



Development of a range-adjusted measure-based common set of weights for dynamic network data envelopment analysis using a multi-objective fractional programming approach

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Abstract

This paper presents a common set of weights (CSWs) method for multi-stage or network structured decision-making units (DMUs). The decision-making approaches proposed here consist of three stages. In the first step, a hybrid dynamic network data envelopment analysis (DNDEA) model is used to determine the efficiency values of the supply chain. Next, a CSW model is developed using the range-adjusted measure (RAM). In the third step, the extracted CSWs are used to compute a separate weight for each component of each DMU. The extracted CSWs are then used in the third step to calculate DMUs weights separately for each component. Then the overall efficiency is obtained by weighted averaging of the efficiency of individual components. Thus, this model evaluates the overall efficiency of a network process as well as the contribution of individual network components. The results of this study demonstrate the model's capability to evaluate the efficiency of dynamic network structures with very high discriminatory power. In an implementation of the model in a case study, only one supplier (KARAN) earned the maximum efficiency value, and the efficiency scores of other suppliers were in the range of 0.6409-0.9983. After applying the CSWs, KARAN remained the most efficient supplier, and the efficiency scores of other suppliers moved to the range of 0.5002-0.9349. The range shifted to 0.4823-0.9921 after applying the stages weights. This weighting method should be considered an integral part of such modeling procedures, Given the enhancement observed in the results of CSW after incorporating the component weights.

Keywords: efficiency assessments; common weight; data envelopment analysis; supply chain.

Paper Type: Original Research

1. Introduction

In a world of economic, political, social, and environmental instability, success belongs to those companies and organizations where managers understand the importance of continuous supply chain performance evaluation (Anisimov et al. 2022). Nowadays, many companies have to rigorously evaluate their suppliers to ensure that they meet their standards (Amiri et al. 2021). There are various methods for such evaluations, one of which is Data Envelopment Analysis (DEA). DEA is a simple but capable method for evaluating the efficiency of a set of alternatives (Decision-Making Units or DMUs) and classifying them into efficient and inefficient units, but it is not without its drawbacks and shortcomings. One of these shortcomings is that classical DEA models ignore the inner workings of DMUs, which means they can only determine whether a unit is efficient or not and cannot identify the source of inefficiency within a unit (Kao and Liu 2022). To address this shortcoming, researchers have introduced a version of DEA called Network DEA or NDEA which can compute not only the overall efficiency but in addition the partial efficiency of units in an integrated framework (Fathi, Karimi, and Saen 2022). But this model also has a shortcoming in that it is static, and does not consider time. Researchers have also developed another version of DEA called Dynamic DEA or DDEA for competitive and dynamic environments with constantly changing variables. This model can compute the efficiency of organizational units in time periods of interest (Bansal and Mehra 2022), but it has the problem of treating units as black boxes and ignoring their inner structures. Another problem of conventional DEA models is that they allow maximum flexibility in selecting input and output weights for DMUs (Tabatabaei et al. 2022), Which means, that each unit allocates the most

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weights to the output and the lowest weights to the inputs to maximize its efficiency (Soltanifar et al. 2022). Under these conditions, different DMUs may be given different sets of weights in efficiency assessments (Ghasemi et al. 2022). This tends to result in most units being classified as efficient, which makes it impossible to compare them. Therefore, one of the most significant issues of DEA is the calculation of weights for input and output indicators. Some researchers have argued that it does not make sense to consider different weights for the same DMUs (Tabatabaei et al. 2022), and have therefore proposed alternative methods of calculating the Common Set of Weights (CSW) for input and output variables. Over the years, this method has been expanded by many researchers, proposing various models, with their drawbacks and strengths.

Considering the multi-stage nature of supply chains and the shortcomings of classic, network, and dynamic DEA models, to avoid the aforementioned problems, this paper uses a Dynamic Network Data Envelopment Analysis (DNDEA) model with the ability to measure network efficiency over multiple time periods (Gharakhani et al. 2018), which makes it more likely to identify the sources of inefficiency in DMUs (Kiaei and Kazemi Matin 2022). However, according to studies (Liu et al. 2022) even DNDEA models may not be able to obtain optimal input and output weights. To overcome DNDEA's weight limitations, in the second stage of the study, a Dynamic Network CSW (DN-CSW) model is developed with a Range-adjusted Measure (RAM) based multi-objective fractional programming approach. This model allows DMUs to be evaluated neutrally on the same scale using CSW. Meanwhile, in many of the existing network DEA models, DMU efficiency is considered to be the arithmetic mean of its components (Chen et al. 2009; Kalantary and Farzipoor Saen 2019; Kalantary, Farzipoor Saen, and Toloie Eshlaghy 2018; Liang, Cook, and Zhu 2008; Moradi et al. 2022). But this approach has a major drawback in that all components will be given the same weight regardless of how individually significant they are to the process. To resolve this issue, in this study, the extracted CSWs are used to compute a weight for each component of each DMU. The efficiency of each DMU is considered to be the weighted average of the efficiency of individual components. To the best of our knowledge, the present study is significant in several respects:

- Development of RAM-based CSW for dynamic network systems
- Proposing a CSW model that deals with determining an assurance value for the non-Archimedean epsilon.
- Investigation of the effect of process component weights on the CSW approach in network structures.

Considering the above issues and the drawbacks of DEA models, the goal of this paper is to expand the CSW model of Jahanshahloo et al. (2005) for dynamic network systems using a RAM-based multi-objective fractional programming approach. This method provides better insights into the common set of weights and is expected to improve the results of the DNDEA models. This method also allows for not only quantifying the efficiency of suppliers but also monitoring dynamic changes over certain periods. Practical cases are further applied to clarify and validate the method concerned. In section two, the research background is reviewed. In section three, the proposed model is formulated and a numerical example is provided to showcase its capability and application. The final section presents the conclusions.

2. Research Background

In this section, we briefly review the background of the methods used in the article.

2.1. Dynamic Network Data Envelopment Analysis

Early DEA models like CCR (Charnes, Cooper, and Rhodes 1978) and BCC (Banker, Charnes, and Cooper 1984), which consider the inputs and outputs of independent decision-making units (DMUs) simultaneously (Pourmahmoud and Sharak 2020), are very helpful tools for relative efficiency evaluations (Ge 2022). But these models have some drawbacks like ignoring the internal mechanisms of activities and DMUs (Shieh et al. 2022). After initial studies of (Färe 1991) and subsequent expansions in Chen et al. (2009); Fare et al. (1995); Färe et al. (1996); Fukuyama & Weber (2010); Tone & Tsutsui (2009) researchers developed DEA models capable of measuring not only the total efficiency but also the partial efficiency of DMUs in an integrated framework. This approach is known as Network Data Envelopment Analysis (NDEA). However, NDEA models are static and do not consider time (Lu et al. 2020), which can cause them to produce misleading results based on short-term analyses (Tone et al. 2018). Later, Nemoto & Goto (2003) introduced the Dynamic Data Envelopment Analysis (DDEA) model to address this issue, but this model treats DMUs as black boxes, completely ignoring their internal structure.

Therefore, a model was needed to consider time as well as DMUs' internal structure. Several reviews of NDEA and DDEA models (Fukuyama and Weber 2010; Hashimoto and Fukuyama 2013; Johnson and Pope 2013) highlighted the need for extending dynamic DEA to network structures. The current literature offers two main non-ratio ways of formulating DEA models with dynamic network structures: the DNDEA model based on Slack-Based Measure (SBM) and Dynamic Network Range-Adjusted Measure (DNRAM). Some of the studies that have been done in the area of DNDEA are listed in table 1:

Table 1: The background of DNDAE models

Researchers (year)	Type of model	objective	Sustainable objective function variables			
			inputs		Carry overs	intermediate
			like, the economic dimension	like, the environment dimension	like, the social dimension	like, the product
(Tone and Tsutsui 2014)	DNSBM	performance evaluation	✓	✓	✓	✓
(Avkiran and Mccrystal 2014)	DNRAM	performance evaluation	✓	✓	✓	✓
(You and Jie 2016)	DNSBM	performance evaluation	✓	×	×	×
(Xie et al. 2018)	DNSBM	Environmental efficiency	✓	×	×	×
(Ramezankhani, Torabi, and Vahidi 2018)	DNSBM	sustainability evaluation	✓	×	×	×
(Kalantary et al. 2018)	DNRAM	sustainability evaluation	✓	×	×	×
(Kalantary and Farzipoor Saen 2019)	DNSBM	sustainability evaluation	✓	×	×	×
(Motevalli and Motamedi 2020)	DNSBM	sustainability evaluation	✓	×	×	×

2.2. Common set of weights

One of the earliest works in the CSW field is the approach presented by Roll et al. (1991). As a first step, they proposed some approaches for deriving the weight control bounds. They then presented a process for determining CSWs for factors based on their strategy. Their method aims to obtain a CSW for all DMUs simultaneously such that the highest (average) efficiency score is obtained. Li and Reeves (1999) introduced a deviation variable for each DMU, representing the deviation of DMU from the efficiency frontier. Then, they proposed three objective functions (minimizing the deviation, minimizing the maximum deviation, and minimizing the sum of the deviations) utilized by other researchers to find CSWs. Some other researchers use different approaches to find CSWs. For example, Jahanshaloo et al. (2005) use the concept of max-min to find the CSWs. Kao and Hung (2005) consider the efficiency scores of DMUs obtained from the classical DEA models as the ideal solution for the DMUs. Then, they derive CSWs closest to the ideal solution based on the generalized measure of distance. Cook and Zhu (2007) develop a nonlinear programming (NLP) model to find CSWs. Jahanshaloo et al. (2010) define an ideal line and determine CSWs for efficient DMUs. Ramón et al. (2012) minimize the deviations of the CSWs from the DEA profiles of weights and consequently derive CSWs for ranking all DMUs. Saati et al. (2012) first define an ideal DMU (IDMU), a hypothetical DMU consuming the least inputs to secure the most outputs. Then, they use the IDMU in an LP model to determine CSWs. Sugiyama and Sueyoshi (2014) propose an approach for determining CSWs based on bargaining games. Hosseinzadeh Lotfi et al. (2013) propose an allocation mechanism based on a common dual weights approach. Rezaie et al. (2014) consider the best and the worst relative efficiencies simultaneously in the form of an interval efficiency over CSWs. Most recently, Arman and Hadi-Vencheh (2021) utilized the fuzzy set theory to control the relative weights in DEA.

In the CSW models, the initial idea is to simultaneously maximize the ratio of the virtual output over the virtual input for the n DMUs. This approach supports the majority of the CSW models. For a better reading, six sub-groups have been considered presenting those CSW procedures based on the multi-objective idea:

- 1- Theoretical foundations: This group studies the fractional MOP model and develops a computational methodology to determine the CSW.
- 2- Procedures based on the ideal (anti-ideal) concepts: Several authors propose to determine the CSW by minimizing the distance from an ideal value.
- 3- Procedures focused on the weighting schemes: The models include those based on pre-weighting, those based on decision maker preferences, and those that minimize the disagreement regarding weighting vector components in order to determine the CSW.
- 4- Procedures that include uncertain or interval values: In this group, fuzzy tools are used to calculate the CSW based on uncertain data and models.
- 5- Statistical-based approaches: in this group, Various statistical techniques have been considered for the determination of the CSW.

6- Procedures focused on the evaluation of a subset of units: this group is formed with those CSW models in which the evaluation is not focused on the complete set of DMUs. Only a subset of the units is considered for the computation of the optimal weighting scheme. Table 2 listed some of the studies that have been conducted in the area of CSW:

Table 2: The background of CSW models

Researchers (year)	CSW procedures based on the multi-objective idea:	Type of model	non-Archimedean epsilon	Does it have an approach to calculating the non-Archimedean epsilon?	Description
(Jahanshahloo et al. 2005)	Theoretical foundations	simple (closed) systems	✓	×	-
(Liu and Hsuan Peng 2008)	Procedures focused on the evaluation of a subset of units	simple (closed) systems	✓	×	-
(Makuei et al. 2008)	Theoretical foundations	simple (closed) systems	×	×	-
(Chiang, Hwang, and Liu 2011).	Theoretical foundations	simple (closed) systems	✓	×	-
(Saati et al. 2012)	Procedures focused on the weighting schemes	simple (closed) systems	✓	✓	draws on the central value between the bounds of the weights
(Sun, Wu, and Guo 2013)	Procedures based on ideal (anti-ideal) concepts	simple (closed) systems	✓	×	-
(Ramezani-Tarkhorani et al. 2014)	Procedures focused on the evaluation of a subset of units	simple (closed) systems	✓	×	-
(Toloo 2013, 2014)	Theoretical foundations	simple (closed) systems	✓	✓	Inverse of the maximum sum of inputs
(Hajiagha et al. 2018)	Procedures that include uncertain or interval values	Dynamic	×	×	-
(Gharakhani et al. 2018).	Procedures based on ideal (anti-ideal) concepts	Dynamic-network	×	×	-
(Omrani, Valipour, and Mamakani 2019)	Statistical-based approaches	simple (closed) systems	✓	×	-
(Mavi and Mavi 2021).	Procedures based on ideal (anti-ideal) concepts	Dynamic	×	×	-

2.3. Research gap

According to Table 1, in the previous research, except for the two studies by (Tone and Tsutsui 2014) and (Avkiran and Mccrystal 2014), which are the basis of many studies, the other DNDEA studies are centered on short-term profits. In other words, this structure is a purely cost-oriented perspective, which made them overlook the environmental and social background of an organization during assessments, despite the fact that environmental and social sustainability are turning into key competitive priorities for many businesses (Longoni and Cagliano 2016). Sustainability can be viewed as the grade to which current decisions in organizations affect the future status of the environmental and socioeconomic viability (Elmsalmi et al., 2021; Salimian et al., 2022; Zhang et al., 2022). It is reasonable to conclude that the sustainability of organizations is influenced by the environmental and socioeconomic decisions taken in the past. So, in this research, the developed DNDEA model is used in such a way as to avoid an excessive focus on short-term profit from a purely cost-oriented perspective and to make sure that the environmental and social effects of supply chain activities are appropriately represented in the model. This helps lay the foundation for making supply chain activities sustainable by taking into account the factors that influence the chain's surrounding environment. The developed DNDEA model used in this article is formulated in such a way that the current efficiency of a business is assumed to be shaped by its former environmental and social activities. The main contributions and advantages of this paper in the field of the DNDEA model are:

- 1- propose a model which measures the direct impact of three pillars of sustainability on efficiency; thereby its discriminating power and reliability are increased and reflect reality.
- 2- Develop a DNDEA model that can rank the suppliers in terms of overall, periodic, partial, and periodic-partial efficiency, and the source of inefficiency of each supplier is identified.

According to Table 2, in many studies in the CSW area, many researchers have tended to concentrate on closed systems. That is, where the outputs from one stage become the inputs to the next stage, and where no other inputs enter the process at any intermediate stage. The problem of interest in this paper is the efficiency evaluation of an open multi-stage process, where, in each stage, some outputs may leave the system and others may turn into inputs for the next stage, and also new inputs may enter the system at any stage. The overall efficiency of this process is defined as the weighted average of the efficiency of all the individual components or stages that make up the process. In this study, the RAM-based DN-CSW method is used to not only evaluate network efficiency over a given period of time but also to determine the weight of each component while taking into account the specific conditions of the DMU under evaluation. Many studies have failed to provide an approach for Archimedean epsilon. In this research, a RAM-based approach is proposed as a non-Archimedean epsilon, which is used in order to avoid the appearance of zero weights. The main contributions and advantages of this paper in the field of the CSW model are:

- 1- The most significant achievement of this study is the development of a quantitative method based on the CSW model for the sustainability assessments of suppliers, which is exploited to select the best DMUs.
- 2- Development of RAM-based CSW for dynamic network systems.
- 3- Propose a CSW model that deals with determining an assurance value for the non-Archimedean epsilon.
- 4- Investigation of the effect of process component weights on the CSW approach in network structure.

2.4. Flowchart Methodology

The flowchart methodology of this research is available in Figure 1

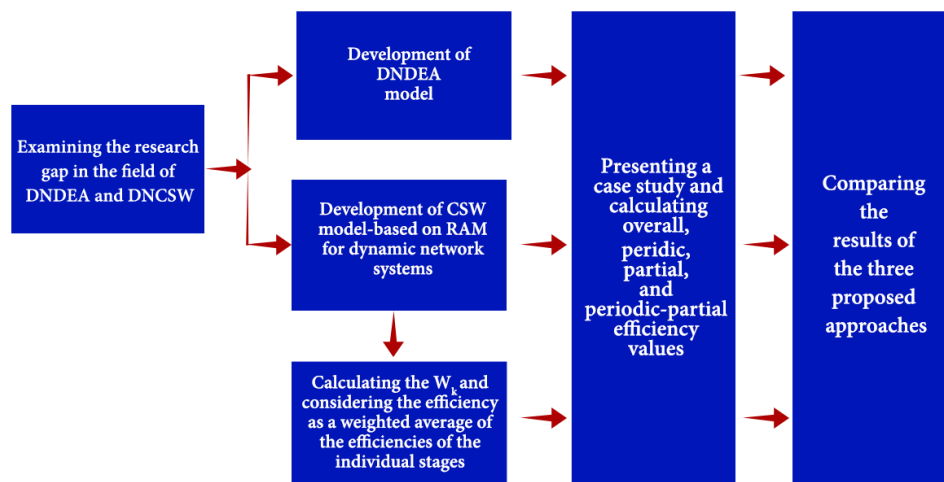


Figure 1: Research Methodology Flowchart

3. Materials and Methods

This section first presents the DNDEA model used in the study, followed by the CSW model of Jahanshahloo et al. (Jahanshahloo et al. 2005) is expanded so that in addition to computing the overall efficiency of DMUs over time, it can also consider dynamic changes in the periodic and partial efficiency of units. Operating based on RAM, the developed model offers better insights into CSWs and enhances the outputs of the DEA method. The extracted CSW is then to compute a weight for each component of the DMU under evaluation. The DMU's efficiency is calculated by weighted averaging of the efficiency of individual components.

3.1. Dynamic Network Data Envelopment Analysis

In this paper, the developed DNDEA model is used based on the RAM model proposed by Moradi et al. (2022). In this model, the input variables, and carry-over variables also have a direct effect on the objective function. According to the classification of Tone and Tsutsui (Tone and Tsutsui 2014), intermediate variables are considered to be fixed, and carry-over variables are considered to be free. Since the objective function of the RAM model calculates inefficiency, which equals one minus efficiency, that study has assumed that:

$$\min q = \frac{1}{T} \sum_{t=1}^T \frac{1}{K} \sum_{k=1}^K \frac{1}{m+u} \sum_{i=1}^m \sum_{u=1}^U \frac{S_{iok}^t}{R_{iok}^t} + \frac{S_{uok}^{t-1,t}}{R_{uok}^{t-1,t}}$$

$$\text{s.t.} \quad \sum_j^n x_{ijk}^t \lambda_{jk}^t + s_{iok}^t = x_{ijk}^t, \quad i = 1, \dots, m, \forall K, T \quad (1-1)$$

$$\sum_j^n I_{wj(k-h)}^t \lambda_{jk}^t = \sum_j^n I_{wj(k-h)}^t \lambda_{jh}^t, \quad w = 1, \dots, W, k = 1, \dots, K-1, \forall T \quad (1-2)$$

$$\sum_j^n C_{ujk}^{t,t+1} \lambda_{jk}^t = \sum_j^n C_{ujk}^{t,t+1} \lambda_{jk}^{t+1}, \quad u = 1, \dots, U, t = 1, \dots, T-1, \forall K \quad (1-3)$$

$$\sum_j^n C_{ujk}^{t,t+1} \lambda_{jk}^t \geq C_{uok}^{t,t+1}, \quad u = 1, \dots, U, t = 1, \dots, T-1, \forall K \quad (1-4)$$

$$\sum_j^n C_{ujk}^{t-1,t} \lambda_{jk}^t + s_{uok}^{t-1,t} = C_{uok}^{t-1,t}, \quad u = 1, \dots, U, \forall K \quad (1-5)$$

$$\sum_j^n \lambda_{jk}^t = 1, \quad \forall K, T, \quad \lambda_{jk}^t, s_{iok}^t, s_{uok}^{t-1,t} \geq 0, \quad \forall i, j, r \quad (1-6) \quad (1)$$

Where:

x_{ijk}^t : The i th input of the j th DMU in the k th stage in time t

$C_{ujk}^{t,t+1}$: The u_{th} ($u = 1, \dots, U$) carry-over of the j th DMU in the k th stage that is transferred from time t to time $t+1$.

$C_{ujk}^{t-1,t}$: $C_{ujk}^{t-1,t}$: The u_{t-1h} ($u = 1, \dots, U$) carry-over of the j th DMU in the k th stage that is transferred from time $t-1$ to time t .

$I_{wj(k-h)}^t$: The w_{in} ($w = 1, \dots, W$) intermediate of the j th DMU that is transferred from the k th stage to the h th stage at time t .

R_{iok}^t : Range of inputs in time t ; $R_{iok}^t = \max(x_{ijk}^t) - \min(x_{ijk}^t)$.

R_{uok}^{t-1} : Range of carry-over variables in time $t-1$; $R_{uok}^{t-1} = \max(C_{ujk}^{t-1,t}) - \min(C_{ujk}^{t-1,t})$.

λ_{jk}^t : Intensity vector of the j th DMU in the k th stage in time t .

In model (1), (1-1) relates to inputs, and (1-2) relates to the fixed link value case. This case corresponds to the situation where the intermediate products are beyond the control of DMUs or the discretion of management. (1-3) refers to carryovers that connect the t th period to the $t+1$ th period. (1-4) and (1-5) refer to the type of carryovers that have a dual function. This means that a free link can be stated as desirable (1-4) or undesirable (5-1). The above model assumes variable returns to scale (VRS) for production. That is, the production frontiers are spanned by the convex hull of the existing DMUs (1-6).

3.2. Multiple objective programming approach for finding a CSW

According to content, the DEA method can evaluate the efficiency of DMUs and classify them as efficient or inefficient units. The model, however, is not without its problems, and limitations. This includes the problem of homogeneous and identical units having different weights. In order to overcome this issue, Jahanshahloo et al. (2005) have proposed a simple yet effective model that has the major advantage of only requiring one problem to determine the CSW of DMUs. However, this model has been developed for simple (closed) systems with limited inputs and outputs. Another shortcoming of this model is that it ignores the internal structure of DMUs. This is while many business entities consist of several interlinked departments, each with its inputs and outputs, the operations of which may fall in different time periods. Multi-objective programming is used in this study to find CSWs for DNDEA models. This approach provides more insights into CSWs, enhances the DEA method, and makes it possible to compare the efficiency scores of DMUs from different perspectives. Furthermore, it enables

us to determine not only a DMU's overall efficiency over time but also to monitor its changes in periodic efficiency and partial efficiency over time. Based on the RAM model, this idea (model (1)) maximizes simultaneously the ratio of outputs to inputs for every DMU:

$$\max \left\{ \frac{\sum_{u=1}^{r_k} u_{ku} c_{ku1}^t + \sum_{w=1}^{W_k} \eta_{kw} I_{kw1}^t + a_k^t}{\sum_{i=1}^{m_k} v_{ki} x_{ki1}^t + \sum_{w=1}^{W_{k-1}} \eta_{k-1w} I_{k-1w1}^t}, \dots, \frac{\sum_{u=1}^{r_k} u_{ku} c_{kuj}^t + \sum_{w=1}^{W_k} \eta_{ku} I_{kwj}^t + a_k^t}{\sum_{i=1}^{m_k} v_{ki} x_{kij}^t + \sum_{w=1}^{W_{k-1}} \eta_{k-1w} I_{k-1wj}^t} \right\}$$

$$\text{s.t.} \quad \frac{\sum_{u=1}^{r_k} u_{ku} c_{kuj}^t + \sum_{w=1}^{W_k} \eta_{kw} I_{kwj}^t + a_k^t}{\sum_{i=1}^{m_k} v_{ki} x_{kij}^t + \sum_{w=1}^{W_{k-1}} \eta_{k-1w} I_{k-1wj}^t} \leq 1 \quad , \quad j=1,2,\dots,n \quad (2-1)$$

$$u_{ku} \geq 1/R_u k[(m_k + w_k) + (u_k + w_{k-1})] \quad , \quad u=1,2,\dots,r \quad (2-2)$$

$$v_{ki} \geq 1/R_i k[(m_k + w_k) + (u_k + w_{k-1})] \quad , \quad i=1,2,\dots,m \quad (2-3)$$

$$I_{kw} \geq R_w k[(m_k + w_k) + (u_k + w_{k-1})] \quad , \quad w=1,2,\dots,W \quad (2-4)$$

$$a_k^t \text{ free on sign} \quad (2-5) \quad (1)$$

Where:

x_{kij}^t : The i th input of the j th DMU in the k th stage in time t

c_{kuj}^t : The u th output of the j th DMU in the k th stage in time t .

I_{kwj}^t : The W th Input intermediates of the j th DMU in the k th stage in time t .

I_{k-1wj}^t : The W th output intermediates of the j th DMU in the k th -1stage in time t .

R_u : Range of outputs; $\max[c_{kuj}] - \min[c_{kuj}] \geq 0$

R_i : Range of inputs variables; $\max[x_{kij}] - \min[x_{kij}] \geq 0$

R_w : Range of intermediates variables; $\max[I_{kwj}] - \min[I_{kwj}] \geq 0$

Where u_{ku} , v_{ki} , η_{kw} and η_{k-1w} are the weights of outputs, inputs, output intermediates and input intermediates, respectively, and m_k , u_k , w_{k-1} and w_k are the number of inputs, outputs, input intermediates and output intermediates in each stage, respectively. Constraint (2-1) is related to the efficiency of j th DMU, which is a given DMU by examining the weighted outputs to weighted inputs of each component. In (1), if u is too large, and v is too small, the value of ratios can be infinite or unlimited. This problem can be avoided by considering all ratios to be less than or equal to one and adding them to the model as constraints. Constraints (2-2)- (2-4) are related to presenting the heuristic approach of the RAM as a non-Archimedean epsilon, which is used to avoid the appearance of zero weights. In this model, the non-Archimedean epsilon plays an imperative role and must be determined correctly. Otherwise, the related model might be infeasible. For solving this problem, the following procedure is suggested. Here we consider the infinite norm, so it tends to be the maximization of the objective function about the DMU will minimum ratio of outputs to inputs:

$$\max \left(\min \left\{ \frac{\sum_{u=1}^{f_k} u_{ku} c_{ku1}^t + \sum_{w=1}^{W_k} \eta_{kw} l_{kw1} + a_k^t}{\sum_{i=1}^{m_k} v_{ki} x_{ki1}^t + \sum_{w=1}^{W_{k-1}} \eta_{k-1w} l_{k-1w1}}, \dots, \frac{\sum_{u=1}^{f_k} u_{ku} c_{kuj}^t + \sum_{w=1}^{W_k} \eta_{kw} l_{kwj} + a_k^t}{\sum_{i=1}^{m_k} v_{ki} x_{kij}^t + \sum_{w=1}^{W_{k-1}} \eta_{k-1w} l_{k-1wj}} \right\} \right)$$

$$\text{s.t.} \quad \frac{\sum_{u=1}^{f_k} u_{ku} c_{kuj}^t + \sum_{w=1}^{W_k} \eta_{kw} l_{kwj} + a_k^t}{\sum_{i=1}^{m_k} v_{ki} x_{kij}^t + \sum_{w=1}^{W_{k-1}} \eta_{k-1w} l_{k-1wj}} \leq 1 \quad , \quad j = 1, 2, \dots, n \quad (3-1)$$

$$u_{ku} \geq 1/R_u k[(m_k + w_k) + (u_k + w_{k-1})] \quad (3-2)$$

$$v_{ki} \geq 1/R_i k[(m_k + w_k) + (u_k + w_{k-1})] \quad (3-3)$$

$$l_{kw} \geq R_w k[(m_k + w_k) + (u_k + w_{k-1})] \quad (3-4)$$

$$a_k^t \text{ free on sign} \quad (3-5) \quad (2)$$

There are no differences between equations (1) and (2) in terms of constraints. Due to this, we refrain from repeating defining their role in the equation again. By introducing a positive variable, z , model (2) can be converted into the model (3):

max : z

$$\left(\sum_{u=1}^{f_k} u_{ku} c_{kuj}^t + \sum_{w=1}^{W_k} \eta_{kw} l_{kwj} \right) - \left(\sum_{i=1}^{m_k} v_{ki} x_{kij}^t + \sum_{w=1}^{W_{k-1}} \eta_{k-1w} l_{k-1wj} \right) + a_k^t \leq 0 \quad (4-1)$$

$$\left(\sum_{u=1}^{f_k} u_{ku} c_{kuj}^t + \sum_{w=1}^{W_k} \eta_{kw} l_{kwj} \right) - z \left(\sum_{i=1}^{m_k} v_{ki} x_{kij}^t + \sum_{w=1}^{W_{k-1}} \eta_{k-1w} l_{k-1wj} \right) + a_k^t \geq 0 \quad (4-2)$$

$$\sum v_{ki} + \eta_{k-1w} + \sum u_{ku} + \eta_{kw} = 1 \quad (4-3)$$

$$v_{ki} \geq 1/R_i k[(m_k + w_k) + (u_k + w_{k-1})] \quad (4-4)$$

$$l_{kw} \geq R_w k[(m_k + w_k) + (u_k + w_{k-1})] \quad (4-5)$$

$$a_k^t \text{ free on sign} \quad (4-6) \quad (3)$$

The model (3) can be solved using a direct search (i.e., derivative-free) algorithm like that of Nelder and Mead (Nelder and Mead 1965). A set of u_{ku}^* , η_{kw}^* , v_{ki}^* and scalar a_k^t , CSW, can be calculated according to model (4). According to this formulation, the third constraint, (4-3) ensures that the CSW for each component is equal to one. Constraints (4-4)- (4-5) are related to presenting the heuristic approach of the RAM as non-Archimedean epsilon, which is used in order to avoid the appearance of zero weights. Equation (4) computes the efficiency of DMUs based on the weights extracted from model (3).

$$\theta_j = \frac{\sum_{u=1}^{f_k} u_{ku} c_{kuj}^t + \sum_{w=1}^{W_k} \eta_{kw} l_{kwj} + a_k^t}{\sum_{i=1}^{m_k} v_{ki} x_{kij}^t + \sum_{w=1}^{W_{k-1}} \eta_{k-1w} l_{k-1wj}}, \quad (4)$$

The weight of each stage is then calculated by considering the common set weights. A rational choice for the weight of a stage (w^k) is the ratio of resources allocated to stage k to all resources consumed in the process, which reflects its relative magnitude. More precisely,

$(\sum_{i=1}^m v_{ij1} X_{ij1} + \sum_{w=1}^w \eta_{wj(1,2)} I_{wj(1,2)} + \sum_{i=1}^m v_{ij2} X_{ij2} + \sum_{w=1}^w \eta_{wj(2,3)} I_{wj(2,3)} + \sum_{i=1}^m v_{ij3} X_{ij3})$ refers to the magnitude or the amount of input spent in the whole process, and w^k indicates the portion of the total input used in stage k (Cook et al. 2010). Thus, there are:

$w_k = (\text{component } k \text{ input}) / (\text{total input across all components})$

$$w_1 = \left(\sum_{i=1}^m v_{i1} X_{ij1} \right) / \left(\sum_{i=1}^m v_{i1} X_{ij1} + \sum_{w=1}^w \eta_{w(1,2)} I_{wj(1,2)} + \sum_{i=1}^m v_{i2} X_{ij2} + \sum_{w=1}^w \eta_{w(2,3)} I_{wj(2,3)} + \sum_{i=1}^m v_{i3} X_{ij3} \right),$$

$$w_2 = \left(\sum_{w=1}^w \eta_{w(1,2)} I_{wj(1,2)} + \sum_{i=1}^m v_{i2} X_{ij2} \right) / \left(\sum_{i=1}^m v_{i1} X_{ij1} + \sum_{w=1}^w \eta_{w(1,2)} I_{wj(1,2)} + \sum_{i=1}^m v_{i2} X_{ij2} + \sum_{w=1}^w \eta_{w(2,3)} I_{wj(2,3)} + \sum_{i=1}^m v_{i3} X_{ij3} \right),$$

$$w_3 = \left(\sum_{w=1}^w \eta_{w(2,3)} I_{wj(2,3)} + \sum_{i=1}^m v_{i3} X_{ij3} \right) / \left(\sum_{i=1}^m v_{i1} X_{ij1} + \sum_{w=1}^w \eta_{w(1,2)} I_{wj(1,2)} + \sum_{i=1}^m v_{i2} X_{ij2} + \sum_{w=1}^w \eta_{w(2,3)} I_{wj(2,3)} + \sum_{i=1}^m v_{i3} X_{ij3} \right),$$

(5)

The core DEA of Equation (5) is to use different weights for different stages of the process depending on the specific conditions of the evaluated supplier. So, the overall efficiency measure of the multistage process can reasonably be represented as a convex linear combination of the k stage measures, namely:

$$\theta_{\text{total}} = \sum_{k=1}^K w_k \theta_k \quad \text{where} \quad \sum_{k=1}^K w_k = 1. \quad (6)$$

Note that weights w_k represent the relative importance of the efficiency of stage k for (or its relative contribution to) the overall efficiency of the process. Here, θ_k is the efficiency of Θ at stage k, say, by solving model (4) is determined.

4. Case study

To validate the proposed model, it is used to examine the sustainability of a company named Nirou Moharekeh Industries (NMI) from 2011 to 2015. NMI is an Iranian manufacturer of auto spare parts and has 12 suppliers. It is assumed that NMI aims to evaluate the overall, Partial, and periodic efficiency of its suppliers. Each supplier has three stages including production, packaging, and distribution. The structure of the input, carry-over, and intermediate variables over the five-year period are shown in Figure 2.

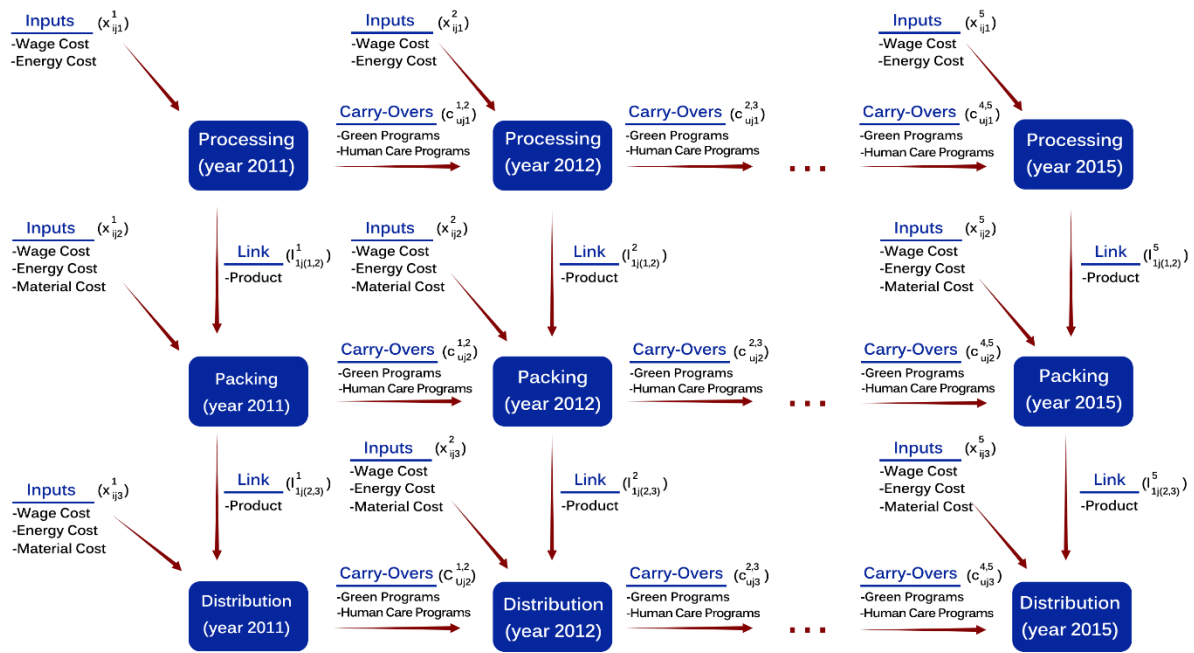


Figure 2. Structure of the suppliers of NMI

Table 3, shows the efficiency (partial, periodic, periodic-partial, and overall) of each NMI supplier based on Model (1).

according to table 3, the developed DNDEA model (model 1) can measure the suppliers in terms of overall (column 3), periodic (column 7-11), partial (column 4-6), and periodic-partial (the subset of the periodic efficiency, column 7-11) efficiency, and then identify the most efficient options

(Column 3). Considering the weights assigned to the stages and periods, the objective function of model **Error! Reference source not found.** undergoes some changes, depending on the kind of efficiency that is being calculated.

Specifically, the objective function is $\left(\min q = 1 - \frac{1}{T} \sum_{t=1}^T \frac{1}{K} \sum_{k=1}^K \frac{1}{m+u} \sum_{i=1}^m \sum_{u=1}^U \frac{S_{iok}^t}{R_{iok}^t} + \frac{S_{uok}^{t-1,t}}{R_{uok}^{t-1,t}} \right)$ for total efficiency,

$\left(\min q = 1 - \frac{1}{K} \sum_{k=1}^K \frac{1}{m+u} \sum_{i=1}^m \sum_{u=1}^U \frac{S_{iok}^t}{R_{iok}^t} + \frac{S_{uok}^{t-1,t}}{R_{uok}^{t-1,t}} \right)$ for periodic efficiency, $\left(\min q = 1 - \frac{1}{T} \sum_{t=1}^T \frac{1}{m+u} \sum_{i=1}^m \sum_{u=1}^U \frac{S_{iok}^t}{R_{iok}^t} + \frac{S_{uok}^{t-1,t}}{R_{uok}^{t-1,t}} \right)$ for partial efficiency, and

$\left(\min q = 1 - \frac{1}{m+u} \sum_{i=1}^m \sum_{u=1}^U \frac{S_{iok}^t}{R_{iok}^t} + \frac{S_{uok}^{t-1,t}}{R_{uok}^{t-1,t}} \right)$ for periodic-partial efficiency. Table 3 shows that KARAN obtained the highest efficiency and SIRIN S. N (0.6409) the lowest. Since only one DMU is identified as efficient, it can be argued that the model has excellent discriminatory power, which enables it to provide a complete ranking. However, for some years (e.g., 2013), several DMUs earned the highest efficiency value, which makes it impossible to produce a periodic ranking. Also, according to Liu et al., DNDEA models cannot compute optimal input and output weights. To overcome this issue, the model of Jahanshahloo et al. (2005) is expanded for dynamic network DEA using a RAM-based approach. But to implement the resulting DN-CSW model, it is first necessary to prepare the data for the model, as there is a notable difference between the largest and the smallest values, and some DMUs have zero inputs in some years. By adding the smallest positive input value to each value, zero values can be eliminated (Caggiani et al. 2021; Gavião et al. 2020). Then the mean normalization method is used to eliminate the imbalance in data (Cheng and Cantore 2020; Gasser et al. 2020). For a detailed explanation of the DEA data preparation process, see (Sarkis 2007). After preparing the data, using model (4), CSW is calculated for the input, intermediate, and output variables.

Table 4: The common set of the weights

variable	CSW	variable	CSW
V ₁₁	0.0403	u ₂₁	0.0204
V ₁₂	0.2643	u ₂₂	0.0063
u ₁₁	0.0460	Z ₂₁	0.3763
u ₁₂	0.0735	V ₃₁	0.1887
Z ₁₁	0.5759	V ₃₂	0.0098
V ₂₁	0.0059	V ₃₃	0.1092
V ₂₂	0.0083	u ₃₁	0.1264
V ₂₃	0.0070	u ₃₂	0.1896

The scalar values of a_k^t are provided in Table 5.

Table 5: The value of a_k^t

	2011	2012	2013	2014	2015
a_1^t	-0.5575	-0.6170	-0.6198	-0.6028	-0.6220
a_2^t	-0.2158	0.1559	0.1610	0.1559	-0.1740
a_3^t	-0.0047	0.0491	0.0790	0.0836	0.0545

Having CSW (Table 4) and scalar a_k^t values (Table 5) obtained from model (4), dynamic network efficiency values of DMUs were calculated using Equation (5). These efficiency values are provided in **Error! Reference source not found.**

Table 6. Efficiency values DN-CSW of the supplier of NMI

DMUs	Rank	Normal efficiency	over-all efficiency	Partial efficiency			Term efficiency															
				Div.1	Div. 2	Div. 3	2011			2012			2013			2014			2015			
							Div.1	Div.2	Div.3	Div.1	Div.2	Div.3	Div.1	Div.2	Div.3	Div.1	Div.2	Div.3	Div.1	Div.2	Div.3	
TECH A. T	2	0.9349	0.6336	0.5567	0.6542	0.6898		0.8089			0.5270			0.5810			0.8668			0.5023		
STEE L. P	10	0.5892	0.3993	0.2416	0.5434	0.4128	1.0000	0.4267	1.0000	0.3216	0.8901	0.3692	0.3476	0.9223	0.4731	0.6261	0.9743	1.0000	0.4714	0.4184	0.6171	
D. L. KAR AN	11	0.5238	0.3549	0.2221	0.5271	0.3156	0.3001	0.3361	0.2312	0.2309	0.8951	0.4400	0.1858	0.8789	0.3236	0.2930	0.9220	0.5597	0.2054	0.3707	0.6701	
PARS HAM	9	0.6321	0.4284	0.2420	0.5426	0.5005	0.2643	0.3233	0.1729	0.2010	0.8856	0.3922	0.2404	0.9652	0.7867	0.2520	0.8543	0.4514	0.1988	0.4137	0.1729	
FA- RA- ZAN	3	0.8658	0.5867	0.4032	0.5401	0.8168	0.3007	0.3443	0.2669	0.2362	0.9009	0.4601	0.1863	0.8857	0.4734	0.3582	0.9754	0.9186	0.2270	0.3805	0.6090	
SI- RIN S. N.	12	0.5002	0.3390	0.2031	0.5457	0.2682	0.4237				0.7054			0.9306			0.7779			0.5682		
PI- ROZ	6	0.7665	0.5195	0.2726	0.4939	0.7920	0.5211	0.2721	0.4778	0.3283	0.9814	0.8063	0.9099	0.9899	0.8919	0.3592	0.9908	0.9837	0.3750	0.3713	0.9583	
AL- SAN	4	0.8627	0.5846	0.4242	0.5423	0.7874	0.2923				0.5567			0.4511			0.3915			0.2762		
KAR AN	1	1.0000	0.6777	0.6877	0.5526	0.7926	0.2907	0.3285	0.2576	0.2312	0.9222	0.5167	0.1729	0.8609	0.3195	0.2012	0.8005	0.1729	0.1729	0.3946	0.2612	
TIR	8	0.6419	0.4350	0.2476	0.5378	0.5195	0.3026				0.6510			0.9102			0.8256			0.4807		
BAR AN	7	0.7247	0.4911	0.3454	0.5328	0.5951	0.2263	0.2553	0.4262	0.2151	0.9650	0.7730	0.7512	0.9923	0.9872	0.4915	0.9909	0.9943	0.2054	0.3638	0.8729	
HAM RAH	5	0.7866	0.5331	0.2495	0.6808	0.6689	0.4180				1.0000			0.9954			0.6376			0.7428		
							0.4770	0.2757	0.5013	1.0000	1.0000	1.0000	0.9998	0.9935	0.9929	0.2999	0.9504	0.6625	1.000	0.3662	0.8620	
							0.5318				0.8346			0.9907			0.7602			0.6140		
							0.6997	0.2913	0.6046	0.6768	0.9928	0.8342	0.9720	1.0000	1.0000	0.6410	0.9724	0.6672	0.5549	0.3754	0.9117	
							0.2814				0.4995			0.6102			0.9984			0.4180		
							0.2783	0.3117	0.2543	0.2095	0.8990	0.3899	0.2781	0.9528	0.5997	1.0000	0.9953	1.0000	0.2253	0.3829	0.6459	
							0.2957				0.6202			0.9789			0.5909			0.6882		
							0.3511	0.2787	0.2574	0.2846	0.9635	0.6123	0.9566	0.9928	0.9873	0.3118	0.9364	0.5244	0.6997	0.3648	1.0000	
							0.2452				0.6860			0.6373			0.6551			0.4509		
							0.1729	0.2541	0.3085	0.1729	0.9882	0.8968	0.3449	0.9672	0.5997	0.1990	0.9798	0.7867	0.2489	0.3651	0.7388	

Table 6 indicates that it is easy to rank DMUs or determine how certain units perform in comparison with others (column 2), using normal overall efficiency (column 3), which helps identify the most efficient supplier in this study. Furthermore, periodic efficiency can be used to monitor the dynamic state of suppliers over time (column 8-12), partial efficiency, can be used to identify the most significant stages of the supply chain (column 5-7), and periodic-partial efficiency can be recruited to find the source of inefficiency in each period (the subset of the periodic efficiency, column 8-12). For example, the first supplier (TECH.A. T) is ranked second with a total efficiency of 0.9349. This supplier has a better performance in the third stage, and in 2014, it achieved a higher efficiency (0.8668). Compared to other stages, in 2014, stage 1 (viz. production) gained the least efficiency (0.6261). As the results of **Error! Reference source not found.** show, the efficiency values computed by the DN-CSW model for all three stages of all DMUs for all years are lower than those obtained from the DNDEA model (Table 3). This is indicative of the higher discriminatory power of the model (4) than model (1). In models (1) and (5), all process components have the same weight in efficiency calculations regardless of whether they are equally significant for the efficiency of the process. In order to address this issue, the extracted CSWs (model 4) were used to calculate an importance weight for each stage of each DMU (supplier).

Table 6. weight of each stage

DMU		TECH. A. T	STEEL P	D. L. KARAN	PARS HAM	FARAZAN	SIRIN S. N.	PIROZ	ALSAN	KARAN	TIR	BARAN	HAMRAH	
Partial Weights	Div.1	0.2352	0.2483	0.3055	0.2542	0.0385	0.2888	0.0563	0.0639	0.0530	0.1771	0.0763	0.0405	
	Div.2	0.2400	0.3300	0.3050	0.3663	0.5591	0.2544	0.5586	0.5317	0.5154	0.4039	0.4717	0.5229	
	Div.3	0.5248	0.4217	0.3895	0.3795	0.4024	0.4568	0.3851	0.4044	0.4316	0.4190	0.4520	0.4366	
Term efficiency	2011	Div.1	0.2393	0.2658	0.2658	0.2855	0.0509	0.2522	0.0203	0.0650	0.0706	0.2256	0.1019	0.0400
		Div.2	0.2095	0.3333	0.3353	0.3384	0.5431	0.3436	0.5829	0.5470	0.5372	0.3715	0.3999	0.5093
		Div.3	0.5512	0.4009	0.3989	0.3761	0.4061	0.4042	0.3968	0.3880	0.3923	0.4029	0.4982	0.4507
	2012	Div.1	0.2403	0.2640	0.2705	0.2652	0.0845	0.2422	0.1220	0.0176	0.0522	0.2353	0.0949	0.0176
		Div.2	0.2036	0.3343	0.3337	0.3360	0.5128	0.3612	0.5072	0.5935	0.5133	0.3275	0.4521	0.5935
		Div.3	0.5561	0.4016	0.3958	0.3988	0.4027	0.3966	0.3708	0.3889	0.4345	0.4372	0.4530	0.3889
	2013	Div.1	0.2471	0.2855	0.1345	0.3309	0.0239	0.3260	0.0258	0.0183	0.0366	0.1324	0.0183	0.0555
		Div.2	0.2194	0.2539	0.4874	0.3160	0.5597	0.2546	0.5876	0.5929	0.5601	0.4185	0.5929	0.4762
		Div.3	0.5336	0.4605	0.3782	0.3531	0.4164	0.4194	0.3866	0.3888	0.4033	0.4491	0.3888	0.4683
2014	Div.1	0.2353	0.2414	0.3437	0.1125	0.0122	0.3099	0.0355	0.1597	0.0584	0.0120	0.1115	0.0111	
	Div.2	0.3360	0.3501	0.3239	0.5128	0.5971	0.1850	0.5758	0.4237	0.4448	0.5886	0.4050	0.5497	
	Div.3	0.4287	0.4084	0.3324	0.3747	0.3907	0.5051	0.3886	0.4166	0.4968	0.3994	0.4835	0.4391	
2015	Div.1	0.2049	0.1190	0.3659	0.1843	0.0120	0.2716	0.0629	0.0132	0.0460	0.1869	0.0121	0.0693	
	Div.2	0.2856	0.4584	0.2183	0.4139	0.5939	0.2381	0.5512	0.5543	0.5403	0.4183	0.5973	0.5046	
	Div.3	0.5095	0.4226	0.4158	0.4018	0.3941	0.4903	0.3858	0.4325	0.4137	0.3948	0.3906	0.4261	

Finally, by using the periodic-partial efficiency value (**Error! Reference source not found.**) and the calculated weights of each stage (Table 6), the periodic and overall efficiency values of the suppliers are recalculated as the total weight of the individual steps.

Table 7. Efficiency values of the suppliers of NMI according to the model 7

DMUs	RANK	Normal efficiency	overall efficiency	Partial efficiency			Term efficiency														
				Div. 1	Div. 2	Div. 3	2011			2012			2013			2014			2015		
				Div. 1	Div. 2	Div. 3	Div.1	Div.2	Div.3	Div.1	Div.2	Div.3	Div.1	Div.2	Div.3	Div.1	Div.2	Div.3	Div.1	Div.2	Div.3
TECH A. T	3	0. 9798	0. 6500	0. 5567	0. 6542	0. 6898	1. 0000	0. 4267	1. 0000	0. 3216	0. 8901	0. 3692	0. 3476	0. 9223	0. 4731	0. 6261	0. 9743	1. 0000	0. 4714	0. 4184	0. 6171
STEEL. P	10	0. 6232	0. 4134	0. 2416	0. 5434	0. 4128	0. 3001	0. 3361	0. 2312	0. 2309	0. 8951	0. 4400	0. 1858	0. 8789	0. 3236	0. 2930	0. 9220	0. 5597	0. 2054	0. 3707	0. 6701
D. L. KARAN	11.	0. 5299	0. 3516	0. 2221	0. 5271	0. 3156	0. 2643	0. 3233	0. 1729	0. 2010	0. 8856	0. 3922	0. 2404	0. 9652	0. 7867	0. 2520	0. 8543	0. 4514	0. 1988	0. 4137	0. 1729
PARS HAM	9	0. 6787	0. 4502	0. 2420	0. 5426	0. 5005	0. 3007	0. 3443	0. 2669	0. 2362	0. 9009	0. 4601	0. 1863	0. 8857	0. 4734	0. 3582	0. 9754	0. 9186	0. 2270	0. 3805	0. 6090
FARA-ZAN	4	0. 9740	0. 6462	0. 4032	0. 5401	0. 8168	0. 5211	0. 2721	0. 4778	0. 3283	0. 9814	0. 8063	0. 9099	0. 9899	0. 8919	0. 3592	0. 9908	0. 9837	0. 3750	0. 3713	0. 9583
SIRIN S. N.	12	0. 4823	0. 3200	0. 2031	0. 5457	0. 2682	0. 2907	0. 3285	0. 2576	0. 2312	0. 9222	0. 5167	0. 1729	0. 8609	0. 3195	0. 2012	0. 8005	0. 1729	0. 1729	0. 3946	0. 2612
PIROZ	6	0. 8987	0. 5962	0. 2726	0. 4939	0. 7920	0. 2263	0. 2553	0. 4262	0. 2151	0. 9650	0. 7730	0. 7512	0. 9923	0. 9872	0. 4915	0. 9909	0. 9943	0. 2054	0. 3638	0. 8729
ALSAN	5	0. 9555	0. 6338	0. 4242	0. 5423	0. 7874	0. 4770	0. 2757	0. 5013	1. 0000	0. 0000	0. 0000	0. 9998	0. 9935	0. 9929	0. 2999	0. 9504	0. 6625	0. 0002	0. 3662	0. 8620
KARAN	1	1. 0000	0. 6634	0. 6877	0. 5526	0. 7926	0. 6997	0. 2913	0. 6046	0. 6768	0. 9928	0. 8342	0. 9720	0. 0000	0. 0000	0. 6410	0. 9724	0. 6672	0. 5549	0. 3754	0. 9117
TIR	8	0. 7217	0. 4787	0. 2476	0. 5378	0. 5195	0. 2783	0. 3117	0. 2543	0. 2095	0. 8990	0. 3899	0. 2781	0. 9528	0. 5997	0. 0000	0. 9953	0. 0000	0. 2253	0. 3829	0. 6459
BARAN	7	0. 8241	0. 5467	0. 3454	0. 5328	0. 5951	0. 3511	0. 2787	0. 2574	0. 2846	0. 9635	0. 6123	0. 9566	0. 9928	0. 9873	0. 3118	0. 9364	0. 5244	0. 6997	0. 3648	0. 0000
HAMRAH	2	0. 9921	0. 6581	0. 2495	0. 6808	0. 6689	0. 1729	0. 2541	0. 3085	0. 1729	0. 9882	0. 8968	0. 3449	0. 9672	0. 5997	0. 1990	0. 9798	0. 7867	0. 2489	0. 3651	0. 7388

As Table 7 shows, using the proposed method changed the efficiency scores of some suppliers, leading to a change in the ranking, which is discussed next section. Note that table 8 has the same overall arrangements compared to table 6.

5. Findings and managerial implications

Our framework and discussion have several managerial implications. To provide an overview of the multitude of factors and relationships involved in our discussion, we used a developed sustainable supply chain model in this paper. With some adjustments in the intervals of analyses and simulations of causal relationships, this method to supply chain analysis can thus aid managers predict the risks and threats that may obstruct the transition of a chain toward sustainability and then devise a plan accordingly. Thus, the method provides managers with a framework for conservative decision-making in this area. Since the proposed model is independent of the criteria utilized in this paper, decision-makers can introduce more criteria to the system or remove those they feel are not appropriate for their specific cases. This enables managers to adjust their supply chain strategies more easily, especially when they feel the chain is exposed to some risks originating from sustainability-related pressures and concerns. Model (1) quantifies efficiency while simultaneously considering process structure, process stages, and time, it can be practiced to accurately trace the source of inefficiency of each decision-making unit (DMU: supplier) each year. For example, HAMRAH became inefficient, with a score of 0.9994 because of inefficiency at Stage 1 (i.e., production) in 2011 while it was efficient packaging and distribution, ranked in second place. Thus, in that year, this supplier should have focused on the production stage. Or the supplier TECH. A.T became inefficient with a score of 0.9631 because of inefficiency in stage 2 (packing) and stage 3 (distribution) in 2015, while it was efficient in the production stage and ranked in third place. Thus, in that year, this supplier should have focused on the packing and distribution stages. As shown in table (1), KARAN and SIRIN.S. N (0.5409) obtained the

highest and lowest efficiency scores, respectively. Taking the common weights of Table 4 and the scalar a_k^t values of Table 5 into account decreased the partial efficiency values of DMUs for all years, with the exception of those that earned the maximum efficiency value (**Error! Reference source not found.**). The greatest and smallest declines in efficiency values compared to the results of model (1) were observed in HAMRAH-2011 (0.752) and KARAN-2014 (0.0016) respectively. After applying common weights, the ranking of six suppliers (TECH A. T, STEEL. P, D. L. KARAN, PARS HAM, FARAZAN, HAMRAH) changed, and the ranking of six suppliers (SIRIN S. N., PIROZ, ALSAN, KARAN, TIR, BARAN) remained constant. By using tables 3, 6, and 8 presented in this study, in addition to the general condition of the suppliers, various analyses can be deduced, including examining the dynamic condition of suppliers. According to table 6, SIRIN S. N, the most inefficient supplier, achieved its best performance in 2012 with a score of 0.5567, and its efficiency declined in the subsequent years. According to partial efficiency, this supplier performed better (0.5557) in the second stage.

Models (1) and (4) give equal weights, $w_1 = w_2 = w_3 = \mathbf{0.33}$, to the components of the production process. In other words, the process efficiency is obtained from the arithmetic mean of the efficiency of components, a mechanism that does not reflect the importance of components for the efficiency of the process. To address this problem, for the first time in the literature for CSW methods, the extraction of the weights by model 4 was used to assign separate weights to each stage of DMU, and the efficiency value of each DMUs was recalculated accordingly. The results showed that the efficiency values were affected by the weight of the stages. After assigning weights to the stages, tables 6 and 8 clearly show that HAMRAH efficiency values increased the most in 2012 (0.2523), while ALSAN decreased most in 2015 (0.1538). These changes also altered the ranking of DMUs (suppliers). The ranking obtained in this way was almost similar to the one obtained from the DNDEA model, as seven suppliers earned exactly the same ranking (TECH. A. T, SIRIN S. N., PIROZ, KARAN, TIR, BARAN, HAMRAH), and the rest were ranked one position higher or lower. For example, STEEL.P was ranked 9th in model (1) but was ranked 10th after applying the component weights in the DN-CSW model. This is because the overall efficiency values obtained in this way are closest to those obtained from the base model (model 1). Supplier HAMRAH, for example, had an efficiency of 0.9994 in the model (1) but changed to 0.7866 in model 2 and returned to 0.9921 after applying the component weighting. Generally, the results of applying three approaches to 12 NMI suppliers within the DNDEA model show that just one unit (i.e., KARAN) obtains an efficient value. But an inefficient score was observed in 11 units, whose technical efficiency value was in the range of 0.6409 to 0.9994 (table 3). After implementing the DN-CSW model, the efficiency scores of other suppliers moved to the range of 0.5002-0.9349 (table 6), and the range shifted to 0.4823-0.9921 after applying the stages weights (table 8). These changes occurred because the choice of weights introduces a kind of value judgment to the DEA model. Therefore, the efficiency values produced by model 1 were higher than those obtained from the other two models. These results

suggest that when using CSW in-network or dynamic network DEA, assigning weight to individual process components will have a positive impact.

6. Conclusion

Due to the importance of structure and time in assessing units, as well as the huge difference between the largest and smallest values, a DNDEA model based on the RAM, was employed. The model allows us to not only calculate the overall efficiency of DMUs throughout time but also consider the dynamic change in the periodic efficiency and the dynamic changes in the partial efficiency of DMUs. The developed model (model 1) was used to assess the efficiency of the suppliers of a company named Nirou Moharekeh Industries (NMI) from 2011-2015. The efficiency scores of each supplier were determined separately for each partial, periodic, and overall, and their periodic-partial efficiency scores were also calculated (Table 3). Subsequently, the source of inefficiency of each supplier was identified. According to Liu et al. (2022), The DNDEA model cannot obtain the optimal input and output weight.

This study extends Jahanshahloo et al. (2005)'s CSW model for dynamic network structure using RAM. According to table 6, the developed DNCSW method has more discriminatory power than the DNDEA model (model 1). Dynamic network DEA can help experts and policymakers better understand the strengths and weaknesses of DMUs. This will enable them to try to enhance their efficiency by working on these strong and weak points. In models (1) and (2), i.e., the DNDEA and DNCSW models, all components of the production process are given the same weight regardless of how individually significant they are for the efficiency of the process. This problem was addressed in this study by using the extracted CSWs (model 6) to compute a separate weight for each stage of each DMU (Table 7). The use of this approach along with the CSW method in the DN-CSW model, caused it to produce efficiency values closer to those obtained from the base model (model 1). In other words, taking this approach led to a more reasonable and fairer ranking of DMUs, capable of offering richer information to decision-makers. Compared to the model by Jahanshahloo et al. (2005) and the previous research, the main contribution and advantages of the DN-CSW model are: Firstly, in addition to calculating the overall efficiency of DMUs over a given period, this model can also determine dynamic changes in the periodic efficiency and, the partial efficiency of DMUs, and periodic-partial efficiency, and then identify the most efficient options. Secondly, the developed model is based on RAM, and through additional constrain defined in the model, considers the weight of each stage to be equivalent to one. Another advantage of this work over previous studies is the examination of the effect of component weights on the CSW approach in network structures. Since the models presented in this article are independent of the number of criteria and their values, they can be applied to any activity in the production or service sectors. Furthermore, this study is expected to assist NMI's management in making better decisions to improve supply chain management and minimize risk in their supply chain to achieve sustainability. Researchers are hoping the study will enrich the theory of DEA and provide more alternative methods for assessing the multi-stage process's performance. Future studies can be devoted to a comparison between the proposed model and complete ranking models such as super-efficiency models. This approach was developed for the BCC model, which has a variable return to scale (VRS). In future studies, a common set of weight models can be developed for the CCR model with the constant return to scale (CRS).

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