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# A bi-objective model for the firefighter problem to maximize fire protection with minimum firefighters

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**Abstract**

The firefighter problem on a graph, depending on the environment, the graph can be continuous or discrete, which includes tree, cubic, regular and irregular graphs, etc., is described in such a way that by starting a fire from a series of vertices, the goal is to contain the fire with the maximum number of vertices saved. Our main innovation is to model the firefighter problem with on a bi- objective model, which simultaneously saves the maximum number of vertices with the minimum number of firefighters. The firefighter problem is a type of Np-hard problem, and because we defined the problem as a bi-objective problem and added three constraints to it, the problem became more difficult, and the weighted bi-objective model is also Np-hard. To solve the NP-hard problem, we used multi-objective optimization4 such as Goal Programming (GP), ε- Constraint, Global Criterion Approach, Weighting Sum Method methods. To prove the performance of our method, we used a randomly generated sample.

**Keywords**: firefighter problem; bi-objective; multi-objective optimization methods.

**Paper Type**: Original Research

## 1. Introduction

Fire is one of the oldest incidents that can endanger human and animal societies in a short period of time. In other words, a fire from a controlled or uncontrollable heat source spreads and spreads with heat or sparks at the moment of the accident. The firefighter problem (FFP) was first presented in 1995 by (Hartnell, 1995) as a deterministic discrete model of fire propagation in graph theoretical terms. One of the studies that has received attention recently is the firefighter problem. In the problem, it is assumed that the spread of fire is on a graph G= (V, E) with V as a vertex and E as an edge. Imagine that at zero hour, a fire occurs on an axis of the G graph. At the next time, the firefighter protects a "G" vertex, and then the fire spreads from each "burning" vertex to all its unprotected neighbors. When a vertex is burned or defended, it is so that the vertex remains burned or protected. This process ends when the fire can no longer spread (Ramos et al. 2019). Extensive research has been done to model spreading and containment phenomena such as diseases, rumors, ideas in social networks and viral marketing (Alvarez et.al 2016). For example, the Firefighter problem is extended to a probabilistic situation where infection is random and a simple policy is formulated that vaccinates only the neighbors of infected nodes (Tennenholtz, 2017). The problem of the firefighter problem can help the forest and human life in controlling fire. Due to the similarity of the spread and contagion of diseases such as influenza and Covid-19 to the firefighter problem, the firefighter problem can be extended to the spread of diseases as well. The weighted bi- objective firefighter model has two objectives. Optimization methods are used to solve dual-objective models. The optimization methods are: Weighted Sum Method, ε-Constraint, Global Criterion Approach, Goal Programming (GP). In large fires, we need to control the fire as quickly as possible to control the fire in sensitive and important areas. Preventing the spread of fire is not recommended at any cost and should be done using the minimum facilities and number of firefighters. For this purpose, we should divide the areas at risk, each of which has a fire station, into safe zones, and considering the minimum number of firemen for each area, we should use the firemen of other areas to spread the fire when necessary. For this purpose, we developed a bi-objective model to maximize the protected areas with the minimum number of firefighters.

## 2. Literature review

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The firefighter problem (FFP) was first presented in 1995 by (Hartnell, 1995) as a deterministic discrete model of fire propagation in graph theoretical terms. A computer approximation of the time-lapse firefighter problem is written by (Anshlevich et al., 2012), we consider two versions of the firefighter problem: a "non-broadcast" model, where the vaccination of a node only means no contamination of this node, and a "diffusion" model in which vaccination is a contagious process. We work with two methods: the first is the Maxsave measurement, in which we want to maximize the number of nodes that are not infected using a fixed budget, and the second is the Min-budget measurement, the purpose of which is to determine a set of nodes that must be protected so that the cost, be minimized. (Bloom et al., 2012), have presented the first metaheuristic to solve this problem. In particular, an ant colony optimization method and a hybrid type of this algorithm are presented. The article, automatic adaptation of genetic operators for multi-objective optimization in the firefighter problem has been worked by Mr. Mishlak. In the article (Ho et al., 2015), with the title of presenting a new solution for the firefighter problem, it is stated that we present a new solution for FFP and it can be used in metaheuristic approaches. In the article, the firefighter problem (Martinez et al., 2015), a deterministic discrete model for spreading and containing fire in graphs is stated. (Tennenholtz et al., 2017) have presented the possible firefighter problem and have stated that the dynamics of the spread of infectious diseases is very important in determining their risk and providing ways to control them. Successive vaccination of people in networks has been studied and the Firefighter problem is the probabilistic situation is extended, where the infection is random and a simple policy is formulated that only vaccinates the neighbors of the infected nodes, and this method is optimal for a regular tree graph and general graphs if there is enough budget. In this study, methods for calculating the upper and lower limits of the expected number of infected people are obtained, as well as the estimation of the budget required for containment, at the expected level. A new intersection operator, in the paper Intersection operator using knowledge transfer for the firefighter problem (Michalak, 2018), is presented, which uses a machine learning model to decide how to combine two FFP parent solutions with children. One of the combined optimization approaches is to use global search methods such as evolutionary algorithms together with local search methods. In the paper (Michalak, 2017) a new crossover operator, SimX is presented, which determines how to combine information from parent samples using the simulation of fire propagation and some problem-specific metaheuristics. In the paper of Mr. Ramos, a computational study of the firefighter problem (Ramos, 2019) (FFP) is carried out, and the main innovation of Mr. Ramos includes improvements to the existing integer linear programming formula, leading to twice the average speed for Exact solutions are calculated. In addition, an innovative Matheuristic method, which is a technique based on the interaction between metaheuristics and mathematical programming, has been prepared. One of the important dimensions of the firefighter problem is the complexity of solving the model, and the complexity of solving the firefighter problem depends more on the degree of the graph than any other factor. In the article, the firefighter problem for tree graphs with maximum degree three, (Finbow et al., 2007), it is shown that the fireman problem for trees with maximum degree three is NP-complete, but for graphs with maximum degree three, if a fire occurs in vertices with maximum degree two, the problem is solved in polynomial time and the problem is of type P. Also, Mr. Duffy says in his paper, for the fireman process with a weight on the vertices, we show that the problem of deciding whether a subset of the total weight of the binary vertices, if restricted to tree graphs, can be saved from burning is NP-complete (Duffy, 2010) and (Bazgan, 2012) investigated the complexity of general versions of the fireman problem in graphs and answered several open questions from Finbow and (2009) MacGillivray (Bazgan, 2012). The firefighter problem in different classes of graphs is surprisingly hard stated by Fomin, even if the input graph is a bipolar graph or a tree of maximum degree 3, it is still an NP-complete problem (Famin et al., 2014). In special and emergency situations, it is necessary to form an emergency group to prevent the fire from spreading to other places, and we need to protect the maximum areas with the minimum firefighter force. The literature review shows that how to form an emergency team to maximize the number of protected points with the minimum number of firefighters has not been studied. We are going to develop the firefighter problem model as a bi-objective of a maximum number of defended nodes with a minimum number of firefighters. The text organization in section 3 the firefighter problem is defined. In the following, tow-objective weighted firefighter model in part 4, Solution methods are presented in Part 5, Section 6 also shows the Computational results and sensitivity analyses.

## 3. Problem definitions

The fire problem occurs in an undirected graph G = (V; E), where it initially reoccurs at the f nodes. At each subsequent time step, two actions are performed: a certain number b of re nodes are placed on top of non-refueling nodes, permanently protecting them from re nodes. Then re is expanded to all non-defensive neighbors of vertices in re. Since the graph is finite, at some point every vertex is either stored in re or state. The process then terminates, as it cannot again expand any further. There are several different targets for the problem. Typically, the goal is to store the maximum possible number of nodes. Other objectives include minimizing the number of iterations (or time steps) until the expansion stops, or determining whether it is possible to avoid burning all the vertices in a given set (Alvarez et.al 2016).

The weighted firefighter model with a specific number of firefighters is as follows.

The notations used in the model are as follows:

1. N(v) denotes neighborhood of a vertex v in G
2. bv,t, ∀v ∈ V and $0 \leq t \leq T$, which indicates whether a vertex v is burned at the end of iteration t
3. dv,t, ∀v ∈ V and $0 \leq t \leq T$ indicates whether a vertex v is defended.
4. B indicates the set of burned nodes.
5. d indicates the total number of firefighters available.$A = \pi r^2$

| | | |
|---|---|---|
| $\max \quad \lvert V \rvert - \sum_{v \in V} v \quad b_{v,T}$ | | (1) |
| $b_{v,t} + d_{v,t} - b_{v',t-1} - 1 \geq 0$ | $\forall\, v \in V, v \in N(V)$ and $1 \leq t \leq T$ | (2) |
| $b_{v,t} + d_{v,t} \leq 1$ | $\forall\, v \in V$ and $1 \leq t \leq T$ | (3) |
| $b_{v,t} - b_{v,t-1} \geq 0$ | $\forall\, v \in V$ and $1 \leq t \leq T$ | (4) |
| $d_{v,t} - d_{v,t-1} \geq 0$ | $\forall\, v \in V$ and $1 \leq t \leq T$ | (5) |
| $\sum_{v \in V} d_{v,t} - d_{v,t-1} \geq 0$ | $1 \leq t \leq T$ | (6) |
| $b_{v,0} = 1$ | $\forall v \in B$ | (7) |
| $b_{v,0} = 0$ | $\forall v \in V \backslash B$ | (8) |
| $d_{v,0} = 0$ | $\forall v \in B$ | (9) |
| $b_{v,t}, d_{v,t} \in \{0,1\}$ | $\forall\, v \in V$ and $1 \leq t \leq T$ | (10) |

In this model, the vertices are discrete and the number of repetitions above T is necessary to control the fire and the defense is staged and T represents the stage of defense. The objective of the problem (z) is to maximize the weight of saved vertices (dv,t) (wv is the weight of vertices). Our innovation is in the objective function of maximizing the total weight of saved vertices, while in the initial model for maximizing the objective function, the number of saved vertices is maximized. The first limitation of the model (2) follows that if the neighboring vertices of vertices are ignited, these vertices can be considered among the ignited vertices (bv,t)) in each iteration (T) or protected vertices (dv,t). That is, all the neighbors of (N(v)) burning node V bv, t-1=1)) in the period t-1 if they are defended, except for the defended ones (d-v, t=1), otherwise, they are among the burnt nodes (bv , t=1) will be. (Constraint (3) prevents a vertex from being considered ignited and protected at the same time. Constraints (4) and (5) are to ensure that a vertex remains ignited or protected until the end of the procedure. Limitation (6) states that the sum of the maximum number of firefighters to protect fv vertices should be equal to the number of firefighters (the number of firefighters to protect each vertex). Our second innovation is given in this constraint (6). In the initial model, the number of the number of firefighters to protect each vertex is fixed and the number is one. In this constraint, the number of firefighters is considered different for each vertex and is multiplied by the binary vertex. Constraint (7) indicates that the vertices of the ignited family B must be considered ignited. Constraints (8) and (9) indicate that other vertices should not be considered burned or defended in the first iteration, and constraint (10) specifies the type of variables.
For example, in figure no. 1, the fire starts from node number one in the first stage (Figure 1). And in the second stage (Figure 2), two nodes number 2 and 3 are exposed to fire. To maximize the number of protected nodes, a firefighter uses node number two to defend and NODE number 3 burns. In the third stage (Figure 3), node number 6 is exposed to fire and fireman defends it. Finally, nodes 2, 5, 6 save and nodes 1, 3, and 6 burn.

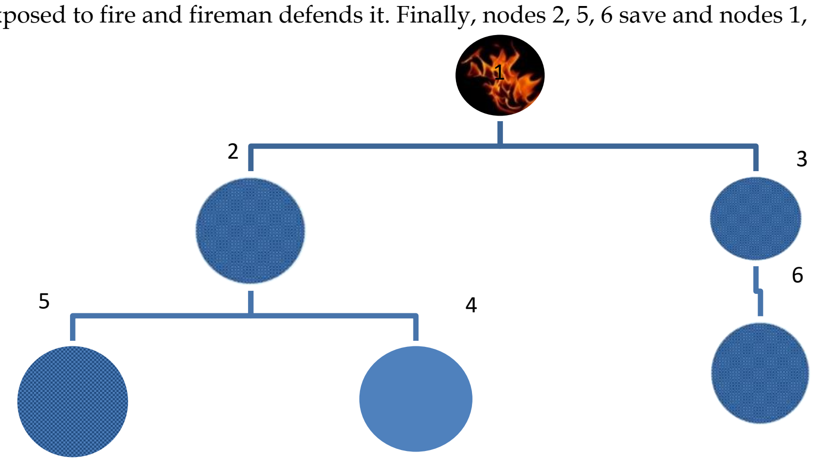


**Figure 1.** First stage

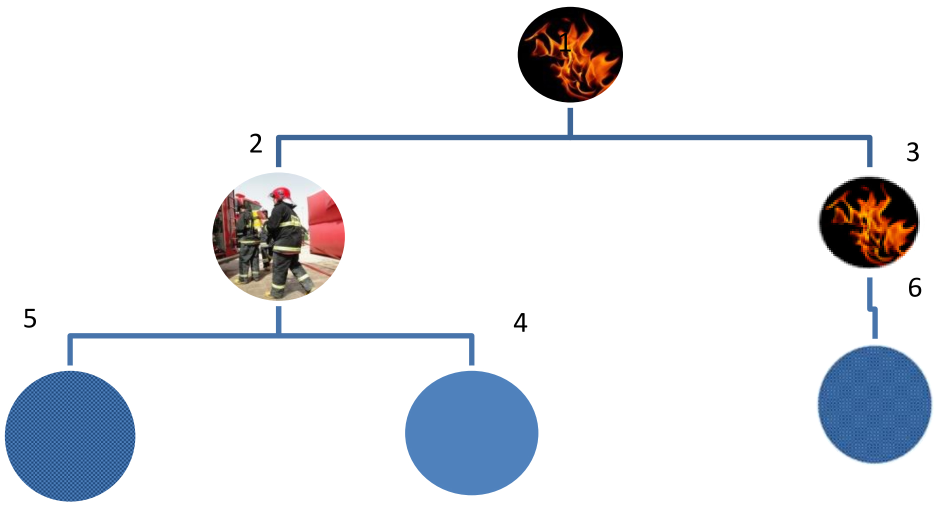


**Figure 2**. Second stage

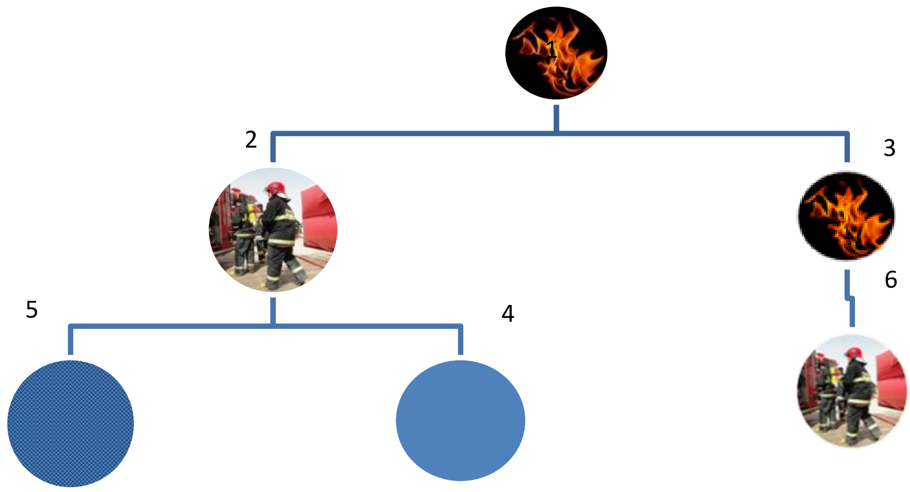


**Figure 3**. Third stage

## 4. Bi-objective model for firefighter problem

In large fires, we need to control the fire as quickly as possible to control the fire in sensitive and important areas. If the regional fire department is not able to control the fire in a short time, then considering the sensitivity of the issue, we need to control the fire for a short time and form an emergency team to prevent the spread of the fire. We changed the model of the firefighter problem to the maximum defended areas with the minimum number of firemen in order to recognize and form a group of aggressors. In this model, the objective function is defined in such a way that maximum areas can be defended against fire with the minimum number of firefighters. Limit number 6 specifies emergency situations. Limitation number 7 controls the minimum number of emergency teams and limit 7 controls the use of emergency teams and available firefighters.
In addition to the previously defined symbols and variables, the following symbols and variables are also defined in our model.
nnkk: The matrix parameter of the number of firefighters who can participate in the emergency group of each station
xxkk,1: Binary variable for participation in each station's emergency group

y: Binary variable of emergency group formation
wv: The weight parameter of each vertex
fff: Variable number of emergency groups

$$maX \quad \sum w_v \quad -(\sum w_v . b_{v,t} - \sum w_v . b_{v,t-1}) \tag{1}$$

$$min \quad \sum_{kk} nn_{kk} xx_{kk,1}$$

$$\text{s.t} \quad b_{v,t} + d_{v,t} - b_{v',t-1} \geq 0 \quad \forall v \in V,\ v' \in N(v) \text{ and } 1 \leq t \leq T \tag{2}$$

$$b_{v,t} + d_{v,t} \leq 1 \quad \forall v \in V \text{ and } 1 \leq t \leq T \tag{3}$$

$$b_{v,t} - b_{v,t-1} \geq 0 \quad \forall v \in V \text{ and } 1 \leq t \leq T \tag{4}$$

$$d_{v,t} - d_{v,t-1} \geq 0 \quad \forall v \in V \text{ and} 1 \leq t \leq T \tag{5}$$

$$Y=1 \exists \ \sum w_v \quad \geq W \quad v \in V \tag{6}$$

$$fff \geq \sum_{kk} nn_{kk} \ .xx_{kk} \quad \forall kk,\ \forall v \in V,\ v' \in N(v) \text{ and } t=1 \tag{7}$$

$$\sum_{v \in V} (f_{v..}d_{v,t} - f_{v}.d_{v,t-1}) \leq y_{..} fff + (1-xx_{kk})(nn_{kk}) \quad \forall v \in V_t,\ \forall kk \in nn \text{ and } 1 \leq t \leq T \tag{8}$$

$$b_{v,0}=1 \forall v \in B \tag{9}$$

$$b_{v,0}=0 \forall v \in V \backslash B \tag{10}$$

$$d_{v,0}=0 \quad \forall v \in V \tag{11}$$

$$xx_{v,1,} y, b_{v,t}, d_{v,t} \in \{0,1\}, W, nn \in n \ \forall v \in V \text{ and } 1 \leq t \leq T \tag{12}$$

## 5. Solution methods

The weighted two- objective firefighter model has two purposes. Optimization methods are used to solve dual-objective models. The category of optimization is as follows: static / dynamic, deterministic / stochastic, constrained / unconstrained, linear / nonlinear, global / local, single-objective / multi-objective. Most realistic optimization problems are usually characterized by MO attributes, i.e . they require the simultaneous optimization of more than one objective function, and it is practically important to investigate multi objective (MO) optimization problems. Often, in this type of optimization, one or more of the objectives are treated as constraints. In (MO) optimization, the goal is to find as many different Pareto-optimal solutions as possible. At present, there are many useful methods to solve these kinds of problems. Classical (or deterministic, or multi-criterion optimization problems related) methods include the weighting sum approach, e-constraint, global criterion method, goal programming, and the complex method. Meanwhile, a number of intelligent methods, which are sometimes called stochastic MO, have emerged in recent years. They are MO genetic algorithms (GAs), MO simulated annealing (SA), MO fuzzy logical approach, etc. Normal optimization is that it finds a solution in the feasible region, which has the minimum (or maximum) value of the object function. In the present published papers about MO, some scholars just discuss the classical methods, such as Ref. and some overview the methods in a specific field in detail but come out the unnecessary repetition, for instance in Ref. Special topics are particularly emphasized in a large number of papers. Classical methods are described in detail in some papers and just give the objective function curtly from the system point of view. Thereinafter, several techniques used for the solution of MO optimization problems are briefly described so that readers can get Therefore the general understanding of them.

### 5.1. Weighting sum method

In this method, each objective is assigned a scalar weight that signifies its relative importance to other objectives. The optimization problem is then turned into optimizing the weighted sum of different objective functions. The MO optimization problem is transformed into a single objective (SO) problem by using a weighted sum of the original multiple objectives as follows .equation (1)

$min \sum w_i f_i$ (1)

where Wi are the weighting coefficients satisfying the following conditions

$\cdot <$ wj<l and $\sum w_i = 1$

If the problem is convex, a complete set of Pareto solutions can be obtained by varying the weighting coefficients .

### 5.2. ε- Constraint

This method is also known as trade-off method ,which means that the decision-maker specifies a trade-off among the multiple objectives. In the ε-constrained method one of the objectives is optimized while the others are treated as constraints. This idea
is express in Eqs. (2) and (3).

min fr(x) (2)

s.t. , fr(x) (χ )<ε „ i = 1,..,N ; i≠ r (3)

where ε, is the limiting value of desired by the decision-maker .

### 5.3. Global criterion approach

The decision-maker uses an approximation solution
f *to formulate a single objective criterion to determine the optimum decision variables by solving the flollwing SO optimization problemEq(4)

$min \sum_{i=1}^{N} (\frac{f^* - f_i(x)}{f_i^*})^P$ (4)

Usually decision-maker sets the value of P equal to 1 or 2

### 5.4. Goal programming (GP)

This approach is accomplished by using deviational variables. The goals are assigned some priority or weighting to show their importance relative to others. The constraints vary within close range of the desired values instead of being strict. The goal criterion can be one of the followings: greater than or equal to, less than or equal to, equal to, or range. The form of objective function can also be seen in the Ref. When decision maker (DM) uses priorities instead of weights to order goals, this problem is known as the lexicographic GP approach .

### 5.5. The complex method

The complex method has been applied to a wide range of problems, which is constraint simplex method developed from the simplex method .The main idea of this algorithm is to replace the worst point by a new and better point. In this method, a complex consisting of several possible problem solutions is manipulated. Each set of parameters represents one single point in the solution space the new point is calculated as the reflection of the worst point. By varying the reflection distance, it is possible for the complex to expand and contract depending on the topology of the objective function. The starting points are generated randomly and it is checked that both the implicit and the explicit constraints are fulfilled. The optimal solution is found when all points in the complex have converged.

### 5.6. MO simulated annealing

Simulated annealing (SA) simulates the natural phenomena of annealing of solids in order to optimize complex systems. Heating up a solid and allowing it to cold down slowly so that thermal equilibrium is maintained to accomplish annealing of solids. In other words, this process consists of two steps. One is to increase the temperature of the heating bath to a maximum value at which the solid melt. The other is to decrease carefully the temperature of the heating bath until the particles arranging themselves in the ground state of the solid. Simulated annealing tries to mimic this process and therefore gets its name. This ensures the atoms possessing a minimum energy state. The algorithm starts with an initial design .New designs are then randomly generated in the neighborhood of the current design according to some algorithm (Xiujuan and Zhongke ,2002).

### 5.7. MO genetic algorithms

The essential idea of genetic algorithms (GAs) is the mechanics of natural selection. These algorithms draw their inspiration from various hypotheses of biological evolution. The hypotheses have proposed that species evolve through a process of survival of the fittest. A population of a species is created. The members of which are allowed to reproduce and recombine to produce new offspring. The fittest offspring are then selected to go on to the next stage, i .e. recombining and producing new offspring (or new solutions .(The ardor of genetic algorithms has grown tremendously in recent years. There is an abundance of different types of genetic algorithms, for example , introduces briefly of multi-objective optimization which include NPGA, VEGA, NSGA, SPEA and NSTEA.

## 6. Computational results and sensitivity analyses

Because the model of Ramos and his colleagues is NP-hard and we developed the firefighter model with two objectives and also added some restrictions to it. So, the dual-purpose model is also NP-hard.
To prove the performance of the two-objective model, we generated 50 random samples and solved the model using Games programming .
To solve the problem, a computer with the processor specifications of AMD A9-9425 RADEON R5, 5 COMPUTE CORES 2C + 3G 3.10GHZ and 8GB of RAM (7/45 GB usable) was used.

### 6.1. To show that the model has two objectives and the two objective functions move in opposite directions

Multi-objective optimization is a field of "Multi-Criteria Decision Making". Multi-objective optimization deals with mathematical optimization problems in which more than one objective function needs to be optimized simultaneously. Multi-objective optimization with other names such as "Multi-Objective Programming", Vector Optimization, Multi-Criteria Optimization, Multi-Attribute Optimization or "Pareto Optimization" is also known.
Multi-objective optimization methods are used in many branches of science and engineering and are used when a trade-off needs to be established between two or more conflicting objectives in order to reach optimal decisions in the system. Undoubtedly, in many engineering applications, designers of engineering processes and systems make decisions based on conflicting goals. For example, in the process of designing a car, in addition to the fact that engineers aim to design a car that has maximum performance, at the same time, they seek to design a car that has the lowest number of emissions and fuel consumption.
In this and similar cases, since more than one objective function must be considered, it is necessary to consider the application of multi-objective optimization methods. The most important feature of such methods is that by using multi-objective optimization models, more than one candidate solution (one possible solution for the desired problem) is available to system designers and engineers. Each of these answers will show the balance between different objective functions.

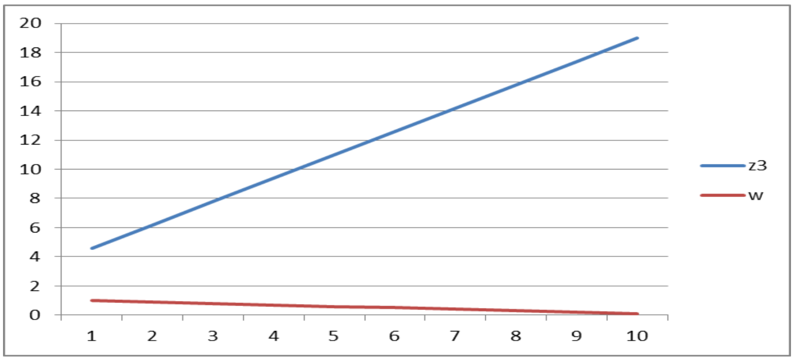


**Figure 1.** First and second objective function diagram

In Figure (4), the second objective function tries to reduce the number of firefighters. So, the desired function of this function is to reduce the number of firefighters, but with the reduction of the number of firefighters, the first objective function, which tries to increase the number of defended points, increasing the weight of defended points, will decrease. The graph shows that the two objective functions move in opposite directions. So, the result is that this model is a two- objective function model.

### 6.2. Determining the number of firefighters to fight the fire at the starting point

Sometimes, due to the sensitivity and importance of the vertices, it is necessary to extinguish the fire at the same starting point. Considering that we have different stations in each area and these areas themselves have the potential to start a fire. It is necessary to maintain the minimum number of firefighters in the same areas, and the emergency team should be formed in such a way that a minimum number of firefighters remain in the areas and the same emergency team can put out the fire at the same point where the fire started. For this purpose, the weight of each of the objective functions is determined so that the solution method can determine the number of firefighters needed to fight the fire at the starting point. The Table 1 shows the number of firefighters required for a graph with different numbers of nodes.

**Table 1**. The number of firefighters required for a graph with different numbers of nodes

| no | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| saved | 19 | 29 | 49 | 39 | 49 | 599 | 699 | 799 | 899 | 999 |
| Number of nodes | 20 | 30 | 50 | 40 | 50 | 600 | 700 | 800 | 900 | 1000 |

### 6.3. In limitation 7, it is shown that a special station for each area and each area is defended with a specific and unique number.

To determine the number of firefighters needed for each station, the number of stations and the importance of stations are taken into consideration. By running the number one model several times and the opinion of experts, the desired number of each station is obtained (Figure 5).

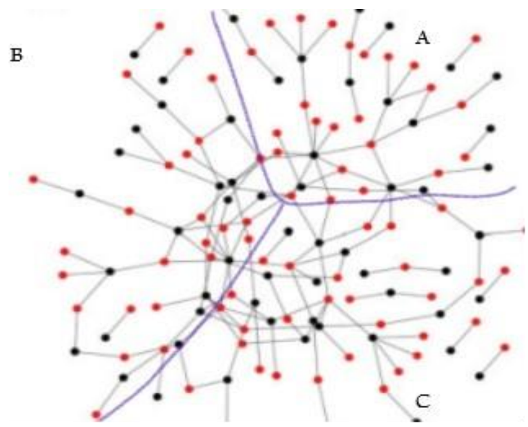


**Figure 2.** The desired number of each station

### 6.4. Results of optimization methods

In (MO) optimization, the goal is to find as many different Pareto-optimal solutions as possible. At present, there are many useful methods to solve these kinds of problems. Classical (or deterministic, or multi-criterion optimization problems related) methods include the weighting sum approach, e-constraint, global criterion method, goal programming, and the complex method.

Weighted sum methods, goal programming, epsilon constraint method and the penalty method are designed to solve multi-objective problems, but according to the conditions of the problem and its limitations, one of the mentioned methods will be suitable for solving the problem. In order to reach the appropriate method, the problem was run with weighted sum methods, the ideal method, epsilon constraint method and the penalty method in gams program. The results are shown in tables (2, 3, 4).

Tables 2, 3 and 4 show that the weighting sum programming provides a better answer than the goal programming method. To solve the firefighter's two-objective model, we used the weighted summation program method and the results of Table 2 were obtained. During the 10 steps of running the method with weighting from 1 to zero, each objective function is observed. If we consider the importance of the defended vertices more, the number of emergency groups will increase as a result of the increase in the weight of the vertices and the first objective function will gain more weight. And if we increase the weight of the second objective function, the number of emergency groups will decrease and the weight of the defended vertices will also decrease. The result is that according to the importance of defending the vertices or at least using the fire brigade, the weight of the functions will be changed to get the best answer, and the Delphi method will be used by experts to determine the weight of the functions. In Table No. 2, with the increase in the weight of the second function, which aims to reduce the number of firefighters, the number of firefighters decreases from 13 to 11, and after that, the weight of the first objective function decreases from 780 to 746.

**Table 1.** Weighting sum programming

| No | Variations of the first objective function (z1) | Variation of the second objective function (z2) | Variations of the third objective function(z3) | Variations in the weight of the first objective function (w1) | Variations in the weight of the first objective function (w2) | The optimal number of firemen of the emergency group (fff) |
|---|---|---|---|---|---|---|
| 1 | 780 | 20 | 780 | 1 | 13 | |
| 2 | 780 | 13 | 700.7 | 0.9 | 0.1 | 13 |
| 3 | 780 | 13 | 621.4 | 0.8 | 0.2 | 13 |
| 4 | 780 | 13 | 542.1 | 0.7 | 0.3 | 13 |
| 5 | 780 | 13 | 462.8 | 0.6 | 0.4 | 13 |
| 6 | 780 | 13 | 383.5 | 0.5 | 0.5 | 13 |
| 7 | 778 | 11 | 304.6 | 0.4 | 0.6 | 11 |
| 8 | 778 | 11 | 225.7 | 0.3 | 0.7 | 11 |
| 9 | 746 | 149.2 | 0.2 | 0.8 | | |
| 10 | 746 | 74.6 | 0.1 | 0.9 | -1 | |
| 11 | 1.39E-16 | 1 | -1 | | | |

In the second stage, the firefighter problem with two functions was solved using the goal programming method and the weighted sum of goal programming, and its results are given in Table No. 3 and TableNo.4. The results show that, the goal programming method and weighted sum of goal programming, two methods are not able to determine the emergency group and the problem cannot be solved by these two methods.

**Table 2**. Goal programming

| No | Variations of the first objective function (z1) | Variations of the second objective function (z2) | Variations of the third objective function(z3) | Variations in the weight of the first objective function (w1) | Variations in the weight of the first objective function (w2) | The optimal number of firemen of the emergency group (fff) |
|---|---|---|---|---|---|---|
| 1 | | 17 | 1 | | | |
| 2 | | 7 | 16 | 0.9 | 0.1 | |
| 3 | | 7 | 15 | 0.8 | 0.2 | |
| 4 | | 7 | 14 | 0.7 | 0.3 | |
| 5 | | 7 | 13 | 0.6 | 0.4 | |
| 6 | | 7 | 12 | 0.5 | 0.5 | |
| 7 | | 7 | 11 | 0.4 | 0.6 | |
| 8 | | 7 | 10 | 0.3 | 0.7 | |
| 9 | | 7 | 9 | 0.2 | 0.8 | |
| 10 | | 7 | 8 | 0.1 | 0.9 | |
| 11 | | 7 | 7 | 1.39E-16 | 1 | |

**Table 3**. Weighted sum of goal planning

| No | z2 | z3 | w1 | w2 | fff |
|---|---|---|---|---|---|
| 1 | 49 | 1 | -1 | | |
| 2 | 8 | 44.9 | 0.9 | 0.1 | |
| 3 | 8 | 40.8 | 0.8 | 0.2 | |
| 4 | 8 | 36.7 | 0.7 | 0.3 | |

| | | | | |
|---|---|---|---|---|
| 5 | 8 | 32.6 | 0.6 | 0.4 |
| 6 | 8 | 28.5 | 0.5 | 0.5 |
| 7 | 8 | 24.4 | 0.4 | 0.6 |
| 8 | 8 | 20.3 | 0.3 | 0.7 |
| 9 | 8 | 16.2 | 0.2 | 0.8 |
| 10 | 8 | 12.1 | 0.1 | 0.9 |
| 11 | 8 | 8 | 1.39E-16 | 1 |

## 7. Conclusion

The firefighter problem was presented by Mr. Hartnell 1995. Then, during those years, it was developed and used for issues such as the spread of diseases, rumors and the spread of computer viruses. Depending on the type of release, different graphs were used for the release method. Other researchers developed it into multi-objective models. Depending on the problem-solving space, the problem was solved in continuous and discrete space. To solve the problem, heuristic and metaheuristic methods were used or new methods were invented and presented. In this research, we developed the problem into a bi-objective model for firefighter problem of maximum rescue with minimum firefighters. In this model, the weight and importance of each vertex is different and each vertex is defended by a different number of firefighters. Also, in this model, the second objective function was added in order to minimize the number of firefighters. Also, some restrictions were added to announce the state of emergency, determine the minimum number of emergency teams and control the use of existing emergency teams and firefighters. To solve the problem, optimization methods such as weighted sum, goal programming, goal sum programming, epsilon limit and metaheuristic methods were considered and researched. To solve the problem, the model was run many times in gams program and the output was analyzed in the Excel program. The results showed that exact solution methods can be used in cases where the number of vertices is not too large. By investigating and solving the problem by the weighted sum programming and with goal programming, we came to the conclusion that the goal programming method is not suitable for solving the problem and the weighted sum programming method provides a better and suitable answer and can defend more vertices by keeping the number of firefighters to a minimum. It is suggested that an application program be written for the weighted two-function model and an application can be prepared for all cities and forests by specifying the importance of each point of the program.

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