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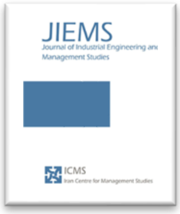

Evaluation of employees' performance by type-2 fuzzy ranking

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Abstract

Today, the proper and effective performance of employees is one of the keys to the success of organizations. Good performance refers to high efficiency, quality, profitability, and customer orientation. One of the most important duties of human resource managers is to design and establish employee performance evaluation systems. Since qualitative indices have a major share of these indices, judgmental methods are generally used for ranking them. Decision makers assign weights to these indices based on their attitudes and rank the employees. Hence, these methods fail to fully explain the performance of organizations' employees and are influenced by some degrees and levels of ambiguity. Fuzzy logic methods are highly useful for resolving the ambiguities in these alternatives. In this paper, we propose an employee performance evaluation method with a type-2 fuzzy ranking approach. In our proposed method, a job ID is designed based on optimal models while an employee ranking method is developed and explained using the trapezoidal interval type-2 fuzzy ranking model introduced by Chen et al. 2012. In the end, the proposed method is utilized for the performance evaluation of employees in a real company.

Keywords: employee performance evaluation; trapezoidal fuzzy number; type-2 fuzzy sets; fuzzy ranking.

Paper Type: Original Research

1. Introduction and preliminaries

Human resources or employees are the most important assets of a company. Evaluation of employees is one of the key duties whereby the current status of the human force is evaluated and the strengths and weaknesses are identified. An employee's performance is influenced by various factors like his/her skills, knowledge, capability, work attitude, leadership quality, working environment, employer's attitude, and company's overall management system (Falsafi et al., 2011). Creating and examining factors and ways to measure the high and low performance of employees is a fundamental way to lead towards efficient employee management (Judge and Ferris, 1993). One of the most important concerns in an employee performance evaluation system is determining the performance evaluation indices. Performance indices are generally classified into quantitative and qualitative indices groups. As for the qualitative indices, performance evaluation is more complicated and the use of these methods is influenced by some degrees of ambiguity due to the type and nature of the job. For instance, in the existing methods, when an employee's performance is evaluated based on the {very good, good, average, weak, and very weak} alternatives, the "good" alternative has an ambiguous meaning. Hence, fuzzy logic methods can be highly useful for removing the ambiguities of these alternatives. Zadeh introduced fuzzy sets (Type-1 Fuzzy Set, T1FS) in 1965 (Zadeh, 1965). These sets formed the basis for a successful method of modeling uncertainty and ambiguity. However, the concept of fuzzy sets could not solve the uncertainty of membership, so in 1975, Zadeh introduced type-2 fuzzy set(T2FS) as an extension of fuzzy sets (Zadeh, 1975). This type of fuzzy set is the generalization of the T1FS, describing membership with fuzzy sets in the interval of [0, 1]. Type-2 fuzzy sets have fuzzy membership degrees, which can mitigate the effect of uncertainties and model them. Type-2 fuzzy number is portrayed by the primary and secondary membership. Therefore, T2FS has a stronger ability to deal with uncertain problems. To determine the T2FS, we need to provide an appropriate fuzzy set for the membership of each element in the domain, which is difficult. To simplify the problem, we have to impose necessary restrictions on the form of T2FS. One approach is to limit the value of 0 or 1 and obtain the interval value fuzzy sets (equivalent to intuitionistic fuzzy sets and vague sets (Takáč, 2013). Another approach is to set the membership function to be the fuzzy number, namely, interval type-2 fuzzy set (IT2FS). IT2FS is a special case of T2FS. The value of the secondary membership is set to 1 and the value of the primary membership is set to a range, thus making it describe uncertainty better than type-1 fuzzy number (Zhao et al., 2015). Human cognition has complexity, uncertainty, and other characteristics that cause difficulty for experts to provide a certain value and allows them to provide only linguistic variables that can be

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represented by fuzzy sets (Chen, 2000). IT2FS has a stronger language explanation ability than ordinary fuzzy sets (Castillo and Melin, 2012).

A type-2 fuzzy set like $\tilde{\tilde{A}}$ with a quadratic membership function is expressed via the following equation.

$$\tilde{\tilde{A}} = \{(x, u), \mu\tilde{\tilde{A}}(x, u) \mid \forall x \in \mathcal{X}, \forall u \in Jx \subseteq [0,1], 0 \leq \mu\tilde{\tilde{A}}(x, u) \leq 1\} \quad (1)$$

In a type-2 fuzzy set, if $\mu\tilde{\tilde{A}}(x, u) = 1$, then $\tilde{\tilde{A}}$ is an interval type-2 fuzzy set. An interval type-2 fuzzy set can be described as a special form of type-2 fuzzy set defined as follows (Mendel et al., 2006).

$$\tilde{\tilde{A}} = \int_{x \in \mathcal{X}}^{0} \int_{u \in J_x}^{0} 1/(x, u)$$

$$Jx \in [0,1] \quad (2)$$

An interval type-2 fuzzy set is called a trapezoidal interval type-2 fuzzy set if its upper and lower membership functions are both trapezoidal fuzzy sets. If it meets this requirement, it is expressed as follows (Chen et al., 2010).

$$\tilde{\tilde{A}} = (\tilde{A}U, \tilde{A}L) = (a_1^U, a_2^U, a_3^U, a_4^U; H1(\tilde{A}U), H2(\tilde{A}U)), (a_1^L, a_2^L, a_3^L, a_4^L; H1(\tilde{A}L), H2(\tilde{A}L)) \quad (3)$$

In the above equation, ÃL and ÃU denote the upper and lower membership functions of $\tilde{\tilde{A}}$, respectively. Besides, H1(ÃL), H2(ÃL) and H1(ÃU), H2(ÃU) show the membership degree, the upper membership, and the lower membership, respectively.
One of the important concepts regarding fuzzy sets is ranking the fuzzy sets. It is possible to select the top alternative by sorting and ranking fuzzy sets.

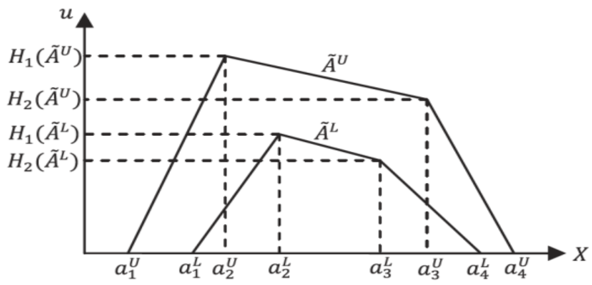

Figure.1. A trapezoidal interval type-2 fuzzy set A⊥≈ (Chen & Lee, 2010b)

Linguistic variables are variables whose values are not numbers. Rather, their values are words or sentences of a natural or artificial language. Although the fuzzy sets theory only deals with mathematical models, it allows for modeling the words and phrases in a natural language using linguistic variables.

In this paper, we propose an employee performance evaluation method. In our proposed method, an employee ranking method is developed and explained using the trapezoidal interval type-2 fuzzy ranking model introduced by Chen et al. (2012).

2. Literature review

One of the most important duties of human resource managers is to design and establish employee performance evaluation systems. Since qualitative indices have a major share of these indices, judgmental methods are generally used for ranking them. Decision makers assign weights to these indices based on their attitudes and rank the employees. Hence, these methods fail to fully explain the performance of organizations' employees and are influenced by some degrees and levels of ambiguity. Fuzzy logic methods are highly useful for resolving the ambiguities in these alternatives. Zadeh introduced fuzzy sets (Type-1 Fuzzy Set, T1FS) in 1965 (Zadeh, 1965). These sets formed the basis for a successful method of modeling uncertainty and ambiguity. However, the concept of fuzzy sets could not solve the uncertainty of membership, so in 1975, Zadeh introduced type-2 fuzzy set(T2FS) as an extension of fuzzy sets (Zadeh, 1975). Mitchell (2006) proposed a method for type-2 fuzzy set ranking. With a statistical approach, he interpreted each type-2 fuzzy set as a weighted set of the ordinary fuzzy set (type-1), which gained the average type-2 fuzzy rank. He stated that the outcome was unexpected but fortunately each type-2 fuzzy set and its rank showed the corresponding degree of uncertainty. Chen and Lee (2010) proposed a model for trapezoidal and triangular fuzzy sets ranking of interval type-2 fuzzy sets. They ranked 13 fuzzy sets based on the proposed model and compared the results of the proposed model with the results of the models previously introduced by Lee and Li's (1988) method, Baas and Kwakernaak's method (1977), and Chang et al. (2006). They concluded that the proposed method was more efficient than the previous methods in ranking. They also introduced a new method for fuzzy multi-criteria group decision-making. Madhooshi et al. (2009) proposed a model for the more effective use of qualitative and subjective information in the employee performance evaluation. First, they classified the evaluation indices based on expert opinions. Thereafter, weights were assigned to the indices by a group of evaluators using linguistic variables. After the employees were evaluated by a group of evaluators, the results were converted into a trapezoidal fuzzy set. Finally, the employees were ranked using the fuzzy TOPSIS technique. Falsafi et al. (2011) proposed decision-making methods for solving the employee performance evaluation and ranking problem. In this study, first, the desired fuzzy method was used to determine the performance evaluation criteria. Afterward, the weights of the criteria were described using linguistic variables, and the employees were ranked using the fuzzy TOPSIS ranking method. In the second proposed method, the weights of the criteria were calculated using Shannon's Entropy and the employees were ranked using TOPSIS. The results of the two methods, the criteria weights, and the final ranks were compared using the advantages of the statistical tests, revealing the homogeneity of the two methods. Ghaderi et al. (2011) defined a process for human resources performance evaluation in banks. In this process, first, the qualitative indices of employee performance evaluation were analyzed and determined. Afterward, a questionnaire was designed and the heads of the bank branches were asked to complete the questionnaire and determine the score of each criterion for their branch personnel. Next, the importance of the qualitative indices for the employee's performance evaluation was determined using the AHP technique and the final score of each qualitative index was calculated based on the score and importance of each index. As a result, the qualitative indices were converted into quantitative values. Finally, the efficiency and rank of the bank branches were determined as compared to each other considering their human resource performance by using the data envelopment analysis (DEA) method and identifying the input and output indices. Chen et al. (2012) proposed an algorithm for solving the fuzzy multi-criteria group decision-making problems based on type-2 fuzzy sets. In this study, they ranked the interval type-2 fuzzy sets through the following three steps. Step one: Forming the weighted decision matrix; step two: forming the average decision matrix; step three: forming the ranking matrix based on the equations defined and finally ranking the alternatives. Mirzaei Nobari et al. (2019) proposed a fuzzy decision support system (FDSS) for the employees of Iran Khodro Company. The researchers selected 12 sub-criteria from the five main criteria. Afterward, they designed the decision matrix as the input for MATLAB software. The average fuzzy set was obtained from the linguistic variables, which were provided by four decision-makers and interviews with three candidates. Thereafter, the mean method was used for defuzzification. Employees were ranked using the Sugeno model by extracting weights through TOPSIS and MATLAB software. A review and some new methods for Rankings and operations for interval type-2 fuzzy numbers are presented by Javanmard and Mishmast Nehi (2019). first, the concept of general interval type-2 trapezoidal fuzzy numbers (GIT2TrFNs) and then arithmetic operations among them introduced. Then, three new ranking methods are suggested for GIT2TrFN. Boral et al., (2021) presented an integrated interval type-2 fuzzy sets and multiplicative half quadratic programming-based MCDM framework for calculating aggregated risk ranking results of failure modes in FMECA. They used the concept of interval type-2 fuzzy sets (IT2FSs), an extended IT2F-DEMATEL method, the concepts of IT2F-MAIRCA, IT2F-MARCOS methods, and modified IT2F-TOPSIS methods. Ranking indicators affecting site selection of vehicle shredding facilities using an interval type-2 fuzzy sets-based Delphi approach proposed by Deveci et al. The introduced methodology consists of four consecutive stages as follows: indicator identification, questionnaire (survey), decision-making analysis, and statistical analysis and indicator classification. The research findings show that the most important indicator for locating vehicle shredding facilities is a financial benefit (Deveci et al., 2022).

3. Methodology

In this paper, an employee performance evaluation system is designed using type-2 fuzzy ranking and the designed system is implemented for the employees in the key positions in a company, which is one of the companies pioneering in the technical and engineering services sector in the Iranian railway industry. The following actions are taken.

Step 1: Preparing and developing a job description for the company's key positions.

Step 2: Determining the linguistic values of the attributes and the corresponding interval type-2 fuzzy values using the model proposed by Chen and Li (2010).

Step 3: Form a team of decision-makers to determine the importance and weight of the evaluation indices.

Step 4: Using the trapezoidal interval type-2 fuzzy ranking model introduced by Chen et al. (2012).

According to the model introduced by Chen et al., the ranking value (RV) of a trapezoidal interval type-2 fuzzy set equals $\tilde{\tilde{A}}$, where in $\tilde{\tilde{A}}$ equals:

$$\tilde{\tilde{A}} = (\tilde{A}U, \tilde{A}L) = ((a_1^U, a_2^U, a_3^U, a_4^U; H1(\tilde{A}U), H2(\tilde{A}U)), ((a_1^L, a_2^L, a_3^L, a_4^L; H1(\tilde{A}L), H2(\tilde{A}L))) \quad (4)$$

Hence, first $RV(\tilde{\tilde{A}}_i)$ is obtained via equation (5) to rank each trapezoidal interval type-2 fuzzy set, and if $1 \leq i \leq n$, then the biggest $RV(\tilde{\tilde{A}}_i)$ value has the highest rank.

$$RV(\tilde{\tilde{A}}_i) = \left[\frac{(a_{11}^U+K_1)+(a_{14}^U+K_1)}{2} + \frac{(H_1(A_i^U)+(H_2(A_i^U)+H_1(A_i^L)+H_2(A_i^L)}{4}\right] \times \left[\frac{(a_{11}^U+K_i)+(a_{12}^U+K_i)+(a_{13}^U+K_i)+(a_{14}^U+K_i)+(a_{11}^L+K_i)+(a_{12}^L+K_i)+(a_{13}^L+K_i)+(a_{14}^L+K_i)}{8}\right] \quad (5)$$

$$K_i = \begin{cases} 0, & if\ Min(a_{11}^U a_{21}^U, \dots, a_{n1}^U) \geq 0, \\ Min(a_{11}^U a_{21}^U, \dots, a_{n1}^U)|, & if\ Min(a_{11}^U a_{21}^U, \dots, a_{n1}^U) < 0, \end{cases} \quad (6)$$

To calculate $RV(\tilde{\tilde{A}}_i)$, the fuzzy multi-criteria group decision-making model that is based on the interval type-2 fuzzy sets ranking method introduced by Chen et al. (2012) is used as follows:

Construct the weighted decision matrix Sp, shown as follows:

$$S_p = DW_p \otimes Y_p = \begin{matrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{matrix} \begin{bmatrix} \tilde{\tilde{w}}_1^p \\ \tilde{\tilde{w}}_2^p \\ \vdots \\ \tilde{\tilde{w}}_m^p \end{bmatrix} \otimes \begin{matrix} & x_1 & x_2 & \cdots & x_n \\ f_1 & \tilde{\tilde{y}}_{11}^p & \tilde{\tilde{y}}_{12}^p & \cdots & \tilde{\tilde{y}}_{1n}^p \\ f_2 & \tilde{\tilde{y}}_{21}^p & \tilde{\tilde{y}}_{22}^p & \cdots & \tilde{\tilde{y}}_{2n}^p \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ f_m & \tilde{\tilde{y}}_{m1}^p & \tilde{\tilde{y}}_{m2}^p & \cdots & \tilde{\tilde{y}}_{mn}^p \end{matrix} = \begin{matrix} & x_1 & x_2 & \cdots & x_n \\ f_1 & \tilde{\tilde{s}}_{11}^p & \tilde{\tilde{s}}_{12}^p & \cdots & \tilde{\tilde{s}}_{1n}^p \\ f_2 & \tilde{\tilde{s}}_{21}^p & \tilde{\tilde{s}}_{22}^p & \cdots & \tilde{\tilde{s}}_{2n}^p \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ f_m & \tilde{\tilde{s}}_{m1}^p & \tilde{\tilde{s}}_{m2}^p & \cdots & \tilde{\tilde{s}}_{mn}^p \end{matrix} \quad (7)$$

Where

$$\tilde{\tilde{s}}_{ij} = \tilde{\tilde{y}}_{ij} \otimes \tilde{\tilde{w}}_i, 1 \leq i \leq m \text{ and } 1 \leq j \leq n. \quad (8)$$

X be a set of alternatives, $X = \{x_1, x_2, \dots, x_n\}$ and F be a set of attributes, $F=\{f_1, f_2, \dots, f_m\}$, Let DW_p be the weighting matrix of the attributes given by decision maker D_p, where $1 \leq p \leq k$ and let Y_p be the evaluating matrix of the alternatives given by decision maker D_p, where $1 \leq p \leq k$.

a) Construct the average decision matrix $\bar{S}$, shown as follows:

$$\bar{S} = \begin{matrix} & x_1 & x_2 & \cdots & x_n \\ f_1 & \tilde{\tilde{s}}_{11}^p & \tilde{\tilde{s}}_{12}^p & \cdots & \tilde{\tilde{s}}_{1n}^p \\ f_2 & \tilde{\tilde{s}}_{21}^p & \tilde{\tilde{s}}_{22}^p & \cdots & \tilde{\tilde{s}}_{2n}^p \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ f_m & \tilde{\tilde{s}}_{m1}^p & \tilde{\tilde{s}}_{m2}^p & \cdots & \tilde{\tilde{s}}_{mn}^p \end{matrix}, \qquad \tilde{\tilde{s}}_{ij} = \left(\frac{\tilde{\tilde{s}}_{ij}^1 \oplus \tilde{\tilde{s}}_{ij}^2 \oplus \dots \oplus \tilde{\tilde{s}}_{ij}^k}{k}\right) \quad (9)$$

$\tilde{\tilde{s}}_{ij}^p$ and $\tilde{\tilde{s}}_{ij}$ are interval type-2 fuzzy sets, $1 \leq i \leq m$, $1 \leq j \leq n$, $1 \leq p \leq k$ and k denotes the number of decision makers. Based on Eq. (5), construct the ranking matrix RS, shown as follows:

$x_1 \quad x_2 \quad \cdots \quad x_n$

$$RS = \begin{matrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{matrix} \begin{bmatrix} RV(\overset{\approx}{s}_{11}) & RV(\overset{\approx}{s}_{12}) & \cdots & RV(\overset{\approx}{s}_{1n}) \\ RV(\overset{\approx}{s}_{21}) & RV(\overset{\approx}{s}_{22}) & \cdots & RV(\overset{\approx}{s}_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ RV(\overset{\approx}{s}_{m1}) & RV(\overset{\approx}{s}_{m2}) & \cdots & RV(\overset{\approx}{s}_{mn}) \end{bmatrix} \quad , 1 \leq i \leq m \text{ and } 1 \leq j \leq n \tag{10}$$

Construct the average agreement degree (AD), shown as follows:

$$AD = \begin{matrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{matrix} \begin{bmatrix} \frac{1}{n}\sum_{j=1}^{n}(RV(\overset{\approx}{s}_{1j})) \\ \frac{1}{n}\sum_{j=1}^{n}(RV(\overset{\approx}{s}_{2j})) \\ \vdots \\ \frac{1}{n}\sum_{j=1}^{n}(RV(\overset{\approx}{s}_{mj})) \end{bmatrix}, AD = \begin{matrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{matrix} \begin{bmatrix} AD_1 \\ AD_2 \\ \vdots \\ AD_m \end{bmatrix}, where 1 \leq i \leq m \text{and } 1 \leq j \leq n \tag{11}$$

Calculate the ranking value R(x_j)of alternative x_j, shown as follows:

$$R(x_j) = \sum_{i=1}^{m} \frac{RV(\overset{\approx}{s}_{ij})}{AD_i}, \quad 1 \leq j \leq n \tag{12}$$

The larger the value of R(x_j), the better the ranking order of alternative x_j.

4. A case study

Step 1: The performance evaluation system currently used in this company is based on a survey of managers using general questionnaires. Considering the feedback provided by the company's management about the employees' dissatisfaction with the current performance evaluation method, a questionnaire was developed to evaluate the employees' satisfaction with the current method. After confirming the reliability and validity of this questionnaire, it was presented to 5 key employees in this company, who took part in the evaluation process using the proposed method. The results of the satisfaction evaluation are listed in Table 1.

Table 1. Measurement of employee satisfaction of company employees from the existing performance evaluation model

Row	Indicator	Scoring (1-100)				
		x_1	x_2	x_3	x_4	x_5
1	Awareness of duties and responsibilities	30	40	30	45	35
2	Awareness of evaluation indicators	35	25	30	25	30
3	Awareness of the evaluation method	55	60	50	60	60
4	The fairness of the evaluation method	30	35	30	40	25
5	Acceptance of evaluation rewards	20	20	15	25	15
6	Trusting the evaluation results correctly	25	20	20	15	25
	Score (%)	32.5%	33.33%	29.16%	35%	31.66%
	Average satisfaction (%)			32.33%		

It is worth stating that letters {x1, x2, x3, x4, x5} are used instead of using the participants' names to ensure the confidentiality of the information.

Considering the results of the pathological analysis of the defects observed in the performance evaluation system used in the company, which led to the employees' dissatisfaction, a committee composed of the human resource manager and the human resource advisor was formed. This committee was in charge of developing a job description for the key jobs such as the project planning expert, project accounting expert, design and engineering expert, project management expert, and HSE expert to develop the description of duties of the employees and the evaluation indices for the jobs. The aforesaid committee held several sessions with the stakeholders to identify the needs and expectations of each job and analyze and adopt the top human resource models including the model proposed by O*NET† and the business process models introduced by APQC‡known as the Process Classification Framework. The committee managed to develop the job description for the selected positions (a sample job description is presented in Table 2). The results were communicated to the employees after winning the CEO’s approval. Thereafter, the human resource department took measures to explain and teach the job descriptions to the appropriate employees. After taking the designated training course, the employees were obliged to fulfill their professional duties and responsibilities based on the related job descriptions. It was decided to evaluate the employees using the proposed method after three months of fulfilling their duties according to the new job description.

Table 2. A sample of a job description

Row	Job Title: Design and engineering expert Job description		
1	Review and analyze the technical requirements and requirements of the product provided by the client		
2	Preparation of technical drawings for product manufacturing and bill of material (BOM)		
3	Obtain final approval of product design from client		
4	Providing technical requests for the purchase or manufacture of products and providing them to suppliers		
5	Review and Selection of Technical Proposals for Suppliers		
6	Quality control of construction process by contractors		
7	Preparation of the Final Book		
	Job evaluation indicators		
Quality	Innovation	Commitment	Systematic thinking

Step 2: The linguistic values of the features and the corresponding interval type-2 fuzzy values were obtained using the model proposed by Chen and Lee (2010) as described in Table 3.

Table 3. Linguistic terms and their corresponding Interval type-2 fuzzy sets

Linguistic terms	Interval type-2 fuzzy sets
Very Low (VL)	((0,0,0,0.1;1,1), (0,0,0,0,0.05;0.9,0.9))
Low (L)	((0,0.1,0.1,0.3;1,1), (0.05,0.1,0.1,0.2;0.9,0.9))
Medium Low (ML)	((0.1,0.3,0.3,0.5;1,1), (0.2,0.3,0.3,0.4;0.9,0.9))
Medium (M)	((0.3,0.5,0.5,0.7;1,1), (0.4,0.5,0.5,0.6;0.9,0.9))
Medium High (MH)	((0.5,0.7,0.7,0.9;1,1), (0.6,0.7,0.7,0.8;0.9,0.9))
High (H)	((0.7,0.9,0.9,1;1,1), (0.8,0.9,0.9,0.95;0.9,0.9))
Very High (VH)	((0.9,1,1,1;1,1), (0.95,1,1,1;0.9,0.9))

Step 3: The decision makers were asked to determine the qualitative importance of the employee performance evaluation indices (attributes) through a questionnaire to determine the importance and weight of the evaluation indices. The results are shown in Table 4.

Table 4. Determining the weight of attributes by Decision Makers

Attributes	Decision Makers		
	D_1	D_2	D_3
Quality	VH	VH	VH
Innovation	H	H	VH
Commitment	VH	H	H
Systematic thinking	H	H	VH

Step 4: The key employee performance questionnaires were provided to the decision makers to conduct the evaluations considering the employees' performance within three months based on Table 3. The results are listed in Table 5.

Table 5. Evaluation of attributes by Decision Makers

Alternatives	Attributes	Decision Makers		
		D_1	D_2	D_3
x_1	Quality	MH	H	MH

† Occupational Information Network
‡ American Productivity and Quality Center

x_2		H	H	VH
x_3		H	H	VH
x_4		H	MH	VH
x_5		VH	VH	MH
x_1		H	M	H
x_2		M	M	H
x_3	Innovation	H	MH	VH
x_4		M	M	MH
x_5		VH	VH	VH
x_1		MH	H	MH
x_2		MH	VH	H
x_3	Commitment	M	VH	MH
x_4		MH	MH	H
x_5		MH	VH	VH
x_1		VH	VH	VH
x_2		M	MH	M
x_3	Systematic thinking	H	M	H
x_4		VH	H	M
x_5		VH	MH	M

Step 5: Construct the DW_p and Y_p matrix, based on Table 4 and Table 5, where $1 \leq p \leq 3$, shown as follows:

$$DW_1 = \begin{matrix} Quality \\ Innovation \\ Commitment \\ Systematic \end{matrix} \begin{bmatrix} VH \\ H \\ VH \\ H \end{bmatrix}, DW_2 = \begin{matrix} Quality \\ Innovation \\ Commitment \\ Systematic \end{matrix} \begin{bmatrix} VH \\ H \\ H \\ H \end{bmatrix}, DW_3 = \begin{matrix} Quality \\ Innovation \\ Commitment \\ Systematic \end{matrix} \begin{bmatrix} VH \\ VH \\ H \\ VH \end{bmatrix}$$

$$\begin{matrix} & x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix}$$

$$Y_1 = \begin{matrix} Quality \\ Innovation \\ Commitment \\ Systematic \end{matrix} \begin{bmatrix} MH & H & H & H & VH \\ H & M & H & M & VH \\ MH & MH & M & MH & MH \\ VH & M & H & VH & VH \end{bmatrix}$$

$$\begin{matrix} & x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix}$$

$$Y_2 = \begin{matrix} Quality \\ Innovation \\ Commitment \\ Systematic \end{matrix} \begin{bmatrix} H & H & H & MH & VH \\ M & M & MH & M & VH \\ H & VH & VH & MH & VH \\ VH & MH & M & H & MH \end{bmatrix}$$

$$\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix}$$

$$Y_3 = \begin{matrix} Quality \\ Innovation \\ Commitment \\ Systematic \end{matrix} \begin{bmatrix} MH & VH & VH & VH & MH \\ H & H & VH & MH & VH \\ MH & H & MH & H & VH \\ VH & M & H & M & M \end{bmatrix}$$

Step 6: Construct the S_p matrix, based on Table 3 and Eq. (7), where $1 \leq p \leq 3$, shown as follows:

$$S_1 = DW_1 \otimes Y_1 = \begin{matrix} Quality \\ Innovation \\ Commitment \\ Systematic \end{matrix} \overset{\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix}}{\begin{bmatrix} \approx{s}^1_{11} & \approx{s}^1_{12} & \approx{s}^1_{13} & \approx{s}^1_{14} & \approx{s}^1_{15} \\ \approx{s}^1_{21} & \approx{s}^1_{22} & \approx{s}^1_{23} & \approx{s}^1_{24} & \approx{s}^1_{25} \\ \approx{s}^1_{31} & \approx{s}^1_{32} & \approx{s}^1_{33} & \approx{s}^1_{34} & \approx{s}^1_{35} \\ \approx{s}^1_{41} & \approx{s}^1_{42} & \approx{s}^1_{43} & \approx{s}^1_{44} & \approx{s}^1_{45} \end{bmatrix}}$$

$\approx{s}^1_{11}$ = $VH \otimes MH$= ((0.9,1,1,1;1,1), (0.95,1,1,1;0.9,0.9))⊗((0.5,0.7,0.7,0.9;1,1),(0.6,0.7,0.7,0.8;0.9,0.9))

= ((0.45,0.7,0.7,0.9;1,1), (0.57,0.7,0.7,0.8;0.81,0.81))

$\approx{s}^1_{12}$ = $VH \otimes H$= ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))

$\approx{s}^1_{13}$ = $VH \otimes H$ = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))

$\approx{s}^1_{14}$ = $VH \otimes H$ = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))

$\approx{s}^1_{15}$ = $VH \otimes VH$ = ((0.81,1,1,1;1,1), (0.9025,1,1,1;0.81,0.81))

$\approx{s}^1_{21}$ = $H \otimes H$ = ((0.49, 0.81,0.81,1;1,1), (0.64,0.81,0.81, 0.9025;0.81;0.81))

$\approx{s}^1_{22}$ = $H \otimes M$ = ((0.21, 0.45,0.45,0.7;1,1), (0.32,0.45,0.45,0.57;0.81,0.81))

$\approx{s}^1_{23}$ = $H \otimes H$ = ((0.49, 0.81,0.81;1,1), (0.64,0.81,0.81, 0.9025;0.81;0.81))

$\approx{s}^1_{24}$ = $H \otimes M$ = ((0.21, 0.45,0.45,0.7;1,1), (0.32,0.45,0.45,0.57;0.81,0.81))

$\approx{s}^1_{25}$ = $H \otimes VH$ = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))

$\approx{s}^1_{31}$ = $VH \otimes MH$ = ((0.45,0.7,0.7,0.9;1,1), (0.57,0.7,0.7,0.8;0.81,0.81))

$\approx{s}^1_{32}$ = $VH \otimes MH$= ((0.45,0.7,0.7,0.9;1,1), (0.57,0.7,0.7,0.8;0.81,0.81))

$\approx{s}^1_{33}$ = $VH \otimes M$ = ((0.27,0.5,0.5,0.7;1,1), (0.38,0.5,0.5,0.6;0.81,0.81))

$\approx{s}^1_{34}$ = $VH \otimes MH$ = ((0.45,0.7,0.7,0.9;1,1), (0.57,0.7,0.7,0.8;0.81,0.81))

$\approx{s}^1_{35}$ = $VH \otimes MH$ = ((0.45,0.7,0.7,0.9;1,1), (0.57,0.7,0.7,0.8;0.81,0.81))

$\approx{s}^1_{41}$ = $H \otimes VH$ = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))

$\approx{s}^1_{42}$ = $H \otimes M$ = ((0.21, 0.45,0.45,0.7;1,1), (0.32,0.45,0.45,0.57;0.81,0.81))

$\approx{s}^1_{43}$ = $H \otimes H$ = ((0.49, 0.81,0.81,1;1,1), (0.64,0.81,0.81, 0.9025;0.81;0.81))

$\approx{s}^1_{44}$ = $H \otimes VH$ = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))

$\approx{s}^1_{45}$ = $H \otimes VH$ = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$

$$S_2 = DW_2 \otimes Y_2 = \begin{matrix} Quality \\ Innovation \\ Commitment \\ Systematic \end{matrix} \begin{bmatrix} \approx s_{11}^2 & \approx s_{12}^2 & \approx s_{13}^2 & \approx s_{14}^2 & \approx s_{15}^2 \\ \approx s_{21}^2 & \approx s_{22}^2 & \approx s_{23}^2 & \approx s_{24}^2 & \approx s_{25}^2 \\ \approx s_{31}^2 & \approx s_{32}^2 & \approx s_{33}^2 & \approx s_{34}^2 & \approx s_{35}^2 \\ \approx s_{41}^2 & \approx s_{42}^2 & \approx s_{43}^2 & \approx s_{44}^2 & \approx s_{45}^2 \end{bmatrix}$$

$\overset{\approx}{s}{}_{11}^2 = VH \otimes H = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))$

$\overset{\approx}{s}{}_{12}^2 = VH \otimes H = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))$

$\overset{\approx}{s}{}_{13}^2 = VH \otimes H = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))$

$\overset{\approx}{s}{}_{14}^2 = VH \otimes MH= ((0.45,0.7,0.7,0.9;1,1), (0.57,0.7,0.7,0.8;0.81,0.81))$

$\overset{\approx}{s}{}_{15}^2 = VH \otimes VH= ((0.81,1,1,1;1,1), (0.9025,1,1,1;0.81,0.81))$

$\overset{\approx}{s}{}_{21}^2 = H \otimes M = ((0.21, 0.45,0.45,0.7;1,1), (0.32,0.45,0.45,0.57;0.81,0.81))$

$\overset{\approx}{s}{}_{22}^2 = H \otimes M = ((0.21, 0.45,0.45,0.7;1,1), (0.32,0.45,0.45,0.57;0.81,0.81))$

$\overset{\approx}{s}{}_{23}^2 = H \otimes MH = ((0.35,0.63,0.63,0.9;1,1), (0.48,0.63,0.63,0.76;0.81,0.81))$

$\overset{\approx}{s}{}_{24}^2 = H \otimes M = ((0.21, 0.45,0.45,0.7;1,1), (0.32,0.45,0.45,0.57;0.81,0.81))$

$\overset{\approx}{s}{}_{25}^2 = H \otimes VH = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))$

$\overset{\approx}{s}{}_{31}^2 = H \otimes H = ((0.49, 0.81,0.81,1;1,1), (0.64,0.81,0.81, 0.9025;0.81;0.81))$

$\overset{\approx}{s}{}_{32}^2 = H \otimes VH = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))$

$\overset{\approx}{s}{}_{33}^2 = H \otimes VH = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))$

$\overset{\approx}{s}{}_{34}^2 = H \otimes MH = ((0.35,0.63,0.63,0.9;1,1), (0.48,0.63,0.63,0.76;0.81,0.81))$

$\overset{\approx}{s}{}_{35}^2 = H \otimes VH = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))$

$\overset{\approx}{s}{}_{41}^2 = H \otimes VH = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))$

$\overset{\approx}{s}{}_{42}^2 = H \otimes MH = ((0.35,0.63,0.63,0.9;1,1), (0.48,0.63,0.63,0.76;0.81,0.81))$

$\overset{\approx}{s}{}_{43}^2 = H \otimes M = ((0.21, 0.45,0.45,0.7;1,1), (0.32,0.45,0.45,0.57;0.81,0.81))$

$\overset{\approx}{s}{}_{44}^2 = H \otimes H = ((0.49, 0.81,0.81,1;1,1), (0.64,0.81,0.81, 0.9025;0.81;0.81))$

$\overset{\approx}{s}{}_{45}^2 = H \otimes MH = ((0.35,0.63,0.63,0.9;1,1), (0.48,0.63,0.63,0.76;0.81,0.81))$

$$S_3 = DW_3 \otimes Y_3 = \begin{matrix} \\ Quality \\ Innovation \\ Commitment \\ Systematic \end{matrix} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ \approx s_{11}^3 & \approx s_{12}^3 & \approx s_{13}^3 & \approx s_{14}^3 & \approx s_{15}^3 \\ \approx s_{21}^3 & \approx s_{22}^3 & \approx s_{23}^3 & \approx s_{24}^3 & \approx s_{25}^3 \\ \approx s_{31}^3 & \approx s_{32}^3 & \approx s_{33}^3 & \approx s_{34}^3 & \approx s_{35}^3 \\ \approx s_{41}^3 & \approx s_{42}^3 & \approx s_{43}^3 & \approx s_{44}^3 & \approx s_{45}^3 \end{matrix}$$

$\overset{\approx}{s}{}_{11}^3 = VH \otimes MH = ((0.45,0.7,0.7,0.9;1,1),(0.57,0.7,0.7,0.8;0.81,0.81))$

$\overset{\approx}{s}{}_{12}^3 = VH \otimes VH= ((0.81,1,1,1;1,1), (0.9025,1,1,1;0.81,0.81))$

$\tilde{\tilde{s}}_{13}^{3} = VH \otimes VH = ((0.81,1,1,1;1,1), (0.9025,1,1,1;0.81,0.81))$

$\tilde{\tilde{s}}_{14}^{3} = VH \otimes VH = ((0.81,1,1,1;1,1), (0.9025,1,1,1;0.81,0.81))$

$\tilde{\tilde{s}}_{15}^{3} = VH \otimes MH = ((0.45,0.7,0.7,0.9;1,1), (0.57,0.7,0.7,0.8;0.81,0.81))$

$\tilde{\tilde{s}}_{21}^{3} = VH \otimes H = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))$

$\tilde{\tilde{s}}_{22}^{3} = VH \otimes H = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))$

$\tilde{\tilde{s}}_{23}^{3} = VH \otimes VH = ((0.81,1,1,1;1,1), (0.9025,1,1,1;0.81,0.81))$

$\tilde{\tilde{s}}_{24}^{3} = VH \otimes MH = ((0.45,0.7,0.7,0.9;1,1), (0.57,0.7,0.7,0.8;0.81,0.81))$

$\tilde{\tilde{s}}_{25}^{3} = VH \otimes VH = ((0.81,1,1,1;1,1), (0.9025,1,1,1;0.81,0.81))$

$\tilde{\tilde{s}}_{31}^{3} = H \otimes MH = ((0.35,0.63,0.63,0.9;1,1), (0.48,0.63,0.63,0.76;0.81,0.81))$

$\tilde{\tilde{s}}_{32}^{3} = H \otimes H = ((0.49, 0.81,0.81,1;1,1), (0.64,0.81,0.81, 0.9025;0.81;0.81))$

$\tilde{\tilde{s}}_{33}^{3} = H \otimes MH = ((0.35,0.63,0.63,0.9;1,1), (0.48,0.63,0.63,0.76;0.81,0.81))$

$\tilde{\tilde{s}}_{34}^{3} = H \otimes H = ((0.49, 0.81,0.81,1;1,1), (0.64,0.81,0.81, 0.9025;0.81;0.81))$

$\tilde{\tilde{s}}_{35}^{3} = H \otimes VH = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))$

$\tilde{\tilde{s}}_{41}^{3} = VH \otimes VH = ((0.81,1,1,1;1,1), (0.9025,1,1,1;0.81,0.81))$

$\tilde{\tilde{s}}_{42}^{3} = VH \otimes M = ((0.27,0.5,0.5,0.7;1,1), (0.38,0.5,0.5,0.6;0.81,0.81))$

$\tilde{\tilde{s}}_{43}^{3} = VH \otimes H = ((0.63,0.9,0.9,1;1,1), (0.76,0.9,0.9,0.95;0.81,0.81))$

$\tilde{\tilde{s}}_{44}^{3} = VH \otimes M = ((0.27,0.5,0.5,0.7;1,1), (0.38,0.5,0.5,0.6;0.81,0.81))$

$\tilde{\tilde{s}}_{45}^{3} = VH \otimes M = ((0.27,0.5,0.5,0.7;1,1), (0.38,0.5,0.5,0.6;0.81,0.81))$

Step 7: Based on Eq. (9), construct the average decision matrix $\bar{S}$, shown as follows:

$$\bar{S} = \begin{matrix} Quality \\ Innovation \\ Commitment \\ Systematic \end{matrix} \overset{\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix}}{\begin{bmatrix} \tilde{\tilde{s}}_{11} & \tilde{\tilde{s}}_{12} & \tilde{\tilde{s}}_{13} & \tilde{\tilde{s}}_{14} & \tilde{\tilde{s}}_{15} \\ \tilde{\tilde{s}}_{21} & \tilde{\tilde{s}}_{22} & \tilde{\tilde{s}}_{23} & \tilde{\tilde{s}}_{24} & \tilde{\tilde{s}}_{25} \\ \tilde{\tilde{s}}_{31} & \tilde{\tilde{s}}_{32} & \tilde{\tilde{s}}_{33} & \tilde{\tilde{s}}_{34} & \tilde{\tilde{s}}_{35} \\ \tilde{\tilde{s}}_{41} & \tilde{\tilde{s}}_{42} & \tilde{\tilde{s}}_{43} & \tilde{\tilde{s}}_{44} & \tilde{\tilde{s}}_{45} \end{bmatrix}}$$

$\tilde{\tilde{s}}_{11} = \left(\tilde{\tilde{s}}_{11}^{1} \oplus \tilde{\tilde{s}}_{11}^{2} \oplus \tilde{\tilde{s}}_{11}^{3}\right) = \frac{1}{3}((0.45,0.7,0.7,0.9;1,1), (0.57,0.7,0.7,0.8;0.81,0.81)) \oplus$

$\frac{1}{3}((0.63,0.9,0.9,1;1,1),(0.76,0.9,0.9,0.95;0.81,0.81)) \oplus$
$\frac{1}{3}((0.45,0.7,0.7,0.9;1,1),(0.57,0.7,0.7,0.8;0.81,0.81)) = ((\frac{045+0.63+0.45}{3}, \frac{0.7+0.9+0.7}{3}, \frac{0.7+0.9+0.7}{3}, \frac{0.9+1+0.9}{3};1,1),$
$(\frac{0.57+0.76+0.57}{3}, \frac{0.7+0.9+0.7}{3}, \frac{0.7+0.9+0.7}{3}, \frac{0.8+0.95+0.8}{3}; 081,0.81))$

$= ((0.51,0.77,0.77,0.93;1,1), (0.63,0.77,0.77,0.85;0.81,0.81))$

$\tilde{\tilde{s}}_{12} = \left(\tilde{\tilde{s}}_{12}^{1} \oplus \tilde{\tilde{s}}_{12}^{2} \oplus \tilde{\tilde{s}}_{12}^{3}\right) = ((0.69,0.93,0.93,1;1,1), (0.81,0.93,0.93,0.98;0.81,0.81))$

$\tilde{\tilde{s}}_{13} = \left(\tilde{\tilde{s}}_{13}^{1} \oplus \tilde{\tilde{s}}_{13}^{2} \oplus \tilde{\tilde{s}}_{13}^{3}\right) = ((0.69,0.93,0.93,0.98;1,1), (0.81,0.93,0.93,0.97,0.81,0.81))$

$\tilde{\tilde{s}}_{14}=\left(\tilde{\tilde{s}}_{14}^{1} \oplus \tilde{\tilde{s}}_{14}^{2} \oplus \tilde{\tilde{s}}_{14}^{3}\right)=$ ((0.63,0.87,0.87,0.97;1,1), (0.74,0.87,0.87,0.92;0.81,0.81))

$\tilde{\tilde{s}}_{15}=\left(\tilde{\tilde{s}}_{15}^{1} \oplus \tilde{\tilde{s}}_{15}^{2} \oplus \tilde{\tilde{s}}_{15}^{3}\right)=$ ((0.69,0.9,0.9,0.97;1,1), (0.79,0.9,0.9,0.93;0.81,0.81))

$\tilde{\tilde{s}}_{21}=\left(\tilde{\tilde{s}}_{21}^{1} \oplus \tilde{\tilde{s}}_{21}^{2} \oplus \tilde{\tilde{s}}_{21}^{3}\right)=$ ((0.44,0.72,0.72,0.9;1,1), (0.57,0.72,0.72,0.81;0.81,0.81))

$\tilde{\tilde{s}}_{22}=\left(\tilde{\tilde{s}}_{22}^{1} \oplus \tilde{\tilde{s}}_{22}^{2} \oplus \tilde{\tilde{s}}_{22}^{3}\right)=$ ((0.35,0.6,0.6,0.5;1,1), (0.47,0.6,0.6,0.7;0.81,0.81))

$\tilde{\tilde{s}}_{23}=\left(\tilde{\tilde{s}}_{23}^{1} \oplus \tilde{\tilde{s}}_{23}^{2} \oplus \tilde{\tilde{s}}_{23}^{3}\right)=$ ((0.55,0.81,0.81,0.9;1,1), (0.67,0.81,0.81,0.89;0.81,0.81))

$\tilde{\tilde{s}}_{24}=\left(\tilde{\tilde{s}}_{24}^{1} \oplus \tilde{\tilde{s}}_{24}^{2} \oplus \tilde{\tilde{s}}_{24}^{3}\right)=$ ((0.29,0.53,0.53,0.77;1,1), (0.40,0.53,0.53,0.65;0.81;0.81))

$\tilde{\tilde{s}}_{25}=\left(\tilde{\tilde{s}}_{25}^{1} \oplus \tilde{\tilde{s}}_{25}^{2} \oplus \tilde{\tilde{s}}_{25}^{3}\right)=$ ((0.69,0.93,0.93,1;1,1), (0.81,0.93,0.93,0.97;0.81,0.81))

$\tilde{\tilde{s}}_{31}=\left(\tilde{\tilde{s}}_{31}^{1} \oplus \tilde{\tilde{s}}_{31}^{2} \oplus \tilde{\tilde{s}}_{31}^{3}\right)=$ ((0.43,0.71,0.71,0.93;1,1), (0.56,0.71,0.71,0.82;0.81,0.81))

$\tilde{\tilde{s}}_{32}=\left(\tilde{\tilde{s}}_{32}^{1} \oplus \tilde{\tilde{s}}_{32}^{2} \oplus \tilde{\tilde{s}}_{32}^{3}\right)=$ ((0.52,0.8,0.8,0.97;1,1), (0.66,0.8,0.8,0.88;0.81,0.81))

$\tilde{\tilde{s}}_{33}=\left(\tilde{\tilde{s}}_{33}^{1} \oplus \tilde{\tilde{s}}_{33}^{2} \oplus \tilde{\tilde{s}}_{33}^{3}\right)=$ ((0.42,0.68,0.68,0.87;1,1), (0.54,0.68,0.68,0.77;0.81,0.81))

$\tilde{\tilde{s}}_{34}=\left(\tilde{\tilde{s}}_{34}^{1} \oplus \tilde{\tilde{s}}_{34}^{2} \oplus \tilde{\tilde{s}}_{34}^{3}\right)=$ ((0.43,0.71,0.71,0.93;1,1), (0.56,0.71,0.71,0.82;0.81,0.81))

$\tilde{\tilde{s}}_{35}=\left(\tilde{\tilde{s}}_{35}^{1} \oplus \tilde{\tilde{s}}_{35}^{2} \oplus \tilde{\tilde{s}}_{35}^{3}\right)=$ ((0.57,0.83,0.83,0.97;1,1), (0.70,0.83,0.83,0.90;0.81,0.81))

$\tilde{\tilde{s}}_{41}=\left(\tilde{\tilde{s}}_{41}^{1} \oplus \tilde{\tilde{s}}_{41}^{2} \oplus \tilde{\tilde{s}}_{41}^{3}\right)=$ ((0.69,0.93,0.93,1;1,1), (0.81,0.93,0.93,0.97;0.81,0.81))

$\tilde{\tilde{s}}_{42}=\left(\tilde{\tilde{s}}_{42}^{1} \oplus \tilde{\tilde{s}}_{42}^{2} \oplus \tilde{\tilde{s}}_{42}^{3}\right)=$ ((0.28,0.53,0.53,0.77;1,1), (0.39,0.53,0.53,0.54;0.81,0.81))

$\tilde{\tilde{s}}_{43}=\left(\tilde{\tilde{s}}_{43}^{1} \oplus \tilde{\tilde{s}}_{43}^{2} \oplus \tilde{\tilde{s}}_{43}^{3}\right)=$ ((0.44,0.72,0.72,0.9;1,1), (0.57,0.72,0.72,0.81;0.81,0.81))

$\tilde{\tilde{s}}_{44}=\left(\tilde{\tilde{s}}_{44}^{1} \oplus \tilde{\tilde{s}}_{44}^{2} \oplus \tilde{\tilde{s}}_{44}^{3}\right)=$ ((0.46,0.74,0.74,0.9;1,1), (0.59,0.74,0.74,0.82;0.81,0.81))

$\tilde{\tilde{s}}_{45}=\left(\tilde{\tilde{s}}_{45}^{1} \oplus \tilde{\tilde{s}}_{45}^{2} \oplus \tilde{\tilde{s}}_{45}^{3}\right)=$ ((0.42,0.68,0.68,0.87;1,1), (0.54,0.68,0.68,0.77;0.81,0.81))

Step 8: Based on Eq. (10), construct the ranking matrix RS, shown as follows:

$$RS=\begin{matrix} Quality \\ Innovation \\ Commitment \\ Systematic \end{matrix}\overset{\begin{matrix} x_1 & \quad x_2 & \quad x_3 & \quad x_4 & \quad x_5 \end{matrix}}{\begin{bmatrix} RV(\tilde{\tilde{s}}_{11}) & RV(\tilde{\tilde{s}}_{12}) & RV(\tilde{\tilde{s}}_{13}) & RV(\tilde{\tilde{s}}_{14}) & RV(\tilde{\tilde{s}}_{15}) \\ RV(\tilde{\tilde{s}}_{21}) & RV(\tilde{\tilde{s}}_{22}) & RV(\tilde{\tilde{s}}_{23}) & RV(\tilde{\tilde{s}}_{24}) & RV(\tilde{\tilde{s}}_{25}) \\ RV(\tilde{\tilde{s}}_{31}) & RV(\tilde{\tilde{s}}_{32}) & RV(\tilde{\tilde{s}}_{33}) & RV(\tilde{\tilde{s}}_{34}) & RV(\tilde{\tilde{s}}_{35}) \\ RV(\tilde{\tilde{s}}_{41}) & RV(\tilde{\tilde{s}}_{42}) & RV(\tilde{\tilde{s}}_{43}) & RV(\tilde{\tilde{s}}_{44}) & RV(\tilde{\tilde{s}}_{45}) \end{bmatrix}}$$

Where

$$RV\left(\tilde{\tilde{s}}_{11}\right)=\left(\frac{0.51+0.93}{2}+\frac{1+1+0.81+0.81}{4}\right)\times\frac{1}{8}(0.51+0.77+0.77+0.93+0.63+0.77+0.77+0.85)=1.21875$$

$RV\left(\tilde{\tilde{s}}_{12}\right) = (\frac{0.69+1}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.69 + 0.93 + 0.93 + 1 + 0.81 + 0.93 + 0.93 + 0.98) = 1.575$

$RV\left(\tilde{\tilde{s}}_{13}\right) = (\frac{0.69+0.98}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.69 + 0.93 + 0.93 + 0.98 + 0.81 + 0.93 + 0.93 + 0.97) = 1.53258$

$RV\left(\tilde{\tilde{s}}_{14}\right) = (\frac{0.63+0.97}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.63 + 0.87 + 0.87 + 0.97 + 0.74 + 0.87 + 0.87 + 0.92) = 1.43646$

$RV\left(\tilde{\tilde{s}}_{15}\right) = (\frac{0.69+0.97}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.69 + 0.9 + 0.9 + 0.97 + 0.79 + 0.9 + 0.9 + 0.93) = 1.51378$

$RV\left(\tilde{\tilde{s}}_{21}\right) = (\frac{0.44+0.90}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.44 + 0.72 + 0.72 + 0.9 + 0.57 + 0.72 + 0.72 + 0.81) = 1.1025$

$RV\left(\tilde{\tilde{s}}_{22}\right) = (\frac{0.35+0.5}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.35 + 0.6 + 0.6 + 0.5 + 0.47 + 0.6 + 0.6 + 0.7) = 0.73482$

$RV\left(\tilde{\tilde{s}}_{23}\right) = (\frac{0.55+0.9}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.55 + 0.81 + 0.81 + 0.9 + 0.67 + 0.81 + 0.81 + 0.89) = 1.27343$

$RV\left(\tilde{\tilde{s}}_{24}\right) = (\frac{0.29+0.77}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.29 + 0.53 + 0.53 + 0.77 + 0.40 + 0.53 + 0.53 + 0.65) = 0.75875$

$RV\left(\tilde{\tilde{s}}_{25}\right) = (\frac{0.69+1}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.69 + 0.93 + 0.93 + 1 + 0.81 + 0.93 + 0.93 + 0.97) = 1.57281$

$RV\left(\tilde{\tilde{s}}_{31}\right) = (\frac{0.43+0.93}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.43 + 0.71 + 0.71 + 0.93 + 0.56 + 0.71 + 0.71 + 0.82) = 1.10553$

$RV\left(\tilde{\tilde{s}}_{32}\right) = (\frac{0.52+0.97}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.52 + 0.8 + 0.8 + 0.97 + 0.66 + 0.8 + 0.8 + 0.88) = 1.28439$

$RV\left(\tilde{\tilde{s}}_{33}\right) = (\frac{0.42+0.87}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.42 + 0.68 + 0.68 + 0.87 + 0.54 + 0.68 + 0.68 + 0.77) = 1.3075$

$RV\left(\tilde{\tilde{s}}_{34}\right) = (\frac{0.43+0.93}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.43 + 0.71 + 0.71 + 0.93 + 0.56 + 0.71 + 0.71 + 0.82) = 1.10553$

$RV\left(\tilde{\tilde{s}}_{35}\right) = (\frac{0.57+0.97}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.57 + 0.83 + 0.83 + 0.97 + 0.7 + 0.83 + 0.83 + 0.9) = 1.35256$

$RV\left(\tilde{\tilde{s}}_{41}\right) = (\frac{0.69+1}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.69 + 0.93 + 0.93 + 1 + 0.81 + 0.93 + 0.93 + 0.97) = 1.57281$

$RV\left(\tilde{\tilde{s}}_{42}\right) = (\frac{0.28+0.77}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.28 + 0.53 + 0.53 + 0.77 + 0.39 + 0.53 + 0.53 + 0.54) = 0.73287$

$RV\left(\tilde{\tilde{s}}_{43}\right) = (\frac{0.44+0.9}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.44 + 0.72 + 0.72 + 0.9 + 0.57 + 0.72 + 0.72 + 0.81) = 1.1025$

$RV\left(\tilde{\tilde{s}}_{44}\right) = (\frac{0.46+0.9}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.46 + 0.74 + 0.74 + 0.9 + 0.59 + 0.74 + 0.74 + 0.82) = 1.13525$

$RV\left(\tilde{\tilde{s}}_{45}\right) = (\frac{0.42+0.87}{2} + \frac{1+1+0.81+0.81}{4}) \times \frac{1}{8}(0.42 + 0.68 + 0.68 + 0.87 + 0.54 + 0.68 + 0.68 + 0.77) = 1.03075$

Step 9: Based on Eq. (11), construct the average agreement degree AD, shown as follows:

$$AD = \begin{matrix} \text{Quality} \\ \text{Innovation} \\ \text{Commitment} \\ \text{Systematic} \end{matrix} \begin{bmatrix} 1.455 \\ 1.088 \\ 1.231 \\ 1.170 \end{bmatrix}$$

Step 10: Based on Eq. (12), we can get:

$$R(x_1) = \sum_{i=1}^{4} \frac{RV(\tilde{\tilde{s}}_{i1})}{AD_i} = \frac{RV\left(\tilde{\tilde{s}}_{11}\right)}{AD_1} + \frac{RV\left(\tilde{\tilde{s}}_{21}\right)}{AD_2} + \frac{RV\left(\tilde{\tilde{s}}_{31}\right)}{AD_3} + \frac{RV\left(\tilde{\tilde{s}}_{41}\right)}{AD_4}$$

$$= \frac{1.2875}{1.455} + \frac{1.1025}{1.088} + \frac{1.10553}{1.231} + \frac{1.57281}{1.170} = 4.14054$$

$$R(x_2) = \sum_{i=1}^{4} \frac{RV(\tilde{\tilde{s}}_{i1})}{AD_i} = \frac{RV(\tilde{\tilde{s}}_{12})}{AD_1} + \frac{RV(\tilde{\tilde{s}}_{22})}{AD_2} + \frac{RV(\tilde{\tilde{s}}_{32})}{AD_3} + \frac{RV(\tilde{\tilde{s}}_{42})}{AD_4} = 3.4276$$

$$R(x_3) = \sum_{i=1}^{4} \frac{RV(\tilde{\tilde{s}}_{i1})}{AD_i} = \frac{RV(\tilde{\tilde{s}}_{13})}{AD_1} + \frac{RV(\tilde{\tilde{s}}_{23})}{AD_2} + \frac{RV(\tilde{\tilde{s}}_{33})}{AD_3} + \frac{RV(\tilde{\tilde{s}}_{43})}{AD_4} = 4.22818$$

$$R(x_4) = \sum_{i=1}^{4} \frac{RV(\tilde{\tilde{s}}_{i1})}{AD_i} = \frac{RV(\tilde{\tilde{s}}_{14})}{AD_1} + \frac{RV(\tilde{\tilde{s}}_{24})}{AD_2} + \frac{RV(\tilde{\tilde{s}}_{34})}{AD_3} + \frac{RV(\tilde{\tilde{s}}_{44})}{AD_4} = 3.55299$$

$$R(x_5) = \sum_{i=1}^{4} \frac{RV(\tilde{\tilde{s}}_{i1})}{AD_i} = \frac{RV(\tilde{\tilde{s}}_{15})}{AD_1} + \frac{RV(\tilde{\tilde{s}}_{25})}{AD_2} + \frac{RV(\tilde{\tilde{s}}_{35})}{AD_3} + \frac{RV(\tilde{\tilde{s}}_{45})}{AD_4} = 4.70224$$

Because$R(x_5) < R(x_3) < R(x_1) < R(x_4) < R(x_2) \Rightarrow x_5 < x_3 < x_1 < x_4 < x_2$

5. Assessing the effectiveness of the proposed method

In this stage, the employees' satisfaction with the evaluation method was evaluated to assess the effectiveness of the proposed method.
Therefore, the satisfaction questionnaire was presented to all the evaluated employees, and the results presented in Table 6 were obtained.

Table 6. Measurement of staff satisfaction of company employees from the proposed method

Row	Indicator	Scoring (1-100)				
		x_1	x_2	x_3	x_4	x_5
1	Awareness of duties and responsibilities	80	80	85	85	90
2	Awareness of evaluation indicators	95	90	90	90	95
3	Awareness of the evaluation method	70	80	75	70	75
4	The fairness of the evaluation method	60	75	60	60	65
5	Acceptance of evaluation rewards	70	80	65	70	75
6	Trusting the evaluation results correctly	85	80	90	90	75
	Score (%)	76.66%	80.83%	77.5%	77.5%	80.83%
	Average satisfaction (%)			78.66%		

A comparison of the overall satisfaction levels listed in Tables 1 and Table 6 reveals that the employees' satisfaction with the proposed performance evaluation method is extremely higher than the previous method (the score increased from 32.33% to 78.66%). Hence, it is inferred and proven that the proposed method increases the effectiveness of the employee performance evaluation system in the company.

6. Conclusion

One of the most important duties in human resource management (HRM) is to design and establish employee performance evaluation systems. One of the most important concerns in an employee performance evaluation system is determining the performance evaluation indices. Since qualitative indices have a major share of these indices, judgmental methods are generally used for ranking them. Fuzzy logic methods are highly useful for resolving the ambiguities in these alternatives. In this paper, we proposed an employee performance evaluation method with type-2 fuzzy ranking approach. In our proposed method, a job ID is designed based on optimal models while an employee ranking method is developed and explained using the trapezoidal interval type-2 fuzzy ranking model. The weight and importance of the qualitative indices with type-2 fuzzy approach were determined and the linguistic terms with the type-2 fuzzy approach were used. In the end, the proposed method is utilized for the performance evaluation of employees in a real company. Using combination indices in type-2 fuzzy numbers ranking for proposing a new type-2 fuzzy numbers ranking method can be considered as an extension of this paper for future research.

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