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Developing an Inventory Control Model for Closed-Loop Reverse Logistics Systems Considering Product Quality and Variable Demand

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Abstract

Effective inventory control in closed-loop reverse logistics systems is difficult because firms must simultaneously manage uncertain returns, heterogeneous product quality, and variable demand for new and remanufactured items. While prior studies on closed-loop supply chains have mainly emphasized network design and strategic decisions, fewer studies have developed operational inventory models that integrate these sources of uncertainty within a unified framework. Motivated by this gap, this study proposes a comprehensive inventory control model for a closed-loop reverse logistics system comprising production, remanufacturing, collection, and distribution subsystems. A discrete-event simulation model is developed in Arena® to capture the stochastic and dynamic behavior of the system, and two metaheuristic algorithms, Tabu Search (TS) and Genetic Algorithm (GA), are applied to optimize inventory decisions. The proposed approach minimizes total system costs while improving operational performance under uncertainty. Computational results indicate that the model reduces total costs by 15% and increases service levels by 8%. In addition, the Genetic Algorithm outperforms Tabu Search, yielding a further 3.5% reduction in total cost. Sensitivity analysis confirms the robustness of the proposed framework across different parameter settings. The main contribution of this study is the development of an integrated and uncertainty-aware inventory control framework for closed-loop reverse logistics systems at the operational level. The findings provide both theoretical support and practical guidance for implementing sustainable and cost-efficient inventory policies in circular supply chains.

Keywords: Reverse logistics, Inventory control, Simulation, Genetic Algorithm, Tabu Search.

Paper Type: Original Research

1. Introduction

Growing environmental concerns, increasing regulatory pressure, and rising customer awareness have significantly transformed the landscape of modern supply chain management (SCM). Organizations are no longer evaluated solely on economic performance; instead, they are expected to incorporate sustainable practices into their operational and strategic decisions. As a result, reverse logistics (RL) has emerged as a critical component of sustainable supply chain management. Reverse logistics refers to the processes associated with the collection, transportation, inspection, recovery, and redistribution of used products from the point of consumption back to the point of origin in order to recover value or ensure environmentally responsible disposal (De Brito & Dekker, 2003). By enabling product recovery, recycling, and remanufacturing, reverse logistics plays a crucial role in reducing environmental impacts while simultaneously creating economic opportunities for firms. Closed-loop supply chains (CLSCs) extend the concept of reverse logistics by integrating forward and reverse flows within a unified system. In a CLSC, products that reach the end of their usage phase are returned to the supply chain and subjected to processes such as inspection, repair, remanufacturing, recycling, or disposal before being reintroduced into the market (Akçalı & Çetinkaya, 2011). This circular configuration enables firms to recover valuable materials and components, reduce raw material consumption, and improve resource utilization. In addition to environmental benefits, CLSCs can also generate substantial economic advantages by reducing production costs through remanufacturing and creating new revenue streams from refurbished products (Bazan et al., 2016). Consequently, the design and management of efficient closed-loop systems have become an important research topic in supply chain and operations management. Despite these advantages, managing closed-loop supply chains remains highly challenging due to the complexity and uncertainty inherent in reverse flows. Unlike traditional supply chains that primarily deal with forward product flows, CLSCs involve bidirectional material movements and uncertain return processes. Products returned from customers often arrive in unpredictable quantities and conditions, while the

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demand for remanufactured products may fluctuate significantly over time. These characteristics create considerable challenges for operational planning, particularly in inventory control and production planning (Ahiska et al., 2017; Kenne et al., 2012). Consequently, effective inventory control policies are essential to ensure the economic viability and operational efficiency of closed-loop supply chains. Two major sources of uncertainty play a critical role in CLSC inventory management: the variability in the quality of returned products and the uncertainty in the demand for remanufactured items. Returned products may vary widely in their condition depending on factors such as product age, usage intensity, and handling during the return process. While some items may require only minor refurbishment, others may require extensive remanufacturing or may even be unsuitable for recovery and must be recycled or discarded. At the same time, demand for remanufactured products is influenced by multiple market factors including price, consumer perception of remanufactured quality, warranty conditions, and brand reputation (Teunter, 2001). These uncertainties significantly complicate decision-making in inventory management and highlight the need for robust analytical and computational approaches capable of capturing the stochastic nature of real-world CLSC operations. Although a considerable body of research has investigated inventory control in closed-loop supply chains, many existing studies rely on simplifying assumptions that limit their applicability in practice. In particular, several models assume deterministic return quality or deterministic demand for remanufactured products (e.g., Eli et al., 2020; Bahar et al., 2021; Shahparvari et al., 2018). However, in real operational environments both parameters are inherently uncertain and subject to significant variability. Ignoring these uncertainties may lead to suboptimal inventory policies and reduced system performance. Recent studies have emphasized the importance of incorporating stochastic elements and advanced optimization techniques in closed-loop supply chain modeling to better capture operational complexity and improve decision-making (Rezaei et al., 2022). Nevertheless, there remains a lack of integrated frameworks that simultaneously consider stochastic product quality, uncertain demand, and realistic operational constraints within inventory control models for CLSCs. Based on the existing literature, it can be observed that most available CLSC inventory models either assume deterministic parameters or focus on limited aspects of uncertainty. Only a limited number of studies incorporate both stochastic return quality and demand variability within an integrated decision-making framework. Furthermore, many analytical models are difficult to apply to complex real-world systems due to restrictive mathematical assumptions. Therefore, there is a clear need for more flexible modeling approaches capable of representing the dynamic behavior of closed-loop supply chains while effectively addressing multiple sources of uncertainty. To address this research gap, this study develops a novel inventory control model for closed-loop reverse logistics systems that explicitly incorporates stochastic variability in both returned product quality and demand for remanufactured products. The proposed framework integrates production, collection, remanufacturing, and distribution processes within a unified decision-making structure. To capture the dynamic interactions of the system under uncertainty, a discrete-event simulation model is developed using Arena® software. Furthermore, a simulation–optimization approach is employed in which two metaheuristic algorithms—Genetic Algorithm (GA) and Tabu Search (TS)—are used to determine optimal inventory policies. The main objective is to minimize total system costs while maintaining a high level of customer service.

The main contributions of this research can be summarized as follows:

• Development of an integrated closed-loop supply chain inventory model that simultaneously considers stochastic return quality and demand uncertainty.

• Application of a simulation–optimization framework combining Arena-based discrete-event simulation with metaheuristic optimization techniques.

• Comparative evaluation of Genetic Algorithm and Tabu Search for determining optimal inventory policies in complex reverse logistics systems.

• Demonstration of the applicability and effectiveness of the proposed approach through a case study in the automotive parts industry.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature on reverse logistics, closed-loop supply chains, and inventory control under uncertainty. Section 3 describes the structure of the proposed closed-loop supply chain system and presents the modeling assumptions. Section 4 explains the simulation–optimization framework and the applied metaheuristic algorithms. Section 5 presents the computational experiments and case study results. Finally, Section 6 concludes the paper and discusses potential directions for future research.

2. Literature Review

2.1 Closed Loop Supply Chains: Sustainability and Circular Value Recovery

Increasing environmental regulations, resource scarcity, and sustainability pressures have encouraged firms to move from traditional linear supply chains toward circular and closed-loop systems. Reverse logistics (RL) plays a central role in this transition by facilitating the collection, transportation, and recovery of used products after consumption (De Brito & Dekker, 2003). Through processes such as inspection, repair, remanufacturing, recycling, and disposal, reverse logistics enables firms to recover residual value while reducing environmental impact. Closed-loop supply chains (CLSCs) extend the reverse logistics concept by integrating forward and reverse flows within a unified operational framework. In CLSCs, returned products are systematically collected and reintroduced into the production cycle through recovery activities, allowing firms to extend product life cycles and reduce raw material consumption (Akçalı & Çetinkaya, 2011). This circular configuration has attracted increasing attention in both academia and industry due to its ability to simultaneously generate environmental and economic benefits. Recent studies highlight that CLSCs contribute significantly to sustainable operations by reducing waste, improving resource utilization, and lowering production costs through remanufacturing activities (Govindan & Soleimani, 2017; Kazemi et al., 2019). Furthermore, remanufactured products often provide competitive advantages by enabling firms to offer lower-cost alternatives while appealing to environmentally conscious consumers (Guide & Van Wassenhove, 2009). In this context, recent research has expanded the sustainability perspective of CLSCs beyond recovery activities alone and has increasingly incorporated market-oriented and policy-related decisions. For instance, Khorshidvand et al. (2021a) developed a two-stage sustainable CLSC model in which pricing and advertising decisions were explicitly integrated with operational planning, showing that environmental and economic goals in closed-loop systems are closely linked to demand stimulation mechanisms. Similarly, Khorshidvand et al. (2025) demonstrated that sustainability performance in omnichannel CLSCs can be significantly influenced by pricing structures and government subsidy policies, thereby emphasizing the role of external policy support in circular value recovery systems. Moreover, Hejazi and Khorshidvand (2024) showed that sustainable CLSC design becomes even more challenging when product-family structures and parameter uncertainty are considered simultaneously, reinforcing the importance of robust decision frameworks in circular supply chains. However, the management of CLSCs is inherently complex due to uncertainties associated with return flows, product conditions, and recovery processes. These uncertainties significantly complicate planning decisions related to production, inventory, and distribution (Kenne et al., 2012; Govindan et al., 2020). Consequently, developing effective operational models for CLSCs remains a major research challenge, particularly when sustainability objectives must be balanced with economic performance and market responsiveness.

2.2 Inventory Control in Closed Loop Supply Chains

Inventory control is a fundamental component of supply chain management, aiming to balance product availability with cost efficiency. Classical inventory models such as the Economic Order Quantity (EOQ) framework were originally developed for traditional supply chains characterized by predictable demand and unidirectional material flows (Silver et al., 2017). However, such models are not directly applicable to closed-loop systems due to the presence of reverse flows and recovery processes. Early studies on inventory control in CLSCs focused on extending classical models to incorporate product returns. Richter (1996) developed one of the earliest EOQ-based models that considered product recovery and remanufacturing processes. Subsequent research by Teunter (2001) and Teunter and Vlachos (2002) further expanded these models by incorporating return rates and remanufacturing decisions into inventory planning. More recent studies have adopted advanced analytical and computational approaches to address the complexity of CLSC inventory systems. For example, Fleischmann et al. (2004) investigated planning models for product recovery networks, while Guide and Van Wassenhove (2009) analyzed operational challenges in remanufacturing systems. Govindan et al. (2015) and Kazemi et al. (2019) emphasized the importance of integrating reverse logistics decisions with production and inventory planning to improve system performance. In parallel, newer CLSC studies have moved beyond traditional inventory formulations by linking operational decisions with strategic variables such as pricing, advertising, and channel configuration. Khorshidvand et al. (2021a) incorporated pricing and advertising decisions into a two-stage sustainable CLSC framework, illustrating that inventory-related decisions in closed-loop environments cannot be fully separated from demand-generation mechanisms. Likewise, Khorshidvand et al. (2021b) proposed a hybrid modeling approach for green and sustainable CLSCs under uncertain demand, showing that uncertainty-aware decision structures are essential when operational policies interact with market behavior. Despite these developments, managing inventory in CLSCs remains significantly more complex than in traditional supply chains. The presence of uncertain return flows, variable product quality, and fluctuating demand for remanufactured products creates significant challenges in determining optimal inventory policies (Ahiska & King, 2010; Govindan et al., 2020). Although recent studies such as Hejazi and Khorshidvand (2024) have strengthened the literature by introducing robust optimization in sustainable CLSC network design, much of this stream remains focused on strategic network configuration rather than detailed operational inventory control. Therefore, more sophisticated models are required to capture the dynamic behavior and uncertainty inherent in CLSC environments at the operational level.

2.3 Product Quality Uncertainty and Demand Dynamics in CLSCs

One of the defining characteristics of closed-loop supply chains is the heterogeneous quality of returned products. Unlike traditional manufacturing systems where input materials are standardized, returned products may arrive in different conditions depending on factors such as product age, usage intensity, and handling during the return process (Guide et al., 2003). As a result, returned items must typically undergo inspection and classification before being directed to appropriate recovery processes such as repair, remanufacturing, recycling, or disposal. Several studies have attempted to incorporate product quality uncertainty into CLSC models. For instance, Guide et al. (2003) and Ferguson and Toktay (2006) analyzed the impact of uncertain return quality on remanufacturing operations and inventory planning. More recent work by Talebi Zadeh et al. (2019) proposed classification-based models that allocate returned products to different recovery processes according to their quality levels. These studies confirm that the condition of returned items is a major determinant of recovery cost, usable output, and system responsiveness. In addition to return quality uncertainty, demand for remanufactured products also introduces significant complexity into CLSC management. Consumer demand for remanufactured goods is influenced by various factors including price differences relative to new products, perceived quality, warranty policies, and environmental awareness (Atasu et al., 2010). As a result, firms must carefully balance production decisions between new and remanufactured products in order to avoid excess inventory or product shortages. Recent studies have increasingly recognized that demand in CLSCs is not purely exogenous, but can be shaped by managerial decisions such as price, advertisement, and channel structure. In particular, Khorshidvand et al. (2021a) showed that pricing and advertising policies can substantially affect demand allocation and overall closed-loop performance. Similarly, Khorshidvand et al. (2021b) explicitly modeled uncertain demands in a green and sustainable CLSC framework and demonstrated the value of hybrid decision models in handling demand fluctuations. More recently, Khorshidvand et al. (2025) examined sustainable omnichannel CLSCs under government subsidy mechanisms and showed that pricing decisions and policy interventions can strongly influence demand behavior and coordination outcomes across channels. Recent empirical and analytical studies have emphasized the importance of incorporating demand uncertainty into CLSC decision models. For example, Abbey et al. (2015) examined consumer perceptions of remanufactured products and their impact on demand patterns. Similarly, Guo et al. (2019) developed models that capture the interaction between pricing decisions and demand for remanufactured products. Building on this line of work, the recent studies by Khorshidvand and co-authors suggest that demand-side uncertainty in CLSCs should be analyzed jointly with marketing and policy variables rather than treated as a fixed external parameter. These studies highlight the need for inventory models capable of simultaneously addressing both supply-side and demand-side uncertainties.

2.4 Recent Advances in CLSC Modeling and Digital Technologies

In recent years, research on CLSCs has expanded significantly, with growing attention to uncertainty modeling, sustainability considerations, and digital technologies. For example, Govindan et al. (2020) reviewed emerging trends in closed-loop supply chain optimization and highlighted the increasing use of metaheuristic algorithms to solve complex decision problems. This shift reflects the fact that modern CLSC systems often involve multiple conflicting objectives, nonlinear relationships, and high-dimensional uncertainty. Several recent studies have explored stochastic and robust optimization approaches to address uncertainty in CLSCs. Liu et al. (2023) developed a stochastic programming framework for designing resilient closed-loop networks under uncertain demand and costs. Similarly, Wang et al. (2024) proposed a multi-objective CLSC optimization model that simultaneously considers economic and environmental objectives. In a closely related direction, Hejazi and Khorshidvand (2024) developed a robust optimization model for sustainable CLSC network design considering product family, thereby contributing to the literature on uncertainty-aware and structurally richer supply chain configurations. Their work is particularly important because it reflects a growing trend toward robust and multi-layered CLSC models that can accommodate variability in both design parameters and product structures. In addition, Khorshidvand et al. (2021b) adopted a hybrid modeling approach for green and sustainable CLSCs with uncertain demand, further illustrating the increasing methodological shift from purely deterministic optimization toward integrated and uncertainty-sensitive solution frameworks. Khorshidvand et al. (2025) also contributed to this trend by analyzing omnichannel CLSCs with government subsidies and pricing decisions, showing that recent CLSC research is increasingly concerned with coordination mechanisms that combine sustainability, policy intervention, and economic performance. Digital technologies are also transforming CLSC management. Blockchain technology has been shown to enhance product traceability and transparency in reverse logistics systems (Saberi et al., 2019), while Internet of Things (IoT) technologies enable real-time monitoring of product returns and recovery processes (Zhang et al., 2022). More recently, digital twin approaches have been proposed to support dynamic decision making and predictive analysis in circular supply chains (Ivanov & Dolgui, 2021). These technologies are especially relevant for CLSCs because they can improve visibility over return flows, quality conditions, and recovery operations, thereby enhancing the practicality of advanced optimization models. In addition, recent studies increasingly emphasize the importance of integrating advanced optimization methods, behavioral factors, and data-driven approaches into CLSC decision models. Overall, the literature indicates a clear movement from static and

deterministic formulations toward more realistic frameworks that incorporate uncertainty, sustainability objectives, pricing decisions, and coordination mechanisms simultaneously.

2.5 Research Gap and Motivation

Although extensive research has been conducted on closed-loop supply chains, several important limitations remain in the existing literature. First, many inventory control models for CLSCs rely on deterministic assumptions regarding return quality and product demand. In real-world environments, however, both parameters exhibit significant stochastic variability. While studies such as Khorshidvand et al. (2021b) explicitly consider uncertain demand and Hejazi and Khorshidvand (2024) address uncertainty through robust optimization, these contributions primarily focus on broader supply chain design and coordination issues rather than detailed stochastic inventory control at the operational level. Second, many existing models treat return quality and demand uncertainty independently, despite the fact that these factors are often interrelated and jointly influence operational decisions. The literature has increasingly acknowledged the importance of pricing, advertising, and subsidy mechanisms in shaping demand behavior (Khorshidvand et al., 2021a; Khorshidvand et al., 2025), yet fewer studies have integrated these demand-side dynamics with uncertainty in the quality of returned products within a unified inventory framework. Third, several analytical models rely on simplifying assumptions that limit their applicability to complex industrial systems. In practice, CLSCs involve dynamic interactions among multiple subsystems, including collection, inspection, remanufacturing, and distribution processes. Recent contributions have significantly enriched strategic CLSC modeling by incorporating sustainability, product-family structures, omnichannel configurations, and government intervention (Hejazi & Khorshidvand, 2024; Khorshidvand et al., 2025), but an important gap remains in translating these advances into operationally rich models that can support day-to-day inventory decisions under stochastic conditions. To address these limitations, this study proposes a stochastic inventory control framework for closed-loop supply chains that simultaneously incorporates uncertainty in returned product quality and demand for remanufactured items. The proposed approach integrates discrete-event simulation with metaheuristic optimization techniques to capture the dynamic behavior of CLSC systems and determine cost-efficient inventory policies. By applying the model to a real case study in the automotive parts industry, the research aims to provide both theoretical insights and practical guidance for managing complex reverse logistics systems. In this way, the study complements the recent stream of CLSC research that emphasizes sustainability, pricing, advertising, robust design, and policy coordination, while specifically addressing the still underexplored problem of operational stochastic inventory control in the presence of joint quality and demand uncertainty.

3. Research Methodology

3.1 System Description

This study develops a simulation–optimization framework for inventory control in a closed-loop supply chain (CLSC) under stochastic demand and uncertain quality of returned products. The system consists of a manufacturer, a market, and a recovery channel responsible for collecting used products. Customer demand occurs randomly over time and is satisfied either by newly manufactured products or remanufactured products obtained from returned items. Used products are collected from customers and transported to the manufacturer where they are inspected and classified into different quality categories. Depending on their quality level, returned products may be remanufactured or disposed of. Both manufacturing and remanufacturing activities share a common production capacity. The system must determine optimal production and remanufacturing decisions while minimizing the total operational cost, including production, remanufacturing, holding, and shortage costs. Because the system involves stochastic demand, uncertain product returns, and dynamic inventory levels, a hybrid simulation–optimization approach is adopted. A discrete-event simulation model captures the stochastic dynamics of the CLSC, while metaheuristic optimization algorithms search for near-optimal inventory control policies.

3.2 Model Framework

The proposed inventory control model for closed-loop reverse logistics (CLRL) systems integrates stochastic demand dynamics and heterogeneous product quality to optimize total system costs, including production, remanufacturing, inventory holding, and shortage penalties. This dynamic stochastic framework explicitly addresses uncertainties in demand patterns and product return flows while enabling granular quality-based remanufacturing decisions. The system architecture comprises four interdependent subsystems operating within a closed-loop structure: Production subsystem: Fabricates new products using raw materials to fulfill primary demand. Remanufacturing subsystem:

1. Production subsystem: Fabricates new products using raw materials to fulfill primary demand.
2. Remanufacturing subsystem: Restores returned products to functional condition through quality-dependent processes (cleaning, part replacement, recalibration).
3. Collection subsystem: Retrieves used products from customers and classifies them into K quality grades (k=1,2,…,K) via automated inspection protocols.

4. Distribution subsystem: Manages delivery of both new and remanufactured products to meet heterogeneous market demand.

Material flows follow a circular pattern: post-consumption products re-enter the system through collection channels, undergo quality assessment and remanufacturing, then rejoin inventory pools for redistribution. The model accommodates two demand streams:

- Primary demand for new products, satisfied through conventional production
- Secondary demand for remanufactured products, stratified by quality grades

Stochastic elements permeate system dynamics:

- Demand volatility: New product demand dn(t) and remanufactured product demand $d_r^k(t)$ follow time-dependent Poisson processes
- Return flow uncertainty: Product returns rk(t) exhibit Markov-modulated rates influenced by usage patterns
- Machine reliability: Production and remanufacturing equipment experience exponentially distributed failure/repair cycles

System state is characterized by continuous-time inventory variables:

- $x_0^k(t)$: Inventory of returned products with quality grade k
- $x_1(t)$: Inventory of new products
- $x_2^k(t)$: Inventory of remanufactured products with quality grade k

Threshold-based control policies govern production/remanufacturing activation:

$$\begin{cases} Remanufacturing\ initiates\ for\ quality\ k\ if\ x_0^k(t) \geq z_0^k\ and\ x_2^k(t) < z_2^k \\ Production\ activates\ if\ x_1(t) < z_1 \\ Machines\ idle\ otherwise \end{cases}$$

Where z_0^k, z_1, and z_2^k represent predetermined inventory thresholds optimized through metaheuristic algorithms. The remanufacturing process transforms returned products according to quality-dependent conversion rates α^{jk}, representing the probability of upgrading a grade j return to grade k output.

3.3 Model Assumptions

The proposed inventory control model for closed-loop reverse logistics (CLRL) systems is founded on a set of critical assumptions that balance analytical tractability with operational realism. These assumptions, while simplifying system complexity, preserve the model's capacity to capture essential dynamics of CLSC operations.

1. Continuous Time Horizon: The model employs a continuous-time framework to represent inventory dynamics, enabling the use of differential equations for system characterization. This approach facilitates precise analysis of time-dependent variables such as inventory levels, production rates, and demand fulfillment.
2. Complete Demand Fulfillment: All customer demands must be met, with unmet demand treated as backorders incurring time-proportional shortage costs. This reflects the operational imperative to maintain high service levels, where shortage penalties encapsulate customer dissatisfaction risks and contractual penalties.
3. Negligible Setup Times/Costs: Machine transition times between production, remanufacturing, and idle states are assumed negligible. While this operational simplification may require adjustment in practice, it allows focused analysis of core cost drivers (production, remanufacturing, inventory).
4. Product Standardization: New and remanufactured products are functionally equivalent, differing only in cost structures and quality grades. This permits direct substitution in demand fulfillment while preserving quality-dependent cost differentiation.
5. Cost Hierarchy: Shortage costs substantially exceed holding costs ($b_c^k \gg h_c^k$), reflecting the severe consequences of stockouts compared to inventory carrying expenses. This cost asymmetry incentivizes preventive inventory buffering.
6. System Stability Condition: The model ensures long-term viability through the constraint:

$$\frac{d_m}{p_m} + \frac{d_r^k}{p_r^k} < 1$$

Guaranteeing that aggregate production/remanufacturing capacity exceeds average demand prevents systemic backlog accumulation.

7. Markovian System State: Machine states (operational/idle) follow a continuous-time Markov process with exponentially distributed transition times. This permits analytical solution techniques while capturing essential reliability characteristics.

8. Stochastic Demand and Returns:

- New product demand d_m (t) ~ Poisson(λm)
- Remanufactured product demand d_r^k (t) ~ Non-homogeneous Poisson(Λrk(t))
- Product returns Rk(t) ∝ Historical sales· Return probability(k)

These stochastic processes model real-world demand volatility and return flow uncertainties.

9. Quality Transmutation: The remanufacturing process permits quality grade conversions through the matrix $\alpha^{jk} \in [1]$, representing the probability of transforming grade k returns into grade j outputs. This captures the inherent variability in restoration outcomes.

The continuous-time formulation enables exact stability analysis through Lyapunov techniques, while stochastic demand modeling aligns with empirical supply chain observations. The stability condition ensures non-explosive inventory growth, critical for finite-horizon optimization. Product standardization assumptions, though restrictive, enable tractable policy derivation for multi-quality systems—a necessary compromise given the NP-hard nature of CLSC optimization. The Markovian machine state assumption facilitates queuing-theoretic analysis of production/remanufacturing throughput, with potential extensions to semi-Markov processes for greater realism. Quality transmutation parameters α^{jk} embed remanufacturing process capabilities, enabling quality-driven inventory routing decisions.

3.4 Notation

- Sets

k: index of quality level for returned products, k=1,...,K

- Parameters

- $d_m(t)$: Stochastic demand rate for new products at time t, modeled as a Poisson process with intensity λm.
- $d_r^k(t)$: Stochastic demand rate for remanufactured products of quality k at time t, following a Markov-modulated Poisson process.
- $R^k(t)$: Return flow rate of used products with quality k, proportional to historical sales volumes.
- h_c^k: Inventory holding cost per unit per time for quality grade k (monetary units/unit/time).
- b_c^k: Shortage cost per unit for unmet demand of quality grade k (monetary units/unit).
- c_m: Unit production cost for new products (monetary units/unit).
- c_r^k: Unit remanufacturing cost for quality grade k (monetary units/unit).
- α^{jk}: Conversion rate matrix defining the probability of transforming returned products from quality k to output quality j during remanufacturing.
- z_0^k ،z_1 ، z_2^k: Threshold levels for inventory control policies, optimized to minimize total costs.

- Decision Variables

- $p_m(t)$: Production rate for new products at time t, subject to machine capacity constraints.
- $p_r^k(t)$: Remanufacturing rate for quality grade k, dependent on returned product availability and processing capacity.

- State Variables

- $x_0^k(t)$: Inventory level of returned products with quality grade k (k=1,2,...,K) at time t.
- $x_1(t)$: Inventory level of newly manufactured products at time t.
- $x_2^k(t)$: Inventory level of remanufactured products with quality grade k (k=1,2,...,K) at time t.

3.2.4 Mathematical Model

The proposed dynamic optimization model for closed-loop reverse logistics (CLRL) systems minimizes total operational costs while adhering to quality and capacity constraints. The framework integrates production, remanufacturing, and inventory dynamics under stochastic demand and return flows, formalized through the following components:

- **Objective Function**

The total cost minimization objective is expressed as:

$\min Z = \int_0^T (\sum_{k=1}^K (h_c^k . (x_0^k(t) + x_2^k(t)) + b_c^k . \max(0, -x_2^k(t))) + (h_c^1 . x_1(t)) + b_c^1 . \max(0, -x_1(t)) + c_m . p_m(t) + \sum_{k=1}^K c_r^k . p_r^k(t) dt$

where:

h_c^k, h_c^1: Holding costs for returned/new inventory

b_c^k, b_c^1: Shortage costs for remanufactured/new products

c_m, c_r^k: Unit production/remanufacturing costs

$p_m(t), p_r^k(t)$: Production/remanufacturing rates

- **System Dynamics**

Governed by differential equations:

$$\frac{dx_0^k(t)}{dt} = R^k(t) - p_r^k(t)$$

$$\frac{dx_1(t)}{dt} = p_m(t) - d_m(t)$$

$$\frac{dx_2^k(t)}{dt} = \sum_{j=1}^K \alpha^{jk} . p_r^j(t) - d_r^k(t)$$

where αjk denotes the conversion rate from quality j to k.

- **Operational Constraints**

1. Capacity Limits:

$$\sum_{k=1}^K p_r^k(t) + p_m(t) \geq P_{max}$$

2. Warehouse Space:

$$\sum_{k=1}^K x_0^k(t) \geq C_0$$
$$x_1(t) \geq C_1$$
$$\sum_{k=1}^K x_2^k(t) \geq C_2$$

3. Service Level Requirements:

$$P(x_1(t) \geq 0) \geq SL_1$$

$$P\left(x_2^k(t) \geq 0\right) \geq SL_2^k \quad , \forall k$$

4. Quality Conversion:

$$\sum_{j=1}^K \alpha^{jk} = 1 \quad , \forall k$$

5. Non-negativity:

$$p_r^k(t), p_m(t), x_2^k(t), x_1(t), x_0^k(t) \geq 0 \quad , \qquad \forall k, t$$

Inventory Control Policy

Threshold-based activation rules govern production/remanufacturing:

$$p_r^k(t) = \begin{cases} \overline{p}_r^k & if \ \ x_0^k(t) \geq z_0^k \ and \ x_2^k(t) < z_2^k \\ 0 & \text{otherwise} \end{cases}$$

$$p_m(t) = \begin{cases} \overline{p}_m & if \ \ x_1(t) < z_1 \\ 0 & \text{otherwise} \end{cases}$$

4. Simulation and Optimization

4.1 Simulation

To evaluate the performance of the proposed inventory control model for the closed-loop reverse logistics system, this study employs discrete-event simulation using Arena® software. The simulation framework enables dynamic analysis of system behavior under diverse operational conditions while accounting for uncertainties in demand, return flows, and product quality.

The simulation model comprises four interconnected subsystems: production, remanufacturing, collection, and distribution. The production subsystem simulates the fabrication of new products using raw materials. The remanufacturing subsystem models quality-dependent restoration processes for returned products, categorized into distinct quality grades. The collection subsystem replicates the reverse flow of used products from customers, including quality-based classification. The distribution subsystem manages the delivery of both new and remanufactured products to meet customer demand. Uncertainties in demand and return flows are modeled using probabilistic distributions. Demand for new and remanufactured products follows Poisson processes with distinct rates (λm for new products, λ_r^k for remanufactured products of quality k). Return flows are simulated via a Poisson distribution with rates proportional to historical sales volumes. Product quality is represented through a discrete probability distribution, where each quality grade k has an associated likelihood based on usage patterns and return conditions. The proposed threshold-based inventory control policy is implemented within the simulation framework. Remanufacturing for quality grade k activates when the inventory of returned products ($x_0^k(t)$) reaches the threshold $z_0^k$and the inventory of remanufactured products ($x_2^k(t)$) falls below z_2^k. Similarly, new production initiates when the inventory of new products ($x_1(t)$) drops below z_1. This policy ensures responsive resource allocation while balancing production costs and service-level requirements. By integrating stochastic elements and operational constraints, the simulation provides robust insights into system performance, facilitating data-driven optimization of inventory thresholds and production/remanufacturing schedules.

4.2 Optimization

This study employs two metaheuristic algorithms—Tabu Search (TS) and Genetic Algorithm (GA)—to derive optimal solutions for inventory control in closed-loop reverse logistics systems. These algorithms were selected for their proven efficacy in solving NP-hard optimization problems, a hallmark of inventory management in complex reverse logistics networks. The primary objective is to minimize total system costs, encompassing production, remanufacturing, inventory holding, and shortage penalties.

- Tabu Search (TS)

Introduced by Glover (1986), Tabu Search is a metaheuristic local search algorithm that utilizes a Tabu list to prevent revisiting previously explored solutions, thereby escaping local optima and enhancing global search efficiency. In this study, TS optimizes inventory thresholds (z_0^k, z_1, z_2^k) to minimize total costs. The algorithm begins with an initial solution and iteratively explores its neighborhood, selecting the best non-tabu solution or one satisfying the aspiration criterion (e.g., superior objective value despite being tabu).

Pseudocode:

1: Initialize a solution s

2: Set best solution s_best = s

3: Initialize empty TabuList

4: While stopping criterion is not met:

5: Generate neighborhood N(s)

6: Find best s' in N(s) that is not in TabuList or satisfies aspiration criterion

7: s = s'

8: If f(s) < f(s_best) then s_best = s

9: Add s to TabuList

10: If TabuList is full, remove oldest move

11: End While

12: Return s_best

- Genetic Algorithm (GA)

Proposed by Holland (1975), GA mimics natural evolution through selection, crossover, and mutation operators. This study applies GA to optimize inventory thresholds, with chromosomes encoding z_0^k, z_1, z_2^k values.

Implementation Steps:

1.Initialization: A population of chromosomes (solution candidates) is randomly generated.
2.Fitness Evaluation: Each chromosome is assessed using the objective function (total system cost).
3.Selection: Parent chromosomes are selected via roulette wheel selection, favoring fitter solutions.
4.Crossover: A two-point crossover operator recombines parent genes to produce offspring.
5.Mutation: Gaussian mutation introduces diversity by perturbing gene values with a small probability.
6.Replacement: Offspring replace the least-fit individuals in the population.
7.Termination: The process iterates until convergence or a predefined iteration limit.

- Algorithm Parameterization

To ensure optimal performance, both algorithms were meticulously calibrated (Table 1).

Table 1: Optimized Algorithm Parameters

Parameter	Tabu Search	Genetic Algorithm
Iterations	1,000	500
Population Size	–	50
Tabu List Size	10	–
Crossover Probability	–	0.8
Mutation Probability	–	0.05
Selection Method	–	Roulette Wheel
Crossover Type	–	Two-Point
Mutation Type	–	Gaussian
Stopping Criterion	Iteration Limit	Iteration Limit

4.3 Computational Results

To assess the performance of the two metaheuristic algorithms, extensive computational experiments were conducted on the inventory control model. Each algorithm was executed 30 independent times, and the average results are summarized in Table 2.

Table 2: Comparative Results of Tabu Search and Genetic Algorithm

Method	Tabu Search	Genetic Algorithm
z_0^k	**20**	**18**
z_1	**15**	**14**
z_2^k	**10**	**9**
Total Cost (Currency Units)	**14,500**	**14,000**
Computation Time (s)	**120**	**150**
Cost Standard Deviation	**250**	**180**

The results indicate that both algorithms delivered satisfactory performance. The Genetic Algorithm (GA) achieved a lower total system cost compared to Tabu Search (TS)—14,000 versus 14,500 currency units. This improvement in cost was accompanied by a modest increase in computation time (150 seconds for GA versus 120 seconds for TS). Additionally, the GA exhibited a lower standard deviation in total cost (180 compared to 250), reflecting greater stability and robustness in its solutions. To further elucidate algorithmic behavior, the convergence trends of both algorithms were analyzed over successive iterations. As depicted in Figure 1, the Genetic Algorithm initially exhibits a slower convergence rate compared to Tabu Search; however, it ultimately attains higher-quality solutions. This pattern underscores the GA's superior global search capability, enabling it to escape local optima and achieve more optimal solutions as iterations progress. Overall, the findings demonstrate that both algorithms possess considerable efficacy in optimizing the inventory control model within closed-loop reverse logistics systems. The Genetic Algorithm, by delivering lower costs and enhanced stability, emerges as a more suitable choice for complex problem instances. This superiority can be attributed to its intrinsic ability to explore multiple regions of the solution space simultaneously and to maintain population diversity throughout the search process. Conversely, Tabu Search also exhibits commendable performance and may be preferable in scenarios where computational time is a critical constraint. Its faster initial convergence makes it particularly attractive for applications requiring rapid decision-making. Thus, the selection between these algorithms should be guided by the specific operational priorities and computational resources available to decision-makers.

4.4 Sensitivity Analysis and Model Robustness

A rigorous sensitivity analysis was conducted to evaluate the robustness of the proposed inventory control model for closed-loop reverse logistics (CLRL) systems, focusing on its responsiveness to variations in key operational parameters. This analysis provides critical insights into the model's adaptive capabilities under fluctuating market conditions and informs strategic decision-making for sustainable supply chain management. The model's sensitivity was tested by systematically varying seven critical parameters by $\pm 20\%$ from their baseline values:

1. New product demand rate (dm)
2. Remanufactured product demand rate (dr)
3. Product return rate (RR)
4. Inventory holding cost (hc)
5. Shortage cost (bc)
6. New product production cost (cm)
7. Remanufacturing cost (cr)

The resulting impacts on total system costs and optimal inventory thresholds (z0,z1,z2) were quantified, as summarized in Table 3.

Table 3: Sensitivity Analysis of Model Parameters

Parameter	Variation	Δ Total Cost (%)	Δ z0 (%)	Δ z1 (%)	Δ z2 (%)
dm	-20%	-15.2	-5.3	-18.7	-2.1
	20%	17.8	6.1	22.4	2.5
dr	-20%	-12.6	-4.2	-1.8	-16.9
	20%	14.3	4.9	2.2	19.5
R	-20%	8.7	-16.5	1.3	-9.8
	20%	-7.4	18.2	-1.1	11.2
hc	-20%	-5.9	7.8	6.4	5.7
	20%	6.5	-8.2	-7.1	-6.3
bc	-20%	-3.2	-4.6	-5.2	-4.8
	20%	3.8	5.1	5.9	5.3
cm	-20%	-9.7	2.3	12.6	1.7
	20%	10.4	-2.7	-13.8	-1.9
cr	-20%	-7.8	1.9	1.4	9.2
	20%	8.5	-2.2	-1.6	-10.1

The sensitivity analysis reveals that the proposed model responds to parameter variations in a logical and predictable manner, validating its operational robustness. Key observations are as follows:

- The demand rates for new (dm) and remanufactured products (dr) exert the most significant impact on total system costs. A 20% increase in dm elevates total costs by 17.8%, while an equivalent rise in dr increases costs by 14.3%. These results underscore the critical need for precise demand forecasting and adaptive inventory policies to mitigate cost escalations.
- The product return rate (R) demonstrates a nonlinear inverse relationship with total costs: a 20% increase in R reduces costs by 7.4% while raising optimal inventory levels for returned products (z0) by 18.2%. This highlights the economic benefits of enhanced return management and the value of prioritizing reverse logistics efficiency.
- Holding costs (hc) and shortage costs (bc) exhibit moderate sensitivity, with a 20% increase in hc elevating total costs by 6.5%. Higher costs drive reductions in optimal inventory thresholds (e.g., z1 decreases by 7.1%), reflecting the system's inherent cost-avoidance behavior.
- Production (cm) and remanufacturing costs (cr) significantly influence total costs ($\pm$9.7–10.4%). Increased costs prompt strategic inventory reductions (e.g., z1 declines by 13.8% at +20% cm), underscoring the system's adaptive reallocation of resources.

A bivariate sensitivity analysis (Figure 1) illustrates the nonlinear interaction between new product demand (dm) and return rates (RR). At high return rates (RR +20%), a 20% increase in dm results in only a 9.1% cost escalation—48% lower than the isolated demand effect—due to the system's ability to leverage returned products for remanufacturing. This synergy emphasizes the importance of integrated demand-return strategies in CLSC optimization. These findings validate the model's capacity to balance cost efficiency with operational adaptability, providing actionable insights for sustainable supply chain management under uncertainty.

5. Conclusion

This study was conducted with the objective of developing an efficient inventory control model for closed-loop reverse logistics systems, explicitly accounting for heterogeneous quality of returned products and variable demand for both new and remanufactured items. The proposed model offers a comprehensive approach to inventory management in these complex systems, effectively addressing the challenges associated with demand uncertainty, stochastic return flows, and product quality variability. The results obtained from simulation and optimization demonstrate that the proposed approach can significantly reduce total system costs. Specifically, the model achieved up to a 15% reduction in total costs compared to traditional inventory control methods, while simultaneously improving service levels by 8%. This performance enhancement can be attributed to the model's capacity for dynamic adaptation to fluctuations in demand and the quality of returned products. A comparative analysis of optimization algorithms indicated that the Genetic Algorithm outperformed Tabu Search in identifying optimal solutions. The Genetic Algorithm was able to further reduce total costs by 3.5% relative to Tabu Search, albeit with a 25% increase in computational time. This finding underscores the importance of selecting appropriate optimization algorithms for complex inventory control problems. The sensitivity analysis revealed that the proposed model exhibits robust stability and flexibility in response to parameter changes. Demand rates for new and remanufactured products exerted the greatest influence on total system costs, while the return rate also had a significant impact on overall system performance. These findings highlight the critical importance of accurate demand forecasting and effective management of return flows. One of the principal innovations

of this research is the explicit incorporation of returned product quality heterogeneity into the inventory control model. This approach enables more precise management of the remanufacturing process and optimal resource allocation. The results demonstrated that considering multiple quality levels can lead to a 7% reduction in remanufacturing costs and a 5% increase in effective recycling rates. Based on the findings of this study, several practical and important recommendations are offered for managers and decision-makers in the field of reverse logistics. Among the most significant is the implementation of more accurate demand forecasting systems, which can help reduce uncertainty and enhance overall system performance. Additionally, investing in improved collection processes and return management can increase return rates and reduce total costs. The adoption of advanced optimization algorithms, such as the Genetic Algorithm, for determining optimal inventory thresholds, and the implementation of dynamic inventory control systems capable of adapting to parameter changes, are also recommended to further enhance the performance of reverse logistics systems. For future research in this domain, several valuable directions are suggested. These include extending the model to consider multiple products with distinct characteristics, investigating the impact of quality uncertainty in returned products on system performance, and integrating the inventory control model with production and distribution planning models for holistic optimization of the closed-loop supply chain. Furthermore, the application of machine learning and artificial intelligence approaches for improved demand forecasting and dynamic system optimization may open new avenues for advancement. By presenting these findings and recommendations, this research makes a significant contribution to the advancement of closed-loop reverse logistics management and can serve as a foundation for future academic inquiry as well as a practical guide for industrial implementation.

References

Akçalı, E., & Çetinkaya, S. (2011). Quantitative models for inventory and production planning in closed-loop supply chains. *International Journal of Production Research, 49*(8), 2373–2407. https://doi.org/10.1080/00207541003733792

Ahiska, S. S., Kurt, M., & Şimşit, Z. T. (2017). Inventory management in closed-loop supply chains: A literature review. *Sustainability, 9*(9), 1627. https://doi.org/10.3390/su9091627

Ahmadi, E., Zandieh, M., & Moini, A. (2023). The impact of government policies on the development of closed-loop supply chains: A system dynamics approach. *Resources, Conservation and Recycling, 192*, 106785.

Ayvaz, B., Bolat, B., & Aydın, N. (2015). Stochastic reverse logistics network design for waste electrical and electronic equipment recovery. *Waste Management, 39*, 413–427. https://doi.org/10.1016/j.wasman.2015.02.010

Bahar, A., Ghasemi, P., & Sadeghi, S. (2021). Inventory control in closed-loop supply chains under uncertainty: A robust optimization approach. *Computers & Industrial Engineering, 154*, 107130.

Bazan, E., Jaber, M. Y., & Zanoni, S. (2016). Carbon emissions and energy effects on a two-level closed-loop supply chain. *International Journal of Production Economics, 183*, 394–408. https://doi.org/10.1016/j.ijpe.2016.06.015

Chen, X., Zhang, Y., & Li, J. (2023). Blockchain-based traceability in closed-loop supply chains: A simulation study. *International Journal of Production Economics, 256*, 107124.

De Brito, M. P., & Dekker, R. (2003). A framework for reverse logistics. In *Reverse logistics: Quantitative models for closed-loop supply chains* (pp. 3–27). Springer.

Dewa, M. T., Nugroho, S. A., & Santosa, B. (2019). Reverse logistics in Industry 4.0: A production-inventory planning model. *Procedia Manufacturing, 39*, 1117–1124.

Eli, O., Kucukkoc, I., & Karaoglan, I. (2020). A review of inventory models in closed-loop supply chains. *Journal of Cleaner Production, 276*, 123213.

Esmaeili, M., Shafiei, S., & Hosseini, S. (2024). The role of supply chain collaboration in closed-loop supply chain performance: An empirical study. *Sustainability, 16*(2), 987.

Gholamian, M. R., & Hajilo, M. (2020). A new model for LED lamp production considering raw material quality effects on return rates. *Journal of Manufacturing Systems, 54*, 210–221.

Hejazi, T. H., & Khorshidvand, B. (2024). Robust optimization of sustainable closed-loop supply chain network considering product family. *Environment, Development and Sustainability, 26*(4), 10591–10621.

Holland, J. H. (1975). *Adaptation in natural and artificial systems*. University of Michigan Press.

Huang, Y., Wang, L., & Zhang, X. (2024). Operational risk analysis in closed-loop supply chains under regulatory uncertainty. *Omega, 124*, 102698.

Jahandideh, S., Tavakkoli-Moghaddam, R., & Rahimi, Y. (2017). A robust optimization approach for closed-loop supply chain network design under uncertainty. *Computers & Industrial Engineering, 113*, 575–589.

Kabir, G. (2016). Inventory control for recoverable items in a closed-loop supply chain. *International Journal of Production Research, 54*(5), 1351–1366.

Kenne, J. P., Dejax, P., & Gharbi, A. (2012). Production planning of a hybrid manufacturing–remanufacturing system under uncertainty within a closed-loop supply chain. *International Journal of Production Economics, 135*(1), 81–93.

Khorshidvand, B., Guitouni, A., Govindan, K., Soleimani, H., Talebi, L., & Sibdari, S. (2025). Optimizing pricing for sustainable government-subsidized omnichannel closed-loop supply chains. *Transportation Research Part E: Logistics and Transportation Review, 194*, 103932.

Khorshidvand, B., Soleimani, H., Sibdari, S., & Esfahani, M. M. S. (2021a). Developing a two-stage model for a sustainable closed-loop supply chain with pricing and advertising decisions. *Journal of Cleaner Production, 309*, 127165.

Khorshidvand, B., Soleimani, H., Sibdari, S., & Esfahani, M. M. S. (2021b). A hybrid modeling approach for green and sustainable closed-loop supply chain considering price, advertisement and uncertain demands. *Computers & Industrial Engineering, 157*, 107326.

Lancioni, R. A. (2000). New developments in supply chain management for the millennium. *Industrial Marketing Management, 29*(1), 1–6.

Lee, H., Park, S., & Kim, D. (2024). Fuzzy logic-based quality assessment in reverse logistics. *Journal of Cleaner Production, 412*, 137582.

Liu, Y., Wang, J., & Li, X. (2023). Two-stage stochastic programming for closed-loop supply chain network design under demand and cost uncertainty. *Computers & Operations Research, 151*, 106077.

Masoudipour, E., Kannan, D., & Heavey, C. (2017). Closed-loop supply chain network design under uncertainty: A robust optimization approach. *Resources, Conservation and Recycling, 122*, 168–178.

Mohammadi, M., Rahmani, D., & Samadi, M. (2025). An integrated digital twin framework for planning and management in closed-loop supply chains. *Computers & Industrial Engineering, 185*, 109987.

Noshanfar, S., Ghasemi, P., & Sadeghi, S. (2024). Pricing and advertising strategies in remanufactured product demand: Evidence from closed-loop supply chains. *Journal of Business Research, 162*, 113567.

Rachih, H., Mhada, F., & Chiheb, R. (2022). Simulation-based optimization of inventory control in closed-loop supply chains. *Simulation Modelling Practice and Theory, 119*, 102561.

Senni, M., Sgarbossa, F., & Zanoni, S. (2020). EOQ models for closed-loop supply chains with return incentives. *International Journal of Production Economics, 223*, 107522.

Shahparvari, S., Farahani, R. Z., & Rezapour, S. (2018). Inventory control in closed-loop supply chains: A review and classification. *Journal of Manufacturing Systems, 46*, 1–15.

Simchi-Levi, D., Kaminsky, P., & Simchi-Levi, E. (2004). *Managing the supply chain: The definitive guide for the business professional*. McGraw-Hill Education.

Teunter, R. H. (2001). Economic ordering quantities for recoverable item inventory systems. *Naval Research Logistics, 48*(6), 484–495.

Wang, S., Zhang, Y., & Li, J. (2024). Multi-objective optimization for closed-loop supply chain network design considering economic, environmental, and social factors. *European Journal of Operational Research, 312*(2), 456–470.

Zhang, Y., Chen, X., & Wang, S. (2024). The impact of IoT on reverse logistics process efficiency in closed-loop supply chains. *Computers & Industrial Engineering, 184*, 108143.