Vol. 9, No 2, 2022, Pages 45-63

JIEMS
Journal of Industrial Engineering and Management Studies

journal homepage: www.jiems.icms.ac.ir

Perishable food supply chain coordination with multiplicative demand function of price, shelf-life duration, and green technology investment under revenue-sharing contract

Mohammad Reza Gholamian *[1], Maryam Noroozi[1]

[1] Department of Logistics and Supply Chain Engineering, School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran.

Received: May 2022-25/ Revised: Sep 2022-06/ Accepted: Sep 2022-18

Abstract

In the proposed study, an inventory model of a two-echelon green perishable supply chain which consists of one manufacturer and one retailer is investigated. The produced items have a deterministic shelf life and will be removed from the shelves when they reach to their expiration date. A novel formulation of the demand function is also presented, which is a multiplicative function of time after replenishment, retail price, and green improvement level. The formulated demand function increases with the time to expiration and the green improvement degree; it also decreases with the retail price. The mentioned characteristics in this supply chain are derived from the industries of selling fruits, vegetables, meat and poultry, as well as dairy products. The manufacturer is considered the leader of the Stackelberg game, and three approaches are proposed to solve the inventory model: Decentralized, Centralized, and Coordinated by the revenue and green technology investment cost sharing contract, which guarantees more profit for each member than the decentralized decision-making approach. The numerical results demonstrate that the proposed revenue-sharing contract could successfully help the supply chain members to achieve the potential supply chain profit in the centralized approach. A comparative study is also conducted to show the differences between implementing and not implementing green technology investments, which shows the profitability of making green technology investments when consumers have green preferences. It was observed that as the reservation cost increases, the importance of investments in green technology will increase for both parties. Also, with high potential market demand, it is more beneficial to invest in green technology. This study deals with a contribution to the formulation of the demand function, as a novel multiplicative function of time after replenishment in the form of power function, and retail price and green improvement level in the form of complementary linear function.

Keywords: supply chain coordination; revenue sharing contract; perishability; green technology investment; Stackelberg game.

Paper Type: Original Research

1. Introduction

The arisen environmental issues have become significant concerns for every society, and it led to the generation of the concept of the green supply chain. The main goal of the green supply chain management is to maximize the profits of the supply chain members while encouraging them to produce items with the least harms for the environment in every stage of producing and distributing cycle (Barari et al., 2012). Considering environmental issues, government policies, and customer's preference to buy green products, many manufacturing companies attempt to invest in green technology to gain their share from the market demand. Green technology is referred to a technology that prevents human's harmful effects on the environment during the production process. Some green technologies make improvements that will lead to resource efficiency, while others deal with producing environmental-friendly items. Employment of green technology facilities in the perishable food supply chain will reduce waste and energy consumption; Moreover, using this technology will help the supply chain to prevent the transmission of bacteria and viruses to food products.
As a result of the widespread use of the fresh food supply chain in people's lives, it has received a lot of attention. Perishable product supply chain management, widely defined as items with a limited shelf life or that deteriorate over a period of time (foods, Pharmaceutical and medical items and etc.), is a famously tough subject with several unique problems. Those problems arise as a result of perishable items losing their quality and quantity over time, even if they are protected and preserved properly along the supply chain. Nonetheless, as the standard of living

*Corresponding Author: gholamian@iust.ac.ir

DOI: 10.22116/jiems.2022.344170.1489

rises and customers seek for high-quality fresh meals, the necessity for efficient rules to regulate perishable product supply chains becomes more pressing.
Although fresh food supply chains are similar to conventional supply chains, but they have unique characteristics that make their management more complex. In particular, the supply chains of agricultural and fresh food products are characterized by food perishability, seasonality, uncertainty in product demand, and the need for special transportation and holding equipment. Therefore, food supply chain management is associated with many challenges in the sectors of suppliers, producers, warehouses, and retailers so that the product can be produced in the right quantity and distributed at the right time and place, and this is only possible if a set of approaches were used among all members of the supply chain to effectively integrate the decisions of supply chain members and to ensure proper coordination between them.
Product perishability affects inventory levels in two different ways. The first one is called "the direct impacts" that change the inventory level by spoiling, which occurs when unsold perishable items become useless after a certain length of time and vanish from the inventory. The other one is called "the indirect impacts," which is related to consumers' preference for fresher items and will lead to a demand reduction over time. Sarker et al. (1997) argued that consumers are less confidence in purchasing perishable items with approaching expiration dates, they regularly check the expiration date before placing an order, according to Tsiros and Heilman (2005), and their desire to buy a perishable product diminishes over time.
This research focused on a perishable green supply chain with only one manufacturer and one retailer. A novel demand function which demonstrates the indirect effect of perishability, consumer green preferences, shelf-life duration and retail price is also formulated. The proposed demand function increases with the time to expiration and the green improvement degree; it also decreases with the retail price. The manufacturer produces a single perishable item with a deterministic shelf-life. The manufacturer is the Stackelberg game's leader and makes decisions about wholesale price, green improvement level and also green technology investments while the retailer is responsible for decision making about replenishment policy using economic order quantity (EOQ) and retail price. Like many coordination models, three decision-making approaches to solving the mathematical model are offered, 1- Centralized: the retailer and the manufacturer are considered as a whole unit that makes decisions jointly on all decision variables, which leads the supply chain to achieve the maximum potential profit. 2- Decentralized: Each member of the supply chain seeks to improve his/her own objective function separately, and 3- Coordinated by the novel revenue-sharing contract: an incentive contract which motivates the supply chain members to make decisions jointly, such that the profit for each supply chain member in this approach is more than those in the decentralized approach. The rest of this paper is about the finding proper answers to following questions.

1- Is the proposed novel contract able to coordinate the supply chain?
2- Is the proposed contract able to achieve the potential profit in the centralized decision-making approach?
3- Is it profitable for the supply chain to invest in green technology?
4- How does investing in green technology affect other decision variables such as wholesale and retail prices?
5- What effect does a variation in problem parameters such as potential market demand (λ_0) and reservation cost (α^ (-1)) have on decision variables and supply chain profit?

This paper's remainder is organized as follows: In section 2, we discuss the background literature for this paper in three parts. In the first part, recent researches on coordinated supply chains under the revenue-sharing contract were briefly reviewed. The second part is about the supply chains that consider the demand function dependent on time. The last part is a review of supply chains with green technology investment. In section 3, the mathematical model is formulated, and three decision-making approaches containing centralized, decentralized, and coordination by the revenue-sharing contract are demonstrated. Section 4 is about solving numerical examples to show the effects of different decision-making approaches on each supply chain member's profits. In section 5 a sensitivity analysis is prepared. Managerial insights for the supply chains similar to this case are presented in section 6. Finally, the conclusion of the whole paper is discussed in section 7.

2. Literature review

Because of the unique characteristics of the perishable food supply chain, several studies have been proposed in this area; a selection of relevant studies are included below. So far, some researches have been done on the impact of various factors on perishable products inventory, such as holding costs and failure rates, preservation technology costs (Mohammadi et al., 2019), time-sensitive demand and holding costs (Rani et al., 2017), unstable demand (Janssen et al., 2018), and green policy (Toktas-Palut, 2021) Other researches have looked into perishable product pricing techniques (Kaya and ghahroodi, 2018). Some studies have addressed pricing and discount (Azadi et al., 2019) or the quantity discount of perishable products (Zhang et al., 2016). A model is proposed by Amiri et al. (2020) for finding the optimal perishable product sales level in a two-stage supply chain with one producer and a number of retailers using VMI policy. This model attempts to maximize profits using exact methods and metaheuristics such as the genetic algorithm and particle swarm optimization. Liu et al. (2021) explore a two-level supply chain for perishable products like fruits and milk, which have price-sensitive demand. The producer, as the leader, determines the product's processing time and wholesale price in order to optimize profit, while the retailer, determines the selling price and quantity of orders. During the pandemic (Covid-19), Kumar et al. (2021) developed and assessed perishable food supply chain risk reduction measures. According to research proposed by Jouzdani and Govindan (2021), a multi-objective mathematical planning model has been developed in order to minimize the cost, energy usage, and traffic related to a perishable food supply chain.

A brief summary of studies on the proposed supply chain model is provided in this part. In supply chain (SC) contexts, numerous coordination models have been addressed by different researchers. Since each supply chain member has different objectives and prefers to increase its own profit, the entire SC performance may decrease, so the SC performance will greatly improve if we determine decision variables under the coordination approach (Govindan et al., 2013). Tsay et al. (1999) and Cachon (2003) gathered many pieces of researches related to different coordination models and provided a review article in this area. This study falls within the scope of coordinating the supply chain by revenue sharing contract, which includes a lot of studies in this area. Sarathi et al. (2014) proposed a mathematical model for a two-stage supply chain with a demand function dependent on the stock and retail price. Various combinations of contracts have been used for supply chain coordination; for example, Raza (2018) proposed a coordination model for a supply chain including a manufacturer and a retailer with a demand function sensitive to price and corporate social responsibility, the coordination model is analyzed by deterministic and stochastic situation, the inventory model is also investigated under the centralized, decentralized and revenue sharing coordination scenarios. He et al. (2009) developed a mathematical model for a supply chain with uncertain demand that is sensitive to the sales effort and the retail price to achieve channel coordination. They analyzed three different combined contracts: a joint revenue-sharing with sales rebate and penalty (SRP) contract, the joint SRP with return policy contract and revenue sharing with the return policy contract. Finally, the results showed that channel coordination could be obtained through a joint return policy with the SRP contract. To coordinate a two-stage supply chain with a demand function dependent on multi factors, two contracts was investigated by Bai et al. (2016), They analyzed the revenue-sharing contract and the revised one. The numerical results in their work show that both contracts can achieve supply chain coordination but the revised contract always makes a higher profit than the other one. Hu et al. (2019) developed a four-stage agricultural supply chain including an agricultural producer, a processing firm, a distributor, and numerous customers based on the Stackelberg game and a combination of contracts, (revenue-sharing, quality commitment, and risk-sharing) with the aim of coordinating the food and agriculture supply chain. Malekitabar et al. (2019) investigated a mathematical inventory model for growing-mortal products. They considered a two-echelon supply chain consists of a farmer and a supplier. The growth function and mortality rate are also considered in the proposed mathematical model. Finally, they used revenue-sharing and revenue and cost-sharing to coordinate the proposed supply chain. Liu et al. (2020) analyzed the effects of implementing the revenue-sharing and cost-sharing contracts on improving product greenness under retailer's and manufacturer's Stackelberg games. Yan et al., (2020) developed two revenue-sharing and wholesale price coordination contracts for decision-making in the fresh agricultural supply chain based on the characteristics of the fresh agricultural supply chain. khorshidvand et al(2021b), offered a two-stage approach that, optimal decisions on pricing, greenness, and advertising are made in the first stage, while in the second stage, a fuzzy multi-objective mixed integer linear programming (MILP) model is used to maximize the supply chain profit. An augmented ϵ-constraint method is used to solve small-scale instances, and Lagrangian relaxation algorithm is used for large-scale instances. Also, Khorshidvand et al (2021a), proposed a sustainable closed-loop supply chain, in which in the first stage, the optimal decision variables are derived and, in the second stage, multi-objective model is extended to reach into Pareto solutions. Besides, a Lagrangian relaxation algorithm is developed based on the weighted-sum method to solve the developed model.

In many deterministic inventory models, there are studies developed under the assumption that an item's demand rate is either constant or dependent on price, time, and many other factors. The market demand for perishable products is positively affected by the remaining duration of their shelf life; however, there are not many kinds of researches in this field. Generally, the age of perishable items inventory negatively affects the demand rate because customers tend to purchase products whose expiration dates are not approaching. In the following, literature related to on time-dependent demand rates was provided. Schlosser (2016) proposed a series of stochastic dynamic pricing and advertising models for a finite number of items with time-dependent demand elasticities. For deteriorating products, KavithaPriya and Senbagam (2018) studied an (EOQ) inventory model with a quadratic time-sensitive demand rate. Maihami and Nakhai (2012) suggested an inventory control model, with a time-related demand function. In the proposed research by Karuppasamy and Uthayakumar (2017) a deterministic inventory model in the pharmaceutical industry has been considered, which formulated the time-dependent demand function and holding cost. Prasad and Mukherjee (2016) established an inventory model for deteriorating items, which face time-dependent demand. Tripath et al. (2017) studied an inventory model with time-sensitive deterioration rate and also exponential time-dependent demand.

Toktas-Palut (2021) proposed a model for coordinating a three-stage green supply chain. As a result, he adopted a two-part tariff contract. The flow of items in the developed supply chain was forward and reverse; in fact, the producer and remanufacturer invested in green technologies to increase demand for new and repaired items. Huang et al. (2020) proposed a two-stage supply chain that investigates carbon emissions during the stages of manufacturing, shipping, and storage. They studied at how investments in green technologies and various carbon emission strategies would affect the proposed supply chain. A green supply chain consists of one manufacturer who is committed to producing green products and two competitive retailers, was offered by Li et al. (2021), who used a two-part tariff contract to establish coordination among supply chain members. Khorshidvand et al. (2021d) developed a hybrid method to optimize green sustainable closed-loop supply chain which makes pricing, greenness and advertising decisions. Robust scenario-based stochastic programming model is used to overcome stochastic demand in this work. In other study, Khorshidvand et al (2021c) investigated a green supply chain that is formulated a demand function dependent on price, greenness and advertisement. Two kinds of selling channels (i.e., online and in-person) are considered to enable all the parties to sell the product through their own channels. Gao et al. (2020) investigated a green supply chain management problem that sells its products in two channels. The government offers subsidies in green technology investment to the manufacturer for maintaining the environmental protocols. Using mathematical modeling and game theory, they optimized prices in both channels considering green standards. Ghosh and Shah (2012) evaluate the impact of greening levels on demands, prices, and supply chain members' profits. They also developed a mathematical model for channel coordination using a two-part tariff contract. A supply chain including one manufacturer and two competitive retailers is considered by Shi et al. (2019) they analyzed the effects of three clean investment scenarios on market demands. The results showed that revenues and carbon pollution are better in the scenario when both retailers invest in a clean supply chain than in the other scenarios. A supply chain with the strategy of green investment for suppliers and manufacturers was proposed by Sun et al. (2019). In this study, a game theory model was investigated between suppliers and manufacturers. They analyzed the effects of government subsidy mechanisms and green technology investment on the supply chain objectives. Wang and Song (2020) proposed a stochastic demand dual-channel supply chain that included sales effort and green technology investment. The manufacturer makes two sorts of items: green and non-green, and distributes non-green products through retailers while selling green products directly to consumers.

3. Mathematical Model

We have investigated a two-stage perishable food supply chain with a single manufacturer and a single retailer. The proposed mathematical model estimates the total profit for the manufacturer and the retailer over an infinite planning period. The manufacturer produces a single item which is perishable, within a fixed period. By decreasing the remaining shelf-life duration, the demand rate decreases too. One of the main ideas in the proposed supply chain is about producing an environmentally friendly product. To accomplish this idea, the manufacturer invests in green technology and the green technology investment is assumed as a quadratic function $\frac{1}{2}\eta\theta^2$, where η is the cost coefficient of the green investment, $\eta > 0$, and θ is the resulting improvement in green degree by investing in green technology, $\theta \geq 0$. In this case, $\theta = 0$ means that the manufacturer does not make any investment in green improvement degree. Many kinds of researches in the literature have used this cost function for green technology investment ($\frac{1}{2}\eta\theta^2$) (He et al, 2009). The manufacturer's decision variables are the optimal green improvement degree (θ) and the optimal wholesale price. Items are produced at the green degree θ and unit cost $c'$and the retailer buys each item at a unit wholesale price c. Eventually, each item will reach to the final customer at a retail price p. The retailer determines replenishment period T and sells products to the customer at a retail price p. The manufacturer and retailer determine their decision variables with manufacturer Stackelberg approach. The model is solved under three decision-making approaches: decentralized, centralized, and revenue sharing contract. The assumptions of the model are summarized below.

1. The shelf-life duration of an item is constant and known to the consumers.
2. There is no backlogged order.
3. The demand for products that passed their shelf-life is equal to zero.

The following notations are used in the mathematical modeling:

Notation

- E - duration of shelf-life
- c'-the production cost of an item.
- c-the wholesale price.
- p-the retail price of a unit.

- h-the holding cost per unit of time
- T-the replenishment period.
- q- the order quantity.
- t- the time passed from replenishment.
- $\lambda(t, p, \theta)$- the demand rate at time t, retail price p and greenness degree θ.
- λ_0- the potential market demands.
- I(q, p, t)- the inventory level at time t.
- θ - the green improvement degree, ($\theta \geq 0$)
- η- green investment cost coefficient, ($\eta > 0$)
- α - price coefficient, ($\alpha > 0$)
- β- coefficient of additional demands caused by green technology investment
- $n -$ coefficient of decreasing demand
- k_b-retailer's fixed ordering cost
- k_s-manufacturers fixed ordering cost
- TP_R^{dec}- The retailer's profit in decentralized structure.
- TP_M^{dec}- The manufacturer's profit in decentralized structure.
- TP^{cen}- The supply chain's profit in the centralized structure
- TP_R^{coo}- The retailer's profit in the coordinated structure.
- TP_M^{coo}- The manufacturer's profit in the coordinated structure.

There are many factors that affect the product demand, in this paper the effects of three main factors including: age of products, the degree of green improvement, and the retail price, have been studied. In the perishable food industry, the product demand is significantly affected by its freshness. As the product reaches into its expiration date, some degradations in flavor, aroma, color and texture would happen, and at result, the consumer desire to buy the product decreases. Another factor affecting the food demand is the process of producing foods considering the amount of organic raw materials used in the food production which has been reflected as product greenness degree. Considering customer environmental awareness, the fewer harmful substances are used in the production of the food products leads to the more food demand. Therefore, in food industry, investing in green products has positive effects on increasing the perishable products demand. Finally, the last factor that is considered in the most demand functions is the inverse relationship between the product price and demand.

Eq.1 demonstrates that demand function has direct relation with green improvement degree and simultaneously has inverse relation with the time after replenishment and retail price.

$$\lambda(t, p, \theta) = \begin{cases} \lambda_0(1 - \alpha p_x)\left(1 - \left(\frac{t}{E}\right)^n\right)(1 + \beta\theta), t \leq E, p \leq \alpha^{-1}, 0 < \theta < 1 \\ 0, \ \ ow \end{cases} \quad (1)$$

The retailer's profit function consists of three elements: the gross margin per unit, the inventory holding costs, and the fixed ordering cost; which can be expressed as follows:

$$TP_R^{dec} = (p - c)Q - h\int_0^T I(Q, P, t)dt - k_b \quad (2)$$

Where:

$$Q = \int_0^T \lambda(t, p, \theta)dt \quad (3)$$

$$I(t) = \int_t^T \lambda(p, \theta, \tau)d\tau \quad (4)$$

Then, the retailer's profit function would be:

$$TP_R^{dec} = (p-c)Q - h\int_0^T \int_t^T \lambda(p,\theta,\tau)d\tau\, dt - k_b \quad (5)$$

$$TP_R^{dec} = (p-c)Q - h\int_0^T t\lambda(p,\theta,t)\, dt - k_b \quad (6)$$

And hence:

$$TP_R^{dec} = \lambda_0(1+\beta\theta)(1-\alpha p)\left[(p-c)\left(1-\frac{(T/E)^n}{n+1}\right) - \frac{hT}{2}\left(1-\frac{2(T/E)^n}{n+2}\right)\right] - \frac{k_b}{T} \quad (7)$$

The manufacturer's profit function consists of three elements: the gross margin per unit, the green technology investment, and the fixed ordering revenue ($k_b > k_s$); which can be represented as follows:

$$TP_M(c,\theta) = \frac{Q(c-c')}{T} - \frac{k_s}{T} - \frac{1}{2}\eta\theta^2 \quad (8)$$

3.1. Decentralized supply chain

Each supply chain member tends to optimize his/her own objective function in a supply chain with a decentralized decision-making structure. The manufacturer Stackelberg game is used in this section to evaluate the optimal value of decision-making variables for both the manufacturer and the retailer in order to achieve maximum total profit for each member of the supply chain. First the optimized decision-making variables of the retailer, such as the retail price (p) and the replenishment period (T) are determined, and then, we try to find the manufacturer's best decision-making variables (i.e., the wholesale price (c) and the green improvement degree (θ)).

3.1.1. The optimal value of retailer's decision variables in decentralized decision-making approach

To optimize the retailer's total profit, the retail price (p) and the replenishment period (T) should be determined. Following equations show the mathematical formulations to find the optimal retail price in the decentralized decision-making structure. Accordingly, the first and second order derivatives of retailer's objective function Eq.7 (TP_R^{dec}) with respect to the retail price (p) are formulated in Eqs (9) and (10).

$$\frac{\partial TP_R^{dec}}{\partial p} = \lambda_0(1+\beta\theta)\alpha[(\alpha^{-1}+c-2p)\left(1-\frac{\left(\frac{T}{E}\right)^n}{n+1}\right) + \frac{hT}{2}\left(1-\frac{2(T/E)^n}{n+2}\right)] \quad (9)$$

$$\frac{\partial^2 TP_R^{dec}}{\partial p^2} = -2\lambda_0(1+\beta\theta)\alpha\left(1-\frac{\left(\frac{T}{E}\right)^n}{n+1}\right) \quad (10)$$

By considering the negativity of second derivative of retailer's objective function to, the optimal retail price (p) is demonstrated as:

$$p = \frac{(\alpha^{-1}+c)}{2} + \frac{hT}{2}\frac{n+1}{n+2}\left(1-\frac{n}{2(n+1-(T/E)^n)}\right) \quad (11)$$

To obtain the optimal value of replenishment time, the replenishment period (T) is replaced by ($T = xE$) to simplify the equations; where x is the shelf-life duration coefficient. Then, the optimal retail price is shown as:

$$p_x = \frac{(\alpha^{-1}+c)}{2} + \frac{hx\,E}{2}\frac{n+1}{n+2}\left(1-\frac{n}{2(n+1-x^n)}\right) \quad (12)$$

Lemma 1. In a profitable Inventory system of a perishable item, the feasible replenishment period T must be equal or less than the shelf-life duration E; consequently, the feasible value for x is in the range of [0,1] (i.e., $0 < x < 1$).

Lemma 2. In a profitable inventory system, the result of subtracting expenses from revenues for every item is positive, the expenses of the proposed inventory system for the retailer include the holding cost and the wholesale price of each item, and the retailer's income is obtained by selling the item (p_x). Accordingly, for every item in a profitable inventory system we have $x < (p_x - c)/(hE)$.(See appendix A.)

According to lemma 1 and lemma 2 the optimal value for x would be:

$$x < min\{1, (p_x - c)/(hE)\} \quad (13)$$

Proposition 1. The value of $x_{max} = min\{1, (p_x - c)/(hE)\}$, is obtained by following constraints:

x_{max}=1 if $\vartheta > \frac{3n+7}{2n+4}$;

$x_{max} = \varepsilon$ if $\vartheta < \frac{3n+7}{2n+4}$; that ε is attained by solving $\varepsilon\left[\frac{n+3}{n+2} + \frac{n(n+1)}{2(n+2)(n+1-\varepsilon^n)}\right] - \vartheta = 0$ in the interval of (0,1].

Proof. By replacing the value of p_x (Eq. 12) into $x < \frac{(p_x-c)}{hE}$ and using mathematical calculations, the expression $A(x) \leq \vartheta$ is obtained. Let define $\vartheta = (\alpha^{-1} - c)/(hE)$ as the time that makes the potential profit of each inventory unit equal to zero, then:

$$A(x) = x\left[\frac{n+3}{n+2} + \frac{n(n+1)}{2(n+2)(n+1-x^n)}\right]$$

$A(x)$ is an increasing function of x in the interval of (0,1].

$$0 = A(0) < A(x) < A(1) = \frac{3n+7}{2n+4}$$

if $\vartheta < \frac{3n+7}{2n+4}$ then x ≤ 1 is the redundant. In this case, $A(x) \leq \vartheta$ can be replaced by x ≤ ε, where ε satisfies $A(\varepsilon) = \vartheta$. $A(x)$ is an increasing function of x in the interval of (0,1], hence ε is unique in this interval, and it will be obtained using one of the root-finding methods (e.g., bisection method, Newton Raphson method, etc.). If $\vartheta > \frac{3n+7}{2n+4}$, then $x < \frac{(p_x-c)}{hE}$ is redundant and so x_{max}= 1.

If $TP_R^{dec}(x_{max}) \leq 0$, the inventory system is not profitable for the retailer and he/she will avoid remaining in the supply chain. Again, considering the retailer's objective function by replacing $T = xE$:

$$TP_R^{dec} = \lambda_0(1+\beta\theta)(1-\alpha p_x)\left[(p_x - c)\left(1 - \frac{x^n}{n+1}\right) - \frac{hEx}{2}\left(1 - \frac{2x^n}{n+2}\right)\right] - \frac{k_b}{Ex} \tag{14}$$

The derivative of Eq. (15) with respect to x would be:

$$TP_R^{dec'} = \frac{\lambda_0}{x^2}\left\{\frac{k_b}{\lambda_0 E} - (1+\beta\theta)(1-\alpha p_x)\left[\frac{(p_x-c)nx^{n+1}}{n+1} + \frac{hx^2E}{2}(1 - \frac{2(n+1)x^n}{n+2})\right]\right\} \tag{15}$$

Let:

$$f(x) = \frac{k_b}{\lambda_0 E} - (1+\beta\theta)(1-\alpha p_x)\left[\frac{(p_x-c)nx^{n+1}}{n+1} + \frac{hx^2E}{2}(1 - \frac{2(n+1)x^n}{n+2})\right] \tag{16}$$

Then by solving the equation $f(x) = 0$ and searching for the optimal value of replenishment period (x^*) in the interval of $[0, x_{max}]$ using one of the root-finding methods, the concavity of the retailer's profit function has been proven as shown in Appendix B. So, we will have:

$$Q^* = \lambda_0(1+\beta\theta)(1-\alpha p^*)x^*E(1 - \frac{(x^*)^n}{n+1}) \tag{17}$$

$$p^* = \frac{(\alpha^{-1}+c)}{2} + \frac{hx^*E}{2}\frac{n+1}{n+2}\,(1 - \frac{n}{2(n+1-x^{*n})}) \tag{18}$$

3.1.2. The optimal value of manufacturer's decision variables in decentralized decision-making approach

Considering the optimal values obtained for the retail price and replenishment period in the previous section, in this part, we try to search for the optimal values of manufacturer decision variables (i.e., the green improvement degree and the wholesale price) that maximizes the manufacturer objective function.

$$TP_M^{dec}(\theta, c) = \frac{Q^*(c-c')}{xE} - \frac{k_s}{xE} - \frac{1}{2}\eta\theta^2 \tag{19}$$

By replacing the optimal value of order quantity (Q^*) from (17) into (19) the manufacturer objective function in decentralized decision-making approach (TP_M^{dec}) would change to (20).

$$TP_M^{dec} = \lambda_0(1+\beta\theta)(1-\alpha p)\left(1 - \frac{(x^*)^n}{n+1}\right)(c - c') - \frac{k_s}{xE} - \frac{1}{2}\eta\theta^2 \tag{20}$$

Then:

$$\frac{\partial TP_M^{dec}}{\partial\theta} = \lambda_0\beta\left(1-\frac{(x^*)^n}{n+1}\right)(1-\alpha p)(c-c')-\eta\theta \quad (21)$$

Hence, the optimal value for green improvement degree (θ) is shown by:

$$\theta^* = \lambda_0\frac{\beta}{\eta}(1-\alpha p)(1-\frac{(x^*)^n}{n+1})\ (c-c') \quad (22)$$

By replacing the optimal value of green improvement degree (θ^*) from (22) into (20) we obtain:

$$TP_M^{dec} = {\lambda_0}^2\frac{\beta^2}{2\eta}\left(1-\frac{(x^*)^n}{n+1}\right)^2(1-\alpha p)^2(c-c')^2+\lambda_0\left(1-\frac{(x^*)^n}{n+1}\right)(1-\alpha p)(c-c')-\frac{k_s}{xE} \quad (23)$$

By $\frac{\partial TP_M^{dec}}{\partial c}=0$ the optimal whole sale price (c) is given as:

$$c^* = \frac{1}{2\alpha}-\frac{hx^*E}{2}\frac{n+1}{n+2}\ (1-\frac{n}{2(n+1-{x^*}^n)})+\frac{c'}{2} \quad (24)$$

The concavity conditions of the manufacturer profit function are given in the Appendix C.

3.2. Centralized supply chain

In a centralized supply chain, the retailer and manufacturer are merged into a single entity that makes decisions together on all decision variables (the green technology investment, the retail pricing, and replenishment period) in order to optimize total supply chain profit. In a centralized decision-making system, the wholesale price (c) payment from the retailer to the producer is encountered as an internal transfer payment, therefore it will not be optimized. The objective function for the centralized decision-making structure is expressed as Eq. (25)

$$TP^{cen} = \lambda_0(1-\alpha p_x)(1+\beta\theta)\left[(p_x-c')\left(1-\frac{x^n}{n+1}\right)-\frac{hEx}{2}\left(1-\frac{2x^n}{n+2}\right)\right]-\frac{k_s}{xE}-\frac{k_b}{xE}-\frac{1}{2}\eta\theta^2 \quad (25)$$

The first term of Eq. (25) represents supply chain revenue from the sale of products, the holding costs for the supply chain is shown in the second term of Eq. (25), the third and fourth terms represent fixed ordering costs, and the last term shows the investments on green technology improvement.

The optimal values of the green improvement degree and retail price in a supply chain with a centralized structure will be determined using the same procedure utilized to extract the optimal green improvement degree and retail price in a decentralized decision-making approach, by optimizing Eq. (25). As a result, in a centralized decision-making process, the optimized retail price (p^{cen}) is offered as follows. (Eq. (26))

$$p^{cen} = \frac{(\alpha^{-1}+c')}{2}+\frac{hxE}{2}\frac{n+1}{n+2}\ (1-\frac{n}{2(n+1-(x)^n)}) \quad (26)$$

The optimal value for green improvement degree (θ^{cen}) in the centralized structure supply chain would be as follows:

$$\theta^{cen} = \lambda_0\frac{\beta}{\eta}(1-\alpha p_x)\left[\frac{(p-c')}{2}\left(1-\frac{x^n}{n+1}\right)-\frac{hxE}{2}\left(1-\frac{2(x)^n}{n+2}\right)\right] \quad (27)$$

The replenishment period in the centralized decision-making approach is optimized, similar to the presented procedure in Eqs. (12) to (16).

$$x < min\{1,(p_x-c')/(hE)\} \quad (28)$$

$$\vartheta' = (\alpha^{-1}-c')/(hE) \quad (29)$$

$$x_{max}\text{=1 if } \vartheta' > \frac{3n+7}{2n+4} \quad (30)$$

$$x_{max} = \varepsilon \text{ if } \vartheta' < \frac{3n+7}{2n+4} \quad (31)$$

$\varepsilon\left[\frac{n+3}{n+2}+\frac{n(n+1)}{2(n+2)(n+1-\varepsilon^n)}\right]-\vartheta'=0$. Again, the equation is solved by and searching for the ε in the interval of [0,1] using root-finding methods.

By substituting θ^{cen} from Eq. (26) in Eq. (25), the supply chain profit function will change into a two-variable function. (i.e., Eq. (32)). The concavity of Eq. (32) has been proven in Appendix D.

$$TP^{cen} = \lambda_0(1-\alpha p_x)\left[(p_x - c')\left(1-\frac{x^n}{n+1}\right) - \frac{hEx}{2}\left(1-\frac{2x^n}{n+2}\right)\right] + \left(\frac{1}{2\eta}\right)\beta^2\left(\lambda_0(1-\alpha p_x)\left[(p_x - c')\left(1-\frac{x^n}{n+1}\right) - \frac{hEx}{2}\left(1-\frac{2x^n}{n+2}\right)\right]\right)^2 - \frac{k_s}{xE} - \frac{k_b}{xE} \quad (32)$$

We can get the following result by calculating the first derivative of Eq. (32) with regard to x:

$$\frac{\partial TP^{cen}}{\partial x} = \frac{\lambda_0}{x^2}\left[\left\{\frac{k_b+k_s}{\lambda_0 E} - (1-\alpha p_x)\left(\frac{(p_x-c')nx^{n+1}}{n+1} + \frac{hx^2E}{2}\left(1-\frac{2(n+1)x^n}{n+2}\right)\right)\right\} + 2\left(\frac{1}{2\eta}\right)\beta^2\left\{\lambda_0(1-\alpha p_x)\left[(p_x-c')\left(1-\frac{x^n}{n+1}\right) - \frac{hEx}{2}\left(1-\frac{2x^n}{n+2}\right)\right]\right\}\left\{-(1-\alpha p_x)\left(\frac{(p_x-c')nx^{n+1}}{n+1} + \frac{hx^2E}{2}\left(1-\frac{2(n+1)x^n}{n+2}\right)\right)\right\}\right] \quad (33)$$

By solving the equation $\frac{\partial TP^{cen}}{\partial x} = 0$ and searching for the optimal value of (x^*) in the interval of $[0, x_{max}]$ using one of the root-finding methods (e.g., bisection method, Newton Raphson method, etc.), the optimal values will be resulted as follows:

$$Q^* = \lambda_0(1+\beta\theta)(1-\alpha p^*)x^*E\left(1-\frac{(x^*)^n}{n+1}\right) \quad (34)$$

$$p^* = \frac{(\alpha^{-1}+c')}{2} + \frac{hx^*E}{2}\frac{n+1}{n+2}\left(1-\frac{n}{2(n+1-(x^*)^n)}\right) \quad (35)$$

$$\theta^{cen} = \lambda_0\frac{\beta}{\eta}(1-\alpha p_x)\left[\frac{(\alpha^{-1}-c')}{2}\left(1-\frac{x^n}{n+1}\right) - \frac{hx^*E}{4}\left(1-\frac{2x^n}{n+2}\right)\right] \quad (36)$$

3.3. Coordinated approach

All members of the supply chain should benefit from changing the decision-making structure from decentralized to centralized; if not, they will refuse to make decisions in the centralized structure, and it will cause to prevent the supply chain from achieving its maximum profit. But really, this was not happened. Hence, an incentive contract is proposed to motivate the supply chain members to make decisions, jointly. A coordinated decision-making structure will stimulate the supply chain members to improve supply chain performance. In the literature, many coordination contracts have been developed as an incentive mechanism. In this paper, we proposed the revenue sharing (RS) contract because of both simplicity and applicability of this contract. In the proposed revenue-sharing contract, the manufacturer reduces the wholesale price (c) and instead, the retailer shares a fraction $(1-\varphi)$ of his/her revenue with the manufacturer. The retailer also pays a fraction $(1-\varphi)$ of investment in green technology. Consequently, the retailer benefits from the decreased wholesale price proposed by the manufacturer, and the manufacturer benefits from sharing the fraction $(1-\varphi)$ of green technology investment and receiving a fraction $(1-\varphi)$ of the retailer's revenue. The manufacturers and retailer's objective functions in a coordinated decision-making structure are formulated as follows. (Eqs.37 and 38)

$$TP_R^{coo} = \lambda_0(1-\alpha p_x)(1+\beta\theta)\left[(\varphi p_x - c)\left(1-\frac{x^n}{n+1}\right) - \frac{hEx}{2}\left(1-\frac{2x^n}{n+2}\right)\right] - \frac{k_b}{xE} - \varphi\frac{1}{2}\eta\theta^2 \quad (37)$$

$$TP_M^{coo} = \lambda_0(1-\alpha p_x)(1+\beta\theta)\left(1-\frac{x^n}{n+1}\right)(c - c' + (1-\varphi)\,p_x) - \frac{k_s}{xE} - (1-\varphi)\frac{1}{2}\eta\theta^2 \quad (38)$$

Under the RS contract, the retailer and the manufacturer can decide freely on their decision variables. However, to improve the whole supply chain performance, optimal values of the retail price (p_x) and the replenishment period (x) in the coordinated decision-making structure should be equal to those in the centralized structure as follows.

Proposition 2. By considering $p^{cen} = p^{coo}$ and $x^{cen} = x^{coo}$ and implementing the rules of the proposed revenue-sharing contract, the supply chain members will take decisions in the coordinated structure that leads to achieving the profit. Under these conditions, the wholesale price and the green improvement degree are obtained as follows:

$$c^{coo} = \varphi c' - \frac{(1-\varphi)\,hx\,E}{2}\frac{n+1}{n+2}\left(1-\frac{n}{2(n+1-x^n)}\right) \quad (39)$$

$$\theta^{coo} = \lambda_0\frac{\beta}{\eta}(1-\alpha p)\frac{(\alpha^{-1}-c')}{2} \quad (40)$$

Proof. To obtain the optimal whole sale price in coordinated structure, we should calculate the optimal retail price in coordinated decision making by solving $\frac{\partial TP_R{}^{coo}}{\partial p} = 0$. The optimal retail price in coordinated structure is presented as Eq. (41). Afterward, considering the coordination conditions (i.e., $p^{cen} = p^{coo}$ and $x^{cen} = x^{coo}$) , the wholesale price in coordinated decision-making structure (c^{coo}) is achieved as shown in Eq. (40).

The optimal value of green improvement degree (θ^{coo}) is obtained by solving $\frac{\partial {TP_M}^{coo}}{\partial \theta} = 0$ and also mathematical manipulation (replacing the optimal value of wholesale price (c^{coo}) and considering the coordination condition $p^{cen} = p^{coo}$). the concavity of manufacturer's profit function (TP_M^{coo}) is presented in Appendix E.

$$p_{coo} = \frac{(\alpha^{-1} + c\varphi^{-1})}{2} + \frac{\varphi^{-1}hxE}{2}\frac{n+1}{n+2}\,(1 - \frac{n}{2(n+1-x^n)}) \tag{41}$$

If we assume that any member in the supply chain will decide rationally, they will only accept a coordination agreement if it leads to an increase in their profits compared with the decentralized decision-making approach.

So, in the proposed revenue-sharing contract, the contract parameter (φ) should be determined such that objective functions of the supply chain members improves compared with the decentralized scenario. At result, in the proposed coordination contract, the conditions $TP_R^{coo} > TP_R^{dec}$ and $TP_M^{coo} > TP_M^{dec}$ must be met. Accordingly, the acceptable range of sharing rate (φ) can be expressed as follows:

$$\varphi > \frac{TP_R^{dec} + \frac{k_b}{xE} + \lambda_0(1-\alpha p_x)(1+\beta\theta)\frac{hEx}{4}\left(1-\frac{2x^n}{n+2}\right)}{[\lambda_0(1-\alpha p_x)(1+\beta\theta)\frac{(\alpha^{-1}-c')}{2}\left(1-\frac{x^n}{n+1}\right)-\frac{1}{2}\eta\theta^2]} \tag{42}$$

$$\varphi < 1 - \frac{TP_M^{dec} + \frac{k_s}{xE}}{\left[\lambda_0(1-\alpha p_x)(1+\beta\theta)\frac{(\alpha^{-1}-c')}{2}\left(1-\frac{x^n}{n+1}\right)-\frac{1}{2}\eta\theta^2\right]} \tag{43}$$

To reach the supply chain coordination by implementing the proposed RS contract, the determined range for sharing rate must satisfy Eqs (42) and (43) simultaneously.

4. Numerical examples

4.1. Case study

Sustainable management of food resources and the supply of high-quality foods are among the most important issues in management of human societies, and hence in this article, the issues and challenges in fruit, vegetable, dairy and meat industries have been considered. The proper function of this supply chain is influenced by several factors such as accurate demand estimation and the existence of coordination between every level of the supply chain. The demand for food products generally depends on the quality, remaining lifetime, and price of the product. Some types of foods, especially fruits and vegetables, have a very short shelf life, and if there is no proper coordination between the producer and the retailer, we will face problems such as supply disruption and lots of losses for the supply chain parties. So, these products should be transferred to the market at the right time, otherwise they would be rotten, the product quality decreases and finally the customers desire to buy them will also decline. On the other hand, what interests' consumers, governments and health organizations into the perishable food supply chain, especially fruits and vegetables, is their organic nature. The more environmentally friendly products, and the less harmful substances used in the production process, the more consumers' attention will be drawn to them.

4.2. Validation approach

This section presents an example of a two-echelon supply-chain for a perishable food in which the data are taken from the numerical examples provided by Avinadav et al. (2013). We use three decision-making approaches (decentralized, centralized, and revenue sharing coordination) to solve the proposed example under two scenarios. In the first scenario, we suppose that the supply-chain members will invest in green technology, while in the second scenario, the green technology investment is equal to zero. The parameters for both scenarios are the same, and are given below:

$\lambda_0 = 200\,; n = 2;\ \alpha = 0.04; h = 0.1;\ k_s = 20;\ k_b = 200; E = 10; c' = 1; \beta = 0.5;\ \eta = 200$

First scenario - with considering green technology investment effects on demand function

In the decentralized decision-making approach, supply-chain members decide separately on their decision variables to maximize their own profit. The manufacturer decides on the wholesale price (c^*) and green improvement degree (θ^*), and the retailer decides on the retail price(p^*) and the replenishment period (x^*), giving that the replenishment period should be in the range of $[0, x_{max}]$.

Table 1. Optimal decentralized solution (First scenario)

Decision making approach	θ^*	c^*	x_{max}	x^*	p^*	TP_M^{dec}	TP_R^{dec}
Decentralized	0.1328	12.8959	0.8636	0.4304	19.0520	553.3725	236.7114

In the centralized decision-making approach, the manufacturer and the retailer are considered as an integrated decision-making unit, and we are about to maximize the supply-chain profit. As can be seen in Table 2, the total profit for the supply-chain in the centralized decision-making approach is more than the sum of total profit for the manufacturer and the retailer in the decentralized decision-making approach ($TP^{cen} > TP_M^{dec} + TP_R^{dec}$).

Table 2. Optimal centralized solution (First scenario)

Decision making approach	x_{max}	x^{*cen}	p^{*cen}	θ^{*cen}	TP^{cen}
Centralized	1	0.2565	13.0634	0.287	1122.4

The revenue-sharing contract is about the sharing of supply-chain profit between supply-chain members. To improve supply-chain performance in the revenue-sharing coordination decision making, the replenishment period and retail price must be equal to those in the centralized decision making i.e., $x^{*coo} = x^{*cen}$ and $p^{*coo} = p^{*cen}$, giving $\theta^{coo} = \lambda_0 \frac{\beta}{\eta}\left(1 - \alpha p^{*coo}\right)\left(1 - \frac{(x^{*coo})^n}{n+1}\right)\frac{(\alpha^{-1} - c')}{2}$ and $c^{coo} = \varphi c' - \frac{(1-\varphi)\, h x^{*coo} E}{2}\frac{n+1}{n+2}\left(1 - \frac{n}{2(n+1-(x^{*coo})^n)}\right)$. The obtained decision variables for the revenue sharing coordination are demonstrated in table 3.

Table 3. Optimal RS solution (First scenario)

Decision making approach	x^{*coo}	p^{*coo}	θ^{*coo}	φ_{min}	φ_{max}
Revenue-Sharing coordination	0.2565	13.0634	0.3035	0.2682	0.5449

By changing the sharing rate (φ) between the acceptable ranges ($\varphi_{min}, \varphi_{max}$), the share of each supply-chain member changes too. The total profit for the retailer (TP_R^{coo}) and wholesale price (c) increases linearly in the sharing rate (φ).

Table 4. Sensitivity analysis on revenue sharing parameter (First scenario)

φ	TP_R^{coo}	TP_M^{coo}	c	$TP_R^{coo} + TP_M^{coo}$
0.27	238.8770	882.8751	0.2237	1121.7521
0.30	274.8391	846.9129	0.2556	1121.7521
0.35	334.7760	786.9760	0.3088	1121.7521
0.40	394.7130	727.0391	0.3620	1121.7521
0.45	454.6499	667.1022	0.4151	1121.7521
0.50	514.5868	607.1652	0.4683	1121.7521
0.54	562.5363	559.2157	0.5108	1121.7521

Second scenario. without considering green technology investment effects on demand function

In this section, we present an example of two echelon supply-chain for a perishable item, in which Contrary to the previous scenario, investing in green technology has no effect on demand.

Table 5. Optimal decentralized solution (Second scenario)

Decision making approach	c^*	x^*	p^*	TP_M^{dec}	TP_R^{dec}
Decentralized	12.9	0.3639	19.0444	547.9456	216.2658

Table 6. Optimal centralized solution (Second scenario)

Decision making approach	x^*	p^*	TP^{cen}
centralized	0.2689	13.0664	1045

Table 7. Optimal RS solution (Second scenario)

Decision making approach	x^*	p^*	φ_{min}	φ_{max}
Revenue sharing coordination	0.2689	13.0664	0.2655	0.5165

Table 8. Sensitivity analysis on revenue sharing parameter (Second scenario)

φ	TP_R^{coo}	TP_M^{coo}	c	$TP_R^{coo} + TP_M^{coo}$
0.27	225.8164	819.0721	0.1731	1044.8885
0.30	259.1713	785.7172	0.207	1044.8885
0.35	314.7627	730.1258	0.2637	1044.8885
0.40	370.3541	674.5344	0.3203	1044.8885
0.45	425.9455	618.9430	0.3770	1044.8885
0.50	481.5369	563.3517	0.4336	1044.8885

Table 9 summarizes the difference between the two scenarios of demand affected by the product's greenness degree and the demand that the greenness degree does not affect it. Since the first case demand is more than the second one, the supply chain's profit in the first scenario is higher either.

Table 9. Comparing first and second scenarios

scenarios	λ	x^*	p^*	φ_{min}	φ_{max}	θ^*_{coo}	$TP_R^{coo}(\varphi_{max})$	$TP_M^{coo}(\varphi_{max})$	θ^*_{cen}	TP^{cen}
Considering green technology investment	106.41	0.2565	13.0634	0.2682	0.5449	0.3035	568.4038	553.3725	0.2787	1122.367
Without considering green technology investment	93.17	0.2689	13.0664	0.2655	0.5165	-	481.5369	563.3517	-	1044.9

4.3. Sensitivity analysis

Sensitivity analysis of the numerical example given in the previous section will be performed in two parts:

In the first part, the potential market demand is analyzed so that the effect of changing λ_0 (the potential market demand) on decision variables (replenishment time, retail price, and greenness degree), and supply chain profit values in centralized and coordinated by the revenue sharing contract, has been reviewed. As shown in Table 10, with the change of λ_0 between the values of 50 to 350, the replenishment time has decreased by 53.4% , and also the supply chain profit values in both cases have increased by 831.8%. Therefore, the mentioned parameter (λ_0) is very effective in determining the profit of the chain. Increasing λ_0, have increased the greenness degree of the product by 640.7% and 643.4% in the coordinated and centralized decision-making approaches, respectively. The results indicate that as the potential market demand varies from 50 to 350, the retail price merely changes from 13.1051 to 13.0504.

Table 10. Sensitivity analysis on potential market demand

λ_0	x^*	p^*	φ_{min}	φ_{max}	θ^*_{coo}	$TP_R^{coo}(\varphi_{max})$	$TP_M^{coo}(\varphi_{max})$	θ^*_{cen}	TP^{cen}
50	0.4351	13.1051	0.2922	0.5797	0.0724	109.2280	118.8874	0.0663	228.1430
100	0.3363	13.0824	0.2774	0.5545	0.1491	252.1357	259.5644	0.1367	511.8528
150	0.2877	13.0709	0.2715	0.5472	0.2262	405.7011	404.7330	0.2076	810.7810
200	0.2565	13.0634	0.2682	0.5449	0.3035	568.4038	553.3725	0.2787	1122.367
250	0.2341	13.0580	0.2660	0.5446	0.3810	739.5520	705.1717	0.3500	1445.688
300	0.2168	13.0538	0.2644	0.5434	0.4586	916.9493	861.9586	0.4214	1780.295
350	0.2028	13.0504	0.2633	0.5467	0.5363	1106.344	1017.7006	0.4929	2125.9329

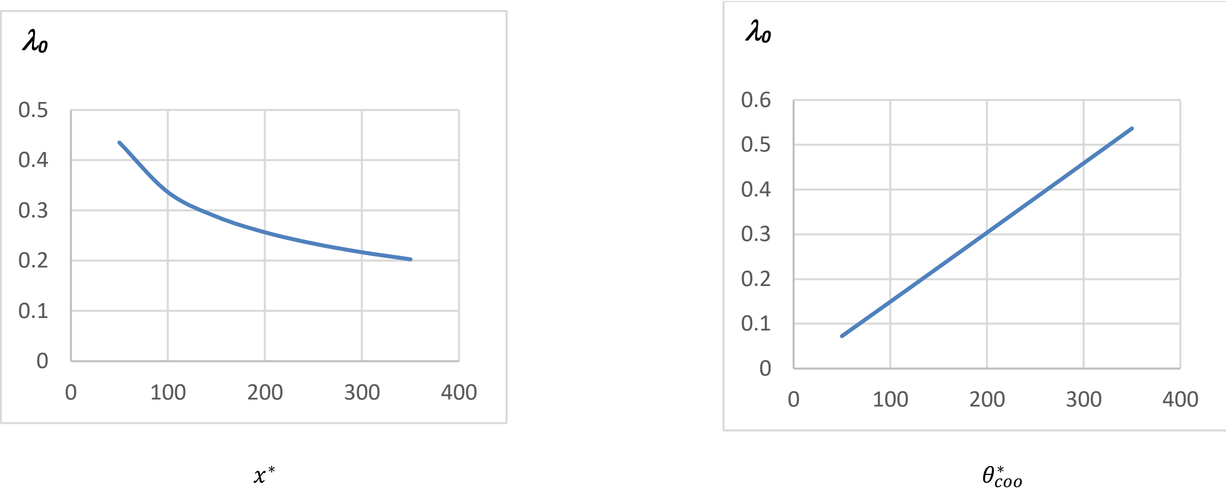

Figure.1. Sensitivity analysis on λ_0 and its effect on x^* and θ^*_{coo}

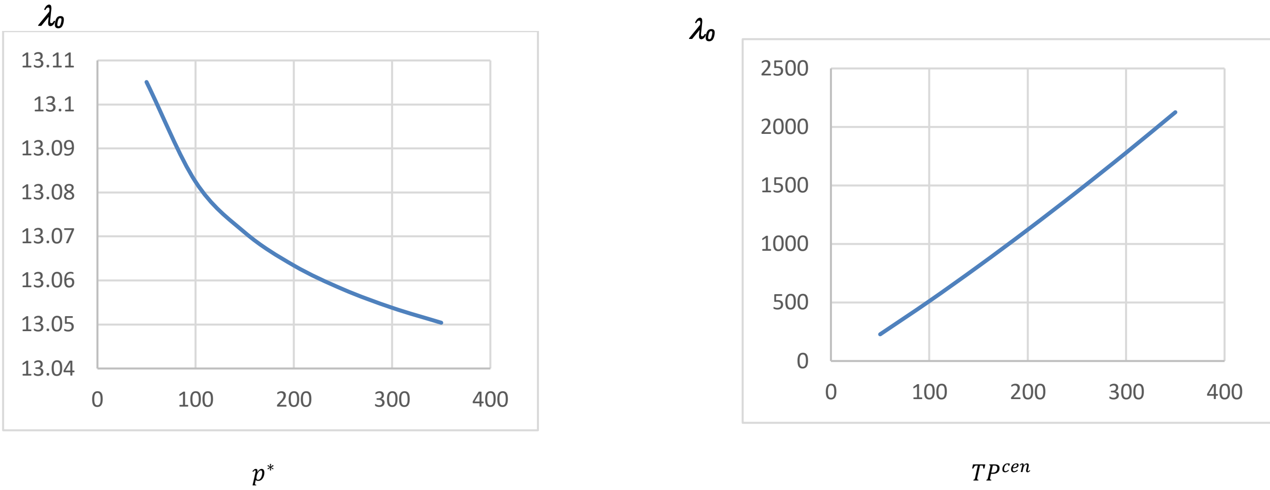

Figure. 2. Sensitivity analysis on λ_0 and its effect on $p^*$and TP^{cen}

In the second part, the reservation cost (α^{-1}) is analyzed so that the effect of changing α^{-1} on decision variables (replenishment time, retail price, and greenness degree), and supply chain profit values in centralized and coordination by the revenue sharing contract, has been reviewed. It can be seen in Table 11, that with the change of α^{-1} between the values of 5 to 45, the replenishment time has decreased by 55.6% , and also the supply chain profit values in both cases have increased by 2000%. Therefore, the mentioned parameter (α^{-1})is very effective in determining the supply chain's profit. Increasing α^{-1}, has increased the greenness degree of the product by 842% and 1323.5% in coordinated and centralized decision-making approach respectively. Also, changing α^{-1} between 5 and 40 has increased the retail price by 560%.

Table 11. Sensitivity analysis on the reservation cost

α^{-1}	x^*	p^*	θ^*_{coo}	φ_{min}	φ_{max}	$TP_R^{coo}(\varphi_{max})$	$TP_M^{coo}(\varphi_{max})$	θ^*_{cen}	TP^{cen}
5	0.4832	3.1157	0.0521	0.3695	0.6744	44.7182	49.6963	0.0327	94.7902
10	0.3600	5.588	0.1161	0.2974	0.5724	158.9650	171.6791	0.0931	331.1742
15	0.3096	8.0761	0.1787	0.2803	0.5541	287.8367	296.1564	0.1547	584.569
20	0.27858	10.5687	0.2412	0.2726	0.5474	424.3062	423.8844	0.2166	848.7911
25	0.2565	13.0634	0.3035	0.2682	0.5449	568.2840	553.4670	0.2787	1122.367
30	0.2396	15.5593	0.3659	0.2653	0.5442	718.3458	685.7405	0.3409	1404.7
35	0.2259	18.0560	0.4283	0.2633	0.5446	874.7729	820.1181	0.4032	1695.5
40	0.2146	20.5532	0.4907	0.2617	0.5455	1035.1107	958.8944	0.4655	1994.6

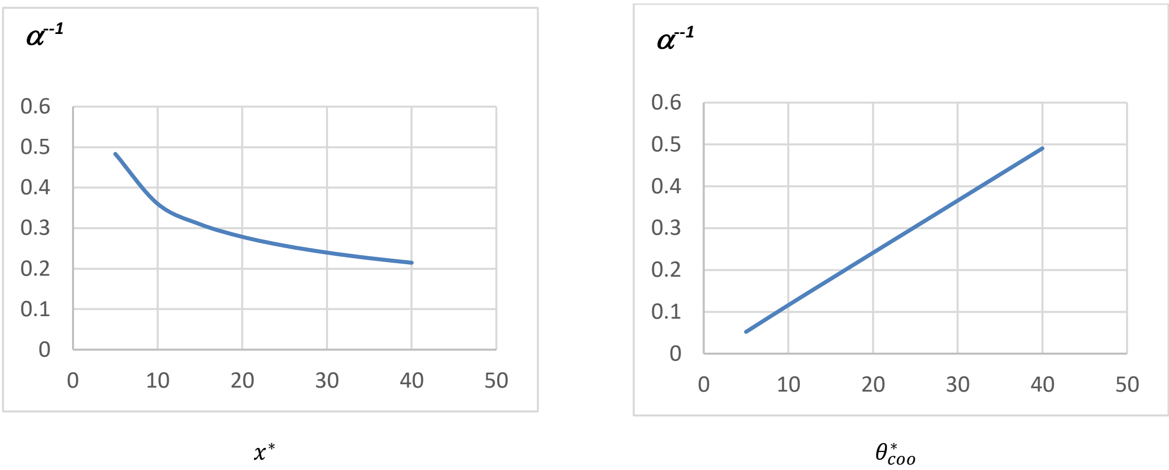

Figure. 3. Sensitivity analysis on $\alpha^{-1}$and its effect on x^* and θ^*_{coo}

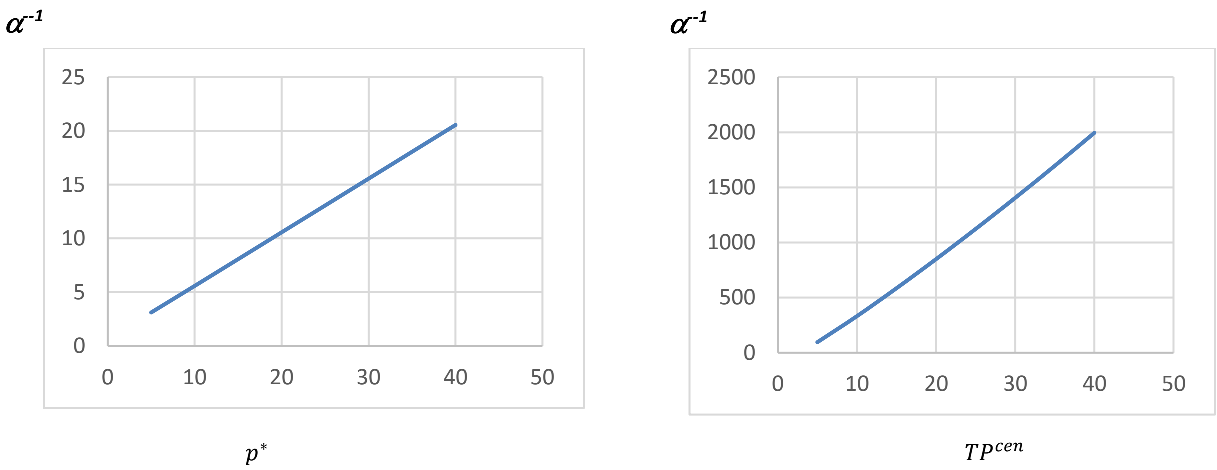

Figure. 4. Sensitivity analysis on $\alpha^{-1}$and its effect on $p^*$and TP^{cen}

4.4. Managerial insights

In this section, some critical managerial insights are provided to improve the supply chain performance for perishable items:

- Deterioration, and special storage conditions of perishable food, will lead to high energy consumption in perishable food supply chains, so employing green technology can be an appropriate option to help the food industries reduce energy costs.
- Considering the government's policies and the existing dependency between consumers demand and the greenness degree of perishable foods, green technology investment in the perishable food supply chain is vital and leads to an efficient food supply chain.

According to the obtained results in the sensitivity analysis, some managerial recommendations are provided in the following:

- Comparing two scenarios (with /without considering green technology investment effects on demand function), it was shown that the value of the demand affected by the greenness degree is higher than the demand that the greenness degree does not affect. The supply chain profit in the first case is higher either, as shown in the numerical example, investment in green technology is beneficial for manufacturers of perishable food because it will lead to a higher profit for both manufacturer and retailer. Generally, managers should analyze that if the net profit of increasing demand due to implementing green technology is greater than the cost of implementing green technology, it is profitable for the supply chain to invest in green technology; otherwise, the investment would be not profitable.

- It was observed that the manufacturer and retailer's profits are positively affected by the potential market demand. So, for perishable items with high potential market demand, it is more beneficial to invest in green technology.
- For perishable foods with low reservation cost (α^{-1}), there is no necessity for high investments in green technology, but as the reservation cost (α^{-1}) of the food increases, the importance of investments in green technology for both manufacturer and retailer will increase as well, because it will favorably affect their profits. It is notable that in the proposed model, the retail price is strongly dependent on the reservation cost (α^{-1}).

5. Conclusion

Due to the growth of environmental challenges, government policies, energy costs, and consumers' attention to environmental issues and their preference to buy green products, investment in green technology has been one of the vital decisions for every industry. The specific characteristics of a perishable food supply chain make it hard to manage, Accordingly, perishable food manufacturers tend to invest in green technologies to preserve foods, reduce energy usage and control wastes.

In this study, we developed a mathematical inventory model for a two-stage supply chain consisting of a single manufacturer and a single retailer, the manufacturer produces a perishable item with a deterministic expiration date. We also formulated a novel multiplicative demand function that is sensitive to three factors: the retail price, the greenness degree of the produced item, and the time after replenishment, and observed that by increasing the greenness degree for the produced items, the demand rate will also increase, but the retail price and market demand are inversely related. There is the same relationship between the time after replenishment and demand function. the proposed model is analyzed under two scenarios which demonstrate the differences between two demand function: 1- demand function is affected by green technology investments. 2- demand function is not affected by green technology investments. Both scenarios are investigated by: 1-Decentralized, 2-Centralized, and 3-Revenue-Sharing coordination decision making approaches. The numerical example in section 4 represents the improvement of demand in first scenario (i.e.,106.41) compared to the second scenario (i.e., 93.17), which results in higher profit of supply chain in first scenario (i.e., 1122.367) compared to the second scenario (i.e., 1045). The revenue sharing contract could successfully coordinate the supply chain by achieving the maximum potential profit (i.e., the resulted profit in the centralized decision-making structure.). The comparative study between two demand functions (demand affected by the greenness degree and demand not affected by the greenness degree) showed that, the total supply chain's profit in every three decision making approaches for the demand affected by the greenness degree is higher than the other one. The results of sensitivity analysis demonstrates that by increasing the potential market demand (λ_0) in a wide range of (50-350), the supply chain profit is increasing in the range of (228- 2126), the green improvement degree increases in the range of (0.07- 0.49), and the replenishment time decreases in the range of (0.44- 0.20). The conducted sensitivity analysis presents that increasing in reservation cost (α^{-1}) in the range of (5- 40) would change the total profit of centralized approach in the range of (95-1995), the green improvement degree in the range of (0.03- 0.47), and would have an inverse effect on the replenishment time that changes in the range of (0.48-0.21).

As recommendations for future studies, the consumer surplus can be added into the model. Meanwhile, the coordination can be performed with integration of other types of contracts such as quantity discount contract. The advertising and goodwill effects on demand function can also be studied in the future researches.

Appendix A.

In a profitable inventory system, the unit revenue of an item exceeds the costs associated with that item.

$$p_x > c + hT \quad \text{(A.1)}$$

By replacing the value of, $T = xE$, the Eq. (A.1), will be changed into:

$$x < (p_x - c)/(hE) \quad \text{(A.2)}$$

Appendix B. concavity of the retailer's profit function in decentralized decision-making approach

The first derivative of $f(x)$ with respect to x is:

$f'(x) = -(1+\beta\theta)\{(1-\alpha p_x)[(hE - 2p_x')x(1-x^n) + (p_x - c - hxE)nx^n)] - \alpha x p_x'(1-x^n)(\alpha^{-1} - c - hxE)\}$

Theorem: $f'(x) < 0$ over $(0, x_{max})$ since the $(2p_x' - hE)$is negative and $(p_x - c - hxE)$ is positive on this interval.

Proof:

Let $p_x' = \frac{hE}{2}\frac{n+1}{n+2}\left(1 - n\frac{n+1+(n-1)x^n}{2(n+1-x^n)^2}\right)$

But $M = \frac{n+1+(n-1)x^n}{(n+1-x^n)^2} = \frac{1}{n+1-x^n} + \frac{nx^n}{(n+1-x^n)^2}$
Then we will have:

$$0 = {p_x}'(x = 1) \leq {p_x}' < {p_x}'(x = 0) = 0.25hE$$

Thus, $-hE < 2{p_x}' - hE < -0.5hE$ over (0,1].
On the other hand, $p_x - c > hxE$ so $p_x - c - hxE > 0$
Appendix C. concavity of the manufacturer's profit function in decentralized decision-making approach

$$TP_M^{dec} = {\lambda_0}^2 \frac{\beta^2}{2\eta}\left(1 - \frac{(x^*)^n}{n+1}\right)^2 (1-\alpha p)^2 (c-c')^2 + \lambda_0\left(1 - \frac{(x^*)^n}{n+1}\right)(1-\alpha p)(c-c') - \frac{k_s}{xE} \tag{C.1}$$

- Let:

$$F = (1-\alpha p)(c-c') \tag{C.2}$$

The first and second derivatives of to is obtained as Eqs. (c.3) and (c.4).

$$\frac{\partial F}{\partial c} = (1-\alpha p) - \frac{\alpha}{2}(c-c') \tag{C.3}$$

$$\frac{\partial^2 F}{\partial c^2} = -\alpha \tag{C.4}$$

Hence the manufacturer's profit function would change into:

$$TP_M^{dec} = {\lambda_0}^2 \frac{\beta^2}{2\eta}\left(1 - \frac{(x^*)^n}{n+1}\right)^2 F^2 + \lambda_0(1 - \frac{(x^*)^n}{n+1})\, F + \frac{k_s}{xE} \tag{C.5}$$

The first and second order derivatives of manufacturer's profit function with respect to c is written as follows:

$$\frac{\partial TP_M^{dec}}{\partial c} = \frac{\partial F}{\partial c}({\lambda_0}^2 \frac{\beta^2}{\eta}\left(1 - \frac{(x^*)^n}{n+1}\right)^2 F + \lambda_0(1 - \frac{(x^*)^n}{n+1})) \tag{C.6}$$

$$\frac{\partial^2 TP_M^{dec}}{\partial c^2} = \frac{\partial^2 F}{\partial c^2}\left({\lambda_0}^2 \frac{\beta^2}{\eta}\left(1 - \frac{(x^*)^n}{n+1}\right)^2 F + \lambda_0\left(1 - \frac{(x^*)^n}{n+1}\right)\right) + {\lambda_0}^2 \frac{\beta^2}{\eta}\left(1 - \frac{(x^*)^n}{n+1}\right)^2 (\frac{\partial F}{\partial c})^2 \tag{C.7}$$

By considering $M = {\lambda_0}^2 \frac{\beta^2}{\eta}\left(1 - \frac{(x^*)^n}{n+1}\right)^2$ and $A = \frac{hx^*E}{2}\frac{n+1}{n+2}\,(1 - \frac{n}{2(n+1-{x^*}^n)})$ the condition of concavity is obtained as Eq. (c.9)

$$\frac{\partial^2 TP_M^{dec}}{\partial c^2} < 0 \tag{C.8}$$

$$-(\frac{2A-\alpha^{-1}-c'}{2}) - \sqrt{\frac{2\lambda_0}{M\alpha}\left(1 - \frac{(x^*)^n}{n+1}\right) - (\frac{\alpha^{-1}-c'}{2} - A)^2} < c < \sqrt{\frac{2\lambda_0}{M\alpha}\left(1 - \frac{(x^*)^n}{n+1}\right) - (\frac{\alpha^{-1}-c'}{2} - A)^2} - (\frac{2A-\alpha^{-1}-c'}{2}) \tag{C.9}$$

Appendix D. Proving the concavity of the profit function in the centralized decision-making

Let:

$$\pi(x) = \lambda_0(1-\alpha p)\left[(p-c')\left(1 - \frac{x^n}{n+1}\right) - \frac{hEx}{2}\left(1 - \frac{2x^n}{n+2}\right)\right] \tag{D.1}$$

Therefore, the TP^{cen} can be rewritten as follows

$$TP^{cen} = \pi + (\frac{1}{2\eta})\beta^2\, \pi^2 - \frac{k_s}{xE} - \frac{k_b}{xE} \tag{D.2}$$

By considering the following properties, the concavity of TP^{cen} could be proven.
Property 1. when f is a quasiconcave (respectively, strictly quasiconcave, semistrictively quasiconcave) function on S and g is an increasing function, the composite function gOf is quasiconcave (respectively, strictly quasiconcave, semistrictively quasiconcave) on S. Even if g is a non-decreasing function, this result still holds for a quasiconcave function.
Hence it follows from property 1 that $\pi(x)^2$ is pseudo-concave respectively.
Property 2. A non-negative linear combination of concave functions on S is a concave function on S.

In Appendix B, the concavity of $\pi - \frac{k_s}{xE} - \frac{k_b}{xE}$ is proved, the concavity of the $\pi(x)^2$ is also proved by the property 1. Accordingly, the non-negative linear combination of these two functions (TP^{cen}) could be concluded.

Appendix E. Proving the concavity of manufacturer's profit function in the coordinated decision-making

By replacing the optimal value of green improvement degree (θ) manufacturer's profit function in the coordinated decision-making, it would change to Eq. (E.1).

$$TP_M^{coo} = \lambda_0\left(1 - \alpha p^{coo}\right)(1-\varphi)\left(1 - \frac{x^n}{n+1}\right)\frac{(\alpha^{-1} - c')}{2} - (1-\varphi)\frac{\beta^2 {\lambda_0}^2}{2\eta}\left[\left(1 - \alpha p^{coo}\right)\frac{(\alpha^{-1} - c')}{2}\right]^2 - \frac{k_s}{xE} \tag{E.1}$$

The first and second derivatives of TP_M^{coo} with respect to c is demonstrated as follows:

$$\frac{\partial TP_M^{coo}}{\partial c} = \lambda_0 \frac{(\varphi^{-1} - 1)}{2}\left(1 - \frac{x^n}{n+1}\right)\frac{(\alpha^{-1} - c')}{2}(-\alpha) + \frac{(\varphi^{-1} - 1)}{2}\frac{\alpha\beta^2 {\lambda_0}^2 (\alpha^{-1} - c')^2}{4\eta}\left(1 - \alpha p^{coo}\right) \tag{E.2}$$

$$\frac{\partial^2 TP_M^{coo}}{\partial c^2} = -\frac{\alpha\varphi^{-1}(\varphi^{-1} - 1)}{4}\frac{\alpha\beta^2 {\lambda_0}^2 (\alpha^{-1} - c')^2}{4\eta} < 0 \tag{E.3}$$

the concavity of manufacturer's profit function in the coordinated decision-making is shown in Eq. (E.3).

References

Amiri, S. A. H. S., Zahedi, A., Kazemi, M., Soroor, J., & Hajiaghaei-Keshteli, M. (2020). Determination of the optimal sales level of perishable goods in a two-echelon supply chain network. Computers & Industrial Engineering, 139, Article 106156.

Avinadav, T., Herbon, A., Spiegel, U., Mahanty, K., Tiwari, M.K. ,2013. Optimal inventory policy for a perishable item with demand function sensitive to price and time, Int. J. Production Economics. 144, 497-506.

Azadi, Z., Eksioglu, S. D., Eksioglu, B., & Palak, G. (2019). Stochastic optimization models for joint pricing and inventory replenishment of perishable products. Computers & Industrial Engineering, 127, 625–642.

Bai, Q., Xu, X., Xu, J., Wang D., 2016. coordinating a supply chain for deteriorating items with multi-factor-dependent demand over a finite planning horizon, Applied Mathematical Modelling. 40, 9342-9361.

Barari, S., Agarwal, G., Zhang, W.J., Mahanty, K., Tiwari, M.K. ,2012. A decision framework for the analysis of green supply chain contracts: An evolutionary game approach, Expert Systems with Applications. 39, 2965-2976.

Cachon, G.P.,2003. Supply Chain Coordination with Contracts, vol.11, Elsevier, pp. 229–340(Chapter6).

Gao, J., Xiao, Z., Wei, H., Zhou, G., 2020. Dual-channel green supply chain management with eco-label policy: A perspective of two types of green products, Computers & Industrial Engineering. 146, 106613.

Ghosh, D., Shah, J., 2012. A comparative analysis of greening policies across supply chain structures, International Journal of Production Economics. 135, 568-583.

Govindan, K., Popiuc, M.N., Diabat, A., 2013. Overview of coordination contracts within forward and reverse supply chains, Journal of Cleaner Production. 47, 319-334.

He, Y., Zhao X., Zhao, L., He, J., 2009. Coordinating a supply chain with effort and price dependent stochastic demand, Applied Mathematical Modelling. 33 ,2777-2790.

Hu, J., Zhang, J., Mei, M., Yang, W., Shen, Q., 2019. Quality control of a four-echelon agri-food supply chain with multiple strategies. Information processing in agriculture.6, 425– 437.

Huang, YS., Fang, CC., Lin, YA., 2020. Inventory management in supply chains with consideration of Logistics, green investment and different carbon emissions policies, Computers & Industrial Engineering. 139, 106207.

Janssen, L., Sauer, J., Claus, T., & Nehls, U. (2018). Development and simulation analysis of a new perishable inventory model with a closing days constraint under non-stationary stochastic demand. Computers & Industrial Engineering, 118, 9–22.

Jouzdani, J., Govindan, K., 2021. On the sustainable perishable food supply chain network design: A dairy products case to achieve sustainable development goals. Journal of Cleaner Production. 278, 123060.

Karuppasamy, S.K. and Uthayakumar, R., 2017. A Deterministic Pharmaceutical Inventory Model for Variable Deteriorating Items with Time-Dependent Demand and Time-Dependent Holding Cost in Healthcare Industries, Innovations in Computational Intelligence. 713, 199-210.

KavithaPriya, R., Senbagam, K., 2018. An EOQ Inventory Model for Two Parameter Weibull Deterioration with Quadratic Time Dependent Demand and Shortages, Journal of Economic Dynamics and Control. 73, 439-452.

Kaya, O., & Ghahroodi, S. R. (2018). Inventory control and pricing for perishable products under age and price dependent stochastic demand. Mathematical Methods of Operations Research, 88(1), 1–35.

Khorshidvand, B., Soleimani, H., Seyyed Esfahani, M.M., Sibdari, S., 2021a. Sustainable closed-loop supply chain network: Mathematical modeling and Lagrangian relaxation, Journal of Industrial Engineering and Management Studies. 80, 240-260.

Khorshidvand, B., Soleimani, H., Sibdari, S., Seyyed Esfahani, M.M., 2021b. Developing a two-stage model for a sustainable closed-loop supply chain with pricing and advertising decisions, Journal of Cleaner Production. 309, 1.

Khorshidvand, B., Soleimani, H., Sibdari, S., Seyyed Esfahani, M.M., 2021c. Revenue management in a multi-level multi-channel supply chain considering pricing, greening, and advertising decisions, Journal of Retailing and Consumer Services. 59, 102425.

Khorshidvand, B., Soleimani, H., Sibdari, S., Seyyed Esfahani, M.M., 2021d. A hybrid modeling approach for green and sustainable closed-loop supply chain considering price, advertisement and uncertain demands, Computers & Industrial Engineering. 157, 107326.

Kumar, A., Mangla, S., Kumar, P., Song, M., 2021. Mitigate risks in perishable food supply chains: Learning from COVID-19. Technological Forecasting & Social Change. 166, 120643.

Li, P., Rao, C., Goh, M., Yang, Z., 2021. Pricing strategies and profit coordination under a double echelon green supply chain, Journal of Cleaner Production. 278, 123694.

Liu, G., Yang, H., Dai, R., 2020. Which contract is more effective in improving product greenness under different power structures: Revenue sharing or cost sharing?, Computers & Industrial Engineering. 148, 106701.

Liu, L., Zhao, Q., Goh, M., 2021. Perishable material sourcing and final product pricing decisions for two-echelon supply chain under price-sensitive demand. Computers & Industrial Engineering. 156, 107260.

Maihami, R., Kamalabadi, I.N., 2012. Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand, International Journal of Production Economics. 136, 116-122.

Malekitabar, M., Yaghoubi, S., Gholamian, M.R., 2019. A novel mathematical inventory model for growing-mortal items (case study: Rainbow trout), Applied Mathematical Modelling. 71, 96-117.

Mohammadi, H., Ghazanfari, M., Pishvaee, M.S., Teimoury, E., (2019). Fresh-product supply chain coordination and waste reduction using a revenue-and-preservation-technology-investment-sharing contract: A real-life case study Journal of Cleaner Production, 213, 262-282.

Prasad, K., Mukherjee, B., 2016. Optimal inventory model under stock and time dependent demand for time varying deterioration rate with shortages, Annals of Operations Research. 243, 323–334.

Rani, S., Ali, R., Agarwal, A., 2017. Green supply chain inventory model for deteriorating items with variable demand under inflation. Int. J. Business Forecasting and Marketing Intelligence 3, 50–77.

Raza, S.A., 2018. Supply chain coordination under a revenue-sharing contract with corporate social responsibility and partial demand information. International Journal of Production Economics 205, 129–145.

Sarathi, G.P., Sarmah, S.P., Jenamani, M, 2014. An integrated revenue sharing and quantity discounts contract for coordinating a supply chain dealing with short life-cycle products, Applied Mathematical Modelling. 38, 4120-4136.

Sarker, B.R., Mukherjee, S., Balan, C.V., 1997. An order-level lot size inventory model with inventory-level dependent demand and deterioration. Int. J. Prod. Econ. 48, 227–236.

Schlosser, R., 2016. Joint stochastic dynamic pricing and advertising with time-dependent demand, Journal of Economic Dynamics and Control. 73, 439-452.

Shi, X., Dong, C., Zhang, C., Zhang, X., 2019. Who should invest in clean technologies in a supply chain with competition, Journal of Cleaner Production, 215, 689-700.

Sun, H., Wan, Y., Zhang, L., Zhou, Z., 2019. Evolutionary game of the green investment in a two-echelon supply chain under a government subsidy mechanism, Journal of Cleaner Production. 235, 1315-1326.

Toktaş-Palut, P., 2021. An integrated contract for coordinating a three-stage green forward and reverse supply chain under fairness concerns, Journal of Cleaner Production. 279, 123735.

Tripathi, R.P., Pareek, S., Kaur, M., 2017. Inventory model with exponential time-dependent demand rate, variable deterioration, shortages and production cost, International Journal of Applied and Computational Mathematics. 3, 1407–1419.

Tsay A.A., Nahmias S., Agrawal N.,1999. Modeling Supply Chain Contracts: A Review. In: Tayur S., Ganeshan R., Magazine M. (eds) Quantitative Models for Supply Chain Management. International Series in Operations Research & Management Science, vol 17. Springer, Boston, MA. https://doi.org/10.1007/978-1-4615-4949-9_10.

Tsiros, M., Heilman, C.M., 2005. The effect of expiration dates and perceived risks on purchasing behavior in grocery store perishable categories. J. Mark. 69 (2), 114–129.

Wang, L., Song, Q., 2020. Pricing policies for dual-channel supply chain with green investment and sales effort under uncertain demand, Mathematics and Computers in Simulation. 171, 79-93.

Yan, B., Chen, X., Cai, C., Guan, S., 2020 Supply chain coordination of fresh agricultural products based on consumer behavior. Computers and Operations Research. 123, 105038.

Zhang, Q., Luo, J., & Duan, Y. (2016). Buyer–vendor coordination for fixed lifetime product with quantity discount under finite production rate. International Journal of Systems Science, 47(4), 821–834.

This article can be cited: Gholamian, M., Noroozi, M., (2022). Perishable food supply chain coordination with multiplicative demand function of price, shelf-life duration, and green technology investment under revenue-sharing contract. Journal of Industrial Engineering and Management Studies, Vol. 9, No.2, pp. 45-63.