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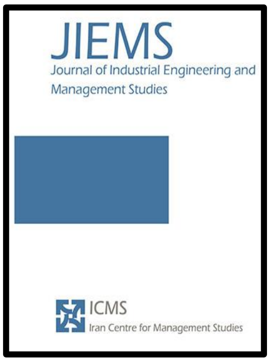

Coordinating pricing and periodic review replenishment decisions in a two-echelon supply chain using quantity discount contract

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Abstract

In this paper, the coordination of pricing and periodic review inventory decisions in a supplier-retailer supply chain (SC) is proposed. In the investigated SC, the retailer faces a stochastic price dependent demand and determines the review period, order-up-to-level, and retail price. On the other hand, the supplier decides on the replenishment multiplier. Firstly, the decentralized and centralized decision making models are established. Afterwards, a quantity discount contract as an incentive scheme is developed to coordinate the pricing and periodic review replenishment decisions simultaneously. The minimum and maximum discount factors, which are acceptable to both SC members, are determined. In addition, a set of numerical examples is conducted to demonstrate the performance of the proposed coordination model. The results demonstrate that the proposed coordination mechanism can improve the profitability of SC along with both the SC members in comparison with the decentralized model. In addition, the results revealed that the proposed incentive scheme is able to achieve channel coordination. Moreover, the coordination model can fairly share the surplus profits between SC members based on their bargaining power.

Keywords: Supply chain coordination, Periodic review replenishment, Pricing, Quantity discount contract.

1. Introduction

In today's competitive market, the supply chain coordination (SCC) is of great significance. In the lack of coordination, the SC members optimize their own decisions independently. Uncoordinated decision making in the SC causes "double marginalization" which is one of the main sources of SC inefficiencies (Spengler 1950). The SC coordination problem seeks for the strategies to persuade the SC members to make optimum decisions from the whole SC viewpoint and improve the SC efficiency by aligning the decisions throughout the SC (Sinha and Sarmah 2010). The pricing decisions as one of the main issues in the supply chain need to be coordinated as well (Xie and Wei 2009). The pricing issue plays an effective role in the success of enterprises in the final market (Azari Khojasteh, Amin-Naseri, and Nakhai Kamal Abadi 2013).

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In the traditional business environment, the retailer often individually decides on the retail price without considering the other SC members. However, under a price-sensitive demand, the decision on the retail price impacts on the profitability of the upstream SC members as it determines the market share of the SC. Therefore, coordinating the pricing decision can greatly improve the efficiency of the SC and solve the "double marginalization" problem (Mokhlesian et al. 2015).

Recently, simultaneous optimization of the inventory and pricing decisions has been vastly investigated in the literature. These models determine the related inventory and pricing decisions in order to maximize the SC member's profit (Sajadieh and Akbari Jokar 2009). In this field of research, firstly, Whitin (1955) incorporated the pricing decision into the newsboy model by considering a linear price sensitive relation for the end customer's demand.

Although there is a vast literature on the coordination of EOQ inventory and pricing decisions (see Table 1), there are no papers which are dealt with coordinating periodic review inventory and pricing decisions in a supply chain. The periodic review inventory systems are one of the main policies for replenishing products which are extensively applied in the grocery stores, supermarkets, pharmacies and so on. However, According to Nematollahi, Hosseini-Motlagh, and Heydari (2017b) there are a few studies on coordinating periodic review inventory decisions.

In this paper, a quantity discount contract as an incentive scheme is developed to coordinate the pricing and replenishment decisions under the periodic review inventory system in a two-echelon supply chain. In the proposed SC model, the retailer faces a price sensitive demand and uses a periodic review inventory system. The pricing and replenishment decisions made by the retailer not only impacts on his/her profitability, but also influence the profitability of the supplier and the whole SC. The investigated SC is modeled under three decision-making structures: (1) the decentralized decision-making structure, (2) the centralized decision-making structure, and (3) the coordinated decision-making structure. Under the decentralized model, the retailer optimizes the review period, order-up-to-level, and retail price decisions individually while the supplier determines the replenishment multiplier based on his/her own objective function. In the centralized model, SC decisions are made from the entire SC perspective and consequently the result of the centralized model can be considered as a benchmark for the whole SC profitability. However, the centralized solution is not necessarily mutually beneficial. In order to transition from the decentralized structure to centralized model, a quantity discount contract as a coordination mechanism is developed to align the decisions of both the SC members. The main contribution of our investigation to the current literature is the coordinating pricing and periodic review inventory decisions simultaneously.

The paper is organized as follows. The literature review is presented in section 2. Section 3 describes the assumptions and notations. Section 4 includes supply chain modeling under three different decision structures and solution procedures. Next, numerical examples are provided in section 5 and, finally, section 6 concludes the paper and discusses future research directions.

2. Literature review

The current study is relevant to the two main categories in the literature: (1) the supply chain coordination and (2) the inventory systems considering pricing policies.

2.1. Supply chain coordination

Coordination strategies aim to encourage SC members to adopt decisions which are optimal for the entire SC (Giannoccaro and Pontrandolfo 2004).

To coordinate the SC members, contracts as a coordination plan are designed effectively among the decentralized SC decision makers to fairly share the achieved profits (Wang and Liu 2007). A well-designed contract can guarantee the participation of all SC members in the coordination model, in addition to the improvement of SC members' performance (Govindan, Diabat, and Popiuc 2012).

Hitherto, various contracts such as collaborative decision making (Nematollahi, Hosseini-Motlagh, and Heydari 2017a) quantity discount (Li, Wang, and Dai 2016; FerhanÇebi 2016; Johari, Hosseini-Motlagh, and Nematollahi 2017), revenue sharing (Panda 2014; Arani Vafa, Rabbani, and Rafiei 2016), buy back (Dutta, Das, and Schultma 2016), two-part tariff (Goering 2012), credit option (Heydari 2013; Hojati et al. 2017), sales rebate (Saha 2013) etc., have been applied in supply chains for resolving the SC members conflicts. Cachon (2003) and Sarmah, Acharya, and Goyal (2007) have provided a comprehensive survey on supply chain coordination.

The quantity discount contracts are more used in practice rather than the other contracts. Under the quantity discount contracts, the supplier improves his/her performance through more sales of products and reduction of his operational costs; on the other hand, the retailer benefits from a discount in wholesale price (Taleizadeh and Pentico 2014). Li and Liu (2006) proposed a discount model for demand uncertainty conditions, dividing the benefits achieved by the discount between two parties equitably based on their bargaining power. Hsu and Lee (2009) developed an inventory model with negotiable backorders between a supplier and a buyer. The supplier offers discounts for backordered items. The model is further extended to include the case of negotiable lead times. Zhang et al. (2016) studied the buyer–vendor coordination of an integrated production-inventory system with quantity discount for a fixed lifetime product under the finite production rate and deterministic demand. In order to improve the supply chain efficiency, they proposed a quantity discount contract to coordinate the buyer–vendor chain. Most researches on the SC coordination models using quantity discount have assumed deterministic demand, zero lead time, and unallowable shortage (Sarmah, Acharya, and Goyal 2006).

2.2. Inventory systems considering pricing policies

The pricing decision has a remarkable effect on the demand as well as optimal inventory policy (Panda et al. 2015). The optimal pricing strategy can be considered as a way of attracting the customers in any business organization (Sana 2011). In addition, the appropriate pricing helps the firms to obtain a revenue with respect to the good's and service's value and keeps the firm's position among its competitors (Esmaeili, Rasti-Barzoki, and Hejazi 2016). Cohen (1977) was the first who examined the interaction effect of pricing and ordering decisions for a retailer who sold a perishable item in a deterministic marketplace. He expanded the classic inventory model by incorporating the issues of pricing, exponentially decaying products, and shortages in his models. Afterward, Emmons et al. (1998) modeled the relationship between a manufacturer and a retailer in a single period setting with price dependent demand uncertainty. They focused on the effect of buy back contracts on both a retailer's and a manufacturer's profits. In another research, Boyacı and Gallego (2002) analyzed the coordination of both pricing and lot sizing decisions in a single wholesaler-multiple retailer system faced with deterministic price sensitive demand. They showed that an inventory consignment selling agreement can maximize the channel profits. Yang (2004) developed an optimal pricing and ordering model for a deteriorating item with price sensitive demand in a vendor-buyer chain. He used a quantity discount contract to convince the buyer to accept the integrated decision making model.

Then, Dumrongsiri et al. (2008) studied a dual channel supply chain where a manufacturer sold to a retailer as well as to consumers directly. The consumers selected the purchase channel according to the price and service qualities. The manufacturer decided on the price of the direct channel and the retailer decided on both the price and order quantity. Sajadieh and Akbari-Jokar (2009) developed an integrated production–inventory marketing model for two-stage supply chain. They found out that supply chain coordination could make a reduction in the selling price, and their analysis showed that it was beneficial for supply chain members to cooperate with each other in competitive environments. Sinha and Sarmah (2010) analyzed the coordination and competition issues in a two-stage supply-chain in which two vendors competed to sell different products through a common retailer in the same market. The demand of a product not only depended on its own price, but also on the price of the other. They focused on the effect of competition on two-stage supply chain coordination under the framework of EOQ and 'Joint Economic Lot Size (JELS)' policy. SeyedEsfahani, Biazaran, Gharakhani (2011) proposed four game-theoretic models including Nash, Stackelberg-manufacturer and Stackelberg-retailer, and cooperative models to coordinate joint pricing and co-op advertising decisions in a manufacturer-retailer chain. The results indicated that under cooperation model the highest amount of profit for the entire SC could be achieved. Panda et al. (2015) investigated pricing and replenishment policies for a high-tech product in a dual-channel supply chain. They assumed that unit cost of the product decreased over its short life cycle. They considered the manufacturer as the Stackelberg leader. They optimized simultaneous pricing and replenishment decisions by profit sharing mechanism through wholesale price adjustment. Seifbarghy et al. (2015) considered a two-echelon supply chain consisting of one manufacturer and one retailer. The demand was assumed constant and depended on the price and quality degree of the final product. They designed a revenue sharing contract and obtained optimal values of whole sale price and quality degree. Heydari and Norouzinasab (2015) proposed a discount model to coordinate pricing and ordering decisions in a supplier-retailer chain. They considered a stochastic demand which depended on the retail price. In their investigated model, the retailer decided on the selling price and order size. Roy et al. (2016) formulated a dual channel model for a manufacturer-retailer chain with a single product. They assumed that manufacturer used the direct online channel and traditional retail channel to increase sell. They analyzed a single-period news vendor type demand in the cases of integrated and Stackelbarg game approaches to obtain optimal stock level, sales prices, promotional effort and service level for both the e-tail and retail channel. They showed that dual channels influence significantly the pricing strategies and effort levels of the supply chain entities.

To sum up, Table 1 illustrates the main differences between aforementioned researches and the current study. As shown, none of the models have studied the coordination of periodic review inventory and pricing decisions simultaneously. To fill this research gap, we have been motivated to address the coordination of the pricing and replenishment decisions under the periodic review inventory system in a two echelon supply chain facing the price sensitive demand.

Table 1. Comparing previous related works and the current study

Reference	Decisions	Inventory system	Supply chain structure	Demand	Coordination mechanism
Emmons et al. (1998)	Pricing, replenishment	EOQ	Manufacturer-retailer	Stochastic, price dependent	Buy back
Boyacı and Gallego (2002)	Pricing, replenishment	-	Single wholesaler-multiple retailers	Deterministic, price dependent	Whole sale price discount
Yang (2004)	Pricing, replenishment	Replenishment period	Vendor-buyer	Deterministic, price dependent	Quantity discount
Dumrongsiri et al. (2008)	Pricing, order quantity	EOQ	Manufacturer-retailer	Stochastic, price and service quality dependent	Whole sale price discount
Sajadieh and Akbari-Jokar (2009)	Pricing, replenishment	EOQ	Vendor-buyer	Deterministic, price dependent	Whole sale price discount
Bin et al. 2010	Pricing, production	-	Manufacturer-retailer	Stochastic, price dependent	Two kind of contracts
Sinha and Sarmah (2010)	Pricing, order quantity	EOQ	Multiple vendors-single retailer	Deterministic, price dependent	Whole sale price discount
SeyedEsfahani, Biazaran, Gharakhani (2011)	Pricing, advertising	-	Manufacturer-retailer	Deterministic, price and advertisement dependent	Game theoretic approach
Panda et al. (2015)	Pricing, replenishment	Replenishment period	Manufacturer-retailer	Deterministic, price dependent	Whole sale price
Seifbarghy et al. (2015)	Pricing, quality degree		Manufacturer-retailer	Price and quality dependent	Revenue sharing
Heydari and Norouzinasab (2015)	Pricing, order quantity	continuous review	Supplier-retailer	Stochastic, price dependent	Whole sale price discount
Roy et al. (2016)	Pricing, order quantity	EOQ	Manufacturer-retailer	Stochastic, price and service quality and promotional effort dependent	Vertical integration
Heydari and Norouzinasab (2016)	Pricing, order quantity, lead time	EOQ	Manufacturer-retailer	Stochastic, price and lead time dependent	Whole sale price discount
Current study	Pricing, review period, order-up-to-level	Periodic review inventory	Supplier-retailer	Stochastic, price dependent	Quantity discount

3. Notations and assumptions

The following notations and assumptions are used throughout this paper.

3.1. Notations

$\mathrm{D(p_r)}$: Expected demand rate per year at retail price $\mathrm{p_r}$

$\mathrm{p_r}$: Retail price (decision variable)

a : Market size

b : Price-elasticity coefficient of demand

T : Length of a review period (decision variable)

R : Order-up-to-level (decision variable)

L : Length of the lead time

X^+: Maximum value of x and 0, that is $X^+ = \max\{x, 0\}$

X : Protection interval $(\mathrm{T+L})$ demand that has a normal distribution function with finite mean $\mathrm{D(T+L)}$ and standard deviation $\sigma\sqrt{\mathrm{T+L}}$

σ: Standard deviation of the demand per unit time

A_r: Retailer's fixed ordering cost per order

h_r: Retailer's inventory holding cost per item per year

n : A positive integer representing the amount of supplier replenishment as a multiple of retailer's order quantity (decision variable)

w : Wholesale price

e : Purchase cost of the supplier per item

A_s: Supplier’s fixed ordering cost per order

h_s: Supplier’s average inventory holding cost per item per year

π : Shortage cost per item short

β : Proportion of the demand during the stock-out period that will be lost, $0 < \beta < 1$

α: Bargaining power of retailer

3.2. Assumptions

1) There is a single-retailer and a single-supplier for a single product in the two-echelon supply chain.

2) The inventory level is reviewed every T units of time. A suitable ordering quantity is ordered up to the level R, and the ordering quantity is obtained after L units of time.

3) The length of the lead time L is less than the cycle length T such that there is never more than a single order outstanding in any cycle.

4) Demand function follows a normal distribution with finite mean $(\mathrm{D(p_r)})$ and standard deviation (σ). The mean of demand $\mathrm{D(p_r)}$ is a linear function of retail price given by $(\mathrm{p_r}) = \mathrm{a} - \mathrm{bp_r}$ similar

to the work of Heydari and Norouzinasab (2015) in which a is market size and b is the price-elasticity coefficient of demand.

5) The order-up-to-level R=expected demand during protection interval + safety stock (SS), which SS = (safety factor)*(standard deviation of protection interval demand), and consequently $R = D(T + L) + k\sigma\sqrt{T + L}$.

6) During the stock out period, a fraction β of the demand will be lost (partial lost sale).

4. Mathematical models and optimal solution procedures

The investigated SC consists of one supplier and one retailer. The supplier uses a lot for lot strategy for replenishing its inventory and decides on the replenishment multiplier (n). The retailer faces a stochastic price-sensitive demand with normal distribution. To this end, mean of demand is considered as a linear function of the retail price. The lead time is deterministic and constant. Moreover, the demand will be partially lost if the customer's needs are not met instantly. The retailer uses the periodic review inventory system (R, T). According to the demand information and its cost structure, the retailer decides on the retail price, review period, and order-up-to-level decisions simultaneously. After the retailer's decision, the supplier determines the replenishment multiplier for replenishing the items. It is assumed that the supplier has enough and finite capacity to meet the retailer's orders. Figure 1 illustrates the investigated two echelon supply chain.

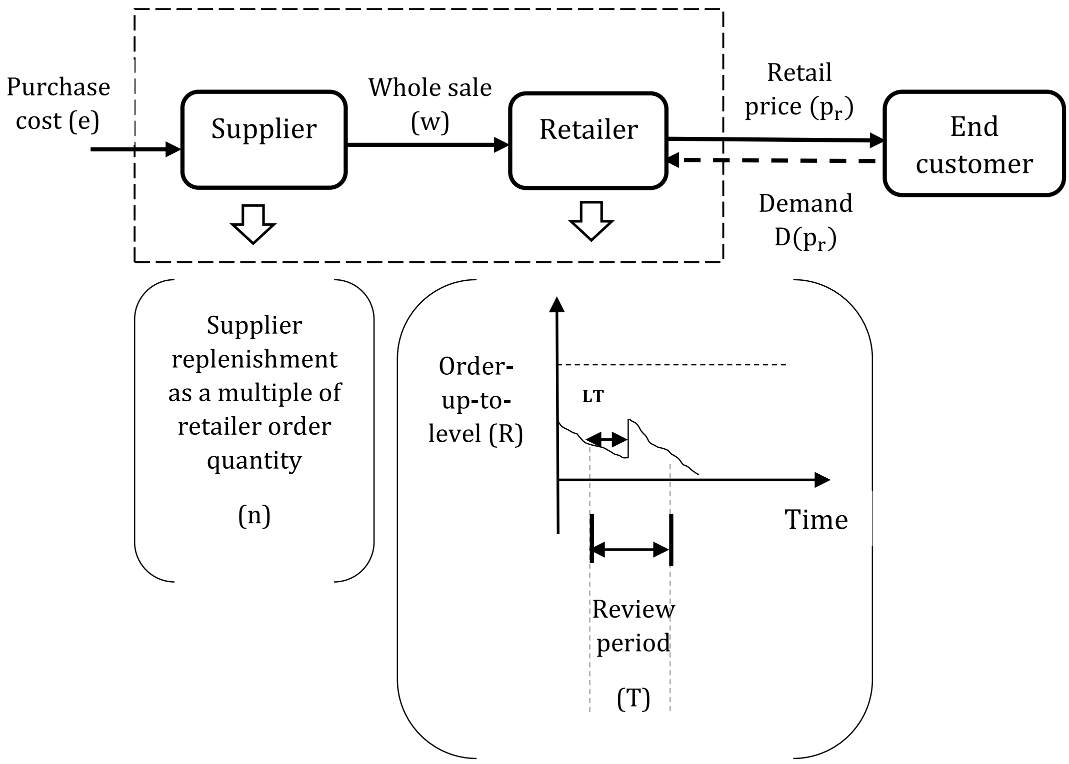

Figure 1. Investigated supplier-retailer chain

4.1. Decentralized structure

Under the decentralized decision making setting, each member tries to maximize its own profit function individually regardless of the other SC members.

4.1.1. Retailer profit function

The expected annual customer demand (mean of demand) is a linear function of the retail price given by $D(p_r) = a - bp_r$. According to Montgomery, Bazaraa, and Keswani (1973) under the periodic review inventory system, the expected holding cost per year is $h_r\left[R - DL - \frac{DT}{2} + \beta E(X-R)^+\right]$ and the expected stock out cost is $\frac{\pi+\beta(P-w)}{T}E(X-R)^+$.

Let $\pi_r(T, R, P_r)$ denotes the expected annual profit function of the retailer, therefore, we have:

$$\pi_r(T,R,P_r) = (P_r - w)(a - bP_r) - \frac{A_r}{T} - h_r\left[R - (a - bP_r)L - \frac{(a - bP_r)T}{2} + \beta E(X-R)^+\right] - \frac{\pi + \beta(P_r - w)}{T}E(X-R)^+ \quad (1)$$

The expected shortage quantity $E(X-R)^+$ at the end of the cycle can be expressed as (Ouyang and Chuang 2000)

$$E(X-R)^+ = \int_R^\infty (X-R)\, f_x dx = \sigma\sqrt{T+L}\psi(k) > 0 \quad (2)$$

In which, $\psi(k) = \varphi(k) - k[1 - \Phi(k)]$ and $\varphi(k)$ and $\Phi(k)$ denote the standard normal p.d.f. and cumulative distribution function $(d.f.)$, respectively. In the rest of this paper, for the sake of simplicity, the safety factor k will be used as a decision variable instead of the order-up-to-level R and therefore the profit function of the retailer can be transferred to:

$$\pi_r(T,k,P_r) = (P_r - w)(a - bP_r) - \frac{A_r}{T} - h_r\left[\frac{(a - bP_r)T}{2} + k\sigma\sqrt{T+L} + \beta\sigma\sqrt{T+L}\psi(k)\right] - \frac{1}{T}(\pi + \beta(P_r - w))\sigma\sqrt{T+L}\psi(k) \quad (3)$$

Where the first term denotes the retailer's expected annual revenue. The second and third terms denote the expected annual ordering cost and annual holding cost, respectively. The last term denotes the expected annual lost sales penalty and opportunity costs. According to Eq. (3), the retailer decides on T, k, and P_r to maximize its own profit function.

Proposition 1. *The retailer profit function is strictly concave with respect to T, k, and P_r under some circumstances*.

Proof. See ''Appendix A''.

By optimizing the retailer profit function with respect to T, the optimal value of T can be obtained through Eq. (4)

$$\frac{A_r}{T^2} = \frac{h_r(a - bP_r)}{2} + \frac{h_r\sigma\big(k + \beta\psi(k)\big)}{2\sqrt{T+L}} - \frac{\big(\pi + \beta(P_r - w)\big)\sigma\psi(k)\sqrt{T+L}}{T^2} + \frac{\big(\pi + \beta(P_r - w)\big)\sigma\psi(k)}{2T\sqrt{T+L}} \quad (4)$$

Similarly, by optimizing the retailer profit function with respect to k, the optimal value of k will be

$$1 - \Phi(k) = \frac{h_r}{h_r\beta + \frac{1}{T}\left(\pi + \beta(P_r - w)\right)} \tag{5}$$

The optimal value of P_r is obtained by optimizing the retailer profit function with respect to P_r

$$P_r = \frac{a}{2b} + \frac{w}{2} + \frac{h_r T}{4} - \frac{\beta\sigma\sqrt{T+L}\psi(k)}{2bT} \tag{6}$$

Three equations (4), (5) and (6) are circularly depending on each other. Hence, a solution procedure is proposed to find the optimal values of T, k, and P_r as follows:

Locally optimal algorithm

Step 1: Set T be equal minimum feasible value.
Step 2: Set P_r be equal minimum feasible value.
Step 3: Calculate k using Eq. (5).
Step 4: Calculate P_r using Eq. (6) and based on obtained k.
Step 5: use a numerical search technique to obtain T which satisfies Eq. (4) and repeat third, fourth, and fifth steps to converge.
Step 6: The obtained T, k, and P_r are optimum.

The optimal policy of the retailer in the decentralized decision making model, which obtained from the above solution procedure, is denoted by T^*, k^*, and ${P_r}^*$.

4.1.2. Supplier profit function

The problem of the supplier under the decentralized model is about his/her ordering strategy individually. The supplier receives the orders from the retailer in stable epochs, based on the retailer's review periods. According to Rosenblatt and Lee (1985) under this situation, the order multiplier n for the supplier must be a positive integer to optimize the supplier's replenishment policy. The supplier incurs an ordering and inventory holding costs. Moreover, in the lost sale inventory systems, the total demand transmitted to the supplier will be less than the total demand of the market. Thus, the supplier's expected profit function is formulated as:

$$\pi_s(n) = (w - e)\left((a - bP_r) - \frac{\beta}{T}\sigma\sqrt{T+L}\psi(k)\right) - \frac{A_s}{nT} - h_s\left[\frac{(n-1)\left(\left((a - bP_r)T\right) - \beta\sigma\sqrt{T+L}\psi(k)\right)}{2}\right] \tag{7}$$

Proposition 2. *The supplier profit function is concave with respect to n.*

Proof. See ''Appendix B''.

n^* is the supplier's lot size multiplier so that it maximizes its profit function under the decentralized model. By releasing the constraint that n is an integer, we have:

$$n^* = \sqrt{\frac{2A_s}{h_s T\left((a - bP_r)T - \beta\sigma\sqrt{T+L}\psi(k)\right)}} \tag{8}$$

Since n must be an integer variable, either the smallest following integer or largest previous integer of n whichever results in larger value of $\pi_s(n)$ will be optimum value of n from the supplier's view point.

4.2. Centralized structure

Under centralized decision making setting, it is assumed that a central decision maker aims to maximize the entire SC profit. In this situation, the pricing and replenishment policies are determined from the entire SC perspective. Let $\pi_{SC}(T, k, P_r, n)$ be the expected annual profit function of SC that is the sum of the retailer and the supplier annual expected profit, then we have:

$$\begin{aligned}\pi_{SC}(T, k, P_r, n) &= \pi_r(T, k, P_r) + \pi_s(n) \\ &= (P_r - e)(a - bP_r) - \frac{1}{T}\left[A_r + \frac{A_s}{n}\right] \\ &\quad - \frac{\sigma\sqrt{T+L}\psi(k)}{T}\left[\big(\pi + \beta(P_r - e)\big) - \frac{h_s(n-1)\beta T}{2}\right] \\ &\quad - \frac{(a - bP_r)T}{2}[h_r + h_s(n-1)] \\ &\quad - h_r\left[K\sigma\sqrt{T+L} + \beta\sigma\sqrt{T+L}\psi(k)\right]\end{aligned} \tag{9}$$

Proposition 3. The *supply chain profit function is concave with respect to T, k, P_r, n under some circumstances.* (For details see ''Appendix C'').

Let $T^{**}, k^{**}, {P_r}^{**}, n^{**}$ denote the values of decision variables that maximize SC profit function by releasing the constraint that n is an integer.

By optimizing the SC profit function $\pi_{SC}(T, k, P_r, n)$ with respect to T, the optimal value of T will be

And also, by optimizing the SC profit function $\pi_{SC}(T, k, P_r, n)$ with respect to k, the optimal value

$$\begin{aligned}\frac{1}{T^{**2}}\left(A_r + \frac{A_s}{n^{**}}\right) &= \left[\big(\pi + \beta({P_r}^{**} - e)\big)\sigma\psi(k^{**})\right]\left(\frac{1}{2T^{**}\sqrt{T^{**}+L}} - \frac{\sqrt{T^{**}+L}}{T^{**2}}\right) \\ &\quad - \frac{h_s(n^{**}-1)\beta\sigma\psi(k^{**})}{4\sqrt{T^{**}+L}} + \frac{(a - b{P_r}^{**})}{2}[h_r + h_s(n^{**}-1)] \\ &\quad + h_r\left[\frac{k^{**}\sigma}{2\sqrt{T^{**}+L}} + \frac{\beta\sigma\psi(k^{**})}{2\sqrt{T^{**}+L}}\right]\end{aligned} \tag{10}$$

of k will be

$$1 - \Phi(k^{**}) = \frac{h_r}{h_r\beta + \frac{1}{T^{**}}\left[\big(\pi + \beta({P_r}^{**} - e)\big) - \frac{h_s(n^{**}-1)\beta T^{**}}{2}\right]} \tag{11}$$

Similarly, the optimal value of P_r is obtained by optimizing the SC profit function $\pi_{SC}(T, k, P_r, n)$ with respect to P_r as follows

$${P_r}^{**} = \frac{a}{2b} + \frac{e}{2} - \frac{\beta\sigma\sqrt{T^{**}+L}\psi(k^{**})}{2bT^{**}} + \frac{T^{**}}{4}[h_r + h_s(n^{**}-1)] \tag{12}$$

And by optimizing the SC profit function $\pi_{SC}(T, k, P_r, n)$ with respect to n, the optimal value of n will be

$$n^{**} = \sqrt{\frac{2A_s}{h_s T^{**}\left[\big((a - b{P_r}^{**})T^{**}\big) - \beta\sigma\sqrt{T^{**}+L}\psi(k^{**})\big)\right]}} \tag{13}$$

Since the values of T^{**}, k^{**}, P_r^{**} , n^{**} are circularly depending on each other, then a solution procedure is established to find the optimal values of T, k, P_r, and n as follows:

Globally optimal algorithm

Step 1: set T be equal minimum feasible value.
Step 2: Set P_r be equal minimum feasible value.
Step 3: Set n = 1 (minimum feasible value for *n*).
Step 4: Calculate k using Eq. (11).
Step 5: Calculate P_r using Eq. (12) and based on obtained k.
Step 6: Calculate n using Eq. (13) and based on obtained k, P_r.
Step 7: use a numerical search technique to obtain T which satisfies Eq. (10) and repeat fourth, fifth, and sixth steps to converge values of T, k, P_r and n.
Step 8: Calculate whole SC expected profit function at the smallest following integer and largest previous integer of n; whichever leads to greater value of $\pi_{SC}(T, k, P_r, n)$ is chosen as optimal n.
Step 9: The obtained T, k, P_r, and n are optimum.

Although centralized decision making improves the SC profitability, it is not capable of making more profitability for all SC members toward the decentralized model. Hence, to encourage the members to accept the centralized solution, an incentive scheme is designed to improve all SC members' profitability.

4.3. Coordination mechanism

Shifting from the local decisions (decentralized model) to the global decisions (centralized solution) causes losses for the retailer. Therefore, without sufficient incentives, the centralized solution cannot be accepted by both SC members. Hence, an incentive mechanism needs to convince the retailer to shift its decisions from (T^*, k^*, P_r^*) to $(T^{**}, k^{**}, P_r^{**})$ and the supplier moves from (n^*) to (n^{**}). In this regard, the retailer should use coefficients $k_T = T^{**}/T^*$, $k_k = k^{**}/k^*$, and $k_{P_r} = P_r^{**}/P_r^*$ on the review period, safety factor, and retail price decisions, respectively and the supplier should use coefficient $k_n = n^{**}/n^*$ on the replenishment multiplier decision. On the other hand, to ensure that both members will gain more profit after applying the mentioned coefficients, a quantity discount policy is considered. In the proposed mechanism, the discount coefficient d_w , which is considered between 0 and 1, will be used to coordinate the channel. Under this strategy, the purchasing price of the retailer from the supplier is reduced from w to wd_w.

The value of the discount coefficient should be set such that it is acceptable for both members. The increased benefit of the retailer by shifting from the decentralized decision making toward centralized decision making will be $\pi_r\left(d_w, k_T T^*, k_k k^*, k_{P_r} P_r^*\right) - \pi_r(T^*, k^*, P_r^*)$. To guarantee the participation of the retailer in the plan, dw must be small enough to ensure a positive value for $\pi_r\left(d_w, k_T T^*, k_k k^*, k_{P_r} P_r^*\right) - \pi_r(T^*, k^*, P_r^*)$.

By putting this equation greater than zero, a maximum acceptable value of dw from the retailer's viewpoint is calculated, which is called $d_w{}^{max}$.

$$d_w^{max} = \left(\frac{1}{\left(w((a\text{-}bk_{P_r} P_r^*) - \frac{\beta\sigma\sqrt{k_T T^* + L}\psi\left(k_k k^*\right)}{k_T T^*}) \right)} \right) \tag{14}$$

$$\times \begin{pmatrix} k_{P_r} P_r^* (a\text{-}bk_{P_r} P_r^*)\text{-}\frac{A_r}{k_T T^*} - h_r(\frac{(a\text{-}bk_{P_r} P_r^*)k_T T^*}{2} \\ +k_k k^* \sigma\sqrt{k_T T^* + L} + \beta\sigma\sqrt{k_T T^* + L}\psi\left(k_k k^*\right)) \\ -\frac{(\pi + \beta k_{P_r} P_r^*)}{k_T T^*}\sigma\sqrt{k_T T^* + L}\psi\left(k_k k^*\right) - (P_r^* - w)(a\text{-}bP_r^*) \\ +\frac{A_r}{T^*} + h_r(\frac{(a\text{-}bP_r^*)T^*}{2} + k^*\sigma\sqrt{T^* + L} + \beta\sigma\sqrt{T^* + L}\psi\left(k^*\right) \\ -\frac{(\pi + \beta(P_r^* - w))}{T^*}\sigma\sqrt{T^* + L}\psi\left(k^*\right) \end{pmatrix}$$

By the similar procedure, the minimum acceptable value of coefficient d_w that ensures more profitability for the supplier can be extracted as:

$$d_w^{min} = \left(\frac{1}{\left(w((a\text{-}bk_{P_r} P_r^*) - \frac{\beta\sigma\sqrt{k_T T^* + L}\psi\left(k_k k^*\right)}{k_T T^*}) \right)} \right) \tag{15}$$

$$\times \begin{pmatrix} e((a\text{-}bk_{P_r} P_r^*)\text{-}\frac{\beta\sigma\sqrt{k_T T^* + L}\psi\left(k_k k^*\right)}{k_T T^*}) \\ +\frac{A_s}{k_n n^* k_T T^*} + \frac{h_s}{2}\left(\left(k_n n^* - 1\right)\left((a\text{-}bk_{P_r} P_r^*)k_T T^* - \beta\sigma\sqrt{k_T T^* + L}\psi\left(k_k k^*\right)\right)\right) \\ +(w - e)\left((a\text{-}bP_r^*)\text{-}\frac{\beta\sigma\sqrt{T^* + L}\psi\left(k^*\right)}{T^*}\right) - \frac{A_s}{n^* T^*} \\ -\frac{h_s}{2}\left(\left(n^* - 1\right)\left((a\text{-}bP_r^*)T^* - \beta\sigma\sqrt{T^* + L}\psi\left(k^*\right)\right)\right) \end{pmatrix}$$

While the interval $[d_w^{min}, d_w^{max}]$ is non-empty, the channel coordination will be achieved and profitability of both members will be increased by shifting from the decentralized to centralized model.

A profit sharing strategy based on the bargaining power of SC members is proposed. Let $\Delta\pi_{SC}$ be the increased profit of entire SC under the centralized decision making in comparison with the decentralized one:

$$\Delta\pi_{SC} = \pi_{SC}(T^{**}, k^{**}, P_r^{**}, n^{**}) - \left(\pi_r(T^*, k^*, P_r^*) + \pi_s(n^*)\right) \tag{16}$$

If the supplier implements ${d_w}^{min}$, the entire $\Delta\pi_{SC}$ will be assigned to the retailer, while implementing ${d_w}^{max}$ assigns all the surplus profits to the supplier. Assume that α is the bargaining power of the retailer against the supplier, therefore the bargaining power of the supplier will be (1-α). Let $\Delta\pi_r$ be the increased profit of the retailer after implementation of the quantity discount policy. Based on the abovementioned analysis:

$$\begin{aligned}\Delta\pi_r\left(d_w\,,\mathrm{k_T},\mathrm{k_k},\mathrm{k_{P_r}}\right) &= \alpha\Delta\pi_{SC} \\ &= \alpha\pi_r\left({d_w}^{min}\,,\mathrm{k_T},\mathrm{k_k},\mathrm{k_{P_r}}\right)+(1-\alpha)\pi_r\left({d_w}^{max},\mathrm{k_T},\mathrm{k_k},\mathrm{k_{P_r}}\right)\end{aligned} \tag{17}$$

Simplifying Eq. (17), d_w can be calculated based on bargaining power α as:

$$\mathrm{d_w} = \alpha\mathrm{d_w}^{\min} + (1-\alpha)\mathrm{d_w}^{\max} \tag{18}$$

Generally, the retailer prefers ${d_w}^{min}$ while the supplier wants to implement ${d_w}^{max}$.

5. Numerical examples

A set of test problems is examined to investigate the performance of the proposed models. To this end, test problems 1 to 4 were designed so that they cover wide range of reasonable parameters. Moreover, in test problems 5 to 7, the parameters were taken from work of Nematollahi, Hosseini-Motlagh, and Heydari (2017b). It is mentioned that for three parameters $(\mathrm{A_r}, \mathrm{b}, \alpha)$ which are not addressed in the work of Nematollahi, Hosseini-Motlagh, and Heydari (2017b) the data were provided under problem conditions. Table 2, indicates data for seven test problems.

Table 3, shows the results of running three models, i.e. decentralized, centralized, and coordinated models for seven test Problems. The values of decision variables and profit functions for the three aforementioned models are compared in Table 3. As shown in Table 3, under the centralized decision-making model, the retailer decreases the retail price toward the decentralized decision-making model. Thus, under joint decision making the retailer undertakes losses due to the decrease in its retail price. Although the SC profit in the centralized decision-making model increases compared to the decentralized decision-making model. Hence, the retailer will refuse to shift its decision variables T, k, and $\mathrm{p_r}$. As a result, designing an incentive scheme is required to guarantee increment of both SC members' profitability.

In the proposed coordination model, the supplier offers a discount to convince the retailer to participate in the coordination plan. Wholesale discount coefficient d_w is calculated based on the SC members' bargaining power using Eq. (18). The minimum and maximum values of wholesale discount coefficient d_w are extracted as well. As can be seen, in all test problems accepting coordination plan improves the profitability of SC as well as both SC members compared to the decentralized decision making. Moreover, the proposed model increases SC profit same as centralized decision making in all test problems. Thus, the proposed model is capable of achieving channel coordination. Also, the proposed incentive mechanism is applicable as it improves profitability of both SC members. Moreover, under identical parameters (Test problems 5, 6, and 7) the proposed model is compared with the work of Nematollahi, Hosseini-Motlagh, and Heydari (2017b). Under the mentioned test problems, our proposed coordination scheme creates greater retail price which in turn leads to more profitability for the retailer compared to the work of Nematollahi, Hosseini-Motlagh, and Heydari (2017b). It implies that our suggested incentive scheme is convincing enough for the retailer to participate in the coordination plan. In other words, in our investigation SC which faces a price dependent demand the developed incentive scheme not only can increase the market demand, but also is able to improve retail price in comparison with the work of Nematollahi, Hosseini-Motlagh, and Heydari (2017b) in which the results is more beneficial for the supplier. Comparing entire SC profitability of two studies revealed that our proposed incentive scheme is able to achieve channel coordination for three mentioned test problems. Further, the developed coordination mechanism can greatly lead to more SC profitability in test problems 6 and 7 compared to the work of

Nematollahi, Hosseini-Motlagh, and Heydari (2017b). Accordingly, it can be concluded that the proposed incentive scheme can increase customer's demand in our investigated SC which in turn results in improving SC profitability as well as both SC members' profitability.

Table 2. Data for the seven investigated test problems

Parameter	Test Problem 1	Test Problem 2	Test Problem 3	Test Problem 4	Test Problem 5*	Test Problem 6*	Test Problem 7*
w	100	150	200	70	18	38	28
a	3000	6000	10000	5000	18000	10000	7000
b	15	25	34	32	400	130	90
A_r	200	240	300	110	180	400	90
A_s	280	300	370	150	250	500	200
h_r	40	8	20	3	9	10	10
h_s	35	12	30	3	7	8	6
ℓ (Day)	4	5	7	2	2	1.5	1
e	70	120	180	50	13	33	24
π	4	3	5	1	1	2	1
β	0.6	0.2	0.3	0.1	0.3	0.3	0.2
σ	40	800	1500	500	6000	4000	3000
α	0.3	0.6	0.5	0.2	0.4	0.7	0.8

* Source: Nematollahi, Hosseini-Motlagh, and Heydari (2017b)

Table 3. Results of decentralized, centralized and coordinated models

	Test Problem 1	Test Problem 2	Test Problem 3	Test Problem 4	Test Problem 5	Test Problem 6	Test Problem 7
Decentralized model							
T	40.49	55.50	27.20	65.18	12.11	27.33	10.33
k	1.17	1.28	1.42	1.28	1.56	1.31	1.66
p_r	150.96	194.90	246.49	113.17	31.23	56.87	52.55
n	1	1	2	2	4	3	6
πr	33118.10	43781.82	52818.83	57397.05	45469.49	22870.77	41220.94
πs	19410.89	31246.76	26886.50	26669.84	22467.18	8360.90	6677.40
πsc	52529.00	75028.59	79705.33	84066.90	67936.68	31231.67	47898.35
$D(p_r)$	735.60	1127.5	1619.34	1378.56	5508.00	2606.90	2270.50
Centralized model							
T	56.05	83.90	44.31	100.51	12.76	27.99	10.46
k	1.12	1.17	1.24	1.13	1.60	1.34	1.69
p_r	136.39	180.05	236.61	103.23	28.90	54.75	50.79
n	1	1	1	1	3	3	6
πr	29557.78	37955.48	48786.24	54109.38	43153.13	22224.01	40929.71
πs	26674.11	43046.50	34626.22	33261.03	27101.62	9661.23	7261.10
πsc	56231.90	81001.98	83412.46	87370.41	70254.75	31885.24	48190.81
$D(p_r)$	954.15	1498.75	1955.26	1696.64	6440.00	2882.50	2428.90
Coordinated model							
${d_w}^{min}$	0.92353	0.94679	0.97945	0.94429	0.95847	0.98731	0.99116
${d_w}^{max}$	0.96251	0.97372	0.98929	0.97221	0.97924	0.99369	0.99559
d_w	0.95082	0.94679	0.984375	0.96662	0.97093	0.98922	0.99205
πr	34228.97	47365.86	54672.39	58057.75	46396.72	23328.27	41454.91
πs	22002.93	33636.12	28740.07	29312.66	23858.03	8556.97	6735.89
πsc	56231.90	81001.98	83412.46	87370.41	70254.75	31885.24	48190.81
Gain	3702.90	5973.39	3707.13	3303.51	2318.06	653.56	292.46

A set of sensitivity analysis is carried out to evaluate the effect of key parameters (a, b, β) on the proposed coordination scheme. A set of sensitivity analysis is conducted to investigate the effect of parameter a on the profitability of SC and both SC members. The required data for this sensitivity analysis are taken from test problem 1. Figure 2 illustrates the changes in the profitability of the retailer as a changes. As shown in Figure 2, by increasing a, the profitability of the retailer under coordinated decision making model is greater than decentralized decision-making model which implies the applicability of the proposed coordination scheme from retailer's perspective. Moreover, in high values of a, the difference between the profitability of retailer under coordination scheme and decentralized decision making becomes wider. As a result, it can be concluded that the proposed coordination scheme could be of great importance when an SC faces high market size.

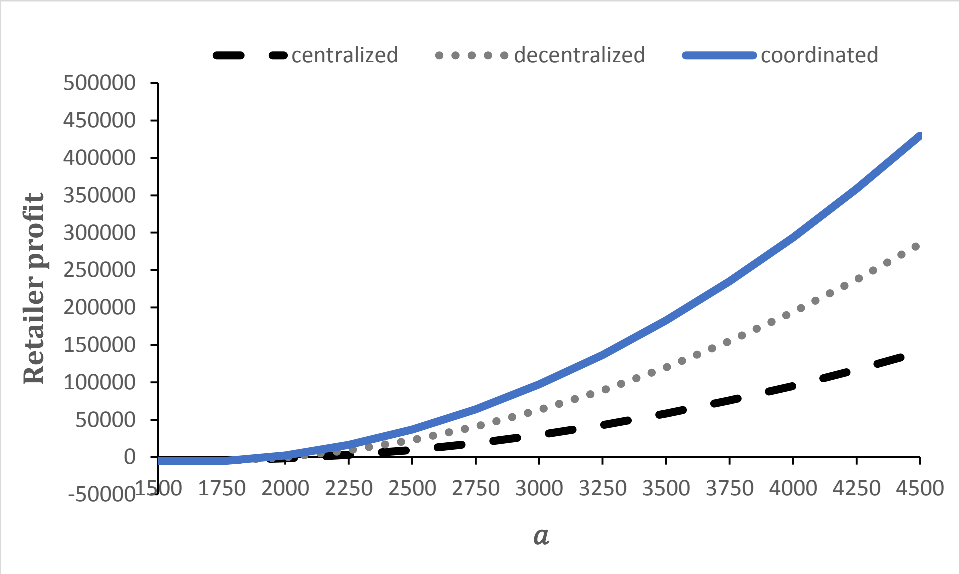

Figure 2. The effect of a on the profitability of the retailer

Moreover, Figure 3 depicts the changes in the profitability of the supplier by changing a. As a grows, under coordinated scheme the supplier's profitability is greater than decentralized decision-making model which reveals the applicability of the developed incentive scheme from supplier's stand point. Thus, market size under the coordination scheme can be challengeable.

Figure 4 shows the changes in the SC profitability as a changes. As can be seen, by increasing a, SC profitability under coordination scheme improves toward the decentralized model which demonstrates the applicability of the proposed coordination scheme from SC view point. Furthermore, in high values of a, the difference between the SC profit under the coordination scheme and decentralized decision making becomes wider. Thus, it can be concluded that the proposed coordination scheme could be of high significance when an SC faces high market size.

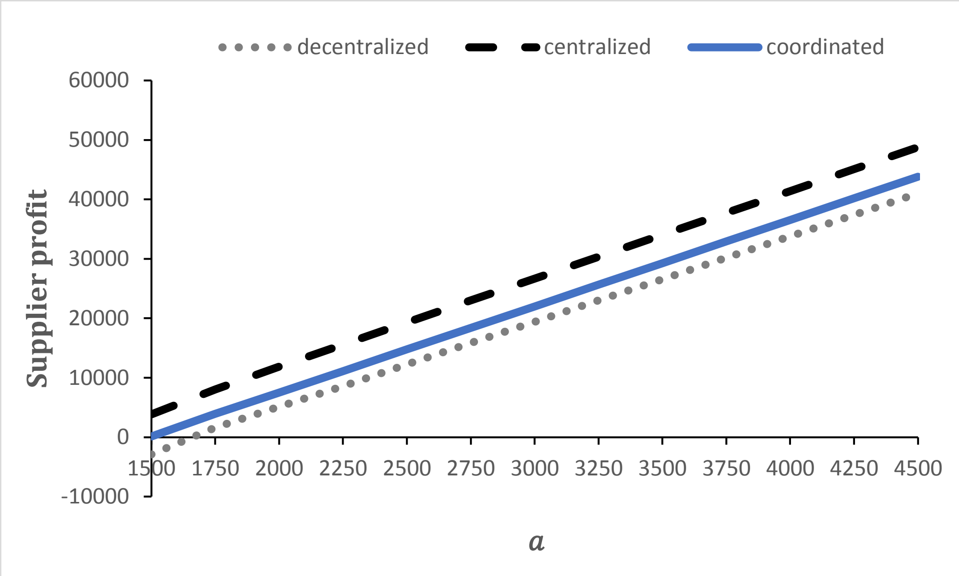

Figure 3. The effect of a on the profitability of the supplier

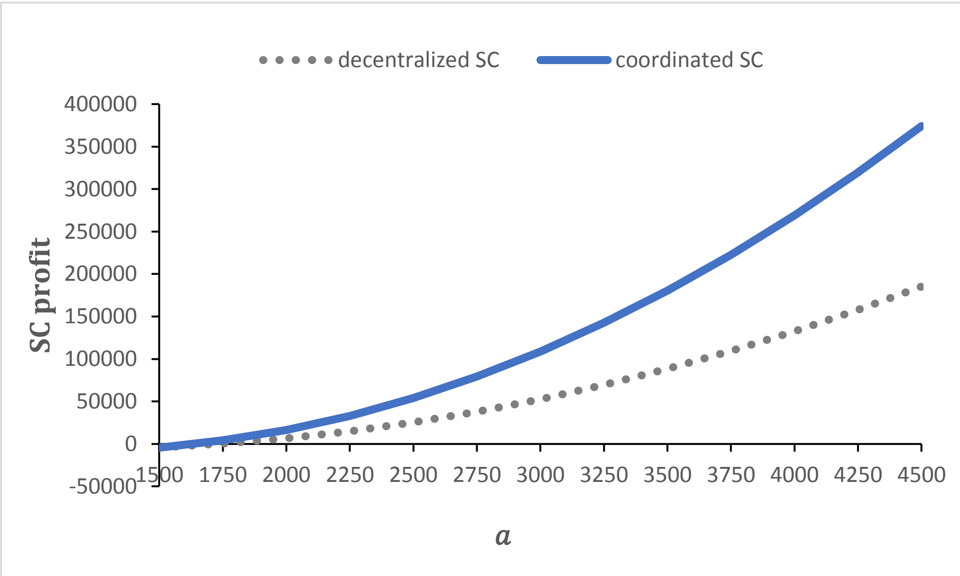

Figure 4. The effect of a on the SC profitability

Moreover, a set of sensitivity analysis is conducted to investigate the effect of the parameter b on the profitability of SC and its members. The required data for this sensitivity analysis are used from test problem 2. Figures 5 and 6 show changing of the retailer and the supplier profits in the decentralized, centralized, and coordinated models over increasing b, repectively. As shown, both members' profitability under coordinated scheme is greater than decentralized decision-making model which indicates that the coordination model is applicable from both members' point of view. According to Figures 5 and 6, there is a threshold of b beyond which the profitability will be less than zero. The proposed coordination scheme is capable of increasing this threshold for both members with respect to the decentralized decision-making model. Therefor, it can be concluded that coordination model is of high benefit for the supply chain when it faces a price sensitive demand.

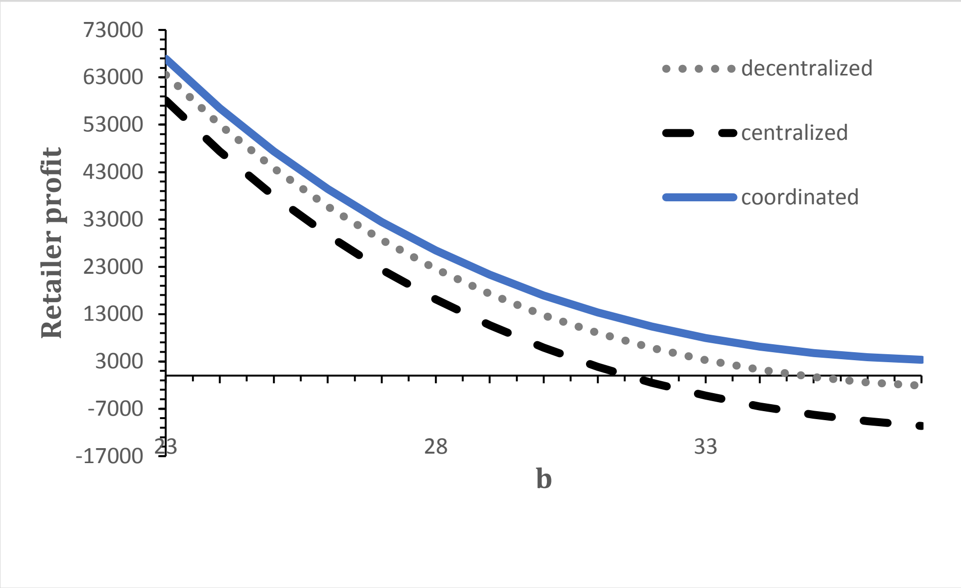

Figure 5. The effect of b on the retailer profit

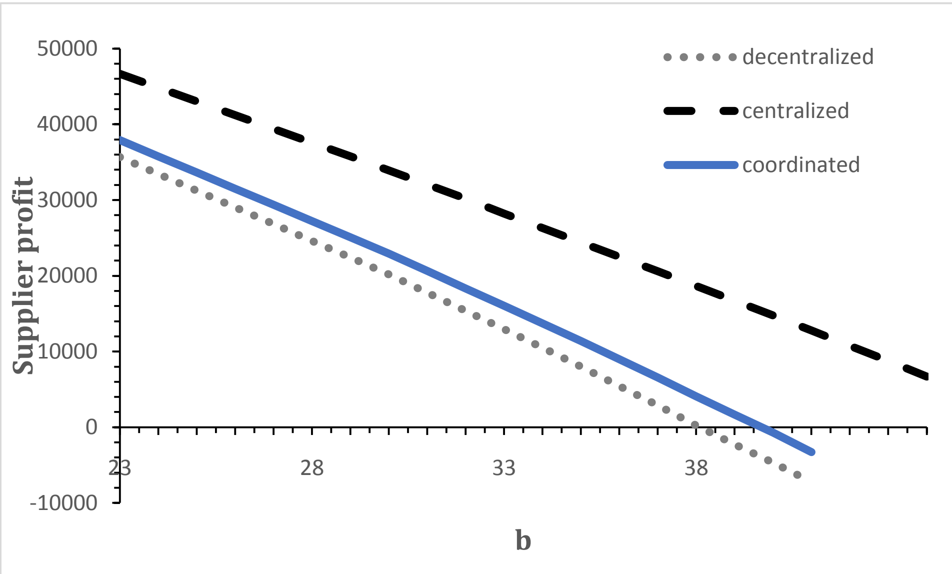

Figure 6. The effect of b on the supplier profit

Figure 7 indicates changes in the whole SC profit by increasing b. According to Figure 7, the coordinated model improves the profitability of the whole SC compared to the decentralized model. As shown in the Figure, by increasing b, SC profitability decreases under both decentralized and coordinated structures. However, the proposed coordination model reduces negative impacts of increasing b on SC profitability with respect to decentralized decision-making model. In addition, there is a threshold of b beyond which the profitability will be less than zero. The developed coordination model is able to increase the threshold of b for entire SC in comparison with the decentralized model.

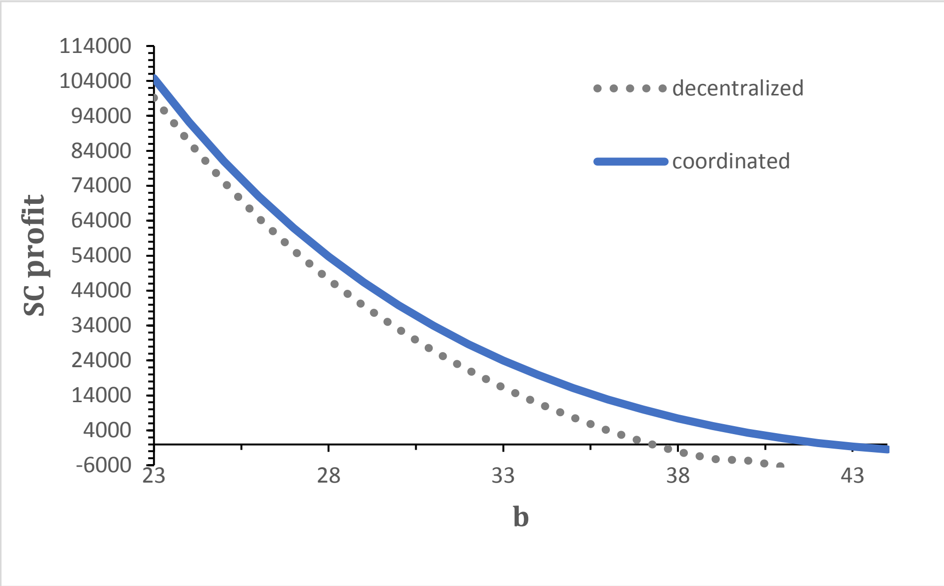

Figure 7. The effect of b on the SC profit

To investigate the capability of the proposed quantity discount contract in achieving channel coordination, changes of ${d_w}^{min}$, ${d_w}^{max}$, and d_w by increasing b are shown in Figure 8. As shown in Figure 8, as b increases the difference between ${d_w}^{min}$ and ${d_w}^{max}$ gets larger which makes the model more applicable. This means the expectations of the retailer and the supplier finding more overlaps by increasing b. In addition, the calculated d_w can motivate both SC members to accept coordination policy. Thus, the proposed model is sufficient when SC faces a price sensitive demand. Moreover, Figure 8 illustrates that under various values of b, there is a valid interval $[{d_w}^{min}, {d_w}^{max}]$; i.e. ${d_w}^{min}$ is always less than ${d_w}^{max}$, then, channel coordination is achievable.

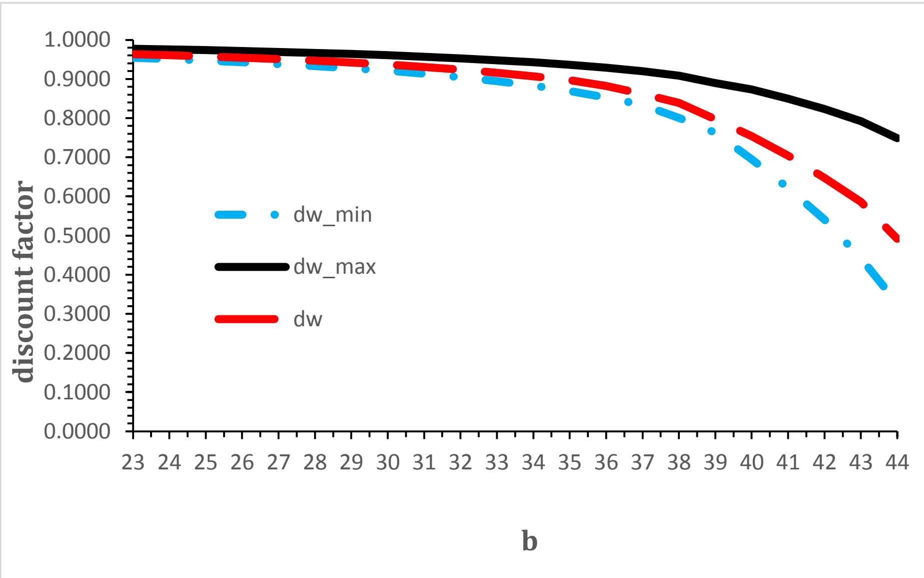

Figure 8. Interval $[{d_w}^{min}, {d_w}^{max}]$ over changing b

Furthermore, a set of sensitivity analysis is conducted to investigate the effect of parameter β on the proposed coordination scheme. Table 4 indicates the results of running model with respect to test problem 2. The changes of the profitability of both SC members as well as SC profitability by increasing β under three decision making models are compared in Table 4. It can be concluded that by growing the lost sale both the SC members' profit besides the SC profitability decrease. While the SC profitability under the proposed coordination scheme increases in comparison with decentralized model which is indicated as "% improvement" item in Table 4. In addition, Figure 9 indicates the changes in the profitability of both SC members along with entire SC profitability versus β. According to Figure 9, as β grows the profitability of both SC members besides SC profit decreases. In other words, by increasing β, reduction in the both retailer and supplier profit functions under coordination model is smaller than decentralized ones. Thus, under lost sale demand applying coordination scheme could be of great significance.

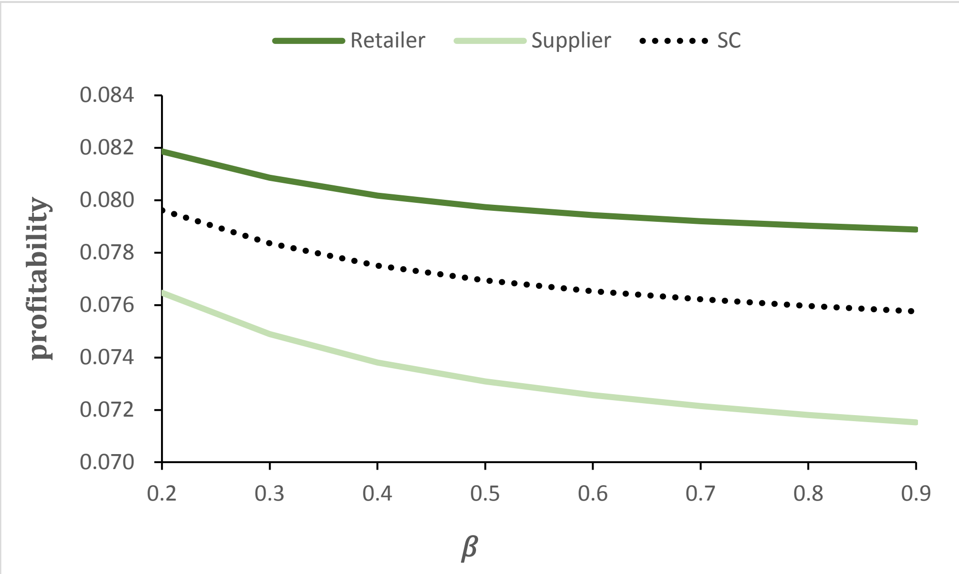

Figure 9. The effect of β on the profitability of two SC members and entire SC

Table 4. Results of the sensitivity analysis considering the lost sale

β	Decentralized SC			Centralized SC			Coordinated SC			% improvement
	π_r	π_s	π_{SC}	π_r	π_s	π_{SC}	π_r	π_s	π_{SC}	
0.2	43781.82	31246.764	75028.59	37955.48	43046.503	81001.98	47365.862	33636.12	81001.9841	7.96%
0.3	43412.69	31247.87	74660.5668	37543.048	42967.9577	80511.0059	46922.958	33588.0477	80511.006	7.84%
0.4	43160.48	31253.185	74413.671	37264.127	42916.6185	80180.74	46620.7307	33560.0153	80180.746	7.75%
0.5	42,970.56	31,253.15	74,223.71	37,055.17	42,879.17	79,934.33	46,396.93	33,537.40	79,934.33	7.69%
0.6	42,819.14	31,251.22	74,070.36	36,889.35	42,849.73	79,739.08	46,220.37	33,518.71	79,739.08	7.65%
0.7	42,693.77	31,248.55	73,942.32	36,752.30	42,825.83	79,578.13	46,075.26	33,502.88	79,578.13	7.62%
0.8	42,587.13	31,245.64	73,832.78	36,635.96	42,805.72	79,441.68	45,952.47	33,489.21	79,441.68	7.60%
0.9	42,494.57	31,242.64	73,737.21	36,535.20	42,788.33	79,323.54	45,846.36	33,477.17	79,323.54	7.58%

6. Conclusion

In this paper, a quantity discount contract was proposed to coordinate the pricing and periodic review inventory decisions in a supplier-retailer chain. In the investigated SC, the retailer used a periodic review inventory system and the supplier applied a lot for lot strategy. The customers' demand was considered to be stochastic and price dependent. Firstly, the investigated SC was modeled under the decentralized structure where each member individually optimized its own objective function. Thereafter, in the centralized structure, the optimal decisions were obtained from the entire SC viewpoint. Although the centralized decision-making improved the profitability of the whole SC, the retailer incurred losses under the centralized model. Therefore, to guarantee more profitability for all SC members, an incentive mechanism based on the quantity discount was developed to motivate the retailer to optimize his/her decisions from the whole SC perspective. The minimum and maximum acceptable discount factors were extracted such that both members have enough incentive to participate in the joint decision-making model. The numerical examples demonstrated that the proposed coordination scheme was able to create small discount factors which made the model more applicable. In addition, a profit sharing strategy was proposed to share the extra profit between two SC members according to their bargaining power fairly. The results revealed that the proposed incentive mechanism was capable of achieving channel coordination. Furthermore, the results demonstrated that the pricing was as important as the inventory decisions and could be considered as a major decision in the supply chain, with great impacts on the SC profitability. As a matter of fact, the coordination model reduced the retail price which in turn led to growing market demand

Some extensions of this study might be of interest. One of the future research directions is to extend the research to other demand functions. In addition to sensitivity of the demand to price, other parameters such as lead time length and product quality could affect the demand. Moreover, in this paper, the demand is assumed to be a linear function. Considering the other possible non-linear demand functions can be considered for future research.

Appendix A:

Proof of Proposition 1. To prove concavity of the retailer profit function with respect to T, k, and P_r, the Hessian matrix of the retailer's expected annual profit function should be calculated. If the principal minors are alternatively negative and positive, i.e., the kth order leading principal minor H_k follows the sign of $(-1)^k$ then the profit function π_r is concave, i.e., maximum at (T^*, k^*, P_r^*). The associated Hessian matrix of π_r is

$$H(\pi_r(T,k,P_r)) = \begin{bmatrix} \partial^2\pi_r(T,k,P_r)/\partial T^2 & \partial^2\pi_r(T,k,P_r)/\partial T\partial k & \partial^2\pi_r(T,k,P_r)/\partial T\partial P_r \\ \partial^2\pi_r(T,k,P_r)/\partial k\partial T & \partial^2\pi_r(T,k,P_r)/\partial k^2 & \partial^2\pi_r(T,k,P_r)/\partial k\partial P_r \\ \partial^2\pi_r(T,k,P_r)/\partial P_r\partial T & \partial^2\pi_r(T,k,P_r)/\partial P_r\partial k & \partial^2\pi_r(T,k,P_r)/\partial P_r^2 \end{bmatrix}$$

Where,

$$\frac{\partial^2\pi_r(T,k,P_r)}{\partial T^2} = \frac{-2A_r}{T^3} + \left[\frac{1}{T^2\sqrt{T+L}} - \frac{2\sqrt{T+L}}{T^3} + \frac{1}{4T(T+L)^{\frac{3}{2}}}\right]\left(\left(\pi + \beta(P_r - w)\right)\sigma\psi(k)\right) + \frac{h_r\sigma\left(k+\beta\psi(k)\right)}{4(T+L)^{\frac{3}{2}}} \quad (1)$$

From Eq. (4) in section 4, this implies that,

$$\frac{A_r}{2T^2(T+L)} + \frac{\left(\pi+\beta(P-w)\right)\sigma\psi(k)\sqrt{T+L}}{2T^2(T+L)} - \frac{\left(\pi+\beta(P-w)\right)\sigma\psi(k)}{4T(T+L)^{\frac{3}{2}}} > \frac{h_r\sigma\left(k+\beta\psi(k)\right)}{4(T+L)} \quad (2)$$

If let

$$E1 = \frac{2A_r}{T^3} - \left[\frac{1}{T^2\sqrt{T+L}} - \frac{2\sqrt{T+L}}{T^3} + \frac{1}{4T(T+L)^{\frac{3}{2}}}\right]\left(\left(\pi+\beta(P_r-w)\right)\sigma\psi(k)\right) \quad (3)$$

and

$$E2 = \frac{A_r}{2T^2(T+L)} + \frac{\left(\pi+\beta(P-w)\right)\sigma\psi(k)\sqrt{T+L}}{2T^2(T+L)} - \frac{\left(\pi+\beta(P-w)\right)\sigma\psi(k)}{4T(T+L)^{\frac{3}{2}}} \quad (4)$$

and

$$E3 = E1 - \frac{h_r\sigma\left(k+\beta\psi(k)\right)}{4(T+L)^{\frac{3}{2}}} \quad (5)$$

Then,

$$\frac{\partial^2\pi_r(T,k,P_r)}{\partial T^2} = -E3 \quad (6)$$

On the other hand,

$$E2 > \frac{h_r\sigma\left(k+\beta\psi(k)\right)}{4(T+L)^{\frac{3}{2}}} \quad (7)$$

From the above are get

$$E3 > E1 - E2 = \frac{Ar(3T+4L)}{2T^3(T+L)} + \frac{\left(\pi+\beta(P-w)\right)\sigma\psi(k)(4L+T)}{2T^3\sqrt{T+L}} > 0 \quad (8)$$

Accordingly,

$$|H_{11}| = \frac{\partial^2\pi_r(T,k,P_r)}{\partial T^2} < 0 \quad (9)$$

Thus, the first principal minor determinant of H is $|H_{11}| < 0$. The second principle minor is positive when:

$$\{\frac{-2A_r}{T^3} + \left[\frac{1}{T^2\sqrt{T+L}} - \frac{2\sqrt{T+L}}{T^3} + \frac{1}{4T(T+L)^{\frac{3}{2}}}\right]((\pi+\beta(P_r-w))\sigma\psi(k)) + \frac{h_r\sigma\left(k+\beta\psi(k)\right)}{4(T+L)^{\frac{3}{2}}}\}$$

$$\times\left\{-h_r\beta\sigma\sqrt{T+L}\psi(k) - \frac{1}{T}\left(\pi+\beta(P_r-w)\right)\sigma\sqrt{T+L}\psi(k)\right\}$$
$$> \{\frac{-h_r\sigma}{2\sqrt{T+L}} - \frac{h_r\beta\sigma(\Phi(k)-1)}{2\sqrt{T+L}} - [(\Phi(k)-1)\left(\pi+\beta(P_r-w)\right)](\frac{1}{2T\sqrt{T+L}} - \frac{\sqrt{T+L}}{T^2})\}^2 \quad (10)$$

And the third principle minor is negative when:

$$\{(\frac{-2A_r}{T^3} + \left[\frac{1}{T^2\sqrt{T+L}} - \frac{2\sqrt{T+L}}{T^3} + \frac{1}{4T(T+L)^{\frac{3}{2}}}\right]((\pi+\beta(P_r-w))\sigma\psi(k)) + \frac{h_r\sigma\left(k+\beta\psi(k)\right)}{4(T+L)^{\frac{3}{2}}})$$

$$\times((-2b)(-h_r\beta\sigma\sqrt{T+L}\psi(k) - \frac{1}{T}\left(\pi+\beta(P_r-w)\right)\sigma\sqrt{T+L}\psi(k)) - \left(-\frac{\beta\sigma\sqrt{T+L}(\Phi(k)-1)}{T}\right)^2)\}$$

$$+\{\left(\frac{h_rb}{2} - \left(\beta\sigma\psi(k)\right)\left(\frac{1}{2T\sqrt{T+L}} - \frac{\sqrt{T+L}}{T^2}\right)\right)\times\left(-\frac{\beta\sigma\sqrt{T+L}(\Phi(k)-1)}{T}\right)\left(\frac{-h_r\sigma}{2\sqrt{T+L}} - \frac{h_r\beta\sigma(\Phi(k)-1)}{2\sqrt{T+L}} - \left[(\Phi(k)-1)\left(\pi+\beta(P_r-w)\right)\right]\left(\frac{1}{2T\sqrt{T+L}} - \frac{\sqrt{T+L}}{T^2}\right)\right) - \left(-h_r\beta\sigma\sqrt{T+L}\psi(k) - \frac{1}{T}\left(\pi+\beta(P_r-w)\right)\sigma\sqrt{T+L}\psi(k)\right)\left(\frac{h_rb}{2} - \left(\beta\sigma\psi(k)\right)\left(\frac{1}{2T\sqrt{T+L}} - \frac{\sqrt{T+L}}{T^2}\right)\right))\} < \{(\frac{-h_r\sigma}{2\sqrt{T+L}} - \frac{h_r\beta\sigma(\Phi(k)-1)}{2\sqrt{T+L}} - \left[(\Phi(k)-1)\left(\pi+\beta(P_r-w)\right)\right]\left(\frac{1}{2T\sqrt{T+L}} - \frac{\sqrt{T+L}}{T^2}\right))$$

$$\times ((-2b)\left(\frac{-h_r\sigma}{2\sqrt{T+L}}-\frac{h_r\beta\sigma(\Phi(k)-1)}{2\sqrt{T+L}}-\left[(\Phi(k)-1)\left(\pi+\beta(P_r-w)\right)\right]\left(\frac{1}{2T\sqrt{T+L}}-\frac{\sqrt{T+L}}{T^2}\right)\right)$$
$$-\left(\frac{h_r b}{2}-\left(\beta\sigma\psi(k)\right)\left(\frac{1}{2T\sqrt{T+L}}-\frac{\sqrt{T+L}}{T^2}\right)\right)\left(-\frac{\beta\sigma\sqrt{T+L}(\Phi(k)-1)}{T}\right))\} \qquad (11)$$

These conditions are tested numerically and observed that it would be satisfied for reasonable parameter values. Then, by satisfying conditions (10) and (11) Hessian matrix of the retailer expected annual profit function is negative definite.

Appendix B:

Proof of Proposition 2: It is enough to show that its second-order derivative of $\pi_s(n)$ with respect to n is negative. To show concavity, it is temporarily assumed that the variable *n* is a continuous variable.

$$\frac{\partial^2\pi_s(n)}{\partial n^2}=-\frac{2A_s}{Tn^3}<0 \qquad (12)$$

This completes the proof of the concavity of $\pi_s(n)$.

Appendix C:

Details of proposition 3. To show concavity of SC profit with respect to variables, T, k, P_r, and n, the Hessian matrix for the SC profit function with respect to T, k, P_r, and n variables should be calculated as follows. If the Hessian matrix is negative definite, the proposition will be proved. To show concavity, it is temporarily assumed that the variable n is a continuous variable.

$$H(\pi_{SC}) = \begin{bmatrix} \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial T^2 & \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial T\partial k & \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial T\partial P_r & \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial T\partial n \\ \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial k\partial T & \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial k^2 & \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial k\partial P_r & \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial k\partial n \\ \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial P_r\partial T & \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial P_r\partial k & \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial P_r^2 & \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial P_r\partial n \\ \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial n\partial T & \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial n\partial k & \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial n\partial P_r & \partial^2\pi_{SC}(\mathrm{T},\mathrm{k},P_r,n)/\partial n^2 \end{bmatrix}$$

Where,

$$\frac{\partial^2\pi_{SC}}{\partial \mathrm{T}^2} = \frac{-2}{T^3}\left(\mathrm{A_r}+\frac{\mathrm{A_s}}{\mathrm{n}}\right) - [(\pi+\beta(P_r-\mathrm{e}))\sigma\psi(\mathrm{k})]\left(\frac{-2}{2\mathrm{T}^2\sqrt{\mathrm{T+L}}}-\frac{1}{4\mathrm{T}(\mathrm{T+L})^{\frac{3}{2}}}+\frac{2\sqrt{\mathrm{T+L}}}{\mathrm{T}^3}\right)-\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\sigma\psi(\mathrm{k})}{8(\mathrm{T+L})^{\frac{3}{2}}} - \mathrm{h_r}\left[\frac{-\mathrm{K}\sigma}{4(\mathrm{T+L})^{\frac{3}{2}}}-\frac{\beta\sigma\psi(\mathrm{k})}{4(\mathrm{T+L})^{\frac{3}{2}}}\right] \qquad (13)$$

The first element of the main diagonal is negative under below condition. However, with respect to rational values of the model parameters, this condition would be satisfied.

$$\frac{-2}{T^3}\left(\mathrm{A_r}+\frac{\mathrm{A_s}}{\mathrm{n}}\right) - [(\pi+\beta(P_r-\mathrm{e}))\sigma\psi(\mathrm{k})]\left(\frac{-2}{2\mathrm{T}^2\sqrt{\mathrm{T+L}}}-\frac{1}{4\mathrm{T}(\mathrm{T+L})^{\frac{3}{2}}}+\frac{2\sqrt{\mathrm{T+L}}}{\mathrm{T}^3}\right) - \mathrm{h_r}\left[\frac{-\mathrm{K}\sigma}{4(\mathrm{T+L})^{\frac{3}{2}}}-\frac{\beta\sigma\psi(\mathrm{k})}{4(\mathrm{T+L})^{\frac{3}{2}}}\right] < \frac{\mathrm{h_s}(\mathrm{n}-1)\beta\sigma\psi(\mathrm{k})}{8(\mathrm{T+L})^{\frac{3}{2}}} \qquad (14)$$

The first principal minor of the above Hessian matrix is the same as the first element of the main diagonal that has a negative value under condition 14. The second principle minor is positive when:

$$\{(\frac{-2}{T^3}\left(\mathrm{A_r}+\frac{\mathrm{A_s}}{\mathrm{n}}\right) - [(\pi+\beta(P_r-\mathrm{e}))\sigma\psi(\mathrm{k})]\left(\frac{-2}{2\mathrm{T}^2\sqrt{\mathrm{T+L}}}-\frac{1}{4\mathrm{T}(\mathrm{T+L})^{\frac{3}{2}}}+\frac{2\sqrt{\mathrm{T+L}}}{\mathrm{T}^3}\right)-\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\sigma\psi(\mathrm{k})}{8(\mathrm{T+L})^{\frac{3}{2}}} - \mathrm{h_r}\left[\frac{-\mathrm{K}\sigma}{4(\mathrm{T+L})^{\frac{3}{2}}}-\frac{\beta\sigma\psi(\mathrm{k})}{4(\mathrm{T+L})^{\frac{3}{2}}}\right])(-\frac{\sigma\sqrt{\mathrm{T+L}}\psi(\mathrm{k})}{T}((\pi+\beta(P_r-\mathrm{e}))-\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\mathrm{T}}{2})-\mathrm{h_r}\beta\sigma\sqrt{\mathrm{T+L}}\psi(\mathrm{k})) > \left(-\sigma(\Phi(\mathrm{k})-1)(\pi+\beta(P_r-\mathrm{e}))\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)+\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\sigma(\Phi(\mathrm{k})-1)}{4\sqrt{\mathrm{T+L}}}-\frac{\mathrm{h_r}\sigma}{2\sqrt{\mathrm{T+L}}} - \frac{\mathrm{h_r}\beta\sigma(\Phi(\mathrm{k})-1)}{2\sqrt{\mathrm{T+L}}}\right)^2 \qquad (15)$$

And the third principle minor is negative when:

$$-\frac{\partial^2\pi_r(\mathrm{T,k},P_r)}{\partial T\partial \mathrm{k}}[\left(\frac{\partial^2\pi_r(\mathrm{T,k},P_r)}{\partial {P_r}^2}\right)\left(\frac{\partial^2\pi_r(\mathrm{T,k},P_r)}{\partial k\partial T}\right)-\left(\frac{\partial^2\pi_r(\mathrm{T,k},P_r)}{\partial P_r\partial T}\right)\left(\frac{\partial^2\pi_r(\mathrm{T,k},P_r)}{\partial k\partial P_r}\right)] + \frac{\partial^2\pi_r(\mathrm{T,k},P_r)}{\partial T\partial P_r}[\left(\frac{\partial^2\pi_r(\mathrm{T,k},P_r)}{\partial P_r\partial k}\right)\left(\frac{\partial^2\pi_r(\mathrm{T,k},P_r)}{\partial k\partial T}\right)-\left(\frac{\partial^2\pi_r(\mathrm{T,k},P_r)}{\partial k^2}\right)\left(\frac{\partial^2\pi_r(\mathrm{T,k},P_r)}{\partial P_r\partial T}\right)]$$

$$\{(\frac{-2}{T^3}\left(\mathrm{A_r}+\frac{\mathrm{A_s}}{\mathrm{n}}\right) - [(\pi+\beta(P_r-\mathrm{e}))\sigma\psi(\mathrm{k})]\left(\frac{-2}{2\mathrm{T}^2\sqrt{\mathrm{T+L}}}-\frac{1}{4\mathrm{T}(\mathrm{T+L})^{\frac{3}{2}}}+\frac{2\sqrt{\mathrm{T+L}}}{\mathrm{T}^3}\right)-\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\sigma\psi(\mathrm{k})}{8(\mathrm{T+L})^{\frac{3}{2}}}-\mathrm{h_r}[\frac{-\mathrm{K}\sigma}{4(\mathrm{T+L})^{\frac{3}{2}}}-\frac{\beta\sigma\psi(\mathrm{k})}{4(\mathrm{T+L})^{\frac{3}{2}}}])((-\frac{\sigma\sqrt{\mathrm{T+L}}\psi(\mathrm{k})}{T}((\pi+\beta(P_r-\mathrm{e}))-\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\mathrm{T}}{2})-\mathrm{h_r}\beta\sigma\sqrt{\mathrm{T+L}}\psi(\mathrm{k}))(-2b)-\left(-\frac{\beta\sigma\sqrt{\mathrm{T+L}}(\Phi(\mathrm{k})-1)}{T}\right)^2)\}$$

$$+\{(-\beta\sigma\psi(\mathrm{k})\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)+\frac{\mathrm{b}}{2}[\mathrm{h_r}+\mathrm{h_s}(\mathrm{n}-1)])((-\frac{\beta\sigma\sqrt{\mathrm{T+L}}(\Phi(\mathrm{k})-1)}{T})(-\sigma(\Phi(\mathrm{k})-1)(\pi+\beta(P_r-\mathrm{e}))\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)+\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\sigma(\Phi(\mathrm{k})-1)}{4\sqrt{\mathrm{T+L}}}-\frac{\mathrm{h_r}\sigma}{2\sqrt{\mathrm{T+L}}}-\frac{\mathrm{h_r}\beta\sigma(\Phi(\mathrm{k})-1)}{2\sqrt{\mathrm{T+L}}})-(-\frac{\sigma\sqrt{\mathrm{T+L}}\psi(\mathrm{k})}{T}((\pi+\beta(P_r-\mathrm{e}))-\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\mathrm{T}}{2}) - \mathrm{h_r}\beta\sigma\sqrt{\mathrm{T+L}}\psi(\mathrm{k}))(-\beta\sigma\psi(\mathrm{k})\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)+\frac{\mathrm{b}}{2}[\mathrm{h_r}+\mathrm{h_s}(\mathrm{n}-1)]))\}$$

$$<\Bigg\{\left(-\sigma(\Phi(\mathrm{k})-1)(\pi+\beta(P_r-\mathrm{e}))\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)+\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\sigma(\Phi(\mathrm{k})-1)}{4\sqrt{\mathrm{T+L}}}-\frac{\mathrm{h_r}\sigma}{2\sqrt{\mathrm{T+L}}}-\frac{\mathrm{h_r}\beta\sigma(\Phi(\mathrm{k})-1)}{2\sqrt{\mathrm{T+L}}}\right)\Bigg((-2b)\left(-\sigma(\Phi(\mathrm{k})-1)(\pi+\beta(P_r-\mathrm{e}))\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)+\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\sigma(\Phi(\mathrm{k})-1)}{4\sqrt{\mathrm{T+L}}}-\frac{\mathrm{h_r}\sigma}{2\sqrt{\mathrm{T+L}}}-\frac{\mathrm{h_r}\beta\sigma(\Phi(\mathrm{k})-1)}{2\sqrt{\mathrm{T+L}}}\right)-\left(-\beta\sigma\psi(\mathrm{k})\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)+\frac{\mathrm{b}}{2}[\mathrm{h_r}+\mathrm{h_s}(\mathrm{n}-1)]\right)\left(-\frac{\beta\sigma\sqrt{\mathrm{T+L}}(\Phi(\mathrm{k})-1)}{T}\right)\Bigg)\Bigg\} \qquad (16)$$

And the forth principle minor is positive when:

$$\{(\frac{-2}{T^3}\left(\mathrm{A_r}+\frac{\mathrm{A_s}}{\mathrm{n}}\right)-[(\pi+\beta(P_r-\mathrm{e}))\sigma\psi(\mathrm{k})]\left(\frac{-2}{2\mathrm{T}^2\sqrt{\mathrm{T+L}}}-\frac{1}{4\mathrm{T}(\mathrm{T+L})^{\frac{3}{2}}}+\frac{2\sqrt{\mathrm{T+L}}}{\mathrm{T}^3}\right)-\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\sigma\psi(\mathrm{k})}{8(\mathrm{T+L})^{\frac{3}{2}}}-\mathrm{h_r}\left[\frac{-\mathrm{K}\sigma}{4(\mathrm{T+L})^{\frac{3}{2}}}-\frac{\beta\sigma\psi(\mathrm{k})}{4(\mathrm{T+L})^{\frac{3}{2}}}\right][(-\frac{\sigma\sqrt{\mathrm{T+L}}\psi(\mathrm{k})}{T}((\pi+$$

$$\beta(P_r-\mathrm{e}))-\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\mathrm{T}}{2})-\mathrm{h_r}\beta\sigma\sqrt{\mathrm{T+L}}\psi(\mathrm{k}))\left((-2b)\left(\frac{-2\mathrm{A_s}}{\mathrm{nT}^3}\right)-\left(\frac{\mathrm{h_s bT}}{2}\right)^2\right)-(-\frac{\beta\sigma\sqrt{\mathrm{T+L}}(\Phi(\mathrm{k})-1)}{T})((\frac{-2\mathrm{A_s}}{\mathrm{nT}^3})(-\frac{\beta\sigma\sqrt{\mathrm{T+L}}(\Phi(\mathrm{k})-1)}{T})-$$

$$(\frac{\mathrm{h_s}\beta\sigma\sqrt{\mathrm{T+L}}(\Phi(\mathrm{k})-1)}{2})(\frac{\mathrm{h_s bT}}{2}))]\}$$

$$+\{(-\beta\sigma\psi(\mathrm{k})\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)$$

$$+\frac{\mathrm{b}}{2}[\mathrm{h_r}+\mathrm{h_s}(\mathrm{n}-1)])((-\sigma(\Phi(\mathrm{k})-1)(\pi+\beta(P_r-\mathrm{e}))\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)+\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\sigma(\Phi(\mathrm{k})-1)}{4\sqrt{\mathrm{T+L}}}$$

$$-\frac{\mathrm{h_r}\sigma}{2\sqrt{\mathrm{T+L}}}-\frac{\mathrm{h_r}\beta\sigma(\Phi(\mathrm{k})-1)}{2\sqrt{\mathrm{T+L}}})\left(\left(-\frac{\beta\sigma\sqrt{\mathrm{T+L}}(\Phi(\mathrm{k})-1)}{T}\right)\left(\frac{-2\mathrm{A_s}}{\mathrm{nT}^3}\right)-\left(\frac{\mathrm{h_s bT}}{2}\right)^2\right)$$

$$-(-\frac{\sigma\sqrt{\mathrm{T+L}}\psi(\mathrm{k})}{T}\left((\pi+\beta(P_r-\mathrm{e}))-\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\mathrm{T}}{2}\right)$$

$$-\mathrm{h_r}\beta\sigma\sqrt{\mathrm{T+L}}\psi(\mathrm{k}))(\left(\frac{-2\mathrm{A_s}}{\mathrm{nT}^3}\right)\left(-\beta\sigma\psi(\mathrm{k})\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)+\frac{\mathrm{b}}{2}[\mathrm{h_r}+\mathrm{h_s}(\mathrm{n}-1)]\right)$$

$$-\left(\frac{-\mathrm{A_s}}{\mathrm{n}^2\mathrm{T}^2}+\frac{\mathrm{h_s}\beta\sigma\psi(\mathrm{k})}{4\sqrt{\mathrm{T+L}}}-\frac{(\mathrm{a}-\mathrm{b}P_r)\mathrm{h_s}}{2}\right)\left(\frac{\mathrm{h_s bT}}{2}\right))$$

$$+\left(\frac{\mathrm{h_s}\beta\sigma\sqrt{\mathrm{T+L}}(\Phi(\mathrm{k})-1)}{2}\right)(\left(\frac{\mathrm{h_s}\beta\sigma\sqrt{\mathrm{T+L}}(\Phi(\mathrm{k})-1)}{2}\right)\left(-\beta\sigma\psi(\mathrm{k})\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)+\frac{\mathrm{b}}{2}[\mathrm{h_r}+\mathrm{h_s}(\mathrm{n}-1)]\right)$$

$$-\left(-\frac{\beta\sigma\sqrt{\mathrm{T+L}}(\Phi(\mathrm{k})-1)}{T}\right)\left(\frac{-\mathrm{A_s}}{\mathrm{n}^2\mathrm{T}^2}+\frac{\mathrm{h_s}\beta\sigma\psi(\mathrm{k})}{4\sqrt{\mathrm{T+L}}}-\frac{(\mathrm{a}-\mathrm{b}P_r)\mathrm{h_s}}{2}\right)))\}$$

$$>\{(-\sigma(\Phi(\mathrm{k})-1)(\pi+\beta(P_r-\mathrm{e}))\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)+\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\sigma(\Phi(\mathrm{k})-1)}{4\sqrt{\mathrm{T+L}}}-\frac{\mathrm{h_r}\sigma}{2\sqrt{\mathrm{T+L}}}$$

$$-\frac{\mathrm{h_r}\beta\sigma(\Phi(\mathrm{k})-1)}{2\sqrt{\mathrm{T+L}}})((-\sigma(\Phi(\mathrm{k})-1)(\pi+\beta(P_r-\mathrm{e}))\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)+\frac{\mathrm{h_s}(\mathrm{n}-1)\beta\sigma(\Phi(\mathrm{k})-1)}{4\sqrt{\mathrm{T+L}}}$$

$$-\frac{\mathrm{h_r}\sigma}{2\sqrt{\mathrm{T+L}}}-\frac{\mathrm{h_r}\beta\sigma(\Phi(\mathrm{k})-1)}{2\sqrt{\mathrm{T+L}}})\left((-2b)\left(\frac{-2\mathrm{A_s}}{\mathrm{nT}^3}\right)-\left(\frac{\mathrm{h_s bT}}{2}\right)^2\right)$$

$$-\left(-\frac{\beta\sigma\sqrt{\mathrm{T+L}}(\Phi(\mathrm{k})-1)}{T}\right)(\left(\frac{-2\mathrm{A_s}}{\mathrm{nT}^3}\right)\left(-\beta\sigma\psi(\mathrm{k})\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)+\frac{\mathrm{b}}{2}[\mathrm{h_r}+\mathrm{h_s}(\mathrm{n}-1)]\right)$$

$$-\left(\frac{-\mathrm{A_s}}{\mathrm{n}^2\mathrm{T}^2}+\frac{\mathrm{h_s}\beta\sigma\psi(\mathrm{k})}{4\sqrt{\mathrm{T+L}}}-\frac{(\mathrm{a}-\mathrm{b}P_r)\mathrm{h_s}}{2}\right)\left(\frac{\mathrm{h_s bT}}{2}\right))$$

$$+\left(\frac{\mathrm{h_s}\beta\sigma\sqrt{\mathrm{T+L}}(\Phi(\mathrm{k})-1)}{2}\right)(\left(\frac{\mathrm{h_s bT}}{2}\right)(-\beta\sigma\psi(\mathrm{k})\left(\frac{1}{2\mathrm{T}\sqrt{\mathrm{T+L}}}-\frac{\sqrt{\mathrm{T+L}}}{\mathrm{T}^2}\right)+\frac{\mathrm{b}}{2}[\mathrm{h_r}+\mathrm{h_s}(\mathrm{n}-1)]$$

$$-(-2b)\left(\frac{-\mathrm{A_s}}{\mathrm{n}^2\mathrm{T}^2}+\frac{\mathrm{h_s}\beta\sigma\psi(\mathrm{k})}{4\sqrt{\mathrm{T+L}}}-\frac{(\mathrm{a}-\mathrm{b}P_r)\mathrm{h_s}}{2}\right))))\}$$

$$+\Bigg\{\Bigg(\frac{-A_s}{n^2T^2}+\frac{h_s\beta\sigma\psi(k)}{4\sqrt{T+L}}$$

$$-\frac{(a-bP_r)h_s}{2}\Bigg)\Bigg(\Bigg(-\sigma(\Phi(k)-1)\big(\pi+\beta(P_r-e)\big)\left(\frac{1}{2T\sqrt{T+L}}-\frac{\sqrt{T+L}}{T^2}\right)+\frac{h_s(n-1)\beta\sigma(\Phi(k)-1)}{4\sqrt{T+L}}-\frac{h_r\sigma}{2\sqrt{T+L}}$$

$$-\frac{h_r\beta\sigma(\Phi(k)-1)}{2\sqrt{T+L}}\Bigg)\Bigg(\left(-\frac{\beta\sigma\sqrt{T+L}(\Phi(k)-1)}{T}\right)\left(\frac{h_sbT}{2}\right)-(-2b)^2\Bigg)$$

$$-\Bigg(-\frac{\sigma\sqrt{T+L}\psi(k)}{T}\left(\big(\pi+\beta(P_r-e)\big)-\frac{h_s(n-1)\beta T}{2}\right)$$

$$-h_r\beta\sigma\sqrt{T+L}\psi(k)\Bigg)\Bigg(\left(\frac{h_sbT}{2}\right)\left(-\beta\sigma\psi(k)\left(\frac{1}{2T\sqrt{T+L}}-\frac{\sqrt{T+L}}{T^2}\right)+\frac{b}{2}[h_r+h_s(n-1)]\right)$$

$$-\left(\frac{-A_s}{n^2T^2}+\frac{h_s\beta\sigma\psi(k)}{4\sqrt{T+L}}-\frac{(a-bP_r)h_s}{2}\right)(-2b)\Bigg)$$

$$+\left(-\frac{\beta\sigma\sqrt{T+L}(\Phi(k)-1)}{T}\right)\Bigg(\left(\frac{h_s\beta\sigma\sqrt{T+L}(\Phi(k)-1)}{2}\right)\Bigg(-\beta\sigma\psi(k)\left(\frac{1}{2T\sqrt{T+L}}-\frac{\sqrt{T+L}}{T^2}\right)$$

$$+\frac{b}{2}[h_r+h_s(n-1)]\Bigg)\left(-\frac{\beta\sigma\sqrt{T+L}(\Phi(k)-1)}{T}\right)\left(\frac{-A_s}{n^2T^2}+\frac{h_s\beta\sigma\psi(k)}{4\sqrt{T+L}}-\frac{(a-bP_r)h_s}{2}\right)\Bigg)\Bigg)\Bigg\}\qquad(17)$$

These conditions are tested numerically and observed that it would be satisfied for reasonable parameter values. Then, by satisfying conditions (14), (15), (16), and (17) Hessian matrix of the SC expected annual profit function is negative definite.

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