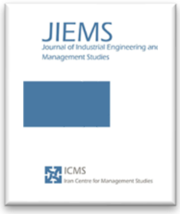
Vol. 10, No. 2, 2024, Pages 90-105

JIEMS

Journal of Industrial Engineering and Management Studies

journal homepage: www.jiems.icms.ac.ir



Two-level warranty contract between service agent and consumer by considering Stackelberg equilibrium (Case study : A truck after-sales service agent in Iran)

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Received: Jun 2023-25/ Revised: Nov 2023-06/ Accepted: Dec 2023-06

Abstract

Warranty and maintenance contracts play an important role in product life cycle. Different failures which occur during useful life cycle of products in additional reducing reliability, make expense for consumer and service agent. This study considers warranty periods and maintenance services costs from service agent and consumer viewpoints under two-dimensional warranty policy. By regarding agent service and consumer decisions, the interactions between them are modeled during the base warranty and extended warranty periods. Maintenance policies are performed as preventive maintenance (PM) in specific interval with fixed level, and corrective maintenance (CM) is carried out as home and road repairs. Finally, for making equilibrium between profits of consumers with different usage rates and agent services, preventive maintenance number and warranty services price are investigated by the Stackelberg equilibrium. A real case study from a truck after sales services agent of Iran is presented to illustrate the application of the proposed model.

Keywords: Two-Dimensional Warranty; Base Warranty; Extended Warranty; Services Agent; Stackelberg equilibrium

Paper Type: Original Research

1. Introduction

Selling products with warranty services is a way to make insurance among consumers and increase manufacturers' profits. A warranty is a contractual agreement provided by the seller for carrying out corrective actions and resolving any failure (the system's inability to act as expected or break down its pieces) for consumers in a warranty coverage. Today, due to the development of a worldwide market, especially in industries such as the automotive industry, consumers prefer to get warranty services from service providers such as agents, who make all warranty services instead of manufacturers. Warranties are divided into two groups: base warranty (BW) and extended warranty (EW). Base warranty is simply called warranty. Warranty is based on the contractual agreement between the manufacturer (service agent) and the buyer (consumer) that is entered into the product sale. This warranty is part of the sale, and its cost is usually included in the sale price of the product. The extended warranty is optional and can be purchased separately. EW terms can be similar to the BW provided by the manufacturer. A large number of products are sold with long-term warranty policies which are in the form of a lifetime warranty, extended warranty, second-hand product warranty, and maintenance contracts. Manufacturers or service agents consider a few key variables when providing a warranty. These variables include the type of warranty, how to repair the damage, how to provide the warranty service (in terms of cost), the warranty periods (BW, EW), the warranty dimensions, and so on. The dimension of warranty policy means the number of variables to determine warranty limits. In a one-dimensional warranty, a policy is described by a single variable, age or usage. For example, preventive maintenance is provided after every 2000 km of vehicle operation or in six-month age cycles. In two-dimensional (2D) warranties, the warranty is determined by a two-dimensional area in which, usually one axis indicates the product age and the other indicates the amount of usage. Typically, maintenance strategies can be divided into two categories: corrective maintenance and preventive maintenance. In corrective maintenance, repairs in terms of improving the failure rate of the failed items are divided into three categories: perfect, imperfect, and minimal. In the first case, the repair restores the failed item's operating condition as good as new: In minimal repair,

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the failure rate after the repair is equal to its before failure. In imperfect repair, the failure rate after the repair is somewhere between its as good as new and as bad as old states, while preventive maintenance is performed as planned before failure. The role of warranty varies from customer to manufacturer. For the customer, the warranty helps to reduce product failures and increase product quality and reliability. From a manufacturer's viewpoint, it helps to sell more in a competitive environment and reduce customer complaints. In recent years, many manufacturers have outsourced maintenance in warranty service, in other words, sales service agents are responsible for the warranty sales and sales back services. Many studies and articles have defined warranty policies from customer, manufacturer, or after-sales service agent viewpoints. Several articles have also optimized the decisions of each of the above parties simultaneously. They determined warranty policies from the manufacturer-customer, customer-service agent, manufacturer-service agent, or ultimately from the customer-manufacturer and service agent viewpoints. In the following, the reporter literature which has studied warranty scope are reviewed by considering maintenance costs and warranty term from the manufacturer, service agent, or consumer viewpoints. Rinsaka and Sandoh modelled the profits of the manufacturer and consumer based on the maintenance costs and obtained warranty prices from the manufacturer and consumer viewpoints. Manna et al. estimated warranty servicing costs for repair products sold with a two-dimensional warranty which is practical for customers, dealers, and producers. Su and Shen proposed two types of extended warranties: one-dimensional extended warranties and two-dimensional extended warranties with different maintenance strategies and found warranty costs from the manufacturer's viewpoint. According to Esmaeili et al. Interactions between manufacturers, agent service, and consumers which contain warranty price and maintenance services were modelled (in three levels) by the game theory approach in a specific period of the product. As a result, the equilibrium warranty price and cost of maintenance were estimated. Jung et al examined the optimal warranty period in two phases from the consumer's viewpoint. In the first phase it was assumed that if the product failed, both minimal maintenance and replacement would be carried out, and in the second phase, only minimal maintenance would be applied. By minimizing costs, they found the optimal maintenance period after the expiry of the extended warranty period. Hamidi et al considered two-dimensional warranty policy from lease and lessor-viewpoints. Interactions between the equipment lease and lessor were modeled by the game theory approach in two modes of cooperation and non-cooperation. Finally, they estimated equilibrium rental time and usage rates. Chen and Weng found optimal warranty price and warranty period from the manufacturer's viewpoint. Maintenance policies involved replacement and CM. At the suggested model all of the costs were paid by the manufacturer. Wang et al modeled interactions between producer and consumer simultaneously with the game theory approach. In this research, they estimated the optimal profit and costs, the number of optimal maintenances, and the level of maintenance. Alqahati and Ammar determined warranty costs for second-hand products under different usage rates from both manufacturer and consumer. Salmasniya and Yazdankhasti modeled manufacturer costs and found the optimal PMs number in the warranty period, so for increasing consumer satisfaction, made an equilibrium between the manufacturer and consumer decisions. Giri et al. Determined the equilibrium warranty period and warranty price in a specific period. This study was carried out in a closed-loop supply chain with just one manufacturer and one consumer. They explored the optimal timing for a manufacturer to bargain a wholesale price with a retailer in a dual-channel supply chain environment. He et al investigated the win-win EW price in a model by taking the purchasing time of EW. This model is beneficial for both customers and manufacturers under different PM policies and usage rates. Zhang et al by considering stable and dynamic markets obtained optimal warranty-reliability-price. Total expected profits over the product’s life cycle are maximized under two-dimensional a warranty policy with heterogeneous usage rates. Hashemi et al. Considered optimal maintenance costs from both manufacturer viewpoints in warranty periods and also, customer viewpoint when the warranty expired, as a result, they estimated optimal PM cost. Khorshidvand et al presented a hybrid modeling approach for a Closed-loop supply chain network and found optimal levels of pricing, greening, and advertising and then maximizing profit and minimizing CO2 emission under uncertain demand. Li et al proposed three types of warranty policies-free replacement, a full refund, and a partial refund, that take into account the random failure threshold based on the degradation model and find the optimal price and warranty period. Khorshidvand et al proposed a multi-level, dual-channel green closed-loop supply chain that integrates a circular economy and determines optimal equilibrium prices, greening levels, and advertising strategies for manufacturers and consumers. Khorshidvand et al presented a two-stage model for a sustainable closed-loop supply chain with multi-objective mixed-integer linear programming then maximizing the profit of the whole chain, minimizing the environmental impacts due to CO2 emissions, and maximizing employee safety for finding optimal product price. Khorshidvand et al determined the optimal price based on: advertising costs, greening costs, and other members' pricing decisions and considered three models including a centralized supply chain, decentralized supply chain, and modified centralized supply chain from producer, consumer, and distributer decisions.

Table 1. shows the summary of past studies.

	Warranty periods			Maintenance strategy		Viewpoints			Decision variables
	base	extended	Life of product	CM	PM	manufacturer	consumer	agent	
Rinsaka and Sandoh (2006)			*		*	*	*		PM number
Monna et al. (2007)	*				*	*	*	*	servicing cost
Su and Shen (2012)		*		*	*	*			Extended warranty cost
Esmaeili et al. (2014)	*			*		*	*	*	price/cost warranty
Jung et al. (2015)			*		*	*	*		warranty/maintenance period
Hamidi et al. (2016)	*		*		*	*			usage rate/maintenance level
Chen and Weng (2017)	*			*	*	*			warranty price/warranty period
Wang et al. (2017)	*	*			*	*	*		expected cost
Alqahati and Ammar (2017)			*	*		*	*		PM number
Salmasniya and Yazdankhasti (2017)	*			*	*	*			PM number
Giri et al. (2018)	*				*	*	*		warranty price/period
Hea et al. (2020)		*		*		*	*		win-win warranty price
zhang et al. (2021)			*		*	*	*		warranty-reliability-pric
Hashemi et al. (2022)					*				PM cost
Li et al(2022)	*	*				*	*		
			*		*				price and warranty period
present study	*	*		*	*		*	*	PM number/ warranty price

One of important problems in industry is related to finding back sales services price and maintenance policies, in useful life of products. In this article, by considering policy makers such as consumers, manufacturers or sellers try to find optimal back sales services price and number of maintenances in specific periods for products such as trucks. Price and number of services should cover all of maintenance costs, so the amount of services price for satisfying all participators because of more quality services and paying lest costs can be so essential. should respond to policy makers expectations. Regarding Table 1, the interactions between agent and customer in base and extended warranty periods for reaching equilibrium and optimal maintenance strategy with a practical method have been investigated less than other subjects. Therefore, the equilibrium warranty price and PM number will be found under agent and consumer strategies with different usage rates. The agent and consumer strategies in one of the trucks back sales service agent in Iran are as follow: In addition, corrective maintenance costs, some trucks fail on the road. Therefore, the agent should pay part of the cost to repair or move them. This maintenance is named road corrective maintenance (which is performed for vehicles that failed on the road, for this type of failure the agent should pay a fraction of the repair costs). In contrast to these services, the consumer can use all the benefits of the repaired product and achieve their level of satisfaction from maintenance quality by paying warranty serveries price. The agent and consumer are two players of the game and each of them tries to rise their profits in two periods: base warranty and extended warranty periods. These interactions are considered dynamic games in two stages: agent Stackelberg and consumer Stackelberg. In section 2, all of the parameters and assumptions are presented. In section 3, the costs of maintenance such as preventive maintenance and corrective maintenance in base and extended warranty are modelled. In section 4, their profit functions are estimated by equilibrium Stackelberg. Finally, the proposed model is investigated by using the real data from a truck service agent, and equilibrium PMs number and warranty price are estimated.

2. Model Description

In this study, consumer and service agent profits are calculated by the game theory approach under a two-dimensional warranty. Maintenance activists involve PM and CM. CMs are divided into two groups: home and road maintenance. In the base warranty, all CMs costs and specific parts of PM costs are paid by the agent. On another side, the customer pays the rest of the PM cost. In the extended warranty period, the agent in turn receiving the extended warranty price, pays just CM costs, and all PM costs are paid by the customer. In the following equilibrium warranty price and PM number are estimated by Stackelberg equilibrium under low and high usage rates.

2.1. Model Assumptions and Notations

The assumptions, decision variables, and parameters used in this paper are given as follows:

2.2. Decision variables

m: The number of preventive maintenances during the base warranty period
n: The number of preventive maintenances during the extended warranty period
P_W: Base warranty price
P_E: Extended warranty price

2.3. Assumptions

The model assumptions are as follows:

- The Cub Dogloss demand function is used for estimating sales volume, and sales price is considered as warranty price in this function.
- Consumer usage rates are divided into two groups: high and low.
- For low and high usage rates, the type of consumer is specified.
- Warranty in base and extended warranty is not free because the repair cost of this kind of product is expensive.
- The base warranty and extended warranty price are fixed over the lifetime of the product.
- Corrective maintenance is performed as minimal repair, so this act does not affect improving the failure rate.
- The warranty policy is non -renewable warranty.
- The participants in the game theory include agent service and consumer, and each of them is perfectly rational to increase the profits, and only these two participants are making a decision.
- Each participant in the game provides just one strategy for decision-making.
- Participants make decisions in order, so they are aware of each other's decisions and adhere to each other's decisions.
- It is assumed that a certain number of trucks require road maintenance and the rest are repaired in the agent’s department.
- The parameters have no uncertainty.

2.4. Input parameters

Table 2 shows the input parameters.

Table 2. Input parameters

T_B	Time to end of 2D in base warranty	r	Usage rate
T_E	Time to end of 2D in the extended warranty	s	Satisfaction with warranty services
U_B	Usage to end of 2D in base warranty	γ	Accelerate frailer time
U_E	Usage to end of 2Din extended warranty	C_p	Expected PM cost
r_0	Nominal usage rate	C_{M1}	Expected home CM cost
r_l	light usage rate	C_{M2}	Expected road CM cost
r_h	High usage rate	G(r)	Intensity function of r
τ_1	The interval between PM acts for light usage rate in the base warranty	χ	Elastic price
τ_2	The interval between PM acts for light usage rate in the extended warranty	k	Amplitude factor for the price
τ_1'	The interval between PM acts for heavy usage rate in the base warranty	α	Scale parameter in Weibull distribution
τ_2'	The interval between PM acts for heavy usage rate in the extended warranty	ε	Shape parameter in Weibull distribution
δ	PM level	ψ	Scale parameter in the gamma distribution
θ	Shape parameter in the gamma distribution	Π_{AB}	The profit of the agent in base warranty

q	Percentage of road maintenance	Π_{CB}	The profit of consumer in base warranty
g(r|r∈ $[r_l, r_0]$)	The conditional intensity function for r_h	Π_{AE}	The profit of the agent in the extended warranty
g(r|r∈ $[r_h, r_0]$)	The conditional intensity function for r_l	Π_{CE}	The profit of consumers in the extended warranty
h(t|r)	Failure intensity function under usage rate	h(t|r)	Failure intensity function under usage rate
θ	Shape parameter in the gamma distribution		
q	Percentage of road maintenance		
g(r|r∈ $[r_l, r_0]$)	The conditional intensity function for r_h		
g(r|r∈ $[r_h, r_0]$)	The conditional intensity function for r_l		

3. Modelling

3.1. Low usage rate

Two-dimensional warranty coverage is defined as a rectangular shape and coverage is shown like [0, T_B)* [0,U_B). The base warranty starts from the sale of the product time and is continued until T_B . U_B is the usage limit and it is equal to consumer usage This value for vehicles is defined by kilometer. It should be noted, each of the two factors of age or usage, reaches its limit sooner, the warranty period is finished. The nominal usage rate is r_0 and it is defined as $r_0=\frac{U_B}{T_B}$. For light users, the usage rates are lower than r_0 ($r_l < r_0$) so the warranty coverage will be finished at the point of (T_B ,$T_B r$) . This area is shown in Figure (1).

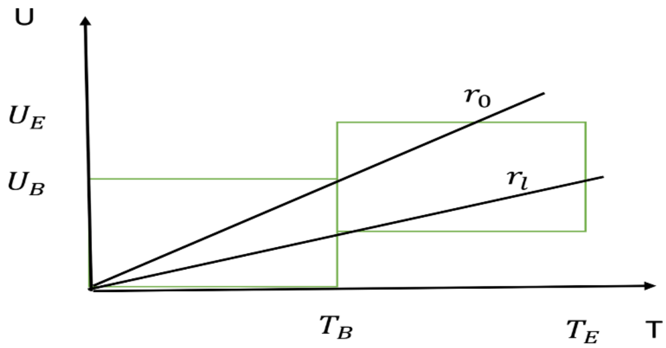

Figure 1. Warranty coverage for light users

In base warranty, periodic preventive maintenance is assumed to be carried out at a constant time interval τ_1 , This value is obtained from the warranty period ratio on the number of periodic preventive maintenance.τ_1 is obtained from equation (1).

$$\tau_1 = \frac{T_B}{m+1} \qquad m \geq 1 \qquad (1)$$

m is the number of preventive maintenances. This value for consumers with different usage rates can be varied and it is between {m_l, m_{l+1},.., m_h} [1]. m_l is the number of preventive maintenances for light users and m_h is the number of preventive maintenances for heavy users? In equation (1), the mean of m+1, is the PM is down at the time of T_B.

After the base warranty, the extended warranty would be started (it displays with T_E), and it continues until the end of the useful life of the product. The extended warranty coverage shown by [T_B ,T_E) * (U_B ,U_E] in which T_E is age limit and U_E is usage limit in the extended warranty period. For consumers whose rates of usage are lower than the nominal usage rate (light users), the extended warranty period ends at the point of (T_E, $T_E r$) [23]. Preventive maintenance is performed in fixed period τ_2 . The value of τ_2 is obtained from equation (2).

$$\tau_2 = \frac{T_E - T_B}{n} \qquad n \geq 1 \qquad (2)$$

In equation (2), n is the number of preventive maintenances, and $(T_E - T_B)$ is the extended warranty period.
The number of preventive maintenances for consumers with different usage rates can be between {$n_l, n_{l+1} \ldots, n_h$ }, where n is an instant value. n_l is the number of preventive maintenance for light users n_h is the number of preventive maintenance for heavy users.

3.1.1. Failure Rate

According to the Kijima model, when preventive maintenance is performed on the product, the failure rate will be younger about δ, which is a continuous value and varies between zero and one (0<δ<1). Therefore, when these maintenances are applied, the failure rate is reduced to the original state. The failure rate is shown in Figure (2).

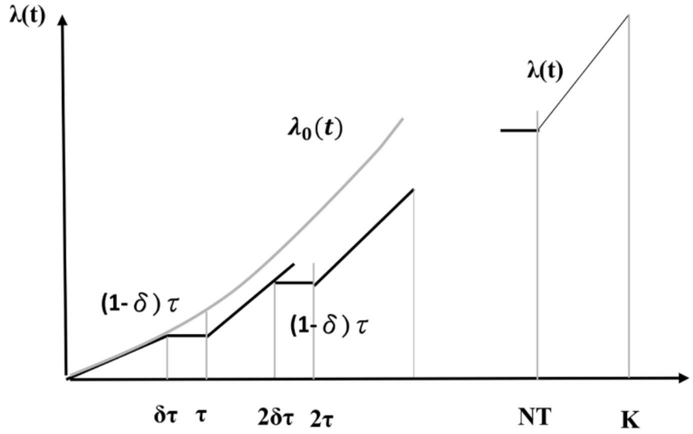

Figure 2. Failure intensity function under PM actions.

Minimal maintenance levels are assumed to be constant over the lifetime of the warranty. Therefore, by performing these maintenances, the lifetime of the product from t is reduced to iδτ, which is referred to as the virtual age and is represented with V. Virtual age between the two maintenances i and i + 1 are evaluated as follows:

V(t)= $i\delta\tau + t - i\tau$ iτ $\leq t <$ (i+1) τ (3)

In this stage, decisions are made about the reliability of the component with a specific nominal usage rate of r_0.The impact of the usage rate on the failure is modelled by the AFT model. According to this model, T_0 [T_r] represents the first failure time under the rate of usage r_0 [r], So:

$\frac{T_r}{T_0}$=($\frac{r_0}{r}$)$^\gamma$ (4)

The failure distribution function for T_0 is shown with F (x: α_0), since α is a scale parameter, and also the failure distribution function for the T_r is similar to T_0 , and the scale parameter is:

α_I= ($\frac{r_0}{r}$)$^{\gamma\alpha}$ (5)

γ ≥1

F (x: α(r)) = F(($\frac{r_0}{r}$)$^\gamma$x: α) (6)

The failure density function is shown as follows:

h(x: α_I) = $\frac{f(x:\alpha(r))}{1-F(x:\alpha(r))}$ (7)

Therefore, the failure density function is h(x: α_I). By considering the virtual age, it can be indicated as $h(\, i\delta\tau + t - i\tau \mid r\,)$. In addition to the failure rate, the failure density function of the usage rate is affected by various factors. By having a history usage rate of products, the density function of the usage rate can be estimated and is represented as g (r). The conditional density function or light and heavy users are shown as g (r|r∈ [r_l, r_0]) and g (r|r∈ [r_0, r_h]), respectively.

3.1.2. Sales Volume

Gilicman and Burger presented the demand function in the form of Cub Dogloss. In this demand function, k_1 is an amplitude constant factor and k_2 is an amplitude time factor ($k_1, k_2 > 0$), it is assumed P is the product price and W is the warranty period,χ is price elasticity ($\chi > 0$). By considering Esmaeili et al and Chen and Weng, the sales volume is influenced by the product price and the warranty period:

P(W)=[$\frac{K_1(K_2+W)^b}{d}$]$^{\frac{1}{\alpha}}$ (8)

It is assumed, in this model, product price is defined as the warranty price, and the warranty term is fixed (b=0). The demand function in the base is as follows:

D (P_w)=k_1 $P_w^{-\chi}$ (9)

The price of the extended warranty services is equal to P_E. Demand function in the extended warranty period is as follows:

D (p_E)=k_1 $P_E^{-\chi}$ (10)

The parameters of this function can be estimated by using regression calculations.

3.1.3. Cost Modelling

Maintenance in the base warranty includes periodic preventive maintenance and corrective maintenance. Corrective maintenance is carried out between iτ and (1+ i) τ of preventive maintenance periods.

It is assumed, during the base warranty period, the consumer pays a fraction of periodic preventive expenses as $(\frac{i}{m})$, and the rest of these costs as $(1-\frac{i}{m})$ are paid by the agent. The average cost of period preventive maintenance is shown by C_p in the useful life. Therefore, the expected cost of preventive maintenance from the consumer's viewpoint is estimated as follows:

$$C_p= \sum_{i=1}^{m} c_{pm}\ (\frac{i}{m}) \quad (11)$$

And the expected cost from the agent's viewpoint is estimated from (12) as follows:

$$C_p=\sum_{i=1}^{m} c_{pm}(1-\frac{i}{m}) \quad (12)$$

Minimal maintenance time between two preventive maintenance is negligible.

The average failure number between two preventive actions is obtained from equation (13):

$$E(m)=\int_{i\tau_1}^{(i+1)\tau_1} \sum_{i=1}^{m} h(\, i\delta\tau_1 + t - i\tau_1 \mid r\,)dt \quad (13)$$

In this function, $h(\, i\delta\tau_1 + t - i\tau_1 \mid r\,)$ is the conditional failure density function of usage rate. C_{r1} is a fixed cost as minimal maintenance cost, so the expected cost of corrective maintenance (C_{CM1}) is equal to (14):

$$C_{CM1}=(1-q)\ C_{r1} \int_{i\tau_1}^{(i+1)\tau_1} \sum_{i=1}^{m} h(\, i\delta\tau_1 + t - i\tau_1 \mid r\,)dt \quad (14)$$

Many trucks sometimes are disrupted on the road, so they cannot move. Regarding this, agents should move them to agent s' departments. It creates an expense for agents as a road CM cost. It is assumed, during the warranty period, each truck would be destroyed on the road with a probability of q and also about (1-q) percentage of trucks repaired in agents' departments. In this study, road corrective maintenance is displayed with C_{CM2} and is modelled from function (15):

$$C_{CM2} = q\ C_{r2} \int_{i\tau_1}^{(i+1)\tau_1} \sum_{i=1}^{m} h(\, i\delta\tau_1 + t - i\tau_1 \mid r\,)dt \quad (15)$$

3.1.4. Agent's profit in the base warranty period

The profits of agent and consumer are based on their policies:

In general, an agent against getting warranty price, pays a fraction of PM cost, home and the road CM costs, so the profit for an agent under a low usage rate can be modelled as follow:

Agent's profit = Sales Volume (warranty price - expected home corrective maintenance – expected road corrective maintenance - expected cost of preventive maintenance)

$$\Pi_A= k_1\ P_w^{-\chi}\ (P_W - \int_{r_l}^{r_0}(\,(1-q)C_{r1} \int_{i\tau_1}^{(i+1)\tau_1} \sum_{i=1}^{m} h(\, i\delta\tau_1 + t - i\tau_1 \mid r\,)dt + \sum_{i=1}^{m} c_{pm} \left(1-\frac{i}{m}\right) + q\ C_{r2} \int_{i\tau_1}^{(i+1)\tau_1} \sum_{i=1}^{m} h(\, i\delta\tau_1 + t - i\tau_1 \mid r\,)dt) g(r|r\in [r_l, r_0])dr\,) \quad (16)$$

3.1.5. Consumer's profit in the base warranty period

In this section, the consumer pays the warranty price and part of the PM cost. The benefit of warranty services can be considered by examining consumer satisfaction during the warranty period [13], which is obtained by a questionnaire and it can estimate the level of consumer satisfaction from agent warranty services quality. Thus, the profit of the consumer is modelled as follows:

Agent's profit = Sales Volume (the percentage of satisfaction * warranty period for light users - prevention cost - warranty price)

$$\Pi_C = k_1\ P_w^{-\chi}\ (S^*(r_l T_B) - \sum_{i=1}^{m} c_{pm}(\frac{i}{m}) - P_W) \quad (17)$$

3.1.6. Agent's profit in the extended warranty period

At this period, the agent only pays CM costs including home and road maintenance. On the other side, all PM costs are paid by consumers.

The profit is represented by the equation (18) as follows:

Agent's profit= warranty demand (extended warranty price- home corrective maintenance- road corrective maintenance)

$$\Pi_A=k_1 P_E^{-\alpha}(P_E - \int_{r_l}^{r_0}((1-q)\ C_r \int_{T_B+i\tau_2}^{T_B+(i+1)\tau_2} \sum_{i=1}^{n-1} h(\, i\delta\tau_2 + t - i\tau_2 \mid r\,)dt + q\ C_r \int_{T_B+i\tau_2}^{T_B+(i+1)\tau_2} \sum_{i=1}^{n-1} h(\, i\delta\tau_2 + t - i\tau_2 \mid r\,)dt)\ g(r|r\in [r_l, r_0])dr\,) \quad (18)$$

3.1.7. Consumer's profit in extended warranty period

Consumer pays extended warranty price and all of PM costs. The consumer's benefit from warranty services can be considered by examining consumer satisfaction which is obtained by a questionnaire. Thus, the profit of the consumer is modelled as follows:

Consumer's profit= warranty demand (satisfaction* warranty period for light users - preventive maintenance- warranty price)

$$\Pi_c = k_1\, P_E^{-\chi}\,(S^*r(T_E - T_B) - \sum_{i=1}^{n-1} c_p(i) - P_E) \qquad (19)$$

3.2. High usage rates

Under a high usage rate, the usage reaches to U_B earlier than the expected time (T_B) and also age equal to $\frac{U_B}{r}$. As a result, the base warranty period ends at the point ($\frac{U_B}{r}$,U_B) ,as shown in Figure3) .The high usage rate is more than r_0 ($r_h \geq r_0$). The fixed time interval between two PMs is equal to τ_1', and it is estimated as follows

$$\tau_1' = \frac{U_B}{(m+1)r} \qquad (20)$$

Similarity, if the usage rate is high, the extended warranty period ends at the point of ($\frac{U_E}{r}$,U_E) . The time interval between two PMs is equal to τ_2', and is estimated from the following equation:

$$\tau_2' = \frac{U_E}{nr} \qquad (21)$$

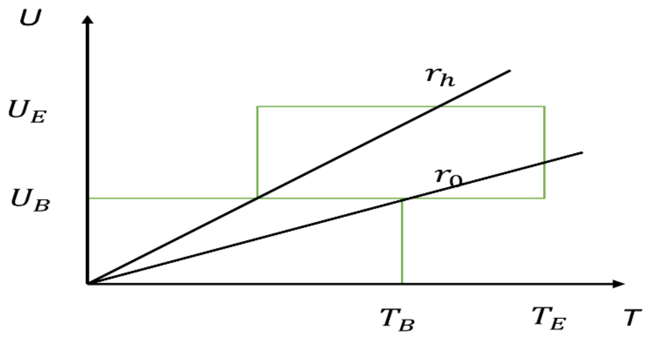

Figure 3. Warranty coverage for high users.

Corrective maintenances are carried out between preventive maintenances within a time interval $\tau_2' = \frac{U_E - U_B}{nr}$, and the number of preventive maintenances in this period is considered n.

The modelling of agent and consumer profits in base and extended warranty under high usage rate are similar to low usage rate as follows:

3.2.1. Agent's profit in the base warranty period

Agent's profit in base warranty for high usage rate is as follows:

$$\Pi_A = k_1 P_W^{-\chi}(P_W - \int_{r_0}^{r_h}((1-q)C_{r1}\int_{i\tau_1'}^{(i+1)\tau_1'} \sum_{i=1}^{m} h(i\delta\tau_1' + t - i\tau_1' \mid r)dt + \sum_{i=1}^{m} c_{Pm}\left(1-\frac{i}{m}\right) + q\; C_{r2}\int_{i\tau_1'}^{(i+1)\tau_1'} \sum_{i=1}^{m} h(i\delta\tau_1' + t - i\tau_1' \mid r)dt)\; g(r|r\in[r_0, r_h])dr\,) \qquad (22)$$

3.2.2. Consumer's profit in the base warranty period

Consumer's profit in base warranty for high usage rate is as follows:

$$\Pi_C = k_1\, P_w^{-\chi}\,(S^*\frac{U_B}{r} - \sum_{i=1}^{m} c_{pm}(\frac{i}{m}) - P_W) \qquad (23)$$

3.2.3. Agent's profit in the extended warranty period

Agent's profit in an extended warranty for a high usage rate is as follow:

$$\Pi_A = k_1 P_E^{-\chi} P_E - \int_{r_0}^{r_h}((1-q)C_{r1}\int_{T_B+i\tau_2'}^{T_B+(i+1)\tau_2'} \sum_{i=1}^{n-1} h(i\delta\tau_2' + t - i\tau_2' \mid r)dt + q\; C_{r2}\int_{T_B+i\tau_2'}^{T_B+(i+1)\tau_2'} \sum_{i=1}^{n-1} h(i\delta\tau_2' + t - i\tau_2' \mid r)dt)\; g(r|r\in[r_0, r_h])dr) \qquad (24)$$

3.2.4. Consumer's profit in the extended warranty period

Consumer's profit in an extended warranty for a high usage rate is as follow:

$$\Pi_C = k_1 P_E^{-\chi} (S*\frac{U_E-U_B}{r} - \sum_{i=1}^{n-1} c_{pm}(i) - P_E) \quad (25)$$

4. Game theory

In this research, agents and consumers are considered as two players of the game. They make decisions in base and extended warranty under dynamic game (in the dynamic game both players have information about their decisions and they make decisions subsequently) conditions.

By considering consumer usage rate, these games are solved in two stages with Stackelberg equilibrium under cooperation combination: (a) Agent-Stackelberg; (b) Consumer-Stackelberg. In this research, the agent and consumer (two players) [3], that each of them presents only one strategy.

4.1. Game solution for light users

4.1.1. Game solution for light users in base warranty

The models are considered in two senses: agent and Consumer -Stackelberg:

A) Agent-Stackelberg

In this way, agent and consumer decisions are made sequentially, and both try to maximize their profits. At first, the agent as a leader enters the game and chooses its strategy by determining any selected amount of warranty price (P_W). Then, the consumer as a follower of the leader's decision, provides the best reaction against each amount of warranty price which is selected by the agent.

If the agent as a leader chooses the warranty price (P_W), the best reaction for consumer is $m^*(P_W)$. In this case, consumer's profit must be derived from several maintenance and this value should be a function of warranty price (like $m'(P_w)$) ,but it is assumed the optimal number of maintenances is an integer value under different usage rate, and can be somewhere in the interval $\{m_l, m_{l+1}, ..., m_h\}$. In the following, the game for different amounts of PMs number is solved and the equilibrium point is found.

• If $m^*=m'(P_W) = m_l$, by replacing this expression in function (22) the optimal warranty price is derived from the agent's profit and its value to be considered P'_W (according to equation (26)) as follow:

$$\frac{\partial \Pi_A(P_w, m_l)}{\partial P_w}=0 \quad (26)$$

Thus, the optimal agent's profit is obtained at point (P'_W, m_l)as follows:

$$\Pi_A(P'_W, m_l)=k_1 P_w'^{-\chi}(P'_W - \int_{r_l}^{r_0} \Big((1-q)C_r \int_{i\tau_1}^{(i+1)\tau_1} \sum_{i=1}^{m_l} h(i\delta\tau_1 + t - i\tau_1 \mid r)dt + \sum_{i=1}^{m_l} c_{pm} \left(1-\frac{i}{m_l}\right) + q\ C_r \int_{i\tau_1}^{(i+1)\tau_1} \sum_{i=1}^{m_l} h(i\delta\tau_1 + t - i\tau_1 \mid r)dt\Big) g(r|r\in [r_l, r_0])dr) \quad (27)$$

It is clear, by decreasing the number of PMs, the agent's profit will be increased. Agent prefers to do fewer PMs in the warranty period, so this is an equilibrium point for agent and it is shown as$(P_W^*, m_l\ )$.

• If $m^*= m'$ $(P_w) = m_{l+1}$, according to equation (22) optimal warranty price (P'_W) is derived from agent's profit as follow:

$$\frac{\partial \Pi_A(P_w, m_{l+1})}{\partial P_w}=0 \quad (28)$$

In this regard, the agent's profit is estimated at *point* (P'_W, m_{l+1}) as follows:

$$\pi_A(P'_W, m_{l+1})= k_1 P_w'^{-\chi}(P'_W - \int_{r_l}^{r_0} \Big((1-q)C_{r1} \int_{i\tau_1}^{(i+1)\tau_1} \sum_{i=1}^{m_{l+1}} h(i\delta\tau_1 + t - i\tau_1 \mid r)dt + \sum_{i=1}^{m_{l+1}} c_{pm} \left(1-\frac{i}{m_{l+1}}\right) + q\ C_{r2} \int_{i\tau_1}^{(i+1)\tau_1} \sum_{i=1}^{m_{l+1}} h(i\delta\tau_1 + t - i\tau_1 \mid r)dt\Big) g(r|r\in [r_l, r_0])dr) \quad (29)$$

• If $m^*= m'$ $(P_w) = m_h$, the warranty price is derived from the agent's profit (22) and it is shown in equation (30) :

$$\frac{\partial \Pi_A(P_w, m_h)}{\partial P_w}=0 \quad (30)$$

Optimal warranty price is as P'_W and *the* agent's profit is estimated at point $(P'_W, m_h\ )$ as follows:

$$\Pi_A(P'_W, m_h\ )= k_1 P_w'^{-\chi}(P'_W - \int_{r_l}^{r_0} ((1-q)C_{r1} \int_{i\tau_1}^{(i+1)\tau_1} \sum_{i=1}^{m_h} h(i\delta\tau_1 + t - i\tau_1 \mid r)dt + \sum_{i=1}^{m_h} c_{pm} \left(1-\frac{i}{m_h}\right) + q\ C_{r2} \int_{i\tau_1}^{(i+1)\tau_1} \sum_{i=1}^{m_h} h(i\delta\tau_1 + t - i\tau_1 \mid r)dt) g(r|r\in [r_l, r_0])dr) \quad (31)$$

B) Consumer-Stackelberg

It is assumed consumer as a leader enters the game and determines its strategy (defines the number of preventive maintenances (m)) then the agent optimizes the best reaction to any consumer decision. The best agent reaction is evaluated based on any selected value of m. Thus, the warranty price is derived from the agent's profit and this value is evaluated as a function of PM number ($P'_W(m)$) as follows:

$$\text{MAX}\Pi_A=k_1 P_W^{-\chi}(P_W - \int_{r_l}^{r_0} ((1-q)C_{r1} \int_{i\tau_1}^{(i+1)\tau_1} \sum_{i=1}^{m} h(i\delta\tau_1 + t - i\tau_1 \mid r)dt + \sum_{i=1}^{m} c_{pm} \left(1-\frac{i}{m}\right) + q\ C_{r2} \int_{i\tau_1}^{(i+1)\tau_1} \sum_{i=1}^{m} h(i\delta\tau_1 + t - i\tau_1 \mid r)dt) g(r|r\in [r_l, r_0])dr) \quad (32)$$

s.t: $P_w > 0$
The agent's profit model is optimal (Appendix A).
By putting $p'_w(m)$ in consumer's profit function, m' is obtained, so the equilibrium point is considered at point of $(p'_w(m), m')$.
As a result, by considering model (23), the optimal number of PM is derived from the consumer's profit.

$$\Pi_C(p'_w(m), m') = k_1\, p'_w(m)^{-\chi}\,(S^*(rT_B) - \sum_{i=1}^{m'} c_{pm}(\frac{i}{m'}) - p'_w(m)) \quad (33)$$

Finally, the consumer prefers to pay a lower price during the warranty period, the equilibrium point is when the price of the warranty service is reduced.

4.1.2. Game solution for light users in the extended warranty period

In the extended warranty period, as well as the base warranty, the consumer's profit is investigated based on a dynamic game. In this period, the extended warranty price is chosen by the agent and the number of maintenances will be decided by the consumer.

A) Agent-Stackelberg

Agent as a leader enters the game and chooses its strategy by determining any selected amount of warranty price (P_E). Then, the consumer as a follower of the leader's decision provides the best reaction against each amount of warranty price which is selected by the agent.
In this case, the agent first enters the game as a leader and specifies the price of extended warranty as P_E. Due to the light usage rates and high usage rate, it is assumed the optimal number of maintenances is an integer value under different usage rates and can be somewhere in the interval $\{n_l, n_{l+1}, \ldots, n_h\}$. The best reaction for the consumer to the decision of agent is $n^*(P_E)$.

• If $n^* = n'(P_E) = n_l$, the optimal extended warranty price is driven from the agent's profit and it is shown with P'_E as follows:

$$\frac{\partial \Pi_A(P_E, n_l)}{\partial P_E} = 0 \quad (34)$$

Now, the agent's profit is obtained at point (P'_E, n_l):

$$\Pi_{AE}(P'_E, n_l) = k_1 P'^{\,-\chi}_E (P'_E - \int_{r_l}^{r_0} ((1-q) C_{r1} \int_{T_B+i\tau_2}^{T_B+(i+1)\tau_2} \sum_{i=1}^{n_l-1} h(i\delta\tau_2 + t - i\tau_2 \mid r) dt + q\; C_{r2} \int_{T_B+i\tau_2}^{T_B+(i+1)\tau_2} \sum_{i=1}^{n_l-1} h(i\delta\tau_2 + t - i\tau_2 \mid r) dt)\, g(r \mid r \in [r_l, r_0]) dr) \quad (35)$$

The value of P_E is obtained from equation (35), the profit is estimated at point (P'_E, n_l). Because agent prefers to provide less maintenance to optimize their profit, this point can be an equilibrium point for an agent.

• If $n^* = n'(P_E) = n_{l+1}$, the optimal extended warranty price is driven from agent's profit and it is shown as P'_E as follow:

$$\frac{\partial \Pi_A(P_E, n_{l+1})}{\partial P_E} = 0 \quad (36)$$

Agent's profit is obtained at point(P'_E, n_{l+1}):

$$\Pi_{AE}(P'_E, n_{l+1}) = k_1 P'^{\,-\chi}_E (P'_E - \int_{r_l}^{r_0} ((1-q) C_{r1} \int_{T_B+i\tau_2}^{T_B+(i+1)\tau_2} \sum_{i=1}^{n_{l+1}-1} h(i\delta\tau_2 + t - i\tau_2 \mid r) dt + q\; C_{r2} \int_{T_B+i\tau_2}^{T_B+(i+1)\tau_2} \sum_{i=1}^{n_{l+1}-1} h(i\delta\tau_2 + t - i\tau_2 \mid r) dt)\, g(r \mid r \in [r_l, r_0]) dr) \quad (37)$$

The value of P_E from equation (37) is estimated at point(P'_E, n_{l+1}).

• If $n^* = n'(P_E) = n_h$, the optimal extended warranty price is driven from agent's profit and it is shown as P'_E:

$$\frac{\partial \Pi_A(P_E, n_h)}{\partial P_E} = 0 \quad (38)$$

Agent's profit is obtained at point (P'_E, n_h):

$$\Pi_{AE}(P'_E, n_h) = k_1 P'^{\,-\chi}_E (P'_E - \int_{r_l}^{r_0} ((1-q) C_{r1} \int_{T_B+i\tau_2}^{T_B+(i+1)\tau_2} \sum_{i=1}^{n_h-1} h(i\delta\tau_2 + t - i\tau_2 \mid r) dt + q\; C_{r2} \int_{T_B+i\tau_2}^{T_B+(i+1)\tau_2} \sum_{i=1}^{n_h-1} h(i\delta\tau_2 + t - i\tau_2 \mid r) dt)\, g(r \mid r \in [r_l, r_0]) dr) \quad (39)$$

B) Consumer-Stackelberg

In this part, the consumer enters the game as a leader and indicates the number of PMs in the extended warranty period, the optimal extended warranty price is obtained from equation (40) and it is calculated as a function of the PM number as follows:

$$\frac{\partial \Pi_A(P_E, n)}{\partial P_E} = 0 \quad (40)$$

So, is displayed as $p'_E(n)$.By putting $p'_E(n)$ in consumer 's profit function, n' is obtained and finally, the equilibrium point is considered at point of $(p'_E(n), n')$.

$$\Pi_C(p'_E(n), n') = k_1\, p'_E(n)^{-\chi}\,(S^* r(T_E - T_B) - \sum_{i=1}^{n} c_{pm}(i) - p'_E(n)) \quad (41)$$

4.2. Game solution for high users

4.2.1. Game solution for high users in base warranty period

A) Agent-Stackelberg

Like the light user, the agent as a leader, determines the warranty price and the consumer provides the best response to the agent's decision. Then, the price of the warranty agent is evaluated under different m.

B) Consumer-Stackelberg

It is assumed, the consumer as a leader enters the game and defines its strategy (the number of preventive maintenances is m), in this case, the best agent's response is determined based on consumer decisions. In this regard, warranty price is driven by the agent's profit and it is written as a function of the number of PMs as P'_W(m).

4.2.2. Game solution for high usage rates in during extended warranty period

For high usage rates: Agent-Stackelberg and consumers are investigated.

A) Agent-Stackelberg

In this case, the number of PMs is derived from the consumer's profit and it is written as the function of the warranty price, which is displayed as n' (P_E).
To determine optimal extended warranty price, the model will be solved for different numbers of PMs.

B) Consumer-Stackelberg

In this mode, P_E is derived from agent 's profit model and it is presented by $p'_E(n)$, the consumer's profit is determined by the number of maintenances and the optimal warranty price.

5. Case study

The first failure time of 192 trucks that came to the agent department for fixing was observed and the age(days) and usage (Kilometers) of each of them were recorded in one year, so by considering the usage rate function(X/t), the usage of all trucks in specific time were obtained. For example, the age of the first truck is 9 days and the usage of it is 6104 (Km), so =2.475(km/year). Regarding this subject, the usage rate of all trucks is estimated and if the usage rate is more than standard usage rate it is called high usage rate and if it is less than standard usage rate, it is called low usage rate. These trucks have a standard warranty of one year or 2*10^5 kilometres This means, base warranty services end after one year or after two 2*10^5 kilometers, so the nominal usage rate is equal to (10^5km / year) r_0=2.
In this way, the average of these values is evaluated individually to determine the amount of usage rate of both high and light users. The results are presented in Table 3.

Table 3. Two usage types.

light usage rate	1.44
heavy usage rate	2.87

The average PM cost, for home and road corrective maintenance are 4.6, 3.8, and 4.2 respectively. The virtual age should be estimated by observing the trucks failure rate, before and after each PM, the maintenance level is estimated (as section 2-3) and it is equal δ =0.32.
The average of satisfaction from quality services in base and extended warranty periods is estimated by questioner with likert scale and the average of results is about 0.64 (this value is determined by questioner). According history of road failures in a specific period of a year, about 0.24% of trucks are disrupted on the road and require road maintenance also rest of them are repaired at the agent department.
As mentioned, for estimating sales volume, the Cub doglass demand function is used. For finding parameters of this function, the number of sales and the sales price in 8 periods collecting and with regression method κ and α are estimated, that they are shown in Table 5.

Table 4. The number of sales and price services

Period	The Number of Sales	Price Services
1	42	61.5
2	61	61.5
3	36	63.9
4	25	64.5
5	34	68.4
6	44	70
7	22	7.5
8	23	7.5

Tabe 5. Parameters of cobdogloss demand with Regression Method

Parameters	Value
k	4505142.1
α	2.79

By considering the failure history in base and extended warranty and also the result of Minitab software, the distribution function of failure time and usage rate obey from Wiebull and Gama distribution respectively as follows:

In the Wiebull failure density function: ε and α_0 are the parameters of scale and shape and in this study, these parameters are equal to 2.063 and 0.625. By considering the AFT model, the Weibull density function is as follows:

$$h(x;\alpha_0)=\varepsilon(\frac{r}{r_0})^{\beta\varepsilon}\frac{x^{\varepsilon-1}}{\alpha_0^{\varepsilon}} \quad (42)$$

β is accelerator failure and this value is about 8.9, and the parameter of the actual usage scale is as follows:

$$\alpha_r= (\frac{r}{r_0})^{\beta}/\alpha_0 \quad (43)$$

Usage rates are also a function of several factors. Therefore, to estimate the density function of the usage rate, the usage rate of 192 deadly trucks was considered and as a result, the Gamma density function was also chosen.

$$g(r)=\frac{r^{\theta-1}e^{-r/\vartheta}}{\int_0^{\infty} t^{\theta-1}e^{-t}dt\,\vartheta^{\theta}} \quad (44)$$

Due to the results of the Minitab software, the scale and shape parameters of Gamma distribution are to 15 with its 0.1 respectively.

5.1. Light usage rate result:

A) Agent-Stackelberg in base warranty

The best consumer's response to the decision of the agent is displayed as $m'(P_w)$. According to the information available from the truck service agent, the number of periodic maintenances in the base warranty period can be valued from the interval {7, 8, ..., 14}, depending on the type of consumers this amount can be various. The game has been evaluated for {7, 8, 10, …., 14} (Table 4). The results of the game are calculated in MATLAB software.

Table 4. The results of Agent-Stackelberg in base warranty

m	P_w^*	Π_A	Π_E
7	48	0.044	-0.0854
8	50	0.0438	-0.0884
10	64	0.0435	-0.0944
14	78	0.0430	-0.1064

Agent and consumer profits are estimated for various quantities of preventive maintenance. According to agent-Stackelberg and under cooperative sense, the profits will be optimized if the number of maintenances is equal to 7, and the base warranty service cost is 48 million, so the profit of the agent at the equilibrium point (48,7)is equal to 0.044, and the agent selects this policy to maximize the profit.

B) The Consumer-Stackelberg in base warranty

Now, the consumer is considered to be the leader in the static game, and the PMs number is equal to m.

Table 5. The results of Consumer-Stackelberg in base warranty.

P_w	m^*	Π_A	Π_E
36	8	0.01	-0.1584
40	8	0.0085	-0.1276
54	10	0.0051	-0.0696
64	13	0.0039	-0.0490

According to Table 5, profit is optimized only at m_h and it is obvious the price of these PMs is increasing, but under light usage rate, the best consumer choice is (13 ,64). If the number of maintenances is equal to 13 and the price of the warranty service is equal to 64 million, the profit is equal to -0.0490. As it can be seen, consumer profit has been negative, and these results are confirmed by experts of truck service agents using actual data from the results.

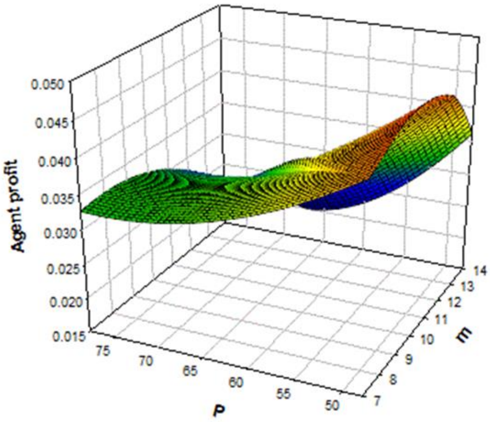

Figure 4. Agent profit in base warranty period.

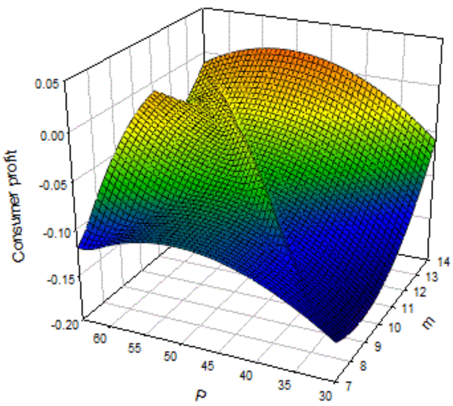

Figure 5. Consumer profit in base warranty period

Figure 4 shows the Π_{AB} is a quadratic convex surface and it shows optimal points in boundaries. Figure 5 shows that Π_{CB} is a quadratic convex surface and the profit is maximized in boundary points.

C) The Agent-Stackelberg in extended warranty period:

The number of periodic maintenances in the extended warranty period can be an integer value from the range of {64, 65, …, 90}. With the desired parameters, the price of the optimal warranty service is estimated at various n values. According to Table 6, the profit of the agent at the equilibrium point is (90.64), and it is equal to 0.051.

Table 6. The results of Agent-Stackelberg an extended warranty.

n	P_E^*	Π_A	Π_E
64	90	0.0051	-0.0022
72	110	0.0049	-0.0024
85	122	0.0045	-0.0027
90	164	0.0041	-0.0028

If the number of maintenances is equal to 90 and the price of the warranty service is equal to 64 million, the optimal profit will be 0.0051.

D) The Consumer-Stackelberg an extended warranty

The results of Consumer-Stackelberg in the extended warranty are as follows:

Table 7. The results of Consumer-Stackelberg an extended warranty.

P_E	n^*	Π_A	Π_C
120	64	0.0082	-0.0487
130	73	0.0071	-0.04017
150	85	0.0055	-0.0285
164	110	0.0049	-0.0230

According to Table 7, the consumer's profit is optimized only at the equilibrium point (164,110). If the number of maintenances is equal to 110 and the price of the warranty service is equal to 164 million, the optimal profit will be - 0.0230.

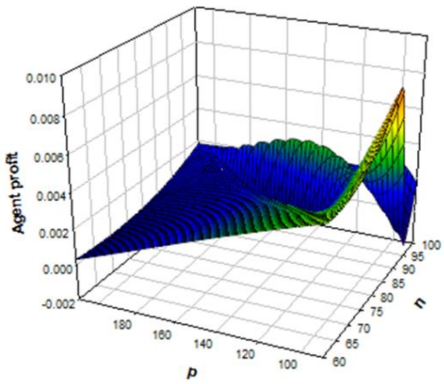

Figure 6. Agent profit in the extended warranty. Period

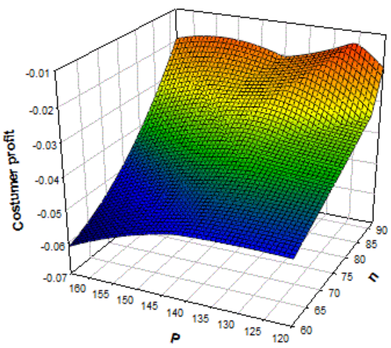

Figure 7. Consumer profit in extended warranty period.

Figure 6 shows,Π_{AE} is a quadratic convex surface and it is increasing by rising warranty price. Figure 7 shows, Π_{AC} is a quadratic convex surface and it is increasing by rising determining optimal warranty and maintenance strategies from agent and consumer viewpoints(policymakers) is the purpose of this research, so by considering services price and maintenance costs in warranty and extended warranty periods, the optimal strategy will be done. The warranty services price and the number of PMs are decision variables that the agent and consumer make decisions about. In the base warranty period, all CM costs and the ratio of PM costs are paid by the agent, so the agent prefers the number of PM maintenance be fewer with a high level of quality because he tends to pay fewer maintenance costs, according to Agent-Stackelberg's method of the game theory he as a leader sets the warranty price, and the consumer by considering this warranty price and rich high level of service quality tries to choose the best number of PM, agent profit estimates by these values. In this case study the number of PMs is 7 and 48$. The equilibrium point is (48,7) and the optimal agent profit is about 0. 044.In contrast, in the Consumer-Stackelberg method, the consumer as a leader prefers the number of PMs to be more, so sets the PM number, and the agent as a follower by considering this, chooses the best service price so it is about 64$ and the PM number is 13. The equilibrium point is at (64,13) and consumer profit is about -0. 049.In the extended warranty period, all CM costs are paid by the agent, and all PM costs are paid by the consumer. The agent as a leader of Stackelberg method sets extended warranty price and the number of PMs chosen by a consumer, so the equilibrium point is at (90,64) and the optimal agent profit is 0.051. If the consumer as a leader sets the number of PMs and the agent tries to choose the best extended warranty price equilibrium point is at (164,110) and optimal consumer.

5.2. High usage rates result:

In base warranty, Agent-Stackelberg solution: the optimal equilibrium policy for consumers is equal to (53, 9), which means if consumers be as a high user, the maintenance price should be equal to 53, and the number of maintenances is 9. Hence, the optimal profit level is 0.068, and both players can choose their best policy. In the case of the Consumer-Stackelberg: the optimal usage policy is equal to (73, 15) and the proceeds from this policy are equal to 0.068. In the extended warranty period, under Agent-Stackelberg s solution: the equilibrium point is (110, 93) and the optimal point from this equilibrium point is 0.0050. The Agent-Stackelberg, equilibrium point is (168, 91), and the optimal point is equal to 0.032.

6. Conclusion

The purpose of this study was to determine the equilibrium policy warranty from service agent and consumer viewpoints. Maintenances were carried out in the form of periodic PMs with fixed intervals and corrective maintenance which did between preventive maintenance as home and road. According to their decisions, their profits were modelled by considering maintenance costs and warranty price in the base warranty period and extended warranty period under different usage rates. In the base and extended warranty period, decision variables are warranty price and number of PMs. In the following, the equilibrium number of period PM and warranty service sales prices for low and heavy users were obtained Stackelberg - approach. These equilibrium points are the best strategy for the agent and consumer. An interesting direction for future research is to develop these models for finding the optimal upgrade level of maintenance under the cooperative and non-cooperative sense of game approach for second-hand products.

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