost (Kalpakam and Shanthi, 2001), and 4) for spare parts system, when a failed part is exchanged by a new one. Likewise, if the failure rate is low, this inventory policy will be more desirable (Schultz, 1990).

The organization of this paper is as follows. The literature of location-inventory models considering queuing theory approaches are presented in Section 2. Section 3 elaborates the model description and mathematical formulation of the concerned problem. In Section 4, solution approach to solve the proposed problem is surveyed. In Section 5, the computational results and sensitivity analysis are carried out. Lastly, some concluding remarks are presented in Section 6.

2. Literature Review

Here, the literature review has been categorized into two sub-sections comprising location-inventory problems and inventory control models with queuing theory approaches.

2.1. Location-inventory problem

Baumol and Wolfe (1958) are the first authors who considered integrated inventory with location costs. They have studied a procedure to locate the variable number of warehouses. Two-level facility location where the second level has limited capacity has been considered by

Tragantalerngsak et al. (2000). Daskin et al. (2002) have proposed a location-inventory problem considering deterministic lead-time. They also have implemented a number of heuristics to finding near optimal solutions. A routing-inventory network design problem considering risk-pooling has been considered by Shu et al. (2005). This research is the developed model of Shen et al. (2003), where two different algorithms to solve the general model have been proposed. Chew et al. (2007) have studied a stochastic integrated location-inventory model aimed to minimize the total costs including the transportation, inventory and ordering costs in multiple capacitated distribution centers. They have used GA to solve their problem. A capacitated integrated location-inventory model with risk pooling has been investigated by Ozsen et al. (2009). They have used normal distribution function to approximate Poisson demand process. Park et al. (2010) have presented a stochastic network design problem with regard to economic order quantity (EOQ) to approximate the optimal solution of the *(r, Q)* inventory policy. Ahmadi-Javid and Seddighi (2012) have formulated a stochastic distribution network design problem in a SC as a mixed integer convex programming model, which minimizes the total costs of location, inventory and delivery delay. They have developed a heuristic solution approach based on SA and Tuba search to solve large-sized problems. An integrated location-inventory model has been studied by Berman et al. (2012). They have assumed each open DC has a periodic-review and they have modeled their problem as a nonlinear integer-programming problem and used an approximate procedure for determining inventory policy. Diabat, Abdallah, et al. (2015) have provided a closed-loop location-inventory model as a mixed integer nonlinear programming, where the lead-time is deterministic. Shahabi et al. (2014) have developed a stochastic SC considering the correlated demands across the retailers. They have developed a solution approach based on an outer approximation strategy and they have illustrated that their solution procedure performs effective and efficient. Jindal, and Solanki (2016) have presented a two single-vendor single-buyer integrated SC inventory model with inflation and time value of money. Rabbani et al. (2016) have considered a basic mixed-integer non-linear programming model to lease new products. A three-level SC network has been introduced by Sadjadi et al. (2016), where the location, allocation and inventory replenishment decisions are simultaneously optimized. They have analyzed the inventory systems by a Markov process and then the location-inventory problem has been formulated based on obtained results. AmalNick and Qorbanian (2017) have assumed dynamic pricing strategy based on demand value. They have combined neural network and evolutionary algorithms to optimize pricing policies. Applying a queuing approach, Rabbani et al. (2017) have considered a reliable location – inventory problem for a supply chain system. Last but not least, a mixed-integer linear programming model for an integrated SC has been proposed by Alshamsi and Diabat (2018). Díaz-Mateus et al. (2018) have used the constrained multinomial logit for discrete choices to estimate demand levels for a non-linear optimization model. To solve the presented model, they have implemented a metaheuristic approach based on particle swarm optimization.

2.2. Inventory control models considering queuing theory approaches

In this subsection due to the use of queuing approach in our research, the literature of inventory control models considering queuing theory has been reviewed. One of the earliest researches in this field has been proposed by Sigman and Simchi-Levi (1992). They have proposed a light traffic heuristic for the M/G/1 queue system. Berman and Kim (2001) have addressed an optimal inventory control, in which the demand and lead-time are uncertain with Poisson and exponential distributions, respectively. They have applied a Markov decision problem to determine the replenishment policy and then they have proposed a numerical study to evaluate the optimal performance. An integrated queuing-inventory model has been investigated by Schwarz and Daduna (2006), where lead time is exponentially distributed. They have derived explicit performance measures for M/M/1 systems considering inventory under continuous review and backordering. Appling a queuing approach, Krishnamoorthy et al. (2013) have investigated *(s,Q)* and *(s, S)* inventory policies and they have analyzed numerically the per unit time expected cost for these systems. Baek and Moon (2014) have extended a queuing system incorporated with a production-inventory system and they have assumed the arrival of customers with a Poisson processes. Table 1 shows the specifications of the related literature reviews in the field of location-inventory.

In this research, we have proposed a joint stochastic location-inventory problem in a three-level SC. A queuing approach has been implemented to cope with inherent uncertainty and to study the characteristics of the inventory policy. With respect to traditional inventory models, there are enormous advantages when inventory systems are described as a Markov process (see, Frizelle and Jaber (2009) and Saffari et al. (2013)). Owing to NP-hardness of the proposed problem, commercial solvers such as BARON are not able to solve the model efficiently. Accordingly, two meta-heuristic algorithms have been proposed to solve the model. Also, we could achieve the optimal inventory policy for open DCs as a closed-form with regard to the convexity of the objective function.

In accordance with the literature review and Table 1, our main contributions for this research can be considered as follows. 1) Modeling queuing approach in location-inventory problems with stochastic demands and lead time. 2) Introducing a closed-form optimal inventory control for a stochastic supply chain network design problem. In this manner, the number of reorder, the mean of inventory level, backlogged demands and the base stock level are obtained. It is worthy to note that this procedure in addition to determining the optimal inventory of DCs, it can enhance the performance of the proposed algorithms in both terms of solutions' quality and CPU Time.

Table 1. Specification of the researches in the field of location-inventory

Reference	Type of modeling	Objective function	Demand distribution	Lead-time distribution	Inventory Policy	Identifying inventory policy	Type of shortage	Solution procedure
Daskin et al. (2002)	MINLP	Single	NO	Det.	*(R,Q)*	AP	BA	LA
Shu et al. (2005)	MINLP	Single	NO	Det.	*(R,Q)*	AP	BA	LA
Ozsen et al. (2008)	MINLP	Single	NO	Det.	*(R,Q)*	AP	BA	LA
Ahmadi Javid and Azad (2010)	MINLP	Single	NO	Det.	*(R,Q)*	AP	BA	ME
Berman et al. (2012)	MINLP	Single	NO	Det.	*(R,T)*	AP	BA	LA
Nekooghadirli et al. (2014a)	MINLP	Multi	NO	Det.	*(R,Q)*	AP	BA	ME
Sadjadi et al. (2016)	MINLP	Single	PO	Det.	*(S-1,S)*	-	BA	CO
Taki et al. (2016)	MINLP	Multi	NO	NO	*(R,Q)*	AP	LO	ME
Rabbani et al. (2016)	MINLP	Single	NO	Det.	*(R,Q)*	AP	BA	ME
Jindal and Solanki (2016)	MINLP	Single	NO	NO	*(R,Q)*	AP	BA	LA
AmalNick, and Qorbanian (2017)	MINLP	Single	NO	Det.	*(R,Q)*	AP	BA	ME
Díaz-Mateus et al. (2018)	MINLP	Single	NO	Det.	*(R,Q)*	AP	BA	ME
This study	**MINLP**	**Single**	**PO**	**EX**	***(S-1,S)***	**OPT**	**BA**	**ME**

3. Model Development

3.1. Problem and notations definition

A multi-echelon SC distribution system including one manufacturer, multiple DCs and multiple retailers is considered. Figure 1 shows the configuration of the proposed supply chain. The demand of retailers is stochastic with Poisson probability distribution function with regard to be independent of retailers' demand from each other. The (S-1, S) inventory policy is taken into account for DCs. Unsatisfied demand will be considered as backordered. The input demand rate is equal to λ that is summation of Poisson rates of allocated retailers. To response the delivered demands those ones that received earlier have priority rather the older ones. An exponentially probability distribution function with the parameter μ has been considered for lead-time of manufacturers. It is assumed to continue serving the demands $\lambda \leq \mu$.

In this paper, a set of integrated strategic and tactical decisions are determined in order to minimize the total costs of the supply chain network design as follows.

- Determining the number of open DCs.
- Identifying the location of open DCs from a predefined set of potential locations.
- Allocating the retailers to open DCs by considering single-sourcing concept.
- Specifying the inventory policy of each open DC.

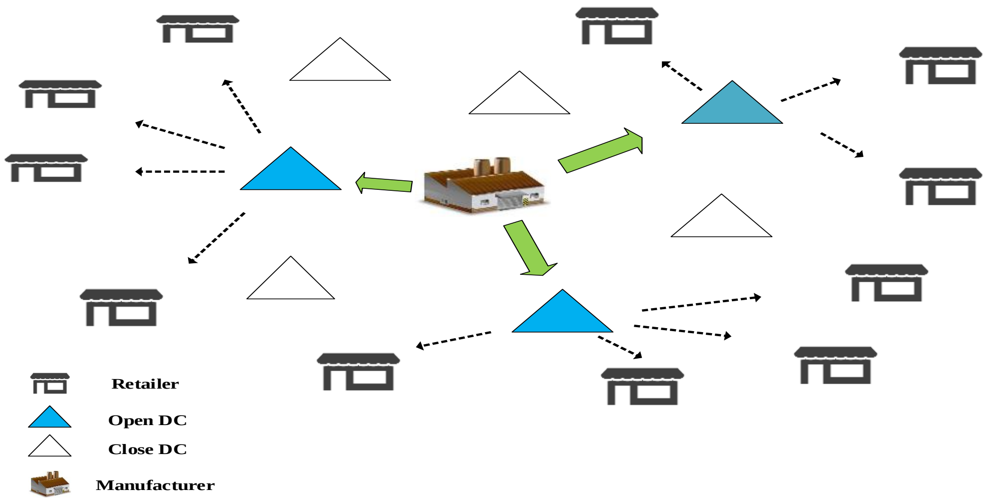


Figure 1. The configuration of the proposed supply chain

In the following, the notations of our proposed model are introduced.

Sets

K: Index for the set of potential DCs

I: Index for the set of retailers

Parameters

C_k: Per unit purchase cost for each product from the manufacturer ($\forall k \in K$)

π_k: Per unit shortage cost of each product for DC k ($\forall k \in K$)

F_k: Fixed cost of opening DC k

T_{ki} : Unit transportation cost from DC k to retailer i ($\forall k \in K$) and ($\forall i \in I$)

A_k: Unit order cost for DC k ($\forall k \in K$)

h_k: Unit holding cost for DC k ($\forall k \in K$)

λ'_i: Mean of annual demand for retailer i ($\forall i \in I$)

μ: The stochastic lead-time for the manufacturer

Decision variables

z_k: 1 if DC k is open, 0 otherwise ($\forall k \in K$)

y_{ki}: 1 if retailer i is assigned to DC k, 0 otherwise ($\forall k \in K$) ($\forall i \in I$)

S_k: Base stock level at DC k ($\forall k \in K$)

λ_k: Mean of annual demand at DC k ($\forall k \in K$)

OR_k: The expected value of reorders at DC k ($\forall k \in K$)

MI_k: The expected inventory level at DC k ($\forall k \in K$)

SH_K: The expected value of backlogged demands at DC k ($\forall k \in K$)

Using queuing approach in the proposed location-inventory model, the inventory policy has been characterized in sub-section 3.2.

3.2. Inventory policy for each open DC

To determine the inventory policy for each open DC, the number of reorders, the mean of inventory level and backlogged demands are obtained as follows.

Let assume $X_k(t)$ denotes the level of inventory at time $t \geq 0$ for DC k. The process $\{X_k(t); t \geq 0\}$ is a Markov process on the state space $\Phi_k = \{S_k, S_k - 1, \dots, 1, 0, -1, -2, -3, \dots.\}$. Thus, the infinitesimal generator matrix is as Eq. (3.1).

$$Q = \begin{bmatrix} \lambda_k & -\mu & & & \\ -\lambda_k & \lambda_k + \mu & -\mu & & \\ & -\lambda_k & \lambda_k + \mu & -\mu & \cdots \\ & & \vdots & \ddots & \end{bmatrix} \tag{3.1}$$

Also, let to consider that $\pi = (\pi_{s_k}, \pi_{s_{k-1}}, \pi_{s_{k-2}}, \dots \pi_{j_k}, \dots)$ denotes the stationary probability vector for the state space components. Then, Eq. (3.2) must be satisfied by the stationary probability vector π.

$$\pi Q = 0 \tag{3.2}$$

Meanwhile, regarding to that $\sum_{j_k \in \Phi_k} \pi_{j_k} = 1$, the stationary probabilities can be computed by Eqs. (3.3) and (3.4).

$$\pi_{j_k} = \left(1 - \frac{\lambda_k}{\mu}\right)\left(\frac{\lambda_k}{\mu}\right)^{S_k - j_k} \qquad j \leq S_k - 1 \tag{3.3}$$

$$\pi_{S_k} = 1 - \frac{\lambda_k}{\mu} \tag{3.4}$$

Given the (*S-1*, *S*) inventory policy and the shortage of demands are backlogged, the number of reorders can be obtained by Eq. (3.5).

$$OR_k = \lambda_k \tag{3.5}$$

The steady state of expected inventory level is determined by Eqs. (3.6) and (3.7).

$$MI_{k=} \sum_{j=0}^{S_k} j \times \pi_{j_k}, \tag{3.6}$$

$$MI_k = \frac{\left(\lambda_k{}^{S_k+1} - \lambda_k \mu^{S_k} - \lambda_k S_k \mu^{S_k} + S_k \mu^{S_k+1}\right)\mu \left(\frac{\lambda_k}{\mu}\right)^{S_k} \left(-1 + \frac{\lambda_k}{\mu}\right)}{\lambda_K{}^{S_k}(\lambda_k - \mu)^2} \tag{3.7}$$

Besides, the retailers may confront shortage when the level of inventory is zero or smaller. Thereupon, the value of expected backlogged demands in each open DC is given by Eq. (3.8).

$$SH_K = \lambda_k \sum_{j_k \le 0} \pi_{j_k} = \lambda_k \left(\frac{\lambda_k}{\mu}\right)^{S_k} \tag{3.8}$$

3.3. Mathematical formulation

According to the defined notations, our proposed location-inventory problem can be formulated as follows.

$$\begin{aligned} Min = {} & \sum_{k \in \mathrm{K}} F_k z_k + \sum_{\mathrm{k \in K}} \sum_{\mathrm{i \in I}} T_{ki}\, y_{ki} \lambda'_i \\ & + \sum_k h_k z_k \left[\frac{\mu \left(\frac{\lambda_k}{\mu}\right)^{S_k} \left(-1 + \frac{\lambda_k}{\mu}\right) \left(\lambda_K{}^{S_k+1} - \lambda_K \mu^{S_k} - \lambda_k S_k \mu^{S_k} + S_k \mu^{S_k+1}\right)}{\lambda_k{}^{S_k}(\lambda_k - \mu)^2} \right] \\ & + \pi_k z_k \left[\lambda_k \left(\frac{\lambda_k}{\mu}\right)^{S_k} \right] + (A_k + C_k) z_k \lambda_k \end{aligned} \tag{3.9}$$

$$\sum_k y_{ki} = 1 \qquad \forall i \in I \tag{3.10}$$

$$y_{ki} \le z_k \qquad \forall i \in I, \forall k \in K \tag{3.11}$$

$$\sum_i \lambda'_i y_{ki} = \lambda_k \qquad \forall k \in K \tag{3.12}$$

$$\lambda_k \le \mu \qquad \forall k \in K \tag{3.13}$$

$$y_{ki}, z_k \in \{0,1\} \qquad \forall i \in I, \forall k \in K \tag{3.14}$$

$$S_k \in \{0,1,\dots\} \qquad \forall k \in K \tag{3.15}$$

The Eq. (3.9) as the objective function minimizes the costs. The first term computes the fixed cost of opening DCs. The second term shows the transportation cost from DCs to the retailers and the last term illustrates the expected inventory costs including holding, shortage, ordering and purchase costs.

Eq. (3.10) shows single-sourcing for retailers. Eq. (3.11) demonstrates the assignment constraint for DCs and retailers and provides the opening condition for DCs. Eq. (3.12) shows the demand rate of each DC which is obtained by the summation of demand rate of assigned retailers. Eq. (3.13) guarantees the stability of the inventory system to avoid the shortage. Eqs. (3.14) and (3.15) are binary and integer variables, respectively.

4. Solution Approach

Our proposed model is defined as well-known capacitated facility location problems which are known as NP-hard problems (Mirchandani and Francis, 1990). Therefore, exact solution methods are mainly inefficient to solve the proposed problem. Thus, to solve our model, two meta-heuristic algorithms which have been successfully used to tackle complex models in the literature of integrated location-inventory problems are implemented (e.g., Ahmadi Javid and Azad (2010) and Nekooghadirli et al. (2014)). Here, the convexity of the objective function is proved in Proposition 1 and then the optimal value of the base stock level is calculated as a closed-form expression in Proposition 2.

Proposition 1. *The objective function of our proposed model is strictly convex considering the base stock level variables are continues.*

Proof. The linear parts of the objective function are convex. Therefore, only the convexity of nonlinear parts of the objective function needs to be proved. The nonlinear parts are presented by Eq. (4.1).

$$\begin{aligned} &g(S_1, S_2, \dots, S_n) \\ &= \sum_k h_k z_k \left[\frac{\mu \left(\frac{\lambda_k}{\mu}\right)^{S_k} \left(-1 + \frac{\lambda_k}{\mu}\right) \left({\lambda_k}^{S_k+1} - \lambda_k \mu^{S_k} - \lambda_k S_k \mu^{S_k} + S_k \mu^{S_k+1}\right)}{{\lambda_k}^{S_k} (\lambda_k - \mu)^2} \right] \\ &\quad + \pi_k z_k \left[\lambda_K \left(\frac{\lambda_k}{\mu}\right)^{S_k} \right] \end{aligned} \tag{4.1}$$

Eq. (4.2) shows the second derivative of g.

$$\frac{\mathrm{d}^2 g(S_1, S_2, \dots, S_n)}{d{S_k}^2}$$

$$\alpha_k = \lambda_k * \left(\frac{\lambda_k}{\mu}\right)^{S_k} * \pi_k * z_k * \log^2\left(\frac{\lambda_k}{\mu}\right) + \frac{h_k * z_k * \left(\frac{\lambda_k}{\mu}\right)^{S_k+1} * \log^2\left(\frac{\lambda_k}{\mu}\right)}{1 - \left(\frac{\lambda_k}{\mu}\right)} \tag{4.2}$$

Since $\frac{\lambda_k}{\mu} < 1$ and the input parameters (i.e., π_k, h_k) are non-negative, α_k $k = 1,2,3 \dots, n$ are greater than zero. In addition the Hessian matrix can be obtained as Eq. (4.3):

$$H = \begin{array}{c} 1 \\ 2 \\ \vdots \\ k \\ \vdots \\ n \end{array} \begin{bmatrix} \alpha_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & \cdots & 0 & 0 & \vdots \\ 0 & \vdots & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_k & \cdots & 0 \\ 0 & 0 & 0 & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_n \end{bmatrix} \tag{4.3}$$

Regarding to α_k *k=1,2,3…,n* are greater than zero, the aforementioned matrix is positive-definite matrix. Accordingly, the objective function is strictly convex. ☐

Proposition 2. *For an open DC, the optimal value of the base stock level is equal to* $[S_k^*]$ *or* $[S_k^*] + 1$*, where,* $[\blacksquare]$ *operator rounds a real number down to the previous integer. The closed-form for* S_k^* *is obtained from Eq. (4.4).*

$$S_k^* = \frac{\log\left(\dfrac{-h_k}{\lambda_k * \pi_k * log\left(\dfrac{\lambda_k}{\mu}\right) - \dfrac{h_j * \left(\dfrac{\lambda_k}{\mu}\right) * log\left(\dfrac{\lambda_k}{\mu}\right)}{\left(\dfrac{\lambda_k}{\mu}\right) - 1}}\right)}{\log\left(\dfrac{\lambda_k}{\mu}\right)} \tag{4.4}$$

Proof. Suppose that S_k is a continuous variable. Due to the convexity of the objective function (please refer to Proposition 1) the optimal value for the base stock level will be calculated by Eq. (4.5).

$$\frac{\mathrm{D}\ obj(S_1, S_2, \dots, S_n)}{dS_k} = 0 \tag{4.5}$$

By replacing z_k with 1 for open DCs, Eq. (4.4) will be derived. Since the strictly convexity of the objective function (Proposition 1) and being integer the values of the base stock levels for open DCs (see Eq. (3.15)), the best value for the base stock levels can be equal to $\lfloor S_k^* \rfloor$ or $\lfloor S_k^* \rfloor + 1$. ☐

4.1. The encoding schema and evaluation procedures

As representation of the solution method for genotype space, the solutions of algorithms are encoded by arrays with the length of *n*, which *n* represents the number of retailers. Each cell of the array represents the corresponding DC that supplies the demands of related retailer. Figure 2 illustrates the encoding schema for the solving procedure. As it can be seen, DC 3 supplies the retailers 1 and 3 and DC 2 supplies the retailer 2.

DCs	3	2	3	…	1	2
Retailers	1	2	3	…	n-1	n

Figure 2. The encoding schema

Noteworthy, by determining this array, the location-allocation decisions (i.e., opened DCs and assignments of retailers) are specified. Therefore, we can obtain the pertaining fixed cost of locating DCs and the transportation cost from DCs to the retailers. Nonetheless, in order to obtain total costs for evaluating the solution in the proposed algorithms, it is required that inventory decisions of opened DCs are specified too. According to Proposition 2, the optimal inventory policy can be determine for open DCs and consequently we able to obtain the total costs in order to assess the corresponding solution in the proposed algorithms.

4.2. Simulated annealing

Simulated annealing is one of the most famous meta-heuristic techniques that its original idea has been inspired from the annealing process of solids and it has been designed to reach a good solution, not necessarily optimum solution within a reasonable computational time. SA has been introduced by Kirkpatrick et al. (1983) as a stochastic algorithm that escapes from local optima by accepting worse solutions with an acceptance probability with regard to freezing rule. The procedure of the algorithm can be implemented by the following steps.

1. Generate an initial solution and then determine the configuration of neighborhood generation.

2. Compare the values of the fitness functions of current solution $E(c)$ and new neighborhood solution $E(n)$. New neighborhood solution $E(n)$ will be accepted with better fitness function value or if $E(n)$ is worse than $E(c)$, the neighbor solution will be accepted due to the Boltzmann probability distribution $e^{\frac{-\Delta E}{T}}$, where T is the temperature of the current iteration.
3. Decrease the temperature T when the equilibrium process is met.
4. Terminate the algorithm when stopping criteria are satisfied.

In this paper, we have used two neighborhood structures. In the first structure, algorithm randomly selects one retailer and one DC (not necessarily to be open in current solution) and then it assigns them to each other. The second structure, the algorithm randomly selects two retailers and then it exchanges their DCs.
The algorithm stops due to different termination criteria such as reaching the minimum temperature, time limitation, no improvement during specific iterations, running algorithm for constant iteration and a set of above conditions. To solve the proposed model, we have considered a set of two conditions including no improvement during specific iterations and reaching the minimum temperature.

The parameters used in SA are defined as following.

NI_{SA-sub}	Maximum number of solutions with no improvement in the sub-loop
$NI_{SA-main}$	Maximum number of solutions with no improvement in the main loop
NI_{sub}	The counter for solutions with no improvement in the sub- loop
NI_{main}	The counter for solutions with no improvement in the main loop
SL_0	Initial solution
SL	Current solution
$SL^{\prime}$	Solution selected in the neighborhood of SL
SL_{best}	Best solution
$Obj(SL)$	The objective function of solution SL
CR	Cooling rate
TM_0	Initial temperature
TM_f	Final temperature
TM	Current temperature

The pseudo-code of SA algorithm has been demonstrated in Figure 3.

```
Input: TM_0, TM_f, NI_{SA-sub}, NI_{SA-main}, CR.
Generate SL_0 randomly
SL_best = SL_0, SL = SL_0, TM = TM_f, NI_main = 0
 While (NI_main ≤ NI_{SA-main} and TM ≥ TM_f) do
    NI_sub = 0
    BL = False
       While (NI_sub ≤ NI_{SA-sub}) do
          Generate SL' in the neighborhood of SL
          ΔE = Obj(SL') − Obj(SL);
           IfΔE ≤ 0 Then
              SL = SL',
                If Obj(SL) < Obj(SL_best) Then
                   SL_best = SL
                   NI_sub = 0
                   BL = True
                 Else
                   NI_sub = NI_sub + 1
                End if
           Else
              Generate ra → uniform (0,1) randomly
             If ra < e^(−ΔE/T) Then
                SL = SL'
             End if
          End if
       End While
    TM = CR * TM
      If BL = True then
         NI_main = 0
     Else
        NI_main = NI + 1
    End If
End While
```

Figure 3. The pseudocode of SA algorithm

4.3. Genetic Algorithm

Similar to SA, GA is a stochastic global search method so that the emergence of this algorithm was investigated in 1960s by Holland (1975). GA works based on the biological evolution behavior and reaches approximate optimal solutions after its main steps including parent selection, recombination, mutation and survivor selection. To select the individuals for next generation two methods can be taken into account. The first method is to eliminate all individuals in older generation and then, to exchange them with new individuals. The second one is to preserve some individuals in old generation with higher values of fitness function and then, to choose other individuals from the new generation.

In our problem, we have generated initial population randomly. DCs and retailers selected randomly will be linked to each other. To select the parents, roulette wheel selection rule (see, Eiben and Smith (2003)) is used. Parents with better values of fitness function are selected to generate offspring.

In order to develop new individuals from existing ones, crossover and mutation operators have been implemented. The crossover operator is used to create new pairs of offspring by combining a pair of parents. In our problem, the tournament selection method and one point crossover is used (see, Eiben and Smith (2003)). Mutation of each individual will be occurred due to the mutation probability P_m, and it will change the DC linked to a certain retailer. The probability of the mutation is calculated by Eq. (4.5).

$$P_m = {}^{1}\!/_{number\ of\ retailers} \tag{4.5}$$

Survivor selection or replacement procedure will be performed by Elitism method (see, Eiben and Smith (2003)). Finally, the algorithm will be terminated when no improvement in certain iterations occurs.

The following parameters are used for GA algorithm.

$NMor$	The number of members in the origin population
$NMch$	The number of members in the children population
$NMmu$	The number of members in the mutation population
ST	Maximum number of solutions with no improvement
NI	The counter for solutions with no improvement
SL_{best}	Best solution
SL_{ibe}	Best solution in each iteration
$Obj(SL)$	The objective function of the solution SL
Pr_{Cr}	The probability of crossover
Pr_{Mu}	The probability of mutation

In this paper, three populations including origin, children and mutation are considered, where the numbers of their members are $Npop, Nch$ and $Nmu,$ respectively. Parameters Nch and Nmu are equal to $2 \times \left[\frac{Npop \times Pr_{Cr}}{2}\right]$ and $[Npop \times Pr_{Mu}]$respectively, where $[\bullet]$ rounds a real number down to the previous integer.

The pseudo-code of GA algorithm is illustrated in Figure 4.

Input: $NMor, Pr_{Cr}, Pr_{Mu}, ST.$

Generate $NMor$ solutions randomly to build the origin population.

Evaluate candidate solutions of the origin population.

Find the best solution in the origin population. Name it SL_{best}.

$NI = 0$

$\boldsymbol{While}$ $(NI \leq ST)$ $\boldsymbol{do}$

$I = 1$

$\boldsymbol{While}$ $\left(I \leq \frac{NMch}{2}\right)$ $\boldsymbol{do}$

Select a pair of parents from the origin population and recombine them

recombine the selected parents and create offsprings

Insert the created offsprings in the children population

$I = I + 1$

$\boldsymbol{End\ while}$

$I = 1$

$\boldsymbol{While}$ $(I \leq NMmu)$ $\boldsymbol{do}$

Select a parent from the origin population

Modify the selected parent

Insert the modified parent in the mutation population

$I = I + 1$

$\boldsymbol{End\ while}$

Merge the origin, children and mutation populations and generate a new population.

Evaluate solutions of the generated population.
Find the best solution in the generated population. Name it SL_{ibe}

$\boldsymbol{If}\ Obj(SL_{ibe}) < Obj(SL_{Best})\ \boldsymbol{Then}$

$SL_{Best} = SL_{ibe}$

$NI = 0$

$\boldsymbol{Else}$

$NI = NI + 1$

$\boldsymbol{End\ if}$

$\boldsymbol{End\ While}$

Figure 4. The Pseudocode of GA algorithm

4.4. Parameter setting

Parameter setting has significant impacts on the performance of the meta-heuristic algorithms. Therefore, we implement Taguchi method in order to tune the parameters of the proposed algorithms. Taguchi is a powerful method for robust parameter design which is based on reducing the effect of causes of variation and improving the quality of solutions. This method provides analysis of different parameters with no need of high amount of experiments. Taguchi takes advantages from two significant tools, the signal-to-noise ratio (S/NR) and orthogonal array (OA). S/NR is a measure that intends to mix information about the variance and the mean. Variety of response, i.e. the nominal or target value, is measured by S/NR under different noise conditions. By using S/NR, control factors that diminish variability are determined and then those control factors that move the mean to target with small or no effect on the S/NR will be specified. In Taguchi method, OA is implemented in order to evaluate the impact of many parameters on the performance of the algorithm in a compressed set of experiments. The suitable OA can be chosen by determining the number of parameters and the number of levels. Accordingly, first the concerning parameters are specified and then their levels of varying will be determined. The levels of a parameter usually consisting of the minimum, maximum, and current values can be determined by an in-depth knowing of the process. For each parameter, when the difference between the maximum and minimum is large, more levels need to test and as the range of a parameter is small, fewer values can be tested. For more detailed insights, interested readers can refer to Taguchi et al. (2000).
In this research, four factors including TM_0, CR, NI_{SA-sub} and $NI_{SA-main}$ are taken into account for SA and each factor are studied in three levels. Table 2 shows the levels for SA factors. Additionally, three factors including $NMor,\ Pr_{Cr}$ and ST are considered for GA that the concerning levels are given in Table 3. Using MINITAB 16, we have first created Taguchi designs for the solving algorithms and then the experiments are run based on the created designs. For both algorithms, L_9 OA have been chosen. Design factors for SA and GA are displayed in Tables 4 and 5, respectively. After analyzing the performed experiments by Taguchi method, the appropriate level of each factor can be obtained.

Here, that level that increases the S/NR measure is selected. Figures 5 and 6 show the average S/NR for considered factors of algorithms. It can be observed that the best values for SA factors, i.e. TM_0, CR, NI_{SA-sub} and $NI_{SA-main}$, are 950, 0.95, 150 and 300, respectively. Besides, for GA the best values of $NMor$, Pr_{Cr} and ST are 140, 0.8 and 1000, respectively.

Table 2. Levels of SA factors

Levels	TM_0	CR	NI_{SA-sub}	$NI_{SA-main}$
Level 1 (low)	850	0.9	50	100
Level 2 (medium)	950	0.95	100	200
Level 3 (large)	1050	0.99	150	300

Table 3. Levels of GA factors

Levels	$NMor$	Pr_{Cr}	ST
Level 1 (low)	100	0.6	300
Level 2 (medium)	120	0.7	750
Level 3 (large)	140	0.8	1000

Table 4. Taguchi designs for SA

Design	TM_0	CR	NI_{SA-sub}	$NI_{SA-main}$
1	850	0.9	50	100
2	850	0.95	100	200
3	850	0.99	150	300
4	950	0.9	100	300
5	950	0.95	150	100
6	950	0.99	50	200
7	1050	0.9	150	200
8	1050	0.95	50	300
9	1050	0.99	100	100

Table 5. Taguchi designs for GA

Design	$NMor$	Pr_{Cr}	ST
1	100	0.6	800
2	100	0.7	900
3	100	0.8	1000
4	120	0.6	900
5	120	0.7	1000
6	120	0.8	800
7	140	0.6	1000
8	140	0.7	800
9	140	0.8	900

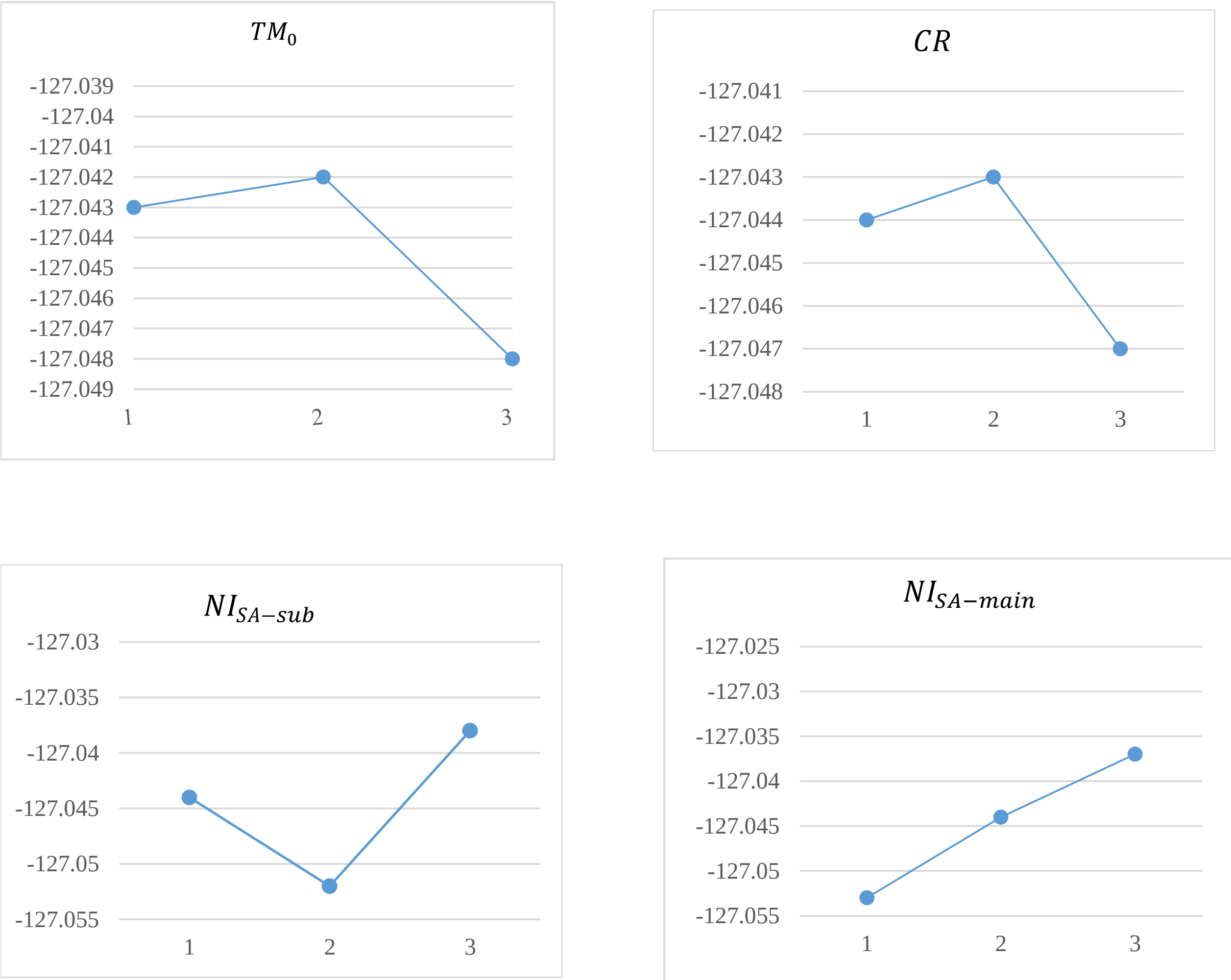
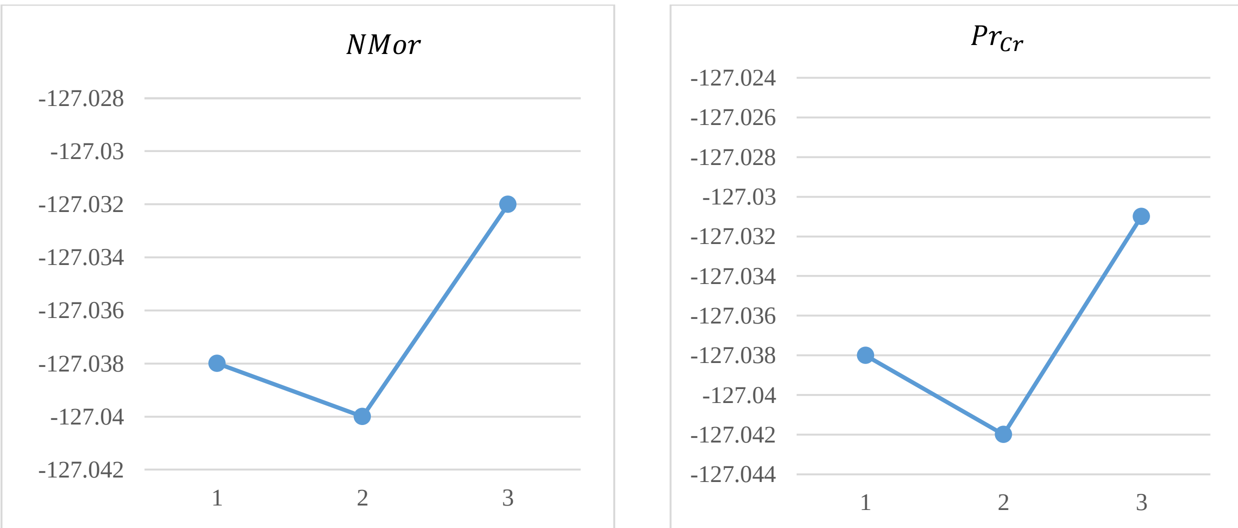


Figure 5. S/NR ratio plot for SA factors



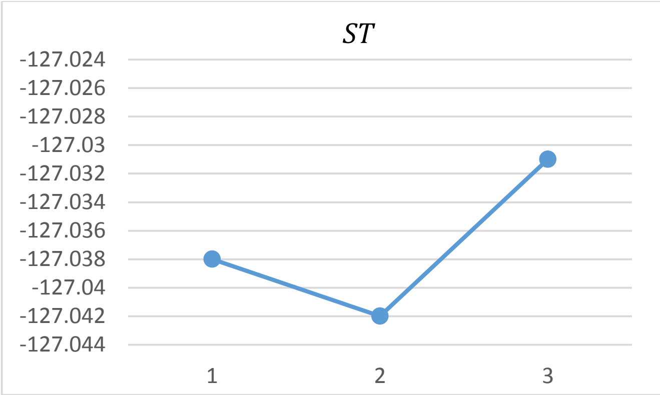


Figure 6. S/NR ratio plot for GA factors

5. Computational Results

In this section, the performance of proposed model and meta-heuristic algorithms are evaluated in terms of the quality of the solutions and computation time for the numerical experiments. Therefore, both solving algorithms are coded in Java programming language and they are run on an Intel$^{(R)}$ $^{(TM)}$ 2.50Hz CPU with 4G memory. The parameters of the model have been generated randomly and they are illustrated in Table 6. We generated three sizes test problems consisting of small, medium and large sizes. In small instances, the performance of the solving algorithms has been compared with General Algebraic Modeling System (GAMS) – version 24.1 - where the BARON solver has been implemented. The obtained results such as objective function, CPU Time and gaps between the algorithms and GAMS are reported in Table 7. The gap is calculated using Eq. (5.1).

$$Gap(\%) = \frac{obj_{GS} - obj_{PA}}{obj_{GS}} \times 100 \qquad (5.1)$$

Where, obj_{GS} and obj_{PA} are the value of objective functions obtained by GAMS and the related meta-heuristic algorithm, respectively. In other words, Gap_1 and Gap_2 are the representatives of GA and SA, respectively. For instances 1 and 2, we have observed that SA and GA are able to obtain same solutions with those obtained by GAMS within a reasonable time. Likewise, our algorithms find the better solutions for instances 4 and 5 with considerably less time than the BARON's run time. Thus, it can be concluded that the solving algorithms perform better than GAMS solver. The algorithms have been run for medium and large instances and the results including the values of objective function, CPU time and the gap are shown in Table 8. The gap of solving algorithms is calculated according to Eq. (5.2).

$$Gap(\%) = \frac{obj_{GA} - obj_{SA}}{obj_{GA}} \times 100 \qquad (5.2)$$

Where, obj_{GA} and obj_{SA} are the value of objective functions for GA and SA, respectively. As the numerical example shows, the quality of solutions of GA is better than those ones obtained by SA, where the maximum gap is 0.853%. On the other hand, in term of elapsed times, SA performs better. Furthermore, elapsed times for both proposed algorithms are increased by increasing the sizes of instances, as expected.

Table 6. The values of the parameters.

Parameter	c_k	A_k	h_k	π_k	F_k	λ_i	T_{ki}
Range	$U[35,60]$	$U[5,15]$	$U[25,40]$	$U[65,90]$	$U[4500,6500]$	$U[550,700]$	[15 25]

Table 7. Computational results for small instances.

	NO.	*# Retailer*	*# Potential DCs*	*GAMS*		*GA*		*SA*			
				Cost ($)	*CPU Time (Sec)*	*Cost($)*	*CPU Time(Sec)*	*Cost($)*	*CPU Time(Sec)*	*Gap$_1$ (%)*	*Gap$_2$ (%)*
Small	1	4	2	182644	1041	182644	0.037	182644	0.023	0	0
	2	10	4	476768	3461	476768	0.094	476768	0.039	0	0
	3	15	6	672898	8751	672652	0.152	672659	0.039	0.036	0.035
	4	20	8	859999	11432	858623	0.179	858651	0.065	0.160	0.156

Table 8. Computational results for medium and large instances.

	NO.	*# Retailer*	*# Potential DCs*	**GA**			**SA**			
				Cost($)	*CPU Time(Sec)*	*Gap GA-GAMS (%)*	*Cost($)*	*CPU Time (Sec)*	*Gap SA-GAMS (%)*	*Gap GA-SA (%)*
Medium	5	25	10	1069945	1.176	0.345	1069945	2.34	0.345	0.0
	6	35	12	1529056	1.323	0.384	1530294	2.868	0.303	-0.081
	7	45	15	1910099	1.528	0.551	1912923	4.99	0.404	-0.148
	8	55	17	2246673	2.562	0.592	2249941	6.15	0.447	-0.145
	9	70	19	2932897	3.443	0.660	2949496	12.59	0.098	-0.566
Large	10	80	21	3194120	13.37	1.224	3197255	12.26	1.127	-0.098
	11	95	28	3671902	21.96	1.551	3684850	16.09	1.204	-0.353
	12	100	35	4067175	25.99	1.990	4073398	25.34	1.840	-0.153
	13	110	40	4255463	36.625	2.045	4258169	27.543	1.983	-0.063
	14	140	45	5548268	51.12	2.511	5586842	30.215	1.833	-0.695
	15	150	50	5833398	85.43	3.192	5852301	35.01	2.878	-0.324

5.1. Sensitivity analysis

In this section, the impact of parameters on the number of open DCs, the objective function and the base stock level is investigated.

5.1.1. The number of open DCs

The effects of the value changing of μ on the number of open DCs and the objective function for two data sets, 10×25 and 21×80 ($n \times m$ means that the test problems include n potential DCs and m retailers), are investigated and the results are reported in Table 9. It can be inferred from Table 9 that the more the value of μ increases, the more the number of open DCs decreases. Because by increasing the value of μ, the expected lead-time decreases and consequently the manufacturers can provide the ordered products for open DCs in a shorter time. Thus, the shortages of each open DC decreases and more demands can be met. Therefore, the SC tends to reduce the number of open DCs in order to decrease the annual fixed cost of locating DCs. Also, it can be observed that the more the value of μ increases, the more the objective function decreases. This fact is consistent to the findings of Sadjadi et al. (2016). Table 10 shows the impacts of the demand rates on the number of open DCs and the value of the objective function.

It can be concluded from Table 10 that the more the demand rates of retailers increases, the more open DCs to meet the demands needs in order to decrease the shortage cost. Furthermore, increasing the values of demand rates enlarges the total costs of SC including the fixed cost of locating DCs, the transportation cost from DCs to retailers and the annual inventory costs.

Table 9. The effects of the value changing of μ on the number of open DCs and the objective function.

	Test problems with 10 potential DCs and 25 retailers		Test problems with 21 potential DCs and 80 retailers	
The value of μ	# open DCs	Cost($)	# open DCs	Cost($)
2000	9	1135738	21	3678297
3000	7	1089615	18	3470915
4000	6	1079730	16	3322724
5000	6	1069991	16	3196149
6000	6	1065491	16	3084758

Table 10. The effects of demand rates on the number of open DCs and the objective function.

	Test problem with 10 potential DCs and 25 retailers		Test problem with 21 potential DCs and 80 retailers	
The mean of annual demand	# open DCs	Cost($)	# open DCs	Cost($)
2000	4	701792.8	5	1214401
3000	6	1049009	9	2474990
4000	6	1396694	13	3777726
5000	7	1748224	15	5101892
6000	8	2107352	21	6480549

5.1.2. Base stock

In his research, we have obtained the optimal quantity of base stock level. Here, we have surveyed the impacts of service rate and demand rate on the base stock level. For this sake, we have assumed h=30, π =75, λ =445 and μ=610. The relationship between the demand rate and base stock level is pictured by Figure 8. It can be concluded from Figure 8 that the more the demand rates of retailers increase, the more the base stock level enlarges. Its reason is due to increasing the probability of shortage and the value of demand rate and consequently the base stock level will enlarge in order to reduce the impact of confronting with shortages.

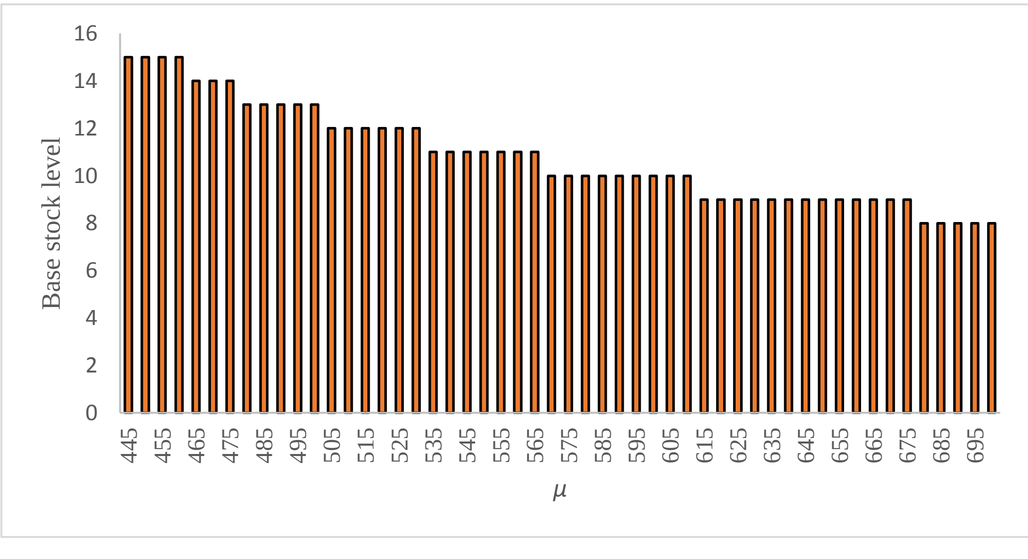


Figure 8. Sensitivity analysis of the base stock level with respect to changes of demand rate

6. Conclusion and Future Research

In this paper, a multi-echelon stochastic SC network design problem in order to minimize the total expected location, inventory and transportation costs was investigated. To bring more reality to the proposed model, the demand and lead-time were assumed to be hemmed in by uncertainty. In an attempt to tackle the proposed supply chain network design problem, a two-phase approach based on queuing and optimization models was devised. Exploiting the queuing approach, the first phase captured the inherent uncertainty of parameters. In the second phase, a mixed-integer nonlinear model was employed to formulate the proposed supply chain network design problem. As such, in the light of the concerned problem was NP-hard, two meta-heuristic algorithms were devised to solve the instances in small, medium and large sizes within a reasonable time. Results endorsed high efficiency and effectiveness of the solving algorithms in comparison with GAMS solver. Meanwhile, we proved the convexity of the objective function and achieved the optimal value for the base stock level as a closed-form.

Various experiments were conducted through which appealing insights were reached. For example, we showed the solving algorithms can solve the proposed problem in efficient ways. An another interesting result was that as the value of μ increases, the expected lead-time decreases and consequently the manufacturers can provide the ordered products for open DCs in a shorter time. As such, with increasing the demand rate of retailers, more open DCs are needed in order to decrease the shortage costs.

For future research, different areas to develop can be accounted as follows. 1) Developing the problem as a multi-product and multi-period model. 2) Utilizing the exact methods to solve the problem. 3) Taking into account multiple inventory policies in DCs and 4) Bringing the concepts of reliability into the model.

References

Ahmadi-Javid, A., and Seddighi, A. H., (2012). "A location-routing-inventory model for designing multisource distribution networks", *Engineering Optimization*, Vol. 44, No. 6, pp. 637-656.

Ahmadi Javid, A., and Azad, N., (2010). "Incorporating location, routing and inventory decisions in supply chain network design", *Transportation Research Part E: Logistics and Transportation Review*, Vol. 46, No. 5, pp. 582-597. doi: 10.1016/j.tre.2009.06.005

Alshamsi, A., and Diabat, A., (2018). "Large-scale reverse supply chain network design: An accelerated Benders decomposition algorithm", *Computers and Industrial Engineering*, Vol. 124, pp. 545-559.

AmalNick, M., and Qorbanian, R., (2017). "Dynamic pricing using wavelet neural network under uncertain demands", *Decision Science Letters*, Vol. 6, No. 3, pp. 251-260.

Baek, J. W., and Moon, S. K., (2014). "The M/M/1 queue with a production-inventory system and lost sales", *Applied Mathematics and Computation*, Vol. 233, pp. 534-544.

Baumol, W. J., and Wolfe, P., (1958). "A warehouse-location problem", *Operations Research*, Vol. 6, No. 2, pp. 252-263.

Berman, O., and Kim, E., (2001). "Dynamic order replenishment policy in internet-based supply chains", *Mathematical Methods of Operations Research*, Vol. 53, No. 3, pp. 371-390.

Berman, O., Krass, D., and Tajbakhsh, M. M., (2012). "A coordinated location-inventory model", *European Journal of Operational Research*, Vol. 217, No. 3, pp. 500-508. doi: 10.1016/j.ejor.2011.09.039

Chao, X., Xu, Y., and Yang, B., (2012). "Optimal policy for a production–inventory system with setup cost and average cost criterion", *Probability in the Engineering and Informational Sciences*, Vol. 26, No. 4, pp. 457-481.

Chen, F. Y., and Feng, Y., (2006). "Optimization and optimality of (s, S) stochastic inventory systems with non-quasiconvex costs", *Probability in the Engineering and Informational Sciences*, Vol. 20, No. 2, pp. 287-306.

Chew, E. P., Lee, L. H., and Rajaratnam, K., (2007). *Evolutionary algorithm for an inventory location problem*, Evolutionary Scheduling (pp. 613-628): Springer.

Daskin, M. S., Coullard, C. R., and Shen, Z.-J. M., (2002). "An inventory-location model: Formulation, solution algorithm and computational results", *Annals of operations research*, Vol. 110, No. 4, pp. 83-106.

Diabat, A., Abdallah, T., and Henschel, A., (2015). "A closed-loop location-inventory problem with spare parts consideration", *Computers and Operations Research*, Vol. 54, pp. 245-256.

Diabat, A., Battaïa, O., and Nazzal, D., (2015). "An improved lagrangian relaxation-based heuristic for a joint location-inventory problem", *Computers and Operations Research*, Vol. 61, pp. 170-178.

Diabat, A., and Theodorou, E., (2015). "A location–inventory supply chain problem: Reformulation and piecewise linearization", *Computers and Industrial Engineering*, Vol. 90, pp. 381-389.

Díaz-Mateus, Y., Forero, B., López-Ospina, H., and Zambrano-Rey, G., (2018). "Pricing and lot sizing optimization in a two-echelon supply chain with a constrained Logit demand function", *International Journal of Industrial Engineering Computations*, Vol. 9, No. 2, pp. 205-220.

Eiben, A. E., and Smith, J. E., (2003). *Introduction to evolutionary computing*, Berlin: Springer.

Federgruen, A., and Zheng, Y.-S., (1993). "Optimal control policies for stochastic inventory systems with endogenous supply", *Probability in the Engineering and Informational Sciences*, Vol. 7, No. 2, pp. 257-272.

Frizelle, G., and Jaber, M., (2009). "Seeing inventory as a queue", *Inventory Management: Non-Classical Views*, pp. 151-172.

Gunasekaran, A., Patel, C., and Tirtiroglu, E., (2001). "Performance measures and metrics in a supply chain environment", *International journal of operations and production Management*, Vol. 21, pp. 71-87.

Hill, R. M., (1999). "On the suboptimality of (S−1, S) lost sales inventory policies", *International Journal of Production Economics*, Vol. 59, No. 1–3, pp. 387-393. doi: http://dx.doi.org/10.1016/S0925-5273(98)00237-0

Holland, J. H., (1975). "Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence", Ann Arbor, Michigan: The University of Michigan Press.

Jindal, P., and Solanki, A., (2016). "Integrated vendor-buyer inventory models with inflation and time value of money in controllable lead time", *Decision Science Letters*, Vol. 5, No. 1, pp. 81-94.

Kalpakam, S., and Shanthi, S., (2001). "A perishable inventory system with modified (S−1,S) policy and arbitrary processing times", *Computers and Operations Research*, Vol. 28, No. 5, pp. 453-471. doi: http://dx.doi.org/10.1016/S0305-0548(99)00131-8

Kirkpatrick, S., Gelatt, C. D., and Vecchi, M. P., (1983). "Optimization by simulated annealing", science, Vol. 220, No. 4598, pp. 671-680.

Krishnamoorthy, A., Manikandan, R., and Lakshmy, B., (2013). "A revisit to queueing-inventory system with positive service time", *Annals of Operations Research*, pp. 1-16.

Miranda, P. A., and Garrido, R. A., (2009). "Inventory service-level optimization within distribution network design problem", *International Journal of Production Economics*, Vol. 122, No. 1, pp. 276-285. doi: 10.1016/j.ijpe.2009.06.010

Mirchandani, P. B., and Francis, R. L., (1990). *Discrete location theory*: Wiley.

Nagurney, A., (2006). *Supply chain network economics: dynamics of prices, flows and profits*: Edward Elgar Publishing.

Nekooghadirli, N., Tavakkoli-Moghaddam, R., Ghezavati, V., and Javanmard, S., (2014). "Solving a new bi-objective location-routing-inventory problem in a distribution network by meta-heuristics", *Computers and Industrial Engineering*, Vol. 76, pp. 204-221.

Ozsen, L., Daskin, M. S., and Coullard, C. R., (2009). "Facility location modeling and inventory management with multisourcing", *Transportation Science*, Vol. 43, No. 4, pp. 455-472.

Park, S., Lee, T.-E., and Sung, C. S., (2010). "A three-level supply chain network design model with risk-pooling and lead times", *Transportation Research Part E: Logistics and Transportation Review*, Vol. 46, No. 5, pp. 563-581.

Rabbani, M., Keyhanian, S., Hasannia, M., Eskandari, M., and Jalali, M., (2016). "Impact of end of lease contracts' option on joint pricing and inventory decisions of remanufacturable leased products", *International Journal of Industrial Engineering Computations*, Vol. 7, No. 2, pp. 191-204.

Rabbani, M., Aliabadi, L., Heidari, R., and Farrokhi-Asl, H., (2017). "Applying queuing theory for a reliable integrated location inventory problem under facility disruption risks", *Journal of Industrial Engineering and Management Studies*, Vol. 4, No. 2, pp. 1-18.

Ramezani, S., and Naderi, K., (2018). "A multi-objective optimization model for locating the distribution warehouses in the logistic network supply chain: A case study of Bonny Chow Co", *Journal of Industrial Engineering and Management Studies*, Vol. 5, No. 1, pp. 26-42.

Sadjadi, S. J., Makui, A., Dehghani, E., and Pourmohammad, M., (2016). "Applying queuing approach for a stochastic location-inventory problem with two different mean inventory considerations", *Applied Mathematical Modelling*, Vol. 40, No. 1, pp. 578-596.

Saffari, M., Asmussen, S., and Haji, R., (2013). "The M/M/1 queue with inventory, lost sale, and general lead times", *Queueing Systems*, No. 75, No. 1, pp. 65-77.

Schultz, C. R., (1990). "On the optimality of the (S — 1,S) policy", *Naval Research Logistics (NRL)*, Vol. 37, No. 5, pp. 715-723. doi: 10.1002/1520-6750(199010)37:5<715::AID-NAV3220370510>3.0.CO;2-U

Schwarz, M., and Daduna, H., (2006). "Queueing systems with inventory management with random lead times and with backordering", *Mathematical Methods of Operations Research*, Vol. 64, No. 3, pp. 383-414.

Shahabi, M., Unnikrishnan, A., Jafari-Shirazi, E., and Boyles, S. D., (2014). "A three level location-inventory problem with correlated demand", *Transportation Research Part B: Methodological*, Vol. 69, pp. 1-18.

Shen, Z.-J. M., Coullard, C., and Daskin, M. S., (2003). "A joint location-inventory model", *Transportation science*, Vol. 37, No. 1, pp. 40-55.

Shu, J., Teo, C.-P., and Shen, Z.-J. M., (2005). "Stochastic transportation-inventory network design problem", *Operations Research*, Vol. 53, No. 1, pp. 48-60.

Sigman, K., and Simchi-Levi, D., (1992). "Light traffic heuristic for anM/G/1 queue with limited inventory", *Annals of Operations Research*, Vol. 40, No. 1, pp. 371-380.

Stevens, G. C., (1989). "Integrating the supply chain", *International Journal of Physical Distribution and Materials Management*, Vol. 19, No. 8, pp. 3-8.

Taguchi, G., Chowdhury, S., and Taguchi, S., (2000). *Robust engineering*: McGraw-Hill Professional.

Tragantalerngsak, S., Holt, J., and Rönnqvist, M., (2000). An exact method for the two-echelon, single-source, capacitated facility location problem, *European Journal of Operational Research*, Vol. 123, Vol. 3, pp. 473-489.

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